Prob.1. The following forces act at point :

(1) 20 kg inclined at 30^0 towards North of East,

(2) 25 kg towards North,

(3) 30 kg towards North-West,

(4) 35 kg inclined at 40^0 towards South of West.

Find the resultant of the forces in magnitude and direction.

(S/2000,

The following forces act at a point :

(1) 20 N inclined at 30^0 towards North of East,

(2) 25 N towards North,

(3) 30 N towards North-West,

(4) 35 N inclined at 40^0 towards South of West.

Find the magnitude and direction of the resultant force.

(2011,

Sol. The system of given forces is shown in fig. 1.30.

Resolving all the forces horizontally and taking their algebraic sum,

$$\begin{aligned}\Sigma H &= 20\cos 30^\circ + 25\cos 90^\circ + 30\cos 135^\circ + 35\cos 220^\circ \\ &= (20\times 0{\cdot}866) + (25\times 0) + 30\,(-0{\cdot}707) + 35\,(-0{\cdot}766) \\ &= -30{\cdot}70\ kg \\ &= -307\ N\end{aligned}$$

Now resolving all the forces vertically and taking their algebraic sum,

$$\begin{aligned}\Sigma V &= 20\sin 30^\circ + 25\sin 90^\circ + 30\sin 135^\circ + 35\sin 220^\circ \\ &= (20\times 0{\cdot}5) + (25\times 1) + (30\times 0{\cdot}707) + 35\times(-0{\cdot}6428) \\ &= 33{\cdot}71\ kg \\ &= 337{\cdot}10\ N\end{aligned}$$

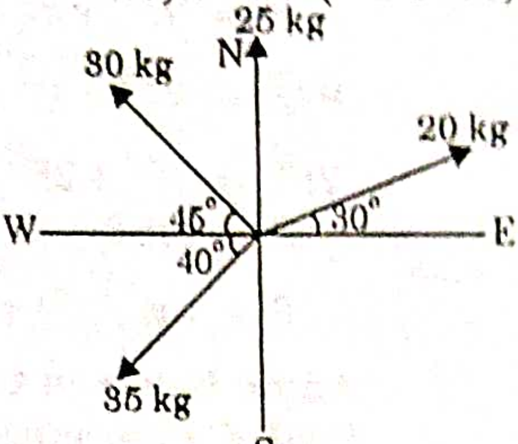


Fig. 1.30

Now resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(-30{\cdot}70)^2 + (33{\cdot}71)^2}$$

Magnitude of resultant force,

$$R = 45{\cdot}59\ kg = 455{\cdot}90\ N$$ **Ans.**

Let α be the angle, which resultant R makes with East, then,

$$\tan\alpha = \frac{\Sigma V}{\Sigma H}$$

$$= \frac{33{\cdot}71}{-30{\cdot}70} = -1{\cdot}098$$

$$\alpha = \tan^{-1}(-1{\cdot}098) = 47^\circ 40'$$

Since, ΣH is negative and ΣV is positive. Therefore, α should lie between 90° and 180°.

$\therefore$ Actual inclination, $\alpha = 180^\circ - 47^\circ 40'$

$= 179^\circ 60' - 47^\circ 40'$

Inclination of resultant with East, $\alpha = 132^\circ 20'$ **Ans.**

Prob.3. *Find the angle between two equal force P, when their resultant is equal to $P\sqrt{2}$.* **(2001)**

Sol. Given,

$$F_1 = P$$

$$F_2 = P$$

$$\text{Resultant, } F = P\sqrt{2}$$

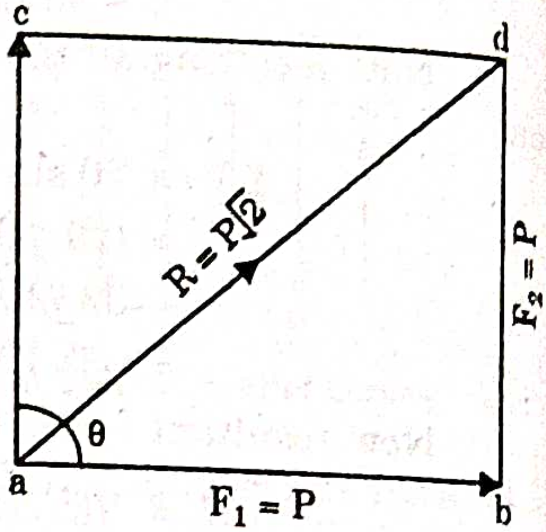


Fig. 1.32

We know that,

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$P\sqrt{2} = \sqrt{P^2 + P^2 + 2PP\cos\theta}$$

$$2P^2 = 2P^2 + 2P^2\cos\theta$$

$$0 = \cos\theta$$

$$\theta = \cos^{-1}(0) = 90^\circ$$

Angle between two equal force, $\theta = 90^\circ$ **Ans.**

Prob.4. *Determine analytically the magnitude and direction of the resultant of the coplanar forces of 8 N, 12 N, 15 N and 20 N acting at a point making angles of 30°, 70°, 120° and 155° respectively with a fixed horizontal line.* **(2002, 09)**

Find the magnitude and direction of the resultant of the coplanar concurrent forces of 8 N, 12 N, 15 N and 20 N making an angle of 30°, 70°, 120° and 155° respectively with a fixed horizontal line. **(S/2016)**

Sol. The system of given forces is shown in fig. 1.33.

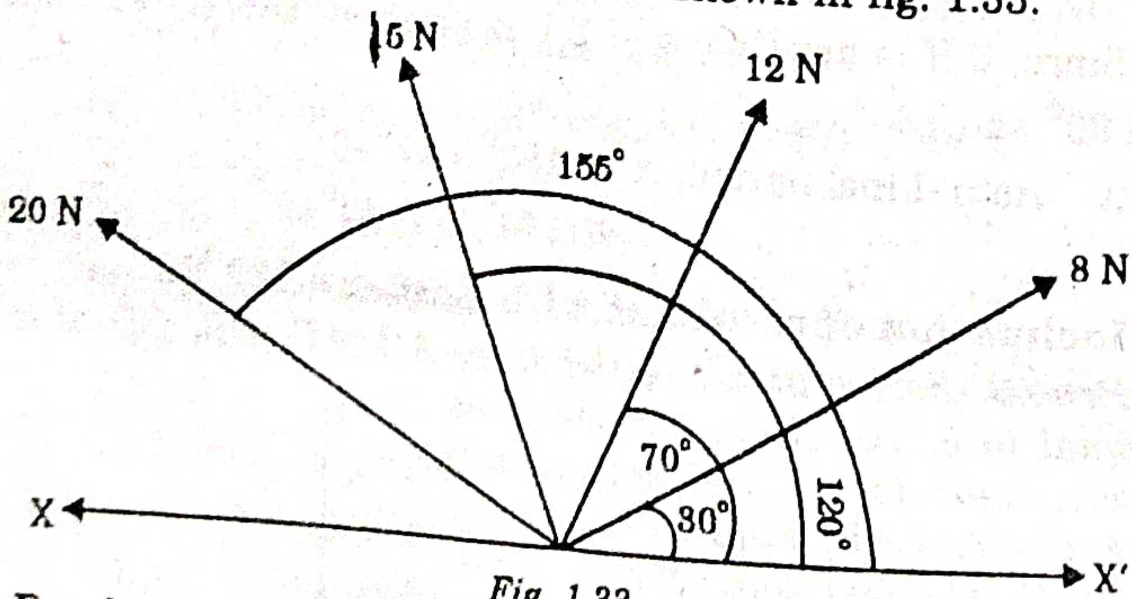


Fig. 1.33

Resolving all the forces horizontally and taking their algebraic sum,

$$\Sigma H = 8\cos 30^\circ + 12\cos 70^\circ + 15\cos 120^\circ + 20\cos 155^\circ$$

$$= (8 \times 0{\cdot}866) + (12 \times 0{\cdot}342) + (15 \times -0{\cdot}5) + (20 \times -0{\cdot}906)$$

$$= 6{\cdot}928 + 4{\cdot}104 - 7{\cdot}5 - 18{\cdot}12$$
$$= -14{\cdot}59\ N$$

Now, resolving all the forces vertically and taking their algebraic sum,

$$\Sigma V = 8 \sin 30^0 + 12 \sin 70^0 + 15 \sin 120^0 + 20 \sin 155^0$$
$$= (8 \times 0{\cdot}5) + (12 \times 0{\cdot}939) + (15 \times 0{\cdot}866) + (20 \times 0{\cdot}422)$$
$$= 4 + 11{\cdot}26 + 12{\cdot}99 + 8{\cdot}44$$
$$= 36{\cdot}69\ N$$

Now resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$
$$= \sqrt{(-14{\cdot}59)^2 + (36{\cdot}69)^2}$$
$$= \sqrt{212{\cdot}86 + 1346{\cdot}15}$$
$$= \sqrt{1559{\cdot}01} = 39{\cdot}48\ N$$

Magnitude of resultant force, $R = 39{\cdot}48\ N$ Ans.

Let α be the angle, which resultant R makes with horizontal then,

$$\tan\alpha = \frac{\Sigma V}{\Sigma H}$$
$$= \frac{36{\cdot}69}{-14{\cdot}59} = -2{\cdot}514$$
$$\alpha = \tan^{-1}(-2{\cdot}514)$$
$$= -68^0 18'$$

Since, ΣH is negative and ΣV is positive. Therefore, α should lie between 90^0 and 180^0.

$\therefore$ Actual inclination,

$$\alpha = 180^0 - 68^0 18'$$
$$= 179^0 60' - 68^0 18' = 111^0 42'$$

Inclination of resultant with horizontal, $\alpha = 111^0 42'$ Ans.

