MODULE 4

Date: 13/5/17

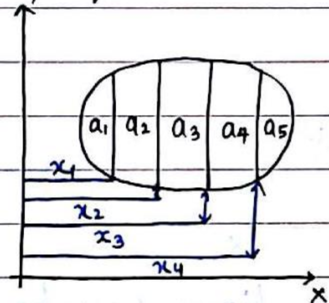
CENTROID AND MOMENT OF INERTIA OF ENGINEERING SECTION...

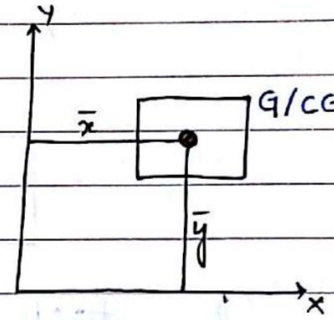
- **CENTRE OF GRAVITY:** Centre of gravity of a body is a point through which the whole weight of the body acts at the centre. It is denoted by centre of gravity (CG) or G.

- **CENTROID:** The point at which the total area of the plane figures is assumed to be concentrated at the centre.

Centroid and centre of gravity are at same point.

- **Determination of centroid by the moment :-**





$$\boxed{\bar{x} = \frac{\Sigma ax}{\Sigma a}} \qquad \boxed{\bar{y} = \frac{\Sigma ay}{\Sigma a}}$$

Let us consider a body of weight of w as shown in figure. The centre of gravity of whole figure is located at a distance $\bar{x}$ from y-axis and $\bar{y}$ from $\bar{x}$-axis.

Let us divide the whole figure into no. of elemental stripe of weight a_1, a_2, a_3, a_4 and whose centroid are located by the distance x_1, x_2, x_3, x_4 from y-axis and y_1, y_2, y_3, y_4 from x-axis.

Now by applying the theorem of moment we have

$$\Sigma ax = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4$$

$$\Sigma ay = a_1y_1 + a_2y_2 + a_3y_3 + a_4y_4$$

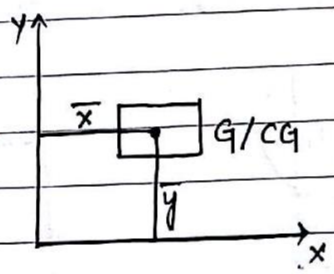
$$\Sigma a = a_1 + a_2 + a_3 + a_4$$

From y-axis, $\bar{x}$ is given by :-

$$\bar{x} = \frac{\Sigma ax}{\Sigma a}$$

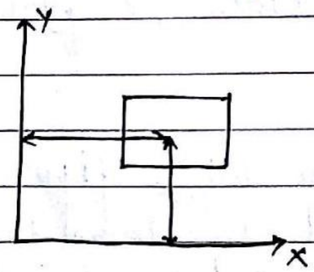
From x-axis, $\bar{y}$ is given by :-

$$\bar{y} = \frac{\Sigma ay}{\Sigma a}$$

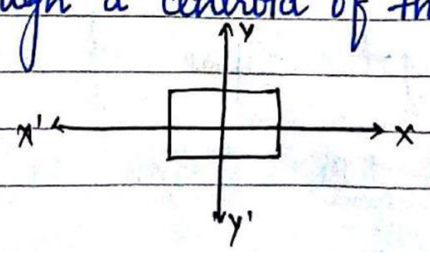


The centroid for any figure can be determined by calculating $\bar{x}$ and $\bar{y}$.

- <u>AXIS REFERENCE</u> : These are the axis with respect to which the centroid of the given figure is determined.



- <u>CENTROIDAL AXIS</u> : This is the axis which passes through a centroid of the given figure.



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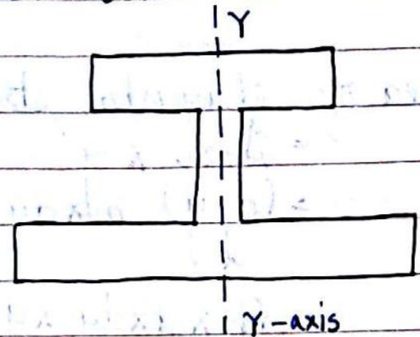
- **SYMMETRICAL AXIS:-** The axis which divides the whole figure into equal parts is known as symmetrical axis.

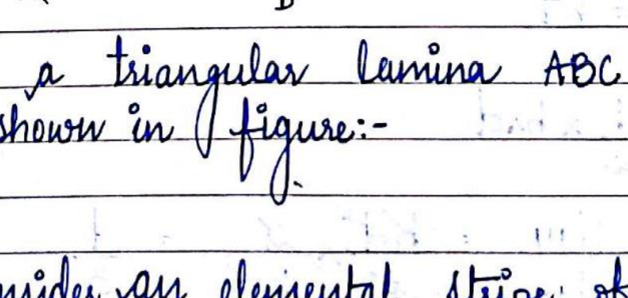


If the section is symmetrical about x-axis then $\bar{y} = 0$ and $\bar{x} = \dfrac{\Sigma ax}{\Sigma a}$

If the section is symmetrical about y-axis then $\bar{x} = 0$ and $\bar{y} = \dfrac{\Sigma ay}{\Sigma a}$

If the section is symmetrical about both axes then $\bar{x} = 0$ and $\bar{y} = 0$

