# ASSIGNMENT-2:

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1A) Given

$M/M/K/\infty$ system

Consider the descriptor $N(t)$ to be

$N(t)$ = Number of jobs at time $t$.

The rate matrix $Q$ is given by

$$Q = \begin{array}{c|cccccc} & 0 & 1 & 2 & \cdots & K & K+1 \cdots \\ \hline 0 & -\lambda & \lambda & 0 & & 0 & 0 \\ 1 & \mu & -(\lambda+\mu) & \lambda & & & \\ 2 & 0 & 2\mu & -(2\mu+\lambda) & \lambda & & \\ \vdots & & & & & & \\ K & & & & & K\mu & -(K\mu+\lambda), \lambda \\ K+1 & & & & & & K\mu, -(K\mu+\lambda), \lambda \\ \vdots & & & & & & \end{array}$$

Note that $N(t)$ ranges from $0$ to $\infty$

Hence dimension of $Q$ is not finite

Let $\pi = [\pi_0 \ \pi_1 \ \pi_2 \ \cdots]$ be the stationary distribution

$$\pi Q = 0$$

$$[\pi_0 \ \pi_1 \ \cdots \ \pi_K \ \cdots] \begin{bmatrix} -\lambda & \lambda & 0 & \cdots \\ \mu & -(\lambda+\mu) & \lambda & \cdots \\ & & & \end{bmatrix} = 0$$

$$\pi_0(-\lambda) + \pi_1(\mu) = 0 \quad \text{--- (1)}$$

Until $i \le K$

$$\pi_i(\lambda + i\mu) = \pi_{i-1}\lambda + \pi_{i+1}((i+1)\mu)$$

From these two eq

$$\pi_i = \left(\frac{\lambda}{\mu}\right)^i \frac{\pi_0}{i!}$$

After $i \ge K$

$$\pi_i(\lambda + K\mu) = \lambda \pi_{i-1} + (K\mu)\pi_{i+1} \quad (3)$$

From (3) (1)

$\forall\ i \ge K$

$$\pi_i = \left(\frac{\lambda}{\mu}\right)^i \times \frac{1}{K!}\left(\frac{1}{K}\right)^{i-K} \pi_0$$

W.K.T

$$\sum \pi_i = 1$$

$$\pi_0 \left[1 + \frac{\lambda}{\mu} + \left(\frac{\lambda}{\mu}\right)^2 \frac{1}{2!} + \cdots + \left(\frac{\lambda}{\mu}\right)^K \frac{1}{K!}\right]$$

$$+ \left(\frac{\lambda}{\mu}\right)^{K+1} \frac{1}{K!} \times \frac{\pi_0}{K}\left[1 + \left(\frac{\lambda}{\mu}\right) + \left(\frac{\lambda}{\mu}\right)^2 \cdots\right] = 1$$

$$\pi_0 \left[\sum_{i=0}^{K} \left(\frac{\lambda}{\mu}\right)^i \frac{K!}{i!} + \left(\frac{\lambda}{\mu}\right)^{K+1} \frac{1}{K!K} \frac{1}{\left(1 - \frac{\lambda}{\mu K}\right)}\right] = 1$$

(4)

Mean number of jobs

$$= \sum_{i=0}^{\infty} i \pi_i$$

$$= \sum_{i=0}^{k} i \times \left(\frac{\lambda}{\mu}\right)^i \times \frac{1}{i!}\pi_0 + \sum_{i=k+1}^{\infty} i \left(\frac{\lambda}{\mu}\right)^i \frac{1}{k!}\left(\frac{1}{k}\right)^{i-k} \pi_0$$

$$= \pi_0 \left[ \frac{\lambda}{\mu} + \frac{2}{2!}\left(\frac{\lambda}{\mu}\right) \cdots \right] + \frac{1}{k!} k^k \pi_0 \sum i \left(\frac{\lambda}{\mu}\right)^i \frac{1}{k^i}$$

$$= \pi_0 \left[ \sum_{i=0}^{k} \frac{1}{(i-1)!}\left(\frac{\lambda}{\mu}\right)^i + \frac{k^k}{k!} \sum i \left(\frac{\lambda}{\mu}\right)^i \frac{1}{k^i} \right]$$

