Fundamental Concepts

Space: It is a region, which extends in all direction and contains everything in it. Examples: Sun, moon, star etc. In space position of body is located with respect to a reference system. The position of aircraft in space found with respect to earth.

Time: It is measure of succession of events. The time is measured in seconds (s) and other related units. An event can be describe by position of point.

Mass: It is an indication of quantity of matter present in a system. The more mass means more matter.

Flexible body: A body, which deform, under the action of applied force, is call flexible body.

Rigid body: A body, which does not deform under the action of applied forces is called rigid body.

Scalar Quantity: Quantities which described by their magnitude known as scalar quantity. Examples are mass, length, time, volume, temperature etc.

Vector Quantity: Quantities, which described by their magnitude and direction (both) known as vector quantity. Examples are velocity, force, acceler-ation, momentum etc.

Significance and Relevance of Mechanics

Mechanics can be defined as branch of science, which deals with behaviour of body under the action of forces. Engineering mechanics refers to practical applications of principles of mechanics to engineering problems. Engineering mechanics is also called as applied mechanics.

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Fundamental units: Length, Mass and time are basic fundamental quantities and unit of these quantities are known as fundamental units.

Derived units: Units of other than fundamental quantities may be derive from the basic units referred as derived units. Examples: (1) Area is result of multiplication of two lengths quantity as L^2. (2) Velocity is length divided by time as $\frac{L}{T}$. (3) Force is product of mass and acceleration as $kg\ m/sec^2$ [N].

SI units: By international agreement in 1960, the international system of units known as S.I. unit is accepted and used all over the world wide. The symbol and ~~units~~ notations of SI units and their derivatives are standardize to avoid any possibility of confusion.

S.no	Fundamental Quantity	Name of SI unit	Symbol
1.	length	Meter	m
2.	Mass	kilogram	kg
3.	Time	Second	s
4.	Electrical Current	Ampere	A
5.	Temperature	Kelvin	K
6.	Luminous Intensity	Candela	cd

Force:

Force is an external agent which tends to changes the state of body at rest or in motion.

Unit of Force:

The force is measured in Newton (N). A Newton is defined as a force which can produce an acceleration of 1 meter per second^2 in a body of 1 kg mass. The longer units of force are-

1 kilo Newton (KN) = 1000 Newton = 10^3 N

1 Mega Newton (MN) = 1000N x 1000 = 10^6 N

1 Giga Newton (GN) = 1000 x 1000 x 1000 N = 10^9 N

Characteristics of force:

To represent force completely it is required following four elements, which are known as characteristics of force.

A. Magnitude
B. Direction
C. Sense - Type of force
D. Point of Application

Effect of Force:

The force when applied, following effects may happen on rigid body:

1. Changes its state of rest or motion.
2. Accelerate or retard its motion.
3. Change its shape and size
4. Turn or rotate it.
5. keep it in equilibrium.

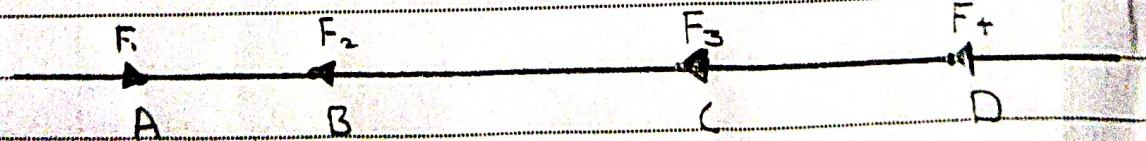
Force system and Classification:

When several forces or group of force act simultaneously on a body, the body is said to be under the action of force system. These force systems are further classify according to the line of action and arrangements of the forces as shown below.

Types of Force system:

1. Collinear force system:

The line of action of all forces lies along the same straight line as shown in fig, then that force system is known as collinear force system.



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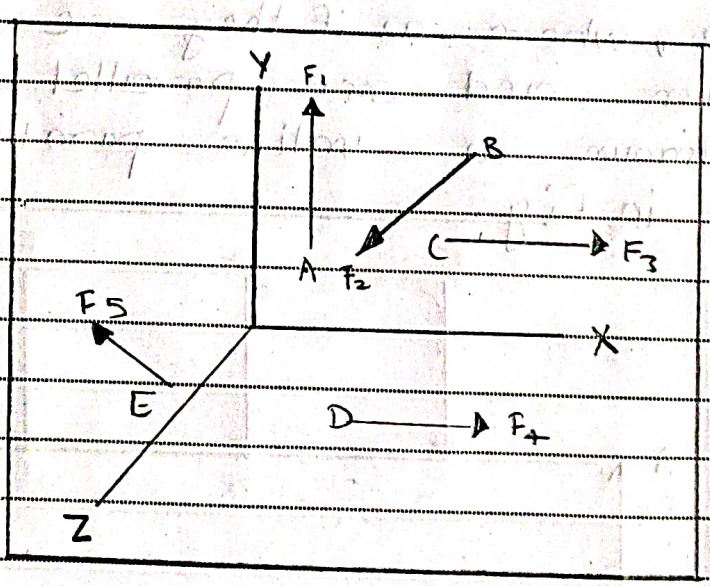
Coplanar force system:

All the forces in this system are lie in one plane, system is known as coplanar force system. Forces F_1, F_2 and F_3 only of force system acting on plan XY i.e., on same plane shown in fig is example of coplanar force system.

