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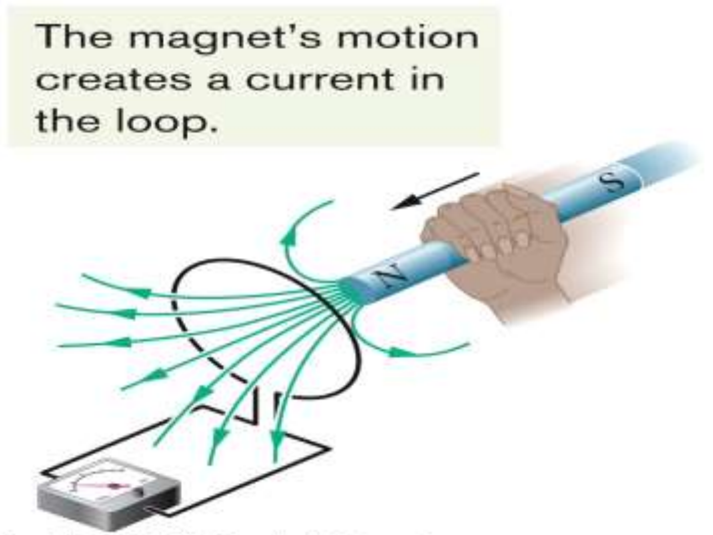
Welcome
to
PHYS 212: Physics

Chapter#30

Induction and Inductance

Experiment-1

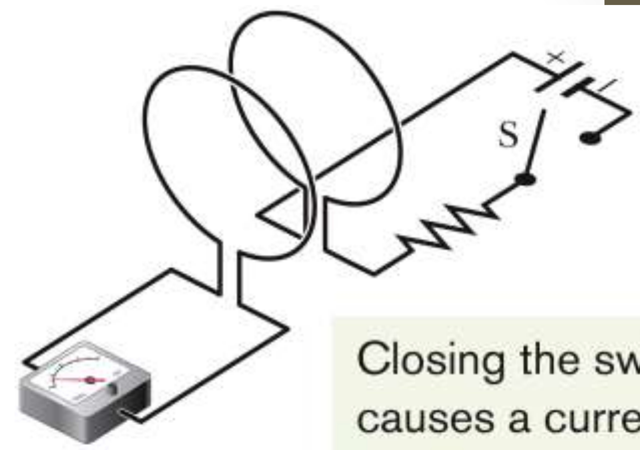
First Experiment. Figure shows a conducting loop connected to a sensitive ammeter. Because there is no battery or other source of emf included, there is no current in the circuit. However, if we move a bar magnet toward the loop, a current suddenly appears in the circuit. The current disappears when the magnet stops moving. If we then move the magnet away, a current again suddenly appears, but now in the opposite direction. If we experimented for a while, we would discover the following:



1. A current appears only if there is relative motion between the loop and the magnet (one must move relative to the other); the current disappears when the relative motion between them ceases.
2. Faster motion of the magnet produces a greater current.
3. If moving the magnet's north pole toward the loop causes, say, clockwise current, then moving the north pole away causes counterclockwise current. Moving the south pole toward or away from the loop also causes currents, but in the reversed directions from the north pole effects.

Experiment-2

Second Experiment. For this experiment we use the apparatus shown in the figure, with the two conducting loops close to each other but not touching. If we close switch **S** to turn on a current in the right-hand loop, the meter suddenly and briefly registers a current—an induced current in the left-hand loop. If the switch remains closed, no further current is observed. If we then open the switch, another sudden and brief induced current appears in the left-hand loop, but in the opposite direction.



Closing the switch causes a current in the left-hand loop.

We get an induced current (from an induced emf) only when the current in the right-hand loop is changing (either turning on or turning off) and not when it is constant (even if it is large). The induced emf and induced current in these experiments are apparently caused when something changes — but what is that "something"? Faraday knew.

Faraday's law of Induction

- An emf is induced in a loop each time the magnetic field changes
- The law states that an emf is induced in the loop each time the number of magnetic field lines that pass through it changes.

So what are these magnetic field lines??

- The magnetic field lines come out of the North pole and go into the South pole.
- The magnetic field lines along with the area of the loop and the magnetic field B combine to give a special quantity called **magnetic flux**.

