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For a CTMC with Q matrix, we have
$P(Y_t > u | X(t) = i) := e^{q_{ii} u}$, *i.e.,* $q_{ii} = -a_i$.
$P\left(X(t + Y_t) = j | X(t) = i\right) = \frac{q_{ij}}{|q_{ii}|}$.

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- How does $P(t) = [[P(N(t) = j|N(0) = i)]]$ look ?
- Entries below the diagonal are zero.
- Diagonal entries have the value $e^{-\lambda t}$
- ijth entry above the diagonal has the value $e^{-\lambda t}\frac{(\lambda t)^{j-i}}{(j-i)!}$

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Example 3: Binomial process as a DTMC

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- DO IT YOURSELF!

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- $\{X_n,\}$ is such that there are no one step transitions from a state to itself, i.e., $p_{ii} = 0$.

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> A chain is said to be irreducible if $i \leftrightarrow j$ for all $i, j \in \mathcal{S}$.

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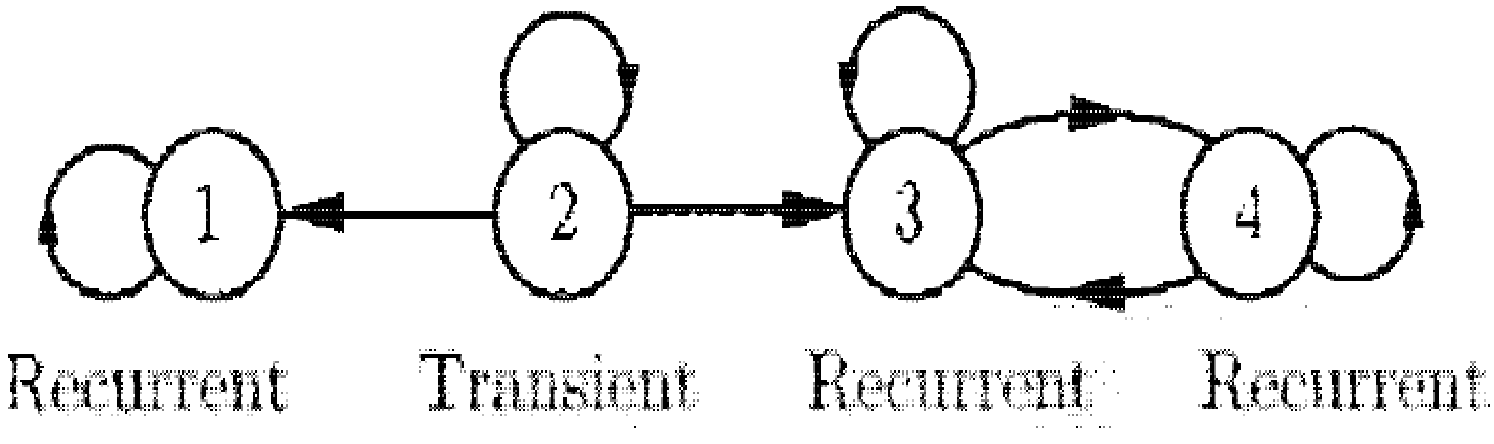
- If $i \leftrightarrow j$ and i is recurrent, then j is recurrent.



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