DSP . Assignment 3

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1A) Given sequences,

$x(n) = \{\underset{\uparrow}{-1}, 1, 0, 1\}$ and $h(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$

$x[0] = -1$ $h[-2] = 1$

$x[1] = 1$ $h[-1] = 2$

$x[2] = 0$ $h[0] = 3$

$x[3] = 1$ $h[1] = 4.$

$h[2] = 5$

Let the linear convolution be $y[m]$

$$y(m) = \sum_{n=-\infty}^{\infty} x(n)\, h[m-n]$$

$$= x(0)\,h[m] + x(1)\,h[m-1] + x(2)\,h[m-2] + x[3]\,h[m-3]$$

(all other values of x are $=0$)

$y[-2] = x(0)\,h[-2] = -1$

$y[-1] = x[0]\,h[-1] + x(1)\,h[-2] = -1\times2+1+1 = -1$

$y[0] = -1\times3 + 1\times2 = -1$

$y(1) = \cancel{-1\times45} \cancel{+1\times04} + \overset{3\times0}{\cancel{2\times03}} + 1\times2 = \cancel{1}$

$y(2) =$

$y(1) = -1\times 4 + 1\times 3 + 0 + 1\times 1 = 0$

$y(2) = -1\times 9 + 1\times 5 + 8 + 1\times 3 = 8\ 1$

$y(3) = 50 + 9 = 19\ 8$

$y(4) = 0 + 4 = 4$

$y(5) = 5$

All other values of y are 0 as h in that range would be 0

Hence the linear convolution

$$y(n) = \{-1, -1, \underset{\uparrow}{-1}, 0, 1, 8, 4, 5\}$$

Circular convolution :

$l_1 = 4$ and $l_2 = 5$

∴ Adding 1 zero to x(n) (zero padding)

i.e $x(n) = \{-1, \underset{\uparrow}{1}, 0, 1, 0\}$

$$h[n] = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$$

In the circular convolution, signals are taken to be periodic, therefore

$$h(n) = \{\underset{\uparrow}{3}, 4, 5, 1, 2\}$$

~~$x(n) \circledast h[n] = \begin{bmatrix} -1 & 1 & 0 & 1 & 0 \end{bmatrix}$~~

$$x(n) \circledast h(n) = \begin{bmatrix} -1 & 0 & 1 & 0 & 1 \\ 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 1 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \\ 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -3+5+2 \\ 3-4+1 \\ 4-5+2 \\ 3+5-1 \\ 4+1-2 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 1 \\ 7 \\ 3 \end{bmatrix}$$

circular convolution $= \{ \underset{\uparrow}{4}, 0, 1, 7, 3 \}$

2A

Given

$$h(n) = a^n u(n)$$

$$x(n) = u(n),$$

W.K.T

$$XY(z) = X(z) \cdot H(z)$$

$\therefore$

$$y(n) = x(n) * h(n)$$

$$= \sum_{k=-\infty}^{\infty} h[k]\, x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} a^k u[k]\, u[n-k]$$

For $k \geq 0$ $\quad u(k) = 1$

$$\therefore y(n) = \sum_{k=\cancel{\infty}0}^{\infty} a^k\, u[n-k]$$

if $n \geq 0$

$$\therefore y(n) = \sum_{k=\cancel{\infty}0}^{\overset{n}{\cancel{\infty}}} a^k \cancel{\infty}$$

if $n < 0$

$$y(n) = 0$$

$\therefore$ if $n \geq 0$ $\quad y(n) = \sum_{k=n}^{\infty} ak$

$$\cancel{\frac{a^n}{1-a}} \quad \frac{1-a^{n+1}}{1-a}$$

$$\therefore \boxed{y(n) = \left(\frac{1-a^{n+1}}{1-a}\right) u(n)}$$

3
6A)

