DSP

SHORTHAND :-

SIGNALS

I Sketching signals

$x(n-2)$	right shift
$x(n+2)$	left shift
$x(-n+2)$	fold and right shift
$x(-n-2)$	fold and left shift
$x(2n)$	divide time axis by 2
$x(\frac{n}{2})$	multiply time axis by 2
$x(-2n+2)$	fold, right shift, divide
$x(-2n-2)$	fold, left shift, divide
$2x(n)$	amplitude doubled
$\frac{1}{2}x(n)$	amplitude halved

II Periodic / Non-periodic

(i) one term CT signal find T
replace t by t+T
$x(t+T) = x(t) \Rightarrow$ Periodic with fundamental period $T = T$ seconds

(ii) two terms CT signal find T_1 & T_2
Ratio $\frac{T_1}{T_2}$ = Rational $\Rightarrow$ Periodic

If $\frac{T_1}{T_2} = \frac{3}{4}$, then $T_1 \times 4 = T_2 \times 3 = T$ (fundamental period)

not rational $\rightsquigarrow$ $\sqrt{2}$, π

(iii) single term DT signal

find f, if rational, then periodic $\rightarrow \frac{k}{N}$

denominator = N = no. of samples

put $n = n+N$

$x(n+N) = x(n) \Rightarrow$ periodic

(iv) two terms DT signal

find f_1 & hence N_1

find f_2 & hence N_2

$\frac{N_1}{N_2}$ = Rational $\Rightarrow$ Periodic

if $\frac{N_1}{N_2} = \frac{2}{3}$, then $N = N_1 \times 3 = N_2 \times 2$ samples

III Even and odd signals

$$x(t) = x_e(t) + x_o(t)$$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

even
$\Downarrow$
$x(-t) = x(t)$
$x(-n) = x(n)$

odd
$\Downarrow$
$x(-t) = -x(t)$
$x(-n) = -x(n)$

IV Energy and Power signals

$0 < E < \infty \Rightarrow$ Energy signal (eg:- non-periodic)

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \qquad E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$P = \frac{1}{T}\int_{0}^{T} |x(t)|^2 \, dt \qquad P = \frac{1}{N}\sum_{n=0}^{N-1} |x(n)|^2$$

$0 < P < \infty \Rightarrow$ Power signal (periodic :- eg.)

T can't be determined

$\Downarrow$ $P = 0$, not PS

$$\rightarrow \sum_{n=\square}^{\infty} a^n = \begin{cases} \dfrac{a^{\square}}{1-a}, & a < 1 \text{ converges} \\ \infty, & a \geq 1 \text{ diverges} \end{cases}$$

$$\rightarrow \sum_{n=\square}^{N-1} a^n = \begin{cases} \dfrac{a^{\square} - a^{N-1+1}}{1-a}, & a \neq 1 \\ N-1+1-\square, & a = 1 \end{cases}$$

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SYSTEMS :-

① memoryless (static) ⇒ o/p depends only on present i/p

$\boxed{\text{put } 0, -1, 1 \text{ \& check}}$

memory/dynamic ⇒ past and/or future

② Causal ⇒ present and/or past not future

non-causal ⇒ future

$\boxed{\text{from previous}}$

③ Linearity ⇒ satisfies SP principle

$$x_1(t) \longrightarrow \boxed{H} \longrightarrow y_1(t)$$

$$x_2(t) \longrightarrow \boxed{H} \longrightarrow y_2(t)$$

$$[a x_1(t) + b x_2(t)] \longrightarrow \boxed{H} \longrightarrow [a y_1(t) + b y_2(t)]$$

$$x_3(t) = [a x_1(t) + b x_2(t)]$$

$$\downarrow$$

$$y_3(t) = [a y_1(t) + b y_2(t)]$$

④ Time invariance $y(t) = x(2-t)$

$x(t) \longrightarrow y(t) = x(2-t)$

$x(t) \longrightarrow y(t) = H[x(t)]$

$x(n) \qquad y(n) = H[x(n)]$

shift o/p, $y(n-n_0) = H[x(n-n_0)]$

$y(t-t_0) = H[x(t-t_0)]$

$y = 2x(t)$

$x_2(t) = x(t) \longrightarrow y_2(t) = 2x_2(t) = 2x(t) \longrightarrow ①$

shift i/p, $y_2(t-t_0) = 2x_2(t-t_0) = 2x(t-t_0) \longrightarrow ②$

$x(t-t_0) \longrightarrow y(t-t_0) = H[x(t-t_0)]$

$x(n-n_0) \qquad y(n-n_0) = H[x(n-n_0)]$

$x_1(t) = x(t-t_0) \longrightarrow y_1(t) = 2x_1(t) = 2x(t-t_0) \longrightarrow ②$ $\longrightarrow ①$

if ① = ② ⇒ time invariant

① ≠ ② ⇒ time variant

$y(n) = \sum_{k=n_0}^{n} x(k)$

⑤ Stability $x_1(k) = x(k-k_0) \longrightarrow y_1(n) = \sum_{k=n_0}^{n} x_1(k) = \sum_{k=n_0}^{n} x(k-k_0) \longrightarrow ①$

finite input produces finite output ⇒ stable

(BIBO)
↳ bounded

$x_2(k) = x(k) \longrightarrow y_2(n) = \sum_{k=n_0}^{n} x_2(k) = \sum_{k=n_0}^{n} x(k) \longrightarrow ②$

