ASSIGNMENT - 1

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202110101

1A) (a)

Given $f(x) = \begin{cases} 1 & 0 \le x \le \pi \\ -1 & \pi \le x < 2\pi \end{cases}$

$f(x+2\pi) = f(x)$

Period $= 2\pi$

To find fourier series (Trigonometric Fourier series) representation

$$f(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)]$$

where $a_0 = \frac{1}{T} \int_{t_0}^{t_0+T} f(t)\, dt$

$$a_n = \frac{2}{T} \int_{t_0}^{t_0+T} f(t) \cos(n\omega_0 t)\, dt$$

$\omega_0 = \frac{2\pi}{2\pi} = 1 \qquad T = 2\pi$

$$a_0 = \frac{1}{2\pi} \int_{t_0}^{t_0+T} f(t)\, dt = \frac{1}{2\pi} \int_0^{2\pi} f(t)\, dt$$

$$= \frac{1}{2\pi} \left[\int_0^{\pi} 1\, dt + \int_{\pi}^{2\pi} -1\, dt \right]$$

$$= \frac{1}{2\pi} [\pi - \pi] = 0$$

$$a_n = \frac{2}{2\pi} \int_0^{2\pi} f(t) \cos(nt)\, dt$$

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$$= \frac{1}{\pi}\left[\int_0^{\pi} \cos nt\, dt - \int_{\pi}^{2\pi} \cos nt\, dt\right]$$

$$= \frac{1}{\pi}\left[\left[\frac{\sin(nt)}{n}\right]_0^{\pi} - \left[\frac{\sin(nt)}{n}\right]_{\pi}^{2\pi}\right]$$

$$= \frac{1}{\pi}\left[\sin n\pi + \sin n\pi\right] \times \frac{1}{n}$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\sin(n\omega_0 t)\, dt$$

$$= \frac{2}{2\pi}\int_0^{2\pi} f(t)\sin(nt)\, dt$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} \sin(nt)\, dt - \int_{\pi}^{2\pi} \sin(nt)\, dt\right]$$

$$= \frac{1}{\pi}\left[\left[\frac{-\cos(nt)}{n}\right]_0^{\pi} - \left[\frac{-\cos(nt)}{n}\right]_{\pi}^{2\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{n} - \frac{\cos \pi n}{n} - \left(\frac{\cos \pi n}{n} - \frac{1}{n}\right)\right]$$

$$= \frac{2}{\pi}\left[\frac{1}{n} - \frac{\cos \pi n}{n}\right]$$

Hence The Fourier series is

$$f(t) = \sum_{n=1}^{\infty}\left[\frac{2}{\pi n}\sin n\pi \cos(n\omega_0 t) + \frac{2}{\pi n}(1-\cos \pi n)\sin(nt)\right]$$

(b) Given $f(x) = x \quad -\pi \le x \le \pi$
and $f(x+2\pi) = f(x)$

$\therefore$ for $0 \le x \le \pi \quad f(x) = x$
$\pi \le x \le 2\pi \quad f(x) = x - 2\pi$

Fourier series approximation

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \right]$$

$$a_0 = \frac{1}{T} \int_{t_0}^{t_0+T} f(t)\, dt$$

$$= \frac{1}{2\pi} \int_0^{2\pi} f(t)\, dt = \frac{1}{2\pi} \left[\int_0^{\pi} t\, dt + \int_{\pi}^{2\pi} (t - 2\pi)\, dt \right]$$

$$= \frac{1}{2\pi} \left[\left[\frac{t^2}{2} \right]_0^{\pi} + \left[\frac{t^2}{2} \right]_{\pi}^{2\pi} - \left[2\pi t \right]_{\pi}^{2\pi} \right]$$

$$= \frac{1}{2\pi} \left[\left[\frac{4\pi^2}{2} \right] - 2\pi^2 \right] = 0$$

$$a_n = \frac{2}{T} \int_{t_0}^{t_0+T} f(t) \cos(n\omega_0 t)\, dt$$

$$= \frac{2}{2\pi} \int_0^{2\pi} f(t) \cos nt$$

$$= \frac{1}{\pi} \left[\int_0^{\pi} t \cos nt\, dt + \int_{\pi}^{2\pi} (t - 2\pi) \cos nt\, dt \right]$$

$$= \frac{1}{\pi} \left[\int_0^{2\pi} t \cos nt\, dt - 2\pi \int_{\pi}^{2\pi} \cos nt\, dt \right]$$

$$= \frac{1}{\pi}\left[\left[\frac{t\sin nt}{n} - \frac{1}{n}\int \sin nt\, dt\right]_0^{2\pi} - 2\pi\left[\frac{\sin nt}{n}\right]_\pi^{2\pi}\right]$$

$$= \frac{1}{\pi}\left[\left[\frac{t\sin nt}{n} + \frac{\cos nt}{n^2}\right]_0^{2\pi} - \frac{2\pi}{n}\left(\sin nt\right)_\pi^{2\pi}\right]$$

$$= \frac{1}{\pi}\left[\cancel{0}\ \frac{1}{n^2} - \frac{1}{n^2} + \frac{2\pi}{n}\sin n\pi\right]$$

$$= \frac{2}{n}\sin n\pi = 0$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\sin(n\omega_0 t)\,dt$$

$$= \frac{2}{2\pi}\int_0^{2\pi} f(t)\sin(nt)\,dt$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} t\sin(nt)\,dt + \int_\pi^{2\pi}(t-2\pi)\sin(nt)\,dt\right]$$

$$= \frac{1}{\pi}\left[\int_0^{2\pi} t\sin(nt)\,dt - 2\pi\int_\pi^{2\pi}\sin(nt)\,dt\right]$$

$$= \frac{1}{\pi}\left[\left[-\frac{t\cos nt}{n} + \frac{1}{n}\int_0^{2\pi}\cos nt\,dt\right]_0^{2\pi} + 2\pi\left[\frac{\cos nt}{n}\right]_\pi^{2\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{-2\pi}{n} + \frac{2\pi}{n}\left[1-\cos n\pi\right]\right] = \frac{2}{n}\left[1-\cos n\pi\right]$$

$$= -\frac{2}{n}\cos n\pi$$

(c) $f(x) = e^{x} \quad -\pi \le x \le \pi$

$f(x+2\pi) = f(x)$

$\therefore \quad f(x) = e^{x} \quad 0 \le x \le \pi$

$= e^{x-2\pi} \quad \pi \le x \le 2\pi$

Fourier series representation

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \right]$$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$$

$$a_0 = \frac{1}{T} \int_{t_0}^{t_0+T} f(t)\, dt = \frac{1}{2\pi} \int_0^{2\pi} f(t)\, dt$$

$$= \frac{1}{2\pi} \left[\int_0^{\pi} e^{t}\, dt + \int_{\pi}^{2\pi} e^{t-2\pi}\, dt \right]$$

$$= \frac{1}{2\pi} \left[(e^{\pi} - 1) + \frac{1}{e^{2\pi}} (e^{2\pi} - e^{\pi}) \right]$$

$$a_n = \frac{2}{T} \int_0^{T} f(t) \cos(n\omega_0 t)\, dt$$

$$= \frac{2}{2\pi} \int_0^{2\pi} f(t) \cos nt\, dt = \frac{1}{\pi} \left[\int_0^{\pi} e^{t} \cos nt\, dt + \int_{\pi}^{2\pi} \frac{e^{t}}{e^{2\pi}} \cos nt\, dt \right]$$

$$\int e^{t} \cos nt\, dt = \frac{e^{t} \sin nt}{n} - \int \frac{e^{t} \sin nt}{n}\, dt \quad \text{— (1)}$$

$$\int e^{t} \sin nt\, dt = \frac{-e^{t} \cos nt}{n} + \int \frac{e^{t} \cos nt}{n}\, dt \quad \text{— (2)}$$

From (1) and (2)

