Revision Notes

Class - 11 Physics

Chapter 5- Laws of Motion

Force

A force is something that causes a body's rest or motion to alter. An interaction between two bodies is also referred to as force. Two bodies exert force on each other even if they are not physically in contact, e.g., electrostatic force between two charges or gravitational force between any two bodies.

It is a vector quantity having SI unit Newton (N) and dimension $\left[MLT^{--2}\right]$.

Superposition of force: When many forces are acting on a body then their resultant is obtained by law of vector addition:

$$\overrightarrow{F_{res.}} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} .... + \overrightarrow{F_n}$$

$$\overrightarrow{F_{res.}} = \sqrt{\overrightarrow{F_1}^2 + \overrightarrow{F_2}^2 + 2\overrightarrow{F_1}\ \overrightarrow{F_2} \cos\theta}$$

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$

The resultant of the two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ acting at angle θ is given by:

$$\overrightarrow{F_{res.}} = \sqrt{\overrightarrow{F_1}^2 + \overrightarrow{F_2}^2 + 2\overrightarrow{F_1}\ \overrightarrow{F_2} \cos\theta}$$

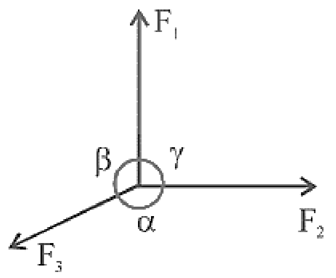
The resultant force is directed at an angle α with respect to force F_1 where:

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$

Lami's Theorem: - If three forces F_1, F_2 & F_3 are acting simultaneously on a body and the body is in equilibrium, then according to Lami's theorem:

$$\frac{F_1}{\sin(\pi - \alpha)} = \frac{F_2}{\sin(\pi - \beta)} = \frac{F_3}{\sin(\pi - \gamma)}$$

Where α, β & γ are the angles opposite to the forces F_1 , F_2 & F_3 respectively.



Basic Forces

There are basically four types of forces: **Weight**, **Contact force**, **Tension** and **Spring Force:**

Weight: It is the force with which earth attracts a body toward itself. It is also called the gravitational force.

Contact Force: When two bodies come in contact, they exert forces on each other that are called contact forces.

- **Normal Force (N)**: This is the contact force component that is normal to the surface. It determines how tightly two surfaces are forced together.
- **Frictional Force (f)**: It is the component of contact force parallel to the surface. It opposes the relative motion (or attempted motion) of the two in contact surfaces.

Tension: It refers to the force exerted by the end of a taut string, rope, or chain. The direction of strain is pulling the body, whereas the natural reaction is pushing it.

Spring Force: It is the resists to change its length; the more you alter its length the harder it resists. The force exerted by a spring is given by $F = -k.x$, where x is the change in length and k is the spring constant (unit Nm^{-1})

<u>NEWTONS LAWS OF MOTION</u>

Newton's First Law

If a body is at rest or in motion in a straight line, it will remain at rest or in motion unless it is acted by any external force. This is known as **Law of Inertia or First law of Newton**

Inertia is the property of inability of a body to change its position of rest or uniform motion in a straight line unless some external force acts on it.

Newton's first law is valid only in a frame of reference of inertial frame, i.e., if a frame of reference is at rest or in uniform motion it is called inertial, otherwise non-inertial.

Newton's Second Law-

According to second law, rate of change of momentum of a body is proportional to the resultant force acting on the body, i.e., $F \propto \frac{dp}{dt}$ or $F \propto ma$ $\left(P = mv = \frac{dp}{dt} = m\frac{dv}{dt}\right)$.

The applied resultant force causes a change in momentum in the direction of the applied resultant force is a measure of the total amount of motion in the body.

$$\vec{F} = m(d\vec{v}/dt) = m\vec{a} = \left(\overrightarrow{p_2} - \overrightarrow{p_1}\right)/t$$

External force acting on a body can accelerate it, either by changing the magnitude of velocity or direction of velocity or both.