Magnetic Flux

A loop of area A is placed in a magnetic field B, so the magnetic flux, when the magnetic field is uniform and is perpendicular to the plane of the loop, is

$$\Phi_B = BA \qquad (\vec{B} \perp \text{area } A,\ \vec{B} \text{ uniform}).$$

The SI unit for magnetic flux is

$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2.$$

Formula for Faraday's Law

The magnitude of the emf $\mathscr{E}$ induced in a conducting loop is equal to the rate at which the magnetic flux Φ_B through that loop changes with time.

As you will see in the next section, the induced emf $\mathscr{E}$ tends to oppose the flux change, so Faraday's law is formally written as

$$\mathscr{E} = -\frac{d\Phi_B}{dt} \qquad \text{(Faraday's law)}, \qquad (30\text{-}4)$$

with the minus sign indicating that opposition. We often neglect the minus sign in Eq. 30-4, seeking only the magnitude of the induced emf.

Formula for Faraday's Law

If we change the magnetic flux through a coil of N turns, an induced emf appears in every turn and the total emf induced in the coil is the sum of these individual induced emfs. If the coil is tightly wound (*closely packed*), so that the same magnetic flux Φ_B passes through all the turns, the total emf induced in the coil is

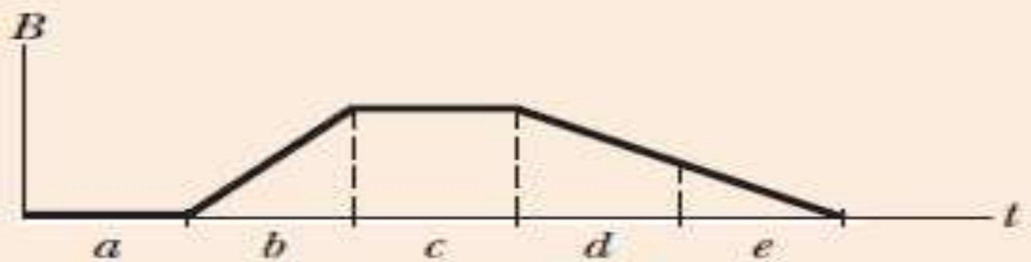
$$\mathscr{E} = -N\frac{d\Phi_B}{dt} \quad \text{(coil of } N \text{ turns).} \tag{30-5}$$

Here are the general means by which we can change the magnetic flux through a coil:

1. Change the magnitude B of the magnetic field within the coil.
2. Change either the total area of the coil or the portion of that area that lies within the magnetic field (for example, by expanding the coil or sliding it into or out of the field).
3. Change the angle between the direction of the magnetic field $\vec{B}$ and the plane of the coil (for example, by rotating the coil so that field $\vec{B}$ is first perpendicular to the plane of the coil and then is along that plane).

CHECKPOINT 1

The graph gives the magnitude $B(t)$ of a uniform magnetic field that exists throughout a conducting loop, with the direction of the field perpendicular to the plane of the loop. Rank the five regions of the graph according to the magnitude of the emf induced in the loop, greatest first.



- *Hint: emf is more where B is more, emf is zero where B is constant or zero.*

Answer: *b*, then *d* and *e* tie, and then *a* and *c* tie (zero)

Problem-1:

A coil consists of 200 turns of wire having a total resistance of 2.0 Ω. Each turn is a square of side 18 cm, and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from 0 to 0.50 T in 0.80 s, what is the magnitude of the induced emf in the coil while the field is changing?

Answer:

$$|\mathcal{E}| = 4.1\ \mathrm{T \cdot m^2/s} = 4.1\ \mathrm{V}$$

Lenz's Law

- This law is used to find the *direction* of the induced current:

> An induced current has a direction such that the magnetic field due to *the current* opposes the change in the magnetic flux that induces the current.

- Also, the direction of the induced emf is the same as the direction of the induced current.