$y_2(n-k_0) = \sum_{k=n_0}^{n-k_0} x(k)$

⑥ Invertibility

i/p of system can be recovered from system o/p

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To compare ① & ② change the summation index in ① so that upper limit becomes $n-k_0$

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LTI

I convolution of two sequences

$x(n) = [1, \underset{\uparrow}{3}, 2]$ $\quad h(n) = [4, \underset{\uparrow}{2}, 1]$

$y(n) = x(n) * h(n)$

$h(n) \backslash x(n)$	1	3 ↓	2
4	4	12	8
→ 2	2	6	4
1	1	3	2

$e^{-\infty} = 0 \because \left(\frac{1}{e^{\infty}}\right)$

$y(n) = [4, 14, \underset{\uparrow}{15}, 7, 2]$

convolution sum

$$x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$$

convolution integral

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \qquad (\tau = k\Delta)$$

step response means i/p $x(n) = u(n)$
$x(t) = u(t)$

find o/p of CT-LTI system for i/p $x(t) = \ldots$ & $h(t) = \ldots$

$\Downarrow$

find $x(t) * h(t)$

Relation b/w step response & impulse response

$s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau \qquad \frac{ds(t)}{dt} = h(t) \qquad s(n) = \sum_{k=-\infty}^{n} h(k)$

$s(n) - s(n-1) = h(n)$

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LTI System properties :-

↳ commutative
↳ associative
↳ distributive

memoryless ⟶ condition is $\boxed{h(k) = 0 \text{ for } k \neq 0}$ DT

causal ⟶ condition is $\boxed{h(k) = 0 \text{ for } k < 0}$

stability ⟶ condition is $\boxed{\sum_{k=-\infty}^{\infty} |h(k)| < \infty}$

$\boxed{\int_{-\infty}^{\infty} |h(t)|\, dt < \infty}$ CT

invertibility & inverse system ⟶ The process of recovering i/p $x(n)$ from system's o/p $y(n)$ is called deconvolution