Special Cases: -

Case-1- If the force is parallel or antiparallel to the motion of the body, then it changes only the magnitude of $\vec{v}$ but not the direction. Therefore, the path will be **straight line**.

Case-2- If the force is acting perpendicular to the motion of body, it changes only the direction but not the magnitude of $\vec{v}$. Therefore, the path will be **Circular**.

Case-3- If the force acts at an angle θ to the motion of a body, it changes both the magnitude and direction of $\vec{v}$. In this case the path of the body may be **elliptical, non-uniform circular, parabolic or hyperbolic.**

Newton's Third Law-

According to this law, for every action there is an equal and opposite reaction. E.g. when two bodies **A** and **B** exerts a force on each other i.e. F_A & F_B. Then the force exerted by any of the body will be same as the force exerted by another body, but in opposite direction.

$$F_{AB} = -F_{BA}$$

The two forces involved in any interaction between two bodies are called **action** and **reaction**.

Linear Momentum-

It is defined as the product of the mass of the body and its velocity i.e.

Linear momentum = mass*velocity

If a body of mass m is moving with a velocity v , its linear momentum $\vec{P}$ is given by:

$$\vec{P} = m\vec{v}$$

It is a **vector quantity** and its direction is the same as the direction of velocity of the body.

SI unit of linear momentum is $kg\ ms^{-1}$ and the **cgs** unit of linear momentum is $g\ cm\ s^{-1}$.

Impulse-

The entire change in linear momentum is used to calculate the force's impulse, which is the product of the average force during impact and the duration of the impact created during the collision.

The force which acts on bodies for short time are called impulsive forces. E.g. hitting a ball with a bat, firing a bullet with a gun etc.

An impulsive force does not remain constant instead, it varies from zero to maximum and then back to zero. Therefore, it is not possible to measure easily the value of impulsive force because it changes with time.

$$\vec{I} = \vec{F}_{av} \times t = \vec{p}_2 - \vec{p}_1$$

Apparent weight on a body in a lift

(a) When the lift is at rest, i.e. a=0: $mg - R = 0$ => $mg = R$

$$R = mg(1 - \frac{a}{g})$$

$W_{app.} = W_o(1 - \frac{a}{g})$

$R + mg - mg = 0$

At $F_{l_{AB}}$

$\Delta t = \; f_{ms} = \mu_s R$

$\angle AOC = \theta$

$W_{app.} = W_o$

(b) When the lift moves upwards with an acceleration a: $R - mg - ma = 0$

=> $R = m(g + a)$

=> $R = mg(1 + \frac{a}{g})$

$\therefore W_{app.} = W_o(1 + \frac{a}{g})$

(c) When the lift moves downwards with an acceleration a: $R + ma - mg = 0$

=> $R = mg(1 - \frac{a}{g})$

$\therefore \; W_{app.} = W_o(1 - \frac{a}{g})$

(d) When the lift falls freely, i.e.,a = g: $R + mg - mg = 0$

=> $R = 0$

=> $\therefore W_{app.} = 0$

Principle of conservation of Linear Momentum

According to this principle, in an isolated system, the vector sum of all the system's linear momenta is conserved and is unaffected by their interactions reciprocal action and response.

Mutual forces between pairs of particles in an isolated system (i.e., a system with no external force) can thus produce changes in linear momentum of individual particles. The linear momentum changes cancel in pairs, and the overall linear momentum remains unaltered, because the mutual forces for each pair are equal and opposing. As a result, an isolated system of interacting particles' total linear momentum is conserved. This principle is a direct result of Newton's second and third laws of motion.

Consider an isolated system that consists of two bodies **A** and **B** with initial linear momenta P_A and P_B. Allow them to collide for a short time **t** before separating with linear momenta P_A and P_B, respectively.