- **DERIVATION OF CENTROID OF TRIANGLE :**



Consider a triangular lamina ABC of area $\frac{1}{2}bh$ as shown in figure:-

Now consider an elemental stripe of area of $(b_1 \times dy)$ at a distance of y from AB. Using property of similar Δ, $\dfrac{b_1}{b} = \dfrac{d-y}{d}$

$$\Rightarrow b_1 = \frac{(d-y)b}{d}$$

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Area of elemental strip is given by $b_1 \times dy$

$$= \frac{(d-y)b}{d} dy$$

Moment of Area of elemental strip about AB =

$$= \text{Area} \times y$$

$$= \frac{(d-y)}{d} b \times dy \times y$$

$$= \frac{\cancel{d}b \times \cancel{d} \times dy \times y}{\cancel{d}} - \frac{y \times b \times dy \times y}{d}$$

$$= b \times y \times dy - \frac{y^2 \times b \times dy}{d}$$

Now area of the whole Δ is

$$\int_0^d b \times y \times dy - \int_0^d \frac{y^2 b}{d} dy$$

$$= b\left[\frac{y^2}{2}\right]_0^d - \frac{b}{d}\left[\frac{y^3}{3}\right]_0^d$$

$$= \frac{bd^2}{2} - \frac{b}{d} \cdot \frac{d^3}{3}$$

$$= \frac{bd^2}{2} - \frac{bd^2}{3} = \frac{bd^2}{6}$$

$$\therefore \Sigma ay = \frac{bd^2}{6}$$

$$\Sigma a = \frac{1}{2} \times b \times d$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{\cancel{b}d^{\cancel{2}}}{\cancel{6}_3} \times \frac{\cancel{2}}{\cancel{bd}} = \frac{d}{3} \Rightarrow \bar{y} = \frac{d}{3}$$

from the base $\bar{y} = d/3$

from the apex its $\bar{y} = 2d/3$

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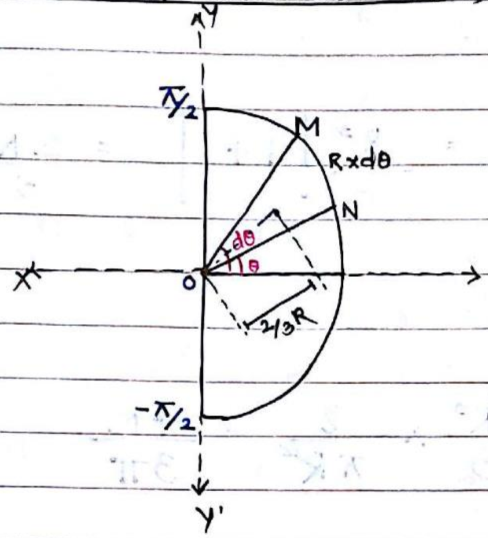
- **DERIVATION OF CENTROID OF SEMICIRCLE :-**

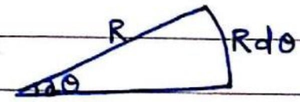


Consider a semicircular lamina of area $\frac{\pi R^2}{2}$ as shown in figure.

Now consider a triangular elemental strip OMN of area $\left(\frac{1}{2} \times R \times R d\theta\right)$ at an angle of ~~dθ~~ θ from x-axis and whose centre of gravity is @at a distance of $\frac{2}{3}R$ from the point O.

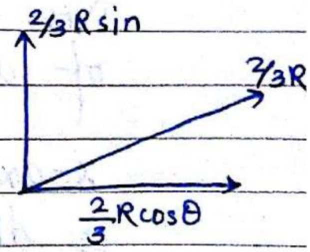
$$CG = \frac{2}{3}R$$

$\Rightarrow$ Area of OMN $= \frac{1}{2} \times R \times R d\theta$



$$\Rightarrow \quad CG = \frac{2}{3}R$$

The projection of centre of gravity on x-axis is given by $\frac{2}{3}R\cos\theta$.



Moment of area of elemental strip about y-axis is given by :-

$$A \times \frac{2}{3} R \times \cos\theta$$

$$\Rightarrow \frac{1}{\cancel{2}} \times R^2 d\theta \, \frac{\cancel{2}}{3} R\cos = \frac{R^3}{3}\cos\theta \, d\theta$$

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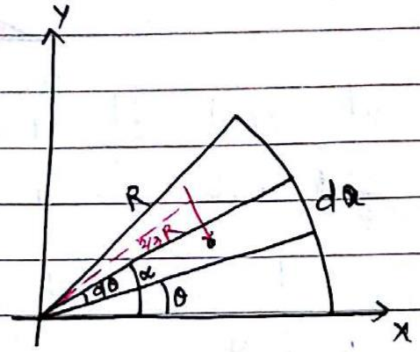
• DERIVATION OF CENTROID OF SECTOR OF A CIRCLE:



Consider a sector of circular lamina as shown in figure.

Consider a Δangular elementary strip $(\frac{1}{2}R^2 d\theta)$ at an angle of θ from x-axis whose centre of gravity is $\frac{2}{3}R$ from point O & Projection is $\frac{2}{3}R\cos\theta$.

Area of strip $= \frac{1}{2} \times R \times R \times d\theta$

Area of sector $= \int_0^{\alpha} \frac{1}{2} R^2 \, d\theta = \frac{1}{2} R^2 \alpha \quad \left\{ \left[\frac{1}{2} R^2 \theta\right]_0^{\alpha} \right\}$

Moment of area of elementary strip $= \frac{\cancel{2}}{3} R\cos\theta \times \frac{1}{\cancel{2}} R^2 d\theta$

$= \frac{R^3 \cos\theta \, d\theta}{3}$

Sum of moment of all strips about y-axis

$= \int_0^{\alpha} \frac{R^3 \cos\theta}{3} \, d\theta = \frac{R^3}{3} \left[\sin\theta\right]_0^{\alpha}$

$\Sigma ax = \frac{R^3}{3} \left\{ [\sin\alpha] - \sin 0 \right\} = \frac{R^3 \sin\alpha}{3}$

$\Sigma a = \frac{R^2 \alpha}{2}$

$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{R^{\cancel{3}} \sin\alpha}{3} \times \frac{2}{\cancel{R^2}\alpha} = \frac{2R}{3\alpha} \sin\alpha$