~~From (4) we can get the mean no of jobs~~

~~Since a new job Define $\rho = \frac{\lambda}{k\mu}$~~

~~In (1) $\rho = \frac{\lambda}{k\mu}$ can be substituted and rearranged as~~

~~$\pi_0$~~

The system utilization is given by $\rho = \frac{\lambda}{k\mu}$

In (4) substituting $\rho$

$$\pi_0 \left[ \sum_{i=0}^{k} \left(\frac{\lambda}{\mu}\right)^i \times \frac{1}{i!} + \left(\frac{\lambda}{\mu}\right)^{k+1} \frac{1}{k!k} \frac{1}{(1-\rho)} \right] = 1$$

If $\rho \geq 1$

$\pi_{k+1} + \cdots$ would be negative

Hence $\rho < 1$ for a stable system

$$\frac{\lambda}{\mu k} < 1$$

$\lambda < \mu k$ for stable system

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a A) Given M/M/2/∞ Queue

↓ arrival rate $\lambda$

↳ of the two servers one server has rate $\mu_1$ other has $\mu_2$

The descriptor of this system would be

$N(t)$ = the number of jobs at time $t$

but for $N(t) = 1$

There are two states possible $1_1$ and $1_2$

$1_1 \rightarrow$ the new job goes to server 1

$1_2 \rightarrow$ the new job goes to server 2

∴ The rate matrix $Q$ =

$$\begin{array}{c|cccc} & 0 & 1_1 & 1_2 & 2 \quad 3 \\ \hline 0 & -\lambda & 0 & \lambda & 0 \quad 0 \\ 1_1 & \mu_1 & -(\mu_1+\lambda) & 0 & \lambda \\ 1_2 & \mu_2 & 0 & -(\mu_2+\lambda) & \lambda \\ 2 \quad 3 & 0 & \mu_2 & \mu_1 & -(\mu_1+\mu_2+\lambda) \\ 3 & & & & \end{array}$$

Let $\pi = [\pi_0 \ \pi_1 \ \pi_2 \ \text{---}]$ be the stationary distribution,

$$[\pi_0 \ \pi_{1_1} \ \pi_{1_2} \ \pi_2 \ \text{---}] \begin{bmatrix} -\lambda & 0 & \lambda & 0 & 0 \ \text{---} \\ \mu_1 & -(\mu_1+\lambda) & 0 & \lambda & 0 \ \text{---} \\ \mu_2 & 0 & -(\mu_2+\lambda) & \lambda & \text{---} \\ 0 & \mu_2 & \mu_1 & -(\mu_1+\mu_2+\lambda) & \lambda \ \text{---} \end{bmatrix} = 0$$

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① — $-\lambda\pi_0 + \mu_1\pi_k + \mu_2\pi_l = 0$

② — $-(\mu_1+\lambda)\pi_k + \mu_2\pi_2 = 0 \Rightarrow \pi_2 = \dfrac{(\mu_1+\lambda)\pi_k}{\mu_2}$ — ⑤

③ — $\lambda\pi_0 - (\mu_2+\lambda)\pi_l + \mu_1\pi_2 = 0$

④ — $\lambda\pi_k + \lambda\pi_l - (\mu_1+\mu_2+\lambda)\pi_2 + (\mu_1+\mu_2)\pi_3 = 0$

① + ② + ③

gives $(\mu_1+\mu_2)\pi_2 = \lambda\pi_k + \lambda\pi_l$

$$\pi_2 = \frac{\lambda(\pi_k+\pi_l)}{\mu_1+\mu_2} \quad \text{— ⑥}$$

⑤ = ⑥

$$\frac{\mu_1+\lambda}{\mu_2}\pi_k = \frac{\lambda(\pi_k+\pi_l)}{\mu_1+\mu_2}$$

From this

$$\pi_l = \frac{\pi_k\,\mu_1(\lambda+\mu_1+\mu_2)}{\lambda\mu_2} \quad \text{— ⑦}$$

From ①

By substituting (7) in (1)

$$\lambda \pi_0 = \mu_1 \pi_k + \mu_2 \left( \frac{\pi_k \mu_1 (\lambda + \mu_1 + \mu_2)}{\lambda \mu_2} \right)$$

$$\pi_k = \frac{\lambda^2 \pi_0}{\mu_1 (\mu_2 + 2\lambda + \mu_1)}$$

$$\pi_l = \frac{\pi_k \mu_1 (\lambda + \mu_1 + \mu_2)}{\lambda \mu_2}$$

$$= \frac{\lambda (\lambda + \mu_1 + \mu_2)}{\mu_2 (\mu_1 + \mu_2 + 2\lambda)} \pi_0$$

from (5)

$$\pi_2 = \frac{(\mu_1 + \lambda) \pi_k}{\mu_2}$$

$$\pi_2 = \frac{\lambda^2 (\mu_1 + \lambda) \pi_0}{\mu_1 \mu_2 (\mu_1 + 2\lambda + \mu_2)}$$

