# DSA Set -2

1) Roll No : 2020101095

$x_1(n) = \{2,0,2,0\} \Rightarrow l_1 = 4$

$x_2(n) = \{1,0,9,5\} \Rightarrow l_2 = 4$

length of linear convolution $= l_1 + l_2 - 1$
$= 4+4-1$
$= 7$

length of circular convolution $= \max(l_1, l_2) = 4$

In order to calculate linear convolution from circular convolution, we pad $x_1(n)$ and $x_2(n)$ with $7-4 = 3$ zeroes

$x_1(n) = \{2,0,2,0,0,0,0\}$

$x_2(n) = \{1,0,9,5,0,0,0\}$

$$x_1(n) \circledast x_2(n) = \sum_{m=0}^{6} x_1(m)\, x_2(n-m)$$

$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 2 \\ 2 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 9 \\ 5 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 20 \\ 10 \\ 18 \\ 10 \\ 0 \end{bmatrix}$$

$\therefore$ linear convolution of $x_1(n)$ and $x_2(n)$ $= \{2,0,20,10,18,10,0\}$

2) Total range of analog signal is $5-(-3) = 8V$

step size $= \dfrac{8}{2^{\text{no. of bits}}} = \dfrac{8}{2^3} = 1V$

$\Rightarrow \{-3,-2\}, \{-2,-1\}, \{-1,0\}, \{0,1\}, \{1,2\}, \{2,3\}, \{3,4\}, \{4,5\}$

can be quantised to 000, 001, 010, 011, 100, 101, 110, 111 respectively

Digital value of 2.2V will be 101.

Quantization error $= \dfrac{\cancel{\text{upper limit} - \text{lower limit}}}{\cancel{2^{n+1}}} = \dfrac{2+3}{2} - 2.2$

$= 0.3$

$\cancel{= \dfrac{8}{16}}$

$\cancel{= 0.5}$

3) Step 1: Sample the signal. Since the frequencies lie from 0 to 2000 Hz, we will use $f_s = 2 \cdot f_{max} = 2 \cdot 2000 = 4000\ Hz$ as sampling frequency

Step 2: Compute the DFT of the sampled signal

$$x(n) \rightarrow x(K)$$

Step 3: Calculate frequency resolution of the signal, N should be such that 200Hz can be seen. $\Delta f = f_s / N$.

$N = 20$

Nearest power of $2 = 2^5 = 32 = N$

Step 4: Calculate the index of the 200Hz frequency component using

$$p = \text{int}\left(\frac{200}{\Delta f}\right)$$

Step 5: Calculate signal strength at 200Hz

$|p(i)|$ is the answer.

If $N=20$, then $p(1)=200 \Rightarrow n=1$

If $N=32$, then 200Hz lie betwee $p(1)$ & $p(2)$. So $n=2$.

4) DTFT for $x(n) = a^n u(n-4)$

$$x(n) = a^n u(n-4)$$

$$= a^4 \left(a^{n-4} \cdot u(n-4)\right)$$

We know that

DTFT of $\alpha^n u(n)$ is

$$\alpha^n u(n) \rightarrow \frac{1}{1-\alpha e^{-j\omega}}, \quad |\alpha| < 1$$

$$\Rightarrow \alpha^{n-4} u(n-4) \longrightarrow \frac{e^{-4j\omega}}{1-\alpha e^{-j\omega}}$$

$$\Rightarrow x(n) = a^4 \left(a^{n-4} u(n-4)\right)$$

DTFT will be

$$X(e^{j\omega}) = a^4 \cdot \frac{e^{-4j\omega}}{1-ae^{-j\omega}}, \quad |a| < 1$$

5) (i) $y(n) = y(n-1) + x(2n-1)$

let $x_1 \to y_1$

$x_2 \to y_2$

$\Rightarrow y_1(n) = y_1(n-1) + x_1(2n-1)$

$y_2(n) = y_2(n-1) + x_2(2n-1)$

Let $y'$ be the output when $a_1x_1 + a_2x_2$ is input

$\Rightarrow y'(n) = y'(n-1) + a_1x_1(2n-1) + a_2x_2(2n-1)$

$\Rightarrow y'(n) - y'(n-1) = a_1(y_1(n) - y_1(n-1)) + a_2(y_2(n) - y_2(n-1))$

$= a_1 y_1(n) + a_2 y_2(n) - (a_1 y_1(n-1) + a_2 y_2(n-1))$

$\Rightarrow y'(n) = a_1 y_1(n) + a_2 y_2(n)$

Linear system

Delay the input by $k$ and let $y'$ be the output

$y'(n) = y'(n-1) + x(2n-1-k) \rightarrow ①$

Now, $y(n-k) = y(n-k-1) + x(2(n-k)-1)$

$\Rightarrow y(n-k) = y(n-k-1) + x(2n-1-2k) \rightarrow ②$

from eq ① & ② we observe that

$y'(n) \neq y(n-k)$

so, not time invariant

$\therefore$ System is linear but time variant

(ii) $y(n) = x(n) + y(n-2) + x(n-1)$

$y_1(n) = x_1(n) + y_1(n-2) + x_1(n-1)$

$y_2(n) = x_2(n) + y_2(n-2) + x_2(n-1)$

let $x = ax_1 + ax_2$

$y'(n) = y'(n-2) + ax_1(n) + a_2x_2(n) + a_1x_1(n-1) + a_2x_2(n-2)$

$\Rightarrow y'(n) - y'(n-2) = a_1(x_1(n) + x_1(n-1)) + a_2(x_2(n) + x_2(n-2))$

$= a_1(y_1(n) - y_1(n-2)) + a_2(y_2(n) - y_2(n-2))$

$\Rightarrow y'(n) = a_1 y_1(n) + a_2 y_2(n)$

Linear system

Delay the input by $K$ and let $y'$ be the output

$$y'(n) = y'(n-2) + x(n-K) + x(n-1-K) \rightarrow (3)$$

Now $$y(n-K) = y(n-2-K) + x(n-K) + x(n-K-1) \rightarrow (4)$$

$$y(n-K) = y'(n) \quad \text{from eq (3) \& (4)}$$

So, time invariant

$\therefore$ System is linear time invariant