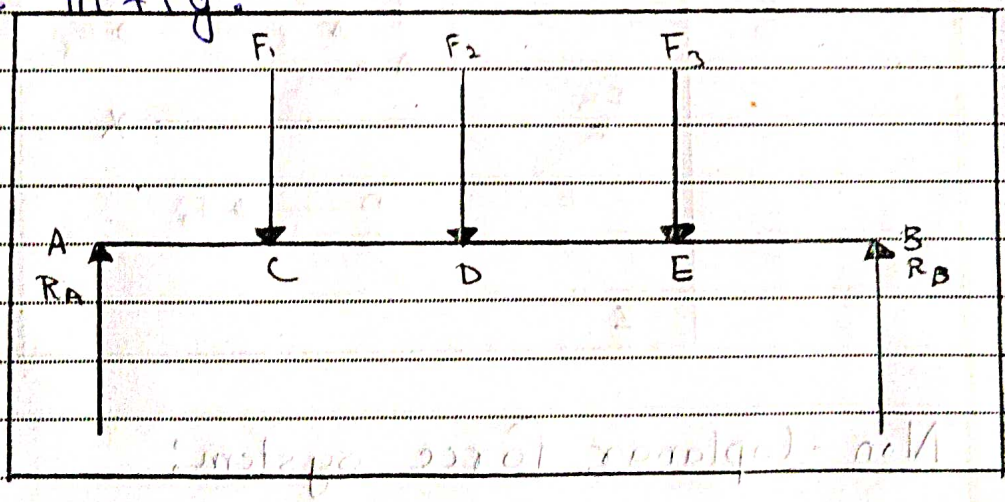


Non-Coplanar force system:

All the forces in the system are not lie in the same plane but act on different plane as shown in fig acting on planes XY, YZ, ZX.

Parallel force system:

The line of action means direction of all the forces are parallel to each other and do not intersect. This system is further sub classified as, Like parallel forces and Unlike parallel forces. If forces acting in the same direction, and are parallel to each other are known as like parallel forces, where as if they are acting in opposite direction and are parallel to each other are known as unlike parallel force as shown in fig.

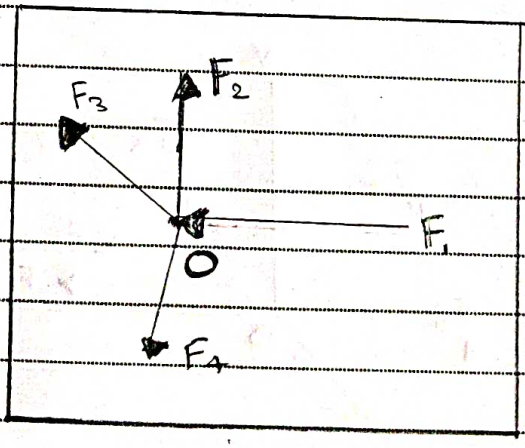


Forces F_1, F_2 and F_3 is like parallel forces for this force system, while all forces F_1, F_2, F_3 and R_A & R_B of force system is unlike parallel forces. Hence all the forces lie in an plane so form Co-planar parallel force

system.

Concurrent force System:

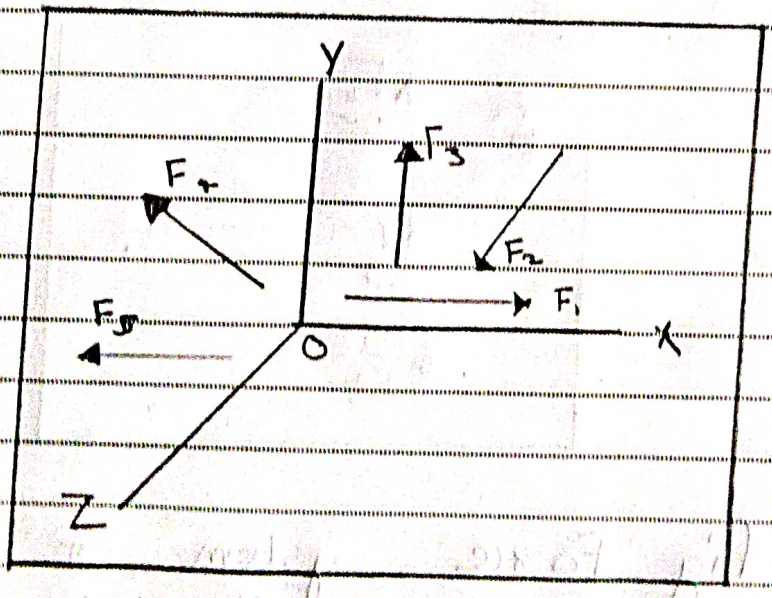
All the forces have different directions but their line of action (Direction) passes through a single common point. Such force system known as concurrent force system. The point, where the line of action of all the forces meet is known as point of concurrency of force system.