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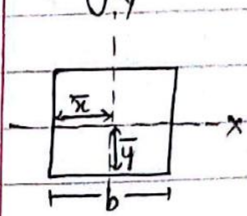
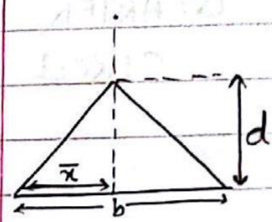
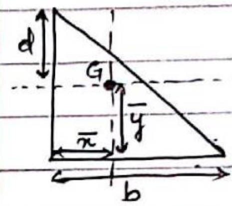
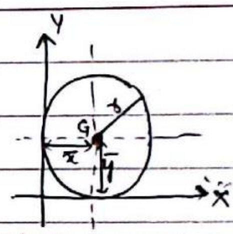
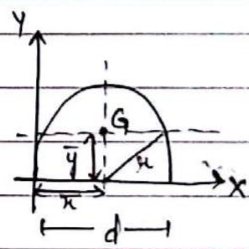
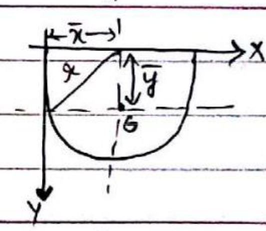
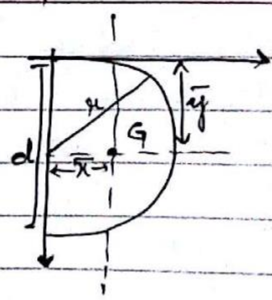
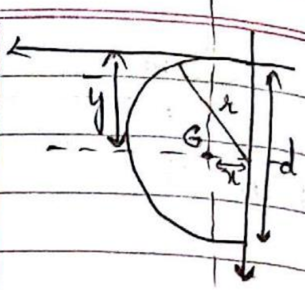
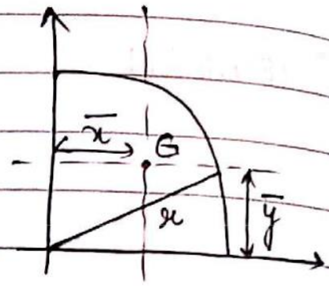
FORMULAE

Shape	Area	x	y	Figure
① RECTANGLE	$b \times d$	$\frac{b}{2}$	$\frac{d}{2}$	
② TRIANGLE	$\frac{1}{2} \times b \times d$	$\frac{b}{2}$	$\frac{d}{3}$	
③ RIGHT-ANGLED TRIANGLE	$\frac{1}{2} \times b \times d$	$\frac{b}{3}$	$\frac{d}{3}$	
④ CIRCLE	πr^2	$\bar{x} = r$	$\bar{y} = r$	
⑤ SEMI-CIRCLE	$\frac{\pi r^2}{2}$	$\frac{d}{2}$	$\frac{4r}{3\pi}$	
		$\frac{d}{2}$	$\frac{-4r}{3\pi}$	
		$\frac{4r}{3\pi}$	$\frac{d}{2}$	

		$-\frac{4r}{3\pi}$	$\frac{d}{2}$	
QUARTER CIRCLE	$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	

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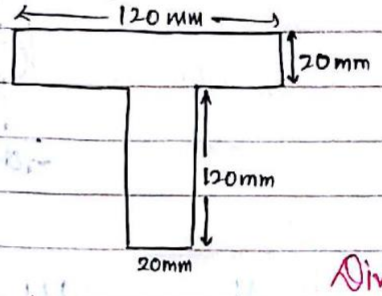
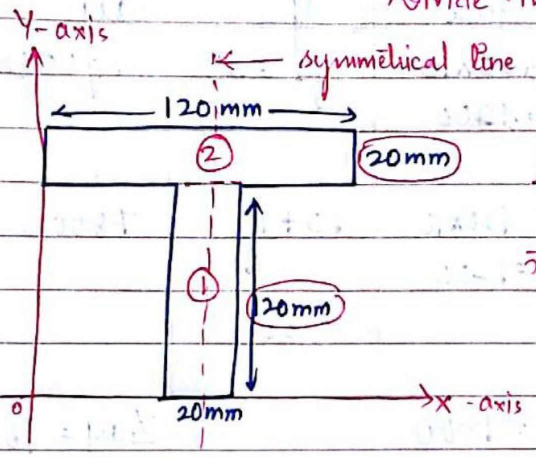
Q.1- Find the centroid for the given figure shown :-



Ans-

Divide the figure into possible no. of units.



symmetrical line about y-axis so $\bar{y}$ we have to calculate.

for calculating y, set the reference axis as (X)

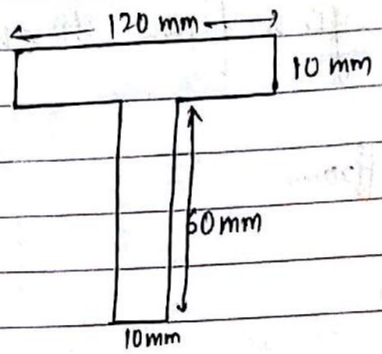
$\bar{x}$ value will be direct.

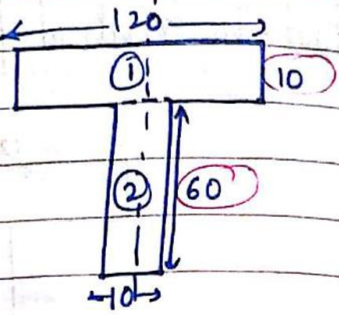
Component	Area (mm^2)	y (mm)	ay (mm^3)
(1) Rectangle	$120 \times 20 = 2400 mm^2$	$\frac{120}{2} = 60 mm$	144000
(2) Rectangle	$120 \times 20 = 2400 mm^2$	$120 + \frac{20}{2} = 130 mm$	312000
	$\Sigma a = 4800$		$\Sigma ay = 456000$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{4560\cancel{00}}{48\cancel{00}} = 95 \text{ mm}$$

$$\bar{x} = (\text{Reference is y-axis}) = \frac{120}{2} = 60 \text{ mm}$$

Q.2-





Ans-

Component	Area	y	ay
② Rectangle	60×10 = 600	$\frac{60}{2} = 30$	18000
① Rectangle	120×10 = 1200	$60 + \frac{10}{2} = 65$	78000