Substituting these values in 4

We get

$$\pi_3 = \left( \frac{\lambda}{\mu_1 + \mu_2} \right) \pi_2 = \frac{\lambda}{\mu_1 + \mu_2} \times \frac{\lambda^2 (\mu_1 + \lambda) \pi_0}{\mu_1 \mu_2 (\mu_1 + \mu_2 + 2\lambda)}$$

Similarly $\pi_k = \left( \frac{\lambda}{\mu_1 + \mu_2} \right)^{k-2} \pi_2$

W.K.T

$$\Sigma \pi_i = 1$$

$$\pi_0 + \pi_b + \pi_a + \pi_2 \cdots 1$$

$$\pi_0 + \pi_b + \pi_a + \pi_2 \left[ 1 + \frac{\lambda}{\mu_1 + \mu_2} \cdots \right] = 1$$

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$$\pi_0 + \pi_1 + \pi_2 + \pi_2 \times \left[\frac{1}{1-\frac{\lambda}{\mu_1+\mu_2}}\right] = 1$$

$$\pi_0 + \frac{\lambda^2}{\mu_1(\mu_1+2\lambda+\mu_2)}\pi_0 + \frac{\lambda(\lambda+\mu_1+\mu_2)\pi_0}{\mu_2(\mu_1+\mu_2+2\lambda)}$$

$$+ \frac{\mu_1+\mu_2}{(\mu_1+\mu_2-\lambda)} \frac{\lambda^2(\mu_1+\lambda)}{\mu_1\mu_2(\mu_1+\mu_2+2\lambda)}\pi_0$$

$$= 1$$

$$\pi_0\left[1 + \frac{\lambda^2}{\mu_1(\mu_1+\mu_2+2\lambda)} + \frac{\lambda(\lambda+\mu_1+\mu_2)}{\mu_2(\mu_1+\mu_2+2\lambda)}\right.$$

$$\left. + \frac{\mu_1+\mu_2}{(\mu_1+\mu_2-\lambda)} \frac{\lambda^2(\mu_1+\lambda)}{\mu_1\mu_2(\mu_1+\mu_2+2\lambda)}\right] = 1$$

From this equation the value of $\pi_0$ can be solved which gives the $\pi_1, \pi_2, \pi_2$

3A) Given an

M/M/K/K Queue

The descriptor for this system would be

$N(t)$ = Number of jobs at time $t$

The rate matrix $Q$ for this system could be:

$$Q = \begin{array}{c|cccc} & 0 & 1 & 2 & \cdots \\ \hline 0 & -\lambda & \lambda & 0 & \cdots \\ 1 & \mu & -(\lambda+\mu) & \lambda & 0 \cdots \\ 2 & 0 & 2\mu & -(\mu+\lambda) & \lambda \cdots \\ \vdots & & & & \end{array}$$

Let $\pi = [\pi_0 \ \pi_1 \ \cdots]$ be the stationary distribution

then $\pi Q = 0$

$$[\pi_0 \ \pi_1 \ \cdots] \begin{bmatrix} -\lambda & \lambda & 0 & \cdots \\ \mu & -(\lambda+\mu) & \lambda & \cdots \end{bmatrix} = 0$$

$$\pi_0(-\lambda) + \pi_1(\mu) = 0$$

$$\pi_i(i\mu+\lambda) = \pi_{i-1}\lambda + \pi_{i+1}((i+1)\mu)$$

From these two equations

$$\pi_K = \left(\frac{\lambda}{\mu}\right)^K \frac{1}{K!} \pi_0$$

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W.K.T

$$\sum \pi_i = 1$$

$$\pi_0 + \frac{\lambda}{\mu}\pi_0 + \left(\frac{\lambda}{\mu}\right)^2 \times \frac{1}{2!}\pi_0 + \cdots + \left(\frac{\lambda}{\mu}\right)^K \frac{1}{K!}\pi_0 = 1$$

$$\pi_0 = \frac{1}{\sum_{i=0}^{K} \frac{1}{i!}\left(\frac{\lambda}{\mu}\right)^i}$$

$$\pi_i = \left(\frac{\lambda}{\mu}\right)^i \times \frac{1}{i!}\pi_0$$

$$= \frac{\left(\frac{\lambda}{\mu}\right)^i \cdot \times \frac{1}{i!}}{\sum_{i=0}^{K} \frac{1}{i!}\left(\frac{\lambda}{\mu}\right)^i}$$

If all the servers are busy, then the incoming jobs is blocked.

