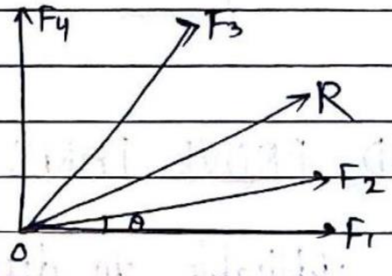
MODULE 2

Analysis of Concurrent Force Systems

1/3/17

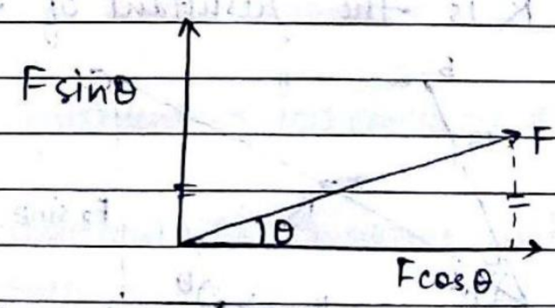
- RESULTANT: Resultant forces, when a no. of concurrent forces acting on a body, it is possible to find a single force which can produce a same effect that can be produced by other forces. Such a force is called as resultant forces.



The process of determining the resultant force of a given system of forces is known as composition of forces.

→ Resultant can be determined by graphical and analytical method.

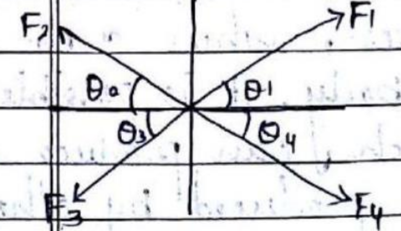
- RESOLUTION OF FORCES: Method of resolving a single force into 2 component is known as resolution of forces



horizontal component = $F\cos\theta$.
vertical component = $F\sin\theta$.

Imp
- METHOD OF RESOLVING NUMBER OF CO-PLANAR CONCURRENT FORCES:

$$\sqrt{\Sigma(HC)^2 + \Sigma(VC)^2}$$



$$\boxed{R = \sqrt{\Sigma H^2 + \Sigma V^2}}$$

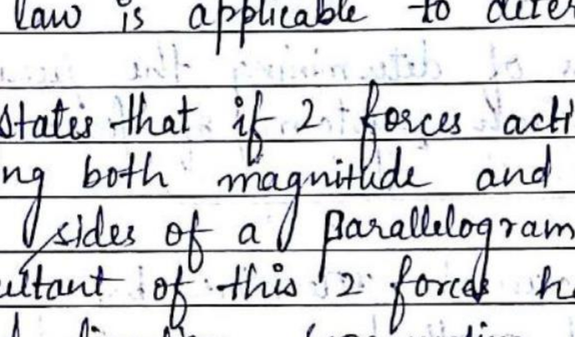
$$\boxed{\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)}$$

Imp • **STATE AND PROVE PARALLELOGRAM LAW OF FORCES:**

This law is applicable to ditermine the

Defn: It states that if 2 forces acting at a point having both magnitude and direction representing two sides of a parallelogram then the resultant of this 2 forces having both magnitude and direction representing diagonal of ||gm. passing through same point.

Proof:- Let F_1 and F_2 are 2 forces acting on line OB & OA and R is the resultant of these 2 forces.



In ΔOCD

$$OC^2 = OD^2 + CD^2$$

$$R^2 = (OA + AD)^2 + F_2^2 \sin^2\theta.$$

$$R^2 = (F_1 + F_2 \cos\theta)^2 + F_2^2 \sin^2\theta$$

$$R^2 = F_1^2 + F_2^2 \cos^2\theta + 2F_1F_2\cos\theta + F_2^2 \sin^2\theta$$

$$R^2 = F_1^2 + F_2^2(\cos^2\theta + \sin^2\theta) + 2F_1F_2\cos\theta.$$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

In ΔOCD :- $\tan\alpha = \dfrac{F_2 \sin\theta}{F_1 + F_2\cos\theta}$

$$\alpha = \tan^{-1}\left(\frac{F_2 \sin\theta}{F_1 + F_2\cos\theta}\right)$$

Case I :- If $\theta = 90^\circ$.

$\alpha = \tan^{-1}\left(\dfrac{F_2}{F_1}\right)$, $R = \sqrt{F_1^2 + F_2^2}$

Case II $\theta = 0$.

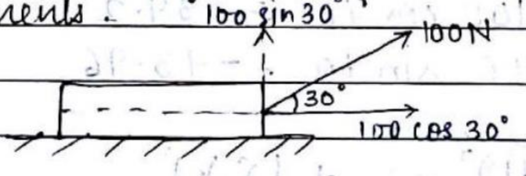
$\alpha = \tan^{-1}(0) = 0$. , $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2}$

Case III : $\theta = 180^\circ$

$R = F_2 - F_1$, $\alpha = 0^\circ$.

Q.1- A force 100 N is acting on the body as shown in fig.
Resolve the force into vertical and horizontal components.

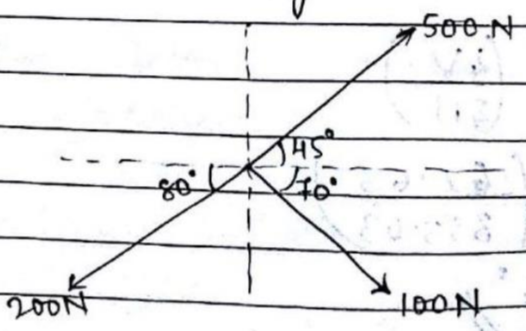
Ans-

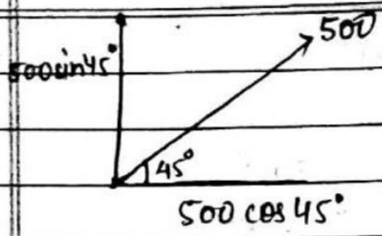


Vertical Component $= 100 \sin 30^\circ = 50 N$
Horizontal Component $= 100 \cos 30 = 86.60 N$

Q.2 Find the resultant of 3 forces acting on a point O' as shown in fig :-

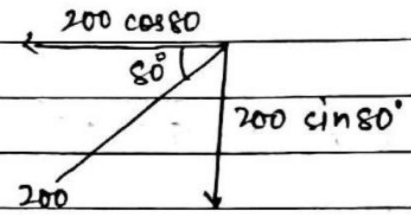
Ans-





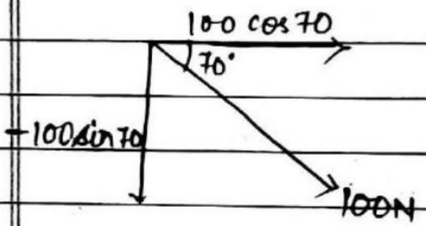
$H = 500 \times \cos 45 = 353.55$

$V = 500 \sin 45 = 353.55.$



$H = -200 \cos 80 = -34.72.$

$V = -200 \sin 80 = -196.96.$



$H = 100 \cos 70 = 34.2$

$V = -100 \sin 70 = -93.96.$

$\therefore R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$\Sigma H = 353.03.$

$\Sigma V = 62.63.$

$R = 35850$ N.

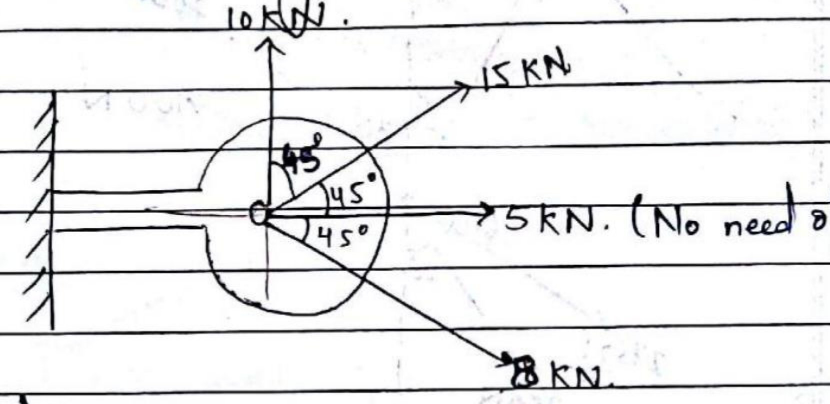
$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$

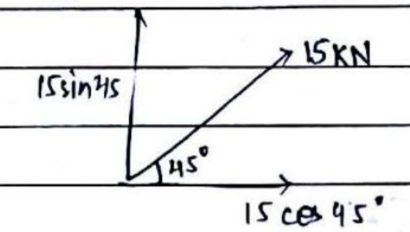
$\alpha = \tan^{-1}\left(\frac{62.63}{353.03}\right)$

$\alpha = 10.06°$ //

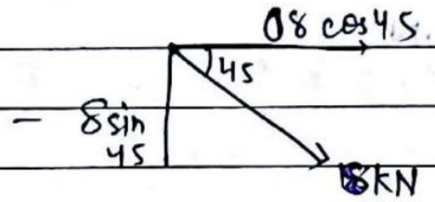
Q3- Find the value of resultant for the system of forces shown in fig :-

Ans-





$H = 15\cos 45° = 10.60$ KN.
$V = 15 \sin 45 = 10.60$ kN



$H = 8\cos 45 = 5.65$ kN
$V = -8\sin 45 = -5.65$ kN

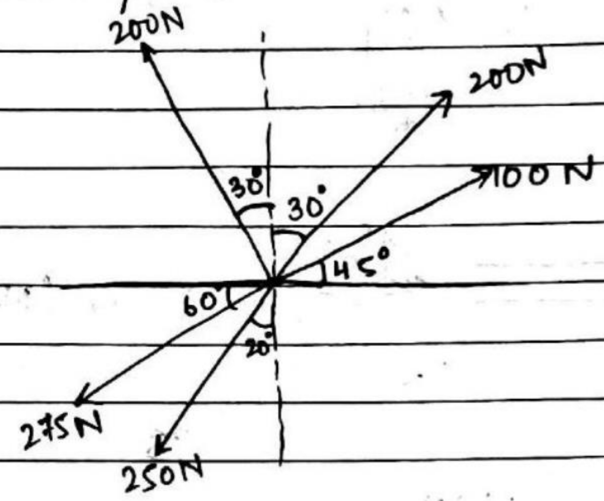
$\therefore \Sigma H = 5 + 10.60 + 5.65 = 21.25$ kN
$\Sigma V = 10 + 10.60 - 5.65 = 14.95$ kN.

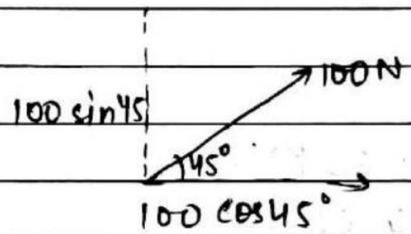
$R = \sqrt{\Sigma H^2 + \Sigma V^2} = 25.98$ KN.

$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = 35.12°$ //

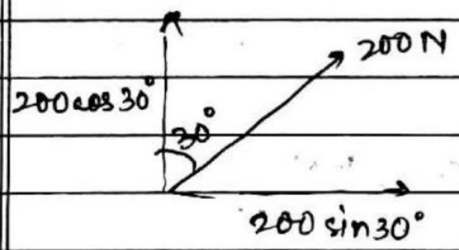
Q. 4- Find the resultant

Ans-

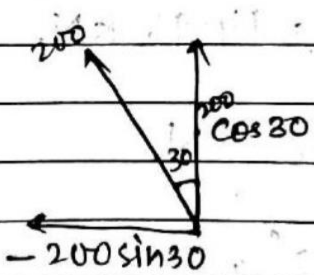




$H = 100 \cos 45 = 70.71 \text{ N}$
$V = 100 \sin 45 = 70.71 \text{ N}$



$H = 200 \sin 30° = 100$
$V = 200 \cdot \cos 30 = 173.21$



$H = -200 \sin 30 = -100$
$V = 200 \cos 30 = 173.21$

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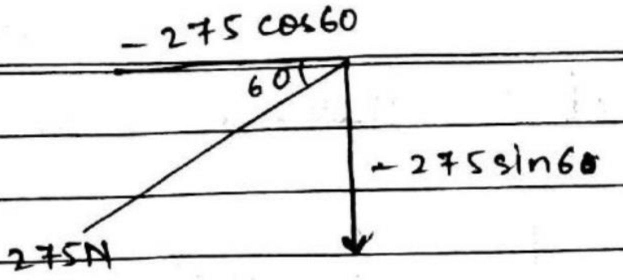
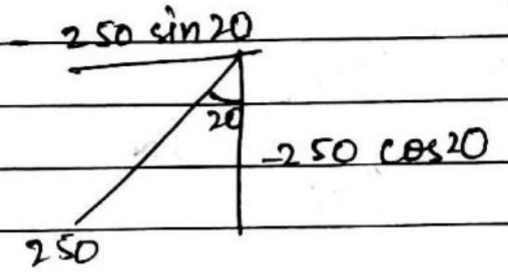


$H = -275\cos 60 = -137.50$

$V = -275 \sin 60 = -238.16.$



$H = -250 \sin 20. = -85.51$

$V = -250 \cos 20 = -234.92$

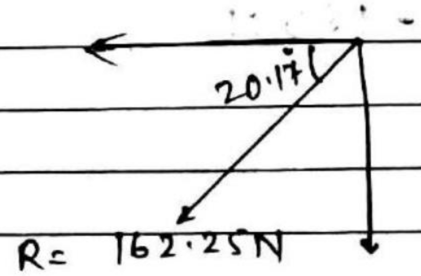
$\therefore \Sigma H = -159.30 \text{ N}$

$\Sigma V = -55.95 \text{ N}.$

$R = \sqrt{\Sigma H^2 + \Sigma V^2} = 162.25\text{N}$

$= \cancel{\sqrt{\quad}}$

$\tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = 20.17^\circ$



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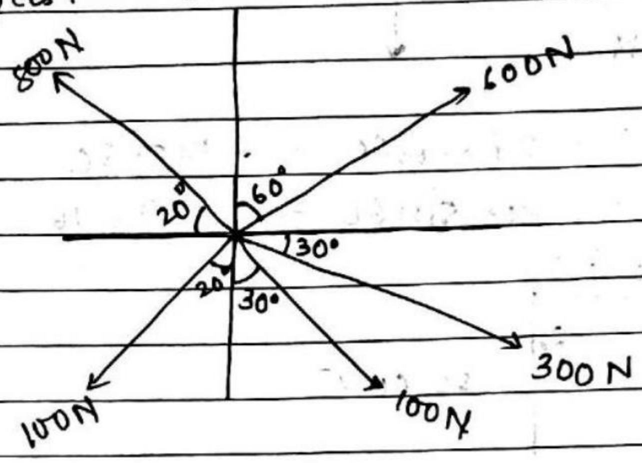
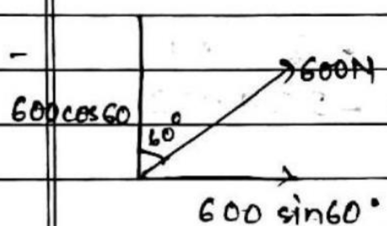
Q.5- Find magnitude of the resultant for the given system of forces.

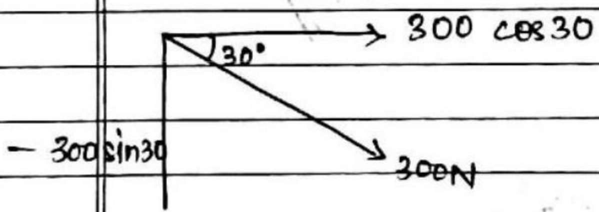


Ans -



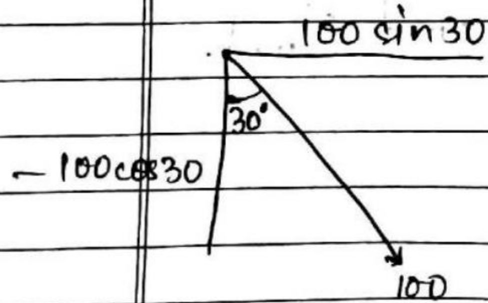
$H = 600 \sin 60 = 519.62$ N

$V = 600 \cos 60 = 300$ N



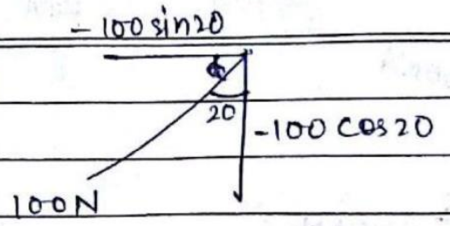
$H = 300 \cos 30 = 259.81$ N

$V = -300 \sin 30 = -150$ N



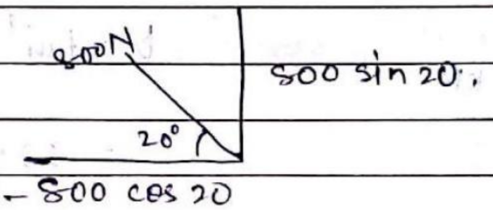
$H = 100 \sin 30 = 50$

$V = -100 \cos 30 = -86.60$.



$H = -100 \sin 20 = -34.20\ N$

$V = -100 \cos 20 = -93.97\ N.$



$H = -800 \cos 20 = -751.75 N$

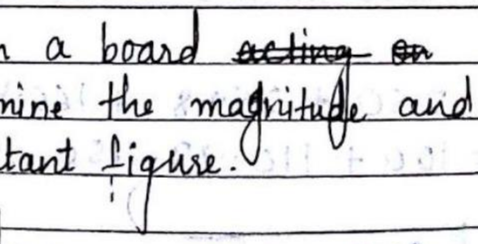
$V = 800 \sin 20 = 273.62\ N.$

$\therefore\ \Sigma H = 43.48\ N.$

$\Sigma V = 243.05\ N.$

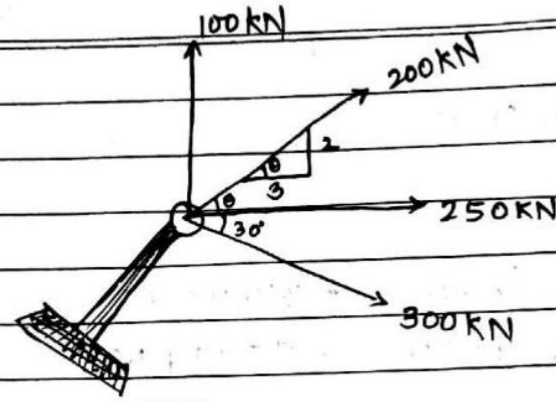
$\therefore\ R = \sqrt{\Sigma H^2 + \Sigma V^2} = 246.91\ N$ //

$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = 79.86°$ //



Q.6- Force acting on a board ~~acting on~~ as shown in figure. Determine the magnitude and direction of the resultant figure.