$\Sigma a = 1800$ $\Sigma ay = 96000$

$$\therefore \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{96000}{1800} = 53.33 \text{ mm.}$$

$$\bar{x} = \frac{120}{2} = 60 \text{ mm}$$

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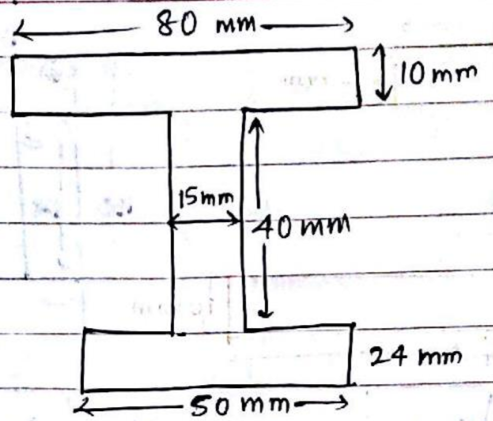
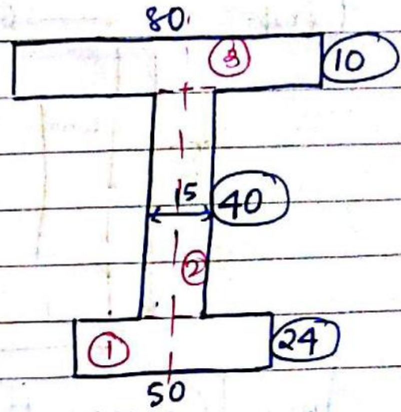
Q.3-





Ans-

Component	Area	y	ay
1 Rectangle	24×50 = 1200	$\frac{24}{2} = 12$	14400
2 Rectangle	15×40 = 600	$24 + \frac{40}{2}$ = 44	26400
3 Rectangle	80×10 = 800	$24+40+\frac{10}{2}$ = 69	55200

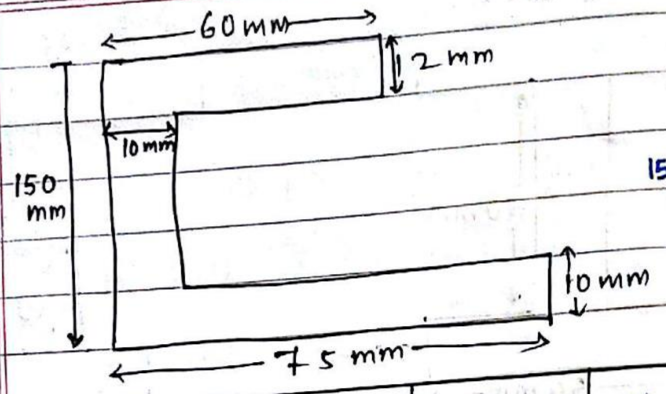
$\Sigma a = 2600$ $\Sigma ay = 96000$

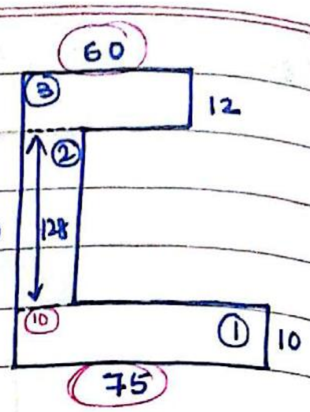
∴ Since the figure is symmetrical about y-axis :-

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{96000}{2600} = 36.923 \text{ mm}$$

$$\bar{x} = \frac{80}{2} = 40 \text{ mm}$$

Q.4-





Ans-

Component	Area	x	y	ax	ay
① Rectangle	$75\times10 = 750$	$\frac{75}{2} = 37.5$	$\frac{10}{2} = 5$	28125	3750
② Rectangle	$128\times10 = 1280$	$\frac{10}{2} = 5$	$10+\frac{128}{2} = 74$	6400	94720
③ Rectangle	$60\times12 = 720$	$\frac{60}{2} = 30$	$10+128+\frac{12}{2} = 144$	21600	103680

$\Sigma a = 2750$

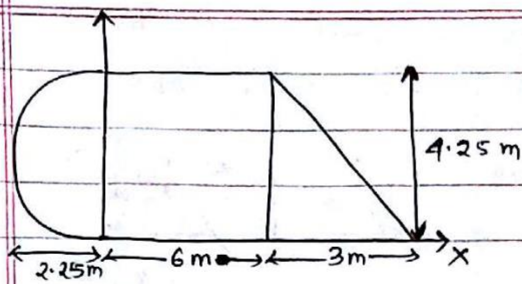
$\Sigma ax = 56125$

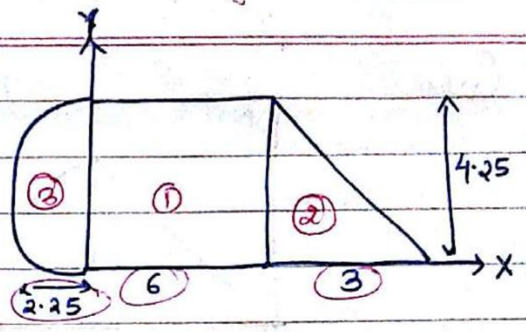
$\Sigma ay = 202150$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{20215\cancel{0}}{275\cancel{0}} = 73.5 \text{ mm}$$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{56125}{2750} = 20.409 \text{ mm}$$

Imp Q.15-





Ans-

Component	Area	x	y	ax	ay
① Rectangle	6×4.25 $= 25.5$	$\frac{6}{2} = 3$	$\frac{4.25}{2}$ $= 2.125$	76.5	54.1875
② Triangle	$\frac{1}{2} \times 3 \times 4.25$ $= 6.375$	$\frac{3}{3} = 1$	$\frac{4.25}{3}$ $= 1.416$	6.375	9.027
③ Semicircle	$\frac{\pi \times (2.25)^2}{2}$ $\frac{3.14 \times (2.25)^2}{2}$ $= 7.9481$	$-\frac{4 \times 2.25}{3 \times 3.14}$ $= -0.95$	$\frac{4.25}{2}$ $= 2.125$	-7.55	16.889

