Proof 1

- $\bar{G}_i(u+v) = P(X(s) = i, s \in [t, t+u+v] | X(t) = i)$
- $\bar{G}_i(u+v) = P(X(s) = i, s \in [t+u, t+u+v]; X(p) = i, p \in [t, t+u] | X(t) = i)$
- $P(AB|C) = P(A|BC)P(B|C)$
- Due to Markov property we have $P(AB|C) = P(A|B)P(B|C)$
- $P(X(s) = i, s \in [t+u, t+u+v] | X(p) = i, p \in [t, t+u]) =$
- $P(X(s) = i, s \in [t+u, t+u+v] | X(t+u) = i) = \bar{G}_i(v)$
- $P(X(p) = i, p \in [t, t+u] | X(t = i)) = \bar{G}_i(u)$
- $\bar{G}_i(u+v) = \bar{G}_i(u)\bar{G}_i(v)$
- Only CCDF function which satisfies this equation is the exponential distribution. This requires a proof. We will skip this part.

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- Since $P(\tau_i > s + t | \tau_i > s) = P(\tau_i > t)$, this implies the distribution has memoriless property and must be exponential.

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- At which step did we use time homogeneity and the Markov property?

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- Given X_0 and P, you can generate n-step probabilities or $P_{X_0}(X_n)$

Chapman Kolmogorov Equations for CTMC

- Let $P(t)$ denote the t-time transition probability matrix.
- CK equation for a CTMC is $P(t+l) = P(t)P(l)$.
- $p_{ij}(t+l) = P(X(t+l) = j | X(0) = i)$
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- Lets look at $\frac{dP(h)}{dh}\Big|_{h=0} = \lim_{h\to 0} \frac{P(h)-P(0)}{h} = \lim_{h\to 0} \frac{P(h)-I}{h}$.

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Theorem

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- Consider Y_t when $X(t) = i$.
- Now use the following theorem for generating the CTMC on a computer

Theorem

(Proposition 2.8 Anderson: we won't see proof)
$P(Y_t > u|X(t) = i) := e^{q_{ii}u}$, *i.e.*, $q_{ii} = -a_i$.
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- This justifies the rate of leaving state i to be $\sum_{j \neq i} q_{ij}$.

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- Equivalently, in state i, you have $M - 1$ exponential(q_{ij}) clocks for $j = 1, 2, \ldots, i-1, i+1, \ldots M$.
- You move to that state whose clock rings first!