Prob.5. *Two forces, whose magnitudes are P and $P\sqrt{2}$ act on a particle in the direction inclined at a angle of 135^0 to each other. Find the magnitude and direction of their resultant.* *(S/2003, 18)*

Sol. Given,

$$F_1 = P$$
$$F_2 = P\sqrt{2}$$
$$\theta = 135^0$$

We know that,

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$
$$= \sqrt{P^2 + (P\sqrt{2})^2 + 2P \times P\sqrt{3} \times \cos 135^0}$$

$$= \sqrt{P^2 + 2P^2 + 2P^2\sqrt{2}\,(-1/\sqrt{2})}$$

$$= \sqrt{P^2 + 2P^2 - 2P^2}$$

Magnitude of resultant, $R = P$ **Ans.**

Now direction of resultant,

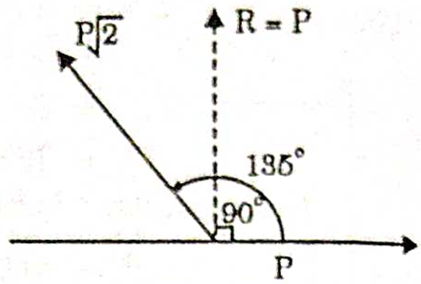


Fig. 1.34

$$\alpha = \tan^{-1}\left[\frac{F_2 \sin\theta}{F_1 + F_2\cos\theta}\right]$$

$$= \tan^{-1}\left[\frac{P\sqrt{2}\sin 135^\circ}{P + P\sqrt{2}\cos 135^\circ}\right]$$

$$= \tan^{-1}\left[\frac{P\sqrt{2}\times\frac{1}{\sqrt{2}}}{P + P\sqrt{2}\times\left(-\frac{1}{\sqrt{2}}\right)}\right]$$

$$= \tan^{-1}(\infty) = 90^\circ$$

Direction of the resultant, $\alpha = 90^\circ$ **Ans.**

Prob.6. *Pull of 10 kg, 20 kg, 30 kg, 40 kg and 30 kg act at a point. The inclination of these forces with the axis OX are 30°, 120°, 180°, 225° and 300°. Find their resultant in magnitude and direction.* **(2003)**

Sol. Resolving all the forces horizontally and taking their algebraic sum,

$$\Sigma H = 10\cos 30^\circ + 20\cos 120^\circ + 30\cos 180^\circ + 40\cos 225^\circ + 30\cos 300^\circ$$

$$= (10\times 0{\cdot}866) + (20\times -0{\cdot}5) + (30\times -1) + (40\times -0{\cdot}707) + (30\times 0{\cdot}5)$$

$$= -44{\cdot}62\ kg$$

$$\Sigma H = -446{\cdot}2\ N$$

Now resolving all the forces vertically and taking their algebraic sum,

$$\Sigma V = 10\sin 30^\circ + 20\sin 120^\circ + 30\sin 180^\circ + 40\sin 225^\circ + 30\sin 300^\circ$$

$$= (10\times 0{\cdot}5) + (20\times 0{\cdot}866) + (30\times 0) + (40\times -0{\cdot}707) + (30\times -0{\cdot}866)$$

$$= -31{\cdot}94\ kg$$

$$\Sigma V = -319{\cdot}4\ N$$

Magnitude of resultant force,

$$R = \sqrt{\Sigma H^2 + \Sigma V^2}$$

$$= \sqrt{(-446{\cdot}2)^2 + (-319{\cdot}4)^2}$$

Magnitude of resultant force = 548·73 N **Ans.**

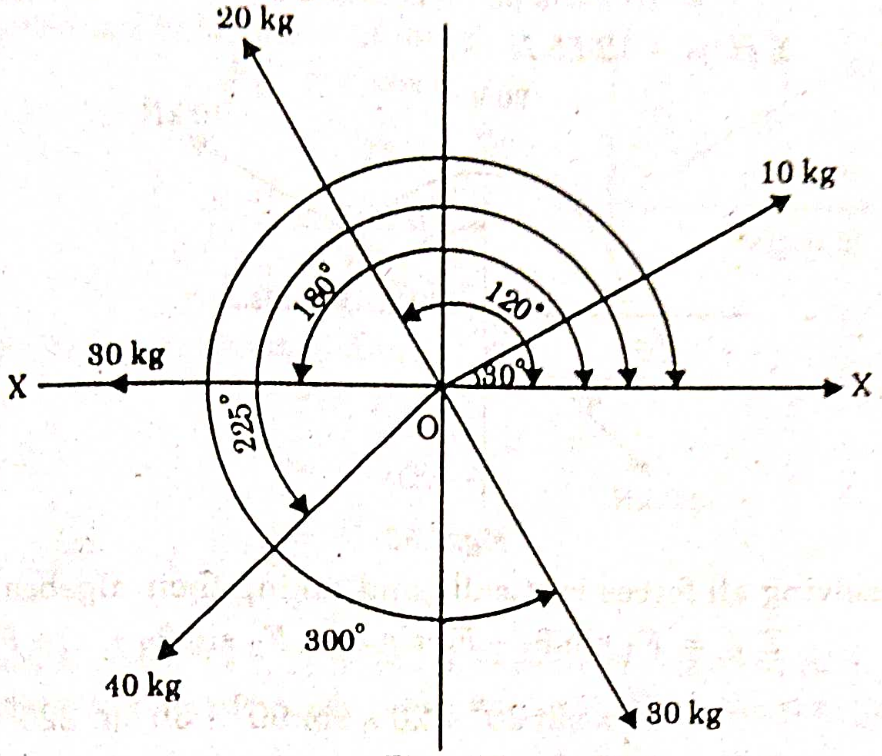


Fig. 1.35

Let α be the angle, which resultant R makes with horizontal then,

$$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$

$$= \tan^{-1}\left(\frac{-319{\cdot}4}{-446{\cdot}2}\right)$$

$$= \tan^{-1}(0{\cdot}72)$$

$$\alpha = 35^{\circ}45'$$

Now ΣH and ΣV both one negative. So α should lie between 180° and 270°.

Hence, actual inclination,

$$\alpha = 180^{\circ} + 35^{\circ}\,45'$$

Inclination of resultant with horizontal, α = 215° 45' **Ans.**

Prob.7. *Three forces of magnitude 10 N, 20 N and 30 N are acting at a point making 30°, 90° and 225° respectively with the horizontal. Calculate the magnitude and direction of the resultant force.* **(2005)**

Three forces of magnitude 10 N, 20 N and 30 N are acting at a point making an angle of 30°, 90° and 225° respectively with the horizontal. Calculate the magnitude and direction of the resultant. **(2019)**

Sol. Resolving all forces horizontally and taking their algebraic sum,

$$\Sigma H = F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3 + \ldots + F_n \cos\theta_n$$

$$= 10 \times \cos 30^\circ + 20 \times \cos 90^\circ + 30 \times \cos 225^\circ$$

$$\Sigma H = -12{\cdot}55\ N$$

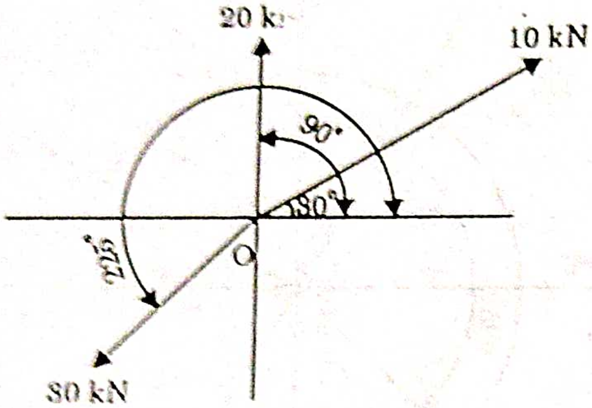


Fig. 1.36

Resolving all forces vertically and taking their algebraic sum,

$$\Sigma V = F_1 \sin\theta_1 + F_2 \sin\theta_2 + F_3 \sin\theta_3 + \ldots + F_n \sin\theta_n$$

$$= 10 \times \sin 30^\circ + 20 \times \sin 90^\circ + 30 \sin 225^\circ$$

$$\Sigma V = 3{\cdot}78\ N$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(-12{\cdot}55)^2 + (3{\cdot}78)^2}$$

$$= \sqrt{171{\cdot}79}$$

$$= 13{\cdot}10\ N$$

Magnitude of resultant force, $R = 13{\cdot}10\ N$ **Ans.**

Let α be the angle, which resultant R makes with horizontal then,

$$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) = \tan^{-1}\left(\frac{3{\cdot}78}{-12{\cdot}55}\right)$$

$$= \tan^{-1}(-0{\cdot}301)$$

$$\alpha = -16^\circ 45'$$

Since, ΣH is negative and ΣV is positive. Therefore, α should lie between 90° and 180°.

$$\therefore \quad \alpha = 180^\circ - 16^\circ 45'$$

Inclination of resultant with horizontal, $\alpha = 163^\circ 15'$ **Ans.**

Prob.8. *A force of 175 N is acting at a point making 60° angle with the vertical. Find horizontal and vertical components of the force.* **(2005)**

Sol. Given, Force, $F = 175\ N$

Angle, $\alpha = 60^\circ$

$\therefore$ Angle, $\theta = 90^0 - 60^0$

$= 30^0$

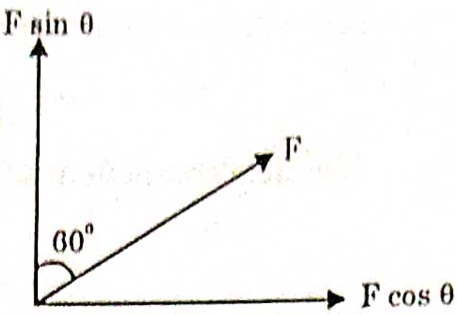


Fig. 1.37

Horizontal component of force

$F_H = F \cos \theta$

$= 175 \cos 30^0$

$= 175 \times \frac{\sqrt{3}}{2}$

$F_H = 151{\cdot}5\ N$ **Ans.**

Vertical component of force

$F_V = F \sin \theta$

$= 175 \sin 30^0$

$= 175 \times \frac{1}{2}$

$F_V = 87{\cdot}5\ N$ **Ans.**

Prob.9. *Force of 1, 2, 3 and 4 newton are applied along the outer side of square body 1 metre side taken in order. Find the resultant force.* ***(S/2006)***

Sol. Resolving all forces horizontally,

$\Sigma H = 1 - 3 = -2\ N$

Resolving all forces vertically,

$\Sigma V = 2 - 4 = -2\ N$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{(-2)^2 + (-2)^2}$

$= \sqrt{8}$

$= 2\sqrt{2}$

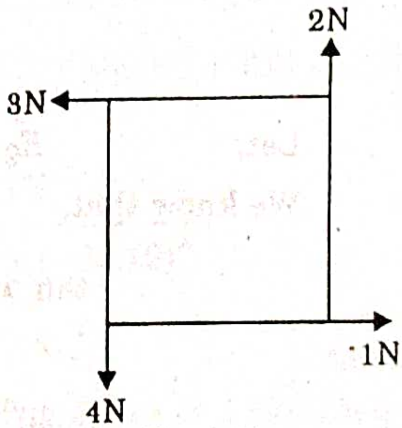


Fig. 1.38

Resultant force, $R = 2{\cdot}828\ N$ **Ans.**

Prob.10. *If a force of 100 N makes 30^0 with the verticle, find its horizontal and vertical components.* ***(2006)***

Sol. Given, Force, $F = 100\ N$

Angle, $\alpha = 30^0$

$\therefore$ Angle, $\theta = 90^0 - 30^0$

$= 60^0$

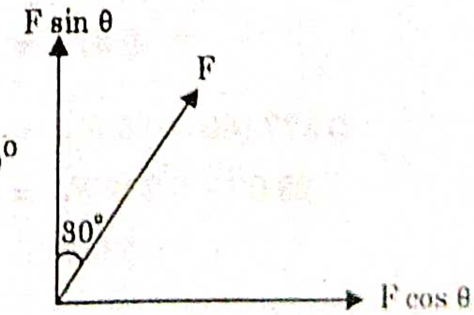


Fig. 1.39

Horizontal component of force,

$F_H = F \cos \theta$

$= 100 \cos 60^0$

$$= 100 \times \frac{1}{2}$$

$$F_H = 50\ N$$ Ans.

