In [1]:
```
# Import libraries

import cv2
import numpy as np
import matplotlib.pyplot as plt
from google.colab.patches import cv2_imshow
from sklearn.decomposition import PCA
```

Read the image file and visualize it.

In [2]:
```
# Load the input image
image = cv2.imread('sat_image_plaksha.jpg')
cv2_imshow(image)
#cv2.waitKey(0)
```

Convert the image into GRAYSCALE and visualize it.

In [3]:
```
# Use the cvtColor() function to convert the image into grayscale from BGR
gray_image = cv2.cvtColor(image, cv2.COLOR_BGR2GRAY)

cv2_imshow(gray_image)
#cv2.waitKey(0)
```

In [4]:
```
# Plot the grayscale image

fig = plt.gcf()
plt.imshow(gray_image, cmap='gray')
plt.axis('off')
plt.show()
```

In [5]:
```
type(gray_image)
```

Out[5]:
```
numpy.ndarray
```

In [6]:
```
gray_image
```

Out[6]:
```
array([[162, 169, 167, ..., 109, 112, 110],
       [168, 169, 164, ..., 106, 107, 104],
       [160, 176, 171, ..., 109, 113, 113],
       ...,
       [191, 196, 159, ...,  74,  73,  70],
       [136, 147, 130, ...,  74,  72,  72],
       [136, 140, 133, ...,  71,  67,  67]], dtype=uint8)
```

Convert the image to double

In [7]:
```
# To perform the mathematical operations accurately

image_double = gray_image.astype(np.float64)
image_double
```

Out[7]:
```
array([[162., 169., 167., ..., 109., 112., 110.],
       [168., 169., 164., ..., 106., 107., 104.],
       [160., 176., 171., ..., 109., 113., 113.],
       ...,
       [191., 196., 159., ...,  74.,  73.,  70.],
       [136., 147., 130., ...,  74.,  72.,  72.],
       [136., 140., 133., ...,  71.,  67.,  67.]])
```

In [8]:
```
image_double.shape
```

Out[8]:
```
(264, 300)
```

Compute the mean of each column and subtract it from the image

In [9]:
```
mean_column = np.mean(image_double, axis = 0)
mean_column.shape
```

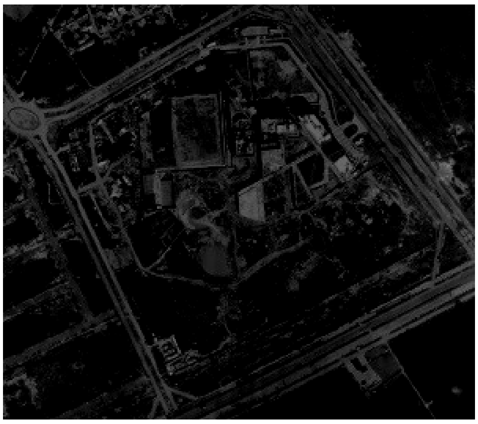
Out[9]:
```
(300,)
```

In [10]:
```
image_mean_subtracted = image_double - mean_column
image_mean_subtracted.shape
```

Out[10]: (264, 300)

In [11]:
```
# Visualize the mean subtracted image

cv2_imshow(image_mean_subtracted)
```



In [12]:
```
# Plot the mean subtracted image

fig = plt.gcf()
plt.imshow(image_mean_subtracted, cmap='gray', vmin = 0, vmax = 255)
plt.axis('off')
plt.show()
```

Compute the covariance matrix using numpy

```
In [13]: covariance_matrix = np.cov(image_mean_subtracted, rowvar = False)
         covariance_matrix
```

```
Out[13]: array([[1518.87589296, 1343.12579214, 1121.83208031, ..., -167.01493548,
                 -126.55036583, -139.92042574],
                [1343.12579214, 1545.76489227, 1265.9413815 , ..., -140.70183777,
                  -89.85392902,  -73.36280101],
                [1121.83208031, 1265.9413815 , 1425.59659811, ..., -144.06704401,
                  -86.40752103,  -38.05082671],
                ...,
                [-167.01493548, -140.70183777, -144.06704401, ..., 1254.67990264,
                 1140.29068729, 1075.81819622],
                [-126.55036583,  -89.85392902,  -86.40752103, ..., 1140.29068729,
                 1184.58519127, 1148.42033932],
                [-139.92042574,  -73.36280101,  -38.05082671, ..., 1075.81819622,
                 1148.42033932, 1233.26545397]])
```

```
In [14]: covariance_matrix.shape

         # Observation - The covariance matrix is symmetrical.
```

```
Out[14]: (300, 300)
```