$\int e^t \cos nt\, dt$

$$= \frac{e^t[\cos nt + n\sin(nt)]}{1+n^2}$$

$$a_0 = \frac{1}{\pi}\left[\int_0^{\pi} e^t \cos nt\, dt + \frac{1}{e^{2\pi}}\int_{\pi}^{2\pi} e^t \cos nt\, dt\right]$$

$$= \frac{1}{\pi}\left[\left(\frac{e^t(\cos nt + n\sin nt)}{1+n^2}\right)_0^{\pi} + \frac{1}{e^{2\pi}}\left(\frac{e^t(\cos nt + n\sin nt)}{1+n^2}\right)_{\pi}^{2\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{e^{\pi} - e^{-\pi}}{1+n^2}\right]\cos n\pi$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\sin(n\omega_0 t)\, dt$$

$$= \frac{1}{\pi}\int_0^{2\pi} f(t)\sin(n\omega_0 t)\, dt$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} f(t)\sin(nt)\, dt + \int_{\pi}^{2\pi} f(t)\sin(nt)\, dt\right]$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} e^t \sin(nt)\, dt + \int_{\pi}^{2\pi} e^{t-2\pi}\sin(nt)\, dt\right]$$

$$\int e^t \sin(nt)\, dt = \frac{e^t\sin(nt) - ne^t\cos(nt)}{1+n^2}$$

from ①, ②

$$\therefore b_n = \frac{1}{\pi}\left[\left[\frac{e^t\sin(nt) - ne^t\cos(nt)}{1+n^2}\right]_0^{\pi} +\right.$$

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(d) Given

$$f(x) = \begin{cases} 0 & -\pi \le x < 0 \\ x & 0 \le x < \pi \end{cases}$$

$$f(x+2\pi) = f(x)$$

For $0 \le x \le \pi$ $\quad f(x) = x$

$\pi \le x \le 2\pi$ $\quad f(x) = 0$

Fourier series representation

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \right]$$

$$a_0 = \frac{1}{T} \int_{t_0}^{t_0+T} f(t)\, dt =$$

$$T = 2\pi \qquad \omega_0 = \frac{2\pi}{2\pi} = 1$$

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(t)\, dt = \frac{1}{2\pi} \left[\int_0^{\pi} f(t)\, dt \right]$$

$$= \frac{1}{2\pi} \left[\int_0^{\pi} t\, dt \right]$$

$$= \frac{1}{2\pi} \left[\frac{\pi^2}{2} \right] = \pi/4$$

$$a_n = \frac{2}{T} \int_0^{T} f(t) \cos(n\omega_0 t)\, dt$$

$$= \frac{1}{\pi} \int_0^{2\pi} f(t) \cos(n\omega_0 t)\, dt = \frac{1}{\pi} \int_0^{\pi} t \cos nt\, dt$$

$$= \frac{1}{\pi} \left[\frac{t \sin(nt)}{n} + \frac{\cos(nt)}{n^2} \right]_0^{\pi}$$

$$= \frac{1 - \cos n\pi}{n^2 \pi}$$

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$$b_n = \frac{2}{2\pi}\int_0^{2\pi} f(t)\sin(nt)\,dt$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} t \sin nt\,dt\right]$$

$$= \frac{1}{\pi}\left[-\frac{t\cos nt}{n} + \frac{\sin(nt)}{n^2}\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{-\pi\cos\pi n}{n}\right] = \frac{-\cos\pi n}{n}$$

$\therefore$ The fourier series expansion

$$f(x) = \pi/4 + \sum_{n=1}^{\infty}\left(\frac{1-\cos n\pi}{n^2\pi}\cos nt - \frac{\cos\pi n}{n}\sin nt\right)$$

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QUESTION-2

(a)

$$f(x) = \begin{cases} x-1 & 0 \le x \le 2 \\ -1-x & -2 < x < 0 \end{cases}$$

$$f(x+4) = f(x)$$

Consider $x \in [0,2]$

$$f(x) = x-1$$

$$f(-x) = -1-(-x) = x-1$$

i.e $f(x) = f(-x)$

i.e $f(x)$ is even in $(-2,2)$ interval

since $f(x+4) = f(x)$

and the graph is symmetric about y-axis thus $f(x)$ is even

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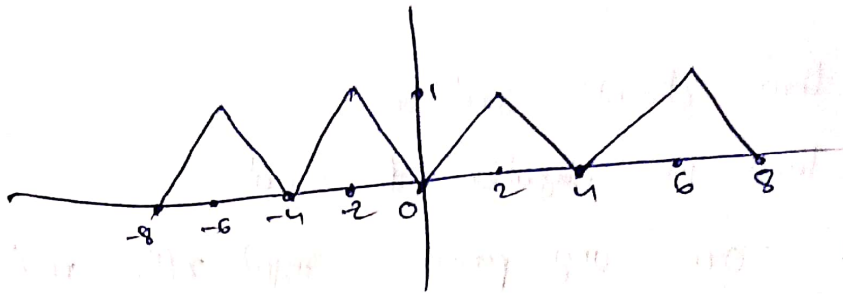


(b) $f(x) = \sin x + \cos x$

$f(-x) = \sin(-x) + \cos(-x)$

$= -\sin x + \cos x$

Clearly $f(x) \neq f(-x)$ for some values of x

i.e when $\sin x \neq 0$

and also $f(x) \neq -f(-x)$ for some values of x

i.e when $\cos x \neq 0$

$\therefore$ $f(x)$ is neither even nor odd.

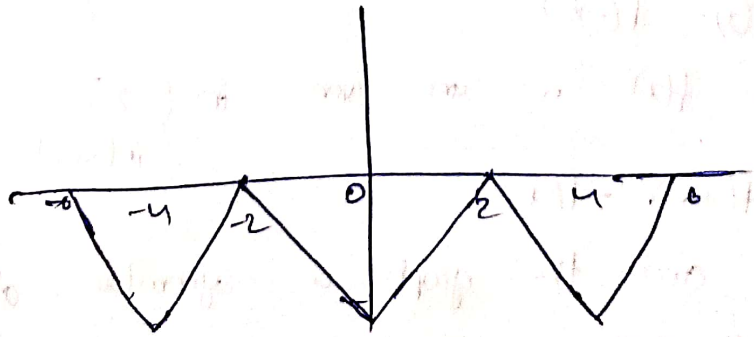
(c) $f(x) = |x| - 2 \qquad -2 \leq x \leq 2$

$f(x+4) = f(x)$

$f(x) = |x| - 2 \qquad x \in [-2, 2]$

$f(-x) = |-x| - 2 = |x| - 2 = f(x)$

Hence $f(x) = f(-x)$ i.e $f(x)$ is even in $(-2, 2)$.



$\therefore$ If the graph is symmetric about y axis, the given function is even.

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QUESTION-3 :

(a)

Ans : There are three dirichlet conditions

1st property :- leads to integration of signal

i.e over any period, signal $x(t)$ must be integrable

2nd property :

In any finite interval $x(t)$ is of bounded variation, that is there are no more than a finite number of maxima and minima during a single period of signal.

