Assignment 1

CS9.312 Introduction to Quantum Information and Computation

Due date of submission: 13/01/2023

1. $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

   (a) Find the eigenvalues and normalized eigenvectors of A.

   (b) Denote the eigenvectors by $|a_1\rangle, |a_2\rangle, |a_3\rangle$. Now show that they form an orthonormal and complete basis, i.e., $\langle a_j|a_k\rangle = \delta_{jk}$ and $\sum_{j=1}^{3} |a_j\rangle\langle a_j| = I$

   (c) State vectors and operators in Quantum mechanics can be represented in different basis. Now consider another set of basis vectors -

   $$|b_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, |b_2\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, |b_3\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

   Find the matrix U of the transformation from the basis $\{|a\rangle\}$ to $\{|b\rangle\}$.

2. If $|\psi\rangle = |\phi_1\rangle + |\phi_2\rangle$ and $|\chi\rangle = |\phi_1\rangle - |\phi_2\rangle$, Show that $\langle\psi|\psi\rangle + \langle\chi|\chi\rangle = 2\langle\phi_1|\phi_1\rangle + 2\langle\phi_2|\phi_2\rangle$ Note that $|\phi_1\rangle$ and $|\phi_2\rangle$ are not orthonormal here.

3. Consider the state $|\psi\rangle = \frac{1}{\sqrt{2}}|\phi_1\rangle + \frac{1}{\sqrt{5}}|\phi_2\rangle + \frac{1}{\sqrt{10}}|\phi_3\rangle$ where $|\phi_1\rangle, |\phi_2\rangle, |\phi_3\rangle$ are orthonormal eigenstates of an operator $\hat{B}$ such that $\hat{B}|\phi_n\rangle = n^2|\phi_n\rangle$.

   (a) Is $|\psi\rangle$ normalized?

   (b) Find the expectation value of $\hat{B}$ for the state $|\psi\rangle$.

   (c) Find the expectation value of $\hat{B^2}$ for the state $|\psi\rangle$

   $\left(\textit{Expectation value is defined as } \langle\hat{B}\rangle = \frac{\langle\psi|\hat{B}|\psi\rangle}{\langle\psi|\psi\rangle} \right)$

4. Prove that for a Hermitian operator ($\hat{A}^\dagger = \hat{A}$), all the eigenvalues are real and the eigenvectors corresponding to different eigenvalues are orthogonal.