$y(n) = 0.1\,x(n) + 0.2\,x(n-1) + 0.3\,x(n-2)$
$+ 0.4\,x(n-4)$

Applying Z transform

$Y[z] = 0.1\,X[z] + 0.2z^{-1}X[z] + 0.3z^{-2}X[z]$
$+ 0.4z^{-4}X[z]$

$X[z]\,[0.1 + 0.2z^{-1} + 0.3z^{-2} + 0.4z^{-4}]$

$$\frac{Y[z]}{X[z]} = 0.1 + 0.2z^{-1} + 0.3z^{-2} + 0.4z^{-4}$$

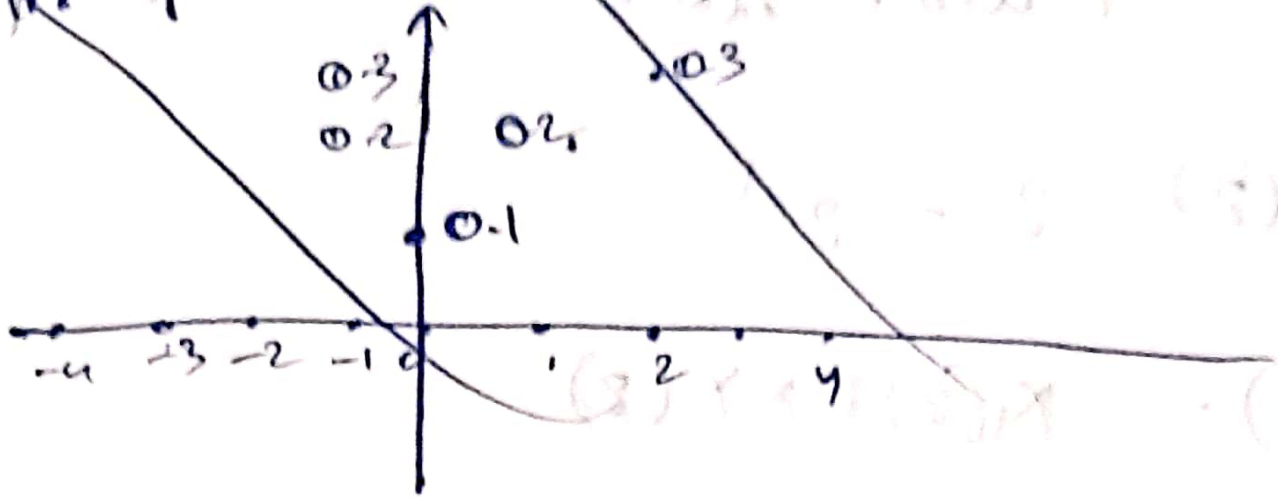
$$H[z] = \frac{Y[z]}{X[z]} = 0.1 + 0.2z^{-1} + 0.3z^{-2} + 0.4z^{-4}$$

$h(n) = 0.1\,\delta(n) + 0.2\,\delta(n-1) + 0.3\,\delta(n-2)$
$+ 0.4\,\delta(n-4)$

$$\delta(n) = \begin{cases} 1 & n=0 \\ 0 & \text{otherwise} \end{cases}$$

$$\therefore\ h(n) = \begin{cases} 0.1 & n=0 \\ 0.2 & n=1 \\ 0.3 & n=2 \\ 0.4 & n=4 \\ 0 & \text{otherwise} \end{cases}$$

∴ The plot of h(n)