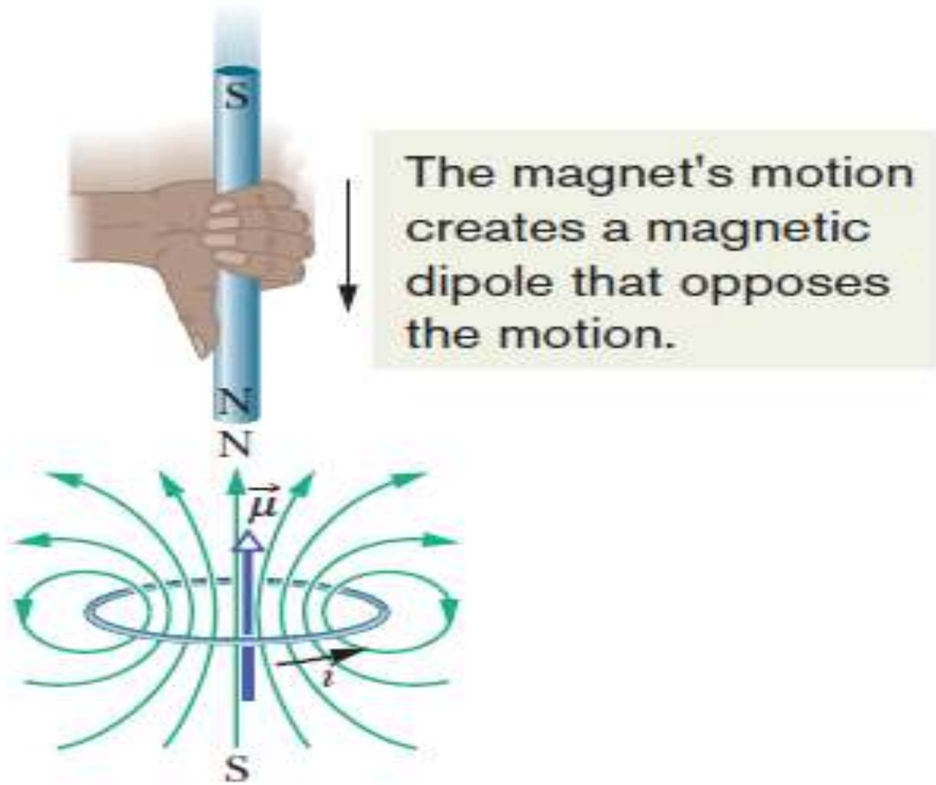


Fig. 30-4 Lenz's law at work. As the magnet is moved toward the loop, a current is induced in the loop. The current produces its own magnetic field, with magnetic dipole moment $\vec{\mu}$ oriented so as to oppose the motion of the magnet. Thus, the induced current must be counterclockwise as shown.

Inductors and Inductance

We found in Chapter 25 that a capacitor can be used to produce a desired electric field. We considered the parallel-plate arrangement as a basic type of capacitor. Similarly, an **inductor** (symbol ℓℓℓℓ) can be used to produce a desired magnetic field. We shall consider a long solenoid (more specifically, a short length near the middle of a long solenoid) as our basic type of inductor.

If we establish a current i in the windings (turns) of the solenoid we are taking as our inductor, the current produces a magnetic flux Φ_B through the central region of the inductor. The **inductance** of the inductor is then

$$L = \frac{N\Phi_B}{i} \qquad \text{(inductance defined),} \qquad (30\text{-}28)$$

in which N is the number of turns. The windings of the inductor are said to be *linked* by the shared flux, and the product $N\Phi_B$ is called the *magnetic flux linkage*. The inductance L is thus a measure of the flux linkage produced by the inductor per unit of current.

The SI Unit of Inductance (L) is $1\ \text{henry} = 1\ \text{H} = 1\ \text{T}\cdot\text{m}^2/\text{A}.$

Inductance of a Solenoid

Consider a long solenoid of cross-sectional area **A**. What is the inductance per unit length near its middle?

To use the defining equation for inductance (Eq. 30-28), we must calculate the flux linkage set up by a given current in the solenoid windings. Consider a length **l** near the middle of this solenoid. The flux linkage there is

$$N\Phi_B = (nl)(BA),$$

in which n is the number of turns per unit length of the solenoid and B is the magnitude of the magnetic field within the solenoid.