If $F_{l_{AB}}$ is force on A exerted by B, and F_{BA} is force on B exerted by A, then according to second law of newton $F_{l_{AB}}$ **.**

Δt Change in linear momentum of A = $\vec{p}_A - \vec{p}_A$

$F_{BA} \times \Delta t =$ Change in linear momentum of B= $\vec{p}_B' - \vec{p}_B$

Now, according to third law of newton: $F_{AB} = -F_{BA}$

$\therefore$ From eqns. $\vec{p}_A' - \vec{p}_A = -(\vec{p}_B - \vec{p}_B)$ or $\vec{p}_A' + \vec{p}_B' = \vec{p}_A + \vec{p}_B$

Which shows that total final linear momentum of the isolated system is equal to its total initial linear momentum. This proves the principle of **conservation of linear momentum**.

<u>Friction</u>

Friction as an **opposing force** that comes into play when one body actually moves (slides or rolls) or even tries to move over the surface of another body.

It is the force that comes into play when two surfaces comes into contact to each other and opposes their relative motion.

I Frictional force is unaffected by contact area. Because with increase in area of contact, force of adhesion also increases.
II. When the surfaces in contact are extra smooth, distance between the molecules of the surfaces in contact decreases, increasing the adhesive force between them. Therefore, the adhesive pressure increases, and so does the force of friction.

Types of friction

There are 3 types of friction: Static, Limiting and Kinetic Friction.

Static Friction- The opposing force that comes into play when one body tends to move over the surface of another body, but the actual motion has yet not started in called Static friction.

Limiting Friction- Limiting friction is the maximum opposing force that comes into play, when one body is just at the verge of moving over the surface of the other body.

Kinetic Friction - Kinetic friction or dynamic friction is the opposing force that comes into play when one body is actually moving over the surface of another body.

Laws of Limiting Friction

i. Friction always works in the opposite direction of relative motion, making it a perverse force.
ii. The maximum static friction force, fms (also known as limiting friction), is proportional to the normal reaction (R) between the two in contact surfaces i.e. $f_{ms} \propto R$
iii. The limiting friction force is tangential to the interface between the two surfaces and is determined by the kind and state of polish of the two surfaces in contact.
iv. As long as the normal reaction stays constant, the limiting friction force is independent of the area of the surfaces in contact.

Coefficient of Static friction-

W.K.T $f_{ms} \propto R => f_{ms} = \mu_s R$

Here, μ_s is a constant of proportionality and is called the **coefficient of static friction** and its value depends upon the nature of the surfaces in contact and is usually less than unity i.e. 1 but never 0.

Since the force of static friction (f s) can have any value from zero to maximum (f_{ms}), i.e. $f_s \leqslant f_{ms}$.

Kinetic Friction-

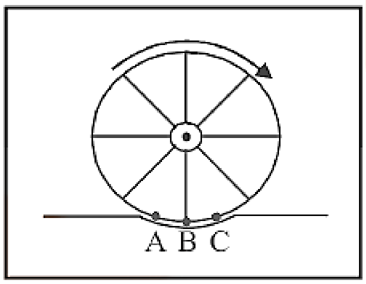
The laws of kinetic friction are exactly the same as those for static friction. Accordingly, the force of kinetic friction is also directly proportional to the normal reaction; $f_k \propto R$

=> $f_k = \mu_k R$

Rolling Friction-

The opposing force that comes into play when a body rolls over the surface of another body is called the rolling friction.

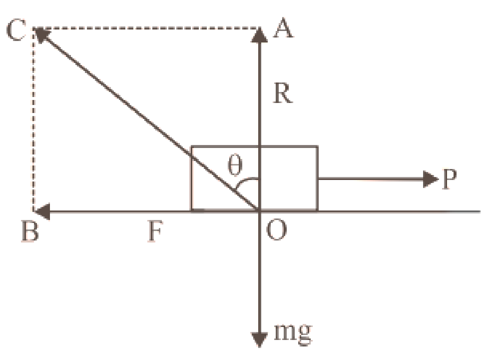
Friction caused by rolling. Consider a wheel that is rolling down a street. As the wheel travels down the road, it presses against the road's surface and is compressed slightly as shown in Fig. :



ANGLE OF FRICTION

The angle of friction between any two surfaces in contact is defined as the angle formed by the resultant of the limiting friction force F and the normal reaction direction R. It is represented by θ.