In fig Force system of forces F_1, F_2, F_3 and F_4 pass through common point O and all lies on same plan such force system known as coplanar concurrent force system.

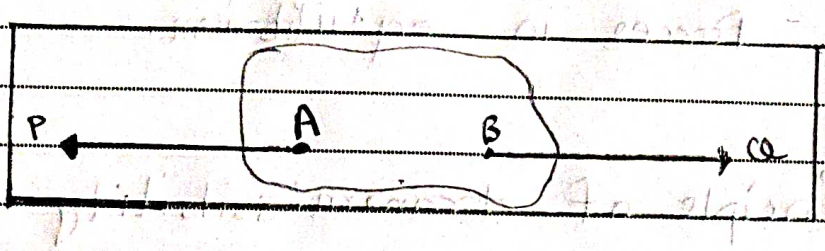
Non-Concurrent and Non-parallel force system.

If the forces of force system are not lie in same line and not pass through common point as well as line of action are not parallel to each other, it means force system which not satisfy the condition of parallel concurrent and linear force system, then such system is known as Non-Concurrent & Non parallel force system.



Equillbrium law of Force :

Two forces can be in equilibrium only, if they are equal in magnitude, opposite in direction and collinear in action.



A body is shown in fig acted upon by two forces P and Q with line of action is same AB at point A and B respectively Now what will happen if (1) The magnitude of P is greater than Q. (2) The magnitude of P is smaller than Q & (3) P and Q are equal. You can say that in case (1) body move in the direction of force P and in case (2) body will more in direction of Q, but in case (3) body ~~no~~ will not move or we can also say that body is at rest, it means body is in equilibrium.

Principle of Superposition of force

The action of a given system of forces on a body will not change, if we add or subtract from it another system of the forces in equilibrium.

Principle of transmissibility of force:

The point of application of force may be transmitted along its line of action without changing the effect of forces on the body Thus the principal of transmissibility is understood by principle of superposition of force.

Resolution of force:

A force can be split up into two given direction such that the resultant of these forces is a given force. These components forces will give the same effect on the body as given by a single force. The procedure is known as resolution of force and resolved forces are known as components of force

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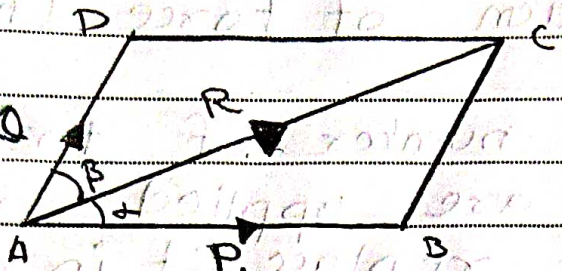
Composition of Force (Resultant force)

If a number of forces in a force system are applied on a body, then we can replace it in a single force, which produce the same effect as force system, then this single replaced single force is known as resultant force and the process by which the resultant force is found is known as composition of force.

Law of Parallelogram of forces:

This method is used to find resultant of two coplanar concurent forces act on a body. Law of parallelogram of force state as below:

Two forces acting simultaneously on a body, if represent in magnitude and direction by two adjacent sides of parallelogram of any scale, the digonal of parallelo-gram from the point of intersection of two forces represent the resultant force in magnitude and direction at same scale.

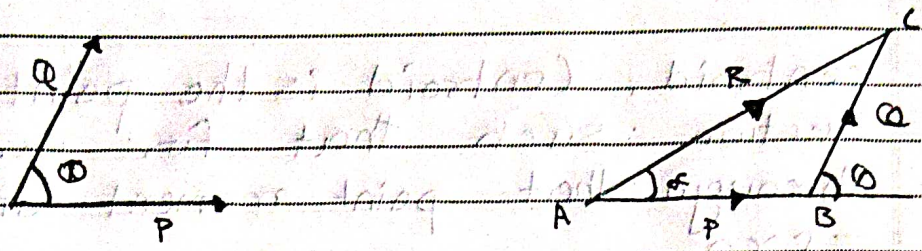


$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

$$\tan\gamma = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

Law of triangle of force:

When only two and only two forces are acting on common point, we can apply this law to find out resultant force of force system. It states "If two forces acting at a point are represented in magnitude and direction taken at a scale by two sides of triangle taken in order, the third side of the triangle taken in opposite order represent the resultant force of two forces in magnitude and direction at same scale.



