CABLE TOWED BODY DYNAMICS

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ABSTRACT

In recent years a large number of towed bodies for military and commerical ocean engineering purposes have been put into service. A general trend in this activity is that towed bodies are getting larger, towing speed is getting faster and the operating depth of towed bodies is getting deeper.

Towing vehicle, tow line and towed body are not only subjected to ocean wave, wind and current but also subjected to vibration excitation force due to flow around tow line and machinery of towing vehicle. In this paper the equilibrium attitude and altitude of towed body, and equilibrium configuration of cable when towing vehicle is advancing at constant speed is discussed. In addition, the dynamic characteristics of tow line due to tow point motion excitation is discussed. Finally, the considerations required to select a right tow line for the towing applications are listed.

SECTION I

TOW CABLE EQUILIBRIUM CONFIGURATION IN A UNIFORM STREAM

In this part, the problem of the equilibrium configuration of the tow cable when the towing ship is advancing at constant forward speed over a smooth ocean surface is treated. The equivalent flow situation is obtained by letting the ship be stationary and introducing a uniform stream in the opposite direction to ship motion.

A. CABLE TENSION AND CONFIGURATION

The sketch of a cable element is shown in Figure 1. We set up the differential equations of the cable element using an S coordinate which represents the arc length along the curved shape of the cable. At the mid-arc-length of this cable element, the tangent line makes an angle ϕ with respect to the horizonal as shown in the figure.

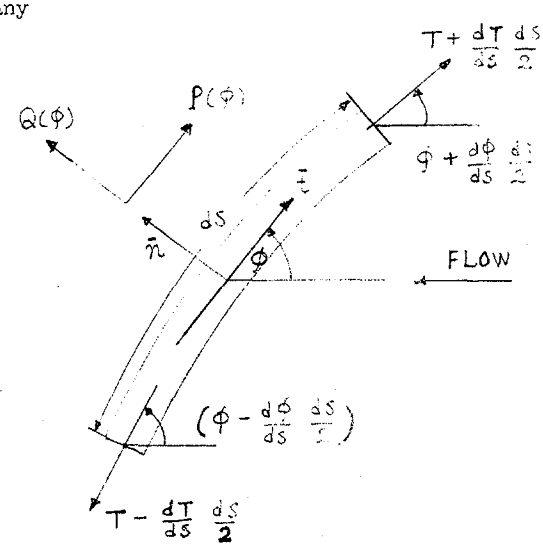

Figure 1. Sketch of Cable Element

The equation of force equilibrium along the unit normal vector $\bar{n}$ is given as:

$$\left(T + \frac{dT}{ds}\frac{ds}{2}\right)\left(\frac{d\phi}{ds}\frac{ds}{2}\right) - \left(T - \frac{dT}{ds}\frac{ds}{2}\right)\left(-\frac{d\phi}{ds}\frac{ds}{2}\right) + Q(\phi)\,ds = 0 \qquad (1\text{-}1)$$

Equation (1-1) is reduced to the following:

$$T d\phi + Q(\phi)\,ds = 0 \qquad (1\text{-}2)$$

where T is the cable tension and $Q(\phi)$ is the sum of the normal components of the hydrodynamic force, gravity force and buoyancy force acting upon the cable element.

The equation of the force equilibrium along the unit tangential vector $\bar{t}$ is given by:

$$dT + P(\phi)\,ds = 0 \qquad (1\text{-}3)$$

where $P(\phi)$ is the sum of the tangential components of the hydrodynamic force, gravity force and buoyancy force acting on the cable element.

Combining Equations (1-2) and (1-3), one obtains

$$\frac{dT}{T} = \frac{P(\phi)}{Q(\phi)}\, d(\phi) \qquad (1\text{-}4)$$

The cable tension is a minimum at the towed body end and is a maximum at the ship's tow point. We designate T_o as the tension at the towed body end of the cable. Integrating Equation (1-4) one obtains the following relationship:

$$T = T_o e^{\int_{\phi_0}^{\phi} \frac{P(\phi)}{Q(\phi)} d\phi} \qquad (1\text{-}5)$$

where ϕ_0 is the cable angle with respect to the horizontal at the towed body end of the cable.