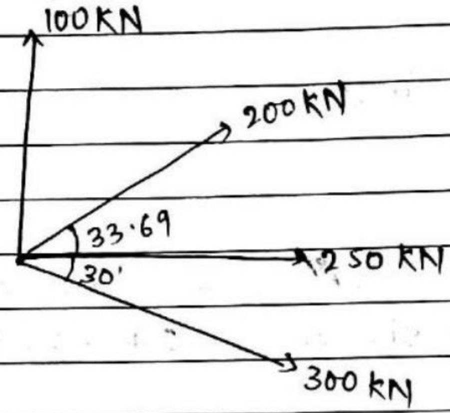
Ans -

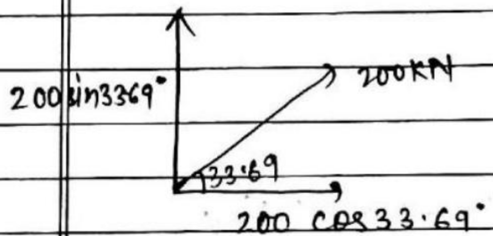


$\tan\theta = \frac{2}{3} \Rightarrow$

$\theta = \tan^{-1}\frac{2}{3}$

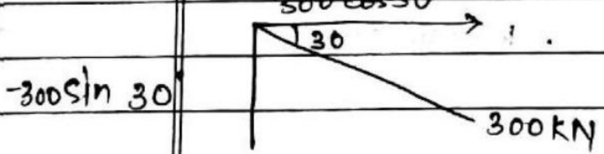
$= 33.69°$.





$H = 200 \cos 33.69 = 166.41$ KN.

$V = 200 \sin 33.69 = 110.93$ KN.



$H = 300 \cos 30 = 259.8$ KN

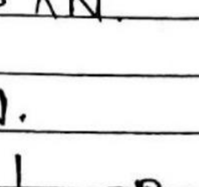
$V = -300 \sin 30 = -150$ KN.

$\Sigma H = 250 + 259.8 + 166.41 = 676.21$ KN.

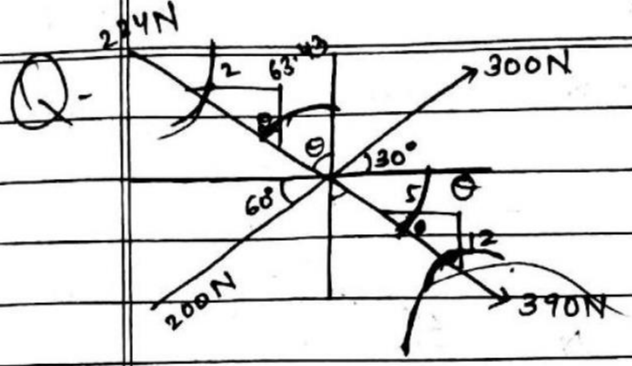
$\Sigma V = 100 + 110.93 - 150 = 60.93$ KN.

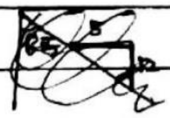
$R = \sqrt{\Sigma H^2 + \Sigma V^2} = 679.09$ KN.

$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = 5.14°$



Q-





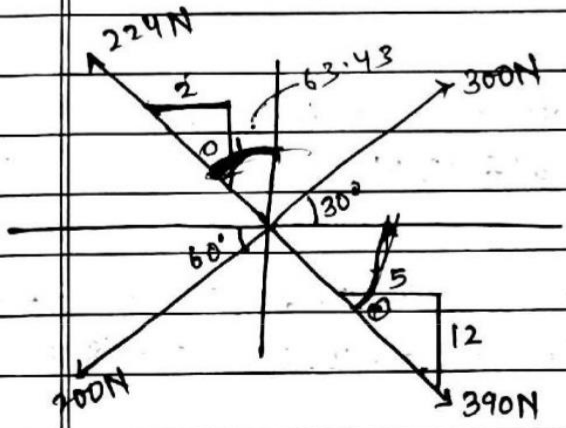
~~$\tan\theta = \frac{5}{12}$~~

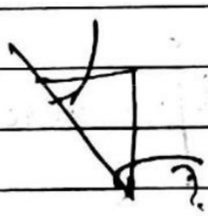
$\theta = 6.5.$

$$\tan\theta = \frac{2}{1}$$

$$\theta = \tan^{-1}(2) = 63.43.$$

$$\tan\theta = \frac{12}{5} = 67.38.$$

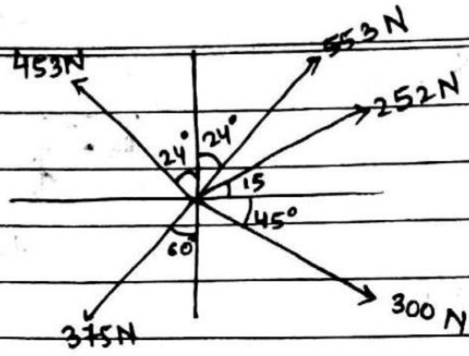


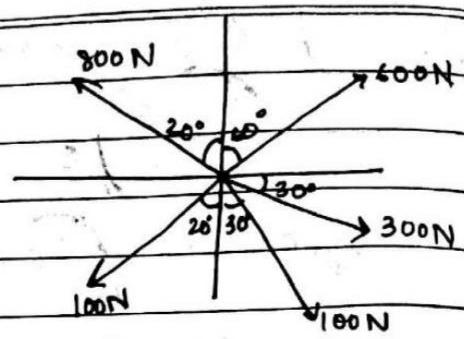


$$\tan\theta = \frac{2}{1}$$

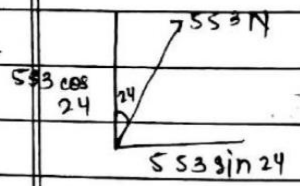
$$\theta = 63.43.$$

Q-



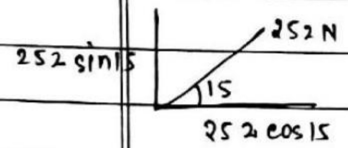


Ans-



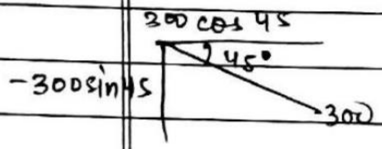
$H = 553 \sin 24 = 224.92\ N$

$V = 553 \cos 24 = 505.19\ N$



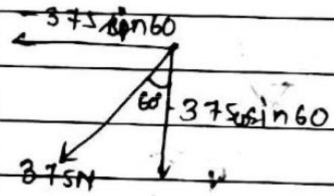
$H = 252 \cos 15 = 243.41\ N$

$V = 252 \sin 15 = 65.22\ N.$



$H = 300 \cos 45 = 212.13\ N$

$V = -212.13\ N.$



$H = -375 \sin 60 = -324.7\ N.$

$V = -375 \cos 60 = -187.5\ N$

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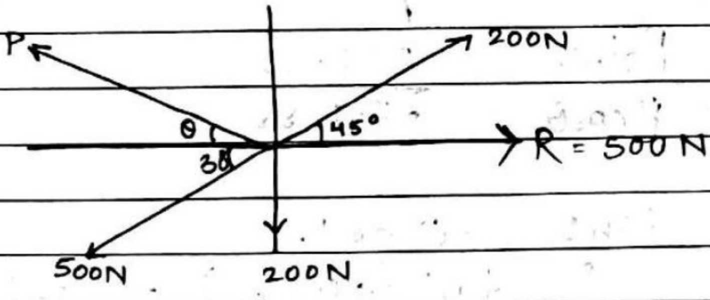
Imp

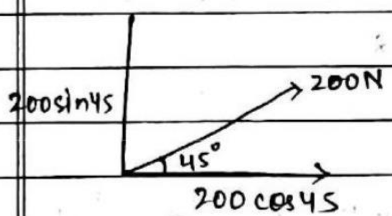
Q.1- 4 - coplanar forces acting @ a point as shown in figure. One of the forces is unknown whose magnitude is P. The resultant has a magnitude of 500 N and is acting along x - axis. Determine the unknown force P.

Ans-



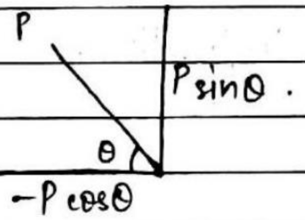
" $\boxed{\Sigma H = R = 500\,N}$ " —— (1)

" $\boxed{\Sigma V = 0\,N}$ " —— (2)



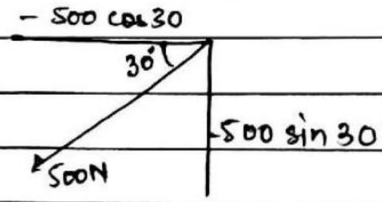
$H = 200\cos 45 = 141.42$ N

$V = 200 \sin 45 = 141.42$ N.



$H = -P\cos\theta$.

$V = P\sin\theta$.



$H = -500\cos 30 = -433.0$

$V = -500 \sin 30 = -250$.

$\Sigma H = 141.42 - P\cos\theta - 433.0$.

$500 = -P\cos\theta - 291.58$. [From eq (1)].

$\Rightarrow +P\cos\theta = -291.58 - 500 = -791.58$

$\Rightarrow P\cos\theta = -791.58$ —— (3)

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$\sum V = 141.42 + P\sin\theta - 250 + 200$

$0 = -308.58 + P\sin\theta.$

$\Rightarrow P\sin\theta = 308.58$ — ④.

Eq ④ ÷ Eq ③.

$\frac{\cancel{P}\sin\theta}{\cancel{P}\cos\theta} = \frac{308.58}{-791.58}$

$\tan\theta = -0.389.$

$\theta = \tan^{-1}(+0.389) = 21.25^\circ$

Squaring and adding eq ③ and ④ :-

$P^2\sin^2\theta + P^2\cos^2\theta = 721820.5128.$

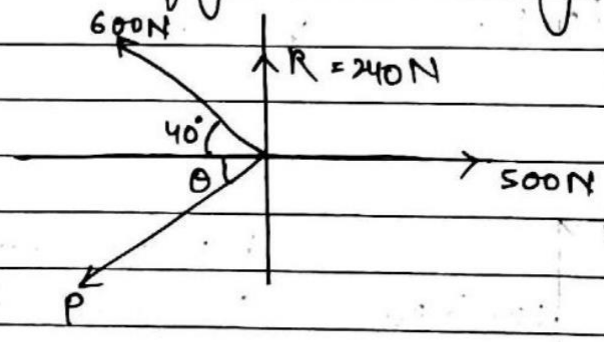
$P^2(1) = 721820.5128$

$\boxed{P = 849.60\text{ N}}$

Imp

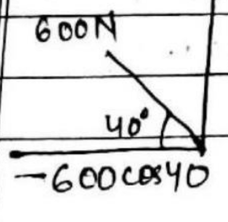
Q.2 A force system as shown in fig. has resultant of 240 N acting upwards along y-axis. Find the value of P and its inclination.

Ans-



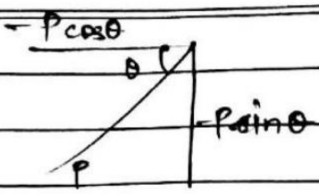
"$\sum V = 240\text{ N} = R$" — ①

$\boxed{\sum H = 0}$ — ②



$H = -600\cos 40 = -459.6\text{ N}$

$V = 600\sin 40 = 385.6\text{ N}$



$H = -P\cos\theta$.

$V = -P\sin\theta$.

$\therefore \Sigma H = 500 - 459.6 - P\cos\theta$

$0 = 40.4 - P\cos\theta$.

$\Rightarrow P\cos\theta = 40.4$ —— (3).

$\Sigma V = 385.6 - P\sin\theta$

$240 = 385.6 - P\sin\theta$.

$P\sin\theta = 385.6 - 240 = 145.6$ —— (4).

$$\frac{P\sin\theta}{P\cos\theta} = \frac{145.6}{40.4}$$

$\tan\theta = 3.603$

$\theta = \tan^{-1}(3.603) = 74.48°$

Squaring and adding eq (8) and (4) :-

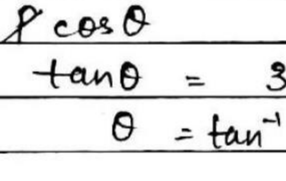
$P^2\cos^2\theta + P^2\sin^2\theta = 22831.52$

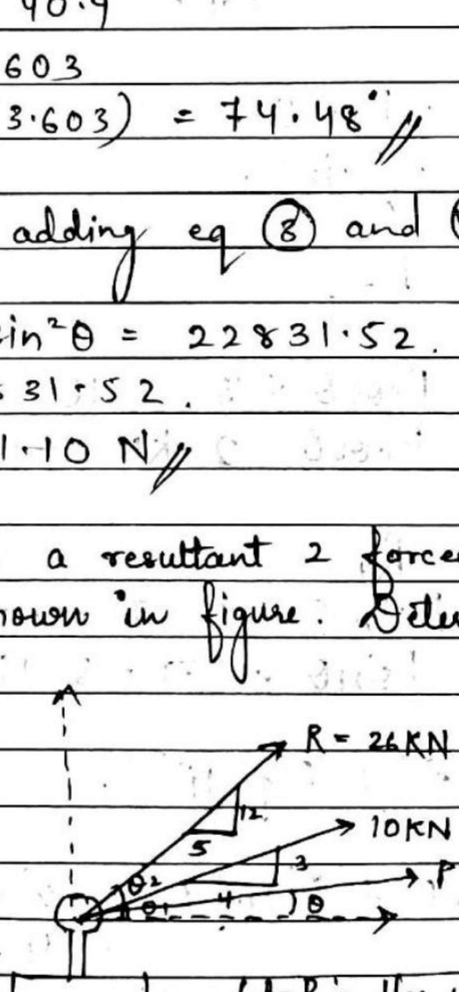
$P^2 = 22831.52$

$P = 151.10\ N$

Q.3 Imp. 26 KN forces is a resultant 2 forcee, one of which is shown in figure. Determine the other force.

Ans.





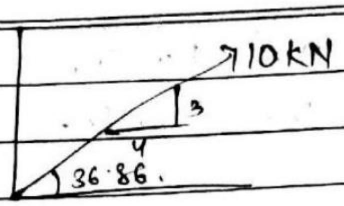
Let P is the unknown forcee

$\theta_2 = \tan^{-1}\left(\frac{12}{5}\right)$

$\theta_2 = 67.38$

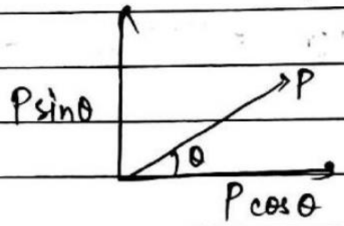
$H = 26\cos 67.38 = 10\ kN = R_x$

$V = 26\sin 67.38 = 23.9\ KN = R_y$



$$\theta_1 = \tan^{-1}\left(\frac{3}{4}\right) = 36.86^\circ$$

$$H = 10\cos 36.86 = 8.00 \text{ kN}$$
$$V = 10\sin 36.86 = 5.99 \text{ KN}.$$



$$H = P\cos\theta.$$
$$V = P\sin\theta.$$

$$\Sigma H = R_x.$$

$$\therefore\ P\cos\theta + 8 = 10$$
$$P\cos\theta = 2 \text{ kN} \quad \text{—— (3)}$$

$$\Sigma V = R_y$$
$$\therefore\ P\sin\theta + 5.99 = 23.9$$
$$P\sin\theta = 23.9 - 5.99 = 17.91 \text{ kN} \quad \text{—— (4)}.$$

$$\text{Eq (3)}^2 + \text{Eq (4)}^2.$$

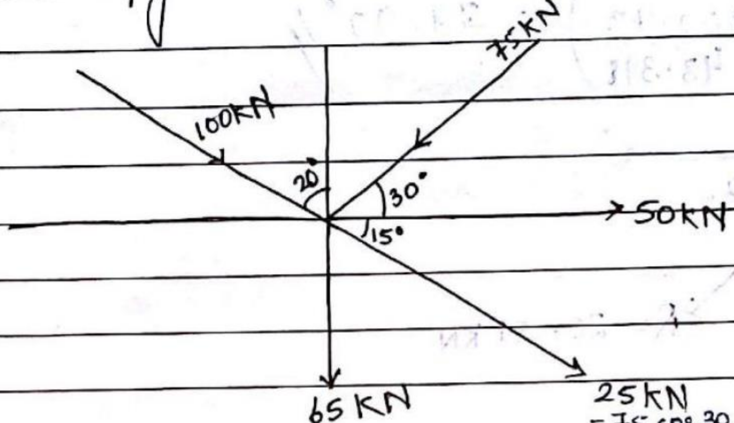
$$\therefore\ P^2\sin^2\theta + P^2\cos^2\theta = 2^2 + 17.91^2$$
$$= 4 + 320.7681.$$
$$P^2 = 324.7681.$$
$$\cancel{P^2} = P = \sqrt{324.7681} = 18.02 \text{ kN}.$$

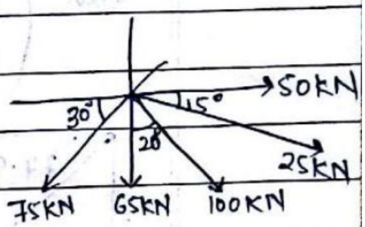
$$\tan\theta = \frac{17.91}{2} = 8.955$$

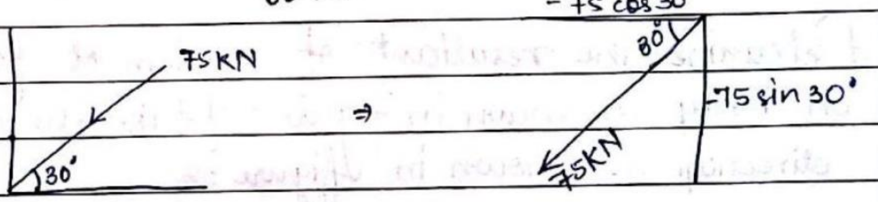
$$\theta = \tan^{-1}(8.955) = 83.62^\circ$$

Q.4 Find the resultant of the given system of forces shown in fig:

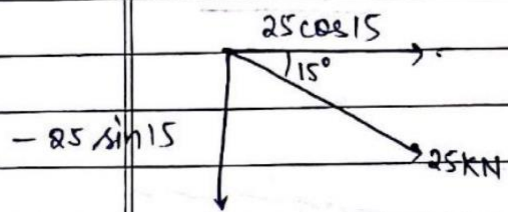
Ans.



