SETS, RELATIONS & FUNCTIONS

SETS

1. SET

A set is a collection of well-defined and well distinguished objects of our perception or thought.

1.1 Notations

The sets are usually denoted by capital letters A, B, C, etc. and the members or elements of the set are denoted by lower-case letters a, b, c, etc. If x is a member of the set A, we write $x \in A$ (read as 'x belongs to A') and if x is not a member of the set A, we write $x \notin A$ (read as 'x does not belong to A,). If x and y both belong to A, we write $x, y \in A$.

2. REPRESENTATION OF A SET

Usually, sets are represented in the following two ways :

(i) Roster form or Tabular form

(ii) Set Builder form or Rule Method

2.1 Roster Form

In this form, we list all the member of the set within braces (curly brackets) and separate these by commas. For example, the set A of all odd natural numbers less that 10 in the Roster form is written as :

$A = \{1, 3, 5, 7, 9\}$

Note...

(i) In roster form, every element of the set is listed only once.

(ii) The order in which the elements are listed is immaterial.

For example, each of the following sets denotes the same set $\{1, 2, 3\}$, $\{3, 2, 1\}$, $\{1, 3, 2\}$

2.2 Set-Builder Form

In this form, we write a variable (say x) representing any member of the set followed by a property satisfied by each member of the set.

For example, the set A of all prime numbers less than 10 in the set-builder form is written as

$A = \{x \mid x \text{ is a prime number less that } 10\}$

The symbol '|' stands for the words 'such that'. Sometimes, we use the symbol ':' in place of the symbol '|'.

3. TYPES OF SETS

3.1 Empty Set or Null Set

A set which has no element is called the null set or empty set. It is denoted by the symbol ϕ .

For example, each of the following is a null set :

(a) The set of all real numbers whose square is –1.

(b) The set of all rational numbers whose square is 2.

(c) The set of all those integers that are both even and odd.

A set consisting of atleast one element is called a non-empty set.

3.2 Singleton Set

A set having only one element is called singleton set.

For example, $\{0\}$ is a singleton set, whose only member is 0.

3.3 Finite and Infinite Set

A set which has finite number of elements is called a finite set. Otherwise, it is called an infinite set.

For example, the set of all days in a week is a finite set whereas the set of all integers, denoted by

$\{\ldots\ldots\ldots -2, -1, 0, 1, 2, \ldots\}$ or $\{x \mid x \text{ is an integer}\}$, is an infinite set.

An empty set ϕ which has no element in a finite set A is called empty of void or null set.

3.4 Cardinal Number

The number of elements in finite set is represented by n(A), known as Cardinal number.

3.5 Equal Sets

Two sets A and B are said to be equals, written as A = B, if every element of A is in B and every element of B is in A.

3.6 Equivalent Sets

Two finite sets A and B are said to be equivalent, if n (A) = n (B). Clearly, equal sets are equivalent but equivalent sets need not be equal.

For example, the sets A = {4, 5, 3, 2} and B = {1, 6, 8, 9} are equivalent but are not equal.

3.7 Subset

Let A and B be two sets. If every elements of A is an element of B, then A is called a subset of B and we write $A \subset B$ or $B \supset A$ (read as 'A is contained in B' or B contains A'). B is called superset of A.

Note...

(i) Every set is a subset and a superset itself.

(ii) If A is not a subset of B, we write $A \not\subset B$.

(iii) The empty set is the subset of every set.

(iv) If A is a set with $n(A) = m$, then the number of subsets of A are 2^m and the number of proper subsets of A are $2^m - 1$.

For example, let A = {3, 4}, then the subsets of A are ϕ, {3}, {4}. {3, 4}. Here, $n(A) = 2$ and number of subsets of $A = 2^2 = 4$. Also, $\{3\} \subset \{3,4\}$ and $\{2,3\} \not\subset \{3,4\}$

3.8 Power Set

The set of all subsets of a given set A is called the power set of A and is denoted by P(A).

For example, if A = {1, 2, 3}, then

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\} \{1,3\}, \{2,3\}, \{1,2,3\}\}$

Clearly, if A has n elements, then its power set P (A) contains exactly 2^n elements.