$\Sigma a = 39.8231$

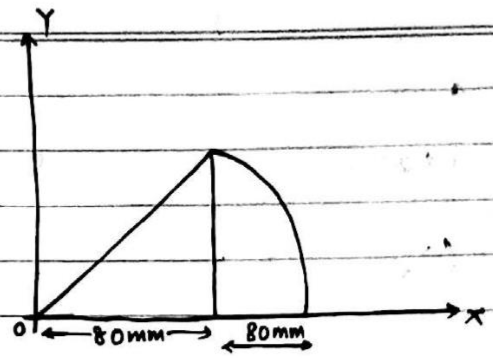
$\Sigma ax = 75.325$

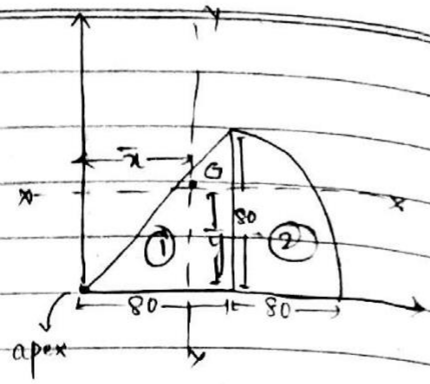
$\Sigma ay = 80.1035$

$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{75.325}{39.8231} = 1.8914$ mm //

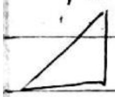
$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{80.1035}{39.8231} = 2.0114$ mm //

Q.6-





Ans-



Component	Area (a)	x	y	ax	ay
1 Triangle	$\frac{1}{2} \times 80 \times 80$ = 3200	$\frac{2}{3} \times 80$ = 53.33	$\frac{80}{3}$ = 26.66	170656	85312
2 Quarter Circle	$\frac{3.14 \times (80)^2}{4}$ = 5024	$80 + \frac{4 \times 80}{3 \times 3.14}$ = 113.970	$\frac{4 \times 80}{3 \times 3.14}$ = 33.97	572585.28	170666

$\Sigma a = 8224$

$\Sigma ay = 255978.66$

$\Sigma ax = 743241.28$

$$\therefore \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{255978.66}{8224} = 31.1258\text{mm}$$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{743241.28}{8224} = 90.374\text{ mm}$$

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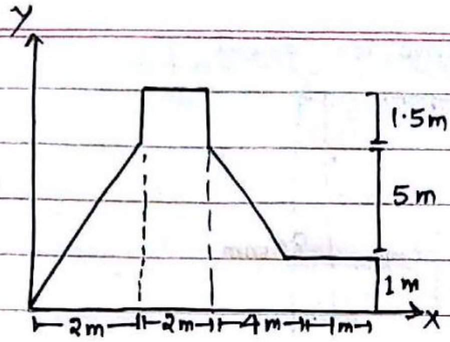
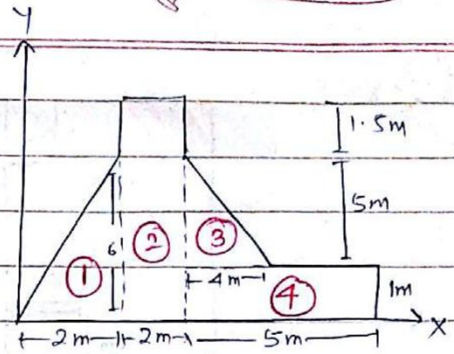
Q.7 -





Ans. Not symmetrical about any of the axis :-

Component	Area (a)	x	y	ax	ay
① Triangle	$\frac{1}{2}\times 2\times 6$ $= 6$	$\frac{2}{3}\times 2$ $= 1.33$	$\frac{6}{3} = 2$	7.98	12
② Rectangle	7.5×2 $= 15$	$2 + \frac{2}{2}$ $= 3$	$\frac{7.5}{2}$ $= 3.75$	45	56.25
③ Triangle	$\frac{1}{2}\times 4\times 5$ $= 10$	$2+2+\frac{4}{3}$ $= 5.33$	$1+\frac{5}{3}$ $= 2.66$	53.3	26.6
④ Rectangle	5×1 $= 5$	$2+2+\frac{5}{2}$ $= 6.5$	$\frac{1}{2}$ $= 0.5$	32.5	2.5

$\Sigma a = 36$

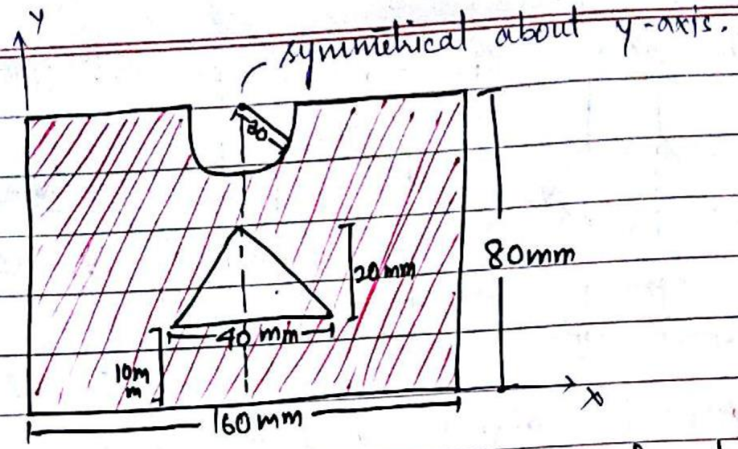
$\Sigma ax = 138.78$

$\Sigma ay = 97.35$

$$\therefore \quad \bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{138.78}{36} = 3.855\ m$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{97.35}{36} = 2.704\ m$$

Q. 8-



Determine the Centroid for the shaded area.

Ans-

Component	Area (mm²)	y (mm)	Σ ay
① Rectangle.	160×80 = 12800	$\frac{80}{2}$ = 40	512000
② Triangle.	$-\frac{1}{2}\times 20\times 40$ = -400	$10+\frac{20}{3}$ = 16.66	-6664
③ Semi-circle.	$-\frac{3.14\times(30)^2}{2}$ = -1413	$80-\frac{4\times 30}{3\times 3.14}$ = 67.261	-95039.793

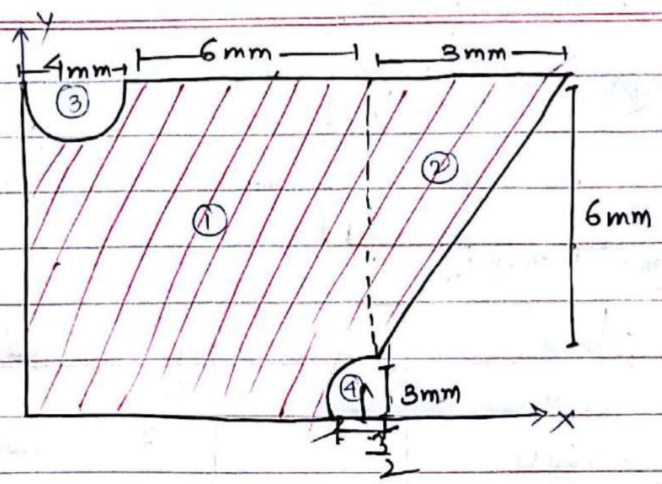
∴ $\Sigma a = 10987$

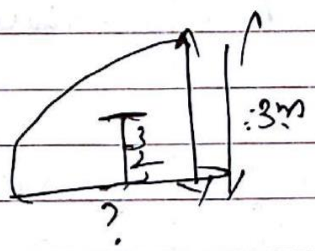
$\Sigma ay = 410296.207$.