The magnitude B is given by Eq. 29-23,

$$B = \mu_0 in,$$

and so from Eq. 30-28,

$$L = \frac{N\Phi_B}{i} = \frac{(nl)(BA)}{i} = \frac{(nl)(\mu_0 in)(A)}{i} = \mu_0 n^2 lA. \tag{30-30}$$

Thus, the inductance per unit length near the center of a long solenoid is

$$\frac{L}{l} = \mu_0 n^2 A \quad \text{(solenoid)}. \tag{30-31}$$

Inductance—like capacitance—depends only on the geometry of the device. The dependence on the square of the number of turns per unit length is to be expected. If you, say, triple n, you not only triple the number of turns (N) but you also triple the flux ($\Phi_B = BA = \mu_0 i n A$) through each turn, multiplying the flux linkage $N\Phi_B$ and thus the inductance L by a factor of 9.

Self-Induction

If two coils—which we can now call inductors—are near each other, a current i in one coil produces a magnetic flux Φ_B through the second coil. We have seen that if we change this flux by changing the current, an induced emf appears in the second coil according to Faraday's law. An induced emf appears in the first coil as well.

> An induced emf $\mathscr{E}_L$ appears in any coil in which the current is changing.

This process (see Fig. 30-13) is called **self-induction,** and the emf that appears is called a **self-induced emf.** It obeys Faraday's law of induction just as other induced emfs do.

Self Induced emf

$$\mathscr{E}_L = -L\frac{di}{dt} \qquad \text{(self-induced emf).} \qquad (30\text{-}35)$$

Thus, in any inductor (such as a coil, a solenoid, or a toroid) a self-induced emf appears whenever the current changes with time. The magnitude of the current has no influence on the magnitude of the induced emf; only the rate of change of the current counts.

You can find the *direction* of a self-induced emf from Lenz's law. The minus sign in Eq. 30-35 indicates that—as the law states—the self-induced emf $\mathscr{E}_L$ has the orientation such that it opposes the change in current i. We can drop the minus sign when we want only the magnitude of $\mathscr{E}_L$.

Problem-2:

(a) Calculate the inductance of an air-core solenoid containing 300 turns if the length of the solenoid is 25.0 cm and its cross-sectional area is 4.00 cm^2.

Answer:

$$L = 0.181\ \text{mH}$$

(b) Calculate the self-induced emf in the solenoid if the current through it is decreasing at the rate of 50.0 A/s.

Answer:

$$\mathcal{E}_L = 9.05\ \text{mV}$$

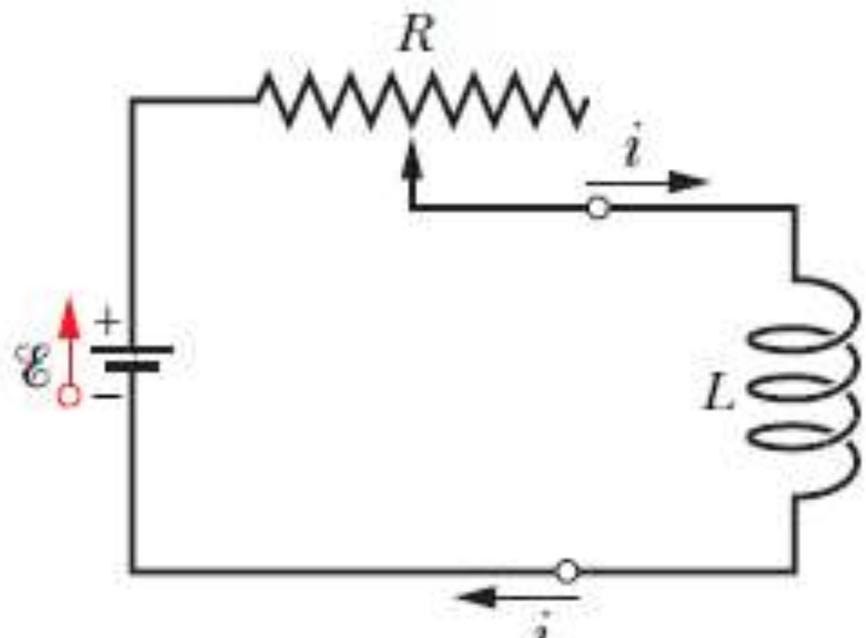


Fig. 30-13 If the current in a coil is changed by varying the contact position on a variable resistor, a self-induced emf $\mathscr{E}_L$ will appear in the coil *while the current is changing.*

