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ASSIGNMENT 2

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Given Matrix

$$U = \begin{pmatrix} \sigma_0 & \sigma_1 \\ -i\sigma_2 & \sigma_3 \end{pmatrix}$$

$$U^* = \begin{pmatrix} \sigma_0 & i\sigma_2 \\ \sigma_1 & \sigma_3 \end{pmatrix}$$

$$UU^* = \begin{pmatrix} \sigma_0^2 + \sigma_1^2 & i\sigma_0\sigma_2 + \sigma_1\sigma_3 \\ -i\sigma_2\sigma_0 + \sigma_3\sigma_1 & \sigma_2^2 + \sigma_3^2 \end{pmatrix}$$

$$= \begin{pmatrix} 2I & i\sigma_2 + \sigma_1\sigma_3 \\ -i\sigma_2\sigma_0 + \sigma_3\sigma_1 & \sigma_2^2 + \sigma_3^2 \end{pmatrix}$$

W.K.T $-i\sigma_1\sigma_2\sigma_3 = I$

$$\sigma_1\sigma_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$= -i\sigma_2$$

Similarly

$\sigma_3\sigma_1 = i\sigma_2$

$$\therefore \quad UU^* = \begin{pmatrix} 2I & 0 \\ 0 & 2I \end{pmatrix}$$

$$U^*U = \begin{pmatrix} \sigma_3^2 + \sigma_1^2 & \sigma_3\sigma_1 + i\sigma_2\sigma_3 \\ \sigma_1\sigma_3 - i\sigma_3\sigma_2 & \sigma_1^2 + \sigma_2^2 \end{pmatrix}$$

$$= \begin{pmatrix} 2I & \sigma_1 + i\sigma_2\sigma_3 \\ \sigma_1 - i\sigma_3\sigma_2 & 2I \end{pmatrix}$$

$$-i\sigma_2\sigma_3 = I \Rightarrow i\sigma_2\sigma_3 = -\sigma_1$$

Similarly $i\sigma_3\sigma_2 = i\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

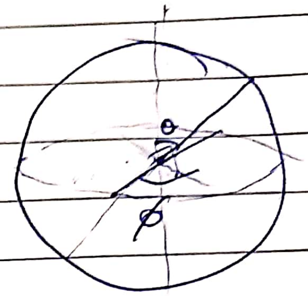
$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \sigma_1$$

$$U^*U = \begin{pmatrix} 2I & 0 \\ 0 & 2I \end{pmatrix}$$

$$\therefore \quad UU^* = U^*U \neq I$$

$\therefore$ U is normal not unitary matrix

2A) The Bloch sphere represents a qubit (2 state system)



If θ, ϕ are the polar and azimuthal angles respectively

and $\theta = 0$ denotes $|0\rangle$ and $\theta = \pi$ denotes $|1\rangle$

the ~~state~~ eigen state of the qubit can be represented as

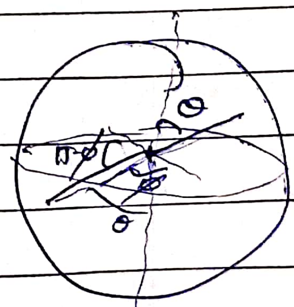
$$|\psi\rangle = \begin{pmatrix} \cos \theta/2 \\ \exp(i\phi) \sin\theta/2 \end{pmatrix}$$

~~Cons~~ let $|a\rangle$ be at an polar angle ~~ka~~ of θ and an azimithal angle of ϕ. (assuming $\theta, \phi \leq \pi/2$)

let $|b\rangle$ be the vector opposite to it.

Then the polar angle $= \pi - \theta$

similarly azimithal angle $= \pi + \phi$



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$$\therefore |a\rangle = \begin{pmatrix} \cos \theta/2 \\ e^{i\phi} \sin \theta/2 \end{pmatrix}$$

$$-\sin^2\theta$$

$$|b\rangle = \begin{pmatrix} \cos \frac{(\pi - \theta)}{2} \\ e^{i(\pi+\phi)} \sin \frac{(\pi - \theta)}{2} \end{pmatrix}$$

$-(\cos\phi + -i\sin\phi)$

$-\cos\phi + i\sin\phi \qquad \cos^2\phi + \sin^2\theta$

$$\langle b| = \begin{pmatrix} \sin \theta/2 & e^{-i(\pi+\phi)} \cos \theta/2 \end{pmatrix}$$

$$\langle b|a\rangle = \cos\theta/2 \sin\theta/2 - \cos\theta/2 \sin\theta/2$$

$$= 0$$

3A) Given

$$\rho = \frac{1}{4}(1-\epsilon) I_4 + \epsilon (|0\rangle \otimes |0\rangle)(\langle 0| \otimes \langle 0|)$$

$$\rho = (1-\epsilon) \frac{1}{2} I_4 \otimes \frac{1}{2} I_4 + \epsilon (|0\rangle \otimes |0\rangle)(\langle 0| \otimes \langle 0|)$$

$$\rho = (1-\epsilon)\left(\frac{1}{2} I_4\right)\left(\frac{1}{2} I_4\right)^{+} + \epsilon (|0\rangle \otimes |0\rangle)(|0\rangle \otimes |0\rangle)^{+}$$

In short ρ can be expressed as

$$\rho = \sum_{i=1}^{2} p_i |\psi_i\rangle\langle\psi_i|$$

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where

$p_1 = (1-\epsilon)$ $\quad$ $p_2 = \epsilon$

$|\psi_1\rangle = \frac{1}{2} I_4$ $\quad$ $|\psi_2\rangle = |0\rangle \otimes |0\rangle$

Also the trace of ρ i.e

$= p_1 + p_2 = 1-\epsilon+\epsilon = 1$

Hence ρ represents a density matrix

4A) Given

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle \otimes |1\rangle\right) - \frac{1}{\sqrt{2}}\left(|1\rangle \otimes |0\rangle\right)$$

$$|\alpha\rangle = \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix} \qquad |\beta\rangle = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

$$\langle\alpha| = \begin{bmatrix} \cos\phi & \sin\phi \end{bmatrix} \qquad \langle\beta| = \begin{pmatrix} \cos\theta & \sin\theta \end{pmatrix}$$

$$\langle\alpha| \otimes \langle\beta| = \begin{bmatrix} \cos\phi\cos\theta & \cos\phi\sin\theta & \sin\phi\cos\theta & \sin\phi\sin\theta \end{bmatrix}$$

$$\left(\langle\alpha| \otimes \langle\beta|\right)|\psi\rangle$$

$$= \begin{bmatrix} \cos\phi\cos\theta & \cos\phi\sin\theta & \sin\phi\cos\theta & \sin\phi\sin\theta \end{bmatrix} \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \sin(\theta - \phi)$$

$$P(\phi, \theta) = \frac{1}{2} \sin^2(\theta - \phi)$$