Assignment 3

CS9.312 Introduction to Quantum Information and Computation

Due date of submission: 27/01/2023

1. Consider the qubit states $\left\{|\psi_k\rangle \langle\psi_k|\right\}_{k=0}^{4}$ where $|\psi_k\rangle$ is defined as

$$|\psi_k\rangle := \cos\frac{2\pi k}{5}|0\rangle + \sin\frac{2\pi k}{5}|1\rangle, k \in 0,1,2,3,4$$

Check whether $\left\{\frac{2}{5}|\psi_k\rangle \langle\psi_k|\right\}_{k=0}^{4}$ is a POVM or not. Also, check whether it is a projective measurement or not.

2. The Walsh-Hadamard transform is a single qubit operation, denoted by H, and performs the following transformation

$$|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$
$$|1\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

(i) Find the matrix representation of H for the basis $\{|0\rangle, |1\rangle\}$, where $|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}, |1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$

(ii) Find the matrix representation of H in the basis $\{|+\rangle, |-\rangle\}$, where

$$|+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}, |-\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$$

(iii) Find the inverse of this operator.

3. Consider a system in the unnormalized state $|\psi\rangle \langle\psi|$ where $|\psi\rangle = \frac{1}{6}\begin{bmatrix}1\\0\\4\end{bmatrix}$ and two observables A and B given by

$$A = \frac{1}{\sqrt{2}}\begin{bmatrix}2 & 0 & 0\\0 & 1 & i\\0 & -i & 1\end{bmatrix}, B = \begin{bmatrix}1 & 0 & 0\\0 & 0 & -i\\0 & i & 0\end{bmatrix} \quad (1)$$

(i) We perform a measurement where A is measured first. Find the probability of obtaining a value 0 for A.

(ii) Now assume you obtain the value 0 as a result of measurement A. Then immediately B is measured. Now find the probability of getting value 1 for B.