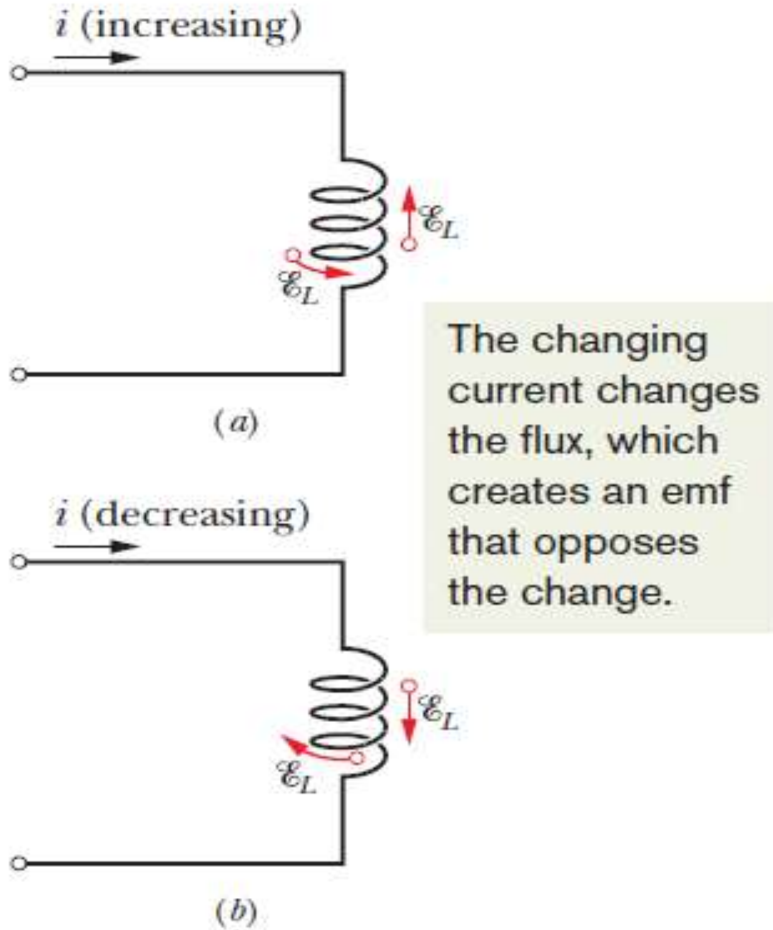
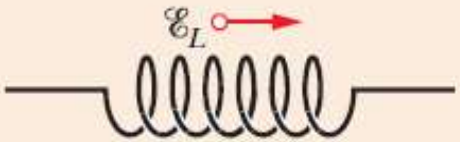


Fig. 30-14 (*a*) The current i is increasing, and the self-induced emf $\mathscr{E}_L$ appears along the coil in a direction such that it opposes the increase. The arrow representing $\mathscr{E}_L$ can be drawn along a turn of the coil or alongside the coil. Both are shown. (*b*) The current i is decreasing, and the self-induced emf appears in a direction such that it opposes the decrease.

CHECKPOINT 5

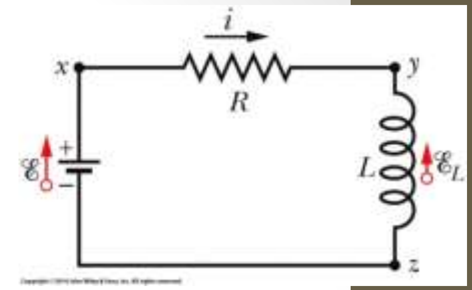
The figure shows an emf $\mathscr{E}_L$ induced in a coil. Which of the following can describe the current through the coil: (a) constant and rightward, (b) constant and leftward, (c) increasing and rightward, (d) decreasing and rightward, (e) increasing and leftward, (f) decreasing and leftward?