3rd property :-

It states that in any finite interval of time there is only a finite number of discontinuous

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3a) (b)

RTP: $\pi/4 = 1 - 1/3 + 1/5 - 1/7 + \cdots$

Proof:

Consider the function

$$f(x) = x/2 \qquad 0 \le x \le 2\pi$$

$$f(x+2\pi) = f(x)$$

The fourier series expansion for this is

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \right]$$

$$\omega_0 = \frac{2\pi}{2\pi} = 1$$

$$a_0 = \frac{1}{T} \int_0^{2\pi} f(t)\,dt$$

$$= \frac{1}{2\pi} \int_0^{2\pi} t/2 \, dt = \frac{1}{4\pi} \left[t^2/2 \right]_0^{2\pi}$$

$$= \frac{1}{4\pi} \times \frac{4\pi^2}{2} = \pi/2$$

$$b_n = 2/T \int_{t_0}^{t_0+T} f(t) \sin(n\omega_0 t)\, dt$$

$$= 2/2\pi \int_0^{2\pi} t/2 \sin(nt)\, dt$$

$$= \frac{1}{2\pi} \int_0^{2\pi} t \sin(nt)\, dt$$

$$= \frac{1}{2\pi} \left[\frac{-t\cos(nt)}{n} + \frac{\sin(nt)}{n^2} \right]_0^{2\pi}$$

$$= \frac{1}{2\pi n} \left(-2\pi \cos(2\pi n) \right) = \frac{-1}{n} \cos(2\pi n)$$

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$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos(n\omega_0 x) + b_n \sin(n\omega_0 x)\right)$$

$$f(x) = \pi/2 + \sum_{n=1}^{\infty}\left(a_n \cos(nx) - \frac{\cos(2\pi n)}{n} \sin(n\right)$$

at $x = \pi/2$

$$f(x) = \pi/4$$

$$\pi/4 = \pi/2 + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{\pi n}{2}\right) - \frac{\cos(2\pi n)}{n} \sin\left(\right.\right.$$

$$-\pi/4 = \sum_{n=1}^{\infty} - \frac{\cos(2\pi n)}{n} \sin\left(\frac{\pi n}{2}\right)$$

$$\pi/4 = \sum_{n=1}^{\infty} \frac{1}{n} \sin(\pi/2\, n)$$

$$= 1 - 1/3 + 1/5 - 1/7 \ldots$$

(c) RTP:

$$\pi^2/8 = 1 + 1/3^2 + 1/5^2 \ldots$$

Proof:

Consider

$$f(x) = x^2 \qquad 0 \le x < 2\pi$$

$$f(x+2\pi) = f(x)$$

Fourier expansion

$$f(x) = a_0 + \sum a_n \cos(n\omega_0 x) + b_n \sin(n\omega_0 x)$$

$$\omega_0 = \frac{2\pi}{2\pi} = 1$$

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$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} x^2\,dx$$

$$= \frac{1}{2\pi}\left[\frac{x^3}{3}\right]_0^{2\pi} = 4\pi^2/3$$

$$a_n = 2/2\pi \int_0^{2\pi} x^2\cos(nx)\,dx$$

$$= 1/\pi \int_0^{2\pi} x^2\cos(nx)\,dx$$

$$\int_0^{2\pi} x^2\cos(nx)\,dx = \frac{x^2\sin(nx)}{n} - \int \frac{2x\sin(nx)}{n}\,dx$$

$$\int x\sin(nx) = \frac{-x\cos(nx)}{n} + \frac{1}{n}\int\cos(nx)\,dx$$

$$= \frac{-x\cos(nx)}{n} + \frac{\sin(nx)}{n^2}$$

$$\int_0^{2\pi} x^2\cos(nx)\,dx = \left[\frac{x^2\sin(nx)}{n} + \frac{2x\cos(nx)}{n^2} - 2/n^3\sin(nx)\right]_0^{2\pi}$$

$$= \frac{4\pi}{n^2}$$

$$a_n = 4/n^2$$

$$f(x) = 4\pi^2/3 + \Sigma\, 4/n^2\cos(nx) + b_n\sin(nx)$$

$$x = \pi$$

$$\pi^2 = 4\pi^2/3 + \Sigma\, 4/n^2\cos(n\pi)$$

$$\pi^2/3 = \sum_{n=1}^{\infty} -4/n^2(-1)^n$$

$$\pi^2/12 = 1 - 1/2^2 + 1/3^2 - \text{——} \quad (1)$$

let $x = 1 + 1/3^2 + \text{——}$

$y = 1/2^2 + 1/4^2 - \text{——}$

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$$x+y = 1^2 + 1/2^2 + 1/3^2 - \cdots$$

But $y = 1/2^2\left(1^2 + 1/2^2 + 1/3^2 \cdots\right)$

$\therefore \quad x+y = 2^2 \times y$

$x = 3y$

From (1)

$$x-y = \pi^2/12$$

$$x - x^2/3 = \pi^2/12 \Rightarrow 2x/3 = \pi^2/12$$

$$x = \pi^2/8$$

$$\boxed{1 + 1/3^2 + 1/5^2 \cdots = \pi^2/8}$$

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SECTION-2

QUESTION-1

(a) Given signal

$$x(t) = t\left(\frac{\sin t}{\pi t}\right)^2$$

let $y(t) = \dfrac{\sin t}{\pi t}$

W.K.T

The fourier transform (FT) of $y(t)$ is $\mathrm{rect}(\omega/2)$

where $\mathrm{rect}(x) = \begin{cases} 0 & \text{if } |x| > 1/2 \\ 1/2 & \text{if } |x| = 1/2 \\ 1 & \text{if} \end{cases}$

$\therefore \mathrm{rect}(\omega/2) = \begin{cases} 0 & \text{if } |\omega| > 1 \\ 1/2 & \text{if } |\omega| = 1 \\ 1 & \text{if } |\omega| < 1 \end{cases}$

Let $z(t) = (y(t))^2$

Fourier transform of $z(t)$ is $= Z(j\omega)$

$= FT(z(t))$

$= FT(y(t) \cdot y(t))$

$= \frac{1}{2\pi} FT(y(t)) * FT(y(t))$

$= \frac{1}{2\pi} X(j\omega) * X(j\omega)$

From convolution theorem.

$$Z(j\omega) = \frac{1}{2\pi} X(j\omega) * X(j\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j(\omega - u)) X(j\omega)\, du$$

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But W.K.T

$$X(j\omega') = \text{rect}\left(\frac{\omega'}{2}\right)$$ (Fourier transform of $x(t)$)

$$\therefore \quad Z(j\omega') = \frac{1}{2\pi}\int_{-\infty}^{\infty} \text{rect}\left(\frac{\omega'-\omega}{2}\right)\text{rect}\left(\frac{\omega}{2}\right) d\omega$$

$$\text{rect}\left(\frac{\omega}{2}\right) = \begin{cases} 0 & |\omega| > 1 \\ 1/2 & |\omega| = 1 \\ 1 & |\omega| < 1 \end{cases}$$

$$\text{rect}\left(\frac{\omega-\omega'}{2}\right) = \begin{cases} 0 & |\omega'-\omega| > 1 \\ 1/2 & |\omega'-\omega| = 1 \\ 1 & |\omega'-\omega| < 1 \end{cases}$$

$$Z(j\omega') = \frac{1}{2\pi}\left[\int_{-\infty}^{-1} \text{rect}\left(\frac{\omega'-\omega}{2}\right)\text{rect}\left(\frac{\omega}{2}\right) d\omega + \int_{-1}^{1} \text{rect}\left(\frac{\omega'-\omega}{2}\right)\text{rect}\left(\frac{\omega}{2}\right) d\omega + \int_{1}^{\infty} \text{rect}\left(\frac{\omega'-\omega}{2}\right)\text{rect}\left(\frac{\omega}{2}\right) d\omega\right]$$

if $|\omega| > 1$ i.e $-\infty < \omega < -1$, $1 < \omega < \infty$

$$\text{rect}\left(\frac{\omega}{2}\right) = 0$$

$$\therefore \quad Z(j\omega') = \frac{1}{2\pi}\int_{-1}^{1} \text{rect}\left(\frac{\omega'-\omega}{2}\right) d\omega$$

for $-1 < \omega < 1$

If $\left|\frac{\omega'-\omega}{2}\right| > 1/2 \Rightarrow \omega' > 1+\omega$, or $\omega' < \omega - 1$