By integrating Equation (1-2) and using relationship (1-5), one obtains the cable length measured along its curved arc as follows:

$$S = \int_{\phi_0}^{\phi} \frac{T_o}{-Q(\phi)} e^{\int_{\phi_0}^{\phi} \frac{P(\phi)}{Q(\phi)} d\phi} d\phi \qquad (1\text{-}6)$$

The horizontal distance between the cable end at the towed body and a point of interest on the cable where it makes an angle ϕ with the horizontal, X, is given by:

$$X = \int_{\phi_0}^{\phi} \frac{T_o \cos\phi}{-Q(\phi)} e^{\int_{\phi_0}^{\phi} \frac{P(\phi)}{Q(\phi)} d\phi} d\phi \qquad (1\text{-}7)$$

The vertical distance between the cable end at the towed body and a point of interest on the cable where it makes an angle ϕ with the horizontal, Z, is given by:

$$Z = \int_{\phi_0}^{\phi} \frac{T_o \sin\phi}{-Q(\phi)} e^{\int_{\phi_0}^{\phi} \frac{P(\phi)}{Q(\phi)} d\phi} d\phi \qquad (1\text{-}8)$$

Whether or not integrals (1-5), (1-6), (1-7) and (1-8) may be solved directly, or if recourse has to be made to numerical integration, will depend upon the form of the functions $P(\phi)$ and $Q(\phi)$. Therefore, the accurate prediction of the cable configuration requires a knowledge of $P(\phi)$ and $Q(\phi)$. There are varieties of cable fairing geometries, cable fairing material, and fabrication methods. It will be impossible to study hydrodynamic characteristics for all faired cables; therefore, only one typical faired cable will be analyzed. Figure 2 shows a hydrodynamic force vector diagram for an element of cable. The lift and drag sum up to become the resultant force F_r. This resultant force is decomposed into the normal component Q_h and the tangential component P_h. P_h and Q_h are usually complicated functions of ϕ.

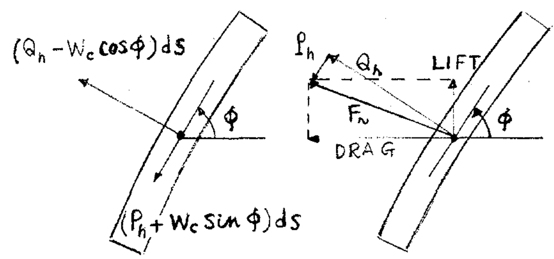

Figure 2. Force Diagram of Cable Element

One approach to guarantee the accuracy of the prediction of the cable configuration is to curve-fit the hydrodynamic cable experimental data in a Fourier series form as follows:

$$Q_h(\phi) = \tfrac{1}{2} A_c V^2 \left[A_o + \sum_{n=1}^{\infty} A_n \cos n(\phi) + \sum_{n=1}^{\infty} B_n \sin n\phi \right] \qquad (1\text{-}9)$$

$$P_h(\phi) = \tfrac{1}{2} A_c V^2 \left[A_o + \sum_{n=1}^{\infty} A_n \cos n(\phi) + \sum_{n=1}^{\infty} B_n \sin n\phi \right] \qquad (1\text{-}10)$$

There is a contribution to Q and P from the weight of the cable in water; i.e.,

$$Q = Q_h - W_c \cos\phi \qquad (1\text{-}11)$$

$$P = -P_h - W_c \sin\phi \qquad (1\text{-}12)$$

When the hydrodynamic cable experimental data for various angles of ϕ are not available, a reasonable estimate of Q_h and P_h will have to be made. For this purpose we consider the pressure force and frictional force separately.

Among the various ways of estimating Q_h and P_h, the following expressions seem logical:

$$Q_h(\phi) = R\left(\frac{D_p}{R} \sin^2\phi + \frac{D_f}{R} \sin\phi\right) \quad (1\text{-}13)$$

$$P_h(\phi) = R\left(\frac{D_f}{R} \cos\phi\right) \quad (1\text{-}14)$$

where R = total drag per unit length of cable when it is normal to the stream

D_p = pressure drag per unit length of cable when it is normal to the stream

D_f = friction drag per unit length of cable when it is normal to the stream

The ratio D_p to D_f still must be found by experiment. When the cable hydrodynamic experimental data is not available, it may be convenient to use the Whicker form of the forces given below:

$$Q_h(\phi) = \frac{t}{c} \sin^2\phi + \left(1 - \frac{t}{c}\right) \sin\phi \quad (1\text{-}15)$$

$$P_h(\phi) = \left(0.386 - 0.303\frac{t}{c}\right) \cos\phi - \left(0.055 - 0.02\frac{t}{c}\right) \cos^2\phi \quad (1\text{-}16)$$

For the actual computation of the cable configuration, GE 635 time-sharing computer was used to solve the following 4 equations by the Runge-Kutta numerical method.

$$\frac{dT(s)}{ds} = -P(\phi) \quad (1\text{-}17)$$

$$\frac{d\phi}{ds} = -\frac{Q(\phi)}{T(s)} \quad (1\text{-}18)$$

$$dx = \cos\phi(s)\, ds \quad (1\text{-}19)$$

$$dy = \sin\phi(s)\, ds \quad (1\text{-}20)$$

B. TOWED BODY EQUILIBRIUM

The sketch of a towed body and the last segment of the tow cable is shown in Figure 3. The weight of the towed body acts vertically downward through the center of gravity of the towed body. The buoyancy B acts vertically upward through the center of buoyancy of the towed body. The coordinates of the tow point are X_t and Z_t.