Hint: i is increasing, it goes against induced emf and i is decreasing, it goes with induced emf (in direction but it is actually still opposing it).

Answer: d and e

30-6 RL Circuits

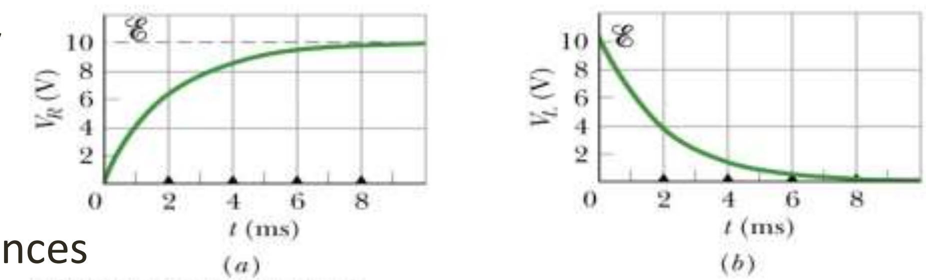


An RL circuit.

If a constant emf $\mathscr{E}$ is introduced into a single-loop circuit containing a resistance *R* and an inductance *L*, the current rises to an equilibrium value of $\mathscr{E}/R$ according to

$$i = \frac{\mathscr{E}}{R}(1 - e^{-t/\tau_L})$$

The resistor's potential difference turns on.
The inductor's potential difference turns off.



Here τ_L, the **inductive time constant**, is given by

$$\tau_L = \frac{L}{R}$$

Plot (a) and (b) shows how the potential differences $V_R(= iR)$ across the resistor and $V_L(=L\, di/dt)$ across the inductor vary with time for particular values of $\mathscr{E}$, *L*, and *R*.

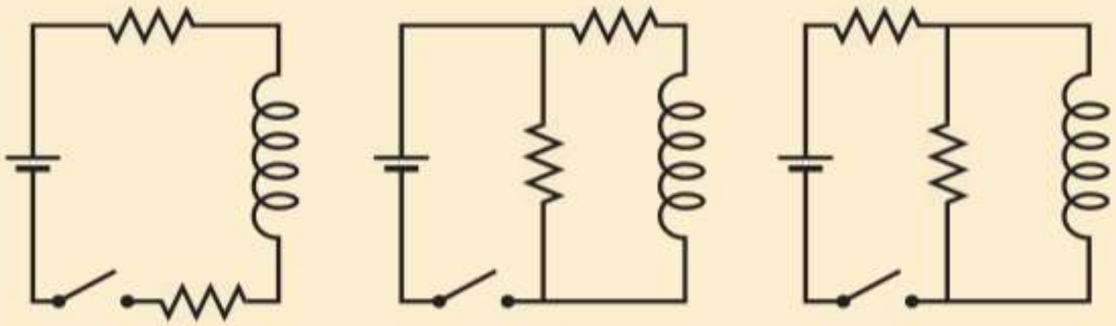
The variation with time of (a) V_R, the potential difference across the resistor in the circuit (top), and (b) V_L, the potential difference across the inductor in that circuit.

When the source of constant *emf* is removed and replaced by a conductor, the **current decays** from a value i_0 according to

$$i = \frac{\mathscr{E}}{R} e^{-t/\tau_L} = i_0 e^{-t/\tau_L}$$

Checkpoint 6

The figure shows three circuits with identical batteries, inductors, and resistors. Rank the circuits according to the current through the battery (a) just after the switch is closed and (b) a long time later, greatest first. (If you have trouble here, work through the next sample problem and then try again.)



(1) (2) (3)

Answer: (a) 2, 3, 1 (zero)
(b) 2, 3, 1

Sample Problem 30.05 *RL* circuit, immediately after switching and after a long time

Figure 30-18*a* shows a circuit that contains three identical resistors with resistance $R = 9.0\ \Omega$, two identical inductors with inductance $L = 2.0$ mH, and an ideal battery with emf $\mathscr{E} = 18$ V.