4. OPERATIONS ON SETS

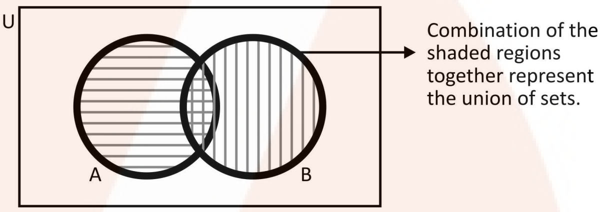
4.1 Union of Two Sets

The union of two sets A and B, written as $A \cup B$ (read as 'A union B'), is the set consisting of all the elements which are either in A or in B or in both Thus,

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

Clearly, $x \in A \cup B \Rightarrow x \in A$ **or** $x \in B$, and

$x \notin A \cup B \Rightarrow x \notin A$ **and** $x \notin B$.



For example, if A = {a, b, c, d} and B = {c, d, e, f}, then $A \cup B$ = {a, b, c, d, e, f}

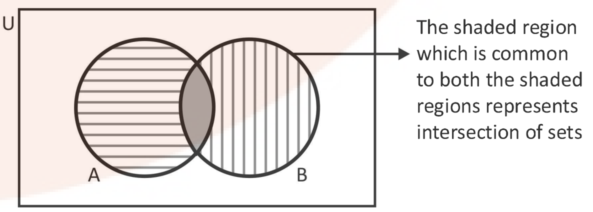
4.2 Intersection of Two sets

The intersection of two sets A and B, written as $A \cap B$ (read as 'A' intersection 'B') is the set consisting of all the common elements of A and B. Thus,

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

Clearly, $x \in A \cap B \Rightarrow x \in A$ **and** $x \in B$, and

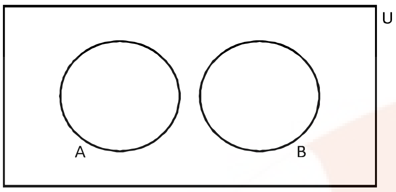
$x \notin A \cap B \Rightarrow x \notin A$ **or** $x \notin B$.



For example, if A = {a, b, c, d) and B = {c, d, e, f}, then $A \cap B$ = {c, d}.

4.3 Disjoint Sets

Two sets A and B are said to be disjoint, if $A \cap B = \phi$, i.e. A and B have no element in common.



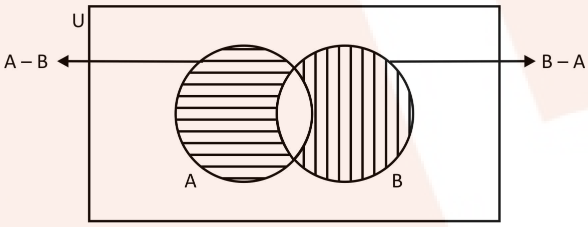
For example, if A = {1, 3, 5} and B = {2, 4, 6}, then $A \cap B = \phi$, so A and B are disjoint sets.

4.4 Difference of Two Sets

If A and B are two sets, then their difference A - B is defined as :

$A - B = \{x : x \in A \text{ and } x \notin B\}$.

Similarly, $B - A = \{x : x \in B \text{ and } x \notin A\}$.



For example, if A = {1, 2, 3, 4, 5} and B = {1, 3, 5, 7, 9} then A - B = {2, 4} and B - A = {7, 9}.

Important Results

(a) $A - B \neq B - A$

(b) The sets A - B , B - A and $A \cap B$ are disjoint sets

(c) $A - B \subseteq A$ and $B - A \subseteq B$

(d) $A - \phi = A$ and $A - A = \phi$

4.5 Symmetric Difference of Two Sets

The symmetric difference of two sets A and B , denoted by $A \Delta B$, is defined as

$A \Delta B = (A - B) \cup (B - A)$.

For example, if A = {1,2,3,4,5} and B = {1, 3,5,7,9} then

$A \Delta B = (A - B) \cup (B - A) = \{2,4\} \cup \{7,9\} = \{2,4,7,9\}$.