$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

Center of Gravity (C.G.)

The point through which the whole weight of the body acts is known as center of gravity (C.G.) irrespective of the position of the body. It may be note that every body has only one & one C.G.. If you balance the body on this point of C.G. it will balance.

Centroid :

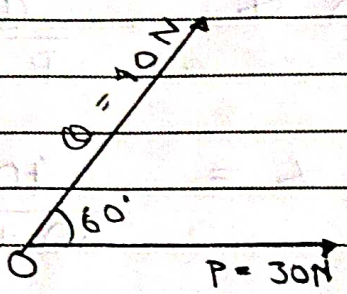
The plane figure like (triangle circle etc) have only area but no mass. The center of area of two dimension figure known as

centroid. Centroid is the point in a plane section such that for any axis through that point moment of area is zero.

Difference between center of gravity and centroid

Center of Gravity	Centroid
Center of gravity is the point where total mass of the object acts.	Centroid is the geometric center of the object where whole area is assume to be concentrate.
Center of gravity is applicable to objects with any density	Centroid is the central point of objects with uniform density.
Generally deals with 3D structure	Generally deals with 2D structure.
Examples: Cube, Cone Cylinder, sphere etc	Examples: Square, triangle Circle, semi-circle etc

Q1 Find the resultant force of two forces 30 N and 40 N acting at a point with an angle of 60° with another.



Solve given,

$P = 30N$, $Q = 40\ N$

$\theta = 60°$

Then resultant force's magnitude

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$R = \sqrt{(30)^2 + (40)^2 + 2(30)(40)\cos 60°}$$

$$R = \sqrt{900 + 1600 + 2\cancel{400}^{1200}\frac{1}{2}}$$

$$R = \sqrt{900 + 1600 + 1200}$$

$$R = \sqrt{3700}$$

$R = 60.83$ N

Direction α,

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$\tan\alpha = \frac{40\sin 60^\circ}{30 + 40\cos 60}$$

$$\tan\alpha = \frac{40 \times 0.86}{30 + 40 \times 0.5}$$

$$\tan\alpha = \frac{34.64}{50}$$

$$\tan\alpha = 0.6928$$

$$\alpha = \tan^{-1}(0.6928)$$

$$\alpha = 34.71^\circ$$

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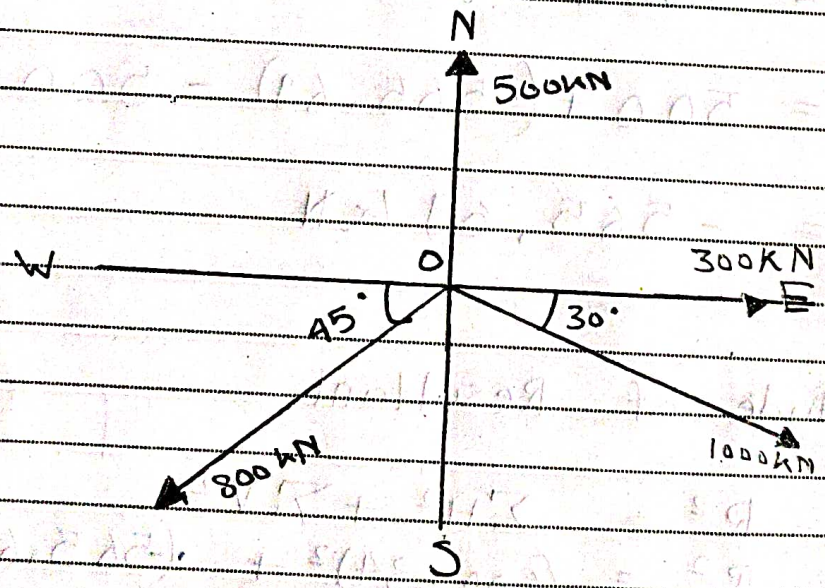
Q2 A system of force four coplanar concurrent forces are acting at a point as given below. Find the magnitude and direction of resultant force.