The equation of force equilibrium along the X axis of the towed body is given by:

$$T_o \sin\gamma + X - (W - B) \sin\alpha = 0 \quad (1\text{-}21)$$

The equation of force equilibrium along the Z axis of the towed body is given by:

$$Z - T_o \cos\gamma + (W - B) \cos\alpha = 0 \quad (1\text{-}22)$$

The equation of moment equilibrium about the center of gravity of the towed body is:

$$BX_b \cos\alpha + M + T_o Z_t \sin\gamma + T_o X_t \cos\gamma = 0 \quad (1\text{-}23)$$

where

T_o = cable tension at the towed body end

γ = angle between the Z axis of the towed body and the cable

X = hydrodynamic force along the X axis of the towed body

W = weight of the towed body in air

B = buoyancy of the towed body in water

α = angle of incidence of the towed body

Z = hydrodynamic force along the Z axis of the towed body

X_b = distance between the C.G. and C.B. along the X axis of the towed body

Z_t = Z coordinate of the tow point

X_t = X coordinate of the tow point

M = hydrodynamic moment about the C.G. of the towed body

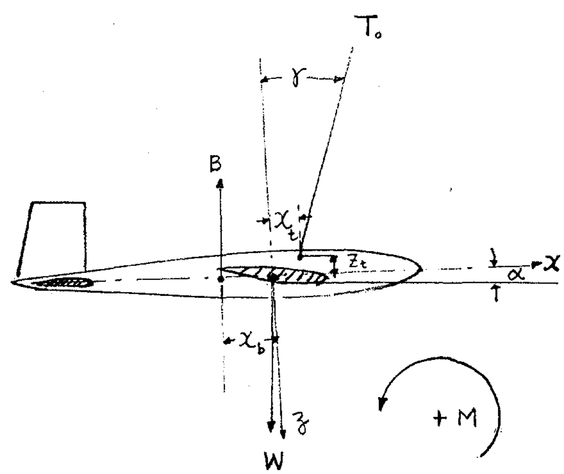

Figure 3. Sketch of Towed Body

Once the geometry of a towed body is selected and its contour is designed, the hydrodynamic forces and moment (X, Z and M) may be determined by a model test in a towing tank or wind tunnel. As a result of this model test, the hydrodynamic forces and moment may be shown as explicit functions of the incidence angle forward velocity V and the wing attack angle:

$$X = f_1 (\alpha, V, \text{Wing attack angle}) \qquad (1\text{-}24)$$

$$Z = f_2 (\alpha, V, \text{Wing attack angle}) \qquad (1\text{-}25)$$

$$M = f_3 (\alpha, V, \text{Wing attack angle}) \qquad (1\text{-}26)$$

After some algebraic reduction and rearrangement of Equations (1-21), (1-22) and (1-23), the equation for solving the incidence angle becomes:

$$P_1 \sin\alpha + Q_1 \cos\alpha = R_1$$

where P_1 is $(W - B)$

$$Q_1 \text{ is } \frac{1}{Z_t} \left[BX_b + X_t \ (W - B) \right]$$

$$R_1 \text{ is } \frac{(M - X_t Z)}{Z_t} + X \qquad (1\text{-}27)$$

Equation (1-27) can be solved by an iteration technique. A computer program has been prepared to solve this equation using the Newton-Raphson method. Once the equilibrium incidence angle α is obtained, substitute it into the following equation to find γ.

$$\gamma = \tan^{-1} \left[\frac{(W-B) \sin\alpha - X}{(W-B) \cos\alpha + Z)} \right] \qquad (1\text{-}28)$$

Having found the equilibrium angles α and γ, substitute them in Equation (1-21) to calculate the cable tension, T_o. The study results of cable equilibrium and towed body equilibrium for 30 and 40 knots speed are shown in Figures 4 and 5 respectively.