Get eigenvalues and eigenvectors using numpy

```
In [15]: eigenvalues, eigenvectors = np.linalg.eig(covariance_matrix)
```

```
In [16]: eigenvalues[10]
```

```
Out[16]: (6679.373299285677+0j)
```

```
In [17]: len(eigenvalues)
```

```
Out[17]: 300
```

```
In [18]: eigenvectors[:, 10].shape
```

```
Out[18]: (300,)
```

```
In [19]: eigenvectors.shape
```

```
Out[19]: (300, 300)
```

Sort eigenvectors by eigenvalues

In [20]:
```
index_of_sorted = np.argsort(eigenvalues)[::-1]
index_of_sorted
```

Out[20]:
```
array([  0,   1,   2,   3,   4,   5,   6,   7,   8,   9,  11,  10,  12,
        13,  14,  15,  16,  17,  18,  19,  20,  21,  22,  23,  24,  25,
        26,  27,  28,  29,  30,  31,  32,  33,  34,  35,  36,  37,  38,
        39,  40,  41,  42,  43,  44,  45,  46,  47,  48,  49,  50,  51,
        52,  53,  54,  55,  56,  57,  58,  59,  60,  61,  62,  63,  64,
        65,  66,  67,  68,  69,  70,  71,  72,  74,  75,  76,  73,  77,
        78,  79,  80,  81,  82,  83,  84,  85,  86,  87,  88,  89,  90,
        91,  94,  93,  92,  95,  96,  97,  98,  99, 100, 101, 102, 103,
       104, 105, 106, 107, 109, 110, 112, 111, 108, 113, 114, 115, 117,
       118, 119, 120, 121, 122, 123, 124, 125, 116, 126, 127, 128, 129,
       130, 131, 133, 132, 134, 135, 136, 137, 138, 140, 139, 141, 142,
       143, 144, 145, 146, 147, 151, 153, 154, 152, 150, 149, 148, 155,
       156, 157, 158, 159, 160, 161, 162, 166, 170, 171, 175, 177, 178,
       176, 174, 173, 172, 168, 169, 167, 165, 164, 163, 179, 181, 182,
       180, 183, 184, 185, 186, 187, 188, 190, 189, 191, 192, 193, 195,
       194, 196, 197, 198, 199, 201, 200, 202, 203, 204, 206, 208, 209,
       211, 210, 212, 213, 219, 220, 221, 222, 223, 224, 225, 218, 216,
       217, 215, 214, 207, 205, 228, 230, 229, 227, 226, 231, 232, 246,
       247, 236, 235, 234, 233, 245, 240, 239, 244, 243, 242, 241, 238,
       237, 252, 253, 257, 258, 256, 255, 254, 251, 250, 249, 248, 259,
       260, 261, 262, 263, 267, 268, 269, 270, 271, 272, 278, 279, 280,
       281, 288, 291, 292, 289, 290, 295, 282, 283, 299, 298, 293, 294,
       265, 266, 284, 285, 296, 297, 286, 287, 277, 274, 275, 276, 273,
       264])
```

In [21]:
```
sorted_eigenvalues = eigenvalues[index_of_sorted]
sorted_eigenvalues.shape
```

Out[21]:
```
(300,)
```

In [22]:
```
principal_components = eigenvectors[:, index_of_sorted]
principal_components.shape
```

