Revision Notes

Class-12 Maths

Chapter 3 – Matrices

Matrix:

- It is an **ordered rectangular array** of collection of numbers or functions arranged in rows and columns is called matrix
- The numbers or functions are known as the elements or entries of the matrix.

  E.g. - $\begin{bmatrix} x & y \\ 1 & 2 \end{bmatrix}$

Row and Column of a Matrix:

- The **horizontal arrangement** of elements or entries are said to form the row of a matrix
- The **vertical arrangement** of elements or entries are said to form the Column of a matrix.

  E.g. - $\begin{bmatrix} x & y \\ 1 & 2 \end{bmatrix}$, This matrix has two rows and two columns.

Order of Matrix:

- It tells us about the **number of rows and columns of a matrix**.
- It is represented by $a \times b$ means a matrix has a rows and b columns.

  For example: $A = \begin{bmatrix} 2 & 8 & 3 \\ 1 & 9 & 8 \\ 0 & 7 & 0 \end{bmatrix}$, there are 3 rows and 3 columns therefore the order of matrix A is 3×3

Types of Matrices

a. Row Matrix: A matrix containing only **one row** is known as row matrix.

For E.g. - $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$

- The order of row matrix is $1 \times b$

b. Column Matrix: A matrix containing only **one column** is known as column matrix.

For E.g. - $\begin{bmatrix} 1 & 2 & 3 & -2 \end{bmatrix}$

- The order of column matrix is $a \times 1$

c. **Square Matrix:** The **number of rows and numbers of columns are equal** in the matrix.

For E.g. - $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 5 \\ 3 & 6 & 8 \end{bmatrix}$

- The order of square matrix is always $a \times a$, where a can be any natural number

d. **Diagonal Matrix:** If the **diagonal elements are non-zero** and all the non-diagonal elements of a matrix are zero, then such type of matrix is known as Diagonal Matrix.

For E.g. - $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

e. **Scalar Matrix:** It is a type of diagonal matrix in which **all diagonal elements are equal.**

For E.g. - $\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}, \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ etc.

f. **Identity Matrix:** It is a type of diagonal matrix in which **all diagonal elements are equal to 1.**

For E.g. - $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

g. **Zero Matrix:** In it all the elements are zero and this is also known as **null matrix.**

For E.g. - $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$ etc.

Equality of Matrices:

- Two matrices are equal if and only if the **order** of both the matrices are **equal** and element of one matrix is equal to the **corresponding element** of another matrix.

  For E.g. - $A = \begin{bmatrix} 1 & 8 \\ 8 & 4 \end{bmatrix}_{2\times2}$ and $B = \begin{bmatrix} 1 & 8 \\ 8 & 4 \end{bmatrix}_{2\times2}$

  All the elements of matrix A is equal to the corresponding elements of matrix B and order of both matrix is same. Hence, $A = B$.

Operations in Matrices:

a. Addition of matrices:

- Addition of two matrices can be done only when they have the **same order.**
- Addition can be done by adding the **corresponding entries** of the two matrices

  For e.g. –

  $A = \begin{bmatrix} 1 & 0 \\ 7 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$

  $C = A + B$

  $C = \begin{bmatrix} 1 & 0 \\ 7 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$

  $C = \begin{bmatrix} 3 & 1 \\ 10 & 9 \end{bmatrix}$

b. Multiplication of a matrix by a scalar:

- When a matrix is multiplied by scalar, then **each element** of matrix is **multiplied by the scalar** quantity and a new matrix is obtained.

  For E.g. –

  $2\begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix}$

  $\begin{bmatrix} 4\times2 & 5\times2 \\ 6\times2 & 7\times2 \end{bmatrix}$

  $\begin{bmatrix} 8 & 10 \\ 12 & 14 \end{bmatrix}$

c. Negative of a matrix:

- **Multiplying a matrix by -1** gives negative of that matrix

For E.g. - $A = \begin{bmatrix} 1 & -1 \\ -1 & -2 \end{bmatrix}$

Negative of Matrix A is

$$-A = (-1)A$$

$$-A = (-1)\begin{bmatrix} 1 & -1 \\ -1 & -2 \end{bmatrix}$$

$$-A = \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}$$

d. Difference of Matrices:

- Two matrices can be subtracted only when they have **same order**
- Subtraction can be done by subtracting the **corresponding entries** of the two matrices

  For e.g. –

  $A = \begin{bmatrix} 1 & 6 \\ 7 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 7 & 9 \end{bmatrix}$

  $$C = A - B$$

  $$C = \begin{bmatrix} 1 & 6 \\ 7 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 7 & 9 \end{bmatrix}$$

  $$C = \begin{bmatrix} -1 & 5 \\ 0 & -5 \end{bmatrix}$$

Properties of Matrix Addition:

1. Commutative Law: Matrix addition is **commutative** i.e., $A + B = B + A$.

2. Associative Law: Matrix addition is **associative** i.e., $(A + B) + C = A + (B + C)$.

3. Existence of Additive Identity: Zero matrix O is the additive identity of a matrix because adding a matrix with zero matrix leaves it unchanged i.e., $X + O = O + X = X$.

4. Existence of Additive Inverse: Additive inverse of a matrix is a matrix which on adding with another matrix **yield** 0 i.e., $X + (-X) = (-X) + X = 0$

Multiplication of Matrices:

- Multiplication of two matrices A and B is defined when **number of columns** of A is **equal to the number of rows** of B.

- Entries in rows is multiplied by corresponding entries in columns i.e., entries in first row are multiplied by entries in first column and similarly for other entries.

