DSA-Assignment-1

Deadline: 25th March 2023

Instructions

1. Deadline for the assignment is **25th March, 2023**
2. Solve all the question and submit a handwritten document
3. Plagiarism will be penalised
4. Submit a pdf of the form <**roll_no**>**_dsa1.pdf**

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1 Fourier Series

1. Plot the following questions and find the fourier series representations of the same.

   (a) Square wave function:

   $$f(x) = \begin{cases} 1 & 0 < x \leq \pi \\ -1 & \pi \leq x < 2\pi \end{cases}$$

   $$f(x + 2\pi) = f(x)$$

   (b) Saw tooth wave function:

   $$f(x) = x, \quad -\pi \leq x < \pi$$

   $$f(x + 2\pi) = f(x)$$

   (c)

   $$f(x) = e^x, \quad -\pi \leq x < \pi$$
   $$f(x + 2\pi) = f(x)$$

   (d)

   $$f(x) = \begin{cases} 0 & -\pi \leq x < 0 \\ x & 0 \leq x < \pi \end{cases}$$

   $$f(x + 2\pi) = f(x)$$

2. Classify the following functions as odd/even/neither.

   (a)
$$f(x) = \begin{cases} x - 1 & 0 \le x \le 2 \\ -1 - x & -2 < x < 0 \end{cases}$$
$$f(x + 4) = f(x)$$

   (b)
$$f(x) = \sin x + \cos x$$

   (c)
$$f(x) = |x| - 2, \;\; -2 \le x < 2$$
$$f(x + 4) = f(x)$$

3. (a) State the Drichlet conditions.

   (b) Show that
$$\pi/4 = 1 - 1/3 + 1/5 - 1/7 + ...$$
   using Fourier series.

   (c) Show that
$$\pi^2/8 = 1 + 1/3^2 + 1/5^2 + ...$$
   using Fourier series.

2 Fourier Transform

1. (a) Determine the transform of the following signal:
   $x(t) = t\left(\frac{sint}{\pi t}\right)^2$

   (b) Use Parseval's Law and the result from previous part to determine the value of
   $A = \int_{-\infty}^{\infty} t^2 \left(\frac{sint}{\pi t}\right)^4 dt$

2. Let $X(\omega)$ denote the Fourier transform of the signal $x(t)$ depicted in the figure below

   (a) Find phase of $X(j\omega)$.

   (b) Find $X(j0)$.

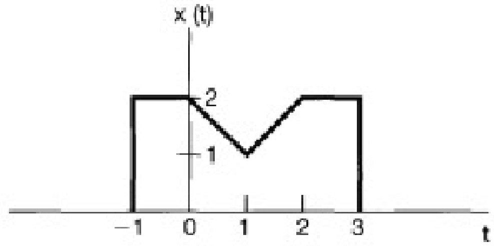
   (c) Find $\int_{-\infty}^{\infty} X(j\omega) d\omega$.

   (d) Evaluate $\int_{-\infty}^{\infty} X(j\omega) \frac{2sim\omega}{\omega} e^{j2\omega} d\omega$.

   (e) Evaluate $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$.

   (f) Sketch the inverse Fourier transform of $Re\{X(j\omega)\}$.

   Note : You should perform all these calculation without explicitly evaluating $X(j\omega)$.



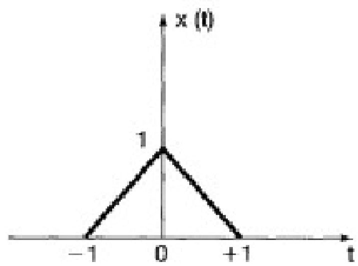
3. Consider the $x(t)$ signal in the figure below

   (a) Find the Fourier transform $X(j\omega)$ of $x(t)$.

   (b) Sketch the signal
   $\tilde{x} = x(t) * \sum_{k=-\infty}^{\infty} \delta(t-4k)$.

   (c) Find another signal $g(t)$ such that $g(t)$ is not same as $x(t)$ and
   $\tilde{x} = g(t) * \sum_{k=-\infty}^{\infty} \delta(t-4k)$.

   (d) Argue that. although $G(j\omega)$ is different from $X(j\omega)$, $G\big(j\frac{\pi k}{2}\big) = X\big(j\frac{\pi k}{2}\big)$ for all integers k. You should not explicitly evaluate $G(j\omega)$ to answer this question.



3 DTFT

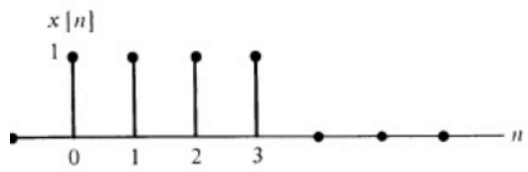
1. Consider a discrete-time signal x[n] of length N. The DTFT of the signal is given by $X(e^{jw})$

   Show that the DTFT is periodic with period 2Π.

2. Consider a discrete-time signal x[n] of length N. The DTFT of the signal is given by $X(e^{jw})$. Let $y[n] = x[n-m]$, where m is an integer.
   Find the DTFT of y[n] in terms of $X(e^{jw})$qq

3. Compute the DTFT for the following signals:

   (a) x[n]=$(1/4)^n u[n+2]$

(b) 

4 DFT

1. Compute the 8-point DFT for the following:
   (a) x(n) = $\frac{1}{4}$ u(4-n) where u(.) is the unit step function.
   (b) x(n) = sin($\frac{\pi n}{4}$)+cos($\frac{\pi n}{4}$)
   (c) x[n] = { 1,-1-j,-1,-1+j }
   (d) x[n] = { 1,1,1,1,1,1,1,1 }
2. Determine the inverse fourier transform of the following:
   (a) $X(e^{jw}) = cos^3 w + cos^2 w$
   (b) $X(e^{jw}) = \frac{e^{-4jw}+e^{-3jw}-e^{-jw}-1}{e^{-jw}+1}$
   (c) $X(e^{jw}) = \frac{3e^{-jw}-1}{3-e^{-jw}}$
3. Given X[k]=k^2 , 0 ≤ k ≤ 7 be 8-point DFT of a sequence x[n] ,find the value of $\sum_{n=0}^{3} x[2n+1]$