In the given fig. OA represents the normal reaction R that balances the weight mg of the body. OB represent F, the limiting force of sliding friction, when the body tends to move to the right. Complete the parallelogram OACB. Join OC. This represents the resultant of R and F. By definition, $\angle AOC = \theta$. is the angle of friction between the two bodies in contact.



The angle of friction is determined by the nature of the materials used on the surfaces in contact as well as the nature of the surfaces themselves.

Relation between μ and θ:

In ΔAOC, $\tan\theta = \frac{AC}{OA} = \frac{OB}{OA} = \mu$ $\tan\theta = \frac{AC}{OA} = \frac{OB}{OA} = \mu$

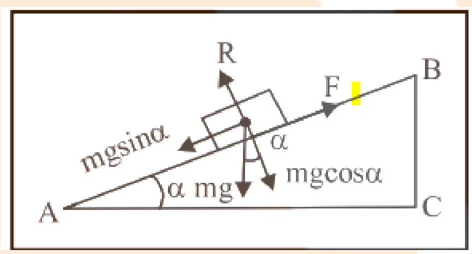
Hence, $\mu = \tan\theta$ ... (6) (i.e. μ is the coefficient of limiting friction)

<u>ANGLE OF REPOSE OR ANGLE OF SLIDING</u>

The minimum angle of inclination of a plane with the horizontal at which a body placed on the plane begins to slide down is known as the angle of repose or angle of sliding.

Represented by α. Its value depends on material and nature of the surfaces in contact.

In fig., AB is an inclined plane such that a body placed on it just begins to slide down. $\angle BAC$ α = angle of repose.



The various forces involved are :

i. weight, mg of the body,
ii. normal reaction, R,
iii. Force of friction F,

Now, mg can be resolved into two rectangular components : mg cosα opposite to R and mg sinα opposite to F**:** $F = mg\sin\alpha$ ... (7)

$R = mg\cos\alpha$ ... (8)

Diving these two eq. we get, $\tan\alpha = \frac{F}{R}$ => $\tan\alpha = \mu$ ... (9)

Hence coefficient of limiting friction between any two surfaces in contact is equal to the tangent of the angle of repose between them.

From (6) and (9): $\mu = \tan\alpha = \tan\theta$

Therefore $\alpha = \theta$ i.e. (Angle of friction = Angle of repose)

METHODS OF CHANGING FRICTION

Some of the ways of reducing friction are:

i. By polishing.
ii. By lubrication.
iii. By proper selection of materials.
iv. By Streamlining.
v. By using ball bearings.

DYNAMICS OF UNIFORM CIRCULAR MOTION CONCETP OF CENTRIPETAL FORCE

The force required to move a body uniformly in a circle is known as centripetal force. This force acts along the circle's radius and towards the centre.

When a body moves in a circle, the direction of motion at any given time is along the tangent to the circle. According to Newton's first law of motion, a body cannot change its direction of motion by itself an external force is needed. This external force is called the centripetal force.

An expression for centripetal force is:

$$F = mv^2 / r = m\omega^2 r$$

$$R - mg = 0 \text{ or } R = mg$$

$$F = \mu_s R = \mu_s mg$$

$$R\ cos\theta = mg + F\ sin\theta$$

$$\text{From } (3),\ R\ (cos\theta - \mu_s\ sin\theta) = mg$$

On account of a continuous change in the direction of motion of the body, there is a change in velocity of the body, an hence it undergoes an acceleration, called **centripetal acceleration** or **radial acceleration**.