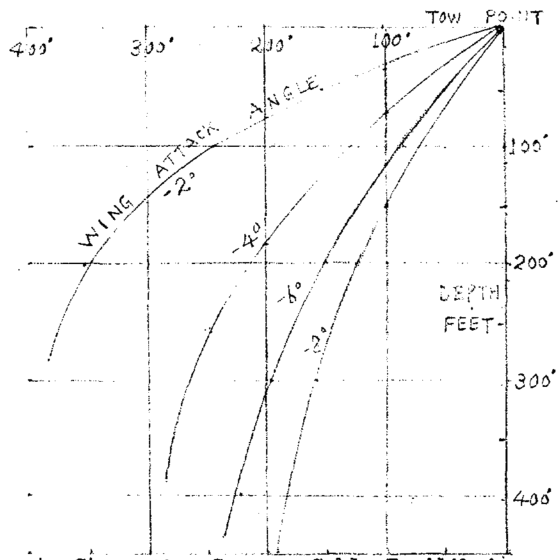

Figure 4. Streamline Section Cable Equilibrium Configuration in 30 Knots Speed

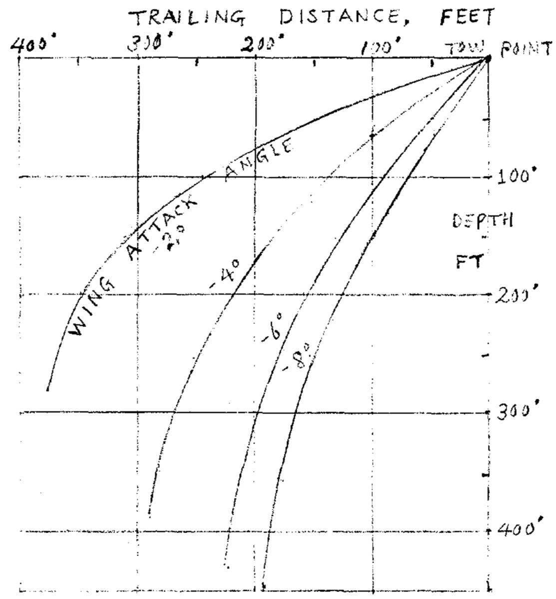

Figure 5. Streamline Section Cable Equilibrium Configuration in 40 Knots Speed

SECTION II

CABLE DYNAMICS

When a towing ship advances in rough seas, the cable deviates from its equilibrium position. In addition the towed body will be disturbed from its equilibrium position and attitude. Exactly speaking, the tow cable is a continuous mechanical system and therefore has an infinite number of degrees of freedom. In practice, however, we will have to select only a finite number of coordinates to describe the motion of the system.

For this purpose, the entire scope of the cable is divided into n straight segments. Each segment is allowed to rotate and stretch. The exciting force on the cable is applied at the top of the first segment at the ship's tow point. There are two generalized coordinates for each segment of the cable when we consider the cable motion in a vertical plane. These coordinates are θ, the angle between the vertical and the cable segment, and u, the stretch of the cable segment. In this study, the Lagrange equations were used to describe the system dynamics. The Lagrange equation for the ith generalized coordinate is given by:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} - Q_{Di} = Q_{Ai} \qquad (2\text{-}1)$$

where

- T: kinetic energy of the system
- U: potential energy of the system
- q_i: ith generalized coordinate
- Q_{Di}: ith generalized damping force
- Q_{Ai}: ith generalized applied force

The kinetic energy of the system consists of two parts. One part is due to motion of the cable segment C.G. and the other part is due to rotational motion of the cable segment as a rigid rod. The potential energy U consists of two parts also. The first part is due to the cable presence in the gravity field potential and the second part is due to cable stretch. For the towed body, the θ coordinate may be included while the u coordinate may be excluded. Because of limited paper space the detailed derivation of the Lagrange equation is omitted. Equation (2-1) may be solved in the frequency domain by a computer handling 2N-1 equations from i = 1 to i = 2N-1. The solution of the stretch coordinate is particularly important because it provides information on the cable dynamic tension. Furthermore a very troublesome cable slack will occur if the sum of smooth water equilibrium tension and the dynamic tension becomes less than zero.

Cable strumming is undesirable not only because it generates vibratory stress but also because of noise generation. When the cable moves in a fluid, Karman vortices form behind the cable and the cable is transversely excited at the Strouhal frequency. The cable transverse vibration is coupled into the longitudinal vibration of twice the transverse vibration frequency. This causes the towed body to vibrate. It has been found that streamline section cable and ribbon faired cable strum less than bare cable.

In summary the following considerations must be given to selecting the right tow line:

1. The cable equilibrium configuration in smooth water which results in a shortest possible scope and minimum equilibrium tension.
2. The tow line should have a least amount of dynamic tension in a rough sea environment. The towed body should be least disturbed by tow line motion.
3. The probability of cable slack and fetch in rough sea must be minimized.
4. Cable strumming must be kept to a minimum in order to reduce high frequency vibratory stresses and spurious noise.
5. Cable must be easily handled by shipboard winch and other launch and retrieval equipment.
6. Cable must have adequate strength in static and dynamic loading.
7. Cable must accommodate electrical conductors and protect them in ocean environment.