

# DSA-Assignment-2

Deadline: 6th April, 2023

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## Instructions

1. Deadline for the assignment is **6th April, 2023**
  2. Solve all the question and submit a handwritten document
  3. Plagiarism will be penalised
  4. Submit a pdf of the form `<roll_no>_dsa2.pdf`
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## 1 Signals

1. Indicate if the following signals are periodic signals. If periodic, find the fundamental period.

- (a)  $x[n] = \sin^2(3n + \pi)$
- (b)  $x[n] = e^{j\pi n/8}$
- (c)  $x[n] = \cos(\pi n/10)\cos(\pi n/30)$
- (d)  $x[n] = \sin(4\pi n + 3)$
- (e)  $x[n] = \cos(\pi n^2/3)$
- (f)  $x[n] = \sum_{k=-\infty}^{\infty} (-1)^k \delta(n - k)$

2. Find the odd and even parts of the following discrete signals

- (a)  $x[n] = \sqrt{2}\cos((an + 1/4)\pi)$
- (b)  $x[n] = e^{jan\pi} + e^{jn\pi/b}$

3. Determine whether the following signals are energy or power signals or neither

- (a)  $x[n] = \begin{cases} 0 & n < 0 \\ n & n \geq 0 \end{cases}$
- (b)  $x[n] = \cos(n\pi/2)$
- (c)  $x[n] = \begin{cases} 3^n & n < 0 \\ (1/2)^n & n \geq 0 \end{cases}$
- (d)  $x[n] = a^n u(n), a \in R$
- (e)  $x[n] = e^n \delta(n - 4)$

## 2 Systems

1. Determine whether or not the following systems are time invariant
  - (a)  $y(t) = t^2 x(t - 1)$
  - (b)  $y[n] = x[n - 1] + x[n + 1]$
  - (c)  $y[n] = \frac{1}{x[n]}$
  - (d) Consider a system  $S$  with input  $x[n]$  and output  $y[n]$  related by  $y[n] = x[n](g[n] + g[n - 1])$ .
    - i. If  $g[n] = 1$  for all  $n$ , show that  $S$  is time invariant.
    - ii. If  $g[n] = n$ , show that  $S$  is not time invariant.
    - iii. If  $g[n] = 1 + (-1)^n$ , show that  $S$  is time invariant.
2. Determine whether or not the following systems are linear
  - (a)  $y(t) = x(\sin t)$
  - (b)  $y(t) = \begin{cases} 0 & t < 0 \\ x(t) + x(t - 2) & t \geq 0 \end{cases}$
  - (c)  $y(t) = \frac{d(x(t))}{dt}$
  - (d)  $y[n] = \sum_{m=0}^M ax[n - m] + \sum_{m=1}^N bx[n - m]$
  - (e)  $y[n] = ax[n] + b\frac{1}{x[n-1]}$
3. Determine whether or not the following systems are causal
  - (a)  $y(t) = x(t - 2) + x(2 - t)$
  - (b)  $y(t) = [\cos(3t)]x(t)$
  - (c)  $y(t) = \int_{-\infty}^{2t} x(k)dk$
  - (d)  $y[n] = \sum_{k=0}^{\infty} x[n + k]$
  - (e)  $y[n] = \sum_{k=0}^{\infty} x[n - k]$

## 3 Sampling Frequency

1. What is aliasing? What can be done to reduce aliasing?  
Let  $x(t) = \frac{1}{2\pi} \cos(4000\pi t) \cos(1000\pi t)$  be a continuous-time signal. Find the Nyquist rate and Nyquist interval for this signal.
2. A waveform,  $x(t) = 10\cos(1000t + \pi/3) + 20\cos(2000t + \pi/6)$  is to be uniformly sampled for digital transmission. What is the maximum allowable time interval between sample values that will ensure perfect signal reproduction? If we want to reproduce 1 hour of this waveform, how many sample values need to be stored?

3. Consider three signals  $x_1(t)$  and  $x_2(t)$  and  $x_3(t)$  with Fourier transforms satisfying:

$$X_1(\Omega) = 0, 120 \leq |\Omega|$$

$$X_2(\Omega) = 0, |\Omega| \leq 60, |\Omega| \geq 100$$

Determine the minimum frequency  $\Omega_s$  at which we must sample the following signals to prevent aliasing.

- (a)  $x(t) = x_1(t) + x_2(t)$
- (b)  $x(t) = x_1(t)x_2(t)$
- (c)  $x(t) = \cos(3.6\pi t + 9.23)$

## 4 Quantization

1. Consider the analog waveform  $x(t)$  and answer the following questions.

$$x(t) = \begin{cases} -2 \sin(\pi x/4) & 0 \leq x < 4 \\ x - 4 & 4 \leq x < 5 \\ 1 & 5 \leq x < 7 \\ 8 - x & 7 \leq x \leq 10 \end{cases}$$

It is sampled at 1000 Hz and quantized with a 2-bit quantizer with input range -2V to 2V.

- (a) Indicate the sample points.
- (b) State the quantization intervals and the corresponding digital words.
- (c) Sketch the digital word assigned to each sample point.
- (d) Indicate the stream of bits generated after the quantization is complete.
- (e) What is the resulting bit rate?
- (f) What is the quantization error?

Answer all of the above questions for a 3-bit quantizer as well.

2. Mention advantages/disadvantages of increasing quantization bits.