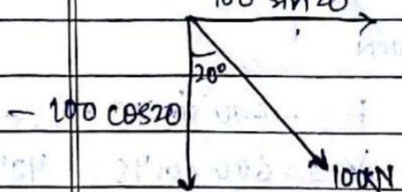




$$H = -75\cos 30 = -64.95\ KN$$
$$V = -75\sin 30 = -37.5\ KN.$$



$$H = 25\cos 15 = 24.148\ KN$$
$$V = -25\sin 15 = -6.47\ KN$$



$$H = 100\sin 20 = 34.20$$
$$V = -100\cos 20 = -93.96.$$

$$\therefore \Sigma H = 50 + 34.20 + 24.148 - 64.95 = 43.398.$$
$$\Sigma V = -65 - 37.5 - 6.47 - 93.96 = -202.93$$

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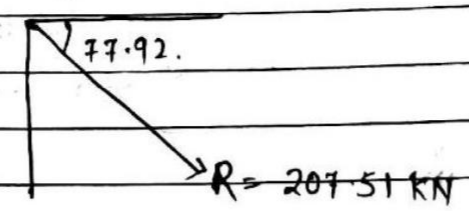
$$R = \sqrt{\Sigma H^2 + \Sigma V^2}$$

$$= \sqrt{(43.398)^2 + (-202.93)^2} = 207.51 \text{ KN}.$$

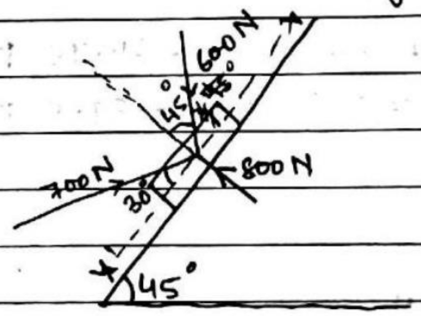
$$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$

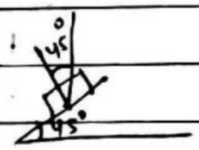
$$= \tan^{-1}\left(\frac{202.93}{43.398}\right) = 77.92°$$

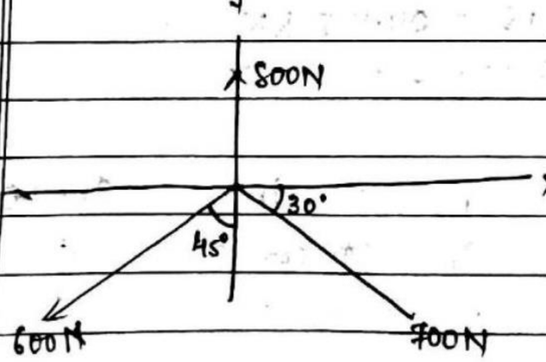


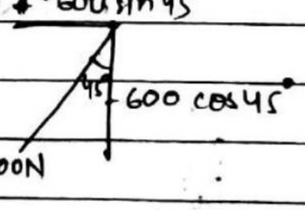
Q.5- Determine the resultant of system of forces acting on body as shown in figure :- take the coordinate direction as shown in figure :-

Ans-









$H = -600 \sin 45 = -424.26$

$V = -600 \cos 45 = -424.26.$

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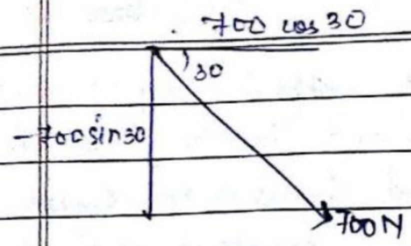


$H = 700 \cos 30 = 606.21$

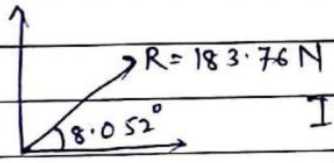
$V = -700 \sin 30 = -350$

$\therefore \Sigma H = 606.21 - 424.26 = 181.95$

$\Sigma V = 800 - 350 - 424.26 = 25.74.$

$R = \sqrt{\Sigma H^2 + \Sigma V^2} = 183.76\ N$

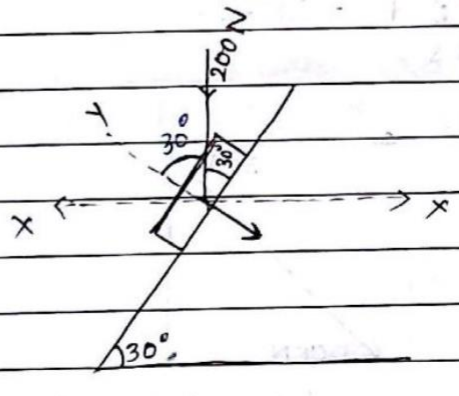
$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = 8.052^\circ$



Ist Quad.

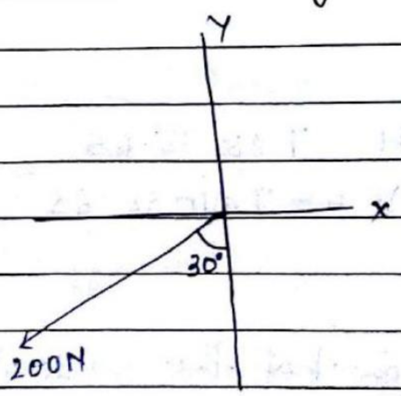
16|3

Q.6-



The force is acting on block. Resolve the force into horizontal and vertical component.

Ans-



$H = -200 \sin 30^\circ = -100\ N$

$V = -200 \cos 30^\circ = -173.20N$

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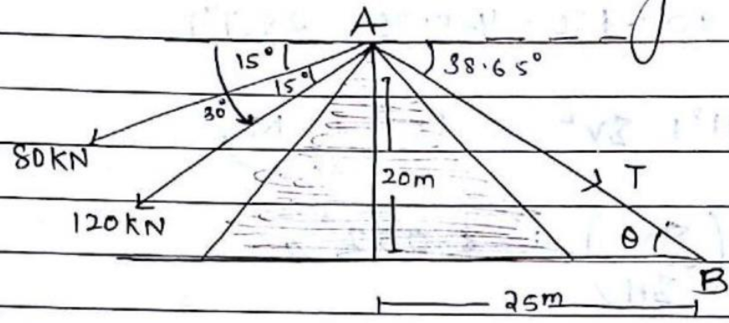
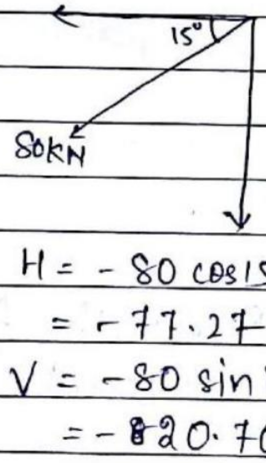
Imp. Q.7- An electrical transmission tower supported by two cables carrying a load of 80 kN and 120 kN as shown in fig. Determine the reqd. tension in cable AB so that all the three cables are extended vertically downwards. Also find the resultant of 3 cables when it (resultant) is acting downwards.

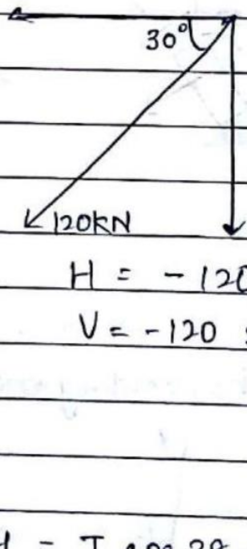
Ans-



$\tan\theta = \frac{20}{25}$

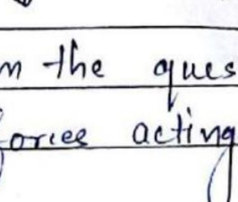
$\theta = \tan^{-1}\left(\frac{20}{25}\right) = 38.65$





$H = -80\cos 15$
$= -77.27$
$V = -80\sin 15$
$= -20.70$

$H = -120\cos 30 = -103.92$
$V = -120\sin 30 = -60$



$H = T\cos 38.65$
$V = -T\sin 38.65$

From the ques, we know that the resultant of all force acting vertically downward so that

$$\boxed{\begin{aligned}\Sigma H &= 0\\ \Sigma V &= R\end{aligned}}$$

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$\Rightarrow \Sigma H = 0$

$\Rightarrow -77.27 - 103.92 + T\cos 38.65 = 0$

$\Rightarrow -181.19 + T(0.78) = 0$

$\Rightarrow T = \dfrac{181.19}{0.78} = 232.29\ \text{kN}$

$\Sigma V = R.$

$\Rightarrow -20.70 - 60 - T(0.624) = R$

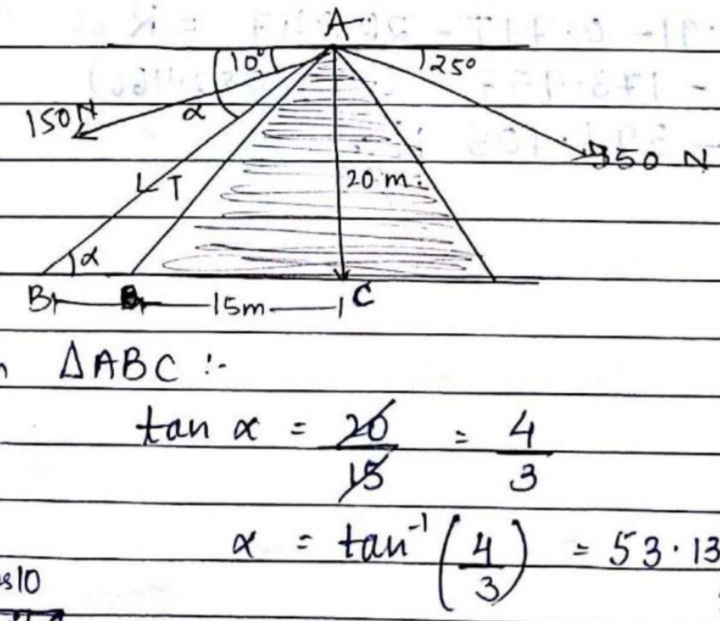
$\Rightarrow -80.7 - 144.94 = R$

$\therefore R = -225.648\ \text{kN}$

(−) indicates the resultant is acting vertically downwards.

Q.8 Two cables attached at the top of the tower carrying an another cable AB. Determine the tension in cable AB such that the resultant of all the forces in 3 cable acts vertically downwards and also determine the resultant of forces.

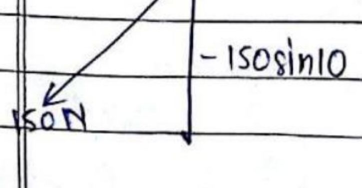
Ans-



In ΔABC :-

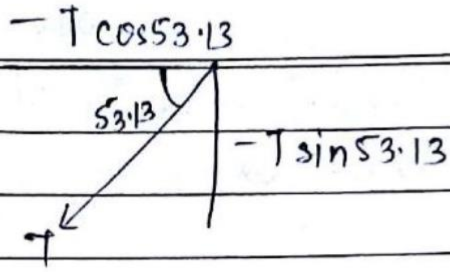
$$\tan\alpha = \frac{20}{15} = \frac{4}{3}$$

$$\alpha = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$$



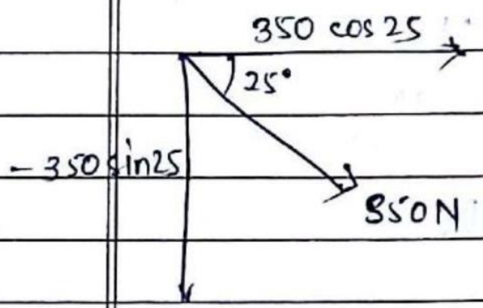
$H = -150\cos 10 = -147.72$

$V = -150\sin 10 = -26.047$



$H = -T\cos 53.13 = -0.600\,T$

$V = -T\sin 53.13 = -0.799\,T.$



$H = 350\cos 25 = 317.20$

$V = -350\sin 25 = -147.91.$

$\Sigma V = R$

$\Sigma H = 0.$

$\therefore \quad \Sigma H = 0$

$\Rightarrow -0.6T + 317.20 - 147.72 = 0$

$\Rightarrow -0.6T = -169.48$

$\Rightarrow T = \dfrac{169.48}{0.6} = 282.466 \,//\, N.$

$\Sigma V = R$

$\Rightarrow -147.91 - 0.79T - 26.047 = R$

$\therefore R = -173.957 - 0.79(282.466)$

$\Rightarrow R = -397.105 \text{ N} \,//$

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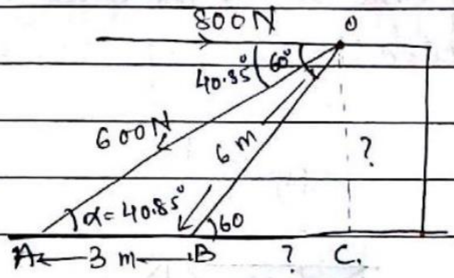
Q.9. Determine the resultant of forces acting at a structure at a point O.

Ans.



$$\cos 60 = \frac{BC}{OB}$$

$$0.5 = \frac{BC}{6}$$

$$BC = 6 \times 0.5 = 3 \text{ m}$$

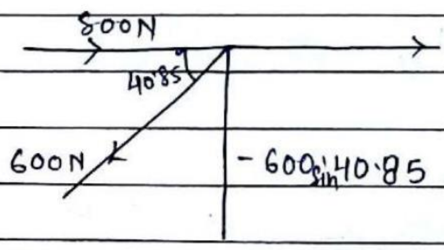
$$\sin 60 = \frac{OC}{OB}.$$

$$0.86 \times 6 = OC$$

$$OC = 5.19 \text{ m}$$

$$\tan \alpha = \frac{OC}{AC} = \frac{5.19}{6}$$

$$\alpha = \tan^{-1} = 40.85°$$



$$H = -600 \cos 40.85 = -453.85.$$

$$V = -600 \sin 40.85 = -392.44.$$

$$R = \sqrt{\Sigma H^2 + \Sigma V^2} = \sqrt{(346.15)^2 + (-392.44)^2}$$

$$= 523.28 \text{ N}$$

$$\tan\theta = \frac{\Sigma V}{\Sigma H} \Rightarrow \theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = 48.58°$$

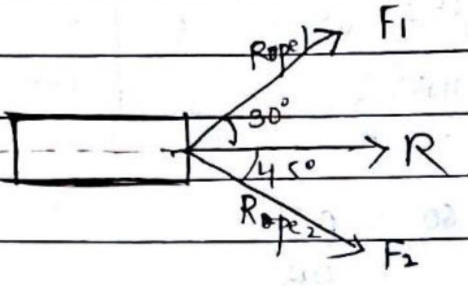
Calcy :- $a_1 = \cos 30$ $\quad a_2 = \sin 30$
$b_1 = \cos 45$ $\quad b_2 = -\sin 45$
$c_1 = 5$ $\quad c_2 = 0$

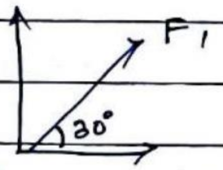
Q.10 A block is pulled by 2 ropes as shown in fig. If resultant of the 2 forces is 5 kN and directed along the axis of the block. Determine the tension in each of the rope.

Ans.

$\Sigma H = R$

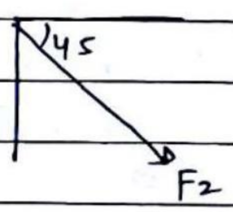
$\Sigma V = 0.$





$H = F_1 \cos 30^\circ = 0.86 F_1$

$V = F_1 \sin 30 = 0.5 F_1.$



$H = F_2 \cos 45 = 0.70 F_2.$

$V = -F_2 \sin 45 = -0.70 F_2.$

$\Sigma H = R$

$\cancel{F_1 \cos 30 + 0}$

$0.86 F_1 + 0.70 F_2 = 5$

$0.5 F_1 - 0.70 F_2 = 0.$

$F_1 = 3.67 \text{ kN}$

$F_2 = 2.62 \text{ KN}$ //

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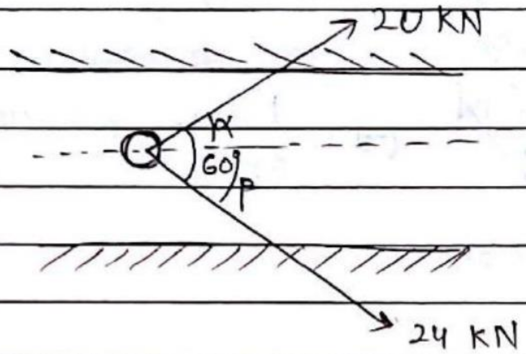
Q.11- Two locomotives moving on opposite bank of a canal and carried by two rope 11 to the banks to pull. a vessel as shown in figure. The force in the ropes are 20 kN and 24 kN by the total angle b/w them is 60°. Find the resultant pull on the vessel and the angles α and β.

Ans-



$F_1 = 20$ kN

$F_2 = 24$ kN

$\theta = 60°$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{20^2 + 24^2 + 2\cdot 20\cdot 24\cos 60}$$

$R = 38.15$ kN

$$\alpha = \tan^{-1}\left(\frac{F_2\sin\theta}{F_1 + F_2\cos\theta}\right)$$

$$= \tan^{-1}\left(\frac{24\sin 60}{20 + 24\cos 60}\right)$$

$$= \tan^{-1}\left(\frac{20.78}{32}\right)$$

$$\alpha = \tan^{-1}\left(\frac{20.78}{32}\right) = 32.99°$$

$\alpha + \beta = 60°$

$\Rightarrow \beta = 60° - \alpha$

$= 27.01°$

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Q.12 - The resultant of 2 forces, one of which is 3 time the other force is 300 N. When the direction of small force is reversed, the resultant is 200 N. Determine the 2 forces and the angle b/w them.

Ans- ~~$F_1 = 3F_2$.~~

~~$F_1 = 300N$~~

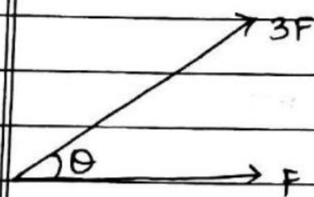
~~$\therefore 3F_2 = 300$~~

~~$\Rightarrow F_2 = 100$ N.~~

~~$R = 200N$~~

$\left.\begin{matrix} F \\ 3F \end{matrix}\right\} R = 300$ N

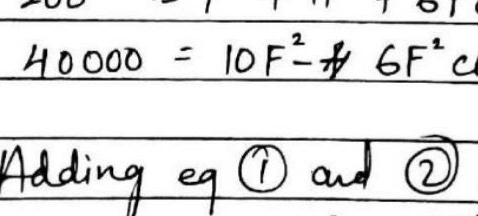
$(-)\left.\begin{matrix} F \\ 3F \end{matrix}\right\} R = 200N$



$R = \sqrt{F^2 + (3F)^2 + 2(F)(3F)\cos\theta}$.

$(300)^2 = F^2 + 9F^2 + \cancel{3F}\ 6F^2\cos\theta$.

$\Rightarrow 90000 = 10F^2 + 6F^2\cos\theta$ —— (1)



$200^2 = F^2 + 9F^2 + 6F^2\cos(180-\theta)$

$\Rightarrow 40000 = 10F^2 - 6F^2\cos\theta$ —— (2)

$\therefore$ Adding eq (1) and (2) :-

$$\begin{aligned} 10F^2 + \cancel{6F^2\cos\theta} &= 90000 \\ 10F^2 - \cancel{6F^2\cos\theta} &= 40000 \\ \hline 20F^2 &= 130000 \end{aligned}$$

$F^2 = \frac{13000\cancel{0}}{2\cancel{0}} = 6500$

$F = 80.6$ N//

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$\therefore\ 3F = 3 \times 80.6$

$= 241.86\ N$

① $\Rightarrow 90000 = 65000 + 39000 \cos\theta$.

$\Rightarrow 39000 \cos\theta = 25000$

$\Rightarrow \theta = \cos^{-1}\left(\frac{25000}{39000}\right) = 50.13°$

18/3/17

EQUILIBRIUM OF FORCES.

- Define Equilibrium. What are the conditions for equilibrium

→ Any system of forces acting on a body is said to be in equilibrium when the resultant of the force is 0 and the algebraic sum of the forces should be equal to zero.