1. 500 KN acting due North
2. 1000 KN acting 30° south of east
3. 800 KN acting south west
4. 300 KN acting from west



Sol^n

$$\Sigma H = 300 + 500\cos 90 + 800\cos 225 + 1000\cos 330°$$

$$\Sigma H = 300 + 0 - 565.69 + 886.03$$

$$\Sigma H = 600.34\ kN$$

$$\Sigma V = 500 + 800\sin 225° + 1000\sin 330°$$

$$= 500 + (-565.69) - 500$$

$$= -565.69\ kN$$

Magnitude of Resultant

$$R^2 = \Sigma H^2 + \Sigma V^2$$

$$R^2 = (600.34)^2 + (-565.69)^2$$

$$R^2 = 360408.11 + 320005.17$$

$$R = \sqrt{680413.28}$$

$$R = 824.87\ kN$$

$$\tan\alpha = \frac{\Sigma V}{\Sigma H}$$

$$\tan\alpha = \frac{-565.69}{600.34}$$

$$\tan\alpha = -0.942$$

$$\alpha = \tan^{-1}(-0.942)$$

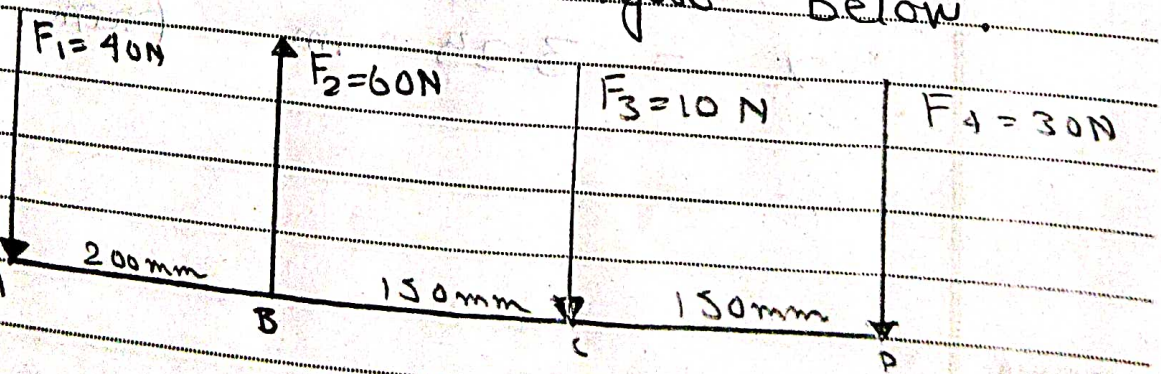
$$\alpha = -43.30$$

Real value of alpha from $\Sigma H \rightarrow$ (East) and $\Sigma V \downarrow$ (south)

$$\alpha = 360 - 43.30$$

Answer. $\alpha = 316.70°$

Q3 Find the resultant of parallel force system as shown in figure below.