$\Rightarrow \omega' > 2$ or $\omega' < -2$

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if $\omega' > 2$, for $-1 < \omega < 1$

$$\omega' - 1 < \omega' - \omega < \omega' + 1$$

$$1 < \omega' - \omega$$

$$\Rightarrow \operatorname{rect}\left(\frac{\omega' - \omega}{2}\right) = 0$$

for $\omega' > 2$,

$$Z(j\omega) = \frac{1}{2\pi}\int_{-1}^{1} \operatorname{rect}\left(\frac{\omega' - \omega}{2}\right) \operatorname{rect}\left(\frac{\omega}{2}\right) d\omega$$

$$= 0$$

if $\omega' < -2$ for $-1 < \omega < 1$

$$\omega' - 1 < \omega' - \omega < \omega' + 1$$

$$\omega' - \omega < -1$$

$$|\omega' - \omega| > 1 \Rightarrow \operatorname{rect}\left(\frac{\omega' - \omega}{2}\right) = 0$$

$$\therefore Z(j\omega) = 0$$

for $-2 < \omega' < 2$,

for $\operatorname{rect}\left(\frac{\omega' - \omega}{2}\right) \neq 0$

$$-1 < \omega' - \omega < 1$$

$$\omega' - 1 < \omega < \omega' + 1$$

for $0 < \omega' < 2 \Rightarrow$ upper limit of $\omega = \max(\omega' + 1, 1)$

lower limit of $\omega' = \max(\omega' - 1, -1)$
$= \omega' - 1$

for $-2 < \omega' < 0 \Rightarrow$ upper limit of $\omega = \max(\omega' + 1, 1)$
$= \omega' + 1$

lower limit of $\omega = \max(\omega' - 1, -1)$
$= \omega' - 1$

$F'(\omega) =$

for $-2 < \omega' < 0$

$$Z(j\omega') = \frac{1}{2\pi}\int_{-1}^{\omega'+1} 1 \, d\omega$$

$$= \frac{1}{2\pi}(\omega)_{-1}^{\omega'+1}$$

$$= \frac{1}{2\pi}(\omega'+1+1) = \frac{\omega'+2}{2\pi}$$

for $0 < \omega' < 2$

$$Z(j\omega') = \frac{1}{2\pi}\int_{\omega'-1}^{1} 1 \, d\omega$$

$$= \frac{1}{2\pi}(\omega)_{\omega'-1}^{1} = \frac{1-(\omega'-1)}{2\pi} = \frac{2-\omega'}{2\pi}$$

$$\therefore \quad Z(j\omega) = \begin{cases} \frac{2+\omega}{2\pi} & -2 \le \omega < 0 \\ \frac{2-\omega}{2\pi} & 0 \le \omega \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

Given

$$x(t) = t\left(\frac{\sin t}{\pi t}\right)^2 = t \cdot Z(t)$$

$$X(j\omega) = FT(x(t)) = FT(t \cdot Z(t))$$

Let us consider Fourier transform

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$\frac{d}{d\omega}(F(\omega)) = \frac{d}{d\omega}\left(\int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt\right)$$

$$= \int_{-\infty}^{\infty} f(t) \times e^{-j\omega t}(-jt) \, dt$$

$$F'(\omega) = -j\int_{-\infty}^{\infty} (f(t) \cdot t) e^{-j\omega t} dt$$

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$F'(j\omega) = -j$ Fourier Transform$(t \cdot f(t))$

$\Rightarrow$ Fourier Transform $(t \cdot f(t)) = j F'(j\omega)$.

$\therefore \quad FT(x(t)) = FT(t \cdot z(t)) = j F'(j\omega)$

$= j \dfrac{d}{d\omega}(Z(j\omega))$

$$Z(j\omega) = \begin{cases} \frac{2+\omega}{2\pi} & -2 \le \omega < 0 \\ \frac{2-\omega}{2\pi} & 0 \le \omega \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{d}{d\omega}(Z(j\omega)) = \begin{cases} 1/2\pi & -2 \le \omega \le 0 \\ -1/2\pi & 0 \le \omega \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$X(j\omega) = \begin{cases} j/2\pi & -2 < \omega < 0 \\ -j/2\pi & 0 < \omega < 2 \\ 0 & \text{otherwise} \end{cases}$$

(b) $x(t) = t\left(\dfrac{\sin t}{\pi t}\right)^2$

$x^2(t) = t^2\left(\dfrac{\sin t}{\pi t}\right)^4$

$$\int_{-\infty}^{\infty} (x(t))^2 \, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 \, d\omega$$

$\hookrightarrow$ Parseval's Law

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \left[\int_{-\infty}^{-2} |X(j\omega)|^2 d\omega + \int_{-2}^{2} |X(j\omega)|^2 d\omega + \int_{2}^{\infty} |X(j\omega)|^2 d\omega \right]$$

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$$= \frac{1}{2\pi}\left[\int_{-2}^{2} X(j\omega)^2\, d\omega + 0 + 0\right]$$

$$= \frac{1}{2\pi}\left[\int_{-2}^{2} \frac{4}{4\pi^2}\, d\omega\right]$$

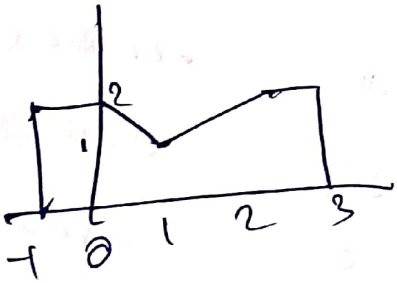
$$= \frac{1}{2\pi} \times \frac{4}{4\pi^2} = \frac{1}{2\pi^3}$$

$$\therefore \quad A = \int_{-\infty}^{\infty} t^2 \left(\frac{\sin t}{\pi t}\right)^4 dt = \frac{1}{2\pi^3}$$

SECTION -2

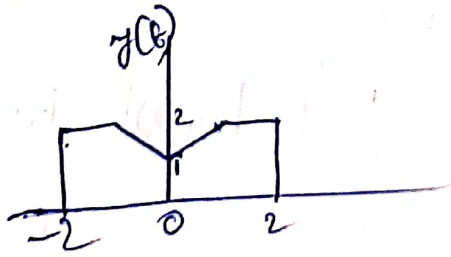
QUESTION -2:

(a) Given $x(t)$:



Consider $y(t) = x(t+1)$

ie the graph of $x(t)$ is shifted to the left by 1 unit.



ie $y(t)$ is even.

Hence

$X(j\omega)$ = Fourier transform ($y(t)$) is also even, real

$x(t+1) = y(t) \Rightarrow x(t) = y(t-1)$

$X(j\omega) = FT(x(t))$

$= FT(y(t-1))$ (time - shifting property)

$= e^{-j\omega(1)} FT(y(t))$

$= X(j\omega)\, e^{-j\omega}$

Since $X(j\omega)$ is real

$\boxed{\text{The phase of } X(j\omega) = -\omega}$

(b) $X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$

$X(j0) = \int_{-\infty}^{\infty} x(t)\, dt$

= Area under the curve

$= 2 \times 4 - \frac{1}{2} \times 1 \times 2$

$= 7$

(c) $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$

By taking $t = 0$,

$x(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, d\omega$

$\therefore \int_{-\infty}^{\infty} X(j\omega)\, d\omega = 2\pi \times x(0)$

$= 2\pi \times 2 = 4\pi$

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(d)

Assume $X(\omega) = \frac{2\sin\omega}{\omega} e^{j(2\omega)}$

W.K.T

FT of $\text{rect}(t/2) \rightarrow \frac{2\sin\omega}{\omega}$

~~Smita~~ By time shifting property

$$\underbrace{\text{rect}\left(\frac{t+2}{2}\right)}_{y(t)} \rightarrow \underbrace{2\sin\omega\, e^{j2\omega}}_{X(\omega)}$$

Hence $y(t) = \begin{cases} 1 & \text{if } -3 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$

