BASICS OF PROGRAMMING ASSIGNMENT - 2

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CHAPTER III EX-IV Q-5

Find the condition that the lines

$$\mathbf{y + t_i x = 2at_i + at_i^3 = 0}$$

where i=1,2,3, are concurrent.

$$\mathbf{l_1 = t_1 x + y - 2at_1 - at_1^3 = 0}$$

$$\mathbf{l_2 = t_2 x + y - 2at_2 - at_2^3 = 0}$$

$$\mathbf{l_3 = t_3 x + y - 2at_3 - at_3^3 = 0}$$

SOLUTION

Considering coefficients of three lines in vector form :

$$\mathbf{L_1} = \begin{pmatrix} a_1 \\ b_1 \\ c_1 \end{pmatrix} = \begin{pmatrix} t_1 \\ 1 \\ -2at_1 - at_1^3 \end{pmatrix} \tag{1}$$

$$\mathbf{L_2} = \begin{pmatrix} a_2 \\ b_2 \\ c_2 \end{pmatrix} = \begin{pmatrix} t_2 \\ 1 \\ -2at_2 - at_2^3 \end{pmatrix} \tag{2}$$

$$\mathbf{L_3} = \begin{pmatrix} a_3 \\ b_3 \\ c_3 \end{pmatrix} = \begin{pmatrix} t_3 \\ 1 \\ -2at_3 - at_3^3 \end{pmatrix} \tag{3}$$

$$\tag{4}$$

Now for three lines

$$\mathbf{a_i x + b_i y + c = 0}$$

where i=1,2,3 to be concurrent,

$$\mathbf{\Delta} = \begin{vmatrix} \mathbf{L_1}^\top \\ \mathbf{L_2}^\top \\ \mathbf{L_3}^\top \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \mathbf{0} \tag{5}$$

$$\mathbf{\Delta} = \begin{vmatrix} t_1 & 1 & -2at_1 - at_1^3 \\ t_2 & 1 & -2at_2 - at_2^3 \\ t_3 & 1 & -2at_3 - at_3^3 \end{vmatrix} = \mathbf{0} \tag{6}$$

On Expanding the determinant we get

$$\begin{aligned} t_1(-2at_3 - at_3^3 + 2at_2 + at_2^3) \\ -t_2(-2at_3 - at_3^3 + 2at_1 + at_1^3) \\ +t_3(-2at_2 - at_2^3 + 2at_1 + at_1^3) = 0 \end{aligned} \tag{7}$$

$$\begin{aligned} t_1(2a(t_2 - t_3) + a(t_2^3 - t_3^3)) \\ -t_2(2a(t_1 - t_3) + a(t_1^3 - t_3^3)) \\ -t_3(2a(t_1 - t_2) + a(t_1^3 - t_2^3)) = 0 \end{aligned} \tag{8}$$

On further simplifying we get

$$\begin{aligned} 2at_1t_2 - 2at_1t_3 + at_1t_2^3 - at_1t_3^3 \\ -2at_1t_2 + 2at_2t_3 - at_2t_1^3 - at_2t_3^3 \\ +2at_1t_3 - 2at_2t_3 + at_3t_1^3 - at_3t_2^3 = 0 \end{aligned} \tag{9}$$

$$\begin{aligned} t_1t_2^3 - t_1t_3^3 - t_2t_1^3 - t_2t_3^3 \\ +t_3t_1^3 - t_3t_2^3 = 0 \end{aligned} \tag{10}$$

$$t_1^3(t_2 - t_3) + t_2^3(t_3 - t_1) + t_3^3(t_1 - t_2) = 0 \tag{11}$$

Equation (11) can be written as

$$\sum t_1^3(t_2 - t_3) = 0 \tag{12}$$

Therefore Equation (12) represents the condition for the three lines to be concurrent.