Ans 1. $i^2 <= n$

$i <= \sqrt{n}$

l	1	2	3	4	.....	l
i	0	1	2	3	.....	$l-1$

for this value of i, l operations are performed

this loop runs as long as the value of

$i <= \sqrt{n}$

Hence,

$$l - 1 = \sqrt{n}$$

↳ we can ignore the constant since n is a very big number

$l = \sqrt{n}$ → these are the number of operations performed by this loop.

Hence time complexity = $\boxed{O(\sqrt{n})}$

Ans 2. Only the 'K' loop is performing a number of print statements.

```
loop i {                          // no operations
        loop j {                  // no operations
                loop K {
                        syso ("K")   // print
                                     // operation
                }
        }
}
```

$i=1$	$i=2$	$i=3$	$i=n$
$j=1$	$j=1,2$	$j=1,2,3$	$j=1,2,\dots n$
1000 operations	2000 operations	3000 operations	$n \times 1000$ operaⁿ

let Total number of operations be l

$$l = 1000 + 2000 + 3000 + 4000 + \dots n \times 1000$$
$$l = 1000\,[1+2+3+4+\dots\dots n]$$
$$l = 1000 \times \left[\frac{n(n+1)}{2}\right]$$
$$l = n^2$$

Time complexity $= \boxed{O(n^2)}$

Ans 3.

$i=1$	$i=2$	$i=3$	$i=n$
$j=1$	$j=1,2,3,4$	$j=1,2,3,4,..9$	$j=1,2,....n$
$\frac{n}{2}$ operations	$4\times\frac{n}{2}$ operations	$9\times\frac{n}{2}$ operations	$n^2\times\frac{n}{2}$ operations

let total no. of operations be l

$$l = \frac{n}{2}\times 1^2 + \frac{n}{2}\times 2^2 + \frac{n}{2}\times 3^2 + \dots \frac{n}{2}\times n^2$$

$$= \frac{n}{2}\left[1^2+2^2+3^2+\dots n^2\right]$$

$$= \frac{n}{2}\times\left[\frac{n(n+1)(2n+1)}{6}\right]$$

$$l = n^4 + \_\_ + \_\_ + \dots$$

The time complexity of a polynomial is its highest degree.

Time complexity = $\boxed{O(n^4)}$

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Ans 4.

l	1	2	3	4	5	...	l
i	1 (2^0)	2 (2^1)	4 (2^2)	8 (2^3)	16 (2^4)	...	2^{l-1}

$$2^{l-1} = n$$

$$2^l = n$$

$$l = \log_2 n$$

Time Complexity = $\boxed{O(\log_2 n)}$

Ans 5. All the loops are independent, Hence we can multiply their time complexities

Time complexity of first loop, second loop & third loo

l	1	2	3	...	l
i	$n/2$	$n/2+1$	$n/2+2$	...	$n/2+(l-1)$
j	1	2	3	...	l
K	2^0	2^1	2^2	...	2^{l-1}

T_i

$\Rightarrow \frac{n}{2} + (l-1) = n$

$\Rightarrow l-1 = \frac{n}{2}$

$\Rightarrow l = \frac{n}{2}$

$T_i = O(n)$

T_j

$\Rightarrow l = \frac{n}{2}$

$T_j = O(n)$

T_k

$\Rightarrow 2^{l-1} = n$

$l = \log_2 n$

$T_K = O(\log_2 n)$

Overall Time Complexity $= T_i \times T_j \times T_k$

$= \boxed{O(n^2 \log_2 n)}$

Ans 6. Similar to Question 5.

All the loops are independent.

$T_i = n$, $T_j = \log_2 n$, $T_k = \log_2 n$

Overall Time Complexity $= T_i \times T_j \times T_k$

$= n \times (\log_2 n)^2$

$\boxed{= O\left(n \times (\log_2 n)^2\right)}$

Ans 7.

$i=1$	$i=2$	$i=3$
$j=1,2,3 \ldots n$	$j=1,3,5,7 \ldots n$	$j=1,4,7,\ldots n$
n operations	$\frac{n}{2}$ operations	$\frac{n}{3}$ operations

$i=n$
$j=1$
$\frac{n}{n}$ operations

$$l = n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \ldots \frac{n}{n}$$

$$= n\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots \frac{1}{n}\right]$$

↳ expansion of log

$$l = n \times \log n$$

$$\text{Time Complexity} = \boxed{O(n \log n)}$$

Ans 8. Both the loops are independent. Hence, we need to find their time complexities individually and multiply them

for loop i :—

l	1	2	3	4	...	l
i	1	$1+k$	$1+2k$	$1+3k$	...	$1+(l-1)k$

$$1+(l-1)k = n$$

$$lk = n$$

$$l = \frac{n}{k}$$

$$T_i = O(n/k)$$

for loop j

$T_j = O(K)$

Overall Time complexity $= \frac{n}{K} \times K$

$= \boxed{O(n)}$