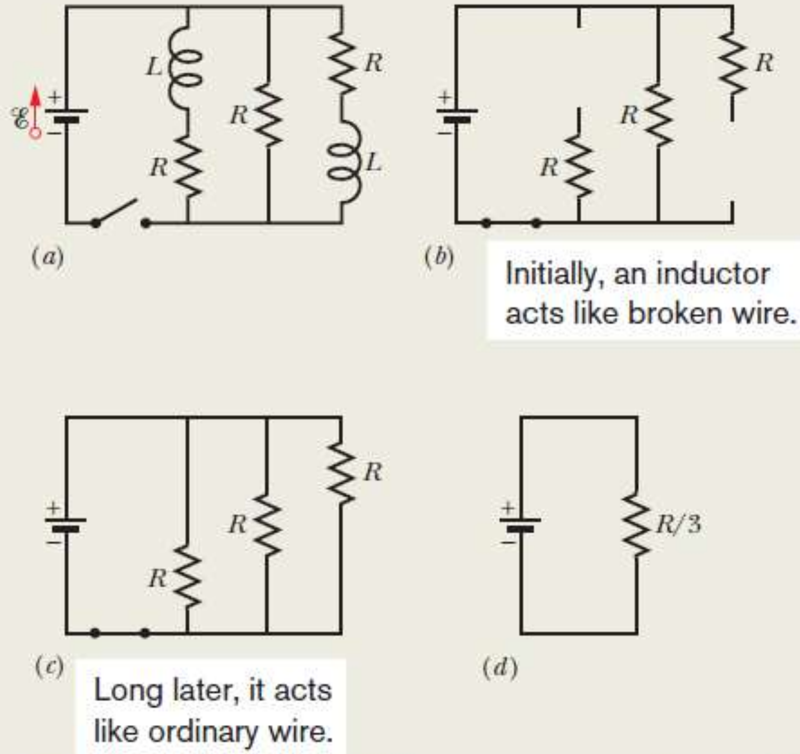


Figure 30-18 (*a*) A multiloop *RL* circuit with an open switch. (*b*) The equivalent circuit just after the switch has been closed. (*c*) The equivalent circuit a long time later. (*d*) The single-loop circuit that is equivalent to circuit (*c*).

(a) What is the current i through the battery just after the switch is closed?

KEY IDEA

Just after the switch is closed, the inductor acts to oppose a change in the current through it.

Calculations: Because the current through each inductor is zero before the switch is closed, it will also be zero just afterward. Thus, immediately after the switch is closed, the inductors act as broken wires, as indicated in Fig. 30-18*b*. We then have a single-loop circuit for which the loop rule gives us

$$\mathscr{E} - iR = 0.$$

Substituting given data, we find that

$$i = \frac{\mathscr{E}}{R} = \frac{18\ \text{V}}{9.0\ \Omega} = 2.0\ \text{A}. \qquad \text{(Answer)}$$

(b) What is the current i through the battery long after the switch has been closed?

KEY IDEA

Long after the switch has been closed, the currents in the circuit have reached their equilibrium values, and the inductors act as simple connecting wires, as indicated in Fig. 30-18*c*.

Calculations: We now have a circuit with three identical resistors in parallel; from Eq. 27-23, their equivalent resistance is $R_{eq} = R/3 = (9.0\ \Omega)/3 = 3.0\ \Omega$. The equivalent circuit shown in Fig. 30-18*d* then yields the loop equation $\mathscr{E} - iR_{eq} = 0$, or

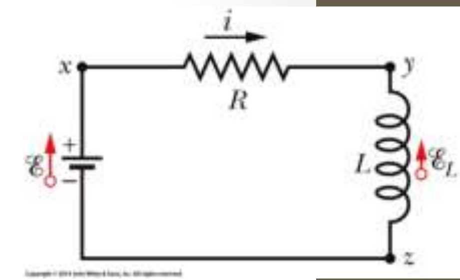
$$i = \frac{\mathscr{E}}{R_{eq}} = \frac{18\ \text{V}}{3.0\ \Omega} = 6.0\ \text{A}. \qquad \text{(Answer)}$$

30-7 Energy Stored in a Magnetic Field

If an inductor *L* carries a current *i*, the inductor's magnetic field stores an energy given by

$$U_B = \tfrac{1}{2}Li^2$$

An RL circuit.



A coil has an inductance of 53 mH and a resistance of 0.35 Ω.