4.6 Complement of a Set

If U is a universal set and A is a subset of U, then the complement of A is the set which contains those elements of U, which are not contained in A and is denoted by A'or A^c. Thus,

$A^c = \{x : x \in U \text{ and } x \notin A\}$

For example, if U = {1,2,3,4 ...} and A {2,4,6,8,...}, then, A^c = {1,3,5,7, ...}

Important Results

a) $U^c = \phi$ b) $\phi^c = U$ c) $A \cup A^c = U$

d) $A \cap A^c = \phi$

5. ALGEBRA OF SETS

1. For any set A , we have

 a) $A \cup A = A$ b) $A \cap A = A$

2. For any set A, we have

 c) $A \cup \phi = A$ d) $A \cap \phi = \phi$

 e) $A \cup U = U$ f) $A \cap U = A$

3. For any two sets A and B, we have

 g) $A \cup B = B \cup A$ h) $A \cap B = B \cap A$

4. For any three sets A, B and C, we have

 i) $A \cup (B \cup C) = (A \cup B) \cup C$

 j) $A \cap (B \cap C) = (A \cap B) \cap C$

5. For any three sets A, B and C, we have

 k) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

 l) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

6. If A is any set, we have $(A^c)^c = A$.

7. Demorgan's Laws For any three sets A, B and C, we have

 m) $(A \cup B)^c = A^c \cap B^c$

 n) $(A \cap B)^c = A^c \cup B^c$

 o) $A - (B \cup C) = (A - B) \cap (A - C)$

 p) $A - (B \cap C) = (A - B) \cup (A - C)$

Important Results on Operations on Sets

(i) $A \subseteq A \cup B, B \subseteq A \cup B, A \cap B \subseteq A, A \cap B \subseteq B$

(ii) $A - B = A \cap B^c$ (iii) $(A - B) \cup B = A \cup B$

(iv) $(A - B) \cap B = \phi$ (v) $A \subseteq B \Leftrightarrow B^c \subseteq A^c$

(vi) $A - B = B^c - A^c$ (vii) $(A \cup B) \cap (A \cup B^c) = A$

(viii) $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$

(ix) $A - (A - B) = A \cap B$

(x) $A - B = B - A \Leftrightarrow A = B$ (xi) $A \cup B = A \cap B \Leftrightarrow A = B$

(xii) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

Example – 1

Write the set of all positive integers whose cube is odd.

Sol. The elements of the required set are not even.

[$\because$ Cube of an even integer is also an even integer]

Moreover, the cube of a positive odd integer is a positive odd integer.

$\Rightarrow$ The elements of the required set are all positive odd integers.

Hence, the required set, in the set builder form, is :

$\{2k + 1 : k \geq 0, k \in Z\}$.

Example – 2

Write the set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}\right\}$ in the set builder form.

Sol. In each element of the given set the denominator is one more than the numerator.

Also the numerators are from 1 to 7.

Hence the set builder form of the given set is :

$\{x : x = n/n+1, n \in N \text{ and } 1 \leq n \leq 7\}$.

Example – 3

Write the set $\{x : x$ is a positive integer and $x^2 < 30\}$ in the roster form.

Sol. The squares of positive integers whose squares are less than 30 are : 1, 2, 3, 4, 5.

Hence the given set, in roster form, is $\{1, 2, 3, 4, 5\}$.

Example – 4

Write the set $\{0, 1, 4, 9, 16, ...\}$ in set builder form.

Sol. The elements of the given set are squares of integers :

$0, \pm 1, \pm 2, \pm 3, \pm 4, ...$

Hence the given set, in set builder form, is $\{x^2 : x \in Z\}$.

Example – 5

State which of the following sets are finite and which are infinite

(i) $A = \{x : x \in N$ and $x^2 - 3x + 2 = 0\}$

(ii) $B = \{x : x \in N$ and $x^2 = 9\}$

(iii) $C = \{x : x \in N$ and x is even$\}$

(iv) $D = \{x : x \in N$ and $2x - 3 = 0\}$.

Sol. (i) $A = \{1, 2\}$.

[$\because x^2 - 3x + 2 = 0 \Rightarrow (x - 1)(x - 2) = 0 \Rightarrow x = 1, 2$]

Hence A is finite.

(ii) $B = \{3\}$.

[$\because x^2 = 9 \Rightarrow x = \pm 3$. But $3 \in N$]

Hence B is finite.

(iii) $C = \{2, 4, 6, ...\}$

Hence C is infinite.

(iv) $D = \phi$. $\left[\because 2x - 3 = 0 \Rightarrow x = \frac{3}{2} \notin N\right]$

Hence D is finite.

