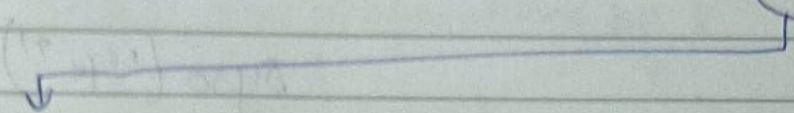


Ans 1.

$$i^2 \leq n$$
$$i \leq \sqrt{n}$$

l	1	2	3	4	...	l
i	0	1	2	3	...	$l-1$



for this value of i , l operations are performed
this loop runs as long as the value of
 $i \leq \sqrt{n}$

Hence,

$$l-1 = \sqrt{n}$$

↳ we can ignore the constant since
 n is a very big number

~~$O(n)$~~ $l = \sqrt{n} \rightarrow$ these are the number of
operations performed by this
loop.

Hence time complexity = $\boxed{O(\sqrt{n})}$

Ans 2. Only the 'K' loop is performing a number of print statements.

loop i { // no operations

loop j { // no operations

loop K {

syso("K") // print
// operation

}
}
}

i = 1	i = 2	i = 3	i = n
j = 1	j = 1, 2	j = 1, 2, 3	j = 1, 2, ..., n
1000 operations	2000 operations	3000 operations	$n \times 1000$ operations

let Total number of operations be l

$$l = 1000 + 2000 + 3000 + 4000 + \dots + n \times 1000$$

$$l = 1000 [1 + 2 + 3 + 4 + \dots + n]$$

$$l = 1000 \times \left[\frac{n(n+1)}{2} \right]$$

$$l = n^2$$

$$\text{Time complexity} = O(n^2)$$

Ans 3.

$i=1$	$i=2$	$i=3$	$i=n$
$j=1$	$j=1, 2, 3, 4$	$j=1, 2, 3, 4, \dots, 9$	$j=1, 2, \dots, n$
$\frac{n}{2}$ operations	$4 \times \frac{n}{2}$ operations	$9 \times \frac{n}{2}$ operations	$n^2 \times \frac{n}{2}$ operations

Let total no. of operations be L

$$L = \frac{n}{2} \times 1^2 + \frac{n}{2} \times 2^2 + \frac{n}{2} \times 3^2 + \dots + \frac{n}{2} \times n^2$$

$$= \frac{n}{2} [1^2 + 2^2 + 3^2 + \dots + n^2]$$

$$= \frac{n}{2} \left[\frac{n(n+1)(2n+1)}{6} \right]$$

$$L = n^4 + \dots + \dots + \dots$$

The time complexity of a polynomial is its highest degree.

$$\text{Time complexity} = \boxed{O(n^4)}$$

Ans 4.

	1	2	3	4	5	...	l
i	1 (2^0)	2 (2^1)	4 (2^2)	8 (2^3)	16 (2^4)	...	2^{l-1}

$$2^{l-1} = n$$

$$2^l = n$$

$$l = \log_2 n$$

$$\text{Time Complexity} = \boxed{O(\log_2 n)}$$

Ans 5. All the loops are independent.
Hence we can multiply their Time complexities

Time complexity of first loop, second loop & third loop

l	1	2	3	...	l
i	$n/2$	$n/2 + 1$	$n/2 + 2$...	$n/2 + (l-1)$
j	1	2	3	...	l
k	2^0	2^1	2^2	...	2^{l-1}

T_i

$$\Rightarrow \frac{n}{2} + (l-1) = n$$

$$\Rightarrow \cancel{n} l - 1 = \frac{n}{2}$$

$$\Rightarrow l = \frac{n}{2}$$

$$T_i = O(n)$$

T_g

$$\Rightarrow l = \frac{n}{2}$$

$$T_g = O(n)$$

T_k

~~\Rightarrow~~

$$\Rightarrow 2^{l-1} = n$$

$$l = \log_2 n$$
$$T_k = O(\log_2 n)$$

$$\text{Overall Time Complexity} = T_i \times T_g \times T_k$$
$$= \boxed{O(n^2 \log_2 n)}$$

Ans 6. Similar to Question 5,

All the loops are independent.

$$T_i = n, \quad T_j = \log_2 n, \quad T_k = \log_2 n$$

$$\text{Overall Time Complexity} = T_i \times T_j \times T_k$$

Φ

$$= n \times (\log_2 n)^2$$

$$= O(n \times (\log_2 n)^2)$$

Ans 7.

$i = 1$	$i = 2$	$i = 3$
$j = 1, 2, 3, \dots, n$	$j = 1, 3, 5, 7, \dots, n$	$j = 1, 4, 7, \dots, n$
n operations	$\frac{n}{2}$ operations	$\frac{n}{3}$ operations
<hr/>		
$i = n$		
$\frac{n}{n}$ operations		

$$l = n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \dots + \frac{n}{n}$$

$$= n \left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n} \right]$$

$$l = n \times \log n$$

$$\text{Time Complexity} = \boxed{O(n \log n)}$$

↳ expansion of
log

Ans 8. Both the loops are independent.
Hence, we need to find their time complexities individually and multiply them.

for loop i :-

l	1	2	3	4	...	l
i	1	1+k	1+2k	1+3k	...	1+(l-1)k

$$1+(l-1)k = n$$

$$lk = n$$

$$l = \frac{n}{k}$$

$$T_i = O(n/k)$$

for loop j

$$T_j = O(K)$$

$$\text{Overall Time Complexity} = \frac{n \times K}{K}$$

$$= \boxed{O(n)}$$