Solve

Magnitude of Resultant force = $R = \sum F$

$R = 40 - 60 + 10 + 30 = 20\ N$

$R = 20\ N$

Point of application

Take moment at point A

$Rx = (40 \times 0) - (60 \times 200) + (10 \times 350) + (30 \times 500)$

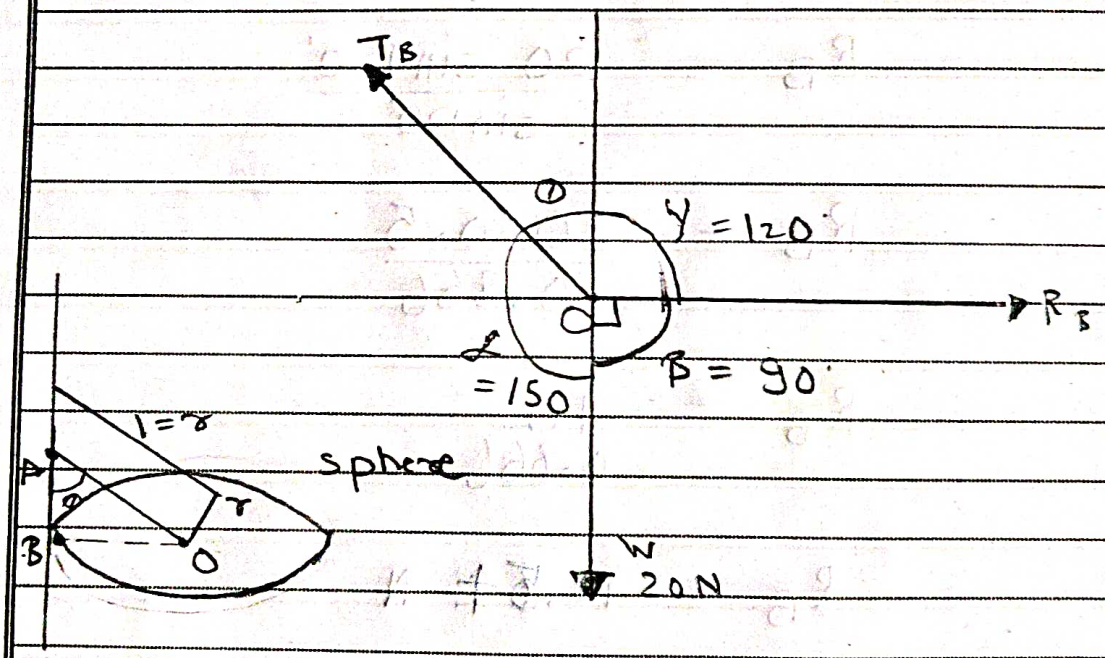
$Rx = -12000 + 3500 + 15000$

$20x = 6500$

$x = \frac{6500}{20}$

$x = 325\ mm$ from point A

Q4 A smooth sphere of radius r 150 mm and weight W 20 N is hung by string whose length equal the radius of sphere with contact to smooth vertical wall. Find inclination and tension in string as well as reaction of wall.



Solve In triangle ABO

$$\sin\theta = \frac{OB}{OA} = \frac{r}{2r} = 0.5$$

$$\theta = 30°$$

Now applying Lami's theorem:

$$\frac{R_B}{\sin\alpha} = \frac{T}{\sin\beta} = \frac{W}{\sin\gamma}$$

$$\frac{R_B}{\sin 150^\circ} = \frac{20}{\sin 120^\circ}$$

$$R_B = \frac{20 \sin 150^\circ}{\sin 120^\circ}$$

$$R_B = \frac{20 \times 0.5}{0.866}$$

$$R_B = \frac{10}{0.866}$$

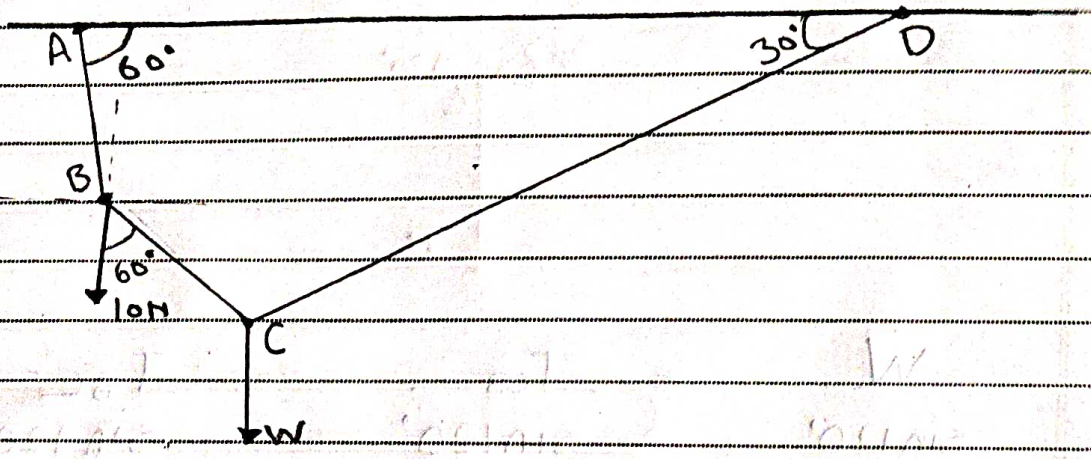
$$R_B = 11.54 \text{ N}$$

$$\frac{T}{\sin 90^\circ} = \frac{20}{\sin 120^\circ}$$

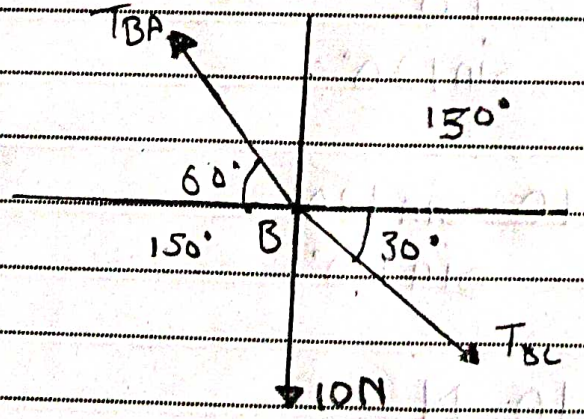
$$T = \frac{20}{\sin 120^\circ}$$

$$T = \frac{20}{0.866} = 23.094 \text{ N}$$

Q5 Find the value of W if a light weight chain ABCD is suspended as shown in figure.