To prove statement ① calculating FT of $y(t)$

$$FT(y(t)) = \int_{-\infty}^{\infty} y(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-3}^{1} e^{-j\omega t}\, dt$$

$$= \frac{e^{3j\omega} - e^{-j\omega}}{j\omega}$$

$$= \frac{e^{-2j\omega}}{\omega}\left[\frac{e^{j\omega} - e^{-j\omega}}{j}\right) = \frac{e^{2j\omega}}{\omega}\, 2\sin\omega$$

$$\therefore \int_{-\infty}^{\infty} X(\omega) \cdot Y(\omega)\, d\omega = 2\pi \left\{x(t) * y(t)\right\}_{t=0}$$

$$= 2\pi \int_{-\infty}^{\infty} x(t')\, y(0 - t')\, dt$$

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$$= 2\pi \int_{-\infty}^{\infty} x(t)\, y(-t)\, dt$$

$$= 2\pi \int_{1}^{3} x(t)\, dt$$

$$= 2\pi \left(\text{Area of the curve from } 1 \text{ to } 3 \right)$$

$$= 2\pi \times 7/2 = 7\pi$$

(2) From Parseval's law

$$\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$x(t) = \begin{cases} 0 & x < -1 \\ 2 & -1 < x < 0 \\ 2-t & 0 < x < 1 \\ t & 1 < x < 2 \\ 2 & 2 < x < 3 \\ 0 & x > 3 \end{cases}$$

$$\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi \left[\int_{-1}^{0} 4\, dt + \int_{0}^{1} (2-t)^2 dt + \int_{1}^{2} t^2 dt + \int_{2}^{3} 4\, dt \right]$$

$$= 2\pi \left[4 + 4 + \left[\frac{(t-2)^3}{3} \right]_0^1 + \left[\frac{t^3}{3} \right]_1^2 \right]$$

$$= 2\pi \left[8 + 7/3 + 8/3 - 1/3 \right]$$

$$= \cancel{\frac{84\pi}{3}} \quad 2\pi \times \frac{38}{3} = \frac{76\pi}{3}$$

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(f) W.K.T

$$X(j\omega) = Y(j\omega)\, e^{-j\omega}$$

where $Y(j\omega)$ is $FT(y(t))$

where $y(t) = x(-t+1)$

$$= Y(j\omega)\,[\cos\omega + j\sin\omega]$$

$$\text{Real}(X(j\omega)) = Y(j\omega)\cos\omega$$

$$= \frac{Y(j\omega)\, e^{j\omega}}{2} + \frac{Y(j\omega)\, e^{-j\omega}}{2}$$

$$= \frac{X(j\omega)\, e^{j\omega}}{2} + \frac{X(j\omega)}{2}$$

Inverse Transform $(\text{Real}(X(j\omega)))$ =

$$\frac{IT(X(j\omega)e^{j\omega})}{2} + \frac{IT(X(j\omega))}{2}$$

$$= \frac{x(t+2) + x(t)}{2}$$

$$= \frac{x(-t) + x(t)}{2}$$ (symmetric about 1)

$$x(t) = \begin{cases} 0 & t<-1 \\ 2 & -1<t<0 \\ 2-t & 0<t<1 \\ t & 1<t<2 \\ 2 & 2<t<3 \\ 0 & \text{otherwise} \end{cases}$$

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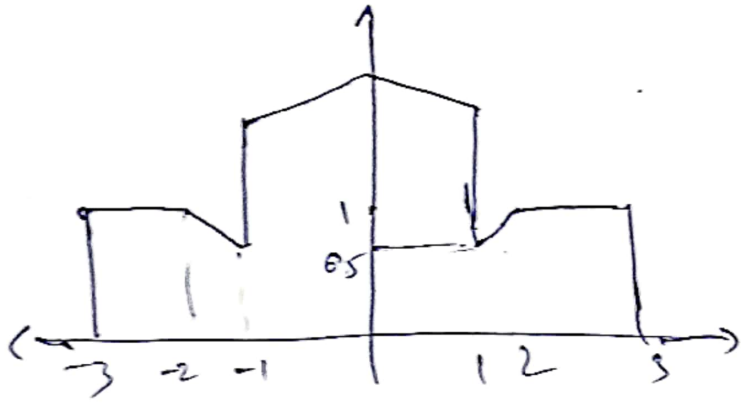
$$x(-t) = \begin{cases} 0 & t < -3 \\ 2 & -3 < t < -2 \\ 2-t & -1 < t < 0 \\ t & -2 < t < -1 \\ 2 & 0 < t < 1 \\ 0 & t > 1 \end{cases}$$

$$\frac{x(t) + x(-t)}{2} = \begin{cases} 1 & t \in (-3, -2) \\ t/2 & t \in [-2, -1] \\ 1 + t/2 & t \in [-1, 0] \\ 2 - t/2 & t \in [0, 1] \\ t/2 & t \in [1, 2] \\ 1 & t \in [2, 3] \\ 0 & \text{otherwise} \end{cases}$$

The sketch is



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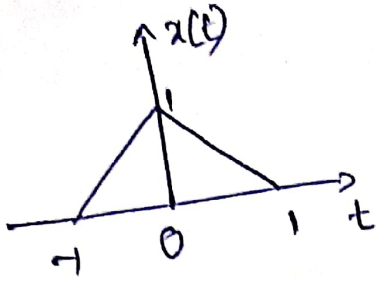
3(i) Given $x(t)$



(a) $$X(j\omega) = \int_{-\infty}^{\infty} x(t) \times e^{-j\omega t}\, dt$$

$$= \int_{-1}^{0} (t+1)\, e^{-j\omega t}\, dt + \int_{0}^{1} (-t+1)\, e^{-j\omega t}\, dt$$

$$\left[\text{as } \begin{array}{ll} x(t) = 0 & \forall\, x \notin [-1,1] \\ x(t) = t+1 & x \in [-1,0] \\ x(t) = 1-t & x \in (0,1] \end{array}\right]$$

$$= \int_{-1}^{0} \left(te^{-j\omega t} + e^{-j\omega t}\right) dt + \int_{0}^{1} -te^{-j\omega t} + e^{-j\omega t}\, dt$$

$$= \left[\frac{te^{-j\omega t}}{-j\omega} + \frac{1}{j\omega}\int_{-1}^{0} e^{-j\omega t}\, dt\right]_{-1}^{0} + \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-1}^{1} + \left[\frac{te^{-j\omega t}}{-j\omega} + \frac{1}{j\omega}\int_{1}^{0} e^{-j\omega t}\, dt\right]_{1}^{0}$$

$$= \left[\frac{te^{-j\omega t}}{-j\omega} - \frac{1}{(j\omega)^2}\left(e^{-j\omega t}\right)_{-1}^{0}\right]_{-1}^{0} + \left[\frac{te^{-j\omega t}}{-j\omega} - \frac{1}{(j\omega)^2}\left(e^{-j\omega t}\right)\right]_{1}^{0}$$

$$+ \frac{e^{j\omega} - e^{-j\omega}}{j\omega}$$

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$$= \frac{1}{(j\omega)^2} - \left(\frac{e^{j\omega}}{j\omega} - \frac{1}{(j\omega)^2} e^{j\omega} \right)$$

$$+ \frac{1}{(j\omega)^2} - \left(\frac{e^{-j\omega}}{j\omega} - \frac{1}{(j\omega)^2} e^{-j\omega} \right)$$

$$+ \frac{e^{j\omega} - e^{-j\omega}}{j\omega}$$

$$= \cancel{\frac{1 - e^{j\omega} - e^{-j\omega}}{(j\omega)^2}} + \cancel{\frac{e^{-j\omega} - e^{j\omega}}{j\omega}} + \cancel{\frac{e^{j\omega} - e^{-j\omega}}{j\omega}} = \frac{2 - 2\cos\omega}{\omega^2}$$

$$X(j\omega) = \cancel{\frac{-(e^{j\omega} - e^{-j\omega})^2}{(j\omega)^2}} \quad \cancel{\frac{2 - 2(e^{j\omega} + e^{-j\omega})}{\omega}} = \frac{2 - 2\cos\omega}{\omega^2}$$

(b) $\tilde{x}(t) = x(t) * \sum_{k=-\infty}^{\infty} \delta(t-4k)$

from convolution theorem

$$= \int_{-\infty}^{\infty} x(t') \left(\sum_{k=-\infty}^{\infty} \delta(t-t'-4k) \right) dt'$$

$$= \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x(t') \, \delta(t-t'-4k) \, dt'$$

$$= \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} x(t') \, \delta(t-t'-4k) \, dt'$$