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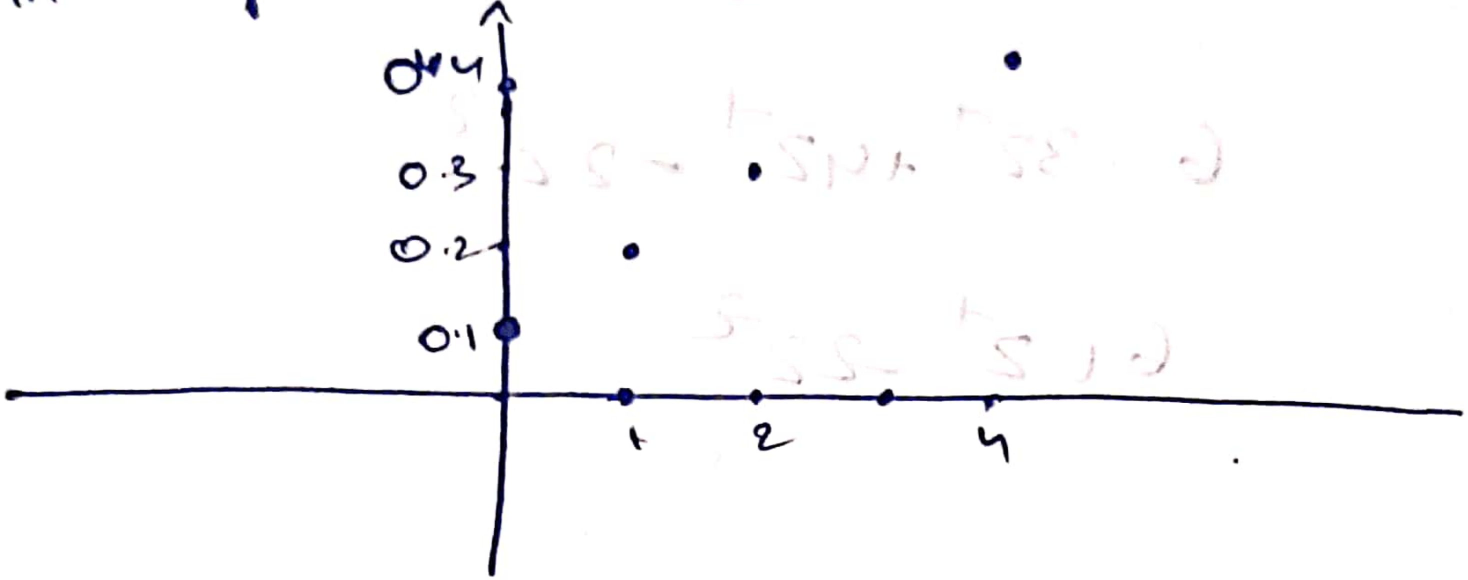


iii) $y(n) = x(n) - 0.5x(n-1) + 0.36x(n-2)$

$Y(z) = X(z) - 0.5 \times z^{-1} X(z) + 0.36 \times z^{-2} \times X(z)$

$Y(z) = X(z) [1 - 0.5z^{-1} + 0.36z^{-2}]$

$\frac{Y(z)}{X(z)} = 1 - (0.5)z^{-1} + (0.36)z^{-2}$

W.K.T the transfer function

$H(z) = \frac{Y(z)}{X(z)}$

$\therefore$ $\boxed{H(z) = 1 - (0.5)z^{-1} + (0.36)z^{-2}}$

$H(z) = \cancel{\frac{A(z)}{B(z)}} \quad \frac{B(z)}{A(z)}$

$\frac{B(z)}{A(z)} = \frac{1 - (0.5)z^{-1} + (0.36)z^{-2}}{1}$

$A(z) = 1$

$B(z) = 1 - (0.5)z^{-1} + (0.36)z^{-1}$

i.e There are no poles and only zeros

Ans (a) $x(n) = \{2, 4, \underset{\uparrow}{5}, 7, 0, 1\}$

$$X(z) = Z \text{ transform } (x(n)) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

$$= \cancel{\sum_{n=-\infty}} 2z^{2} + 4z^{1} + 5 + 7z^{-1} + z^{-2}$$

The ROC of this Z transform is the entire Z-plane except z=0

(b) $x(n) = a^{n}u(n) + b^{n}u(-n-1)$

$$X(z) = Z \text{ transform } (x(n)) = \sum_{n=-\infty}^{\infty} \left(a^{n}u(n) + b^{n}u(-n-1)\right) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} (az^{-1})^{n} u(n) + \sum_{n=-\infty}^{\infty} (bz^{-1})^{n} u(-n-1)$$