Example – 6

Which of the following are empty (null) sets ?

(i) Set of odd natural numbers divisible by 2

(ii) $\{x : 3 < x < 4, x \in N\}$

(iii) $\{x : x^2 = 25$ and x is an odd integer$\}$

(iv) $[x : x^2 - 2 = 0$ and x is rational$]$

(v) $\{x : x$ is common point of any two parallel lines$\}$.

Sol. (i) Since there is no odd natural number, which is divisible by 2.

$\therefore$ it is an empty set.

(ii) Since there is no natural number between 3 and 4.

$\therefore$ it is an empty set.

(iii) Now $x^2 = 25 \Rightarrow x = \pm 5$, both are odd.

$\therefore$ The set $\{-5, 5\}$ is non-emptry.

(iv) Since there is no rational number whose square is 2,

$\therefore$ the given set is an empty set.

(v) Since any two parallel lines have no common point,

$\therefore$ the given set is an empty set.

Example – 7

Find the pairs of equal sets from the following sets, if any, giving reasons :

$A = \{0\}$, $B = \{x : x > 15$ and $x < 5\}$,

$C = \{x : x - 5 = 0\}$, $D = \{x : x^2 = 25\}$,

$E = \{x : x$ is a positive integral root of the equation $x^2 - 2x - 15 = 0\}$.

Sol. Here we have,

$A = \{0\}$

$B = \phi$

$[\because$ There is no number, which is greater than 15 and less than 5$]$

$C = \{5\}$ $\quad [\because x - 5 = 0 \Rightarrow x = 5]$

$D = \{-5, 5\}$ $[\because x^2 = 25 \Rightarrow x = +5]$

and $E = \{5\}$.

$[\because x^2 - 2x - 15 = 0 \Rightarrow (x-5)(x+3) = 0 \Rightarrow x = 5, -3$. Out of these two,

5 is positive integral$]$

Clearly $C = E$.

Example – 8

Are the following pairs of sets equal ? Give reasons.

(i) $A = \{1, 2\}$, $B = \{x : x$ is a solution of $x^2 + 3x + 2 = 0\}$

(ii) $A = \{x : x$ is a letter in the word FOLLOW$\}$,

$B = \{y : y$ is a letter in the word WOLF$\}$.

Sol. (i) $A = \{1, 2\}$, $B = \{-2, -1\}$

$[\because x^2 + 3x + 2 = 0 \Rightarrow (x+2)(x+1) = 0 \Rightarrow x = -2, -1]$

Clearly $A \neq B$.

(ii) $A = \{F, O, L, L, O, W\} = \{F, O, L, W\}$

$B = \{W, O, L, F\} = \{F, O, L, W\}$.

Clearly $A = B$.

Example – 9

Let $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 4, 5, 6, 7\}$, $C = \{6, 7, 8, 9\}$ and $D = \{7, 8, 9, 10\}$. Find :

(a) **(i)** $A \cup B$ **(ii)** $B \cup D$

(iii) $A \cup B \cup C$ **(iv)** $B \cup C \cup D$

(b) **(i)** $A \cap B$ **(ii)** $B \cap D$ **(iii)** $A \cap B \cap C$.

Sol. (a) (i) $A \cup B = \{1, 2, 3, 4, 5\} \cup \{3, 4, 5, 6, 7\}$

$= \{1, 2, 3, 4, 5, 6, 7\}$.

(ii) $B \cup D = \{3, 4, 5, 6, 7\} \cup \{7, 8, 9, 10\}$

$= \{3, 4, 5, 6, 7, 8, 9, 10\}$.

(iii) $A \cup B \cup C = \{1, 2, 3, 4, 5\} \cup \{3, 4, 5, 6, 7\} \cup \{6, 7, 8, 9\}$.

$= \{1, 2, 3, 4, 5, 6, 7\} \cup \{6, 7, 8, 9\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

(iv) $B \cup C \cup D = \{3, 4, 5, 6, 7\} \cup \{6, 7, 8, 9\} \cup \{7, 8, 9, 10\}$.

$= \{3, 4, 5, 6, 7, 8, 9\} \cup \{7, 8, 9, 10\} = \{3, 4, 5, 6, 7, 8, 9, 10\}$.