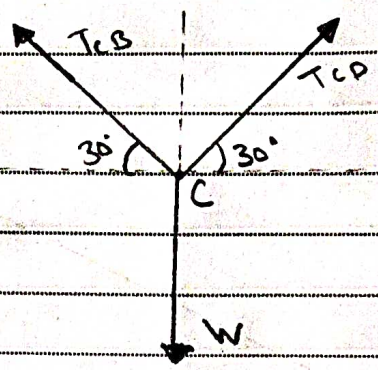
Solve (a) Applying lamis theorem at point B



$$\frac{T_{BC}}{\sin 150^\circ} = \frac{10}{\sin 150^\circ}$$

$T_{BC} = 10\ N$

b) Applying Lami's theorem at point C



$$\frac{W}{\sin 120^\circ} = \frac{T_{CB}}{\sin 120^\circ} = \frac{T_{BC}}{\sin 120^\circ}$$

$$\frac{W}{\sin 120^\circ} = \frac{10}{\sin 120^\circ}$$

$$W = \frac{10 \sin 120^\circ}{\sin 120^\circ}$$

$$W = 10\ N$$

$$Y_1 = 120 + \frac{30}{2} = 135 \text{ mm}$$

$$Y_2 = \frac{120}{2} = 60 \text{ mm}$$

$$\bar{X} = \frac{3000(50) + 3600(50)}{3000 + 3600}$$

$$\bar{X} = \frac{150000 + 180000}{6600}$$

$$\bar{X} = \frac{330000}{6600}$$

$$\bar{X} = 50 \text{ mm}$$

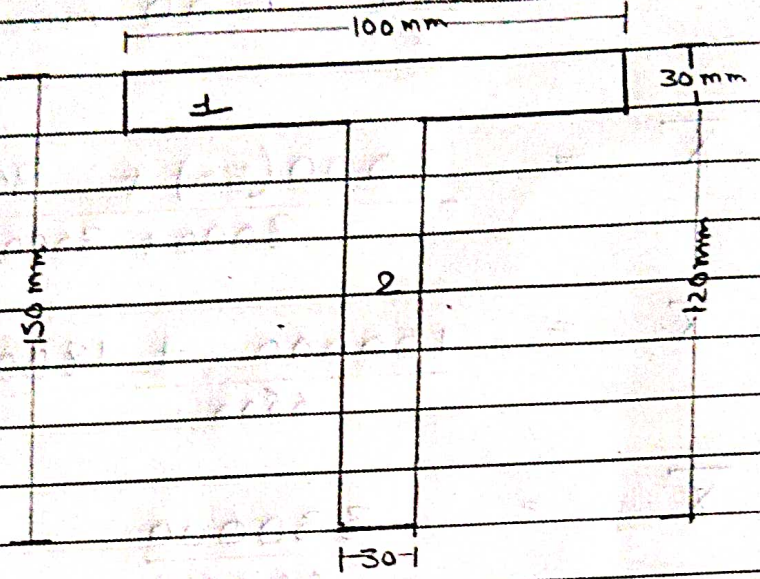
$$\bar{Y} = \frac{3000(135) + 3600(60)}{3000 + 3600}$$

$$= \frac{405000 + 216000}{6600}$$

$$= \frac{621000}{6600}$$

$$= 94.09 \text{ mm}$$

Q6 Find the centroid (CG) of 100 mm x 150 mm x 30 mm T-section as shown in figure.



Solve

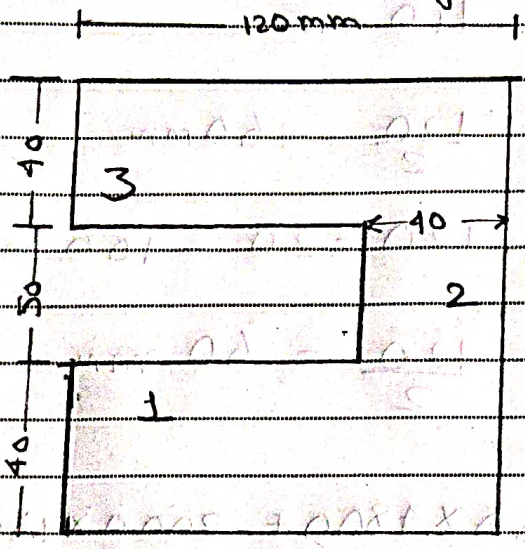
Area of rectangle 1, A_1 = 30 x 100
= 3000 mm^2

Area of rectangle 2, A_2 = 30 x 120
= 3600 mm^2

$\bar{X}_1 = \frac{100}{2} = 50$

$\bar{X}_2$ = 50 from symmetry.