∴ $\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{410296.207}{10987} = 37.34$ mm

$\bar{x} = \frac{160}{2} = 80$ mm

Q.9-





$\frac{4r}{3\pi}$

Ans-

Component	Area	x	y	Σax	Σay
① Rectangle.	$10 \times 9 = 90$	$\frac{10}{2} = 5$	$\frac{9}{2} = 4.5$	450	405
② Triangle	$+\frac{1}{2} \times 3 \times 6 = 9$	$10 + \frac{3}{3} = 11$	$3 + \frac{2}{3} \times 6 = 7$	99	63
③ Semi-circle.	$-\frac{3.14 \times (2)^2}{2} = -6.28$	$\frac{4}{2} = 2.$	$\frac{4 \times 2}{3 \times 3.14} = 0.84$	-12.56	-5.2752
④ Quater circle.	$-\frac{3.14 \times (3)^2}{4} = -7.065$	$10 - \frac{4 \times 3}{3 \times 3.14} = 8.726$	$\frac{3}{2} = 1.5$	-61.649	-10.5975

$\Sigma a = 85.655.$

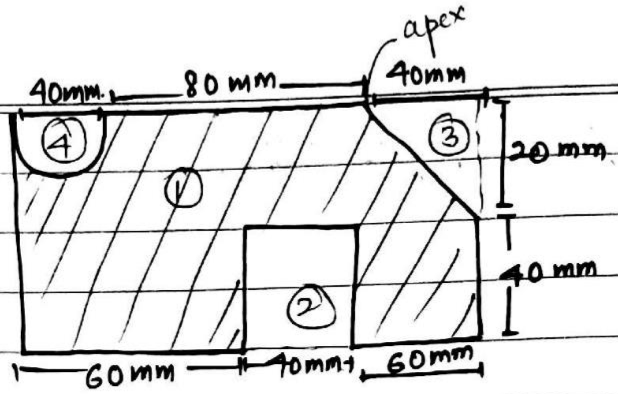
$\Sigma ax = 474.791.$

$\Sigma ay = 452.1273$

$\therefore \bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{474.791}{85.655} = 5.543$ mm

$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{452.1273}{85.655} = 5.278$ mm

Q. 10-



Ans -

Component	Area	x	y	Σax	Σa…
1 Rectangle	$160 \times 60 = 9600$	$\frac{160}{2} = 80$	$\frac{60}{2} = 30$	768000	288…
2 Rectangle	$-40 \times 40 = 1600$	$\frac{40}{2} + 60 = 80$	$\frac{40}{2} = 20$	(-128000)	(-320…
3 Triangle.	$-\frac{1}{\cancel{2}} \times \overset{10}{\cancel{20}} \times 40 = -400$	$120 + \frac{2}{3} \times 40 = 146.66$	$40 + \frac{2}{3} \times 20 = 53.33$	(-58664)	-2133…
4 Semi-circle	$-\frac{3.14 \times (20)^2}{2} = -628$	$\frac{40}{2} = 20$	$60 - \frac{4 \times 20}{3 \times 3.14} = 51.507$	-12560	-3234… 396…

$\Sigma a = 10172.$

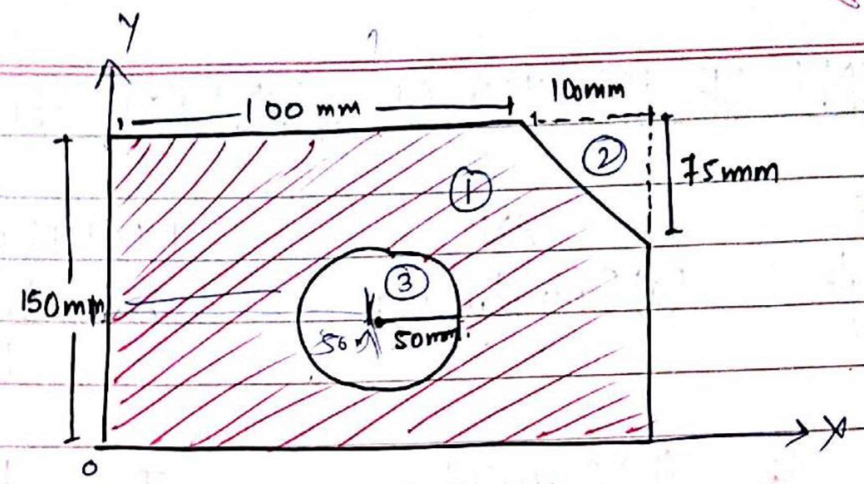
$\Sigma ax = 593896.$

$\Sigma ay = 202321.604$

$\therefore \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{202321.604}{10172} = 19.89 \text{ mm}$ //

$\bar{x} = \frac{\Sigma ax}{\Sigma a} = 58.385 \text{ mm}$ //

Q.11-



Ans-

	Component	Area (mm²)	x (mm)	y (mm)	Σax	ay
1	Rectangle	150×200 = 30000	$\frac{200}{2}$ = 100	$\frac{150}{2}$ = 75	3000000	2250000
2	Triangle	$\frac{1}{2}$ × 75 × 100 = 3750	100 + $\frac{2}{3}$ × 100 = 166.66	75 + $\frac{2}{3}$ × 75 = 125	624975	468750
3	Circle	3.14 × $(50)^2$ = 7850	~~50 + 50 = 100~~			