→ A body is said to be in state of equilibrium, the body should be in rest under action of forces.

Conditions for equilibrium :-

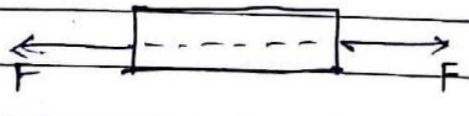
→ If the body should be in equilibrium, it should satisfy the following conditions :-

$$\Sigma H = 0$$
$$\Sigma V = 0$$
$$R = 0$$
$$\Sigma M = 0$$

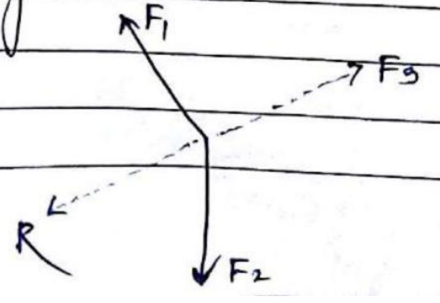
- PRINCIPLE OF EQUILIBRIUM FOR DIFF. FORCE SYSTEM :-

-> 2 Force System

If the body is acted upon 2 forces, then for equilibrium, the forces should be in equal in magnitude and opposite in direction.



→ 3 Force System

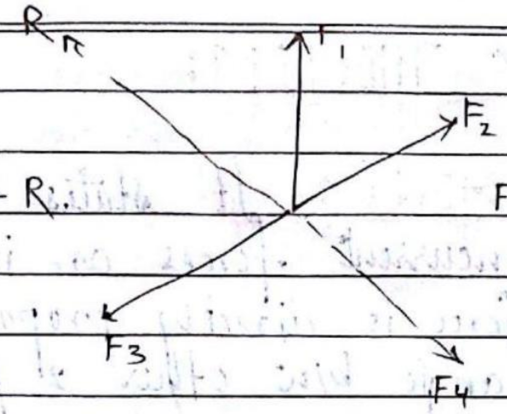


F_1 & F_2 = R

$R = F_3$

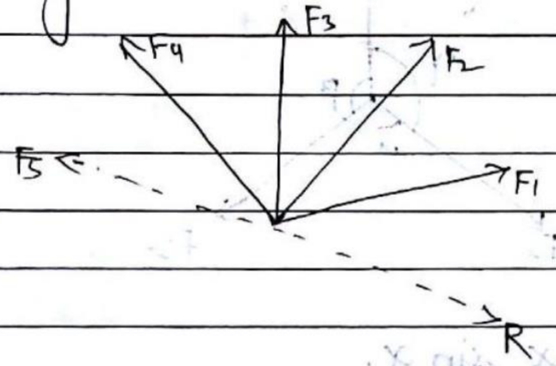
m. → 4 force system

F_1 & F_2 & F_3 = R.
R = F_4. //



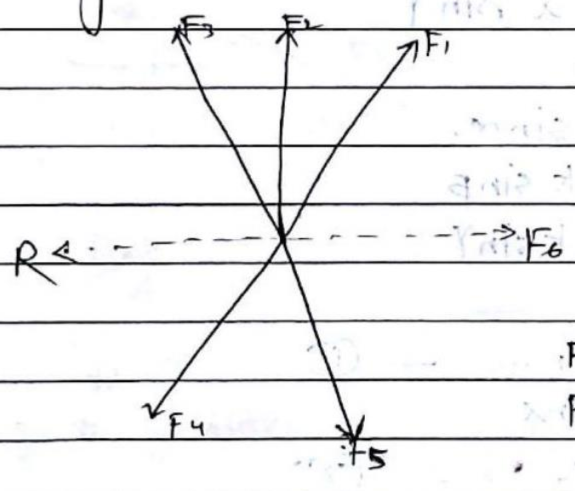
F_1 & F_2 = R_1
F_3 & F_4 = R_2
R_1 = R_2.

→ 5 force system



F_1 & F_2 & F_3 & F_4 = R.
F_5 = R //

- 6 force system:-



F_1 & F_2 & F_3 = R_1
F_4 & F_5 & F_6 = R_2
R_1 = R_2.

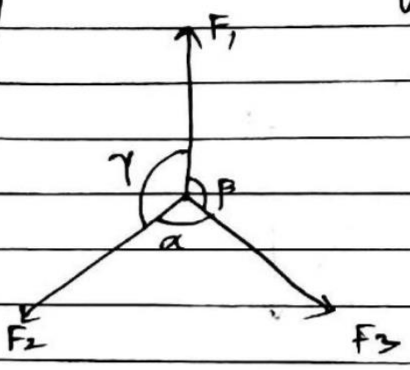
F_1 & F_2 & F_3 & F_4 & F_5 = R
R = F_6 //

Imp

LAMI'S THEOREM:

STATEMENT :- It states that if 3 coplanar concurrent forces are in equilibrium then each force is directly proportional to sine of the angle b/w other 2 forces.

ie



$F_1 \propto \sin\alpha.$

$F_2 \propto \sin\beta$

$F_3 \propto \sin\gamma$

$\Rightarrow F_1 = k\sin\alpha.$

$\Rightarrow F_2 = k\sin\beta$

$\Rightarrow F_3 = k\sin\gamma$

$\Rightarrow k = \dfrac{F_1}{\sin\alpha}$ — (1)

$\Rightarrow k = \dfrac{F_2}{\sin\beta}$ — (2)

$\Rightarrow k = \dfrac{F_3}{\sin\gamma}$ — (3)

From (1), (2) and (3)

$$\boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma} = k}$$

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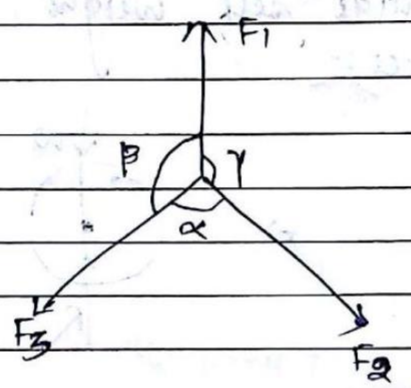
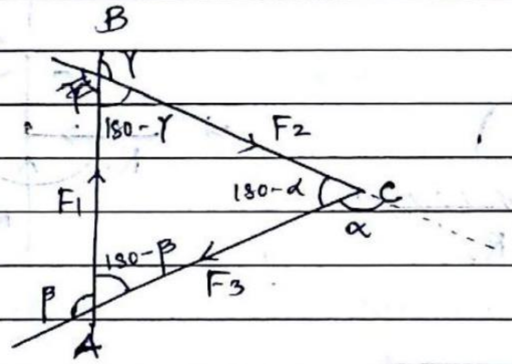
Consider the 3 forces are in equilibrium. Then by converse law of Δ, the forces can be represented by sines of the Δ by both magnitude and direction.

By the Δ law, ΔABC can be drawn with the sides $\parallel$ to the forces F_1, F_2 and F_3





By applying the sine rule to the ΔABC, we have

$$F_1 \propto \sin(180 - \alpha)$$
$$F_2 \propto \sin(180 - \beta)$$
$$F_3 \propto \sin(180 - \gamma)$$

$$\left.\begin{aligned} F_1 &= k\sin(180-\alpha) = k\sin\alpha. \\ F_2 &= k\sin(180-\beta) = k\sin\beta. \\ F_3 &= k\sin(180-\gamma) = k\sin\gamma \end{aligned}\right\}$$ We know that $\sin(180-\theta) = \sin\theta$

$$\therefore \boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma} = k}$$

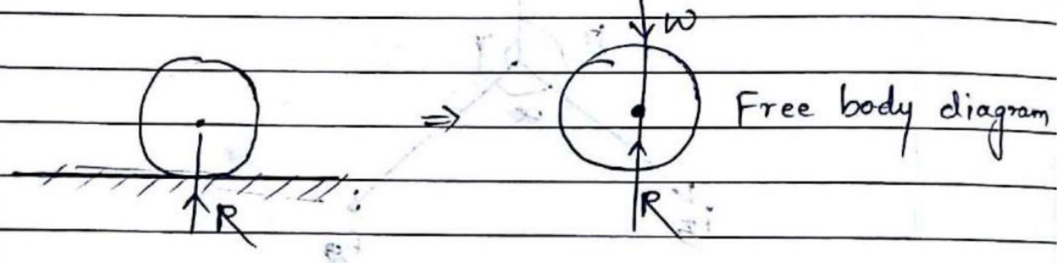
Hence Proved

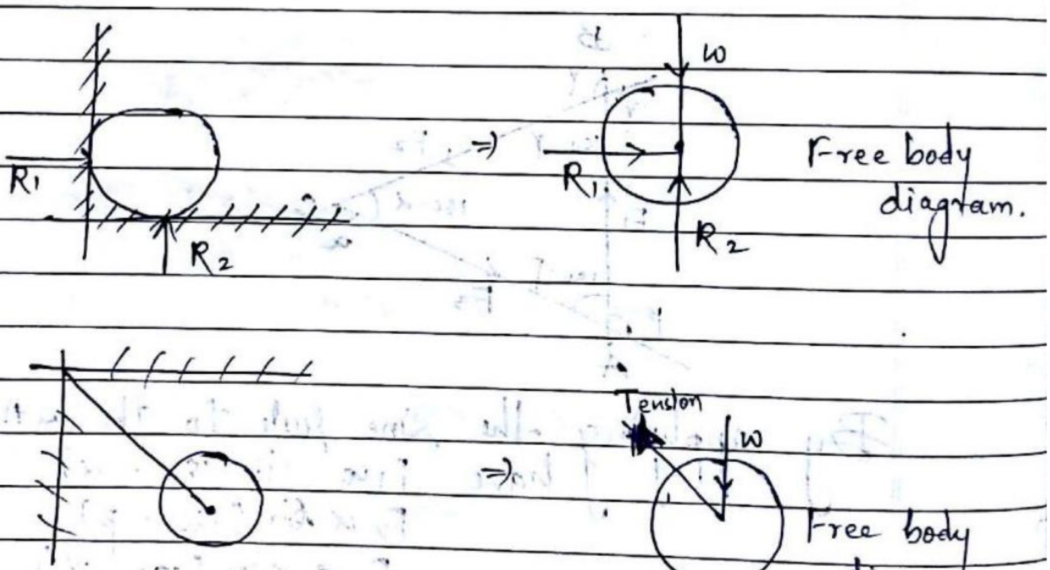
- **Define free body diagram with an example :-**

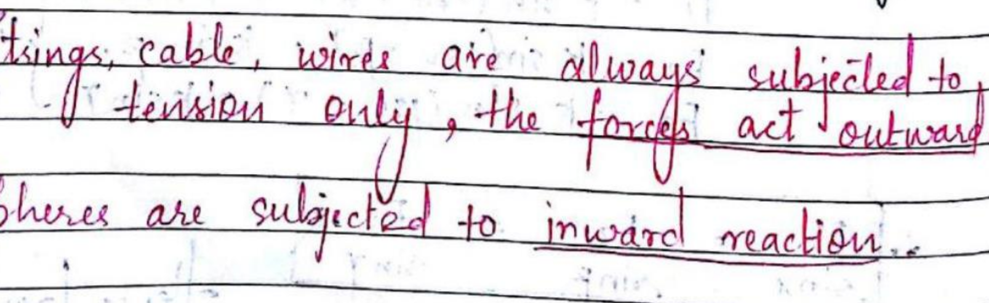
A free body diagram is nothing but a sketch shows the various forces acting on a body keeping in equilibrium by removing the contact surfaces.

The forces include self weight, reaction and other forces :-

Eg :-







NOTE: Strings, cable, wires are always subjected to tension only, the forces act outward.

Spheres are subjected to inward reaction.

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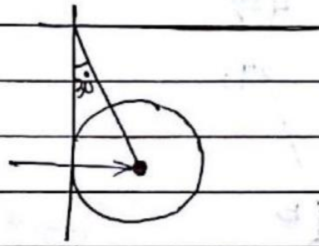
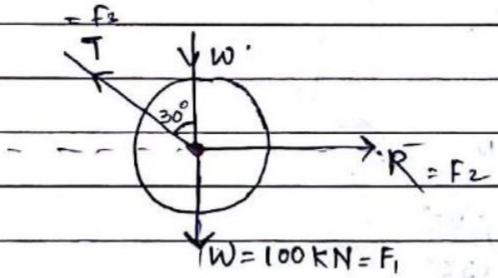
Q.1- A sphere is touching to the wall of weight 100 kN. Find the tension in the wire which is supporting the sphere.

Ans-





Using Lami's Theorem :- $\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$

$\Rightarrow \frac{100}{\sin 120} = \frac{R}{\sin 150} = \frac{T}{\sin 90°}$

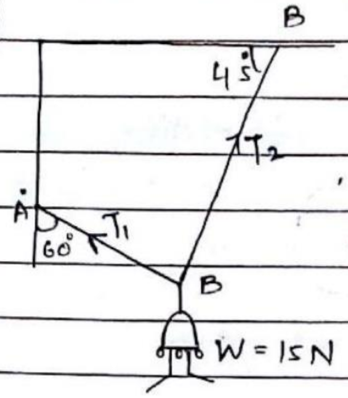
$\Rightarrow \frac{T}{1} = \frac{100}{\sin 120} = 115.47$ KN //

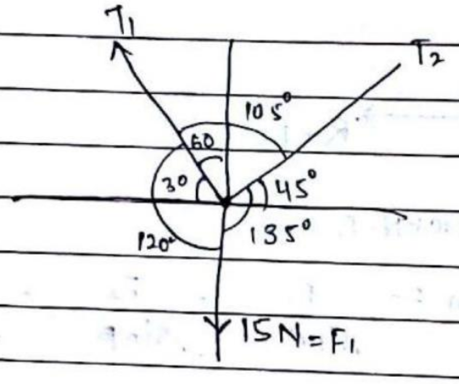
$R = \frac{100 \sin 150}{\sin 120} = \cancel{\frac{50}{57.73}} \cancel{= 0.866 \text{ KN}}$

$= 57.73$ KN //

Q.2- The figure shows, the lamp is fixed with strings. Determine the tension in strings.

Ans -





∴ Using Lami's Rule :-

$$\frac{F_1}{\sin 105} = \frac{T_2}{\sin 120} = \frac{T_1}{\sin 135}$$

$$\Rightarrow \frac{15}{0.96} = \frac{T_1}{\sin 135}$$

$$\Rightarrow T_1 = \frac{15 \sin 135}{\sin 105} \simeq 10.98 \text{ N}$$

$$\Rightarrow T_2 = \frac{F_1 \sin 120}{\sin 105} = \frac{15 \sin 120}{\sin 105} = 13.44 \text{ N}$$

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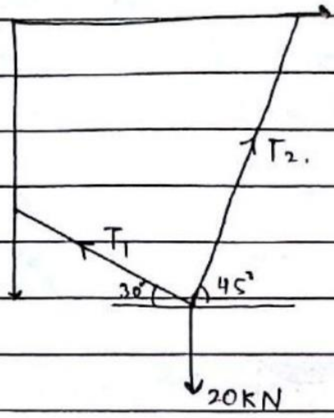
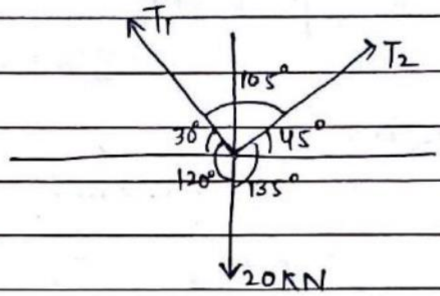
Q.3



Ans-



Using Lami's Rule :-

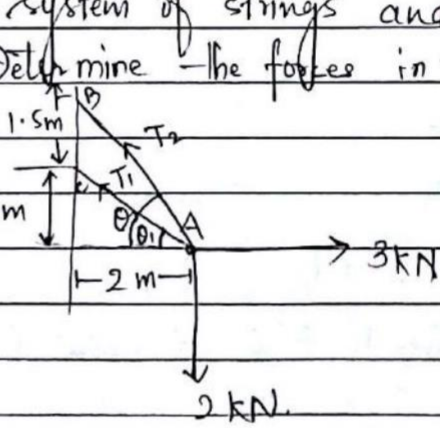
$$\Rightarrow \frac{20}{\sin 105} = \frac{T_1}{\sin 135^\circ} = \frac{T_2}{\sin 120}$$

$\Rightarrow T_1 = 14.64\ kN.$

$\Rightarrow T_2 = 17.93\ kN$ //

Q.4- The system of strings and forces are in equilibrium. Determine the forces in the string.