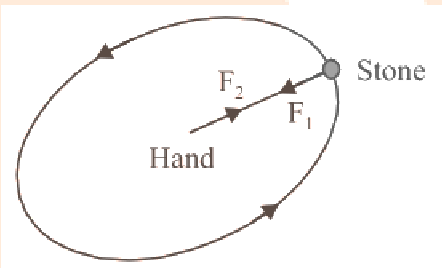
CENTRIFUGAL FORCE

Centrifugal force is a force that arises when a body is moving actually along a circular path, by virtue of tendency of the body to regain its natural straight line path.

When a body is moving in a straight-line centripetal force is applied on the body, it is forced to move along a circle. The body has a natural inclination to return to its normal straight-line course while moving in a circle. This tendency gives rise to a force called **centrifugal force.**

Magnitude of centrifugal force = mv^2/r, which is same as that of centripetal force, but opposite in direction i.e. The centrifugal force acts along the circle's radius, away from the centre.

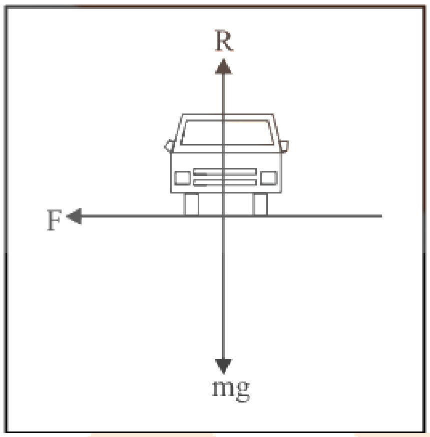
Note: Centripetal and Centrifugal forces, being the forces of action and reaction act always on different bodies. E.g. when a piece of stone tied to one end of a string is rotated in a circle, centripetal force F1 is applied on the stone by the hand. Due to the stone's desire to revert to its natural straight line route, centrifugal force F2 acts on it, pulling the hand outwards.. The centripetal and centrifugal forces are shown in Fig. :



ROUNDING A LEVEL CURVED ROAD

When a vehicle goes round a curved road, it requires some centripetal force. The vehicle's wheels have a tendency to depart the curved path and return to the straight-line path as it rounds the curve. Force of friction between the wheels and the road opposes this tendency of the wheels. This force (friction) therefore, acts, towards the centre of the circular track and provides the necessary centripetal force.

Three forces are acting on the car, fig.



i. The weight of the car, **mg**, acting **vertically downwards**,
ii. **Normal reaction R** of the road on the car, acting **vertically upwards**,
iii. **Frictional Force F**, along the surface of the road, towards the centre of the turn.

As there is no acceleration in the vertical direction,

$R - mg = 0 \text{ or } R = mg$ ...(1)

The centripetal force required for circular motion is applied along the road's surface, toward the turn's centre. As previously stated, static friction is what supplies the required centripetal force. Clearly $F \leqslant mv^2 / r$ …(2)

where v is velocity of car and r is the radius of the curved path

As, $F = \mu_s R = \mu_s mg$ (μ_s is coefficient of static friction between the tyres and the road)

Therefore from (2),

$$\frac{mv^2}{r} \leqslant \mu_s mg \text{ or } v \leqslant \sqrt{\mu_s rg} \therefore \underline{v_{max} = \sqrt{\mu_s rg}} \text{ ……(3)}$$

Therefore, the maximum velocity with which car can move without slipping is: $v = \sqrt{\mu_s rg}$