Verticle component of force,

$$F_V = F \sin \theta$$

$$= 100 \sin 30^\circ$$

$$= 100 \times \frac{\sqrt{3}}{2}$$

$$F_V = 86{\cdot}60\ N$$ Ans.

Prob.11. *Two forces act an angle of 120°. The bigger force is of 40 N and the resultant makes 90° angle with the smaller one. Find the smaller force.* **(2006)**

Sol. Given, $F_1 = 40\ N$

$$\theta = \angle AOC = 120^\circ$$

$$\angle AOB = 90^\circ$$

$$\therefore \quad \alpha = \angle BOC$$

$$= 120^\circ - 90^\circ$$

$$= 30^\circ$$

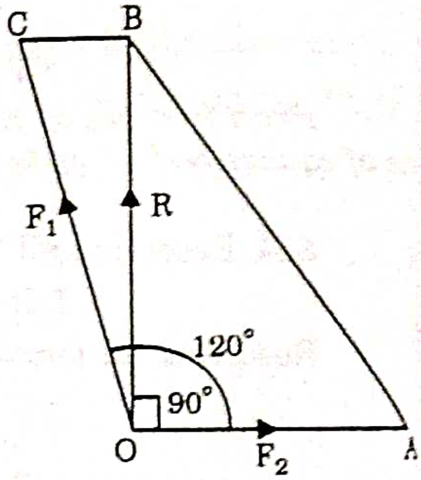


Fig. 1.40

Let, F_2 = Smaller force

We know that,

$$\tan \alpha = \frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta}$$

$$\tan 30^\circ = \frac{F_2 \sin 120^\circ}{F_1 + F_2 \cos 120^\circ}$$

$$0{\cdot}577 = \frac{0{\cdot}866\, F_2}{40 + F_2(-0{\cdot}5)}$$

$$0{\cdot}577 = \frac{0{\cdot}866\, F_2}{40 - 0{\cdot}5\, F_2}$$

$$0{\cdot}577\,(40 - 0{\cdot}5\, F_2) = 0{\cdot}866\, F_2$$

$$23{\cdot}08 - 0{\cdot}288\, F_2 = 0{\cdot}866\, F_2$$

$$23{\cdot}08 = 0{\cdot}866\, F_2 + 0{\cdot}288\, F_2$$

$$F_2 = \frac{23{\cdot}08}{1{\cdot}154}$$

Smaller force, $F_2 = 20\ N$ Ans.

Squaring both sides,

$$1^2 = 2 + 2\cos\theta$$
$$1 - 2 = 2\cos\theta$$
$$-\frac{1}{2} = \cos\theta$$

or
$$\cos\theta = -0.5$$
$$\theta = \cos^{-1}(-0.5)$$
$$\text{Angle, } \theta = 120^\circ$$ Ans

(2) When resultant, $R = \frac{P}{2}$

$$\frac{P}{2} = \sqrt{P^2 + P^2 + 2P.P.\cos\theta}$$
$$\frac{P}{2} = P\sqrt{1 + 1 + 2\cos\theta}$$
$$\frac{1}{2} = \sqrt{2 + 2\cos\theta}$$

Squaring both sides,

$$\left(\frac{1}{2}\right)^2 = 2 + 2\cos\theta$$
$$\frac{1}{4} = 2 + 2\cos\theta$$
$$\frac{1}{4} - 2 = 2\cos\theta$$
$$-\frac{7}{4} = 2\cos\theta$$

or
$$\cos\theta = -\frac{7}{8}$$
$$\cos\theta = -0.875$$
$$\theta = \cos^{-1}(-0.875)$$
$$\text{Angle, } \theta = 151.04^\circ$$ Ans

Prob.14. *The forces 20 N, 30 N, 40 N, 50 N and 60 N are acting at one of the angular point of a regular hexagon towards the other angular points, taken in order. Find the magnitude and direction of the resultant force.* **(2008, S/09, 12, S/15)**

Sol. The system of the given forces is shown in fig. 1.42.

Resolving all the forces horizontally (i.e., along AB),

$$\begin{aligned}\Sigma H &= 20\cos 0^\circ + 30\cos 30^\circ + 40\cos 60^\circ + 50\cos 90^\circ + 60\cos 1 \\ &= (20 \times 1) + (30 \times 0.866) + (40 \times 0.5) + (50 \times 0) + [60 \times (-0.5 \\ &= 35.98\ N\end{aligned}$$

Angle, θ = 101·04

Prob.14. *The forces 20 N, 30 N, 40 N, 50 N and 60 N are acting o[n] one of the angular point of a regular hexagon towards the other angula[r] points, taken in order. Find the magnitude and direction of the resultan[t] force.* ***(2008, S/09, 12, S/15)***

Sol. The system of the given forces is shown in fig. 1.42.

Resolving all the forces horizontally (i.e., along AB),

$$\Sigma H = 20 \cos 0^{\circ} + 30 \cos 30^{\circ} + 40 \cos 60^{\circ} + 50 \cos 90^{\circ} + 60 \cos 120^{\circ}$$

$$= (20 \times 1) + (30 \times 0{\cdot}866) + (40 \times 0{\cdot}5) + (50 \times 0) + [60 \times (-0{\cdot}5)]$$

$$= 35{\cdot}98\ N$$

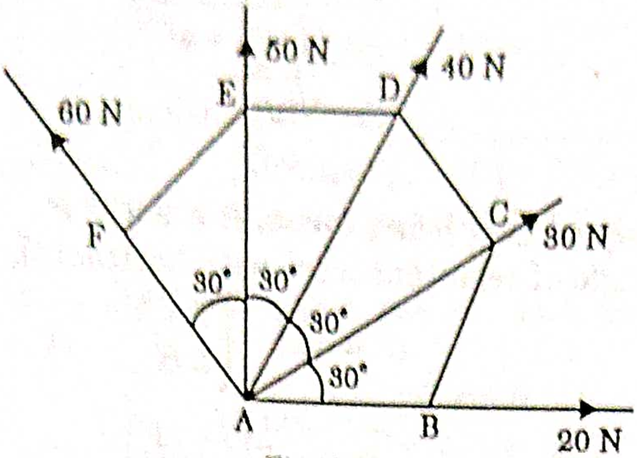


Fig. 1.42

Now resolving all the forces vertically (i.e., at right angle to AB),

$$\Sigma V = 20 \sin 0^{\circ} + 30 \sin 30^{\circ} + 40 \sin 60^{\circ} + 50 \sin 90^{\circ} + 60 \sin 120^{\circ}$$
$$= (20 \times 0) + (30 \times 0{\cdot}5) + (40 \times 0{\cdot}866) + (50 \times 1) + (60 \times 0{\cdot}866)$$
$$= 151{\cdot}60\ N$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$
$$= \sqrt{(35{\cdot}98)^2 + (151{\cdot}60)^2}$$

Magnitude of resultant force, R = 155·81 N **Ans.**

Let α be an angle, which the resultant R makes with the horizontal (i.e., AB)

$$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$
$$= \tan^{-1}\left(\frac{151{\cdot}60}{35{\cdot}98}\right)$$
$$= \tan^{-1}(4{\cdot}21)$$

Inclination of resultant with horizontal, α = 76°38′ **Ans.**

Prob.15. *Four forces equal to P, 2P, 3P and 4P are respectively acting along the four sides of a square ABCD, taken in order. Find the magnitude and direction of the resultant force and also find the position of resultant.* *(S/2010)*

Sol. By resolving all the forces in horizontal direction,

$$\Sigma H = P - 3P = -2P$$

Resolving all the forces in vertical direction,

$$\Sigma V = 2P - 4P = -2P$$

Now, $$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$
$$= \sqrt{(-2P)^2 + (-2P)^2}$$

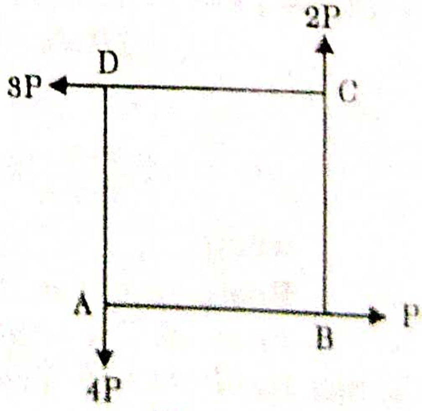


Fig. 1.43

$$= \sqrt{4P^2 + 4P^2}$$

$$= \sqrt{8P^2}$$

$$= 2P\sqrt{2}$$

Magnitude of resultant force, R = 2·828 P An

Let the angle of resultant force with horizontal, be α.

So $$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$

$$= \tan^{-1}\left(\frac{-2P}{-2P}\right)$$

$\therefore$ $$\alpha = \tan^{-1}(1) = 45^0$$

Since, ΣH and ΣV both are negative therefore, the actual valu α will lie between 180^0 and 270^0.