Ans-



$$\theta_1 = \tan^{-1}\left(\frac{1}{2}\right) = 26.56$$

$$\theta_2 = \tan^{-1}\left(\frac{2.5}{2}\right) = 51.34$$

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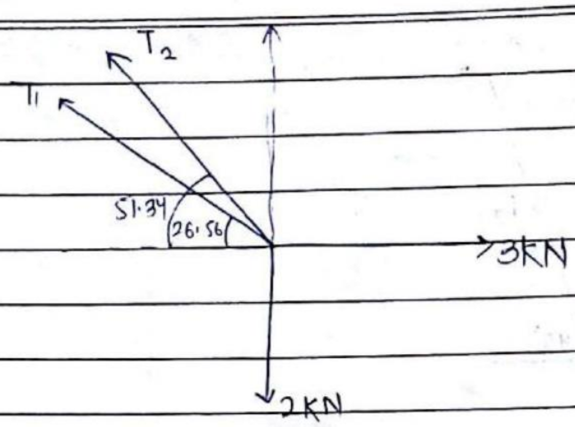
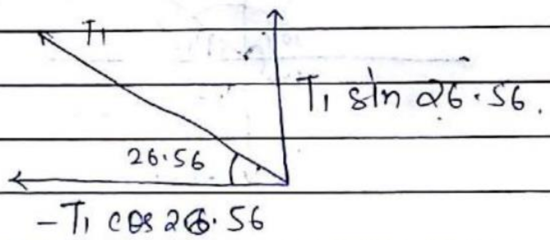
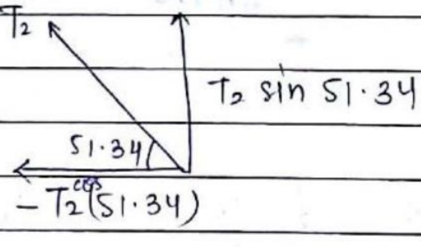


As this is in equilibrium $\Sigma H = 0$

$\Sigma V = 0$



$$H = -T_1(0.86)$$
$$V = T_1(0.44).$$



$$H = -T_2(0.62)$$
$$V = T_2(0.78)$$

$$\Sigma H = -0.86T_1 - T_2(0.62) + 3$$

$$0 / \pm \cancel{8 \cdot 6}\ 0.86T_1 + 0.62T_2 + 2 = 0 \quad — (1)$$

$$0.44T_1 + T_2(0.78) = 0 \quad — (2)$$

$\Sigma H = 0.86T_1 + 0.62T_2 + 3.$

$0.86T_1 + 0.62T_2 = +3$ —— (1)

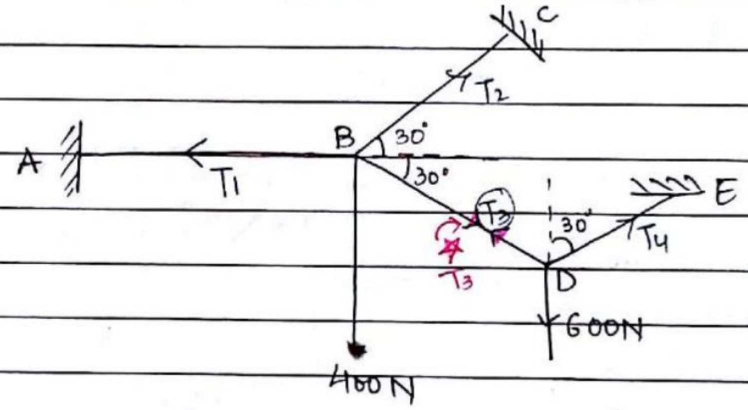
$0.44T_1 + 0.78T_2 = 2$ —— (2)

$T_1 = 2.76\text{ N}$

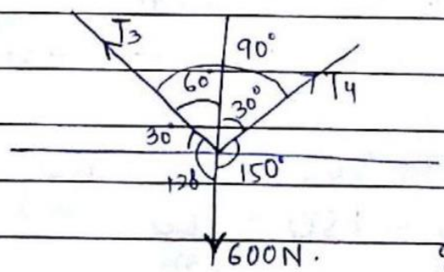
$T_2 = 1.00\text{ N}$

Q.5 (Imp) Determine the tension in diff parts of the strings as shown in figure :-

Ans-



FBD at D :-



By Lami's Theorem :-

$$\frac{T_3}{\sin 150^\circ} = \frac{T_4}{\sin 120^\circ} = \frac{600}{\sin 90}$$

$\Rightarrow$ $T_3 = \frac{600 \times \sin 150}{1} = 300\text{ N}.$

$T_4 = 600 \times \sin 120 = 519.61\text{ N}$

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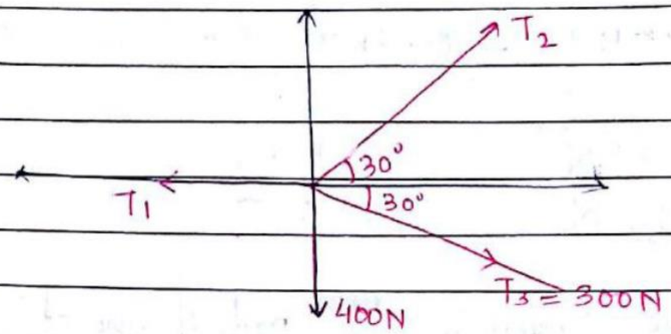
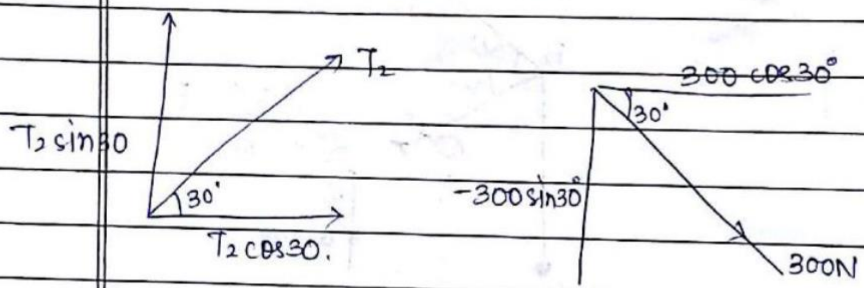
FBD at B :-



$\Sigma H = 0$

$\Sigma V = 0.$



$\Sigma H = -T_1 + T_2 \cos 30° + 300 \cos 30°$

$\Rightarrow 0 = -T_1 + T_2 \cos 30° + 259.8$

$\Rightarrow T_1 - T_2 \cos 30 = 259.8$ ——— (1)

$\Sigma V = T_2 \sin 30 - 300 \sin 30 - 400$

$\Rightarrow 0 = T_2 \sin 30 - 150 - 400$

$\Rightarrow T_2 \sin 30 = 550$ ——— (2)

$T_2 = \cancel{519.6} \ \frac{550}{0.5} = 1100$ //

$T_1 - 1100 \cos 30 = 259.8$

$T_1 = 259.8 + 1212.42 = 1472.2N$ //

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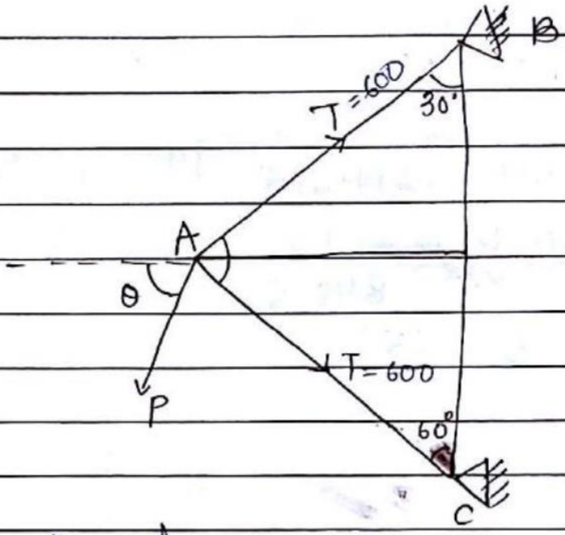
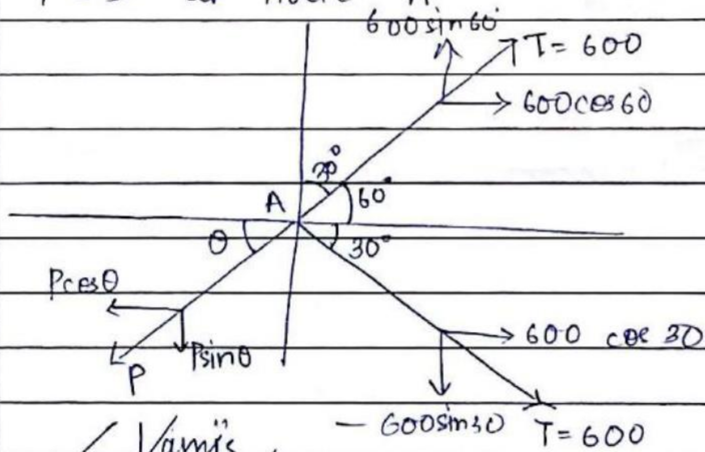
Q.- A force P is applied on a
The tension in strings is equal to 600 N. Determine the magnitude and direction of P

Ans -



FBD at node A



By ~~Snell's~~ Lami's Law :-

$$\Sigma H = 0$$

$$\Sigma V = 0.$$

$\Rightarrow \Sigma H = 0$

$\Rightarrow 600 \cos 60 + 600 \cos 30 - P\cos\theta = 0.$

$\Rightarrow 300 + 519.61 - P\cos\theta = 0.$

$\Rightarrow 819.615 - P\cos\theta = 0$

$\Rightarrow P\cos\theta = 819.615$

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$\Sigma V = 0.$

$600 \sin 60 - 600 \sin 30 - P \sin\theta = 0$

$\Rightarrow P\sin\theta = 519.615 - 300. = 219.615.$

$\Rightarrow P^2(1) = 71999.49$

$\Rightarrow P = 848.52\ N$ //

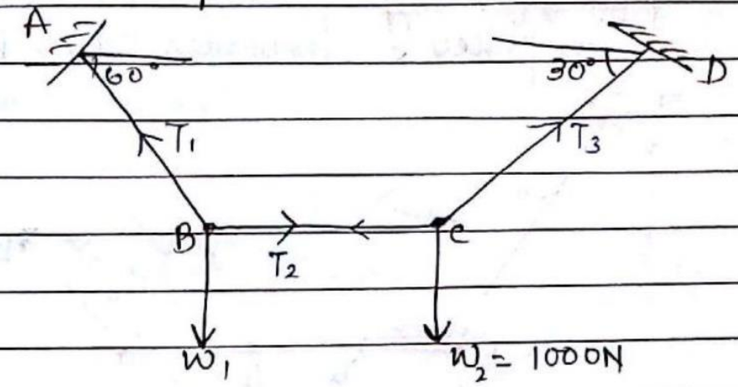
$\cancel{\theta}$ $P\sin\theta = 219.615$

$\sin\theta = \dfrac{219.615}{848.52.} = 0.25$

$\theta = 15^\circ$ //

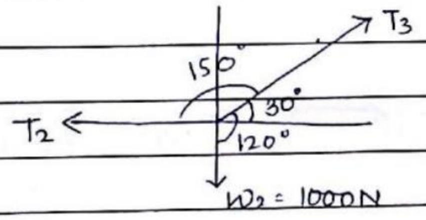
Q- Find the forces in all the bias and the load w_1 to keep the system in equilibrium as shown :-

Ans.



Two nodes :-

FBD at node C :-

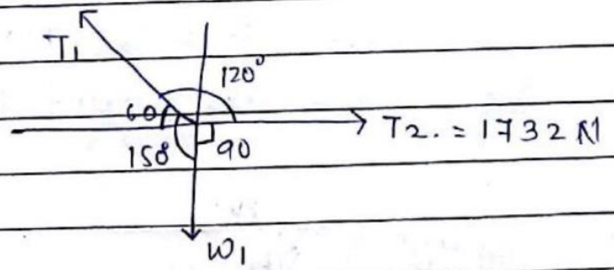


$$\frac{1000}{\sin 150} = \frac{T_2}{\sin 120} = \frac{T_3}{\sin 90°}$$

$\Rightarrow$ $T_2 = \frac{1000 \sin 120}{\sin 150} = 1732.0 \text{ N}$

$\Rightarrow$ $T_3 = \frac{1000 \sin 90}{\sin 150} = 2000 \text{ N}.$

FBD at node B :-



$T_1 = 3464N$
$T_2 = 1732N$
$T_3 = 2000N$
$w_1 = 2999.91N$

By lami's theorem

$$\frac{1732}{\sin 150°} = \frac{T_1}{\sin 90} = \frac{W_1}{\sin 120°}$$

$\Rightarrow$ $w_1 = 2999.91 \text{ N}.$

$\Rightarrow$ $T_1 = 3464 \text{ N}$

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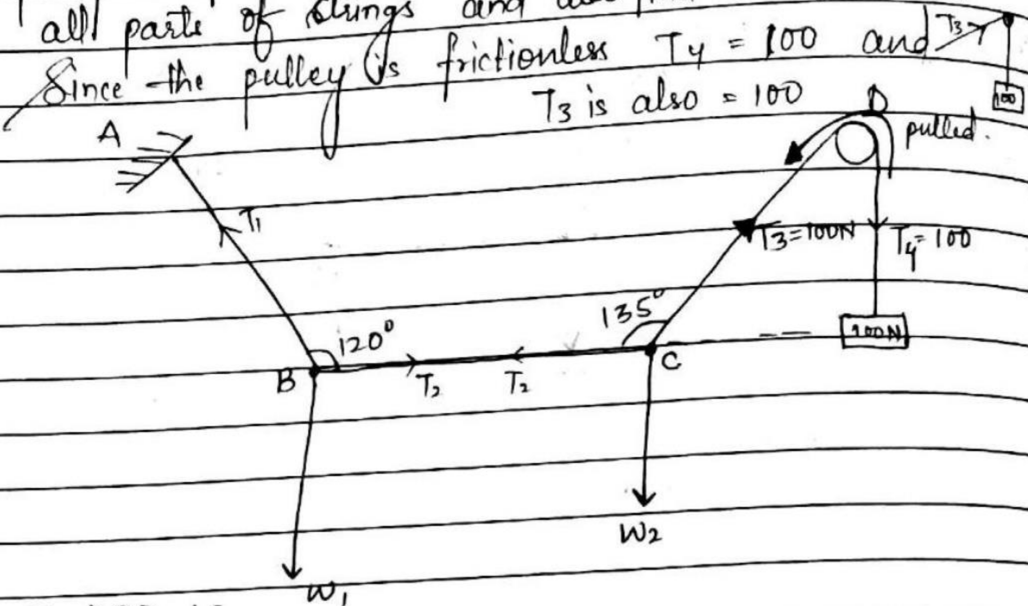
Imp

Q.- In the figure the BC position is horizontal and the pully is frictionless. Determine the tension in all parts of strings and also find w_1 and w_2.

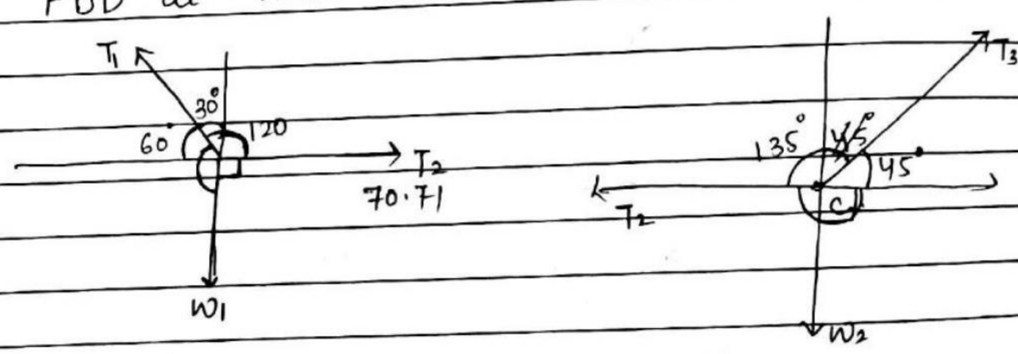
Ans.- Since the pulley is frictionless $T_4 = 100$ and T_3 is also $= 100$



2 FBD :-

FBD at node B :-

FBD at node C :-



$$\frac{100}{\sin 90} = \frac{T_2}{\sin 135} = \frac{W_2}{\sin 135}$$

$T_2 = 70.71 N$

$W_2 = 70.71 N$ //.

$$\frac{70.71}{\sin 150^\circ} = \frac{T_1}{\sin 90^\circ} = \frac{W_1}{\sin 120}$$

$\Rightarrow T_1 = 141.42. N$

$\Rightarrow W_1 = 122.47 N.$

$W_2 = 70.71 N$

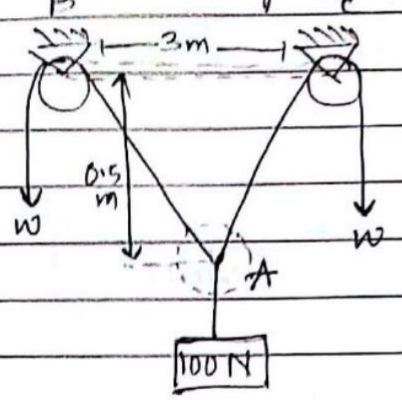
$W_1 = 70.71 N$

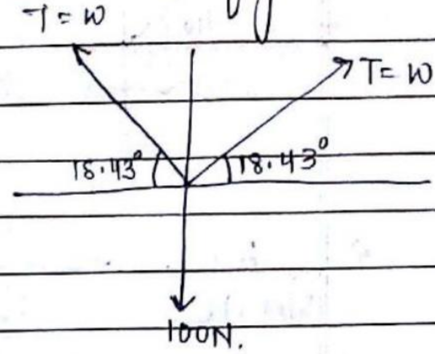
$T_1 = 141.42 N$

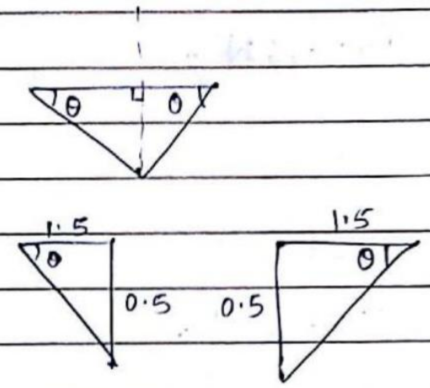
$W_1 = 122.47 N$

Q- Find the value of w which is reqd. to maintain equilibrium configuration as shown in fig :-

Ans-





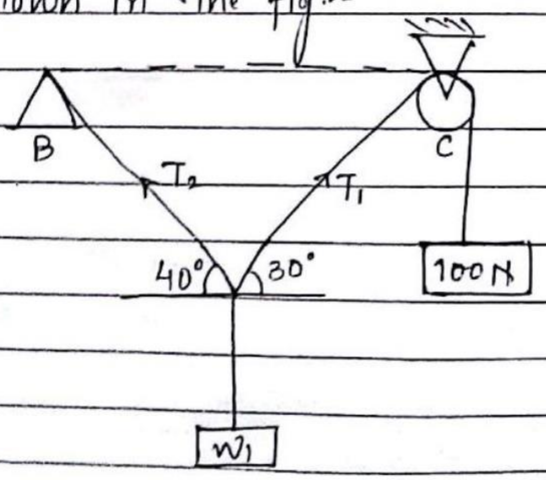


$\theta = \tan^{-1}\left(\frac{0.5}{1.5}\right) = 18.43°$ //

By Lami's theorem :- $\frac{100}{\sin(143.4)} = \frac{w}{\sin(108.43)} = \frac{w}{\sin(108.43)}$

$W = 158.15 N$ //

Q- Find the value of w_1 for the equilibrium condition as shown in the fig:-



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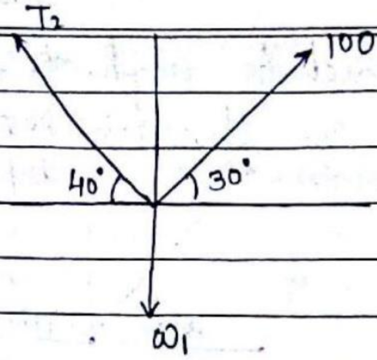
Ans-



$\Rightarrow \dfrac{w_1}{\sin 110} = \dfrac{100}{\sin 130} = \dfrac{T_2}{\sin 120}$

$\Rightarrow w_1 = \dfrac{100 \sin 110}{\sin 130}$

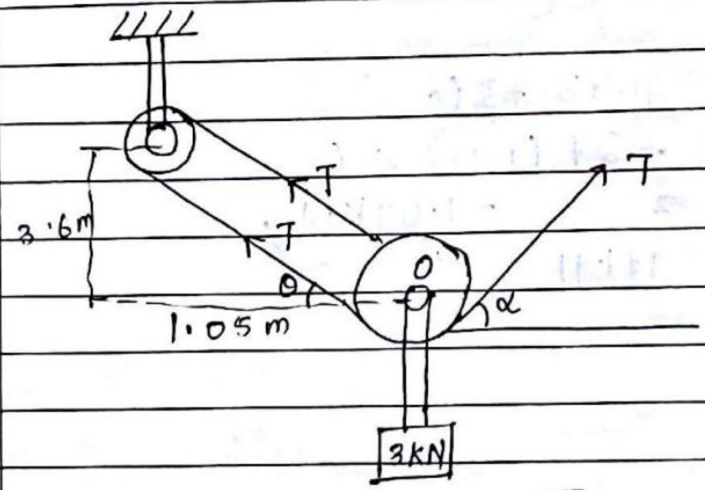
$\Rightarrow w_1 = \dfrac{93.96}{0.766} = 122.66\,N$

$\dfrac{100}{\sin 130} = \dfrac{T_2}{\sin 120}$

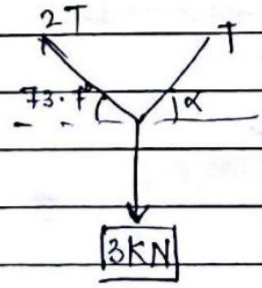
$\Rightarrow T_2 = \dfrac{100 \sin 120}{\sin 130} = 113.05\,N$

Imp

Q.9- A 3KN crate is to be supported by the rope and pulley arrangement shown in figure. Determine the magnitude and direction of the force T, which should be exerted at the free end of the rope.