(a) If a 12 V emf is applied across the coil, how much energy is stored in the magnetic field after the current has built up to its equilibrium value?

KEY IDEA

The energy stored in the magnetic field of a coil at any time depends on the current through the coil at that time, according to Eq. 30-49 ($U_B = \tfrac{1}{2}Li^2$).

Calculations: Thus, to find the energy $U_{B\infty}$ stored at equilibrium, we must first find the equilibrium current. From Eq. 30-41, the equilibrium current is

$$i_\infty = \frac{\mathscr{E}}{R} = \frac{12\text{ V}}{0.35\ \Omega} = 34.3\text{ A}. \qquad (30\text{-}51)$$

Then substitution yields

$$U_{B\infty} = \tfrac{1}{2}Li_\infty^2 = (\tfrac{1}{2})(53 \times 10^{-3}\text{ H})(34.3\text{ A})^2$$
$$= 31\text{ J}. \qquad \text{(Answer)}$$

(b) After how many time constants will half this equilibrium energy be stored in the magnetic field?

Calculations: Now we are being asked: At what time *t* will the relation

$$U_B = \tfrac{1}{2}U_{B\infty}$$

be satisfied? Using Eq. 30-49 twice allows us to rewrite this energy condition as

$$\tfrac{1}{2}Li^2 = (\tfrac{1}{2})\tfrac{1}{2}Li_\infty^2$$

or

$$i = \left(\frac{1}{\sqrt{2}}\right)i_\infty. \qquad (30\text{-}52)$$

This equation tells us that, as the current increases from its initial value of 0 to its final value of i_∞, the magnetic field will have half its final stored energy when the current has increased to this value. In general, we know that *i* is given by Eq. 30-41, and here i_∞ (see Eq. 30-51) is $\mathscr{E}/R$; so Eq. 30-52 becomes

$$\frac{\mathscr{E}}{R}(1 - e^{-t/\tau_L}) = \frac{\mathscr{E}}{\sqrt{2}R}.$$

By canceling $\mathscr{E}/R$ and rearranging, we can write this as

$$e^{-t/\tau_L} = 1 - \frac{1}{\sqrt{2}} = 0.293,$$

which yields

$$\frac{t}{\tau_L} = -\ln 0.293 = 1.23$$

or

$$t \approx 1.2\tau_L. \qquad \text{(Answer)}$$

Thus, the energy stored in the magnetic field of the coil by the current will reach half its equilibrium value 1.2 time constants after the emf is applied.

Checkpoint 7

The table lists the number of turns per unit length, current, and cross-sectional area for three solenoids. Rank the solenoids according to the magnetic energy density within them, greatest first.

Solenoid	Turns per Unit Length	Current	Area
a	$2n_1$	i_1	$2A_1$
b	n_1	$2i_1$	A_1
c	n_1	i_1	$6A_1$

$$u_B = \frac{1}{2}\mu_0 n^2 i^2$$

Answer: *a* and *b* tie, then *c*

Problem-3:

A coil has an inductance of 3.00 mH, and the current through it changes from 0.200 A to 1.50 A in a time of 0.200 s. Find the magnitude of the average induced emf in the coil during this time.

Answer:

$$|\overline{\varepsilon}| = 19.5\ \mathrm{mV}$$

Problem-4:

A coiled telephone cord forms a spiral with 70 turns, a diameter of 1.30 cm, and an unstretched length of 60.0 cm. Determine the self-inductance of one conductor in the unstretched cord.

Answer:

$$L = \boxed{1.36\ \mu\text{H}}$$

Problem-5:

Calculate the magnetic flux through the area enclosed by a 300-turn, 7.20-mH coil when the current in the coil is 10.0 mA.

Answer:

$L = \boxed{240 \text{ nT} \cdot \text{m}^2}$ (through each turn)

Problem-6:

An emf of 24.0 mV is induced in a 500-turn coil at an instant when the current is 4.00 A and is changing at the rate of 10.0 A/s. What is the magnetic flux through each turn of the coil?

Answer:

$$\Phi_B = \boxed{19.2\ \mu\mathrm{T}\cdot\mathrm{m}^2}$$