$$\boxed{x(n) = y(n) * h^{-1}(n)}$$

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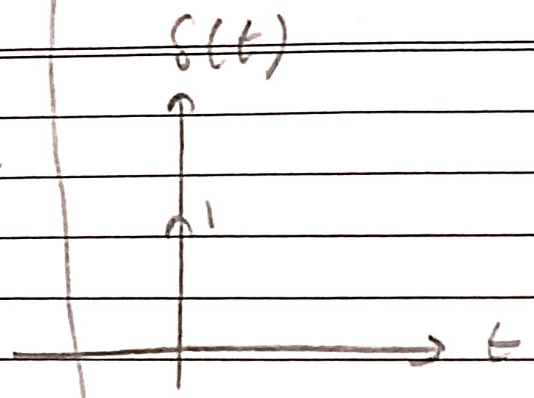
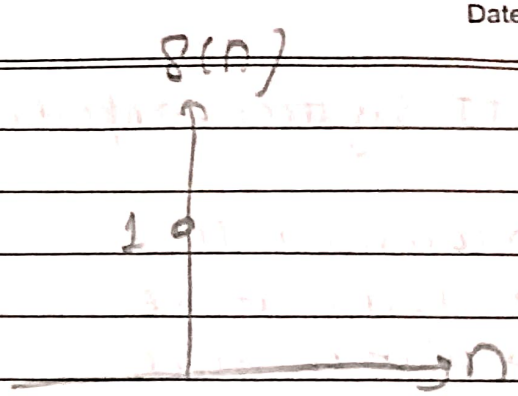
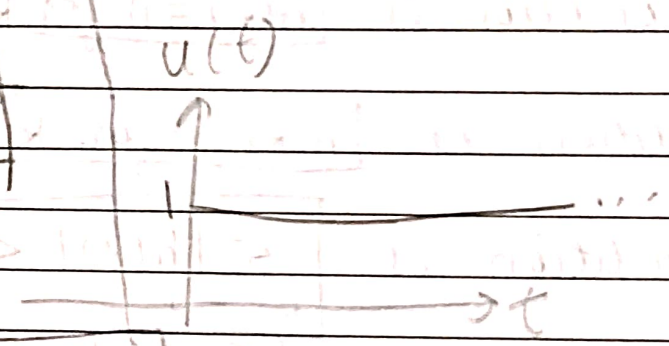
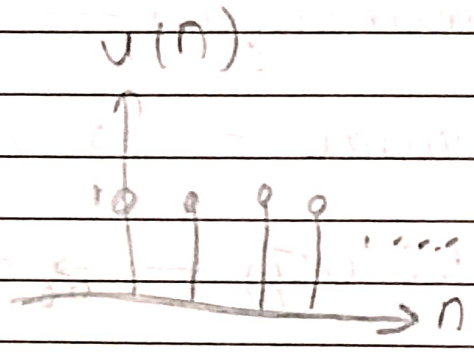
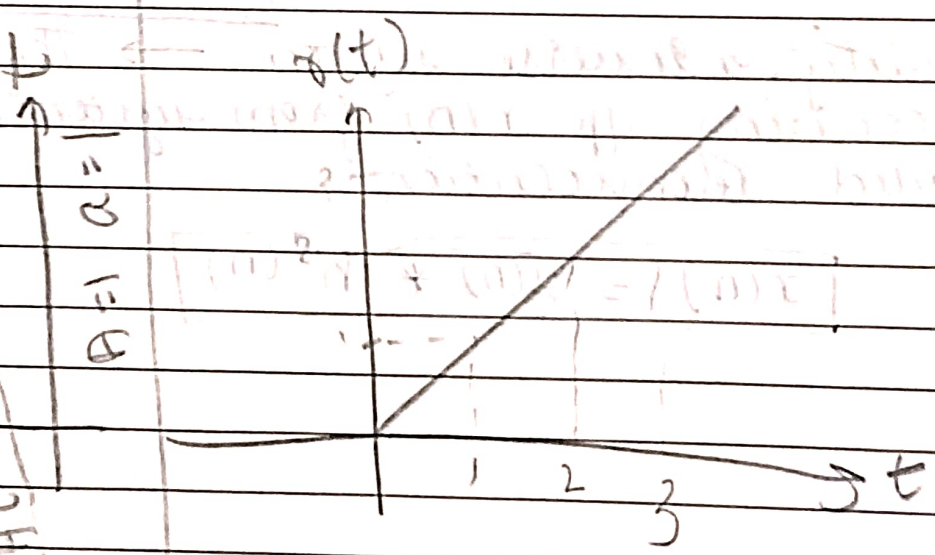
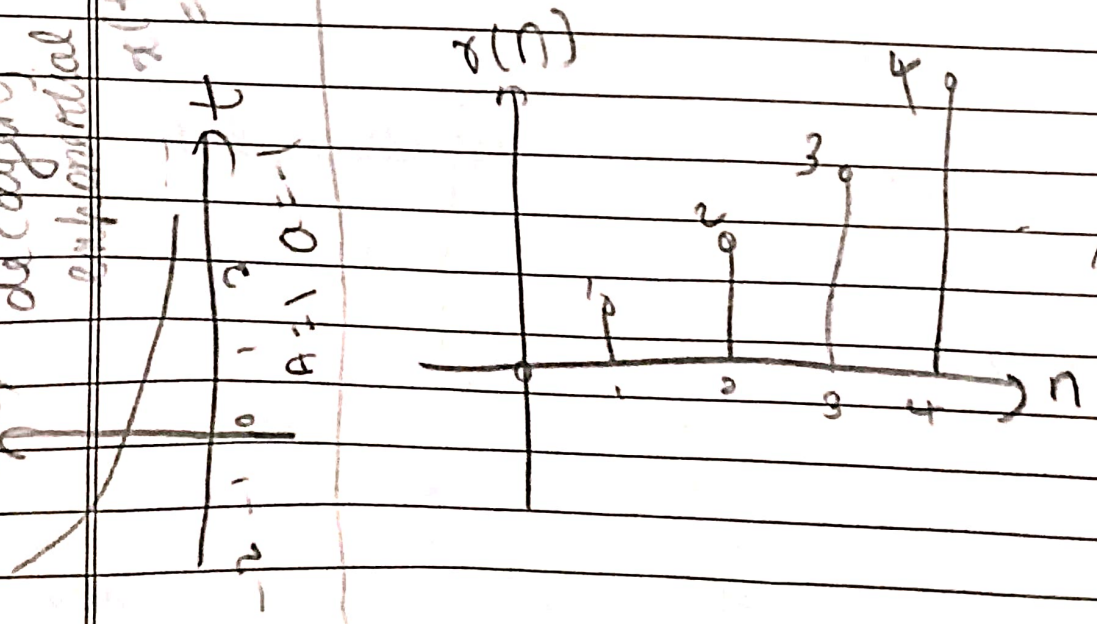








$x(t) = e^{at}$ exponential function



$x(t) = a^t$ exponential function