

Assignment - 1

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PROBLEM

Find the Area of Quadrilateral when four points are given

$$\mathbf{P} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} -2 \\ -2 \end{pmatrix} \quad (1)$$

SOLUTION

Area of a Quadrilateral PQRS is

$$Area(\Delta PQR) + Area(\Delta PRS) \quad (2)$$

$$Area(\Delta PQR) = \frac{1}{2} \|(\mathbf{Q} - \mathbf{P}) \times (\mathbf{Q} - \mathbf{R})\| \quad (3)$$

For two vectors $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$

$$\|\mathbf{a} \times \mathbf{b}\| = |a_1 b_2 - a_2 b_1| \quad (4)$$

$$\mathbf{Q} - \mathbf{P} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$\mathbf{Q} - \mathbf{R} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$$

$$Using Area(\Delta PQR) = \frac{1}{2} \|(\mathbf{Q} - \mathbf{P}) \times (\mathbf{Q} - \mathbf{R})\| \quad (5)$$

$$= \frac{1}{2} |(-23)| \quad (6)$$

Similarly

$$Area of (\Delta PRS) = \frac{1}{2} \|(\mathbf{S} - \mathbf{P}) \times (\mathbf{S} - \mathbf{R})\| \quad (7)$$

$$\mathbf{S} - \mathbf{P} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}$$

$$\mathbf{S} - \mathbf{R} = \begin{pmatrix} 1 \\ -6 \end{pmatrix}$$

$$Using Area of (\Delta PRS) = \frac{1}{2} \|(\mathbf{S} - \mathbf{P}) \times (\mathbf{S} - \mathbf{R})\| = \frac{1}{2} |(27)| \quad (8)$$

So total area of Quadrilateral PQRS is

$$Area(\Delta PQR) + Area(\Delta PRS) \quad (9)$$

$$= \frac{1}{2} |(-23)| + \frac{1}{2} |(27)| \quad (10)$$

$$= (23 + 27)/2 \quad (11)$$

$$= 25 \text{ sq. units} \quad (12)$$

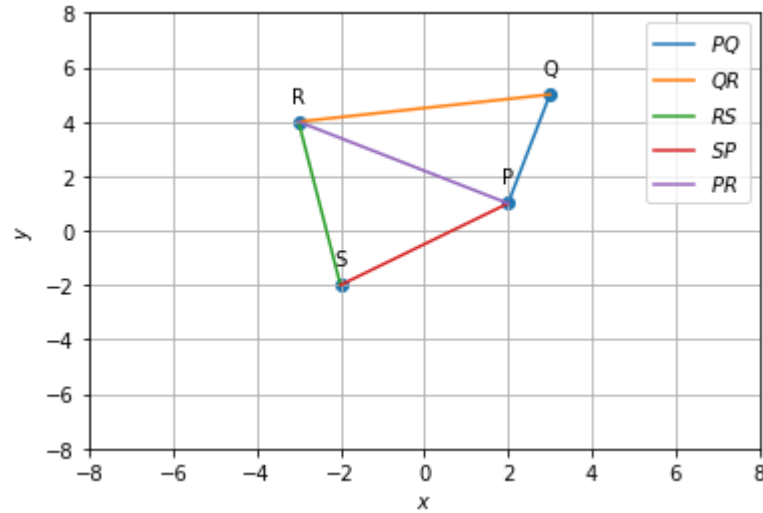


Fig. 1. Quadrilateral PQRS