$\therefore$ Actual inclination, $\alpha = 180^0 + 45^0 = 225^0$

Inclination of resultant with horizontal, α = 225^0 A

... perpendicular distance between point A and line

Prob.17. *Find the magnitude and direction of the resultant of the concurrent forces of 10 N, 15 N, 20 N and 25 N making angles of 30°, 70°, 120°15′ and 155° respectively with a fixed line.* *(S/2010)*

Sol. Resolving all the forces horizontally and taking their algebraic sum,

$$
\begin{aligned}
\Sigma H &= 10 \cos 30^\circ + 15 \cos 70^\circ + 20 \cos 120^\circ 15' + 25 \cos 155^\circ \\
&= 8{\cdot}66 + 5{\cdot}13 + (-10{\cdot}04) + (-22{\cdot}66) \\
&= 8{\cdot}66 + 5{\cdot}13 - 10{\cdot}04 - 22{\cdot}66 \\
&= -18{\cdot}91\ N
\end{aligned}
$$

Now resolving all the forces vertically and taking their algebraic sum,

$$
\begin{aligned}
\Sigma V &= 10 \sin 30^\circ + 15 \sin 70^\circ + 20 \sin 120^\circ 15' + 25 \sin 155^\circ \\
&= 5 + 14{\cdot}09 + 17{\cdot}29 + 10{\cdot}56 \\
&= 46{\cdot}94\ N
\end{aligned}
$$

Now resultant force,

$$
\begin{aligned}
R &= \sqrt{(\Sigma H)^2 + (\Sigma V)^2} \\
&= \sqrt{(-18{\cdot}91)^2 + (46{\cdot}94)^2} \\
&= \sqrt{2560{\cdot}95}
\end{aligned}
$$

Magnitude of resultant force = 50·60 *N* **Ans.**

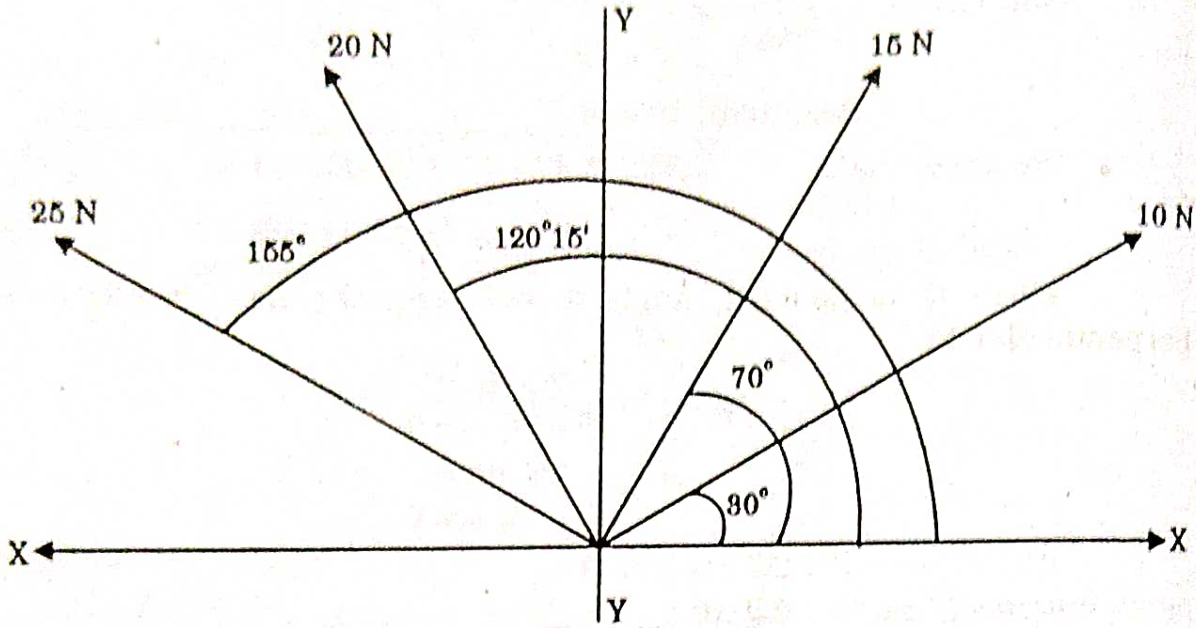


Fig. 1.45

Let α be the angle, which resultant R makes with horizontal then,

$$\tan \alpha = \frac{\Sigma V}{\Sigma H}$$

$$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$

$$= \tan^{-1}\left(\frac{46{\cdot}94}{-18{\cdot}91}\right)$$

$$= \tan^{-1}(-2{\cdot}482)$$

$$\alpha = -68^{0}3'$$

Now ΣH is negative and ΣV is positive. So α should lie between 90^{0} and 180^{0}.

$\therefore$ Actual inclination, $\alpha = 180^{0} - 68^{0}3'$

$$= 179^{0}60' - 68^{0}3' = 111^{0}57'$$

Inclination of resultant with horizontal, $\alpha = 111^{0}57'$ Ans.

Prob.18. *Two forces act at angle of 120^{0}. The bigger force is 50 N and the resultant is perpendicular to smaller one. Find the smaller force.* ***(2010)***

Sol. Given,

Bigger force, $F_1 = 50\,N$

$$\theta = \angle AOC = 120^{0}$$

$$\angle AOB = 90^{0}$$

$\therefore$ $\alpha = \angle BOC = 120^{0} - 90^{0}$

$$= 30^{0}$$

Prob.20. *Find the magnitude of two forces such that if they act at right angle, their resultant is $\sqrt{10}$ N. But if they act at 60°, their resultant is $\sqrt{13}$ N.* **(S/2011, 12, S/13, S/20)**

Sol. Let, the two forces be P and Q.

When, $\theta = 90^\circ$

$$R = \sqrt{P^2 + Q^2}$$

$$\sqrt{10} = \sqrt{P^2 + Q^2}$$

$$10 = P^2 + Q^2 \qquad \text{...(i)}$$

When, $\theta = 60^\circ$

$$R^2 = P^2 + Q^2 + 2PQ \cos 60^\circ$$

$$(\sqrt{13})^2 = P^2 + Q^2 + PQ$$

$$13 = 10 + PQ$$

$$\therefore \quad PQ = 3 \qquad \text{...(ii)}$$

We have, $(P+Q)^2 = P^2 + 2PQ + Q^2$

$$= P^2 + Q^2 + 2PQ$$

$$= 10 + 2 \times 3$$

$$= 16$$

$$\therefore \quad P + Q = 4 \qquad \text{...(iii)}$$

Also, $(P-Q)^2 = P^2 + Q^2 - 2PQ$

$$= 10 - 2 \times 3$$

$$= 4$$

$$\therefore \quad P - Q = 2 \qquad \text{...(iv)}$$

On adding equations (iii) and (iv),

$$P + Q + P - Q = 4 + 2$$

$$2P = 6$$

$$\mathbf{P = 3\,N} \qquad \textbf{Ans.}$$

Putting the value of P in equation (iii),

$$P + Q = 4$$

$$3 + Q = 4$$

$$\mathbf{Q = 1\,N} \qquad \textbf{Ans.}$$

Prob.21. *Forces of 2, 3, 5, $\sqrt{3}$ and 2 Newton's respectively act at one of the angular point of a regular hexagon towards the five other points taken in order. Find the magnitude and direction of their resultant force.* **(S/2012)**

Sol. The system of given forces is shown in fig. 1.48.

We know that each internal angle of a regular hexagon is equal to 120°.

Now resolving all the forces horizontally and taking their algebraic sum,

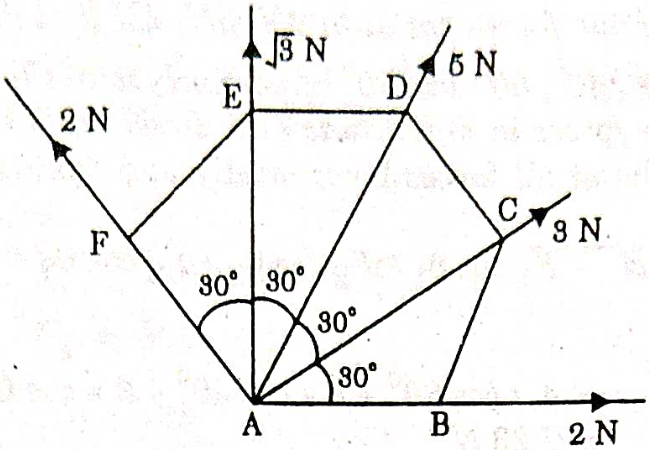


Fig. 1.48

$$\Sigma H = 2\cos 0^0 + 3\cos 30^0 + 5\cos 60^0 + \sqrt{3}\cos 90^0 + 2\cos 120^0$$
$$= 2 + (3 \times 0{\cdot}866) + (5 \times 0{\cdot}5) + (\sqrt{3} \times 0) + (2 \times -0{\cdot}5)$$
$$= 2 + 2{\cdot}598 + 2{\cdot}5 + 0 - 1$$
$$= 6{\cdot}098\ N$$

Resolving all the forces vertically and taking their algebraic sum,

$$\Sigma V = 2\sin 0^0 + 3\sin 30^0 + 5\sin 60^0 + \sqrt{3}\sin 90^0 + 2\sin 120^0$$
$$= 0 + (3 \times 0{\cdot}5) + (5 \times 0{\cdot}866) + (\sqrt{3} \times 1) + (2 \times 0{\cdot}866)$$
$$= 0 + 1{\cdot}5 + 4{\cdot}33 + 1{\cdot}732 + 1{\cdot}732$$
$$= 9{\cdot}294\ N$$

$$\text{Resultant, } R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$
$$= \sqrt{(6{\cdot}098)^2 + (9{\cdot}294)^2}$$
$$= \sqrt{37{\cdot}186 + 86{\cdot}378}$$
$$= \sqrt{123{\cdot}564}$$

Magnitude of resultant force, R = 11·11 N Ans.

The angle α, which resultant R makes with the horizontal (i.e., East direction) is,

$$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$
$$= \tan^{-1}\left(\frac{9{\cdot}294}{6{\cdot}098}\right)$$
$$= \tan^{-1}(1{\cdot}524)$$

Inclination of resultant with horizontal, α = 56°43′ Ans.

Inclination of resultant with horizontal,

Prob.27. *Three forces of 5, 7, 9 N are acting along the consecutive sides of equilateral triangle in order find out the magnitude and direction of resultant force.* ***(S/2019)***

Sol. Let ABC is an equilateral triangle and $5\,N$, $7\,N$ and $9\,N$ forces acts along the consecutive sides of equilateral triangle.