$-n-1 \geq 0$

$$= \sum_{n=0}^{\infty} (az^{-1})^{n} + \sum_{n=-\infty}^{-1} (bz^{-1})^{n}$$

$$= \frac{1}{1-(az^{-1})} + \frac{(bz^{-1})^{-1}}{1-(bz^{-1})^{-1}}$$

$\downarrow$ ROC: $|az^{-1}| < 1$

$\downarrow$ ROC: $|b^{-1}z| < 1$

$\therefore$ ROC: $\left|\frac{a}{z}\right| < 1$ && $\left|\frac{z}{b}\right| < 1$

ie $|a| < |z| < |b|$

6A)

$x_1(n) = 3\delta(n) + 2\delta(n-1)$

$x_2(n) = 2\delta(n) - \delta(n-1)$

(a) Finding z transform of convolution

$X(z) = z(x_1(n) * x_2(n))$

$X_1(z) \overset{\times}{\cancel{*}} X_2(z)$

$X_1(z) = z(3\delta(n) + 2\delta(n-1))$

$3 + 2z^{-1}$

$z(\delta(n)) = 1$

$z(\delta(n-1)) = z^{-1}$ (Time shifting property)

$x_2(n) = 2\delta(n) - \delta(n-1)$

$X_2(z) = 2 - z^{-1}$

$X(z) = X_1(z) \times X_2(z)$

$= (3+2z^{-1})(2-z^{-1})$

$= 6 - 3z^{-1} + 4z^{-1} - 2z^{-2}$

$= 6 + z^{-1} - 2z^{-2}$

(b)

$x(n) = x_1(n) * x_2(n)$

W.K.T

$X(z) = 6 + z^{-1} - 2z^{-2}$

W.K.T Inverse Z Transform $(1) = \delta(n)$

from time shifting property

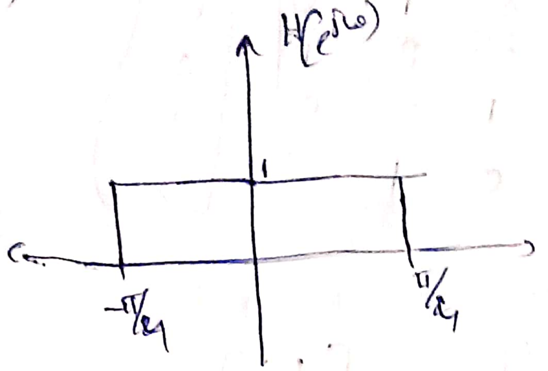
$IZT(z^{-1}) = \delta(n-1)$

$IZT(z^{-2}) = \delta(n-2)$

$\therefore$

$IZT(X(z)) = x(n) = 6\delta(n) + \delta(n-1) - 2\delta(n-2)$

7A) given



$$H(e^{j\omega}) = \begin{cases} 1 & -\pi/4 \le \omega \le \pi/4 \\ 0 & \text{otherwise} \end{cases}$$

DTFT :- $h(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega}) \cdot e^{j\omega n} d\omega$

$$\boxed{h(n) = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} 1 \cdot e^{j\omega n} d\omega} \quad -①$$

$$\frac{1}{2\pi} \times \left[\frac{e^{j\omega n}}{jn}\right]_{-\pi/4}^{\pi/4}$$

$$= \frac{1}{2\pi n}\left[\frac{e^{jn(\pi/4)} - e^{-jn(\pi/4)}}{j}\right]$$

$$\boxed{h(n) = \frac{1}{n\pi} \sin\left(n\pi/4\right)}$$

clearly $h(0)$ is not defined in this formula

From ①

$$h(0) = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} d\omega = \frac{1}{2\pi} \times \pi/2$$

