BASICS OF PROGRAMMING ASSIGNMENT - 2

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CHAPTER III EX-IV Q-5

Find the condition that the lines

$$\mathbf{y + t_i x = 2at_i + at_i^3}$$

where i=1,2,3, are concurrent.

$$\mathbf{t_1 x + y = 2at_1 + at_1^3}$$

$$\mathbf{t_2 x + y = 2at_2 + at_2^3}$$

$$\mathbf{t_3 x + y = 2at_3 + at_3^3}$$

SOLUTION

Considering coefficients of three lines in matrix form :

$$\begin{pmatrix} t_1 & 1 \end{pmatrix} \mathbf{x} = 2at_1 + at_1^3 \tag{1}$$

$$\begin{pmatrix} t_2 & 1 \end{pmatrix} \mathbf{x} = 2at_2 + at_2^3 \tag{2}$$

$$\begin{pmatrix} t_3 & 1 \end{pmatrix} \mathbf{x} = 2at_3 + at_3^3 \tag{3}$$

The above equations form a matrix equation as below:

$$\begin{pmatrix} t_1 & 1 \\ t_2 & 1 \\ t_3 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2at_1 + at_1^3 \\ 2at_2 + at_2^3 \\ 2at_3 + at_3^3 \end{pmatrix} \tag{4}$$

Given the lines are concurrent ,so considering above equations are consistent and are reduced to augmented form as below to find the condition for lines to be concurrent:

$$\begin{pmatrix} t_1 & 1 & -2at_1 - at_1^3 \\ t_2 & 1 & -2at_2 - at_2^3 \\ t_3 & 1 & -2at_3 - at_3^3 \end{pmatrix} \tag{5}$$

Considering Transpose of above augmented form of matrix and performing row operations below: $\begin{pmatrix} t_1 & t_2 & t_3 \\ 1 & 1 & 1 \\ -2at_1 - at_1^3 & -2at_2 - at_2^3 & -2at_3 - at_3^3 \end{pmatrix}$

Performing row operations on the above augmented matrix as follows:

$$\begin{pmatrix} t_1 & t_2 & t_3 \\ 1 & 1 & 1 \\ -2at_1 - at_1^3 & -2at_2 - at_2^3 & -2at_3 - at_3^3 \end{pmatrix}$$

$$R_3 \leftarrow R_3 + 2aR_1$$

$$\begin{pmatrix} t_1 & t_2 & t_3 \\ 1 & 1 & 1 \\ -at_1^3 & -at_2^3 & -at_3^3 \end{pmatrix}$$

$$R_3 \leftarrow R_3/(-a)$$

$$\begin{pmatrix} t_1 & t_2 & t_3 \\ 1 & 1 & 1 \\ t_1^3 & t_2^3 & t_3^3 \end{pmatrix} \leftrightarrow \begin{pmatrix} t_1 & 1 & t_1^3 \\ t_2 & 1 & t_2^3 \\ t_3 & 1 & t_3^3 \end{pmatrix}$$

$$R_2 \leftarrow R_2 - R_1$$

$$R_3 \leftarrow R_3 - R_1$$

$$\begin{pmatrix} t_1 & 1 & t_1^3 \\ t_2 - t_1 & 0 & t_2^3 - t_1^3 \\ t_3 - t_1 & 0 & t_3^3 - t_1^3 \end{pmatrix}$$

$$R_2 \leftarrow R_2/(t_2 - t_1)$$

$$R_3 \leftarrow R_3/(t_3 - t_1)$$

$$(t_2 - t_1)(t_3 - t_1) \begin{pmatrix} t_1 & 1 & t_1^3 \\ 1 & 0 & t_2^2 + t_1^2 + t_1t_2 \\ 1 & 0 & t_3^2 + t_1^2 + t_3t_1 \end{pmatrix}$$

$$R_2 \leftarrow R_2 - R_3)$$

$$(t_2 - t_1)(t_3 - t_1) \begin{pmatrix} t_1 & 1 & t_1^3 \\ 0 & 0 & t_2^2 - t_3^2 + t_1t_2 - t_1t_3 \\ 1 & 0 & t_3^2 + t_1^2 + t_3t_1 \end{pmatrix}$$

$$(t_2 - t_1)(t_3 - t_1) \begin{pmatrix} t_1 & 1 & t_1^3 \\ 0 & 0 & (t_2 - t_3)(t_2 + t_3 + t_1) \\ 1 & 0 & t_3^2 + t_1^2 + t_3t_1 \end{pmatrix}$$

$$R_2 \leftarrow R_2/(t_2 - t_3)$$

$$(t_2 - t_1)(t_3 - t_1)(t_2 - t_3) \begin{pmatrix} t_1 & 1 & t_1^3 \\ 0 & 0 & (t_2 + t_3 + t_1) \\ 1 & 0 & t_3^2 + t_1^2 + t_3t_1 \end{pmatrix} \tag{6}$$

Since system of equations are considered consistent,we get

$$(t_2 - t_1)(t_3 - t_1)(t_2 - t_3)(t_2 + t_3 + t_1) = 0 \quad (7)$$

$$(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)(t_1 + t_2 + t_3) = 0 \quad (8)$$

Therefore Equation (8) represents the condition for the three lines to be concurrent.