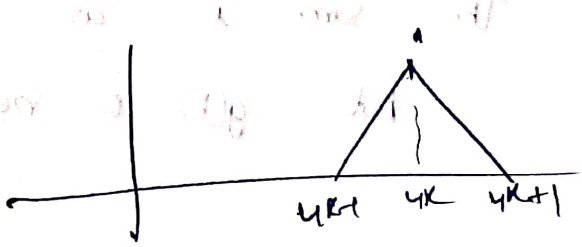
W.K.T $\int_{-\infty}^{\infty} f(t') \, \delta(t-t') \, dt' = f(t)$

$$\therefore \tilde{x}(t) = \sum_{k=-\infty}^{\infty} x(t-4k)$$

$$x(t) = \begin{cases} 0 & t < -1 \\ t+1 & -1 \le t < 0 \\ 1-t & 0 \le t < 1 \\ 0 & t \ge 1 \end{cases}$$

$$x(t-4k) = \begin{cases} 0 & t < 4k-1 \\ (t-4k)+1 & 4k-1 < t < 4k \\ 1-(t-4k) & 4k \le t < 4k+1 \\ 0 & t \ge 4k+1 \end{cases}$$

i.e The graph of $x(t-4k)$



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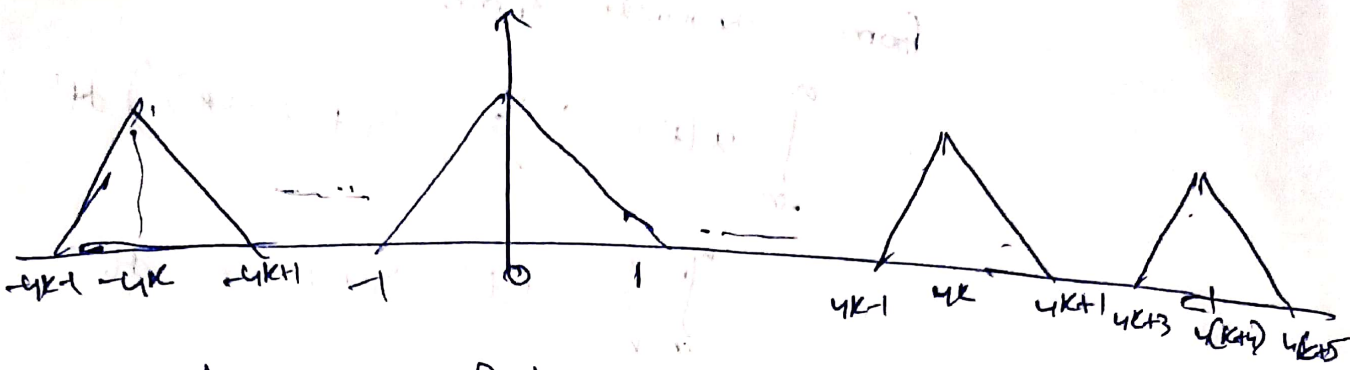
$\therefore \sum_k x(t-4k)$. The sketch of $\sum_k x(t-4k)$



(c) Consider a function

$$g(t) = x(t-4)$$

In the same way as part b

$$\bar{x} = g(t) * \sum_{k=-\infty}^{\infty} \delta(t-4k)$$

$$= \sum_{k=-\infty}^{\infty} g(t-4k)$$

~~But~~ $g(t-4k) = \cancel{x(t-}$ let $k = l-1$

$$= \sum_{l=-\infty}^{\infty} g(t-4l+4)$$

$$= \sum_{l=-\infty}^{\infty} x((t-4l)-4)$$

$$= \sum_{l=-\infty}^{\infty} x(t-4l)$$

$$= \sum_{k=-\infty}^{\infty} x(t-4k)$$

i.e The same $\bar{x}$ as the above

but $g(t)$ is not same as $x(t)$

(d) Since

$$g(t) = x(t-4)$$

$$G(j\omega) = FT(g(t)) = FT(x(t-4))$$

$$= FT(x(t))\, e^{-j\omega(4)}$$

$$= X(j\omega)\, e^{-4j\omega}$$

$$G(j\omega) = X(j\omega)\, e^{-4j\omega}$$

$$G(j\pi/2k) = X(j\pi/2k)\, e^{-4j\,\pi/2k}$$

$$= X(j\pi/2k)\, e^{-(2\pi k)j}$$

$$= X(j\pi/2k)\left(\cos(2\pi k) - j\sin(2\pi k)\right)$$

$$= X(j\pi/2k)$$

i.e. $G(j\pi/2k) = X(j\pi/2k)$

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3. DTFT

1A) Given DTFT of the signal is given by $X(e^{j\omega})$

RTP: DTFT is periodic with period 2π

PROOF:

Let $f(\omega)$ be the DTFT of the signal.

We have to prove that

$$f(\omega) = f(\omega+2\pi)$$

$$f(\omega) = X(e^{j\omega})$$

$$f(\omega+2\pi) = X(e^{j(\omega+2\pi)})$$

$$= X(e^{j\omega} \times e^{2\pi j})$$

$$= X(e^{j\omega}(\cos 2\pi + i\sin 2\pi))$$

$$= X(e^{j\omega}(1))$$

$$= X(e^{j\omega})$$

$$= f(\omega)$$

$$\therefore f(\omega) = f(\omega+2\pi)$$

$\therefore$ f ie DTFT is periodic with ~~periodic~~ period 2π

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2A) W.K.T

$$X(e^{j\omega}) = \sum_n x[n] e^{-j\omega n}$$

Let the DTFT of $y[n]$ be $Y[e^{j\omega}]$.

$$Y[e^{j\omega}] = \sum_n y[n] e^{-j\omega n}$$

$$= \sum_n x[n-m] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} x[n-m] e^{-j\omega n}$$

Let $n-m=k$

$n = k+m$

$$= \sum_{k+m=-\infty}^{\infty} x[k] e^{-j\omega(k+m)}$$

$$= \sum_{k=-\infty}^{\infty} x[k] e^{-j\omega k} (e^{-j\omega m})$$

$$= \left(\sum_{k=-\infty}^{\infty} x[k] e^{-j\omega k}\right) (e^{-j\omega m})$$

$$= X(e^{j\omega}) e^{-j\omega m}$$

$\therefore$ $\boxed{\text{DTFT of } y(n) = X(e^{j\omega}) e^{-j\omega m}}$

3A) (a) $x[n] = \left(\frac{1}{4}\right)^n u[n+2]$

$$\text{DTFT} = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$= \sum_{n=-2}^{\infty} u e^{-j\omega n}$$

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$$\left(\text{as } u(n+2) = 0 \quad \forall\ n \le -2 \right.$$
$$\left. \qquad\qquad\quad = 1 \quad n \ge -2 \right)$$

$$DTFT = \sum_{n=-2}^{\infty} e^{-j\omega n} \left(\frac{1}{4}\right)^n$$

$$= \left(\frac{1}{4}\right)^{-2} e^{-j\omega(-2)} + \left(\frac{1}{4}\right)^{-1} e^{-j\omega(-1)} + 1 + \frac{1}{4} e^{-j\omega} + \cdots$$

$$= 16 e^{2j\omega} + 4 e^{j\omega} + \left(1 + \frac{1}{4} e^{-j\omega} + \cdots\right)$$

$$= 16 e^{2j\omega} + 4 e^{j\omega} + \frac{1}{1 - \frac{e^{-j\omega}}{4}}$$

$$= 16 e^{2j\omega} + 4 e^{j\omega} + \frac{4 e^{j\omega}}{4 e^{j\omega} - 1}$$

$$= \frac{-16 e^{2j\omega} + 16 e^{2j\omega} + 64 e^{3j\omega} - 4 e^{j\omega} + 4 e^{j\omega}}{4 e^{j\omega} - 1}$$

$$= \frac{64 e^{3j\omega}}{4 e^{j\omega} - 1}$$

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(b) $x[n] = \begin{cases} 1 & \text{if } n = 0,1,2,3 \\ 0 & \text{otherwise} \end{cases}$

Therefore

$$DTFT = \sum_n x(n)\, e^{-j\omega n}$$

$$= e^{-j\omega(0)} + e^{-j\omega(1)} + e^{-j\omega(2)} + e^{-j\omega(3)}$$

$$= \frac{e^{j\omega(3)} + e^{j\omega(2)} + e^{j\omega(1)} + 1}{e^{j\omega(3)}}$$