Hence the blocking probability

$$= \pi_K$$

$$= \frac{\left(\frac{\lambda}{\mu}\right)^K \times \frac{1}{K!}}{\sum_{i=0}^{K} \frac{1}{(i)!}\left(\frac{\lambda}{\mu}\right)^i}$$

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4(A) Given

$$N(t) = \max(A(t) - D(t), 0)$$

To prove that $N(t)$ is a markov chain

$$P\left(\frac{N(t) = k}{N(t_k) = n_k, N(t_{k-1}) = n_{k-1} \ldots N(t_0) = n_0}\right)$$

$$= P\left(\frac{N(t) = k}{N(t_0) = n_0}\right)$$

But we know that

$N(t) = A(t) - D(t)$ with a lower bound of 0

and since $A(t)$, $D(t)$ are poisson processes they can be modelled as markov chains.

Hence $N(t) = k = A(t) - D(t)$

only depends on

$N(t_k) = A(t_k) - D(t_k)$

i.e the most recent value.

$\therefore N(t)$ is a markov chain

The range of $N(T) = \max(A(T) - D(T), 0)$ is $\{0, 1, 2, \ldots\}$

Therefore the Rate matrix Q's dimension is not finite.

Rate matrix Q

$$
\begin{array}{c|cccccc}
 & 0 & 1 & 2 & 3 & 4 & \cdots \\
\hline
0 & -\lambda & \lambda & & & & \\
1 & \mu & -(\lambda+\mu) & \lambda & & & \\
2 & & \mu & -(\lambda+\mu) & \lambda & & \\
3 & & & & & & \\
4 & & & & & & \\
\vdots & & & & & &
\end{array}
$$

Three elements of Q can be expressed as follows

Q 17

$Q_{01} = \lambda$ as $N(T)$ increases from 0 to 1 ie a new process happens in $A(T)$ whose rate is $\lambda$

$Q_{00} = -\lambda$ (conservative)

$$
Q_{ij} = \begin{cases} \lambda & j = i+1 \\ & \text{as } N(t) \text{ increases by 1 ie } A(T) \text{ happens whose rate is } \lambda \\ \mu & j = i-1 \\ & \text{in this case } D(T) \text{ happens whose rate } D\mu \\ -(\lambda+\mu) & j = 1, i \end{cases}
$$

$Q_{ij} = \{ 0$ otherwise

This rate matrix is similar to that for the queue $M/M/1/\infty$ with params $\lambda, \mu$ as $A(T)$ denotes the arrival rate, $D(T)$ denotes the departure rate i.e The service time of the job

and $N(t) = \max(A(t) - D(t), 0)$ denotes the number of arrivals - number of departures = no of jobs in the system i.e The descriptor in $M/M/1/\infty$.

To get the stationary distribution

$$\pi Q = 0$$

where $\pi = \{\pi_0, \pi_1, \pi_2 \ldots\}$

$$[\pi_0 \ \pi_1 \ \pi_2 \ldots] \begin{bmatrix} -\lambda & \lambda & \ldots \\ \mu & -(\mu+\lambda) & \ldots \\ & & \end{bmatrix} = 0$$

$$-\lambda\pi_0 + \pi_1\mu = 0 \quad \text{--- (1)}$$

$$\pi_n(\lambda+\mu) = \pi_{n-1}(\lambda) + \pi_{n+1}(\mu) \quad \text{--- (2)}$$

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From ①, ②

$$\pi_i = \left(\frac{\lambda}{\mu}\right)^i \pi_0$$

$$\sum \pi_i = 1$$

$$\sum \left(\frac{\lambda}{\mu}\right)^i \pi_0 = 1$$

$$\pi_0 \times \left[\frac{1}{1-\frac{\lambda}{\mu}}\right] = 1$$

$$\pi_0 = \left(1-\frac{\lambda}{\mu}\right)$$

$$\pi_i = \left(\frac{\lambda}{\mu}\right)^i \left[1-\frac{\lambda}{\mu}\right]$$

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5A) Given

$$S_n = \sum_{i=1}^{n} X_i$$

$X_i = 1$ with probability $p$ and $X_i = -1$ with probability $1-p$.