Out[22]:
```
(300, 300)
```

In [23]:
```
principal_components
```

Out[23]:
```
array([[ 1.64775531e-02+0.j,  1.13838574e-02+0.j,  1.01639798e-01+0.j,
        ..., -2.12193126e-02+0.j,  5.97496350e-02+0.j,
        -8.10008999e-02+0.j],
       [ 2.09573162e-02+0.j,  1.09951118e-02+0.j,  1.19540353e-01+0.j,
        ..., -9.12833544e-03+0.j, -9.20623133e-02+0.j,
         8.52521211e-02+0.j],
       [ 3.12862767e-02+0.j,  2.93602609e-02+0.j,  1.19115524e-01+0.j,
        ..., -3.19015811e-02+0.j,  9.57754300e-02+0.j,
        -1.00467392e-01+0.j],
       ...,
       [ 7.04306480e-02+0.j, -6.39672485e-02+0.j, -5.64373005e-02+0.j,
        ..., -1.03391152e-02+0.j, -4.39811209e-02+0.j,
        -8.14174939e-02+0.j],
       [ 7.27870647e-02+0.j, -5.57765265e-02+0.j, -5.23987058e-02+0.j,
        ..., -5.50023378e-02+0.j,  1.87391332e-01+0.j,
        -1.85193694e-04+0.j],
       [ 7.43567861e-02+0.j, -5.58917639e-02+0.j, -4.34414149e-02+0.j,
        ..., -2.68257214e-02+0.j, -1.23094553e-01+0.j,
         8.86314626e-02+0.j]])
```

Define the number of principal components to keep

In [24]:
```
num_components = [10, 20, 30, 40, 50, 60, 90]
```

For each num_components, compress the image and then reconstruct it. Store all reconstructed images in output_images variable.

In [25]:
```
output_images = [ ]
```

In [26]:
```
def real_part(complex_num):
    '''
      Function to select only the real part from a complex number.
    '''
    return complex_num.real
```

In [27]:
```
for N in num_components:
    # Take N number of components and extract eigenvectors.
    eigen_matrix = principal_components[:, : N]                # Shape = (300, N)

    # Project the data onto the selected components
    compressed_data = np.dot(image_mean_subtracted, eigen_matrix)    # (264, 300) * (300, 10) -> (264, 

    # Reconstruct the image
    reconstructed_image  = np.dot(compressed_data, eigen_matrix.T) + mean_column

    v_func = np.vectorize(real_part)
    reconstructed_image_real  = v_func(reconstructed_image)
    output_images.append(reconstructed_image_real)
```

In [28]:
```
len(output_images)
```

Out[28]: 7

Display the results

In [29]:
```
def compute_explained_variance(n_components):
    '''
      Function to compute the cumulative explained variance for all principal components till the (n_co
    '''
    eigen_sum = np.sum(sorted_eigenvalues)
    var_sum = sum(sorted_eigenvalues[:n_components])

    return real_part(var_sum * 1.0 / eigen_sum)
```

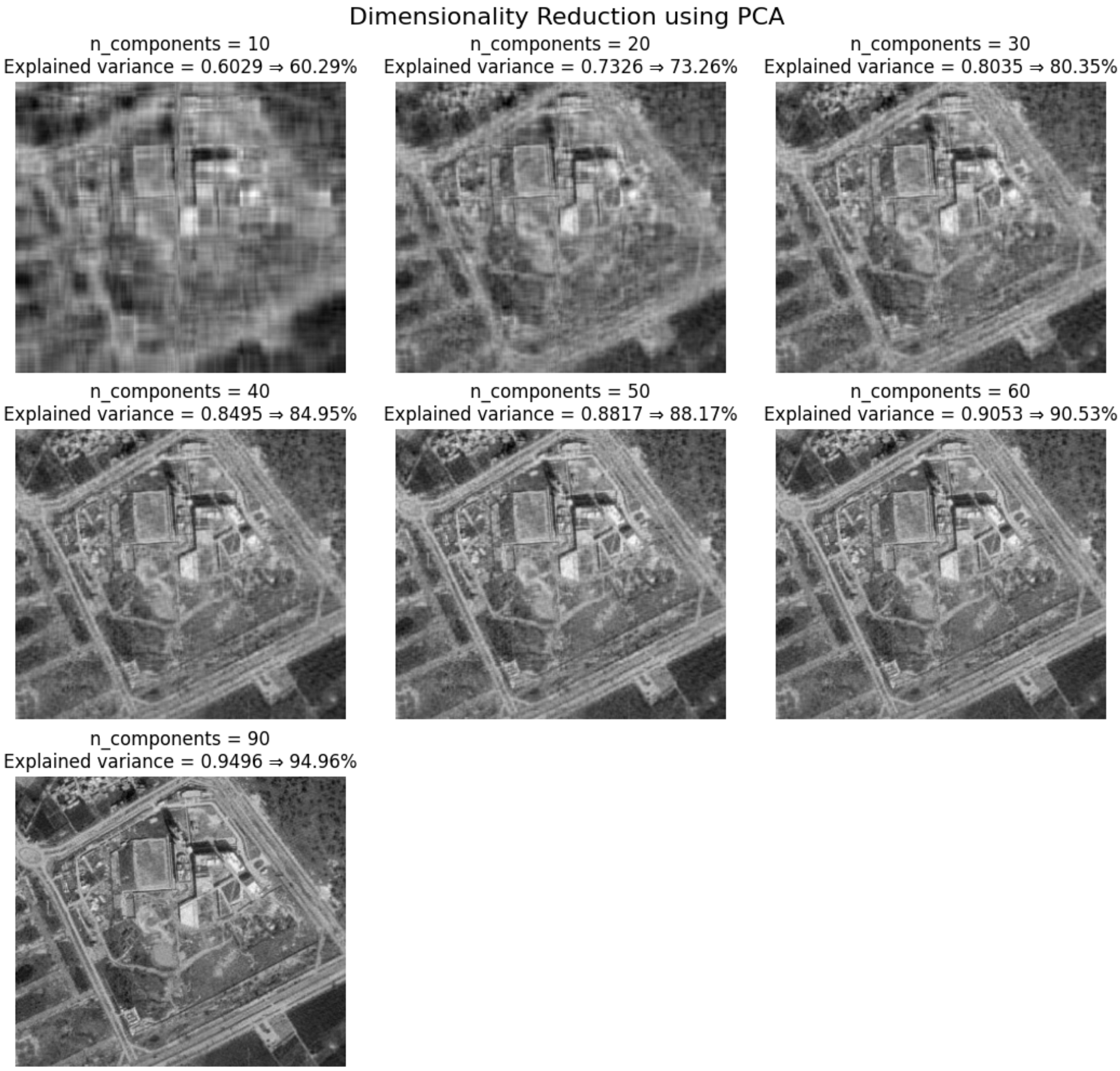
In [30]:

```python
fig, axes = plt.subplots(3, 3, figsize=(10.5, 10.5))
k = 0
n_rows = len(num_components) // 3 + 1

for r in range(n_rows):
    for c in range(3):
        if r == (n_rows - 1) and c != 0:
            axes[r, c].axis('off')
            break
        else:
            n_components = num_components[k]
            explained_variance = compute_explained_variance(n_components)

            axes[r, c].imshow(output_images[k], cmap='gray')
            title_text = f'n_components = {n_components}\nExplained variance = {explained_variance:.4f}
            title_text += f'\u21D2 {100 * explained_variance:.2f}%'
            axes[r, c].set_title(title_text)
            axes[r, c].axis('off')
            k += 1

plt.suptitle('Dimensionality Reduction using PCA', fontsize=16)
plt.tight_layout()
plt.axis('off')
plt.show()
```



Now compute minimum num_components needed to explain 95% variance in data

In [31]:
```
tot_components = len(sorted_eigenvalues)
tot_components
```

Out[31]:
```
300
```

Finding minimum num_components using computed eigenvalues.

In [32]:
```
for num_components in range(tot_components):
    explained_variance = compute_explained_variance(num_components) * 100
    if explained_variance >= 95:
        print('Minimum number of components needed to explain 95% variance in data is:', num_components
        break
```

```
Minimum number of components needed to explain 95% variance in data is: 91
```

Finding minimum num_components using PCA function from sklearn

In [33]:
```
for num_components in range(tot_components):
    pca = PCA(n_components = num_components)
    pca.fit(image_double)

    explained_variance = sum(pca.explained_variance_ratio_) * 100
    if explained_variance >= 95:
        print('Minimum number of components needed to explain 95% variance in data using PCA is:', num_
        break
```

```
Minimum number of components needed to explain 95% variance in data using PCA is: 91
```

In [33]:
```

```