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# CV Final Exam.

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TLP0052

3. Given: $\underset{(2\times1)}{\begin{bmatrix} x \\ y \end{bmatrix}} = \alpha . \underset{(2\times3)}{P} . \underset{(3\times3)}{R_x(\theta)} . \underset{(3\times1)}{\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}} + \begin{bmatrix} x_0 \\ y \end{bmatrix}$ ; $\alpha$ = scaling factor.

$\left.\begin{array}{l} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \to \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \to \begin{bmatrix} 3 \\ 0 \end{bmatrix} \end{array}\right\} \to \alpha, x_0, y_0 = ?$

Derive:
$x = \alpha X + x_0$
$y = \alpha(\cos(\theta)Y - \sin(\theta)Z) + y_0$.

To derive the given ||le projection equation from the series of transformations,

a) Rotation over x-axis.

$$\text{Let } R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$\therefore R_x(\theta)\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X \\ Y\cos\theta - Z\sin\theta \\ Y\sin\theta + Z\cos\theta \end{bmatrix}$$

b) Orthogonal projection:

$$P.R_x(\theta)\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{\text{orthogonal projection matrix}} \begin{bmatrix} X \\ Y\cos\theta - Z\sin\theta \\ Y\sin\theta + Z\cos\theta \end{bmatrix} = \begin{bmatrix} X \\ Y\cos\theta - Z\sin\theta \end{bmatrix}$$

c) Scaling by $\alpha$,

$$\alpha P . R_x(\theta)\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \alpha X \\ \alpha(\cos(\theta)Y - \sin(\theta)Z) \end{bmatrix}$$

Translation by $(x_0, y_0)$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \alpha x \\ \alpha(\cos(\theta)y - \sin(\theta)z) \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

Hence derived.

[From ①]

Consider $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \to \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \underline{x_0 = 0;\ y_0 = 0}.$$

[From ②]

Consider $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \to \begin{bmatrix} 3 \\ 0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} \alpha \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} \Rightarrow \underline{\alpha = 3}.$$

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a) Let $\tilde{x}$ be the 2D point in homogeneous co-ordinates.
Let $\tilde{l}_i$ be the 2D line in homogeneous co-ordinate.

Objective function $D = \sum(\tilde{x}^T \tilde{l}_i)^2$.

To find $\tilde{x}$ that minimizes D, we'll turn the shared quantity to a quadratic form $\tilde{x}^T A \tilde{x}$ where A is a matrix.

$$D = \sum_i (\tilde{x}^T \tilde{l}_i)^2 = \sum_i \tilde{x}^T \tilde{l}_i \tilde{l}_i^T \tilde{x}$$

Let $A = \sum_i \tilde{l}_i \tilde{l}_i^T$

$$\therefore D = \tilde{x}^T A \tilde{x}$$

We can find $\bar{x}$ which minimizes D by solving the system $Ax = 0$ by eigen-value decomposition. The eigenvectors will correspond to the direction of the lines and the eigen value will correspond to the minimum distance to these lines.

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b) Let Centroid $= (\bar{x}, \bar{y}) = \left(\frac{\sum_i x_i}{n}, \frac{\sum_i y_i}{n}\right)$

Co-variance matrix $C = \frac{1}{n} \sum_{i=1}^{n} (x-\bar{x})(y-\bar{y})$

~~Let $v_1$ and~~. The eigenvector corresponding to the largest eigen value of C represents the major axis of the co-variance ellipsoid.

For any line passing through the centroid the sum of squared distances from the points to the line can be expressed as

$$SSE = \sum_{i=1}^{n} \left[(x_i - \bar{x})\cos\theta + (y_i - \bar{y})\sin\theta\right]^2$$

where $\theta$ is the angle between the line and x-axis.

$\therefore$ By aligning the line with the major axis, we minimize SSE by maximizing the spread of the points along the line.

c) The first approach ~~minimizes the distance~~ finds a point that minimizes the distances to the lines and the second approach involves finding a line that minimizes the distance to the points. The underlying geometry results in different ~~obj~~ optimizations to satisfy the different objectives

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8. i) No. of parameters $= ((m \times n \times d) + 1) \times k = p$.

where $m =$ ~~filter~~ kernel width,
$n =$ kernel height
$d =$ no. of channels in input image
$k =$ no. of filters $= 64$

a) For 3x3 kernel.

$$p = (3\times3\times3+1)\times 64 = 28\times 64 = \underline{1792}$$

b) For 5x5.

$$p = (5\times5\times3+1)\times 64 = 76\times 64 = \underline{4864}$$

c) For 7x7

$$p = (7\times7\times3+1)\times 64 = (49\times3+1)\times 64 = 148\times 64 = \underline{9472}$$

d) For 9x9,

$$p = (9\times9\times3+1)\times 64 = 244\times 64 = \underline{15616}$$

2) Output size:

$$\text{output size} = \frac{\text{input size} - \text{kernel size} + 1}{\text{Stride}}$$

a) $\underline{3 \times 3}$

output size $= \dfrac{256-3+1}{1} = \underline{254}$. $\Rightarrow 254 \times 254$.

b) $\underline{5 \times 5}$

output size $= \dfrac{256-5+1}{1} = \underline{252}$. $\Rightarrow 252 \times 252$

c) $\underline{7 \times 7}$

output size $= \dfrac{256-7+1}{1} = \underline{250}$. $\Rightarrow 250 \times 250$.

d) $\underline{9 \times 9}$

output size $= \dfrac{256-9+1}{1} = \underline{248}$ $\Rightarrow 248 \times 248$.

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9. 1) Yes, convolutions are translation invariant. For convolution operations, shifting the input (image) results in the same output inherently.

2) Convolutions are not translation equivariant. For convolutions translating the input results in proportional and not equivalent shift in the output feature map.

3) Convolutions are not scale invariant. Convolutional operations are sensitive to the scale of the features in the input.

4) Convolutions are not scale ~~equivale~~ equivariant since changing the scale of features in the input leads to different scale responses in the layer.

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