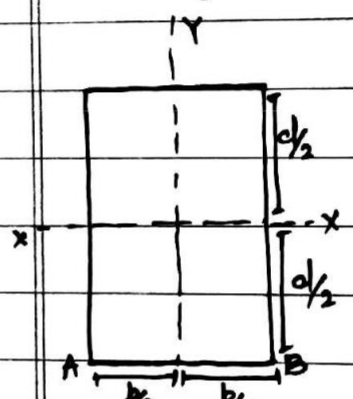
Σa = 41600

Σax

Σay =

- # MOMENT OF INERTIA OF STANDARD SECTIONS:-

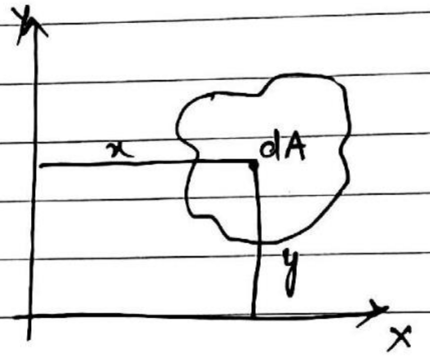
SHAPE	AXIS	MOMENT OF INERTIA
(1) Rectangle	(a) Centroidal axis X - X	$I_{xx} = \frac{bd^3}{12}$
	(b) Centroidal axis Y - Y	$I_{YY} = \frac{db^3}{12}$
	(c) A - B	



Difference b/w centroid & moment of inertia

* Moment of Inertia signifies shape and distribution of area.

The word inertia represent the property of materials by virtue of which it resists any change in the state of rest or uniform motion or rotational motion.



Consider a smaller area dA as shown in the figure of the Lamina with x and y as reference axis. The product of $dA \times y$ represent the moment

If $dA \times y \times y$ it becomes moment of inertia.

$dA \times y$ represent 1st moment of area.
$dA \times y \times y$ represent 2nd moment of area.

The moment of inertia of entire lamina along x-axis is given by

$$\boxed{I_{xx} = \int dA \times y^2} \ (mm)^4$$

Similarly moment of inertia along y-axis is given by

$$\boxed{I_{yy} = \int dA \times x^2} \ (mm)^4$$

The SI unit of moment of inertia is $(mm)^4$

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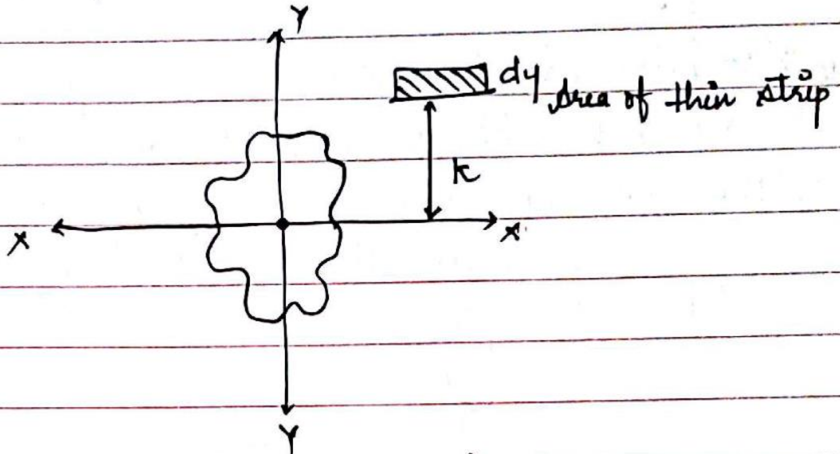
• RADIUS OF GYRATION :



Consider the area as shown in figure whose moment of inertia along x and y - axis are I_{xx} and I_{yy}. In order to maintain the same moment of inertia along the reference axis, the area must be thrown to a new location such that the elementary strips maintains the constant distance k.

Then the distance k can be defined as the distance of area @ which if it squeez and placed so that there is no change in moment of inertia is called as the radius of gyration k ie

$$I_{xx} = \int y^2 \, dA$$

$$I_{yy} = \int x^2 \, dA.$$

$$\therefore I_{xx} = k^2_{xx} \, dA$$

$$\boxed{K_{xx} = \sqrt{\frac{I_{xx}}{A}}} \quad mm$$

$$\boxed{K_{yy} = \sqrt{\frac{I_{yy}}{A}}} \quad mm.$$

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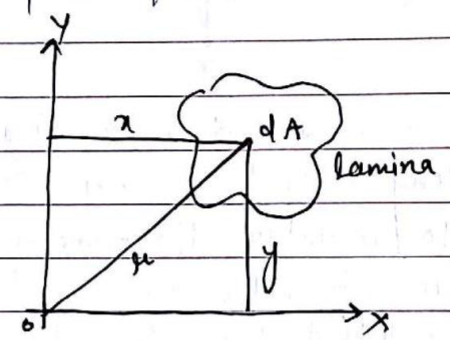
Imp • PERPENDICULAR AXIS THEOREM :-

Statement :- Moment of inertia of a given area about an axis ⊥ to the plane through a point is equal to the sum of moment of inertia of that area about any 2 ⊥ axis through that point. The axis will be in the plane of area.



Let x is the distance of dA from y-axis and y be the distance of dA from x-axis.
Then from fig.

$$r^2 = x^2 + y^2 \quad \text{(Pythagoras)}$$

If z is the axis through a coordinate system at O, then according to the theorem we need to prove

$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$

Proof : Let dA be an elemental area @ a distance r from O.

Then moment of inertia along z-axis is given by

$$I_{zz} = \Sigma dA \times r^2$$

We know that $r^2 = x^2 + y^2$

$$I_{zz} = \Sigma (x^2 + y^2)\, dA$$

$$= \Sigma x^2 dA + \Sigma y^2 dA$$

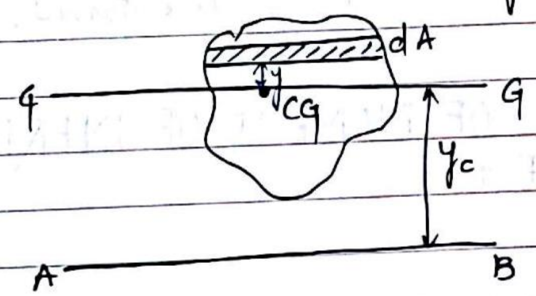
$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$ // Hence proved.