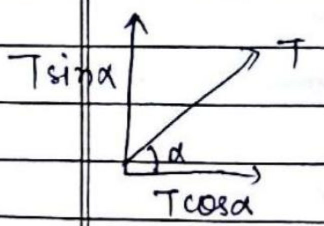
FBD at node O :-



$$\tan\theta = \frac{3.6}{1.05}$$

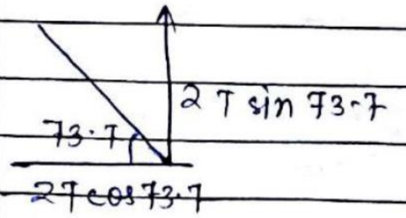
$$\theta = 73.7°$$

$$\Sigma H = 0$$
$$\Sigma V = 0$$



$H = T\cos\alpha$

$V = T\sin\alpha.$



$H = -2T\cos 73.7$

$V = 2T\sin 73.7$

$\Sigma H = 0.$

$\Rightarrow$ $T\cos\alpha - 2T\cos 73.7 = 0$

$\Rightarrow$ $\cancel{T}\cos\alpha = 2\cancel{T}\cos 73.7$

$\Rightarrow$ $\alpha = \cos^{-1}(2\cos 73.7)$

$\Rightarrow$ $\alpha = \cos^{-1}(0.56) = 55.9°$ //

$\Rightarrow$ $\Sigma V = 0$

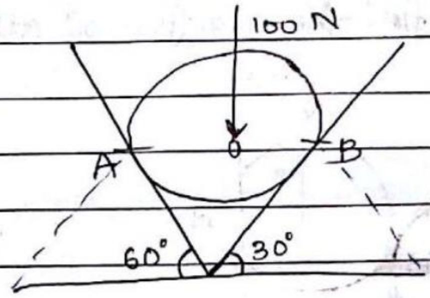
$T\sin\alpha + 27\sin 73.7 - 3 = 0$

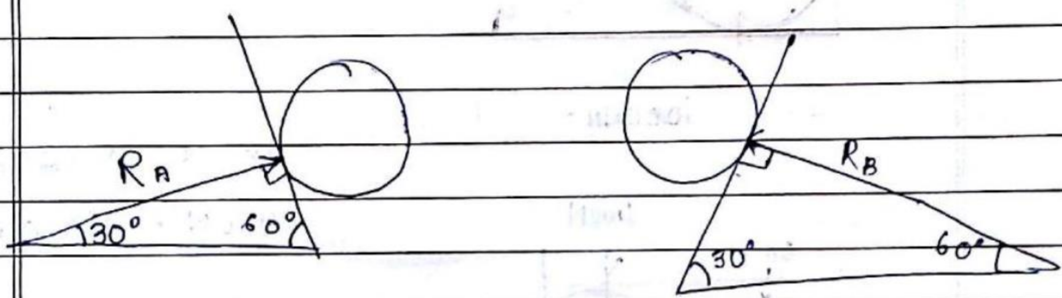
$\Rightarrow$ $T\sin 55.9 + 27(1.91) - 3 = 0$

$\Rightarrow$ $T = \dfrac{3}{\sin 55.9 + 1.91} = 1.09\,KN$ //

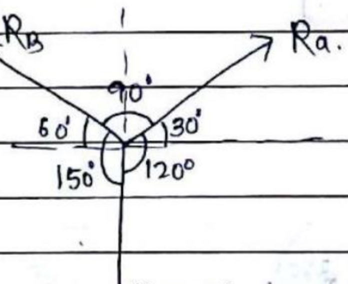
Q- A sphere of 100 N is fitted in a right angle ~~launch~~ notch as shown in fig. Determine the reaction at A and B.

Ans-





FB at node O :-



By Lami's Theorem :-

$$\frac{100}{\sin(90)} = \frac{Ra}{\sin 150} = \frac{R_B}{\sin 120}$$

$\Rightarrow$ $Ra = \dfrac{100 \sin 150}{\sin 90} = 50\ N$ //

$\Rightarrow$ $R_B = \dfrac{100 \sin 120}{\sin 90} = 86.60\ N$ //

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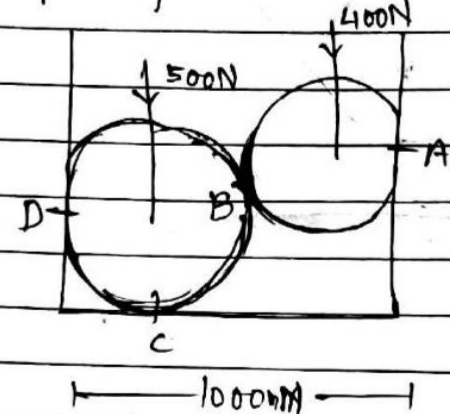
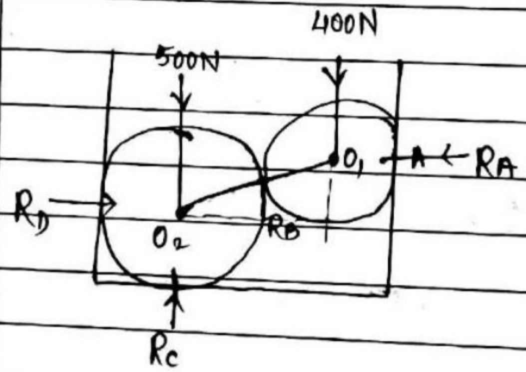
Q. Imp. A horizontal channel of ~~interference~~ inner clearance of 1000 mm carries 2 spheres of radius 350 mm & 250 mm whose weight are 500 N and 400 N respectively. Find the reaction at all the contact surfaces.

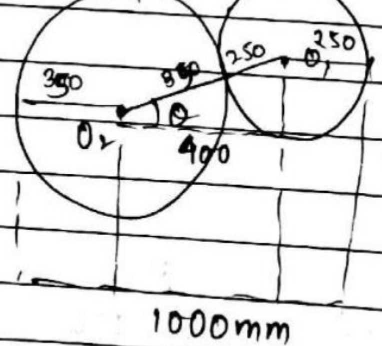
Ans-

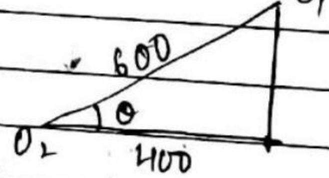


500 N → 350 mm

400 N → 250 mm.





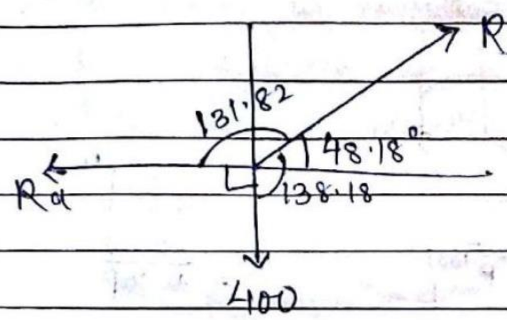


$$\cos\theta = \frac{400}{600}$$

$$\theta = \cos^{-1}\left(\frac{400}{600}\right)$$

$$= 48.18^\circ$$

FBD @ O_1 (min forces)



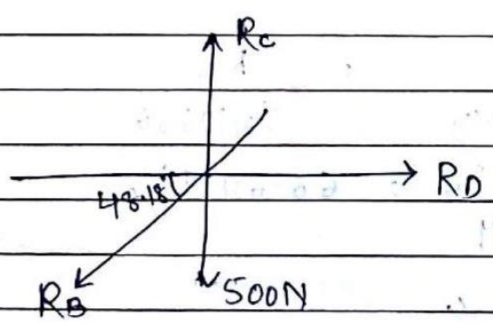
H = By Applying Lami's Theorem.

$$\frac{R_B}{\sin 90} = \frac{R_A}{\sin 138.18} = \frac{400}{\sin 131.82}$$

$\Rightarrow R_A = \dfrac{400 \sin 138.18}{\sin 131.82} = 357\,N$

$\Rightarrow R_B = \dfrac{400 \sin 90}{\sin 131.82} = 536.73\,N$

FBD at O_2



= 53673.

$\Sigma H = 0$

$\Sigma V = 0.$

$\Sigma H = 0$

$\Rightarrow R_D - \cancel{48}.\ 536.73 \cos 48.18 = 0$

$\Rightarrow R_D = 357.88\,N.$

$\Sigma V = 0$

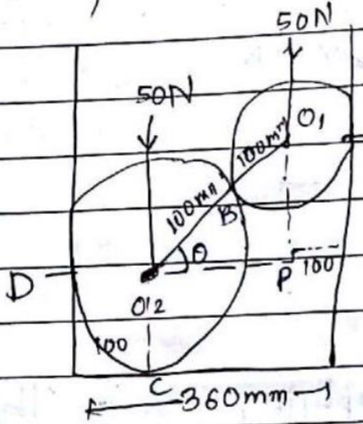
$\Rightarrow -500 + R_c - 536.73 \sin 48.18 = 0$

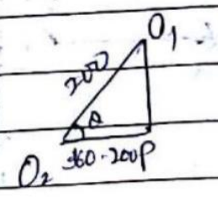
$\Rightarrow R_c = 500 + 536.73 \sin 48.18$

$\Rightarrow R_c = 899.99\,N$

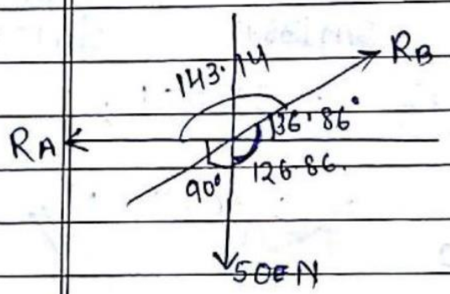
8/4/17

Q- Find the reactions at A, B, C as shown in figure.





$$\theta = \cos^{-1}\left(\frac{160}{200}\right) = 36.86^\circ$$



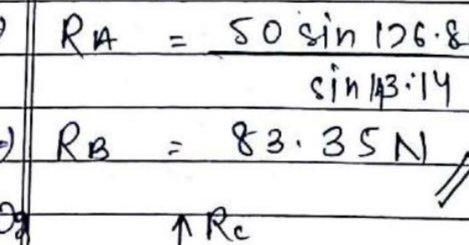
Lami's theorem

$$\frac{R_B}{\sin 90} = \frac{50}{\sin 143.14} = \frac{R_A}{\sin 126.86}$$

$$\Rightarrow R_A = \frac{50 \sin 126.86}{\sin 143.14} = 66.69\ N$$

$$\Rightarrow R_B = 83.35 N$$

At O_2



$\Sigma H = 0$

$\Rightarrow R_D - 83.35 \cos 36.86 = 0$

$\Rightarrow R_D = 66.68\ N$

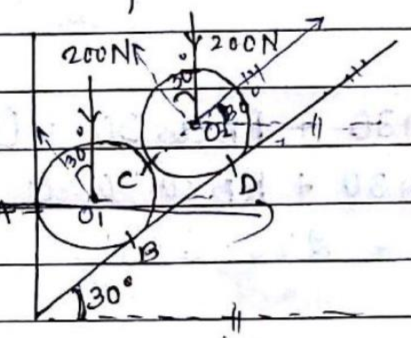
$\Sigma V = 0$

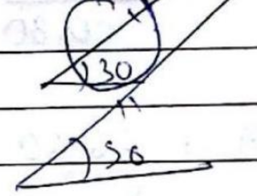
$\Rightarrow R_C - 50 - 83.35 \sin 36.86 = 0$

$\Rightarrow R_C = 50 + 83.35 \sin 36.86 = 99.99\ N$

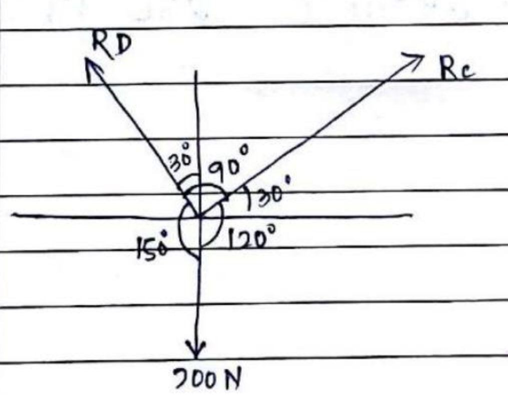
Q. Two identical rollers each of weight 200 N are placed in a notch as shown in figure with all contact surfaces as smooth. Determine the reaction developed at all the contact surfaces.

Ans -





FBD at O_2 :-



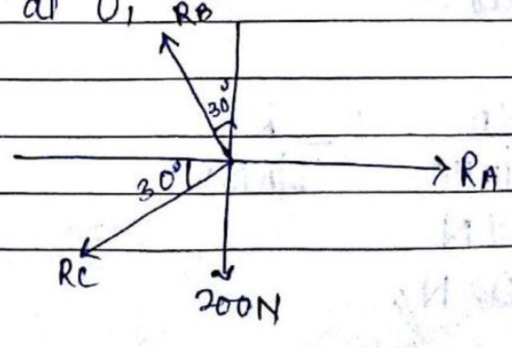
By Lami's theorem:-

$$\frac{200}{\sin 90} = \frac{RD}{\sin 120} = \frac{Rc}{\sin 150}$$

$\Rightarrow$ $RC = \dfrac{200 \sin 150}{\sin 90} = 100\ N$

$\Rightarrow$ $RD = \dfrac{200 \sin 120}{\sin 90} = 173.20\ N$

FBD at O_1



$\Sigma H = 0$

$\Rightarrow RA - Rc \cos 30 - R_B \sin 30 = 0$

$\Rightarrow RA - 100 \cos 30 - R_B \sin 30 = 0$

$\Rightarrow RA - RB \sin 30 = 86.60$

$\Sigma V = 0$

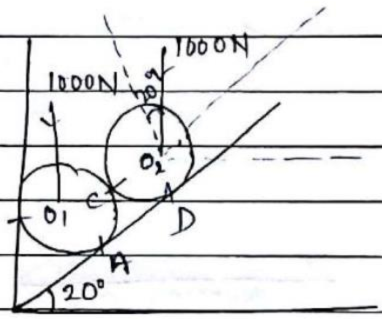
$\Rightarrow -200 - Rc \sin 30 + R_B \cos 30 = 0$

$\Rightarrow -200 - 100 \sin 30 + R_B \cos 30 = 0$

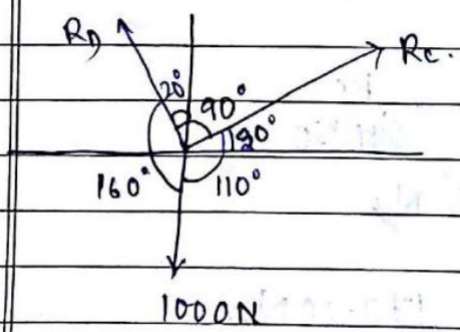
$\Rightarrow R_B = \frac{250}{\cos 30} = 288.67\,N$

$\Rightarrow RA = 86.60 + 288.67 \sin 30.$
$= 86.60 + 288.67 \sin 30 = 230.935\,N$

Q.



FBD at O_2 :-



By lami's theo :-

$$\frac{1000}{\sin 90} = \frac{R_D}{\sin 110} = \frac{R_c}{\sin 160}$$

$RD = 939.69\,N$

$Rc = 342.02\,N$

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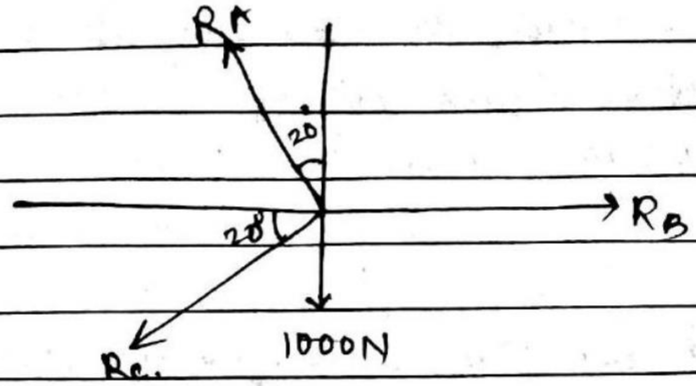
FBD at O_1 :-



$\Sigma V = 0$

$\Rightarrow -1000 - R_A \sin 20 - R_C \cos 20 = 0$

$\Rightarrow -1000 - R_A \sin 20 - 342.02 \cos 20 = 0$

$\Rightarrow R_A \sin 20 = -1321.39$

$\Rightarrow RA = -3863.49N$

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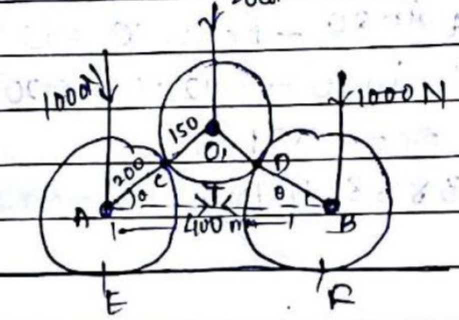
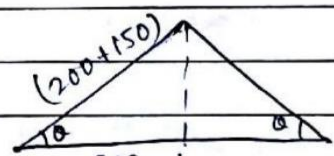
Imp

Q.- 2 smooth circular cylinder each of weight 1000 N and radius 200 mm are ~~collected~~ connected at their centres by the ~~centres~~ string AB of length 400 mm and rest upon the horizontal floor. Supporting above them in an another cylinder of wt. 2000 N and radius 150 mm. Determine the tension in string and reaction developed at contact surfaces

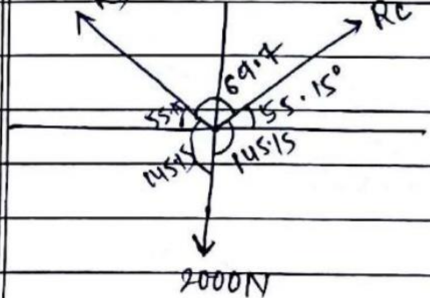
Ans.-





$$\theta = \cos^{-1}\left(\frac{200}{350}\right) = 55.15'$$

FBD at O_1 :-



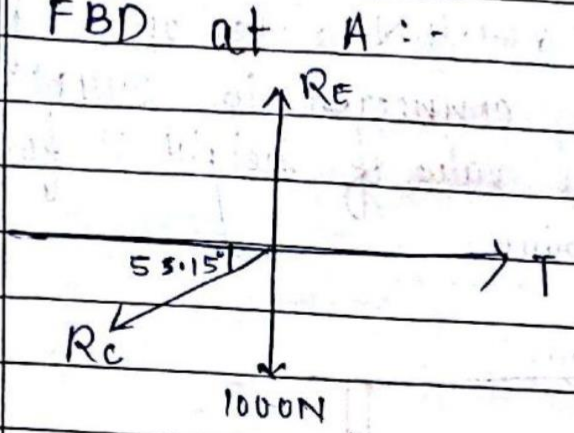
By Lami's theorem:-

$$\frac{2000}{\sin 69.7} = \frac{R_C}{\sin 145.15} = \frac{R_D}{\sin 145.15}$$

$R_C = 1218.54$ N

$R_D = 1218.54$ N

FBD at A :-



$\Sigma H = 0$

$\Rightarrow \cancel{100}\, T - R_C \cos 55.15 = 0.$

$\Rightarrow T = 1218.54 \cos 55.15 = 696.31\ N$ //.

