BASICS OF PROGRAMMING ASSIGNMENT - 2

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CHAPTER III EX-IV Q-5

Find the condition that the lines

$$\mathbf{y + t_i x = 2at_i + at_i^3}$$

where i=1,2,3, are concurrent.

$$\mathbf{t_1 x + y = 2at_1 + at_1^3}$$

$$\mathbf{t_2 x + y = 2at_2 + at_2^3}$$

$$\mathbf{t_3 x + y = 2at_3 + at_3^3}$$

SOLUTION

Considering coefficients of three lines in matrix form :

$$\begin{pmatrix} t_1 & 1 \end{pmatrix} \mathbf{x} = 2at_1 + at_1^3 \tag{1}$$
$$\begin{pmatrix} t_2 & 1 \end{pmatrix} \mathbf{x} = 2at_2 + at_2^3 \tag{2}$$
$$\begin{pmatrix} t_3 & 1 \end{pmatrix} \mathbf{x} = 2at_3 + at_3^3 \tag{3}$$

The above equations form a matrix equation as below:

$$\begin{pmatrix} t_1 & 1 \\ t_2 & 1 \\ t_3 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2at_1 + at_1^3 \\ 2at_2 + at_2^3 \\ 2at_3 + at_3^3 \end{pmatrix} \tag{4}$$

Given the lines are concurrent ,so considering above equations are reduced to augmented form as below to find the condition for lines to be concurrent:

$$\begin{pmatrix} t_1 & 1 & 2at_1 + at_1^3 \\ t_2 & 1 & 2at_2 + at_2^3 \\ t_3 & 1 & 2at_3 + at_3^3 \end{pmatrix} \tag{5}$$

Performing row operations on the above augmented matrix as follows:

$$\begin{pmatrix} t_1 & 1 & 2at_1 + at_1^3 \\ t_2 & 1 & 2at_2 + at_2^3 \\ t_3 & 1 & 2at_3 + at_3^3 \end{pmatrix}$$

$$R_2 \leftarrow R_2 - R_1$$

$$R_3 \leftarrow R_3 - R_1$$

$$\begin{pmatrix} t_1 & 1 & t_1^3 \\ t_2 - t_1 & 0 & 2at_2 - 2at_1 + at_2^3 - at_1^3 \\ t_3 - t_1 & 0 & 2at_3 - 2at_1 + at_3^3 - at_1^3 \end{pmatrix}$$

$$\begin{pmatrix} t_1 & 1 & t_1^3 \\ t_2 - t_1 & 0 & (t_2 - t_1)(2a + a(t_2^2 + t_1^2 + t_1 t_2)) \\ t_3 - t_1 & 0 & (t_3 - t_1)(2a + a(t_3^2 + t_1^2 + t_3 t_1)) \end{pmatrix}$$

$$R_2 \leftarrow R_2/(t_2 - t_1)$$

$$R_3 \leftarrow R_3/(t_3 - t_1)$$

$$(t_2 - t_1)(t_3 - t_1) \begin{pmatrix} t_1 & 1 & t_1^3 \\ 1 & 0 & 2a + a(t_2^2 + t_1^2 + t_1 t_2) \\ 1 & 0 & 2a + a(t_3^2 + t_1^2 + t_3 t_1) \end{pmatrix}$$

$$R_3 \leftarrow R_3 - R_2)$$

$$(t_2 - t_1)(t_3 - t_1) \begin{pmatrix} t_1 & 1 & t_1^3 \\ 1 & 0 & 2a + a(t_2^2 + t_1^2 + t_1 t_2) \\ 0 & 0 & a(t_3^2 - t_2^2 + t_3 t_1 - t_1 t_2) \end{pmatrix}$$

$$(t_2 - t_1)(t_3 - t_1) \begin{pmatrix} t_1 & 1 & t_1^3 \\ 1 & 0 & 2a + a(t_2^2 + t_1^2 + t_1 t_2) \\ 0 & 0 & a(t_3 - t_2)(t_2 + t_1 + t_3) \end{pmatrix}$$

$$R_3 \leftarrow R_3/a(t_3 - t_2)$$

$$a(t_2 - t_1)(t_3 - t_1)(t_3 - t_2) \begin{pmatrix} t_1 & 1 & t_1^3 \\ 1 & 0 & 2a + a(t_2^2 + t_1^2 + t_1 t_2) \\ 0 & 0 & (t_2 + t_1 + t_3) \end{pmatrix} \tag{6}$$

Considering above matrix in echelon form the last row should be zero,so we get the condition for concurrency of lines as follows:

$$(t_2 - t_1)(t_3 - t_1)(t_3 - t_2)(t_2 + t_3 + t_1) = 0 \tag{7}$$

$$(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)(t_1 + t_2 + t_3) = 0 \tag{8}$$

Therefore Equation (8) represents the condition for the three lines to be concurrent.

Now considering lines as follows:

$$\begin{aligned} \mathbf{x+y=3} \\ \mathbf{2x+y=12} \\ \mathbf{-3x+y=-33} \end{aligned} \tag{9}$$

Now considering above lines in the format of lines given in question we get,

$$\begin{aligned} t_1 = 1 \\ t_2 = 2 \\ t_3 = -3 \end{aligned} \tag{10}$$

so from equation (8) we got the condition ,so substituting values from equation (10) in the condition we get:

$$\begin{aligned} (t_1-t_2)(t_2-t_3)(t_3-t_1)(t_1+t_2+t_3) = 0 \\ (1-2)(2-3)(-3-1)(1+2-3) = 0 \end{aligned} \tag{11}$$

In the above equation LHS =RHS .Hence verified.

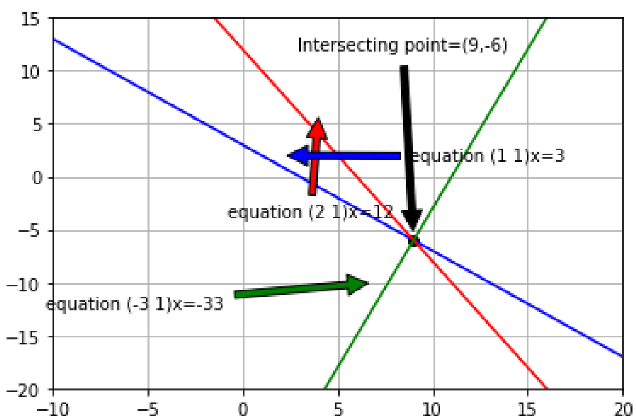

Fig. 0: Representation of lines intersecting at point(9,-6)