(b) (i) $A \cap B = \{1, 2, 3, 4, 5\} \cap \{3, 4, 5, 6, 7\} = \{3, 4, 5\}$.

(ii) $B \cap D = \{3, 4, 5, 6, 7\} \cap \{7, 8, 9, 10\} = \{7\}$.

(iii) $A \cap B \cap C = \{1, 2, 3, 4, 5\} \cap \{3, 4, 5, 6, 7\} \cap \{6, 7, 8, 9\} = \{3, 4, 5\} \cap \{6, 7, 8, 9\} = \phi$.

Example – 10

If $A_1 = \{2, 3, 4, 5\}$, $A_2 = \{3, 4, 5, 6\}$, $A_3 = \{4, 5, 6, 7\}$, find $\cup A_i$ and $\cap A_i$, where $i = \{1, 2, 3\}$.

Sol. (i) $\cup A_i = A_1 \cup A_2 \cup A_3 = \{2,3,4,5\} \cup \{3,4,5,6\} \cup \{4,5,6,7\}$

$= \{2,3,4,5\} \cup \{3,4,5,6,7\} = \{2,3,4,5,6,7\}$.

(ii) $\cap A_i = A_1 \cap A_2 \cap A_3 = \{2,3,4,5\} \cap \{3,4,5,6\} \cap \{4,5,6,7\}$

$= \{2,3,4,5\} \cap \{4,5,6\} = \{4,5\}$.

Example – 11

Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {1, 2, 3, 4}, B = {2, 4, 6, 8}. Find :

(i) A^C (ii) B^C (iii) $(A^C)^C$ (iv) $(A \cup B)^C$

Sol. (i) A^C = Set of those elements of U, which are not in A = {5, 6, 7, 8, 9}.

(ii) B^C = Set of those elements of U, which are not in B = {1, 3, 5, 7, 9}.

(iii) $(A^C)^C$ = Set of those elements of U, which are not in A' = {1, 2, 3, 4} = A.

(iv) $A \cup B = \{1,2,3,4\} \cup \{2,4,6,8\} = \{1,2,3,4,6,8\}$.

$(A \cup B)^C$ = Set of those elements of U, which are not in $(A \cup B) = \{5,7,9\}$.

Example – 12

If U = {x : x is a letter in English alphabet}, A = {x : x is a vowel in English alphabet}.

Find A^C and $(A^C)^C$.

Sol. (i) Since A = {x : x is a letter in English alphabet},

$\therefore$ A^C is the set of those elements of U, which are not vowels = {x : x is a consonant in English alphabet}.

(ii) $(A^C)^C$ is the set of those elements of U, which are not consonants = {x : x is a vowel in English alphabet} = A.

Hence $(A^C)^C = A$.

Example – 13

Let A = {1, 2, 3, 4, 5, 6}, B = {3, 4, 5, 6, 7, 8}. Find (A – B) $\cup$ (B – A).

Sol. We have, A = {1, 2, 3, 4, 5, 6} and B = {3, 4, 5, 6, 7, 8}.

$\therefore$ A – B = {1, 2} and B – A = {7, 8}

$\therefore$ $(A – B) \cup (B – A) = \{1, 2\} \cup \{7, 8\} = \{1, 2, 7, 8\}$.

Some Basis Results about Cardinal Number

If A, B and C are finite sets and U be the finite universal set, then

(i) $n(A^c) = n(U) - n(A)$

(ii) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(iii) $n(A \cup B) = n(A) + n(B)$, where A and B are disjoint non - empty sets.

(iv) $n(A \cap B^c) = n(A) - n(A \cap B)$

(v) $n(A^c \cap B^c) = n(A \cup B)^c = n(U) - n(A \cup B)$

(vi) $n(A^c \cup B^c) = n(A \cap B)^c = n(U) - n(A \cap B)$

(vii) $n(A - B) = n(A) - n(A \cap B)$

(viii) $n(A \cap B) = n(A \cup B) - n(A \cap B^c) - n(A^c \cap B)$

(ix) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

(x) If $A_1, A_2, A_3, \ldots A_n$ are disjoint sets , then $n(A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n) = n(A_1) + n(A_2) + n(A_3) + \ldots + n(A_n)$

(xi) $n(A \Delta B)$ = number of elements which belong to exactly one of A or B.