$$= 1/4$$

h is even

$$h(1) = h(-1) = \frac{1}{\pi} \, {}^{1}\!/\!_{\sqrt{2}} = \frac{1}{\sqrt{2}\,\pi}$$

$$h(2) = h(-2) = \frac{1}{2\pi} \times 1 = \frac{1}{2\pi}$$

$$h(3) = h(-3) = \frac{1}{3\pi} \times \frac{1}{\sqrt{2}} = \frac{1}{3\sqrt{2}\,\pi}$$

$$h(4) = h(-4) = \frac{1}{4\pi} \times 0 = 0$$

ⓐ Let the filter to be designed be $l(n)$

then without any window (~~window size~~ delay $= 9$

$l(-n) = h(n) \qquad -4 \le n \le 4$

$l(-4) = l(4) = 0$

$l(-3) = l(3) = \frac{1}{3\sqrt{2}\pi}$

$l(-2) = l(2) = \frac{1}{2\pi}$

$l(-1) = l(1) = \frac{1}{\sqrt{2}\pi}$

$l(0) = 1/4$

$$\therefore \quad l = \left[0, \frac{1}{3\sqrt{2}\pi}, \frac{1}{2\pi}, \frac{1}{\sqrt{2}\pi}, \underset{\uparrow}{\frac{1}{4}}, \frac{1}{\sqrt{2}\pi}, \frac{1}{2\pi}, \frac{1}{3\sqrt{2}\pi}, 0\right]$$

$$l = \left[0, 0.7502, 0.15915, 0.22507, \underset{\uparrow}{0.25}, 0.22507, \right.$$

$$\left. 0.1595, 0.7502, 0\right]$$

(b) Hamming window $w(n)$

$$= 0.54 - 0.46 \cos\left(2\pi \frac{n}{N-1}\right)$$

where $N =$ window size

$= 8$

$$w(n) = 0.54 - 0.46 \cos\left(\frac{\pi n}{4}\right) \qquad 0 \leq n \leq 7$$

~~(i)~~ Let the filter to be designed be l

$$l(0) = h(0) \times w(0)$$

$$= \frac{1}{4} \times 0.08 = 0.02$$

$$l(1) = h(1) \times w(1)$$

$$= \frac{1}{\sqrt{2}\pi} \times \left[0.54 - \frac{0.46}{\sqrt{2}}\right] = 0.0483$$

$$l(2) = \frac{1}{2\pi} \times 0.54 = 0.0859$$

$$l(3) = \frac{1}{3\sqrt{2}\pi} \times \left[0.54 + \frac{0.46}{\sqrt{2}}\right] = 0.0649$$

$l(4) = 0$

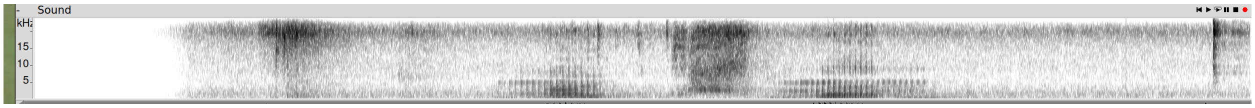
Since l is symmetric

$l(n) = l(-n)$

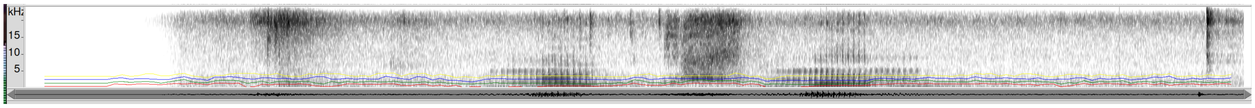
$l = [0, 0.0649, 0.0859, 0.0483, \overset{\uparrow}{0.02},$

$0.0483, 0.0859, 0.0649, 0]$

Spectogram:



Formant plot:



Power Plot: