

# OUTLINE

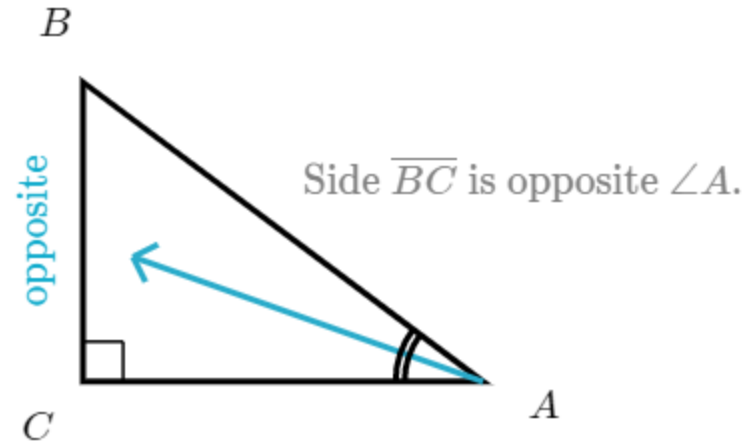
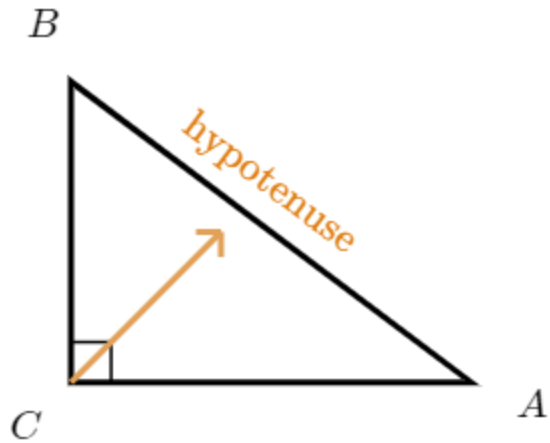
- Pythagoras Theorem
- Trigonometric ratios
- Inverse Trigonometric Functions
- Reciprocal Trigonometric Ratios
- Law of Sines
- Law of Cosines

# PYTHAGORAS THEOREM

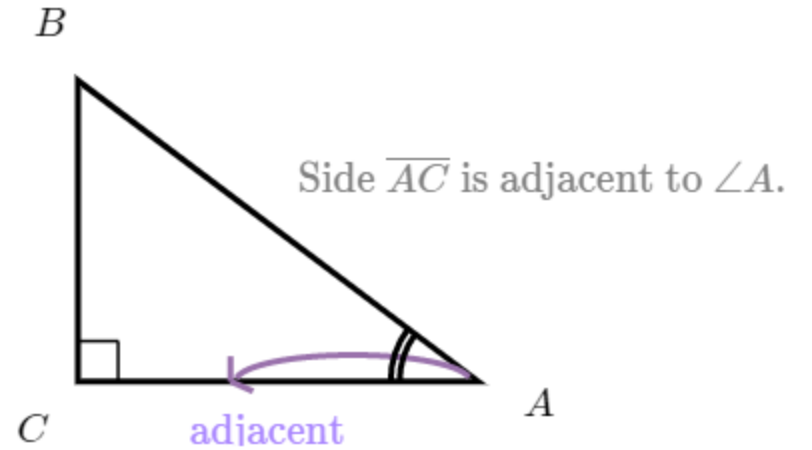
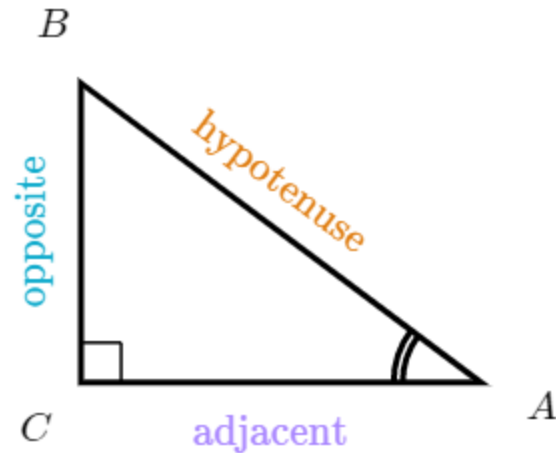
- Hypotenuse

Opposite

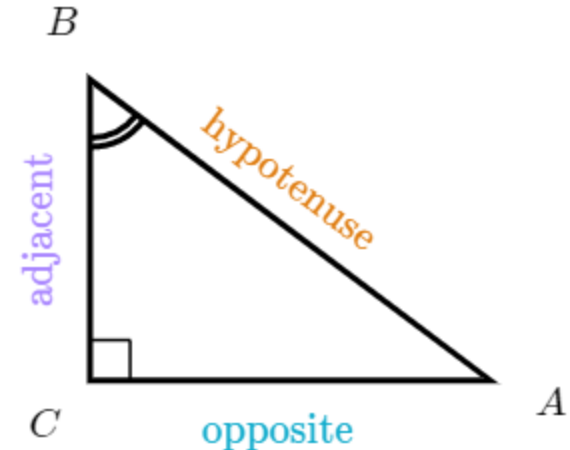
Adjacent



from the perspective of  $\angle A$



from  $\angle B$



- Finding an unknown side in a right angle triangle:

$$h^2 = a^2 + b^2$$

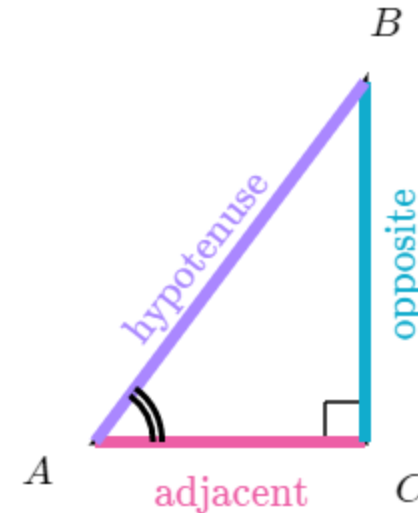
## TRIGONOMETRIC RATIOS:

- The ratios of the sides of a triangle are called trigonometric ratios. Three common trigonometric ratios are the **sine (sin)**, **cosine (cos)**, and **tangent (tan)**.
- Using SOH CAH TOH:

$$\text{SOH, } \sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\text{CAH, } \cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\text{TOA, } \tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$$



$$\sin(A) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos(A) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan(A) = \frac{\text{opposite}}{\text{adjacent}}$$

# INVERSE TRIGONOMETRIC FUNCTIONS

Trigonometric functions input  
angles and output side ratios

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

→

$$\sin^{-1}\left(\frac{\text{opposite}}{\text{hypotenuse}}\right) = \theta$$

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

→

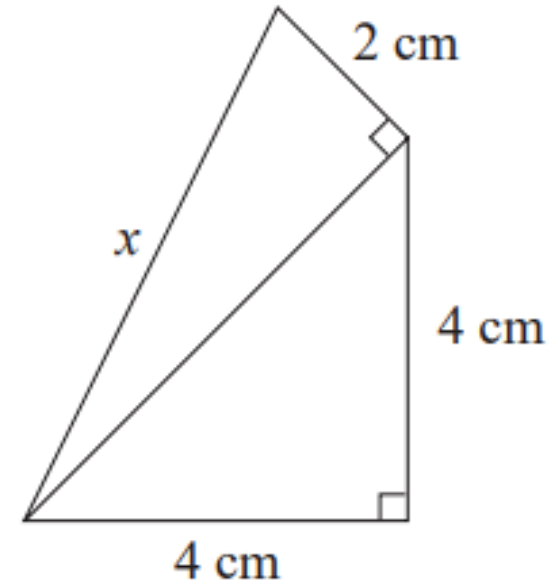
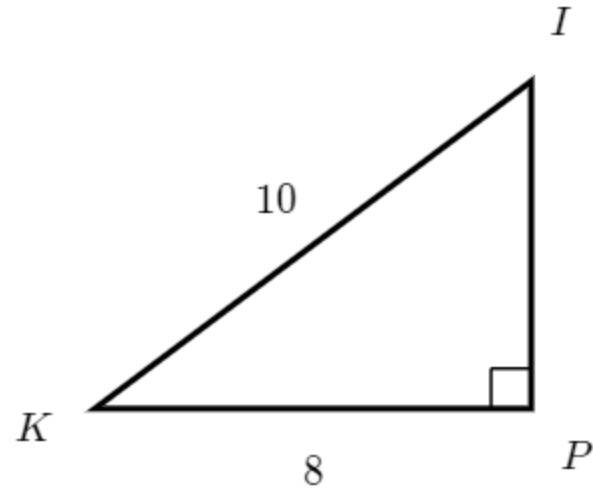
$$\cos^{-1}\left(\frac{\text{adjacent}}{\text{hypotenuse}}\right) = \theta$$

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

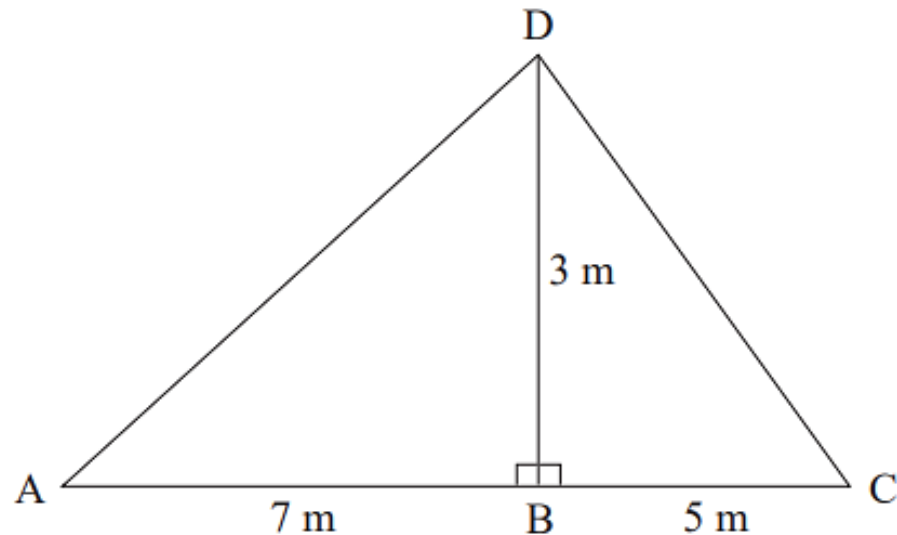
→

$$\tan^{-1}\left(\frac{\text{opposite}}{\text{adjacent}}\right) = \theta$$

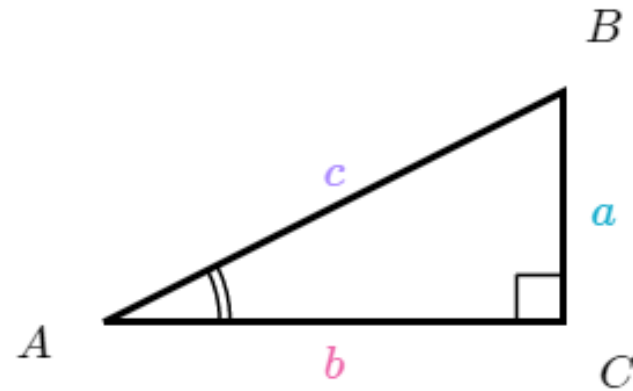
**Example 1:** Find all the unknowns in the following triangles.



**Question 1:** In the following triangle, (a) calculate the length of  $AD$ , (b) calculate the size of angle  $DCB$ .



# RECIPROCAL TRIGONOMETRIC RATIOS



$$\sin(A) = \frac{a}{c}$$

$$\cos(A) = \frac{b}{c}$$

$$\tan(A) = \frac{a}{b}$$

## Cosecant (csc)

$$\sin(A) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\csc(A) = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{c}{a}$$

## Secant (sec)

$$\cos(A) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\sec(A) = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{c}{b}$$

## Cotangent (cot)

$$\tan(A) = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$

$$\cot(A) = \frac{\text{adjacent}}{\text{opposite}} = \frac{b}{a}$$

## Verbal description

## Mathematical relationship

**cosecant**

The *cosecant* is the *reciprocal* of the *sine*.

$$\csc(A) = \frac{1}{\sin(A)}$$

**secant**

The *secant* is the *reciprocal* of the *cosine*.

$$\sec(A) = \frac{1}{\cos(A)}$$

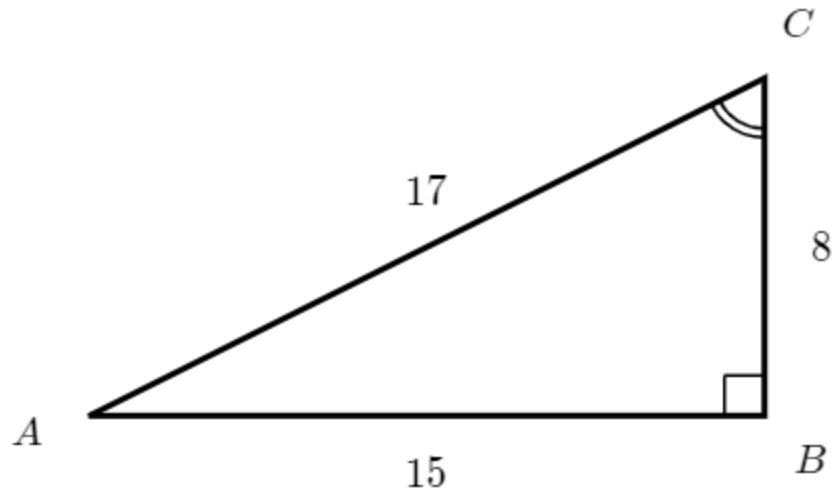
**cotangent**

The *cotangent* is the *reciprocal* of the *tangent*.

