DSA Set-1

1. Total range of analog signal is $4-(-2) = 6V$

Step size $= \frac{6}{2^{\text{no. of bits}}} = \frac{6}{2^3} = 0.75\,V$

$\Rightarrow \{-2, -1.25\}, \{-1.25, -0.5\}, \{-0.5, 0.25\}, \{0.25, 1\},$
$\{1, 1.75\}, \{1.75, 2.5\}, \{2.5, 3.25\}, \{3.25, 4\}$

can be quantised to 000, 001, 010, 011, 100, 101, 110, 111 respectively

Digital value of 1.4 V will be 100

Quantization error $= \cancel{\frac{\text{upper limit} - \text{lower limit}}{2}} = \left|\frac{1+1.75}{2} - 1.4\right|$

$= 0.025$

2. Step 1: Sample the signal. Since the frequencies lie from 0 to 2500 Hz we will use $f_s = 2 \cdot f_{max} = 2 \cdot 2500 = 5000\,Hz$ as sampling frequency.

Step 2: Compute the DFT of the sampled signal

$x(n) \rightarrow x(k)$

Step 3: Calculate frequency resolution of the signal, N should be such that 200 Hz can be seen. $\Delta f = f_s / N$.

$N = 25$

Nearest power of $2 = 2^5 = 32 = N$

Step 4: Calculate the index of the 200 Hz frequency component using

$p = \text{int}\left(\frac{200}{\Delta f}\right)$

Step 5: Calculate signal strength at 200 Hz

$|P(i)|$ is the answer

If $N = 25$, then $P(1) = 200 \Rightarrow n = 1$

If $N = 32$, then 200 Hz lie between $P(1)$ and $P(2)$
so signal strength $n = 2$

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$$x_1(n) = \{2,0,2,0\} \Rightarrow l_1 = 4$$
$$x_2(n) = \{1,0,9,5\} \Rightarrow l_2 = 4$$

length of linear convolution $= l_1 + l_2 - 1$
$= 4+4-1$
$= 7$

length of circular convolution $= \max(l_1, l_2)$
$= 4$

In order to calculate linear convolution from circular convolution, we pad $x_1(n)$ and $x_2(n)$ with $7-4=3$ zeroes

$$x_1(n) = \{2,0,2,0,0,0,0\}$$
$$x_2(n) = \{1,0,9,5,0,0,0\}$$

$$x_1(n) \circledast x_2(n) = \sum_{m=0}^{6} x_1(m)\, x_2(n-m)$$

$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 2 \\ 2 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 9 \\ 5 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 20 \\ 10 \\ 18 \\ 10 \\ 0 \end{bmatrix}$$

$\therefore$ Linear convolution of $x_1(n)$ and $x_2(n)$ is $\{2,0,20,10,18,10,0\}$

4) i, $y(n) = 3x(n) + x(n-2) + 4$

$$y_1(n) = 3x_1(n) + x_1(n-2) + 4$$
$$x_2(n) = x_1(n-n_0)$$
$$y_2(n) = 3x_2(n) + x_2(n-2) + 4 \rightarrow (1)$$
$$y_2(n) = 3x_1(n-n_0) + x_1(n-n_0-2) + 4$$
$$y_1(n-n_0) = 3x_1(n-n_0) + x_1(n-n_0-2) + 4 \rightarrow (2)$$

eq^n (1) & eq^n (2) are same

$\therefore$ The system is time invariant

Linearity :

$$y_1(n) = 3x_1(n) + x_1(n-2) + 4$$

$$y_2(n) = 3x_2(n) + x_2(n-2) + 4$$

$$x(n) = a_1x_1(n) + a_2x_2(n)$$

$$\begin{aligned} y(n) &= 3x(n) + x(n-2) + 4 \\ &= 3\left[a_1x_1(n) + a_2x_2(n)\right] + \left[a_1x_1(n-2) + a_2x_2(n-2)\right] + 4 \\ &= a_1\left[3x_1(n) + x_1(n-2)\right] + a_2\left[3x_2(n) + x_2(n-2)\right] + 4 \\ &= a_1\left[y_1(n) - 4\right] + a_2\left[y_2(n) - 4\right] + 4 \end{aligned}$$

$$\Rightarrow y(n) \neq a_1y_1(n) + a_2y_2(n)$$

System is not linear

$\therefore$ System is time invariant, but not linear

b) $y(n) = -x(n) + y(2n)$

$x_1(n) \to y_1(n)$, $x_2 \to y_2(n)$

y' be the output for $a_1x_1 + a_2x_2$

$$\Rightarrow y'(n) = -a_1x_1 - a_2x_2 + y'(2n)$$

$$\Rightarrow y'(n) - y'(2n) = a_1(y_1(n) - y_1(2n)) + a_2(y_2(n) - y_2(2n))$$

$$= a_1y_1(n) + a_2y_2(n) - a_1y_1(2n) - a_2y_2(2n)$$

$$\Rightarrow y'(n) = a_1y_1(n) + a_2y_2(n)$$

Hence it is linear

$x_1(n) = x(n-k)$ then

$$y_1(n) = -x_1(n) + y_1(2n) \to (3)$$

$$y(n-k) = -x_1(n-k) + y(2(n-k)) \to (4)$$

from eq (3) & (4) we can say that

$$y_1(n) \neq y(n-k)$$

Time variant

$\therefore$ System is linear time variant.

5) DTFT for $x(n) = a^n u(n-2)$

$$x(n) = a^n u(n-2)$$

$$= a^2 \left(a^{n-2} u(n-2)\right)$$

We know that

DTFT of $\alpha^n u(n)$ is

$$\alpha^n u(n) \rightarrow \frac{1}{1-\alpha e^{-jw}} \quad , \ |\alpha| < 1$$

$$\Rightarrow \quad \alpha^{n-2} u(n-2) \longrightarrow \frac{e^{-2jw}}{1-\alpha e^{-jw}}$$

$$\Rightarrow \quad x(n) = a^2 \left(a^{n-2} u(n-2) \right)$$

DTFT will be

$$X(e^{jw}) = a^2 \cdot \frac{e^{-2jw}}{1-ae^{-jw}} \quad , \ |a| < 1$$