Imp. ● PARALLEL - AXIS THEOREM :-

Statement :- Moment of inertia of an area about any axis in a plane of area dA is equal to sum of moment of inertia about the centroidal parallel axis of the area and the product of the area and the square of distance of centroid of the area from the axis.

Then according to the theorem wrt to figure we need to prove:

$$\boxed{I_{AB} = I_{GG} + Ay_c^2}$$



Proof I_{AB} = moment of inertia of the area about the axis AB.

I_{GG} = moment of inertia of the area about the axis GG which is ∥ to AB.

y_c = distance of centroid G from the axis GG

Consider an elementary strip of area dA ∥ to AB and it is at a distance of y from GG axis.

Then acc. to theorem then $I_{AB} = \Sigma (y+y_c)^2 \cdot dA$

$$\Rightarrow I_{AB} = \Sigma(y^2 + 2yy_c + y_c^2)\, dA$$

$$\Rightarrow I_{AB} = \Sigma y^2 dA + \Sigma 2yy_c dA + \Sigma y_c^2 \quad \text{——— (1)}$$

Now, $\Sigma y^2 dA$ = moment of inertia along the centroidal axis GG = I_{GG}.

$$\therefore \Sigma y^2 dA = I_{GG} \quad \text{——— (2)}$$

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- PROCEDURE TO SOLVE THE PROBLEMS :-

Component	Area	x	y	Ax	Ay	Ax^2	Ay^2	I_x	I_y
	ΣA			ΣAx	ΣAy	ΣAx^2	ΣAy^2	ΣI_x	ΣI_y

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} \qquad \bar{y} = \frac{\Sigma Ay}{\Sigma A}$$

$$I_{1-1} = I_x + Ay^2$$

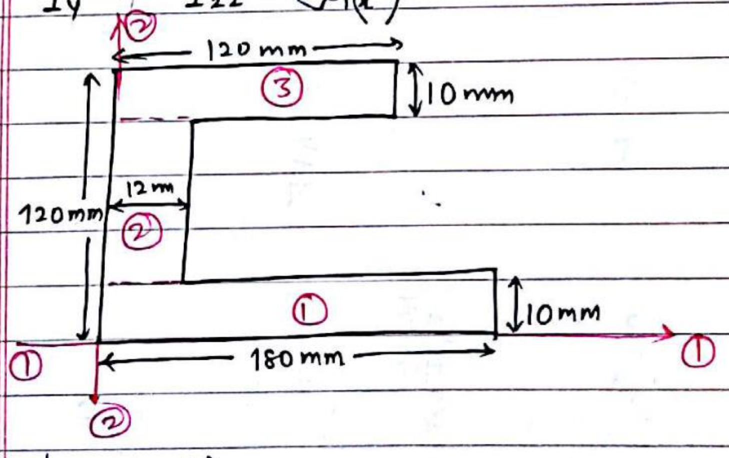
$$I_{1-1} = \Sigma I_x + \Sigma Ay^2$$

$$\bar{I}_x = I_{1-1} - \Sigma A(\bar{y}^2) \Rightarrow I_{1-1} - \Sigma A\left(\frac{\Sigma Ay}{\Sigma A}\right)^2$$

$$I_{22} = \Sigma I_x + \Sigma Ax^2$$

$$\bar{I}_y = I_{22} - \Sigma A(\bar{x}^2)$$

Q.1-



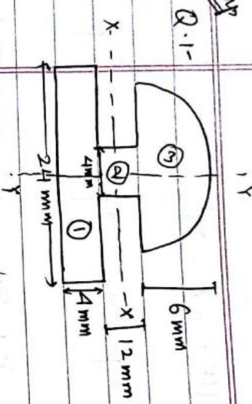
Ans-

	Area (mm^2)	x	y	Ax	Ay	Ay^2	Ax^2	I_x	I_y
(1)	$180\times10 = 1800$	$\frac{180}{2} = 90$	$\frac{10}{2} = 5$	162000	9000	45000	1458×10^4	$\frac{180\times10^3}{12}$	$\frac{10\times180^3}{2}$
(2)	$100\times12 = 1200$	$\frac{12}{2} = 6$	$10+\frac{100}{2} = 60$	7200	72000	4320000	432×10^2	$\frac{12\times100^3}{12}$	$\frac{100\times12^3}{12}$
(3)	$120\times10 = 1200$	$\frac{120}{2} = 60$	$10+100+\frac{10}{2} = 115$	72000	138000	15870000	432×10^4	$\frac{120\times10^3}{12}$	$\frac{10\times120^3}{12}$

Imp

Q.1-

Component	Area	~~x~~	y	~~Ax~~	Ay	Ay^2	I_x (mm)	I_y (mm^4)
Rectangle	$24 \times 4 = 96$		$\frac{4}{2} = 2$		192	384	$\frac{24 \times (4)^3}{12} = 128$	$\frac{(24)^3 \times 4}{12} = 4608$
Rectangle	$4 \times 12 = 48$		$4 + \frac{12}{2} = 10$		480	4800	$\frac{(4)(12)^3}{12} = 576$	$\frac{(4)^3\, 12}{12} = 64$
Semicircle	$\frac{3.14 \times (6)^2}{2} = 56.52$		$4 + 12 + \frac{4 \times 6}{3 \times 3.14} = 16 + \frac{24}{9.42} = 18.547$		1048.276	19442.383	$0.11 \times (6)^4$ ~~$3.14 \times (6)^4$~~ $= 142.56$	$\frac{3.14 \times 6^4}{8} = 508.68$
	$\Sigma A = 200.52$				$\Sigma Ay = 1720.276$	$\Sigma Ay^2 = 24626.383$	$\Sigma I_x = 846.56$	$\Sigma I_y = 5180.68$



$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = 8.579$

$I_{xx} = \Sigma I_x + \Sigma Ay^2$

I_{yy}

$2 > L$

I_y

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