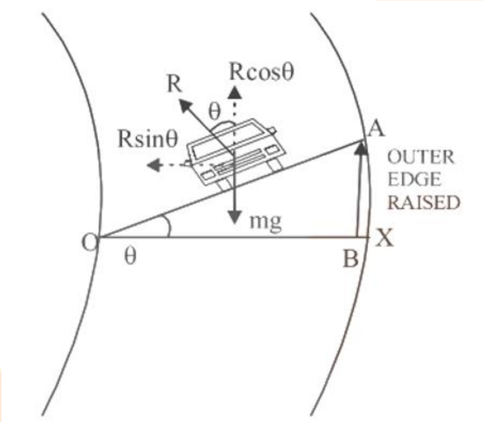
BANKING OF ROADS

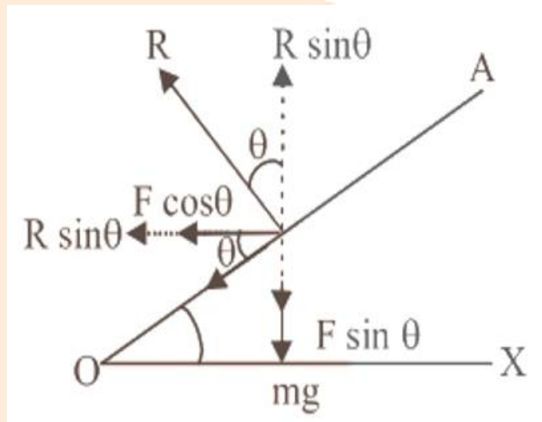
The phenomenon of raising outer edge of the curved road above the inner edge is called banking of roads.

The maximum permissible velocity with which a vehicle can go round a level curved road without skidding depends on **µ (Coefficient of friction between the**

tyre and the road). As a result, frictional force is not a reliable source of the requisite centripetal force for the vehicle.

In Fig., OX is a horizontal line. OA is the level of banked curved road whose outer edge has been raised. $\angle XOA = \theta$ = angle of banking.





Forces are acting on the vehicle as shown in Fig:

i. Weight mg of the vehicle acting vertically downwards.
ii. Normal reaction R of the banked road acting upwards in a direction perpendicular to OA.
iii. Force of friction F between the banked road and the tyres, acting along AO. R can be resolved into two rectangular components-
i. **R cosθ**, along vertically upward direction
ii. **R sinθ**, along the horizontal, towards the centre of the curved road. F can also be resolved into two rectangular components:

(i) **F cosθ**, along the horizontal, towards the centre of curved road

(ii) **F sinθ**, along vertically downward direction.

As there is no acceleration along the vertical direction, the net force along this direction must be zero. Therefore,

$$R\cos\theta = mg + F\sin\theta \quad \ldots\ldots(1)$$

If v is the vehicle's velocity on a banked circular road with radius r, then centripetal force is $= mv^2/r$. This is provided by the horizontal components of **R** and **F** as shown in Fig.

Therefore, $$R\sin\theta + F\cos\theta = \frac{mv^2}{r} \quad \ldots(2)$$

But F < μ_s R, where μ_s is coefficient of static friction between the banked road and the tyres. To obtain $v_{\max}$, we put $F = \mu_s R$ in (1) and (2)

$$R\cos\theta = mg + \mu_s R\sin\theta \qquad \ldots (3)$$

And $R\sin\theta + \mu_s R\cos\theta = \frac{mv^2}{r} \ldots (4)$

From (3), $R\ (cos\theta - \mu_s\ sin\theta) = mg$

$$R = \frac{mg}{\cos\theta - \mu_s \sin\theta} \ldots (5)$$

From (4), $R(\sin\theta + \mu_s\cos\theta) = \frac{mv^2}{r}$

Using(5), $\frac{mg(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)} = \frac{mv^2}{r}$

$$\therefore \quad v^2 = \frac{rg(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)} = \frac{rg\cos\theta(\tan\theta + \mu_s)}{\cos\theta(1 - \mu_s\tan\theta)}$$

$$v = \left[\frac{rg(\mu_s + \tan\theta)}{(1 - \mu_s\tan\theta)}\right]^{1/2} \ldots (6)$$

Discussion-

1. If μ_s= 0, i.e., if banked road is perfectly smooth, then from eqn. (51

   $v_o = (rg\tan\theta)^{\frac{1}{2}}$

Even when there is no friction, this is the speed at which a banked road may be rounded. On a banked road, driving at this speed causes essentially little wear and tear tyres.