Resolving all the forces horizontally and taking their algebraic sum,

$$\Sigma H = 5\cos 0^\circ - 7\cos 60^\circ - 9\cos 60^\circ$$
$$= 5 - 7 \times 0{\cdot}5 - 9 \times 0{\cdot}5$$
$$= -3\,N$$

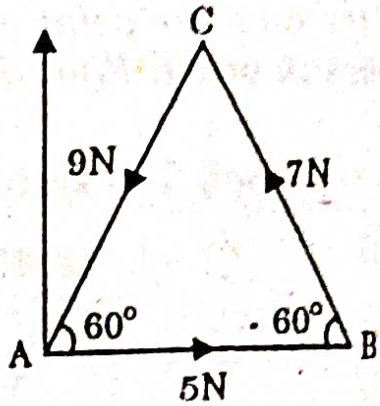


Fig. 1.54

Now, resolving all the forces vertically and taking their algebraic sum,

$$\Sigma V = 5\sin 0^\circ + 7\sin 60^\circ - 9\sin 60^\circ$$
$$= 7 \times 0{\cdot}866 - 9 \times 0{\cdot}866$$
$$= -1{\cdot}732\,N$$

$$\text{Resultant force, } R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$
$$= \sqrt{(-3)^2 + (-1{\cdot}732)^2}$$
$$R = 3{\cdot}46\,N \qquad \textbf{Ans.}$$

Let α be an angle, which the resultant R makes with horizontal

$$\therefore \quad \tan\alpha = \frac{\Sigma V}{\Sigma H}$$
$$= \frac{-1{\cdot}732}{-3}$$
$$= 0{\cdot}577$$
$$\alpha = \tan^{-1}(0{\cdot}577)$$
$$\alpha = 29{\cdot}98 = 30^\circ \text{ (In horizontal direction)} \qquad \textbf{Ans.}$$

Prob.28. *Four forces of magnitude 4 N, 3 N, 2 N and 6 N are acting*

Prob.1. *A body of 65 kg is suspended by two string of length 50 cm and 120 cm, attached to two points in same horizontal line and 130 cm apart. Find the tension in the string.* **(S/2003, S/22)**

Sol. Given,

$$\text{Weight} = 65\ kg = 65 \times 10 = 650\ N$$

$$AC = 50\ cm = 0.5\ m$$

$$BC = 120\ cm = 1.2\ m$$

$$AB = 130\ cm = 1.3\ m$$

Now in triangle ABC,

$$(AB)^2 = (BC)^2 + (AC)^2$$
$$(1{\cdot}3)^2 = (1{\cdot}2)^2 + (0{\cdot}5)^2$$
$$1{\cdot}69 = 1{\cdot}69$$
$$\therefore \quad \angle ACB = 90^0$$
$$\sin A = \frac{BC}{AB}$$
$$\sin A = \frac{1{\cdot}2}{1{\cdot}3}$$
$$\angle A = \sin^{-1}\left(\frac{1{\cdot}2}{1{\cdot}3}\right)$$
$$= 67^0\ 22'\ 48{\cdot}49''$$
$$\angle B = 90^0 - \angle A$$
$$= 90^0 - 67^0\ 22'\ 48{\cdot}49'' = 22^0\ 37'\ 11{\cdot}51''$$

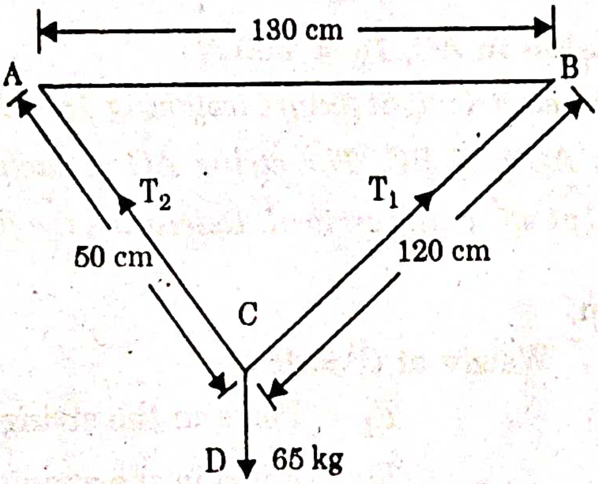


Fig. 2.34

Let, T_1 = Tension in BC

T_2 = Tension in AC

$$\because \quad \angle ACB = 90^0$$
$$\angle ACD = 180^0 - \angle B$$
$$= 180^0 - 22^0\ 37'\ 11{\cdot}51''$$
$$= 157^0\ 22'\ 48{\cdot}4''$$

Similarly,
$$\angle BCD = 180^0 - \angle A$$
$$= 180^0 - 67^0\ 22'\ 48{\cdot}49''$$
$$= 112^0\ 37'\ 11{\cdot}5''$$

Applying Lami's theorem at point C,

$$\frac{T_1}{\sin \angle ACD} = \frac{T_2}{\sin \angle BCD} = \frac{650}{\sin 90^0}$$

$$\frac{T_1}{\sin(157^\circ\ 22'\ 48{\cdot}4'')} = \frac{T_2}{\sin(112^\circ\ 37'\ 11{\cdot}5'')} = \frac{650}{\sin 90^\circ}$$

$$\frac{T_1}{\sin(157^\circ\ 22'\ 48{\cdot}4'')} = \frac{650}{\sin 90^\circ}$$

$$\therefore \quad T_1 = \frac{650 \times \sin(157^\circ\ 22'\ 48{\cdot}4'')}{\sin 90^\circ}$$

$$= 650 \times 0{\cdot}3846 = 249{\cdot}99 \approx 250$$

Tension in BC, T_1 = 250 N **Ans.**

and
$$\frac{T_2}{\sin(112^\circ\ 37'\ 11{\cdot}5'')} = \frac{650}{\sin 90^\circ}$$

$$T_2 = \frac{650 \times \sin(112^\circ\ 37'\ 11{\cdot}5'')}{\sin 90^\circ}$$

$$= \frac{650 \times 0{\cdot}923}{1} = 599{\cdot}95 \approx 600$$

Tension in AC, T_2 = 600 N **Ans.**

Prob.2. *An electric light fixture weighing 15 N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the vertical. Determine the forces in the strings AC and BC.* ***(2009, 14)***

Sol. Given,

Weight at C = 15 N

Let, T_1 = Force in the string AC

T_2 = Force in the string BC

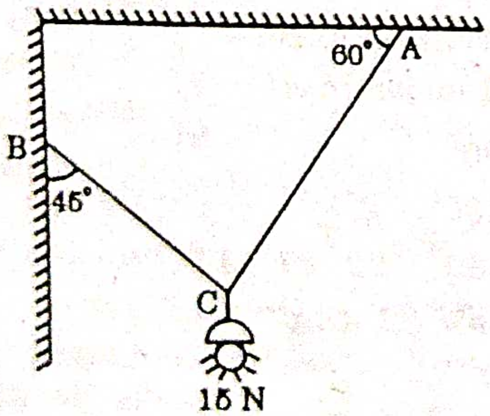


Fig. 2.35

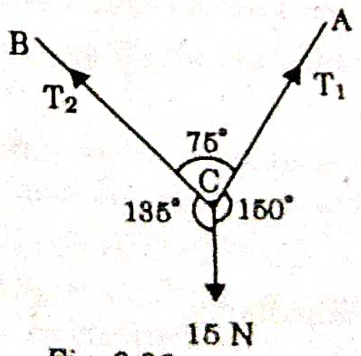


Fig. 2.36

From the geometry of the fig. 2.36, the angle between T_1 and 15 N is 150° and angle between T_2 and 15 N is 135°,

$$\angle ACB = 180^\circ - (45^\circ + 60^\circ) = 75^\circ$$

Applying Lami's theorem at point C,

$$\frac{15}{\sin 75^0} = \frac{T_1}{\sin 135^0} = \frac{T_2}{\sin 150^0}$$

or
$$\frac{15}{\sin 75^0} = \frac{T_1}{\sin (180^0 - 45^0)} = \frac{T_2}{\sin (180^0 - 30^0)}$$

$$\frac{15}{\sin 75^0} = \frac{T_1}{\sin 45^0} = \frac{T_2}{\sin 30^0}$$

$\therefore$
$$T_1 = \frac{15 \sin 45^0}{\sin 75^0}$$

$$= \frac{15 \times 0{\cdot}707}{0{\cdot}9659}$$

Force in the string AC, T_1 = $10{\cdot}98\ N$ **Ans.**

and
$$T_2 = \frac{15 \sin 30^0}{\sin 75^0}$$

$$= \frac{15 \times 0{\cdot}5}{0{\cdot}9659}$$

Force in the string BC, T_2 = $7{\cdot}76\ N$ **Ans.**

Prob.4. *Two men carry a weight of 2 kN by means of two ropes fixed to the weight. One rope is inclined at 45° and other at 30° with their vertices. Find the tension in each rope.* **(S/2011,15)**

Two men carry a load of 2 kN by means of two ropes fixed to the weight. One rope is inclined at 45° and other at 30° to the vertical. Find the tension in each rope. **(S/2020)**

Sol. Suppose, T_1 is the tension in string BC and T_2 in AC.

Applying Lami's theorem at point C,

$$\frac{T_1}{\sin(90^\circ + 60^\circ)} = \frac{T_2}{\sin(90^\circ + 45^\circ)} = \frac{2}{\sin[180^\circ - (60^\circ + 45^\circ)]}$$

$$\frac{T_1}{\sin 150^\circ} = \frac{T_2}{\sin 135^\circ} = \frac{2}{\sin 75^\circ}$$

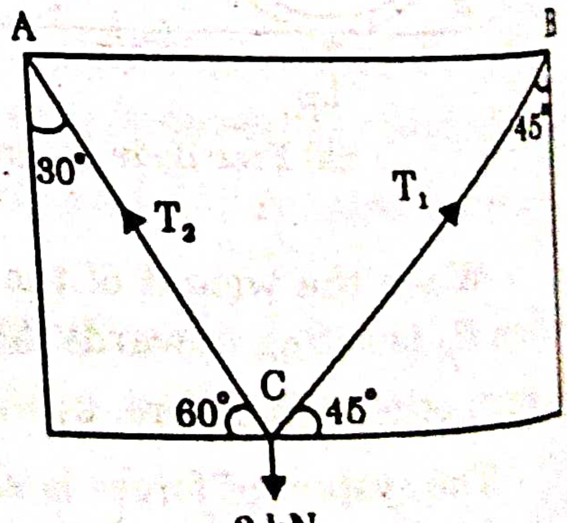


Fig. 2.40

$$\therefore \quad \frac{T_1}{\sin 150^\circ} = \frac{2}{\sin 75^\circ}$$

$$T_1 = \frac{2 \sin 150^\circ}{\sin 75^\circ}$$

$$= \frac{2 \times 0.5}{0.9659}$$

Tension in BC, T_1 = 1.035 kN Ans

$$\text{and} \quad \frac{T_2}{\sin 135^\circ} = \frac{2}{\sin 75^\circ}$$

$$T_2 = \frac{2 \sin 135^0}{\sin 75^0}$$

$$= \frac{2 \times 0{\cdot}707}{0{\cdot}9659}$$

Tension in *AC*, T_2 = 1·464 *kN* **Ans.**

Ans.

