

Assignment - 1

LAKSHMI GAYATHRI GUDIPUDI - SM21MTECH11001

CHAPTER II EX-II Q.9-II

Find the Area of Quadrilateral when four points are given

$$\mathbf{P} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} -2 \\ -2 \end{pmatrix} \quad (1)$$

SOLUTION

Area of a Quadrilateral PQRS is

$$Area(\Delta PQR) + Area(\Delta PRS) \quad (2)$$

$$Area(\Delta PQR) = \frac{1}{2} \|(\mathbf{Q} - \mathbf{P}) \times (\mathbf{Q} - \mathbf{R})\| \quad (3)$$

For two vectors

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \quad (4)$$

$$\|\mathbf{a} \times \mathbf{b}\| = |a_1 b_2 - a_2 b_1| \quad (5)$$

$$\mathbf{Q} - \mathbf{P} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad (6)$$

$$\mathbf{Q} - \mathbf{R} = \begin{pmatrix} 6 \\ 1 \end{pmatrix} \quad (7)$$

$$Area(\Delta PQR) = \frac{1}{2} \|(\mathbf{Q} - \mathbf{P}) \times (\mathbf{Q} - \mathbf{R})\| \quad (8)$$

$$\frac{1}{2} |(-23)| \quad (9)$$

Similarly

$$Areaof(\Delta PRS) = \frac{1}{2} \|(\mathbf{S} - \mathbf{P}) \times (\mathbf{S} - \mathbf{R})\| \quad (10)$$

$$\mathbf{S} - \mathbf{P} = \begin{pmatrix} -4 \\ -3 \end{pmatrix} \quad (11)$$

$$\mathbf{S} - \mathbf{R} = \begin{pmatrix} 1 \\ -6 \end{pmatrix} \quad (12)$$

$$Area(\Delta PRS) = \frac{1}{2} \|(\mathbf{S} - \mathbf{P}) \times (\mathbf{S} - \mathbf{R})\| \quad (13)$$

$$= \frac{1}{2} |(27)| \quad (14)$$

So total area of Quadrilateral PQRS is

$$Area(\Delta PQR) + Area(\Delta PRS) \quad (15)$$

$$= \frac{1}{2} |(-23)| + \frac{1}{2} |(27)| \quad (16)$$

$$= \frac{1}{2} (23 + 27) \quad (17)$$

$$= 25 \text{ sq. units} \quad (18)$$

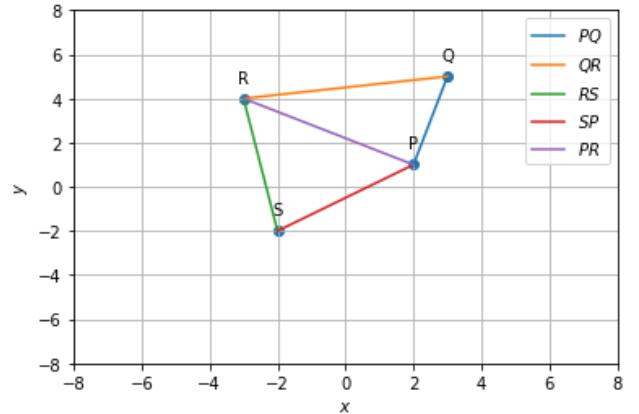


Fig. 0: Quadrilateral PQRS