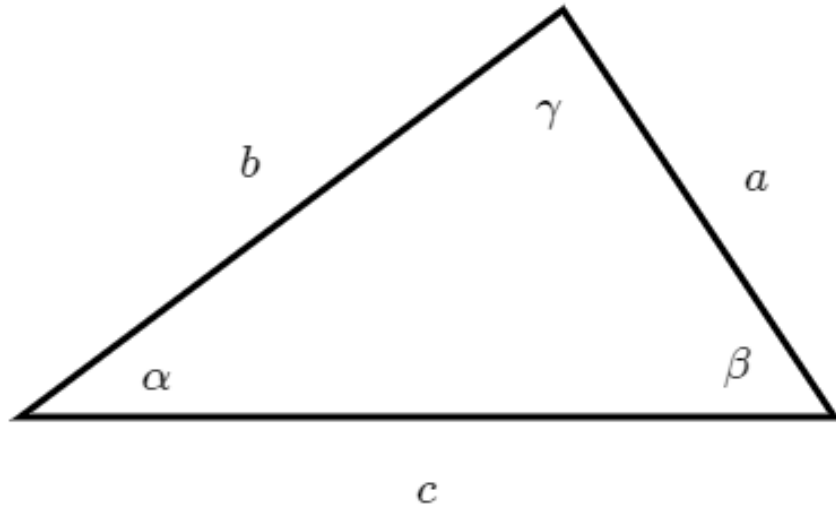
$$\cot(A) = \frac{1}{\tan(A)}$$



**Example 2:** In the triangle below, find  $\csc(c)$ ,  $\sec(c)$ , and  $\cot(c)$ .



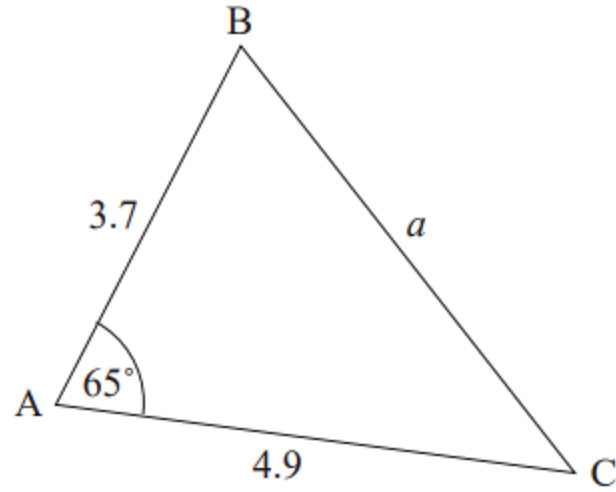
# LAW OF SINES AND COSINES



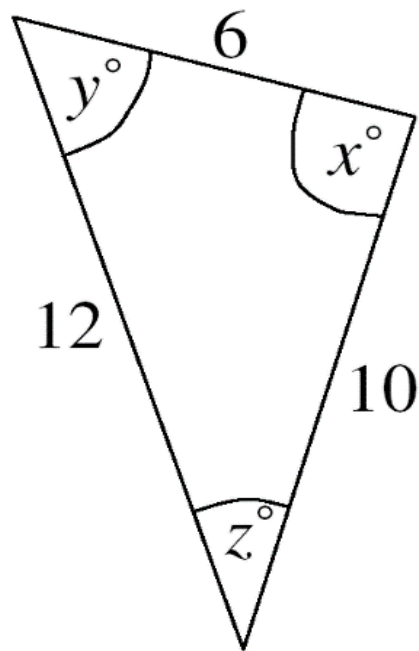
Law of Sines:  $\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$

Law of Cosines:  $c^2 = a^2 + b^2 - 2ab \cos(\gamma)$

**Example 3:** Find all the unknowns of the triangle shown:



**Question 2:** Find all the unknowns of the triangle shown?



**Question 3:** An airplane is approaching point  $A$  along a straight line and at a constant altitude  $h$ . The angle of elevation of the airplane changes from  $20^\circ$  to  $60^\circ$  in 2 minutes. What is the altitude  $h$  of the airplane if the speed of the airplane is constant and equal to 600 km/hr?

Resource:

<https://www.raywenderlich.com/2736-trigonometry-for-game-programming-part-1-2>

42

أبو ظبي  
ABU DHABI