$x = 2{\cdot}75\ m$

Prob.12. *A simply supported beam AB of span 4 m is carrying point loads of 5, 2 and 3 kN at 1, 2 and 3 m respectively from the support A. Calculate the reactions at supports A and B.* *(2008, 10)*

Sol. Let R_A and R_B be the vertical reactions at support A and B respectively.

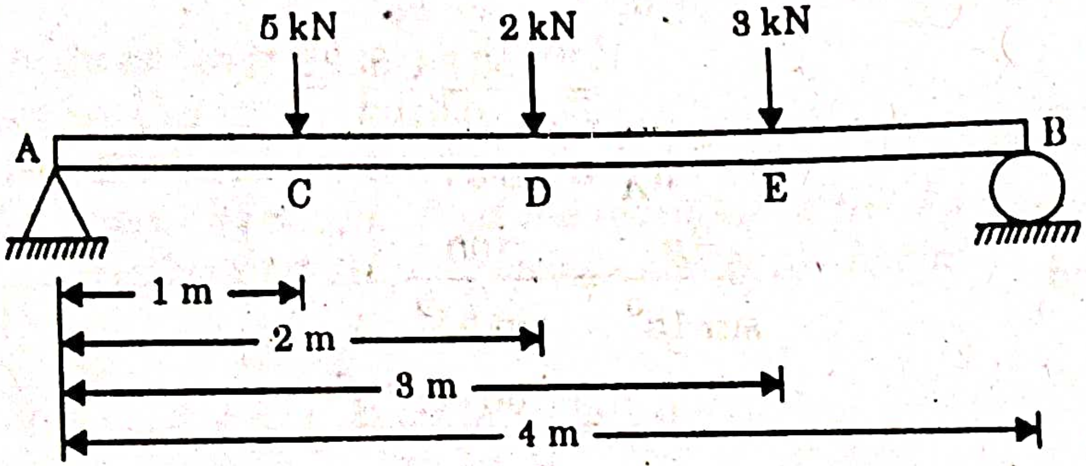


Fig. 2.49 Simply Supported Beam

For the equilibrium of the beam, taking moments of the force on the beam about the support A.

$$R_B \times 4 = 5 \times 1 + 2 \times 2 + 3 \times 3$$

$$4R_B = 5 + 4 + 9 = 18$$

$$R_B = \frac{18}{4}$$

Reaction at B, $R_B = \mathbf{4{\cdot}5\ kN}$ Ans.

$$R_A = \text{Total load on the beam} - R_B$$

$$= (5 + 2 + 3) - 4{\cdot}5$$

$$R_A = 10 - 4{\cdot}5$$

Reaction at A, $R_A = \mathbf{5{\cdot}5\ kN}$ Ans.

Prob.13. *A simply supported beam AB of span 6 m is carrying point load of 5, 2 and 3 kN at 2, 4 and 5 m respectively from the support A. Calculate the reaction at supports A and B.* *(2012)*

Sol. Let, R_A and R_B be the vertical reactions at support A and B respectively.

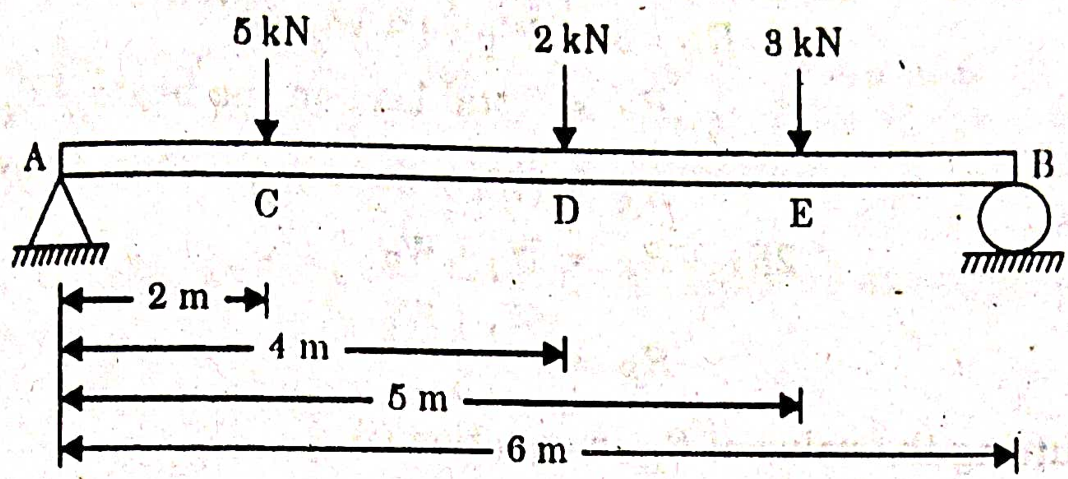


Fig. 2.50 Simply Supported Beam

For the equilibrium of the beam, taking moments of the force on the beam about the support A.

$$R_B \times 6 = 5 \times 2 + 2 \times 4 + 3 \times 5$$

$$6R_B = 10 + 8 + 15 = 33$$

$$R_B = \frac{33}{6}$$

Reaction at B, R_B = 5·5 *kN* **Ans.**

$$R_A = \text{Total load on the beam} - R_B$$

$$= (5 + 2 + 3) - 5{\cdot}5$$

$$R_A = 10 - 5{\cdot}5$$

Reaction at A, R_A = 4·5 *kN* **Ans.**

Prob.1. *A block of metal is placed on a table having a metallic top. The angle of friction for metal on metal is 10^0. It has been found that a push of 200 N inclined at 30^0 to the horizontal is adequate to just move the block. Determine the weight of the block.* **(2000)**

Sol. Given,

$$\text{Angle of friction, } \phi = 10^0$$
$$\text{Load, } P = 200\,N$$
$$\text{Angle of inclination, } \theta = 30^0$$

Resolving the forces on the body horizontally, we get

$$\mu R = 200 \cos 30^0 \qquad (\because \mu = \tan \phi)$$
$$R \tan 10^0 = 200 \cos 30^0$$
$$R = \frac{200 \cos 30^0}{\tan 10^0}$$
$$= \frac{200 \times 0{\cdot}866}{0{\cdot}176}$$
$$R = 982{\cdot}29\,N$$

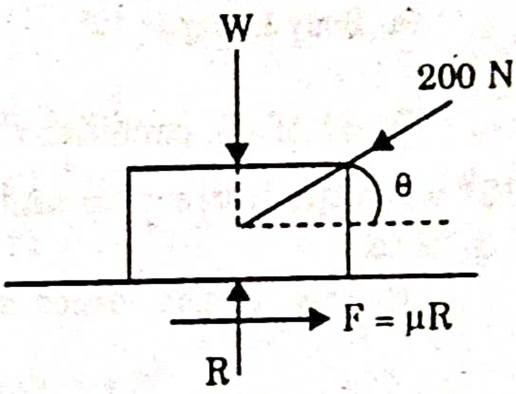


Fig. 3.11

Resolving the forces on the body vertically, we get

$$R = W + 200 \sin 30^0$$
$$W = R - 200 \sin 30^0$$
$$= 982{\cdot}29 - 200 \times 0{\cdot}5$$

Weight of the block, $W = 882{\cdot}29\,N$ **Ans.**

Prob.2. *An effort of 150 N is required just to move a certain body up an inclined plane of angle 12^0, the force acting parallel to the plane. If the angle of inclination of the plane is made 15^0, the effort required, again parallel to the plane, is found to be 172 N. Find the weight of the body and the coefficient of friction.* **(2001)**

An effort of 150 N is required to just move a certain body upon a inclined plane at angle 12^0. The force is acting parallel to the plane. If the angle of inclination changes 15^0, the effort required to drag is 172 N. Find the coefficient of friction. **(S/2008)**

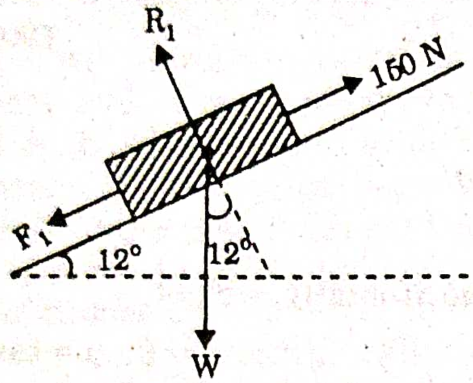
Sol. Given,

$$P_1 = 150\,N$$
$$\theta_1 = 12^0$$

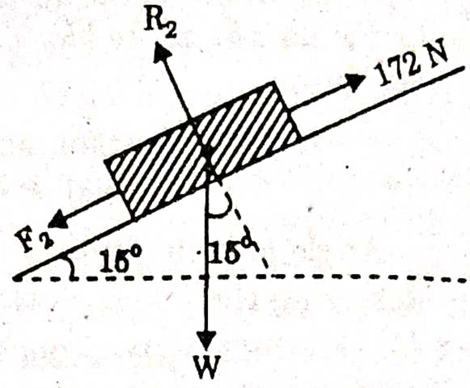
$$P_2 = 172\ N$$

$$\theta_2 = 15^o$$

Let,
μ = Coefficient of friction
W = Weight of the body in Newtons
R = Normal reaction
F = Force of friction



(a) Body Lying at 12°



(b) Body Lying at 15°

Fig. 3.12

First of all, consider the body lying on a plane inclined at an angle of 12^o with the horizontal and subjected to an effort of 150 N as shown in fig. 3.12 (a).

Resolving the forces at right angles to the plane,

$$R_1 = W \cos 12^o \quad \text{...(i)}$$

Now, resolving the forces along the plane,

$$150 = F_1 + W \sin 12^o$$

$$= \mu R_1 + W \sin 12^o \quad (\because F_1 = \mu R_1)$$

$$= \mu W \cos 12^o + W \sin 12^o$$

$$= W (\mu \cos 12^o + \sin 12^o) \quad \text{...(ii)}$$

Now, consider the body lying on a plane inclined at an angle of 15^o with the horizontal and subjected to an effort of 172 N as shown in fig. 3.12 (b).