Example – 14

If A = {1, 2, 3}, B = {4, 5, 6} and C = {7, 8, 9}, verify that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Sol. We have, A = {1, 2, 3}, B = {4, 5, 6} and C = {7, 8, 9}.

$A \cup B = \{1,2,3\} \cup \{4,5,6\} = \{1,2,3,4,5,6\}$ ...(1)

$A \cup C = \{1,2,3\} \cup \{7,8,9\}$

$= \{1,2,3,7,8,9\}$ ...(2)

and $B \cap C = \{4,5,6\} \cap \{7,8,9\} = \phi$ ...(3)

Now $A \cup (B \cap C) = \{1, 2, 3\} \cup \phi = \{1,2,3\}$ ...(4)

and $(A \cup B) \cap (A \cup C) = \{1,2,3,4,5,6\} \cap \{1,2,3,7,8,9\}$

$= \{1,2,3\}$ ...(5)

From (4) and (5), $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, which verifies the result.

Example – 15

Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {2, 4, 6, 8} and B = {2, 3, 5, 7}. Verify that

(i) $(A \cup B)^C = A^C \cap B^C$ **(ii)** $(A \cap B)^C = A^C \cup B^C$.

Sol. We have, A = {2, 4, 6, 8} and B = {2, 3, 5, 7}.

(i) $A \cup B = \{2, 4, 6, 8\} \cup \{2, 3, 5, 7\}$

$= \{2, 3, 4, 5, 6, 7, 8\}$

$(A \cup B)^C = \{1, 9\}$ ...(1)

Also $A^C = \{1, 3, 5, 7, 9\}$

and $B^C = \{1, 4, 6, 8, 9\}$

$A^C \cap B^C = \{1, 3, 5, 7, 9\} \cap \{1, 4, 6, 8, 9\}$

$= \{1, 9\}$ ...(2)

From (1) and (2), $(A \cup B)^C = A^C \cap B^C$, which verifies the result.

(ii) $A \cap B = \{2, 4, 6, 8\} \cap \{2, 3, 5, 7\} = \{2\}$

$(A \cap B)^C = \{1, 3, 4, 5, 6, 7, 8, 9\}$ ...(3)

and $A^C \cup B^C = \{1, 3, 5, 7, 9\} \cup \{1, 4, 6, 8, 9\}$

$= \{1, 3, 4, 5, 6, 7, 8, 9\}$ ...(4)

From (3) and (4), $(A \cap B)^C = A^C \cup B^C$, which verifies the result.

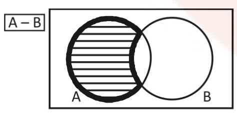
Example – 16

If A and B are any two sets, prove using Venn Diagrams

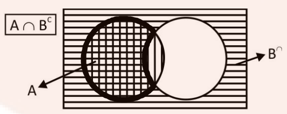
(i) A – B = $A \cap B^C$ **(ii) (A – B)** $\cup$ **B =** $A \cup B$.

Sol.

(i)

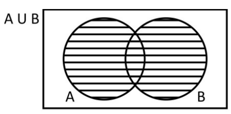


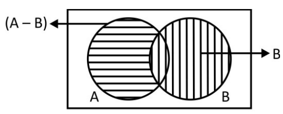
Shaded region in both are the same.



The common shaded area represents the intersection.

(ii)





Combination of all the shaded regions represents the union.

Example – 17

Prove that :

$A \cap (B - C) = (A \cap B) - (A \cap C)$

Sol. Let x be an arbitrary element of $A \cap (B - C)$.

Then $x \in A \cap (B - C)$

$\Rightarrow$ $x \in A$ and $x \in (B - C)$

$\Rightarrow$ $x \in A$ and ($x \in B$ and $x \notin C$)

$\Rightarrow$ ($x \in A$ and $x \in B$) and ($x \in A$ and $x \notin C$)

$\Rightarrow$ $x \in (A \cap B)$ and $x \notin (A \cap C)$

$\Rightarrow$ $x \in \{(A \cap B) - (A \cap C)\}$

$A \cap (B - C) \subseteq (A \cap B) - (A \cap C)$ ... (1)

Let y be an arbitrary element of $(A \cap B) - (A \cap C)$.