~~Consider~~

Consider $P\left(S_{n+1} = k / S_n = x_n, S_{n-1} = x_{n-1} \cdots S_0 = x_0\right)$

The value of $S_{n+1} = k$ depends on $\sum_{i=1}^{n} X_i$

i.e $S_n, X_{n+1}$ $(S_{n+1} = S_n + X_{n+1})$

i.e $S_{n+1}$ depends on $S_n$ and does not depend on $S_{n-1}, \cdots S_0$.

$$\therefore P\left(S_{n+1} = k / S_n = x_n, S_{n-1} = x_{n-1} \cdots S_0 = x_0\right) = P\left(S_{n+1} = k / S_n = x_n\right)$$

Hence it can be modelled as a markov chain.

The values of $S_n$ can take the whole set of Integers.

Therefore the dimension of its step transition matrix $P$ is not finite.

Since $S_{n+1} = S_n + X_{n+1}$

and $X_{n+1}$ takes $1$ with probability $p$

takes $-1$ with probability $1-p$

$$\therefore S_{n+1} = \begin{cases} S_n + 1 & \text{with prob } p \\ S_n - 1 & \text{with prob } 1-p \\ Z - \{S_{n+1}, S_{n-1}\} & \text{with prob } 0 \end{cases}$$

$$\therefore P_{kl} = P\left(S_{n+1} = k / S_n = l\right) = \begin{cases} p & \text{if } k = l+1 \\ 1-p & \text{if } k = l-1 \\ 0 & \text{otherwise} \end{cases}$$

Consider the mean total number of visits to state $i$.

It is equal to $= \sum_{n=1}^{\infty} p_{ii}^{n}$

Consider $p_{ii}^{n}$

$p_{ii}^{n}$ represents the probability of being in $S_{n+1} = i$ state starting from $S_1 = i$ state after $n$ steps.

If $S_1 = i$ and $S_{n+1} = i$

then $S_{n+1} - S_1 = x_{n+1} + x_n + \cdots x_2 = 0$

ie there are equal number of 1's and -1's in $x_i$'s

The probability of this happening

$$= \begin{cases} {}^{n}C_{n/2}\, p^{n/2} (1-p)^{n/2} & \text{when } n = \text{even} \\ 0 & \text{when } n = \text{odd} \end{cases}$$

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Therefore

mean number of visits

$$= \sum P_{ii}^{n} = \sum_{n=1}^{\infty} {}^{2n}C_n \, p^n (1-p)^n$$

Consider ${}^{2n}C_n$

W.K.T

$$\sum_{k=0}^{2n} {}^{2n}C_k = 2^{2n} = 4^n$$

The avg value of ${}^{2n}C_k = \dfrac{4^n}{2n+1}$

The largest term is ${}^{2n}C_n$

$$\therefore \underset{\text{(avg of sequence)}}{\frac{4^n}{2n+1}} < \underset{\text{(largest term in sum)}}{{}^{2n}C_n} < \underset{\text{(Total sum)}}{4^n}$$

$$\sum_{n=1}^{\infty} \frac{4^n}{2n+1} \, p^n (1-p)^n < \sum_{n=1}^{\infty} {}^{2n}C_n \, p^n (1-p)^n$$

$$< \sum_{n=1}^{\infty} {}^{2n}4^n \, p^n (1-p)^n$$

let $L = \sum_n {}^{2n}C_n \, p^n (1-p)^n$

If $p = 1/2$

$$\sum_{n=1}^{\infty} \frac{4^n}{2n+1} \times \frac{1}{4^n} < L$$

W.K.T $\sum_{n=1} \dfrac{1}{2n+1}$ is diverging

Hence $L$ is diverging

$\therefore$ Since $\sum_{n=1}^{\infty} P_{ii}^{n}$ is diverging

the state is recurrent

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Now consider $p \neq 1/2$

$$L < \sum_{n=1}^{\infty} 4^n \times (p(1-p))^n$$

since $p \neq 1/2$ $\quad p(1-p) < 1/4$

$$4p(1-p) < 1$$

$$(4p(1-p))^n < 1$$

$\therefore \sum_{n=1}^{\infty} 4^n (p(1-p))^n$ series is converging

as $n^{th}$ term $\to 0$ as $n \to \infty$

, $n^{th}$ term $= (4p(1-p))^n$

$\therefore$ L is also converging

i.e $\sum p_{ii}^n$ is converging

$\therefore$ the state is transient.