E.g. - $A=\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Product of A and B is

$$AB=\begin{bmatrix} 2(0)+1(1) & 2(2)+1(1) & 2(1)+1(1) \\ 1(0)+2(1) & 1(2)+2(1) & 1(1)+2(1) \end{bmatrix}$$

$$AB=\begin{bmatrix} 1 & 5 & 3 \\ 2 & 4 & 3 \end{bmatrix}$$

Properties of Matrix Multiplication:

1. **Non-Commutative Law:** Matrix multiplication is **not commutative** i.e., $AB \neq BA$ but not in the case of diagonal matrix.
2. **Associative Law:** Matrix multiplication **follow associative law** i.e., $A(BC)=(AB)C$
3. **Distributive Law:** Matrix multiplication **follow distributive law** i.e.,
   a) $A(B+C)=AB+AC$
   b) $(A+B)C=AC+BC$
4. **Existence of Multiplicative Identity: Identity matrix** I is the multiplicative identity of a matrix because multiplying a matrix with I leaves it unchanged.

Transpose of a Matrix:

- It is the matrix obtained by **interchanging the rows and columns** of the original matrix.
- It is denoted by P' or P^T if original matrix is P.

For E.g. –

$$P=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$P^T \text{ or } P' = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Properties of Transpose of Matrix:

1. $(A')'=A$
2. $(kA)'=kA'$ (Where, k is any constant)

3. $(A+B)' = A' + B'$

4. $(AB)' = B'A'$

Special Types of Matrices:

- **Symmetric Matrices:** It is a square matrix in which **original matrix is equal to its transpose.**

  For E.g. –

  $$P = \begin{bmatrix} 1 & -1 & 3 \\ -1 & 2 & 7 \\ 3 & 7 & 3 \end{bmatrix}$$

  Transpose of Matrix P, $P^T = \begin{bmatrix} 1 & -1 & 3 \\ -1 & 2 & 7 \\ 3 & 7 & 3 \end{bmatrix}$

  $\because P = P^T$

  Therefore, it is a Symmetric Matrix.

- **Skew-Symmetric Matrices:** It is a square matrix in which **original matrix is equal to the negative of its transpose**.

  For E.g. –

  $$P = \begin{bmatrix} 9 & 2 & -3 \\ -2 & 0 & 7 \\ 3 & -7 & 0 \end{bmatrix}$$

  Transpose of Matrix P, $P^T = (-1)\begin{bmatrix} 9 & 2 & -3 \\ -2 & 0 & 7 \\ 3 & -7 & 0 \end{bmatrix}$

  $\because P^T = -P$

  Therefore, it is a Skew-Symmetric Matrix.

Elementary Operation (Transformation) of a Matrix

Elementary operations can be performed by three ways

a. By **interchanging any two rows or two columns**.

- Interchange of i^{th} and j^{th} rows is denoted as $R_i \leftrightarrow R_j$
- Interchange of i^{th} and j^{th} columns is denoted by $C_i \leftrightarrow C_j$.

b. By **multiplying any scalar** to each element of any row or column of matrix.

- It is denoted as $R_i \leftrightarrow kR_j$ for rows and $C_i \leftrightarrow kC_j$ for columns

c. By **multiplying any scalar** to each element of any row or column and then **adding** the result to any other row or column.

- It is denoted as $R_i \leftrightarrow R_i + kR_j$ for rows and $C_i \leftrightarrow C_i + kC_j$ for column.

Invertible Matrix:

- A matrix A is **invertible** only when there exists another matrix B such that $AB = BA = I$, where I is identity matrix.
- It is a property of **square matrix**.
- Inverse of matrix is always **unique**.

For E.g. – Let us consider two matrices $A = \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & \frac{3}{2} \\ 1 & -1 \end{bmatrix}$

Now,

$$AB = \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} -1 & \frac{3}{2} \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I$$

And

$$BA = \begin{bmatrix} -1 & \frac{3}{2} \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I$$

Hence, B is inverse of A

Inverse of a matrix by elementary operations

- Inverse of a matrix can be obtained by using elementary operations.
- We know that $A = IA$ on **using elementary operation** on A only which is on the left side of equal to keeping right side one as it is and on I then the identity matrix I will become **inverse** of A

For example: Inverse of $A=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$ using elementary operation is

We know that $A=IA$

$$\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A$$

$$R_1\to\frac{R_1}{3}$$

$$\begin{bmatrix}1 & \frac{2}{3}\\ 1 & 4\end{bmatrix}=\begin{bmatrix}\frac{1}{3} & 0\\ 0 & 1\end{bmatrix}A$$

$$R_2\to R_2-R_1$$

$$\begin{bmatrix}1 & \frac{2}{3}\\ 0 & \frac{10}{3}\end{bmatrix}=\begin{bmatrix}\frac{1}{3} & 0\\ \frac{-1}{3} & 1\end{bmatrix}A$$

$$R_2\to R_2\times\frac{3}{10}$$

$$\begin{bmatrix}1 & \frac{2}{3}\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{3} & 0\\ \frac{-1}{10} & \frac{3}{10}\end{bmatrix}A$$

$$R_1\to R_1-\frac{2}{3}R_2$$

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{2}{5} & \frac{-1}{5}\\ \frac{-1}{10} & \frac{3}{10}\end{bmatrix}A$$

Since, $I=A^{-1}A$

Therefore, $A^{-1}=\begin{bmatrix}\frac{2}{5} & \frac{-1}{5}\\ \frac{-1}{10} & \frac{3}{10}\end{bmatrix}$