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MPL Assignment-1

1A) (a) let the statement "it rains" be denoted by p and "Raju carries an umbrella" be denoted by q

Then given $p \Rightarrow q$

~~Given~~

We have to validate $q \Rightarrow p$

q	p	$q \Rightarrow p$
T	F	F
F	F	T
T	T	T

The case of $q = F$, $p = T$ wont be possible because $p \Rightarrow q$

It is clear that $q \Rightarrow p$ is not true always given $p \Rightarrow q$

Hence ~~we~~ ~~should~~ The statement is not always true

(b) $p \equiv$ "weather is warm"
$q \equiv$ "the sky is clear"
$r \equiv$ "we go swimming"
$s \equiv$ "we go boating"

We have to check the validity of the statement

$$\left[\left((p \wedge q) \rightarrow (r \vee s)\right) \wedge \left(\sim(\sim r \rightarrow \sim q)\right)\right] \longrightarrow p \vee s$$

p	q	r	s	$(p \wedge q \to r \vee s)$	$\sim(\sim q \vee r \to \sim r)$	$p \vee s$	T
T	T	T	T	T	F	T	T
T	T	T	F	T	F	T	T
T	T	F	T	T	T	T	T
T	T	F	F	F	T	T	T
T	F	T	T	T	F	T	T
T	F	T	F	T	F	T	T
T	F	F	T	T	F	T	T
T	F	F	F	T	F	T	T
F	T	T	T	T	F	T	T
F	T	T	F	T	F	F	T
F	T	F	T	T	T	T	T
F	T	F	F	T	T	F	F
F	F	T	T	T	F	T	T
F	F	T	F	T	F	F	T
F	F	F	T	T	F	T	T
F	F	F	F	T	F	F	

Clearly for $p = F$, $q = T$, $r = F$, $s = F$

the statement T is false

Hence the given statement is false

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2A) Given $A = \text{true} = B$

$X = Y = \text{False}$

(a) $\sim(A \vee X) = \sim(T \vee F) = \sim(T)$

$= F$

(b) $A \vee (X \wedge Y) = T \vee (F \wedge F) = T \vee F = T$

$$\boxed{A \vee (X \wedge Y) = T}$$

(c) $A \wedge (X \vee (B \wedge Y)) = T \wedge (F \vee (T \wedge F))$

$= T \wedge (F \vee F)$

$= T \wedge F$

$= F$

$$\boxed{A \wedge (X \vee (B \wedge Y)) = F}$$

(d) $[(A \wedge X) \vee \sim B] \wedge \sim[(A \wedge X) \vee \sim B]$

$= [(T \wedge F) \vee F] \wedge (\sim [(T \wedge F) \vee F])$

$= [F \vee F] \wedge [\sim [F]]$

$= F \wedge T = F$

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(e) $P \wedge Q \wedge (\sim A \vee X)$

$= P \wedge Q \wedge (F \vee F)$

$= (P \wedge Q) \wedge (F) = F$ (Since anything $\wedge F = F$)

(f) $[(X \wedge Y) \rightarrow A] \rightarrow [X \rightarrow (Y \rightarrow A)]$

$= [(F \wedge F) \rightarrow T] \rightarrow [F \rightarrow (F \rightarrow T)]$

$= [F \rightarrow T] \rightarrow [F \rightarrow T]$

$= [T] \rightarrow [T] = T$

3A) (a) Formal Proof:

(a) RTP: $P \to \sim Q,\ \sim Q \to R \Rightarrow P \to R$

Step 1: Setting an axiom or premise as P

1. P Hypothesis
2. $P \to \sim Q$ Simplification
3. $\sim Q$ Modus Ponens rule for 1 and 2
4. $\sim Q \to R$ Simplification
5. R Modus Ponens rule for 3 and 4

∴ We can conclude $P \to R$ is true.

Resolution :

RTP : $P \to \sim Q, \sim Q \to R \Rightarrow P \to R$

Proof :

Converting into CNF

$P \to \sim Q \equiv \sim P \vee \sim Q$ (Premise)

$\sim Q \to R \equiv Q \vee R$ (Premise)

From Resolution

$(\sim P \vee \sim Q), (Q \vee R) \Rightarrow (\sim P \vee R)$

But $(\sim P \vee R) \equiv P \to R$

$\therefore$

$(P \to \sim Q, \sim Q \to R) \Rightarrow P \to R$

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Formal Proof :

(b) $\Rightarrow ((P \vee Q) \wedge (\sim P)) \rightarrow Q$.

$P \vee Q$ can be assumed as $\sim P \rightarrow Q$

As the actual premise is a Null set, Then RHS is to be proved as a Tautology

Set an initial premise as $\sim P \rightarrow Q$

1. $\sim P \rightarrow Q$ Hypothesis
2. $\sim P$ simplification
3. Q Modus Ponens rule for 1 and 2

$\therefore ((P \vee Q) \wedge (\sim P)) \rightarrow Q$

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Resolution:

Resolution method requires ... in LHS to draw conclusions from.
Since there are no boolean equations. We have to prove that RHS is a tautology

i.e $((P \vee Q) \wedge \sim P) \rightarrow Q$ is a tautology

$((P \vee Q) \wedge \sim P) \rightarrow Q \equiv (P \wedge \sim P) \vee (Q \wedge \sim P) \rightarrow Q$

$\equiv (Q \wedge \sim P) \rightarrow Q$

$\equiv (\sim Q \vee P) \vee Q$

$\equiv T$

$\therefore \Rightarrow ((P \vee Q) \wedge \sim P) \rightarrow Q$

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