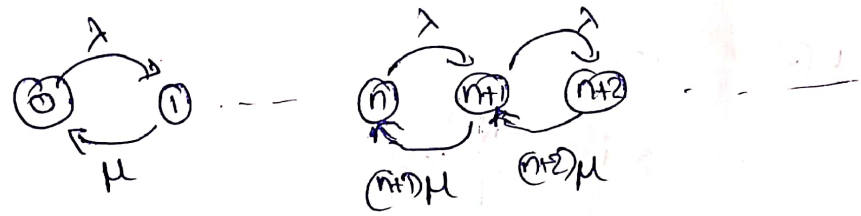
6A) Given

$M/M/\infty$

arrival rate ($\lambda$) — Service rate ($\mu$) — Infinite Servers

$X(t)$ - denote the number of jobs

The state transition diagram for this queue $M/M/\infty$ is




ie The rate matrix $Q$ of $X(t)$ =

$$\begin{bmatrix} -\lambda & \lambda & 0 & \cdots \\ \mu & -(\mu+\lambda) & \lambda & \cdots \\ & & & \\ & & n\mu & -(n\mu+\lambda) & \lambda & \cdots \\ & & & & & \end{bmatrix}$$

The dimension of $Q$ would not be finite as $X(t)$ can take infinite no of jobs.

Let the stationary distribution be $\pi = [\pi_0 \ \pi_1 \ \cdots]$

$\pi Q = 0$

$-\pi_0\lambda + \pi_1\mu = 0$

$\pi_1 = \left(\frac{\lambda}{\mu}\right)\pi_0$

$\pi_1\lambda - (\mu+\lambda)\pi_1 + 2\mu\pi_2 = 0$

$2\mu\pi_2 = \pi_0\left[\frac{\lambda^2}{\mu}\right]$

$\pi_2 = \pi_0\frac{\lambda^2}{2\mu^2} = \frac{\pi_0\lambda^2}{(2!)\mu^2}$

$\pi_1\lambda$ - Similarly

$\pi_1\lambda - (\mu+\lambda)\pi_2 + 3\mu\pi_3 = 0$

$\frac{\lambda}{\mu}\pi_0 - (\mu+\lambda)\frac{\pi_0\lambda^2}{2\mu^2} + 3\mu\pi_3 = 0$

$\Rightarrow \pi_3 = \left(\frac{\lambda}{\mu}\right)^3 \times \frac{1}{3!}\pi_0$

Similarly

$\pi_k = \left(\frac{\lambda}{\mu}\right)^k \times \frac{1}{k!}\pi_0$

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W.K.T

$$\sum_i \pi_i = 1$$

$$\pi_0 + \pi_0 \left(\frac{\lambda}{\mu}\right) \times \frac{1}{1!} + \pi_0 \left(\frac{\lambda}{\mu}\right)^2 \times \frac{1}{2!} \cdots = 1$$

$$\pi_0 \left[ 1 + \left(\frac{\lambda}{\mu}\right) \times \frac{1}{1!} + \left(\frac{\lambda}{\mu}\right)^2 \times \frac{1}{2!} \cdots \right] = 1$$

$$\pi_0 \times e^{\lambda/\mu} = 1$$

$$\pi_0 = e^{-\lambda/\mu}$$

$(1, (0,1))$

$$E(x) = \sum i \pi_i$$

$$= \sum i \, \pi_0 \times \left(\frac{\lambda}{\mu}\right)^i \times \frac{1}{i!}$$

$$= \pi_0 \sum \left(\frac{\lambda}{\mu}\right)^i \times \frac{1}{(i-1)!} = \pi_0 \times \frac{\lambda}{\mu} \sum \left(\frac{\lambda}{\mu}\right)^{i-1} \times \frac{1}{(i-1)!}$$

$$= \pi_0 \times \frac{\lambda}{\mu} \times e^{\lambda/\mu}$$

$$= \lambda/\mu$$

$$\boxed{E(x) = \lambda/\mu}$$

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