$\Sigma V = 0.$

$\Rightarrow -1000 + RE - R_C \sin 55.15 = 0$

$\Rightarrow RE = 1000 + 1218.54 \sin 55.15 = 0$

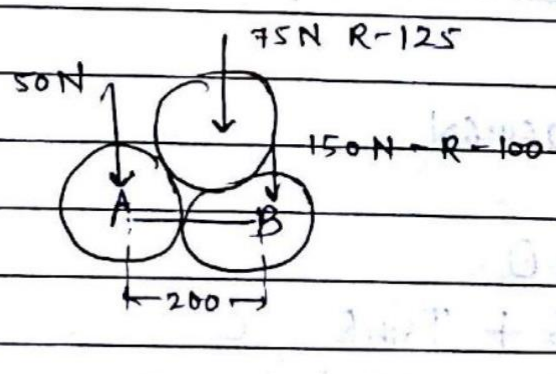
$= 1999.99\ N$ //

$\therefore R_E = R_F = 1999.99\ N.$

$R_C = R_D = 1218.54\ N.$

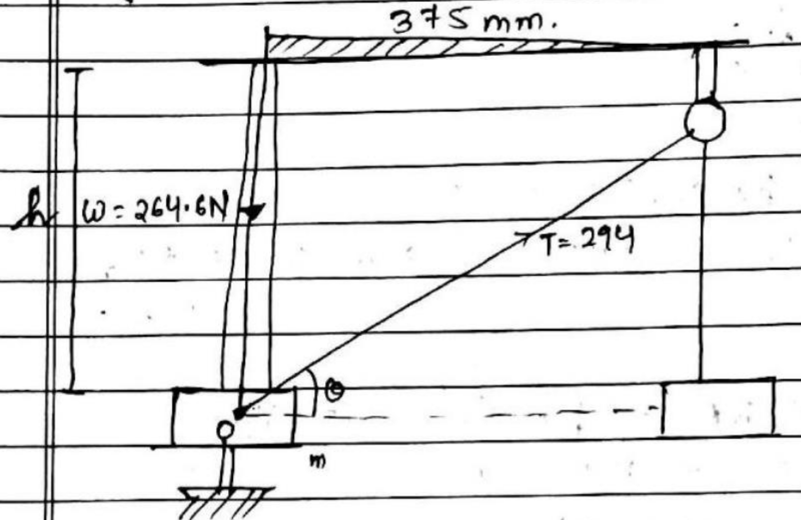
$T = 696.31\ N$ //.

HW

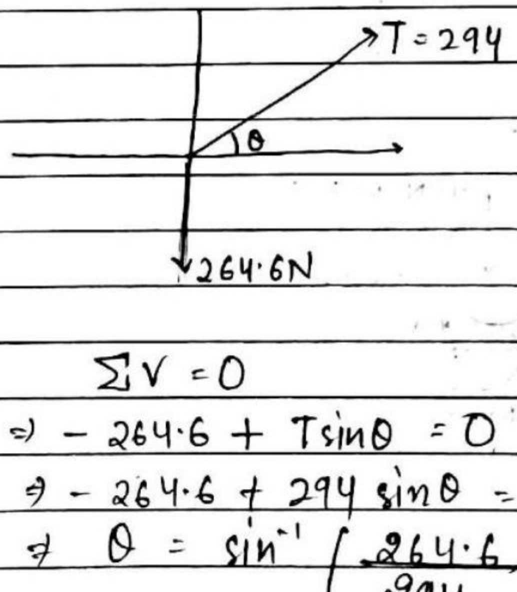


(crane)

Q- The collor of weight 264.4 N slides on a frictionless vertical rod and it's connected to 294N countor weight. Determine the value of height H for which system is in equilibrium.



FBD at O:-

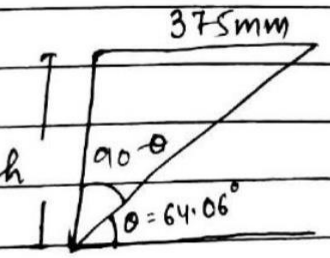


$$\Sigma V = 0$$

$$\Rightarrow -264.6 + T\sin\theta = 0$$

$$\Rightarrow -264.6 + 294\sin\theta = 0.$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{264.6}{294}\right) = 64.06^\circ$$



$$90-\theta = 90 - 64.06 = 25.94$$

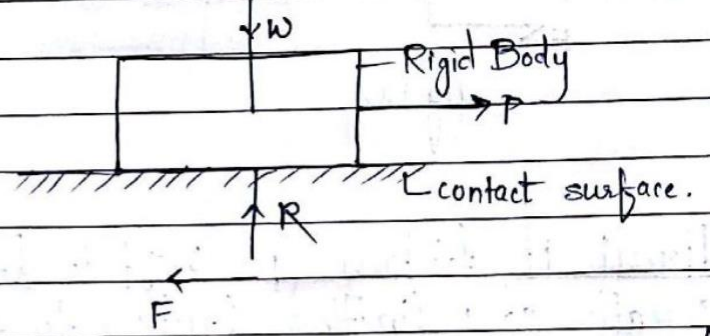
$$\tan\theta = \frac{375}{h} = \cancel{375}$$

$$h = \frac{375}{\tan 25.94} = 774\ m$$

FRICTION

10/4/17 Define friction. explain the types of friction with a neat figure.

Ans. When one body tends to move in contact with another body a resistance to its movement is setup. This resistance to the movement is called as friction or force of friction.



where W is the self weight of the body.
R is reaction.
P is applied force.
F is frictional force.

The frictional force always acts opposite to the body in motion.

Types of Friction :-

(1) Static friction.
(2) Limiting friction.
(3) Dynamic friction.
(4) fluid friction.
(5) dry friction.

- Static friction :- in rest
- Limiting friction :- about to move.
- dynamic friction :- in motion.
- fluid friction :- when body is in contact with fluid.
- dry friction :- when body is in contact with dry surface.

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TM Define coefficient of friction and show that angle of friction is equal to coefficient of friction

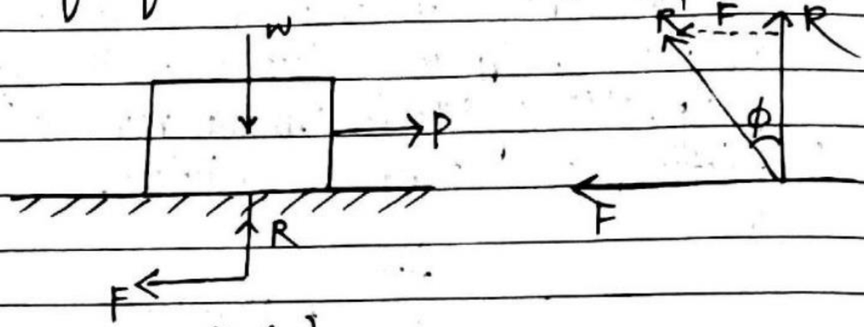


fig (a)

Coefficient of friction (μ) : It is defined as the ratio of limiting friction to the normal reaction b/w 2 surfaces and is given by

$$\boxed{\mu = \frac{F}{R}} \quad \text{—— (1)}$$

Angle of friction (ϕ) : The angle with the resultant reaction R' due to the normal reaction R makes with the normal surface is called as angle of friction ie ϕ.

Let a body of weight w subjected to in the figure(a)

R is the normal reaction which is ⊥ to the frictional force F acting opp to the applied force.

From the figure (b) we can see that the normal reaction R and the frictional force are ⊥ to each other and makes an angle ϕ with R'

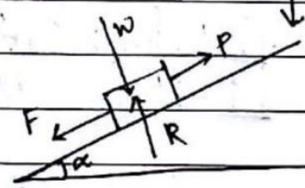
ie $\tan\phi = \frac{F}{R}$ —— (2)

Comparing eq ① and ②

$$\boxed{\tan\phi = \frac{F}{R} = \mu}$$

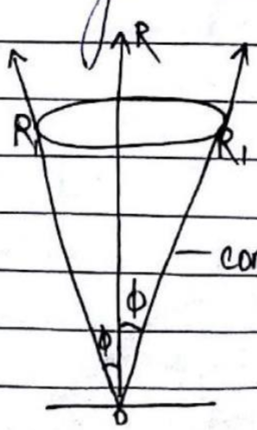
- Angle of Repose (α) :- If a body is placed on a inclined plane then the angle at which body is sliding down is called as angle of repose α. ie $\boxed{\phi = \alpha}$

Imp • Cone of friction :-



Whenever a body is in contact with other tend to move, then the normal reaction OR and friction comes into play. The normal reaction & friction can be replaced by resultant reaction OR_1. When this reaction OR_1 making angle ϕ is resolved around point O, will form a right circular cone.

This cone having the point of contact as the vertex O, the normal OR at the point of contact as its axis and ϕ as the semi-vertex angle is called the Cone of friction.



• EXPLAIN THE LAWS OF DRY FRICTION :-

(1) The force of friction always act in the direction opposite to the body motion.

(2) The magnitude of limiting friction (F) bears a constant ratio R between the two surfaces ie

$$\boxed{\mu = \frac{F}{R}}$$

(3) The magnitude of the force of friction which will be equal to the force applied as long as body is at rest ie $\boxed{P = F}$

(4) The force of friction is independent of the area of contact between the two surfaces.

(5) The force of friction depends upon the roughness of surfaces.

$\boxed{\mu < 1}$

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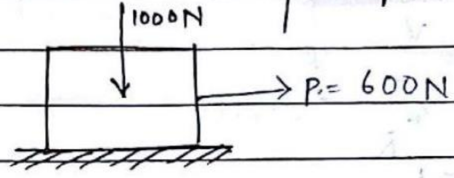
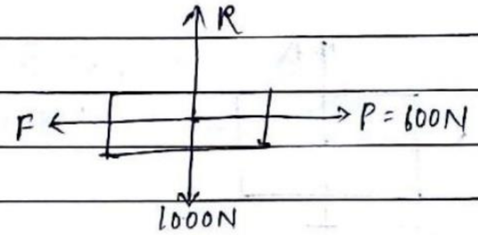
Q.1. A body of weight 1000 N is placed on the rough horizontal plane. Determine the coefficient of friction due to the force of 600 N in horizontal direction.

Ans.



$\mu = \frac{F}{R}$

FBD



$\Sigma H = 0.$

$P = F = 600N.$

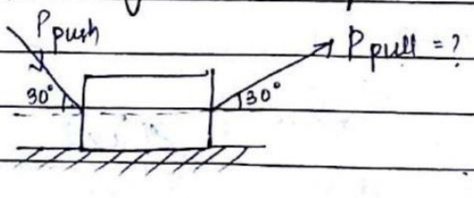
$\Sigma V = 0.$

$R = 1000N.$

$\therefore \mu = \frac{F}{R} = \frac{600}{1000} = 0.6 //$ $\boxed{\mu = 0.6}$

Imp Q.2. A block of weight 5 kN rest on a horizontal rough surface and the coefficient of friction b/w them is 0.4. Show that the magnitude of force required to pull is less than magnitude of force reqd. to push if the angle made by both forces pull and push is 30°.

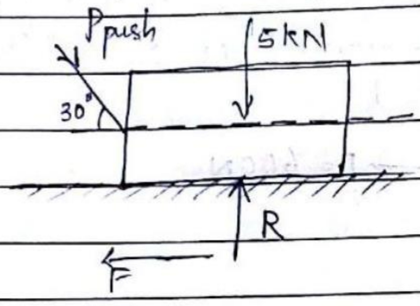
Ans -



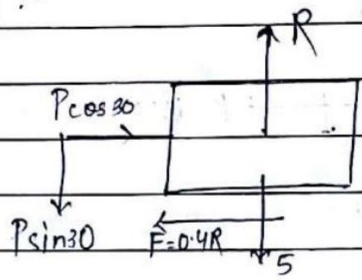
$\mu = 0.4$

$\frac{F}{R} = 0.4$ $\boxed{F = 0.4R}$

Case : I :- P_{push}



FBD :-



$\Sigma H = 0$

$P\cos 30 - 0.4R = 0$ —— ①

$\Sigma V = 0$

$R - 5 - P\sin\theta = 0$

$-P\sin\theta + R = 5$ —— ②

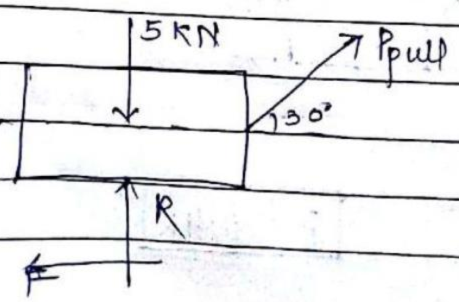
Solving eq ① and ② :-

$P = 3\,kN$

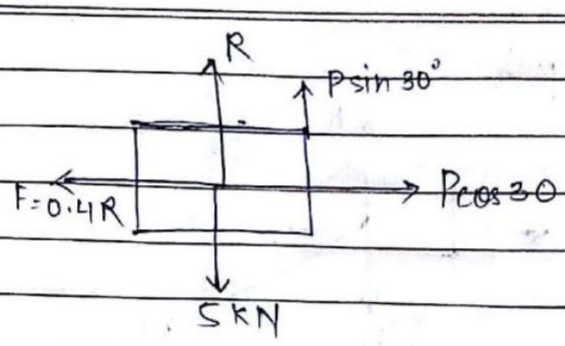
$R = 6.5\,kN$

$\boxed{P_{push} = 3\,kN}$

Case II :- $P_{pull} = ?$



FBD :-



$\Sigma H = 0$

$P\cos 30 - 0.4R = 0$. —— (3).

$\Sigma V = 0$.

$R - 5 + P\sin 30 = 0$

$P\sin\theta + R = 5$ —— (4).

Solving eq (3) and (4) :-

$P = 1.87\ kN$.

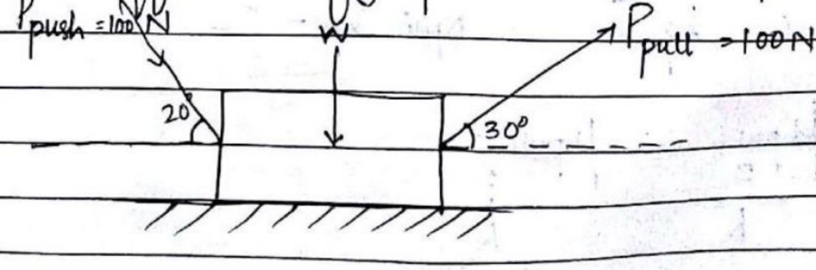
$R = 4.06\ kN$.

$\therefore \boxed{P_{pull} = 1.87 kN}$

$\therefore P_{pull} < P_{push}$. Hence Proved.

Imp

Q. A body is resting on a horizontal plane reqd to pull of 100N make an angle of 30° to the horizontal just to move it. It was also found that a push of 110 N inclined at 20° to the plane just to move the body. Determine the weight of the body and also the coefficient of friction.



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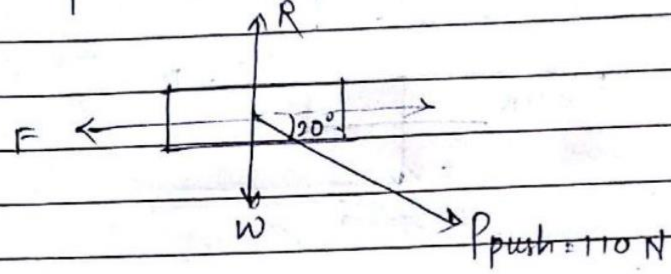
Ans- FBD :- P_{push}



$\Sigma H = 0.$

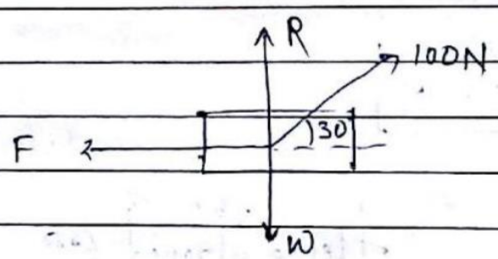
$\Rightarrow -F + 110\cos 20 = 0$

$\Rightarrow F = 103.36\ N$

$\Sigma V = 0.$

$\Rightarrow R - W - 110\sin 20 = 0$ —— (1)

FBD :- P_{pull}



$\Sigma H = 0$

$-F + 100\cos 30 = 0$

$F = 86.60$

$\Sigma V = 0$

$R - W + 100\sin 30 = 0$ —— (2)

From eq (1) :- $R_{push} = W + 37.62$

From eq (2) :- $R_{pull} = W - 50$

$\mu_{push} = \mu_{pull}$

$\frac{F}{R} = \frac{F}{R}$

$$\Rightarrow \frac{103.36}{W + 37.62} = \frac{86.6}{W - 50}$$

$\Rightarrow 103.36\,W - 5168.5 = 86.6\,W + 3257.89.$

$\Rightarrow 16.76\,W = 8422.89$

$\Rightarrow \boxed{W = 504.36\ N}$ //

$R = 504.36 - 50 = 454.36.$

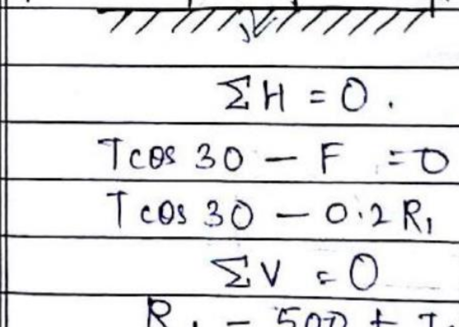
$$\mu = \frac{F}{R} = \frac{86.6}{454.36} = 0.1905 \ //.$$

The weight of body is 504.36 N with the coefficient friction of 0.1905.

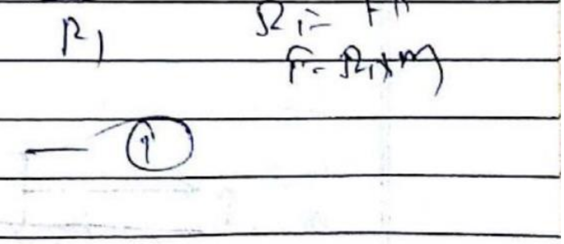
$$\mu = \frac{F}{W + 37.62} = \frac{103.36}{504.36 + 37.62} = 0.19.$$

Q. Find the force P just reqd. to slide the block B in the arrangement shown in the figure. Find also tension in the string, if the weight of block A is 500 N and weight of the block B is 1000 N. Take μ for all the contact surface as 0.2.