Then $y \in (A \cap B) - (A \cap C)$

$\Rightarrow$ $y \in (A \cap B)$ and $y \notin (A \cap C)$

$\Rightarrow$ ($y \in A$ and $y \in B$) and ($y \in A$ and $y \notin C$)

$\Rightarrow$ $y \in A$ and ($y \in B$ and $y \notin C$)

$\Rightarrow$ $y \in A$ and $y \in (B - C)$

$\Rightarrow$ $y \in A \cap (B - C)$

$(A \cap B) - (A \cap C) \subseteq A \cap (B - C)$ ... (2)

Combining (1) and (2).

$A \cap (B - C) = (A \cap B) - (A \cap C)$.

Example – 18

Prove the following :

$A \subset B \Leftrightarrow B^c \subset A^c$

Sol. Let $x \in B^c$, where x is arbitrary.

Now $x \in B^c$

$\Rightarrow$ $x \notin B$

$\Rightarrow$ $x \notin A$ [$\because A \subset B$]

$\Rightarrow$ $x \in A^c$

$B^c \subset A^c$ ... (1)

Conversely : Let $x \in A$, where x is arbitrary.

Now $x \in A$

$\Rightarrow$ $x \notin A^c$

$\Rightarrow$ $x \notin B_c$ [$\because B^c \subset A^c$]

$\Rightarrow$ $x \in B$

$A \subset B$

Combining (1) and (2), $A \subset B \Leftrightarrow B^c \subset A^c$.

Example – 19

Prove the following :

$A - B = A - (A \cap B)$

where U is the universal set.

Sol. Let $x \in (A - B)$, where x is arbitrary.

Now $x \in (A - B)$

$\Leftrightarrow$ $x \in A$ and $x \notin B$

$\Leftrightarrow$ $(x \in A$ and $x \in A)$ and $x \notin B$

[Note this step]

$\Leftrightarrow$ $x \in A$ and $(x \in A$ and $x \notin B)$

[Associative Law]

$\Leftrightarrow$ $x \in A$ and $x \notin (A \cap B)$

$\Leftrightarrow$ $x \in A - (A \cap B)$

Hence $A - B = A - (A \cap B)$.

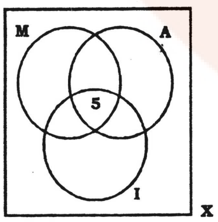
Example – 20

In a class of 200 students who appeared in a certain examination. 35 students failed in MHTCET, 40 in AIEEE, 40 in IIT, 20 failed in MHTCET and AIEEE, 17 in AIEEE and IIT, 15 in MHTCET and IIT and 5 failed in all three examinations. Find how many students

(i) Did not fail in any examination.

(ii) Failed in AIEEE or IIT.

Sol.



$n(M) = 35, n(A) = 40, n(I) = 40$

$n(M \cap A) = 20, n(A \cap I) = 17,$

$n(I \cap M) = 15, n(M \cap A \cap I) = 5$

$n(X) = 200$

$n(M \cup A \cup I) = n(M) + n(A) + n(I) -$

$n(M \cap A) - n(A \cap I) - n(M \cap I) + n(M \cap A \cap I)$

$= 35 + 40 + 40 - 20 - 17 - 15 + 5 = 68$

(i) Number of students passed in all three examination

$= 200 - 68 = 132$

(ii) Number of students failed in IIT or AIEEE

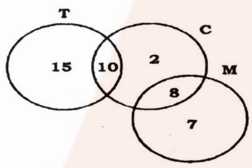
$= n(I \cup A) = n(I) + n(A) - n(I \cap A)$

$= 40 + 40 - 17 = 63$

Example – 21

In a hostel, 25 students take tea, 20 students take coffee, 15 students take milk, 10 students take both tea and coffee, 8 students take both milk and coffee. None of the them take tea and milk both and everyone takes atleast one beverage, find the number of students in the hostel.

Sol.



Let the sets, T and C and set M are the students who drink tea, coffee and milk respectively. This problem can be solved by Venn diagram.

$n(T) = 25; n(C) = 20; n(M) = 15$

$n(T \cap C) = 10; n(M \cap C) = 8$

Number of students in hostel

$= n(T \cup C \cup M)$

$\therefore n(T \cup C \cup M) = 15 + 10 + 2 + 8 + 7 = 42$