Resolving the forces at right angles to the plane,

$$R_2 = W \cos 15^o \quad \text{...(iii)}$$

Now, resolving the forces along the plane

$$172 = F_2 + W \sin 15^o$$

$$= \mu R_2 + W \sin 15^o \quad (\because F_2 = \mu R_2)$$

$$= \mu W \cos 15^o + W \sin 15^o$$

$$= W (\mu \cos 15^o + \sin 15^o) \quad \text{...(iv)}$$

Dividing equation (iv) by (ii) we get,

$$\frac{172}{150} = \frac{W(\mu \cos 15^o + \sin 15^o)}{W(\mu \cos 12^o + \sin 12^o)}$$

$$172\,\mu \cos 12^o + 172 \sin 12^o = 150\,\mu \cos 15^o + 150 \sin 15^o$$

$$\mu\,(172 \cos 12^o - 150 \cos 15^o) = 150 \sin 15^o - 172 \sin 12^o$$

$$\mu = \frac{150 \sin 15^o - 172 \sin 12^o}{172 \cos 12^o - 150 \cos 15^o}$$

$$= \frac{150 \times 0{\cdot}2588 - 172 \times 0{\cdot}2079}{172 \times 0{\cdot}9781 - 150 \times 0{\cdot}9659}$$

$$= \frac{38{\cdot}82 - 35{\cdot}7588}{168{\cdot}23 - 144{\cdot}885}$$

$$= \frac{3{\cdot}0612}{23{\cdot}345} = 0{\cdot}1311$$

Coefficient of friction, μ = 0·1311 **Ans.**

Substituting the value of μ in equation (ii),

$$150 = W(0{\cdot}1311 \cos 12^o + \sin 12^o)$$

$$= W(0{\cdot}1311 \times 0{\cdot}9781 + 0{\cdot}2079)$$

$$150 = 0{\cdot}336\,W$$

$$W = \frac{150}{0{\cdot}336} = 446{\cdot}4\,N$$

Weight of the body, W = 446·4 N **Ans.**

Prob.3. *A metal block weighing 50 kg is placed on a horizontal wooden surface. The coefficient of friction of metal on wood is 0·55. Find the horizontal force which is just sufficient to pull the block.* ***(2002)***

Sol. Given,

Coefficient of friction, $\mu = 0{\cdot}55$

Weight, $W = 50\,kg = 490\,N$

$\because \quad F = \mu R$

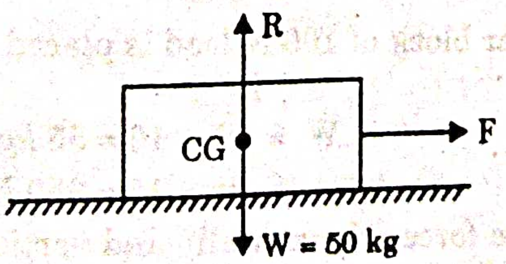


Fig. 3.13

Resolving the force vertically,

$$R = W$$

Hence,

$$F = \mu R = \mu W$$

$$= 490 \times 0{\cdot}55$$

$$= 269{\cdot}5\,N$$

Ans.

$\bar{y} = 6$ cm Ans.

Prob.2. *An I-section has the following dimensions in mm units :*

Bottom flange = 300×100

Top flange = 150×50

Web = 400×50

Determine mathematically the position of centre of gravity of the section. (S/2001, 02)

An I-section has the following dimensions :

(1) top flange = 150×50

(2) web = 400×50 *mm*

(3) bottom flange = 300×100 *mm*

Find CG of the above sections. (2014)

Sol. Given, Top flange = 150×50 *mm*

Web = 400×50 *mm*

Bottom flange = 300×100 *mm*

The section is symmetrical about *Y-Y* axis, therefore its centre of gravity will lie on this axis. Split up the whole section into three rectangles 1, 2 and 3. Let, bottom face of the bottom flange be the axis of reference.

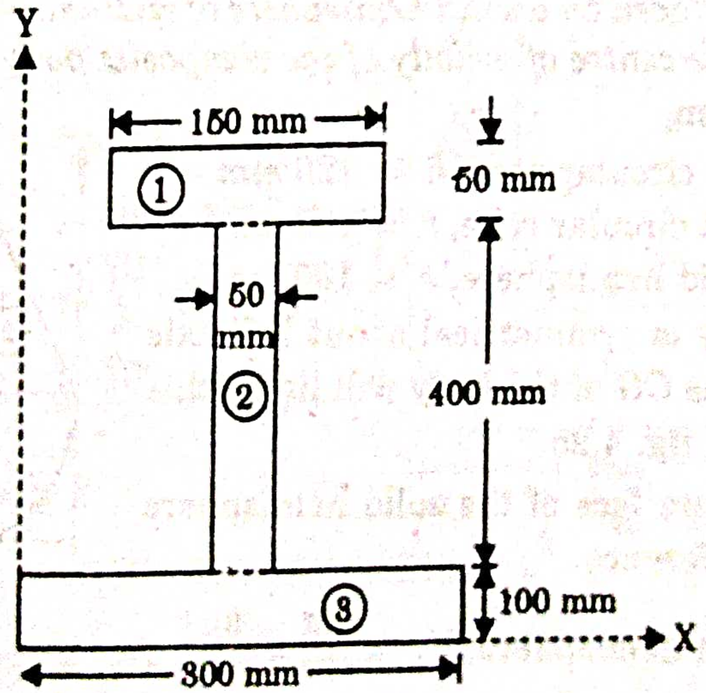


Fig. 4.25

For top flange, $a_1 = 150 \times 50 = 7500\ mm^2$

and $y_1 = 100 + 400 + \dfrac{50}{2} = 525\ mm$

For web, $a_2 = 400 \times 50 = 20000\ mm^2$

and $y_2 = 100 + \dfrac{400}{2} = 300\ mm$

For bottom flange, $a_3 = 300 \times 100 = 30000\ mm^2$

Ans.

Prob.4. Find out the centroid of an unequal angle section 100 mm × 80 mm × 20 mm. **(2005, 10, S/11, S/13)**

Sol. Let split up the section into two rectangles 1 and 2 as shown in fig. 4.27.

Let bottom face of the horizontal section be the axis of reference.

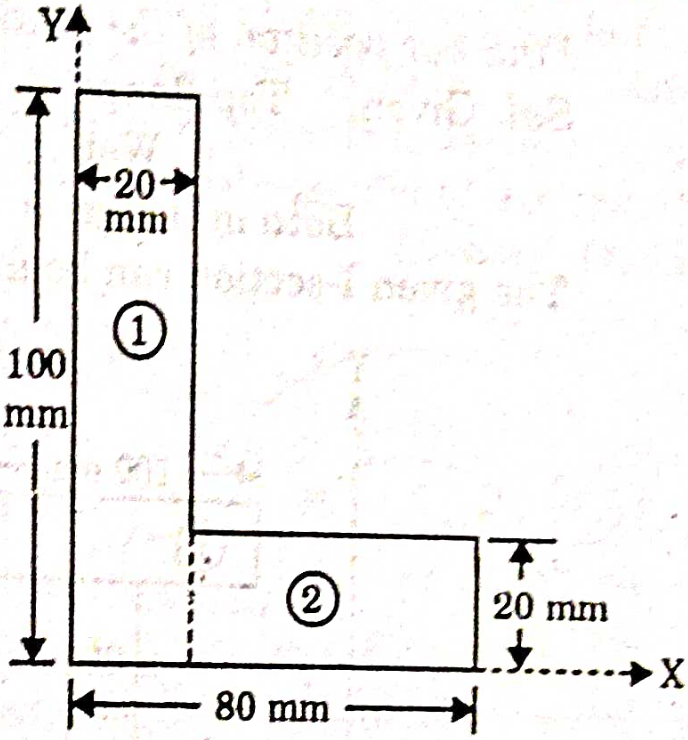


Fig. 4.27

For rectangle 1,

$$a_1 = 100 \times 20$$

$$= 2000\ mm^2$$

$$x_1 = \frac{20}{2} = 10\ mm$$

and $$y_1 = \frac{100}{2} = 50\ mm$$

For rectangle 2,

$$a_2 = (80 - 20) \times 20 = 1200\ mm^2$$

$$x_2 = 20 + \frac{60}{2} = 50\ mm$$

and $$y_2 = \frac{20}{2} = 10\ mm$$

The distance between centre of gravity of the section and left face,

$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2}$$

$$= \frac{(2000 \times 10) + (1200 \times 50)}{2000 + 1200}$$

$$\bar{x} = 25\ mm$$ **Ans.**

The distance between centre of gravity of the section and bottom face,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{(2000 \times 50) + (1200 \times 10)}{2000 + 1200}$$

$$\bar{y} = 35 \ mm$$

Ans

Prob.5. *An I-section is made of three rectangles having following dimensions :*

Top flange $= 100 \times 20 \ mm$

Web $= 180 \times 20 \ mm$

Bottom flange $= 200 \times 40 \ mm$

Find out the position of the centre of gravity of the section. (2006)

An I-section has the following dimensions in mm units :

Top flange $= 100 \times 20 \ mm$

Web $= 180 \times 20 \ mm$

Bottom flange $= 200 \times 40 \ mm$

Find out position of the centre of gravity of the section. (2009)

Sol. Given, Top flange $= 100 \times 20 \ mm$

Web $= 180 \times 20 \ mm$

Bottom flange $= 200 \times 40 \ mm$

The given I-section can be divided into three rectangles.