Ans.



FBD at Block A :-



$\mu = \frac{F}{R_1}$ $\quad R_1 = \frac{F}{\mu}$ $\quad F = R_1 \times \mu$

$\Sigma H = 0.$

$T\cos 30 - F = 0$

$T\cos 30 - 0.2R_1 = 0. \quad — (1)$

$\Sigma V = 0$

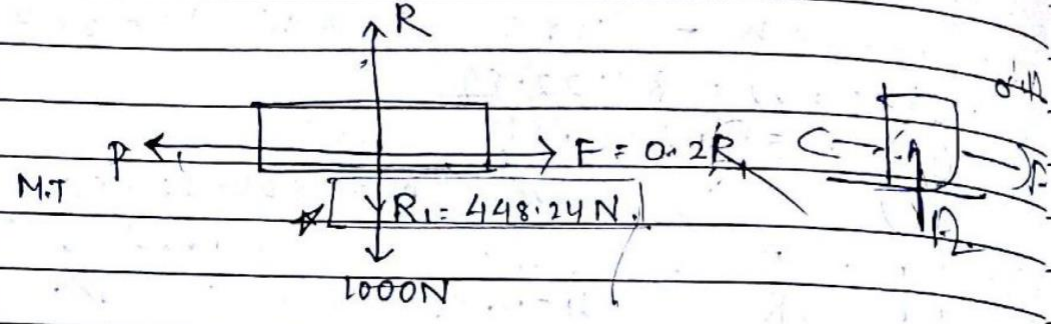
$R_1 - 500 + T\sin 30 = 0$

$T\sin 30 + R_1 = 500 \quad — (2)$

$T = 103.51\ N.$

$R_1 = 448.24\ N$ //

FBD at B :-



$\Sigma H = 0.$

$\Rightarrow -P + F$

$\Rightarrow -P + 0.2R = 0$ ——— (1)

$\Sigma V = 0.$

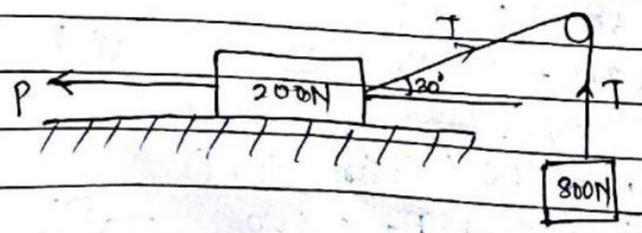
$\Rightarrow R - 448.24 - 1000 = 0$

$\Rightarrow R = 1448.24$ N.

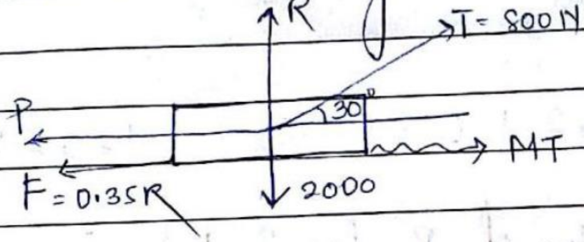
$\Rightarrow P = 0.2R = 289.64$ N //.

Q. A block of wt. 2000 N is attached to one end of the cord which passes round of frictionless pulley as shown in fig. Carries a load of 800N at the other end. Find the value of P. if
(a) motion of the body is impending towards left.
(b) if motion of body is impending towards right.

$\mu = 0.35$ //



Case (i) :-
Motion trend → Right.



$\therefore \Sigma H = 0$

$-P + 800\cos 30 - 0.35R = 0$

$P + 0.35R = 800\cos 30$

$P + 0.35R = 692.82$ —— (1)

$\Sigma V = 0$

$R + T\sin 30 - 2000 = 0.$

$R = 2000 - 800\sin 30$

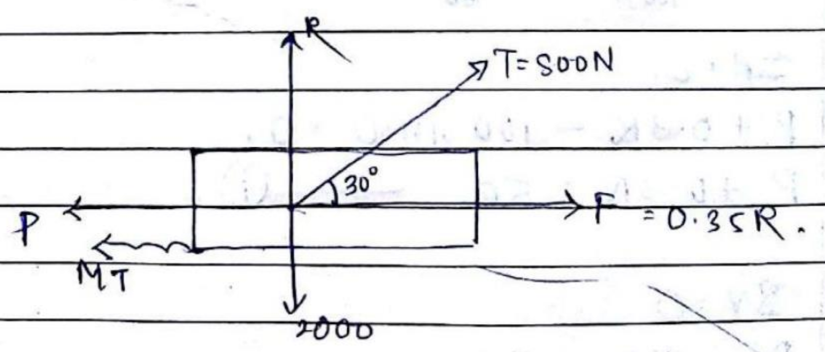
$R = 1600\ N$ //

$P = 692.82 - 0.35(1600)$

$= 692.82 - 560$

$= 132.82\ N$ //

Case (ii) :- Motion Trend → Left.



$\Sigma H = 0$

$-P + 0.35R + 800\cos 30 = 0$

$P - 0.35R = 692.82$ —— (1).

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$\Sigma V = 0$

$R + 800 \sin 30 - 2000 = 0.$

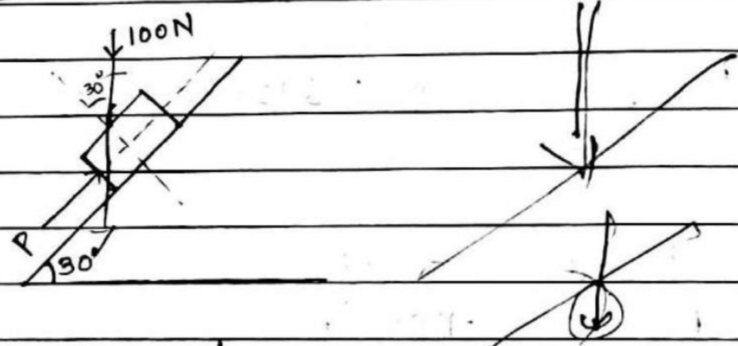
$R = 1600\ N$

$P = 692.82 + 0.35(1600) = 1252.82\ N$

Q - Find the value of P for the body impending down the plane and also find P for the body impending up the plane.

$\mu = 0.3$

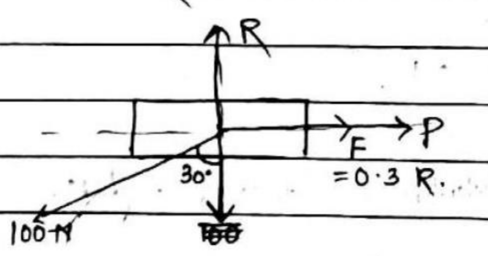
Ans -



Case (i) impending down :-

$\mu = 0.3$

$F = 0.3R$



$\Sigma H = 0.$

$\Rightarrow P + 0.3R - 100 \sin 30 = 0.$

$\Rightarrow P + 0.3R = 50 \quad \text{——} \quad (1).$

$\Sigma V = 0$

$\Rightarrow R - 100 \cos 30 =$

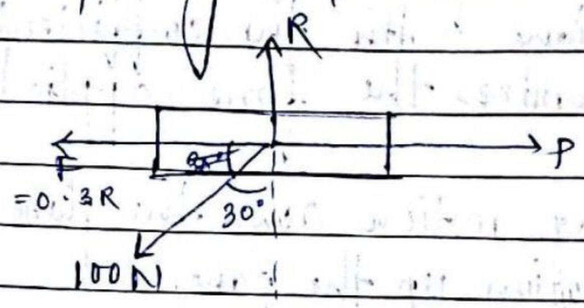
$\Rightarrow R = 86.60\ N$

$\Rightarrow P = 50 - 0.3(86.60) = 24.02\ N$

papergrid

Date: / /

Case (ii) Impending upwards :-



$\Sigma H = 0$

$\Rightarrow$ ~~$P - 0.3R - 100 \cancel{\cos 30} = 0$~~ (sin 30) $\Rightarrow P = 0.3R + 50$.

~~$\Rightarrow P - 0.3R = 86.60 N$ — (1)~~

$\Sigma V = 0$

$\Rightarrow R - 100 \cancel{\sin} 30 = 0$ (cos)

$\Rightarrow R = \cancel{50 N} \; 86.60$

$\Rightarrow P = \cancel{86.60}$ (50) $+ 0.3\cancel{(50)}(86.60)$

~~$= 86.60 + 15$~~ $25.98 + 50 = 75.98 N$

~~$= 101.6 N$~~

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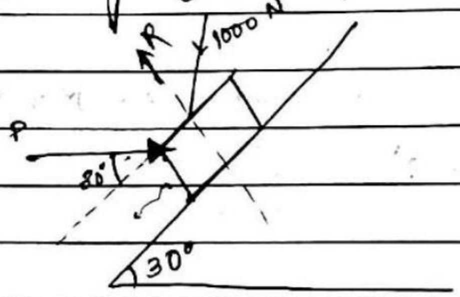
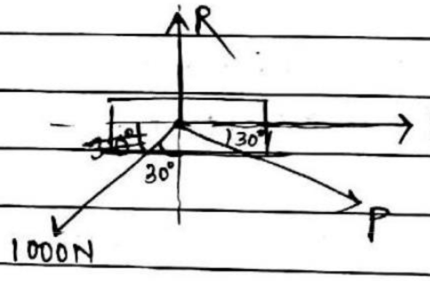
Q. A small block of 1000 N is placed on a 30° inclined plane with the coefficient friction of 0.25. Determine the force applied for.

(a) an impending motion down the plane.
(b) impending motion up the plane.



(a) Case (i) :-



$\Sigma H = 0.25R - 1000 \sin 30 + P\cos 30°$.

$0 \Rightarrow 0.25R + P\cos 30 - 500$

$\Rightarrow P\cos 30 + 0.25R = 500$ // —— ①.

$\Sigma V = 0$

$\Rightarrow R - P\sin 30 - 1000\cos 30 = 0$

$\Rightarrow P\sin 30 - R = -866.02$ —— ②

$P = 286.06$ N

$R = 1009.05$ N

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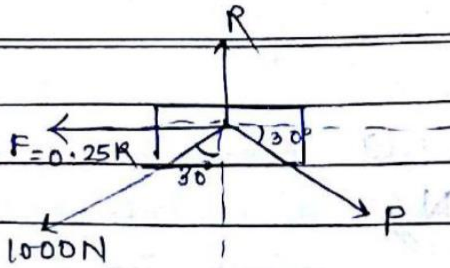
Case (ii)



$$\Sigma H = -0.25R - 1000\sin 30 + P\cos 30^\circ$$

$$0 = P\cos 30 - 0.25R - 500$$

$$P\cos 30 - 0.25R = 500 \quad \text{---} \ (1).$$

$$\Sigma V = R - 1000\cos 30 - P\sin 30$$

$$0 = -P\sin 30 + R - 866.06$$

$$\Rightarrow P\sin 30 \mp R = -866.06 \quad \text{---} \ (2)$$

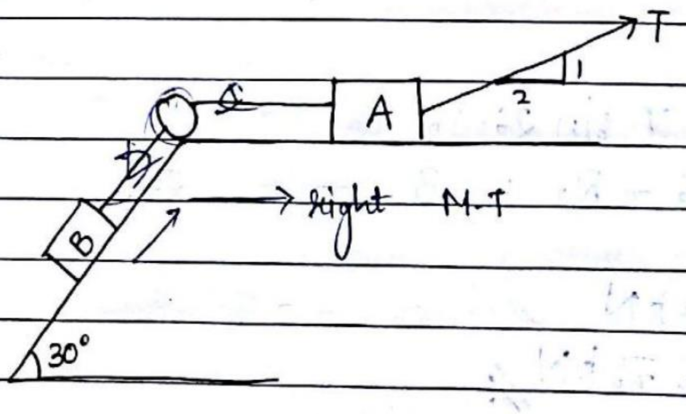
$P = 966.92 N$

$R = 1349.52 N$.

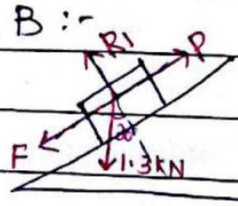
Imp

Q- 2 blocks A and B 3 kN and 1.3 kN resp. are connected by a string over a frictionless pulley as shown in the figure:-
Find the min. value of T to generate an impending motion towards right.
Take $\mu = 0.2$ for block A and $\mu = 0.3$ for block B

Ans-



FBD of block B :-



$F = 0.3 R_1$

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$\sum V = 0$

$\Rightarrow R_1 - 1.3\cos 30 = 0$

$R_1 = 1.125\ kN$ //

$\sum H = 0$

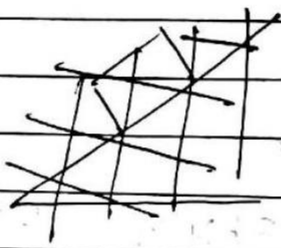
$\Rightarrow P - F - 1.3\sin 30 = 0$

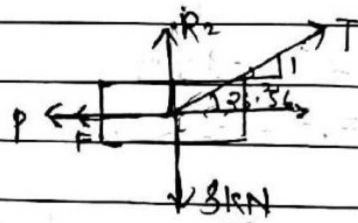
$\Rightarrow P - 0.3R_1 = 1.3\sin 30$

$\Rightarrow P = 1.3\sin 30 + 0.3 \times 1.125$

$\Rightarrow P = 0.9875\ kN$ //

FBD at block A:





$\tan^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow 26.56$

$\sum H = 0$

$\Rightarrow T\cos 26.56 - P - 0.2R_2 = 0.$

$\Rightarrow T\cos 26.56 - 0.9875 - 0.2R_2 = 0$

$\Rightarrow T\cos 26.56 - 0.2R_2 = 0.9875$ —— (1)

$\sum V = 0$

$\Rightarrow R_2 - 3 + T\sin 26.56 = 0$

$\Rightarrow T\sin 26.56 + R_2 = 3$ —— (2)

$T = 1.612\ kN$

$R_2 = 2.278\ kN$ //

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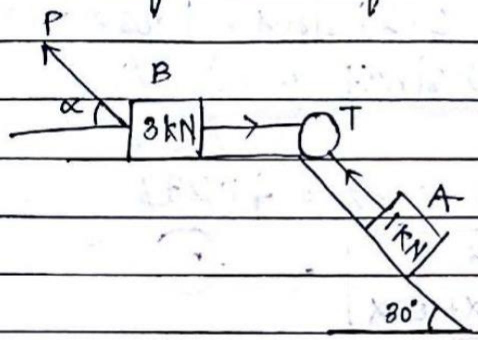
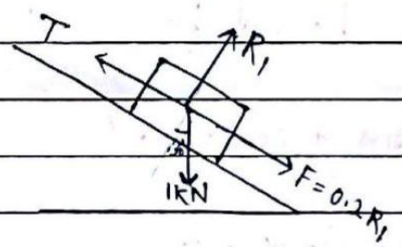
Imp

Q- Find the least value of P reqd. to ~~form~~ cause the ~~system~~ block as shown in figure to have impending motion to the left, $\mu = 0.2$ for all the contact surface.

Ans-



FBD at A



$\Sigma V = 0$

$\Sigma V = R_1 - 1\cos 30$

$0 = R_1 - \cos 30$

$R_1 = 0.866 \text{ kN}$

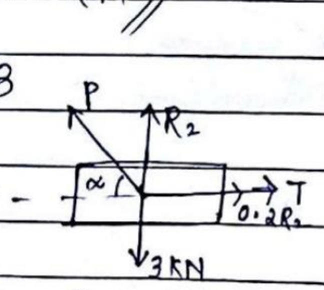
$\Sigma H = 0$

$\Sigma H = -T + 0.2R_1 + 1\sin 30$

$0 = -T + 0.2 \times 0.866 + 0.5$

$\Rightarrow T = 0.6732 \text{ kN}$

FBD at B



$\Sigma V = R_2 - 3 + P\sin\alpha = 0$

$\Rightarrow R_2 + P\sin\alpha = 3$ —— (1)

$\Rightarrow R_2 = 3 - P\sin\alpha$

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$\Sigma H = 0$

$0.673 + 0.2 R_2 - P\cos\alpha = 0$ —— (2)

$\Rightarrow 0.673 + 0.2(3 - P\sin\alpha) - P\cos\alpha = 0$

$\Rightarrow 0.673 + 0.6 - 0.2P\sin\alpha - P\cos\alpha = 0$

$\Rightarrow 1.273 - P(0.2\sin\alpha + \cos\alpha) = 0.$

$\Rightarrow +P(0.2\sin\alpha + \cos\alpha) = +1.273$

$\Rightarrow \boxed{P = \dfrac{1.273}{0.2\sin\alpha + \cos\alpha}}$ —— (3)

Max means directly put $\alpha = 0$.

Since the value of P should be minimum, denominator should be maximum in eq (3).

$\therefore \dfrac{d}{d\alpha}(0.2\sin\alpha + \cos\alpha) = 0$

$\Rightarrow 0.2\cos\alpha - \sin\alpha = 0$

$\Rightarrow 0.2 = \dfrac{\sin\alpha}{\cos\alpha}$

$\Rightarrow \tan\alpha = 0.2$

$\Rightarrow \alpha = \tan^{-1}(0.2) = 11.309°$

$P = \dfrac{1.273}{0.2\sin 11.309 + \cos 11.309}$

$P = 1.24$ KN

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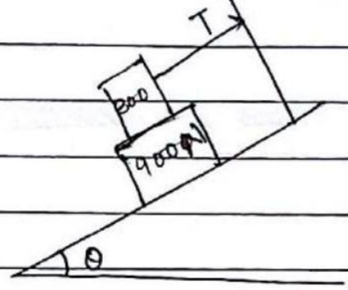
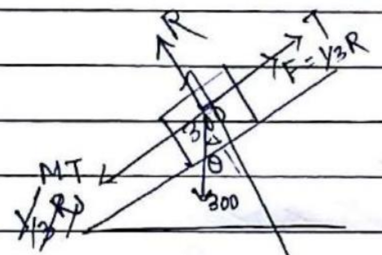
Q- What should be the value of θ which will make the motion of the block 900 N down the plane.
Take $\mu = 1/3$ for all the contact surfaces.

Ans -





$\Sigma V = 0$

$\Rightarrow R - 300\cos\theta = 0$

$\Rightarrow R = 300\cos\theta.$

$\Sigma H = 0$

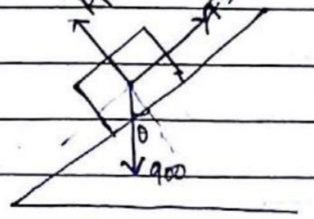
$\Rightarrow T + \frac{1}{3}R - 300 \cancel{\cos\theta}\sin\theta = 0.$

$\Rightarrow T + \frac{1}{3}(300\cos\theta) - 300\cancel{\cos\theta}\sin\theta = 0$

$\Rightarrow 300\cos\theta\left(\frac{1}{3} - 1\right) + T = 0$

$\Rightarrow T = -300\cos\theta(-0.66).$

$= T = 200\cos\theta$



$\Sigma V = 0$

$R_1 - 900\cos\theta = 0.$

$\Sigma H = 0.$

$+\frac{1}{3}R_1$