$v_o^{\,2} = rg\ \tan\theta$ =>

$$\vec{F}_{real} + \vec{F}_{pseudo} = m_p a_{P,O}$$

$$\vec{F}_{real} - m_p\vec{a}_o = m_p\vec{a}_{P,O}$$

.

2. If speed of vehicle is less than v_o, frictional force will be up the slope. Therefore, the vehicle can be parked only if $\tan\theta \leqslant \mu_s$. The average speed of vehicles passing through a road is usually banked. However, if a vehicle's speed is slightly less or more than this, the self-adjusting static friction between the tyres and the road will operate, and the vehicle will not slide.

Note that curved railway tracks are also banked for the same reason. The level of outer rail is raised a little above the level of inner rail, while laying a curved railway track.

BENDING OF A CYCLIST

When a cyclist turns, he must also exert centripetal force. His weight is balanced by the usual reaction of the ground if he keeps himself vertical while spinning. In that case, he must rely on the friction between the tyres and the road to generate the required centripetal force. Because the force of friction is modest and unpredictable, relying on it is risky.

In order to avoid relying on friction to generate centripetal power, the cyclist must bend slightly inwards from his vertical position while turning. A component of normal reaction in the horizontal direction produces the required centripetal force in this way.

To calculate the angle of bending with vertical, suppose

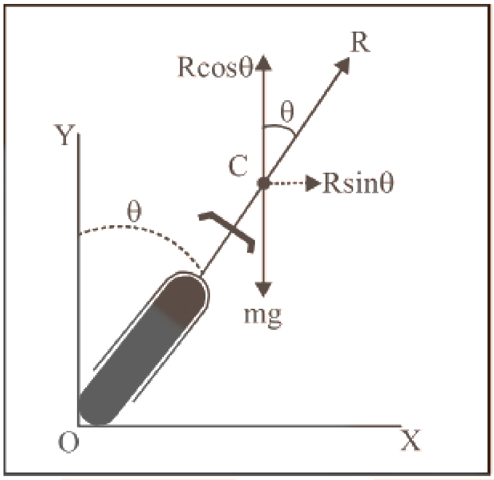
m = mass of the cyclist,

v = velocity of the cyclist while turning,

r = radius of the circular path,

θ = angle of bending with the vertical

In the fig. shown below weight of the cyclist(**mg**) acts vertically downwards at the centre of gravity **C**. **R** is the normal reaction on the cyclist. It acts at an angle θ with the vertical.



R can be resolved into two rectangular components: R cosθ, along the vertical upward direction, R sinθ, along the horizontal, towards the centre of the circular track.

In equilibrium, $R\ cos\theta = mg$

$R\ cos\theta = mg$ ….(1)

$R\ \sin\theta = mv^2/r$ ….(2)

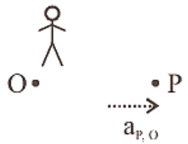
Dividing (2) by (1), we get, $\tan\theta = \dfrac{v^2}{rg}$

For a safe turn, θ should be small, for which v should be small and r should be large i.e. Turning should be done slowly and on a greater radius track. This means, a safe turn should neither be fast nor sharp.

PSEUDO FORCE

$\vec{F}_{pseudo} = -m_p\vec{a}_0$

If observer O is non-inertial and still wants to apply Newton's Second Law on particle P, observer must add a "Pseudo force" to the real forces on particle P.



$\vec{F}_{real} + \vec{F}_{pseudo} = m_p a_{P,O}$

i.e $\vec{F}_{real} - m_p\vec{a}_o = m_p\vec{a}_{P,O}$

Where $\vec{a}_{P,O}$ is acceleration of P with respect to observer O.