Q7 Find the centroid of given C-section as shown in figure:



Solve

Area of rectangle 1 A_1 = 120 X 40
= 4800 mm^2

Area of rectang 2 A_2 = 40 X 50
= 2000 mm^2

Area of rectangle 3 A_3 = 120 X 40
= 4800 mm^2

$\bar{X}_1 = \frac{40}{2} = 20$ mm

$\bar{X}_2 = 40 + \frac{50}{2} = 65$

$Y_3 = \frac{40+50+40}{2}$

$= 110$ mm

$X_1 = \frac{120}{2} = 60$mm

$X_2 = 120 - 20 = 100$ mm

$X_3 = \frac{120}{2} = 60$ mm

$$\bar{X} = \frac{60 \times 4800 + 2000 \times 100 + 60 \times 4800}{4800 + 4800 + 2000}$$

$$= \frac{288000 + 200000 + 288000}{11600}$$

$$= \frac{776000}{11600}$$

$$= 66.89 \text{ mm}$$

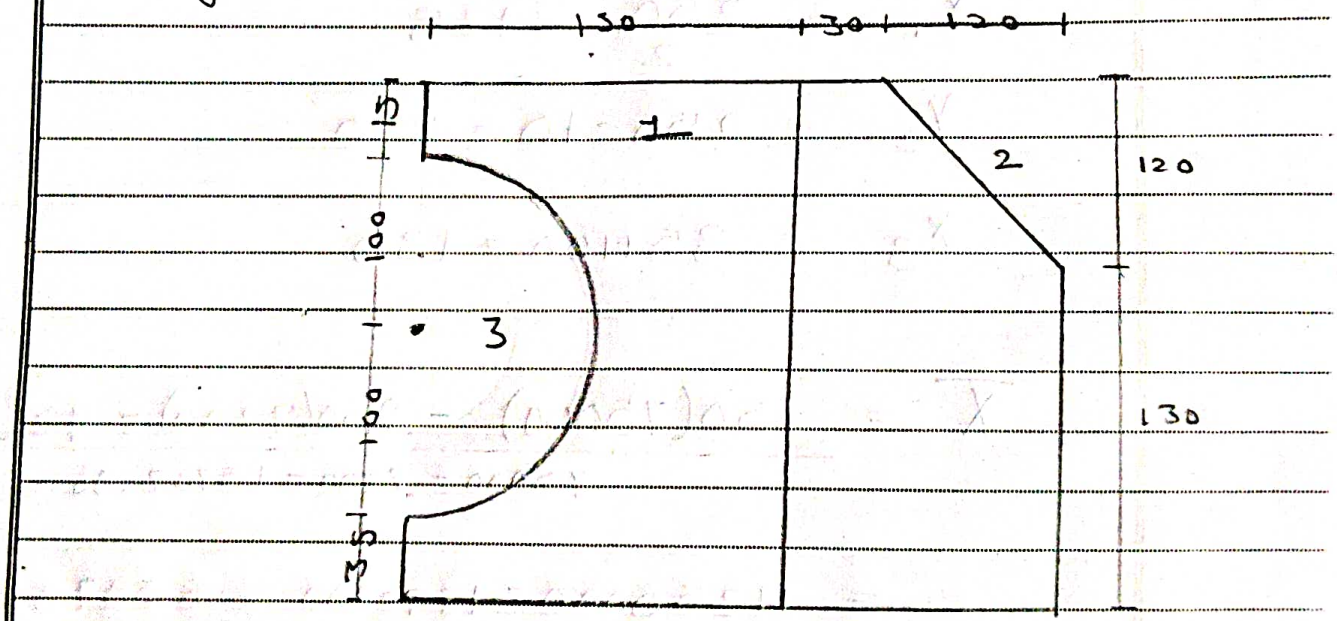
$$\bar{Y} = \frac{20 \times 4800 + 2000 \times 65 + 110 \times 4800}{4800 + 2000 + 4800}$$

$$= \frac{96000 + 130000 + 528000}{11600}$$

$\overline{Y} = \frac{754000}{11600}$

$\overline{Y} = 65$ mm

Q8 Find the centroid of the given composite figure :



Solu

Area of rectangle 1, $A_1 = 300 \times 250 = 75000$

Area of triangle, $A_2 = \frac{1}{2} \times 120 \times 120 = 7200$

Area of Circle $A_3 = \frac{1}{2} \pi 100^2 = 15707.96$

$$X_1 = \frac{300}{2} = 150$$

$$X_2 = 300 - 40 = 260$$

$$X_3 = \frac{4 \times 100}{3\pi} = 42.44$$

$$Y_1 = \frac{250}{2} = 125$$

$$Y_2 = 250 - 40 = 210$$

$$Y_3 = 35 + 100 = 135$$

$$\bar{X} = \frac{150(75000) - 260(7200) - 42(15707.96)}{75000 - 7200 - 15707.96}$$

$$= \frac{11250000 - 1872000 - 659734.32}{52092.04}$$

$$= \frac{8711354.18}{52092.04}$$

$$= 167.23 \text{ mm}$$

$$\bar{Y} = \frac{250(75000) - 210(7200) - 135(15707.96)}{75000 - 7200 - 15707.96}$$

$$= \frac{18750000 - 1512000 - 2120571.6}{52092.04}$$

$$= \frac{5742425.40}{52092.04}$$

$$= 110.23 \text{ mm}$$