---

4 DFT

(A) : (a) $x(n) = \frac{1}{4} u(4-n)$

if $n \geq 4$

$x(n) = \frac{1}{4}$

$\frac{1}{8}(2) + 0 + 0 + 0 + 0 + 0 + 0 + 0)$

It's 8 point DFT is

$[2, 0, 0, 0, 0, 0, 0, 0]$

if $n \leq 4$

$x(n) = 0$

$= \frac{1}{8}[0 + 0 + 0)$

It's 8 point DFT is

$[0, 0, 0, 0, 0, 0, 0, 0]$

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(b) $x(n) = \sin\left(\frac{\pi}{4} n\right) + \cos\left(\frac{\pi}{4} n\right)$

$$\cos\left(\frac{\pi}{4} n\right) = \frac{e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n}}{2}$$

$$= \frac{e^{\left(j\frac{2\pi}{8}\right)\times(1\times n)} + e^{\left(j\left(\frac{2\pi}{8}\right)\right)\times(7\times n)}}{2} \quad (1)$$

$$i \sin\left(\frac{\pi}{4} n\right) = \frac{e^{j\frac{\pi}{4}n} - e^{-j\left(\frac{\pi}{4}n\right)}}{2}$$

$$\sin\left(\frac{\pi}{4} n\right) = \left(\frac{e^{\left(j\frac{2\pi}{8}\right)\times 7n} - e^{\left(j\times\frac{2\pi}{8}\right)\times 1n}}{2}\right) i \quad (2)$$

$x(n) = (1) + (2)$

$$= \frac{1}{2}\left[e^{j\left(\frac{2\pi}{8}\right)\times(1\times n)}[1-i] + i\, e^{j\left(\frac{2\pi}{8}\right)\times(7\times n)}[1+i]\right]$$

$$= \frac{1}{8}\left[4(1-i)\, e^{j\frac{2\pi}{8}(1n)} + 4(1+i)\, e^{j\frac{2\pi}{8}(7n)}\right]$$

$\therefore$ $x(1) = 4(1-i)$

$x(7) = 4(1+i)$

all other points : $x(i) = 0$

The 8 point DFT is

$$[0,\ 4(1-j),\ 0,\ 0,\ 0,\ 0,\ 0,\ 4(1+j)]$$

(1) $$X[k] = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

$$\sum_{n=0}$$

Given $x(n) = \{1, -1-j, -1, -1+j\}$

$$X[k] = 1\, e^{-j\,2\pi/8\,(n)\times 0}$$

$$+ (-1-j)\, e^{-j\,2\pi/8\,0\times k}\, 1$$

$$+ (-1)\, e^{-j\,2\pi/8\times k\times 2} + (-1+j)\, e^{-j\,2\pi/8\times k\times 3}$$

$$X[k] = 1 + (-1-j)\, e^{-j\,\pi/4\,k} + (-1)\, e^{-j\,\pi/4\,2k}$$

$$+ (-1+j)\, e^{-j\,\pi/4\,3k}$$

$$X[0] = 1 + (-1-j) + (-1) + (-1+j)$$

$$= -2$$

$$X[1] = 1 + (-1-j)\, e^{-j\pi/4} + (-1)\, e^{-j\pi/2} + (-1+j)\, e^{-j\,3\pi/4}$$

$$= 1 + (-1-j)\frac{(1-j)}{\sqrt{2}} + (-1)(-j) + (-1+j)\frac{(-1-j)}{\sqrt{2}}$$

$$= 1 + \frac{(-1-1)}{\sqrt{2}} + j + \frac{(1+1)}{\sqrt{2}} = 1 + j$$

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$$x[2] = 1 + (-1-j)\, e^{-j\pi/4 \cdot 2} + (-1)\, e^{-j\pi/4 \cdot 4} + (j-1)\, e^{-j\pi/4 \times 6}$$

$$= 1 + (-1-j)(-j) + (-1)(-1) + (j-1)\, j$$

$$= 1 + (1+j)(j) + 1 + (j-1)\, j$$

$$= 2 + j - 1 + 1 - j$$

$$= 1$$

$x[3]$:

$$1 + (-1-j)\, e^{-j\pi/4 \cdot 3} + (-1)\, e^{-j\pi/4 \cdot 6} + (j-1)\, e^{-j\pi/4 \times 9}$$

$$= 1 + (-1-j)\left(\frac{-1-j}{\sqrt{2}}\right) + (-1)(j) + (j-1)\left(\frac{1-j}{\sqrt{2}}\right)$$

$$= 1 + \left(\frac{1-1+2j}{\sqrt{2}}\right) + (-j) + \left(\frac{2j}{\sqrt{2}}\right)$$

$$= 1 - j + (2\sqrt{2})\, j = 1 + (2\sqrt{2} - 1)\, j$$

$x[4]$:

$$1 + (-1-j)\, e^{-j\pi/4 \times 4} + (-1)\, e^{-j\pi/4 \times 8} + (j-1)\, e^{-j\pi/4 \times 12}$$

$$= 1 + (-1-j)(-1) + (-1)(1) + (j-1)(-1)$$

$$= 1 - 1 + (-2)(-1) = 2$$

$x[5]$:

$$1 + (-1-j)\, e^{-j\pi/4 \times 5} + (-1)\, e^{-j\pi/4 \times 10} + (j-1)\, e^{-j\pi/4 \times 15}$$

$$= 1 + (-1-j)\left[\frac{-1+j}{\sqrt{2}}\right] + (-1)(-j) + (j-1)\left[\frac{1+j}{\sqrt{2}}\right]$$

$$= 1 + j + \frac{1+1}{\sqrt{2}} + \frac{1-1-1}{\sqrt{2}}\, j + 1 + j$$

$$X[6] = 1 + (-1-j)e^{-j\pi/4 \times 6} + (-1)e^{-j\pi/4 \times 12} + (j-1)e^{-j\pi/4 \times 18}$$

$$= 1 + (-1-j)e^{-j3\pi/2} + (-1)e^{-j\pi(3)} + (j-1)e^{-j9\pi/2}$$

$$= 1 + (-1-j)\,j + (-1)(-1) + (j-1)(-j)$$

$$= 1 - j + 1 + 1 + 1 + j = 4$$

$$X[7] = 1 + (-1-j)e^{-j\pi/4 \times 7} + (-1)e^{-j\pi/4 \times 14} + (j-1)e^{-j\pi/4 \times 21}$$

$$= 1 + (-1-j)e^{j\pi/4} + (-1)e^{j\pi/2} + (j-1)e^{3\pi/4}$$

$$= 1 + (-1-j)\left(\frac{1+j}{\sqrt{2}}\right) + (-1)(j) + (j-1)\left(\frac{-1+j}{\sqrt{2}}\right)$$

$$= 1 + -\left(\frac{1-1+2j}{\sqrt{2}}\right) \cdot -j + \left(\frac{-2j}{\sqrt{2}}\right)$$

$$= 1 - j - 2\sqrt{2}j = 1 + j(-1-2\sqrt{2})$$

$\therefore$ The 8 point DFT =

$$\left[-2,\ 1+j,\ 1,\ 1+2(\sqrt{2}-1)j,\ 2,\ 1-j,\ 4,\ 1+j(-1-2\sqrt{2})\right]$$

(d) $x(n) = \{1,1,1,1,1,1,1,1\}$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\,2\pi/N\, k(n)}$$

$$= \sum_{n=0}^{N-1} e^{-j\,\pi/4\, k(n)}$$

$$\cancel{X(k) = \frac{1 - e^{-j\pi}}{1 - e^{-j\pi/4\, k}}} \quad \text{if } k \neq 0$$

$$X[k] = \frac{1 - e^{j\pi/4\, 8k}}{1 - e^{-j\pi/4\, k}} = \frac{1 - e^{-2j\pi k}}{1 - e^{-j\pi/4\, k}} \quad \text{if } e^{-j\pi/4\, k} \neq 1$$