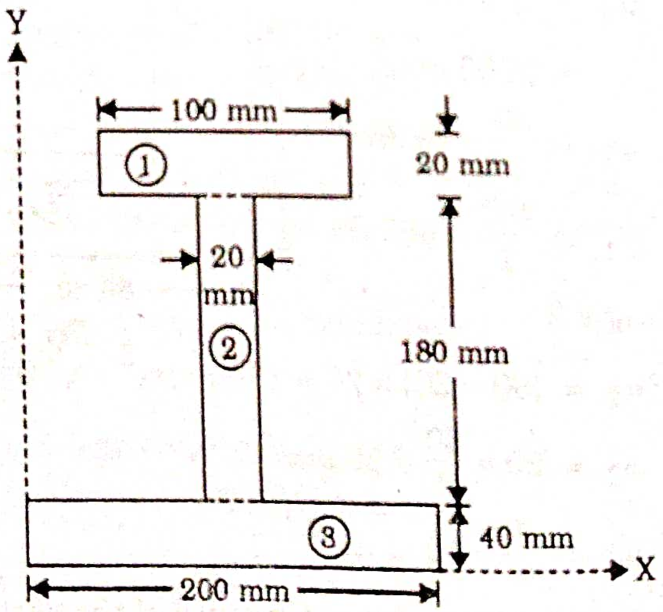


Fig. 4.28

For top flange, $a_1 = 100 \times 20 + 2000 \ mm^2$

and $y_1 = 40 + 180 + \frac{20}{2} = 230 \ mm$

For web, $a_2 = 180 \times 20 = 3600 \ mm^2$

and $$y_2 = 40 + \frac{180}{2} = 130\ mm$$

For bottom flange, $$a_3 = 200 \times 40 = 8000\ mm^2$$

and $$y_3 = \frac{40}{2} = 20\ mm$$

The distance between centre of gravity of the section and bottom

...ce,

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(2000 \times 230) + (3600 \times 130) + (8000 \times 20)}{2000 + 3600 + 8000}$$

$$\bar{y} = 80\ mm$$

The given I-section is symmetrical about vertical Y-axis therefore ...e distance of CG from Y-axis or x-coordinate of CG will be

$$\bar{x} = \frac{200}{2} = 100\ mm$$

The position of centre of gravity of a given I-section is at a ...istance of 80 *mm* from *Y*-axis and 100 *mm* from *X*-axis. Ans.

$$= \frac{(30\pi r^2 \times 15) + (40\pi r^2 \times 60)}{(30\pi r^2 + 40\pi r^2)}$$

$$= \frac{2850}{70}$$

$$\bar{y} = \mathbf{40{\cdot}71\ mm} \qquad \text{Ans.}$$

Prob.13. *Find the centroid of an inverted T-section with flange 60 mm × 10 mm and web 50 mm × 10 mm.* **(S/2018)**

Sol. Given, Flange = $60\ mm \times 10\ mm$

Web = $50\ mm \times 10\ mm$

The section is symmetrical about Y-Y axis, therefore its centre of gravity will lie on this axis. Split up the whole section into two rectangles. Let bottom of the flange be the axis of reference.

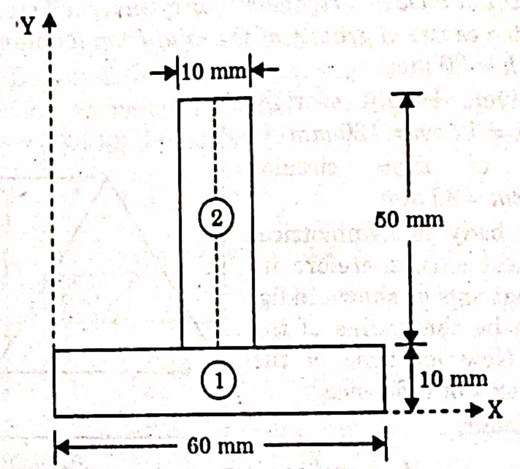


Fig. 4.36

For flange, $a_1 = 60 \times 10 = 600\ mm^2$

and $y_1 = \frac{10}{2} = 5\ mm$

For Web, $a_2 = 50 \times 10 = 500\ mm^2$

and $y_2 = 10 + \frac{50}{2} = 35\ mm$

The distance between centre of gravity of the section and bottom face,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{(600 \times 5) + (500 \times 35)}{600 + 500}$$

$$\bar{y} = \mathbf{18{\cdot}63\ mm} \qquad \text{Ans.}$$

Prob.14. *Find the centroid of a 100 mm × 150 mm × 30 mm T-section.* **(2015)**

Find the centre of gravity of a 100 mm × 150 mm × 30 mm T-section. **(S/2019)**

Sol. The section is symmetrical about Y-Y axis, therefore its centre of gravity will lie on this axis. Split up the whole section into two rectangles. Let, bottom of the web be the axis of reference.

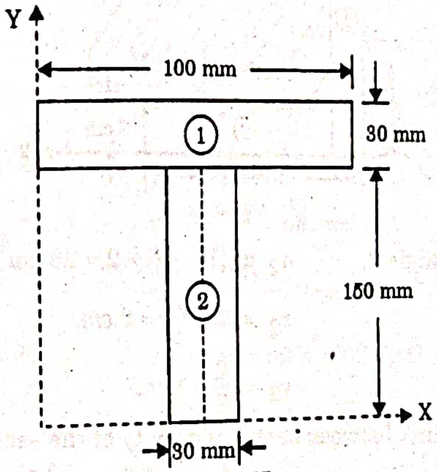


Fig. 4.37

For flange, $a_1 = 100 \times 30 = 3000\ mm^2$

and $y_1 = 150 + \frac{30}{2} = 165\ mm$

For web, $a_2 = 150 \times 30 = 4500\ mm^2$

and $y_2 = \frac{150}{2} = 75\ mm$

The distance between centre of gravity of the section and bottom of the web,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{(3000 \times 165) + (4500 \times 75)}{3000 + 4500}$$

$$\bar{y} = \mathbf{111\ mm} \qquad \text{Ans.}$$

Prob.15. *Find the position of the centre of gravity of an unequal angle section 10 cm × 16 cm × 2 cm.* **(S/2017)**

Sol. Let, split up the section into two rectangles 1 and 2 as shown in fig. 4.38.

Let bottom face of the horizontal section be the axis of reference.

For rectangle 1, $a_1 = 10 \times 2 = 20\ cm^2$

$$x_1 = \frac{2}{2} = 1\ cm$$

and $$y_1 = \frac{10}{2} = 5\ cm$$

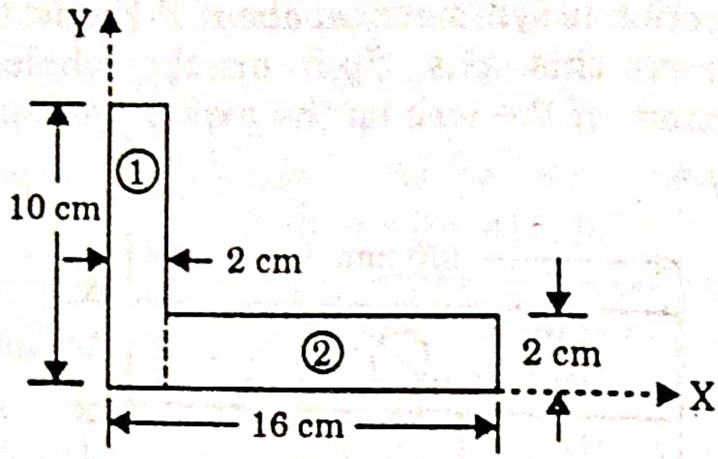


Fig. 4.38

For rectangle 2, $$a_2 = (16 - 2) \times 2 = 28\ cm^2$$

$$x_2 = 2 + \frac{14}{2} = 9\ cm$$

and $$y_2 = \frac{2}{2} = 1\ cm$$

The distance between centre of gravity of the section and left fac

$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2}$$

$$= \frac{(20 \times 1) + (28 \times 9)}{20 + 28}$$

$$\bar{x} = 5.67\ cm$$ An

The distance between centre of gravity of the section and botto

face,

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2}$$

$$= \frac{(20 \times 5) + (28 \times 1)}{20 + 28}$$

$$\bar{y} = 2.67\ mm$$ Ar

Prob.16. *A square disc of 50 mm ...*

Ans.

Prob.19. *Calculate the position of centroid of unequal angle section 100 × 80 × 10 in mm.*

(S/2022)

Sol. Let split up the section into two rectangles 1 and 2 as shown Fig. 4.42.

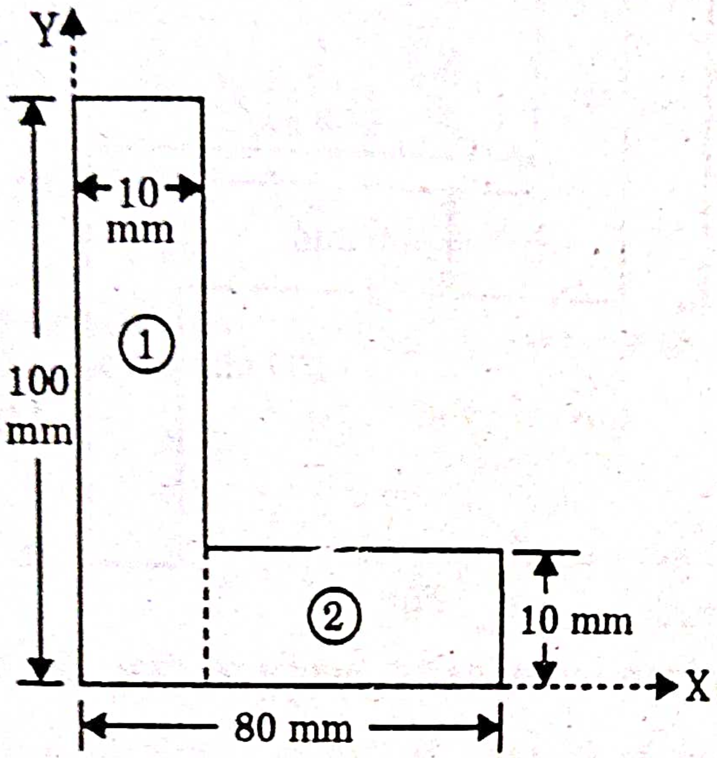


Fig. 4.42

Let bottom face of the horizontal section be the axis of reference.

For rectangle 1, $a_1 = 100 \times 10 = 1000\ mm^2$

$$x_1 = \frac{10}{2} = 5\ mm$$

and $$y_1 = \frac{100}{2} = 50\ mm$$

For rectangle 2, $a_2 = (80 - 10) \times 10 = 700\ mm^2$

$$x_2 = 10 + \frac{70}{2} = 45\ mm$$

and $$y_2 = \frac{10}{2} = 5\ mm$$

The distance between centre of gravity of the section and left face

$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2}$$

$$= \frac{(1000 \times 5) + (700 \times 45)}{1000 + 700}$$

$$\bar{x} = \mathbf{21 \cdot 47}\ \boldsymbol{mm}$$ Ans

The distance between centre of gravity of the section and bottom face,

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2}$$

$$= \frac{(1000 + 50) + (700 \times 5)}{1000 + 700}$$

$$\bar{y} = \mathbf{31 \cdot 47}\ \boldsymbol{mm}$$ Ans