BASICS OF PROGRAMMING ASSIGNMENT - 2

LAKSHMI GAYATHRI GUDIPUDI - SM21MTECH11001

CHAPTER III EX-IV Q-5

Find the condition that the lines

$$\mathbf{y + t_i x = 2at_i + at_i^3 = 0}$$

where i=1,2,3, are concurrent.

$$\mathbf{t_1 x + y = 2at_1 + at_1^3 = 0}$$

$$\mathbf{t_2 x + y = 2at_2 + at_2^3 = 0}$$

$$\mathbf{t_3 x + y = 2at_3 + at_3^3 = 0}$$

SOLUTION

Considering coefficients of three lines in matrix form :

$$\begin{pmatrix} t_1 & 1 \end{pmatrix} \mathbf{x} = 2at_1 + at_1^3 \tag{1}$$

$$\begin{pmatrix} t_2 & 1 \end{pmatrix} \mathbf{x} = 2at_2 + at_2^3 \tag{2}$$

$$\begin{pmatrix} t_3 & 1 \end{pmatrix} \mathbf{x} = 2at_3 + at_3^3 \tag{3}$$

The above equations form a matrix equation as below:

$$\begin{pmatrix} t_1 & 1 \\ t_2 & 1 \\ t_3 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2at_1 + at_1^3 \\ 2at_2 + at_2^3 \\ 2at_3 + at_3^3 \end{pmatrix} \tag{4}$$

Given the lines are concurrent ,so considering above equations are consistent and are reduced to augmented form as below to find the condition for lines to be concurrent:

$$\begin{pmatrix} t_1 & 1 & -2at_1 - at_1^3 \\ t_2 & 1 & -2at_2 - at_2^3 \\ t_3 & 1 & -2at_3 - at_3^3 \end{pmatrix} \tag{5}$$

Splitting the above augmented matrix as shown below to perform column operations:

$$\begin{gathered} \begin{pmatrix} t_1 & 1 & -2at_1 \\ t_2 & 1 & -2at_2 \\ t_3 & 1 & -2at_3 \end{pmatrix} + \begin{pmatrix} t_1 & 1 & -at_1^3 \\ t_2 & 1 & -at_2^3 \\ t_3 & 1 & -at_3^3 \end{pmatrix} \\ \xleftrightarrow{C_3 \leftarrow C_3/(-2a)} \xleftrightarrow{C_3 \leftarrow C_3/(-a)} \\ \begin{pmatrix} t_1 & 1 & +t_1 \\ t_2 & 1 & +t_2 \\ t_3 & 1 & +t_3 \end{pmatrix} + \begin{pmatrix} t_1 & 1 & +t_1^3 \\ t_2 & 1 & +t_2^3 \\ t_3 & 1 & +t_3^3 \end{pmatrix} \\ \xleftrightarrow{C_3 \leftarrow C_3 - C_1} \xleftrightarrow{C_1, C_2, C_3 \leftarrow C_1, C_2, C_3} \\ \begin{pmatrix} t_1 & 1 & 0 \\ t_2 & 1 & 0 \\ t_3 & 1 & 0 \end{pmatrix} + \begin{pmatrix} t_1 & 1 & +t_1^3 \\ t_2 & 1 & +t_2^3 \\ t_3 & 1 & +t_3^3 \end{pmatrix} \\ 0 + \begin{pmatrix} t_1 & 1 & +t_1^3 \\ t_2 & 1 & +t_2^3 \\ t_3 & 1 & +t_3^3 \end{pmatrix} \end{gathered} \tag{6}$$

Since system of equations are considered consistent,we get

$$t_1^3(t_2 - t_3) + t_2^3(t_3 - t_1) + t_3^3(t_1 - t_2) = 0 \tag{7}$$

Equation (7) can be written as

$$\sum t_1^3(t_2 - t_3) = 0 \tag{8}$$

Therefore Equation (8) represents the condition for the three lines to be concurrent.