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If $e^{-j\pi/4 k} = 1$ $\quad X(k) = 8$

else

$X(k) = 0$ as

$$e^{-j(2\pi k)} = 1$$

$$1 \; \forall \; k \in \mathbb{Z}$$

$\therefore$ 8 point DFT =

$$[8, 0, 0, 0, 0, 0, 0, 0]$$

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QUESTION-2:

(a) $X(e^{j\omega}) = \cos^3\omega + \cos^2\omega$

$$= \frac{3\cos\omega + \cos 3\omega}{4} + \frac{1+\cos 2\omega}{2}$$

$$= \frac{1}{2} + \frac{3}{4}\cos\omega + \frac{1}{2}\cos 2\omega + \frac{1}{4}\cos 3\omega$$

$$= \frac{1}{2} + \frac{3}{4}\left(\frac{e^{j\omega}+e^{-j\omega}}{2}\right) + \frac{1}{2}\left(\frac{e^{2j\omega}+e^{-2j\omega}}{2}\right) + \frac{1}{4}\left(\frac{e^{3j\omega}+e^{-3j\omega}}{2}\right)$$

$$= \frac{1}{2}(1) + \frac{3}{8}(e^{j\omega}) + \frac{3}{8}(e^{-j\omega}) + \frac{1}{4}(e^{2j\omega}) + \frac{1}{4}(e^{-2j\omega}) + \frac{1}{8}(e^{3j\omega}) + \frac{1}{8}(e^{-3j\omega})$$

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IFT(1) =

$$IFT\{1\} = \int_{-\infty}^{\infty} e^{j\omega t}\,dt$$

$$= \delta(t)$$

$$IFT\{e^{aj\omega}\} = \int_{-\infty}^{\infty} e^{aj\omega} e^{-j\omega t}\,d\omega$$

$$= \delta(t-a)$$

$$\therefore \quad IFT(X(e^{j\omega})) = \tfrac{1}{2}\delta(t) + \tfrac{3}{8}\delta(t-1) + \tfrac{3}{8}\delta(t+1) + \tfrac{1}{4}\delta(t-2) + \tfrac{1}{4}\delta(t+2) + \tfrac{1}{8}\delta(t-3) + \tfrac{1}{8}\delta(t+3)$$

(b)

$$X(e^{j\omega}) = \frac{e^{-4j\omega} + e^{-3j\omega} - e^{-j\omega} - 1}{e^{-j\omega}+1}$$

$$= \frac{e^{-3j\omega}(e^{-j\omega}+1) - (e^{-j\omega}+1)}{e^{-j\omega}+1}$$

$$= \frac{(e^{-3j\omega}-1)(e^{-j\omega}+1)}{e^{-j\omega}+1} = e^{-3j\omega}-1$$

$$X(e^{j\omega}) = e^{-3j\omega}-1$$

$$IFT(X(e^{j\omega})) = IFT\{(e^{-3j\omega}-1)\}$$

$$= IFT(e^{-3j\omega}) - IFT(1)$$

$$= \delta(t+3) - \delta(t)$$

$$\therefore \quad IFT(X(e^{j\omega})) = \delta(t+3) - \delta(t)$$

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2(c) :- $X(e^{j\omega}) = \dfrac{3e^{j\omega} - 1}{3 - e^{j\omega}}$

$$= \frac{3e^{j\omega} - 9 + 8}{3 - e^{j\omega}}$$

$$= -3 + \frac{8}{3 - e^{j\omega}}$$

$$= -3 + \frac{8}{3} \frac{e^{j\omega}}{e^{j\omega} - \left(\frac{1}{3}\right)}$$

$$\text{IFT}\left\{\frac{e^{j\omega}}{e^{j\omega} + a}\right\} = a^t u(t)$$

$$\therefore \text{IFT}\left(X(e^{j\omega})\right) = -3\,\text{IFT}(1) + \frac{8}{3}\,\text{IFT}\left(\frac{e^{j\omega}}{e^{j\omega} - \left(\frac{1}{3}\right)}\right)$$

$$= -3\,\delta(t) + \frac{8}{3}\left(\frac{1}{3}\right)^t u(t)$$

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Section-4

Question-3:

Given

$X(k) = k^2$; $0 \le k \le 7$ be 8' point DFT of sequence $x(n)$.

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi kn}{N}}$$

$$x(n) = \frac{1}{8} \sum_{k=0}^{7} k^2 e^{j\frac{2\pi kn}{8}} = \frac{1}{8} \sum_{k=0}^{7} k^2 e^{j\frac{\pi kn}{4}}$$

$$x[0] = \frac{1}{8} \sum_{k=0}^{7} k^2 e^{j\frac{2\pi k}{8}}$$

$$= \frac{1}{8}\Big[0 + 1e^{j\pi/4} + 4e^{j\pi/2} + 9e^{j3\pi/4} + 16e^{j4\pi/4} + 25e^{j5\pi/4} + 36e^{j6\pi/4} + 49e^{j7\pi/4}\Big]$$

$$x[3] = \frac{1}{8}\Big[\sum_{k=0}^{7} k^2 e^{j\frac{3\pi k}{4}}\Big]$$

$$x[5] = \frac{1}{8}\Big[\sum_{k=0}^{7} k^2 e^{j\frac{5\pi k}{4}}\Big]$$

$$x[7] = \frac{1}{8}\Big[\sum_{k=0}^{7} k^2 e^{j7\pi k/4}\Big]$$

$$x[1] = x[3] + x[5] + x[7]$$

$$= \frac{1}{8}\Big[1\left(e^{j\pi/4} + e^{j3\pi/4} + e^{5\pi/4} + e^{7\pi/4}\right) + 4\left(e^{j\pi/2} + e^{j3\pi/2} + e^{j5\pi/2} + e^{j7\pi/2}\right) + 9\left(e^{j3\pi/4} + e^{j9\pi/4} + e^{j15\pi/4} + e^{j21\pi/4}\right) + 16\left(e^{j\pi} + e^{j3\pi} + e^{j5\pi} + e^{j7\pi}\right) + 25\left(e^{j5\pi/4} + e^{j15\pi/4} + e^{j25\pi/4} + e^{j35\pi/4}\right) + 36\left(e^{j6\pi/4} + e^{j18\pi/4} + e^{j30\pi/4} + e^{j42\pi/4}\right)$$

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$$+49\left[e^{7j\pi/4} + e^{21j\pi/4} + e^{35j\pi/4} + e^{49j\pi/4}\right]\Big]$$

$$\sum_{n=0}^{3} x(2n+1) = \frac{1}{8}\left[\sum_{k=1}^{7} k^2 \left[e^{j\theta} + e^{3\theta j} + e^{5\theta j} + e^{7\theta j}\right]\right]$$

where $\theta = \frac{\pi}{4}k$

$$\underbrace{e^{j\theta} + e^{3\theta j} + e^{5\theta j} + e^{7\theta j}}_{①} = (\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta)$$

$$+ j(\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta)$$

$$= 4\cos\theta\cos 2\theta\cos 4\theta + j(4\sin 4\theta\cos 2\theta\cos\theta)$$

$\theta = \pi/4$ ① $= 4\times 0 + j\times 0 = 0$

$\theta = 2\pi/4$ ① $= 4\times 0 + j\times 0 = 0$

$\theta = 3\pi/4$ ① $= 4\times 0 + j\times 0 = 0$

$\theta = \pi$ ① $= 4\times(-1) + j\times 0 = -4$

$\theta = 5\pi/4$ ① $= 4\times 0 + j\times 0 = 0$

$\theta = 6\pi/4$ ① $= 4\times 0 + j\times 0 = 0$

$\theta = 7\pi/4$ ① $= 4\times 0 + j\times 0 = 0$

$$\therefore \sum_{n=0}^{3} x(2n+1) = \frac{1}{8}(16\times -4) = -8$$

$$\boxed{\sum_{n=0}^{3} x(2n+1) = -8}$$

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