Imam Abdulrahman Bin Faisal University
College of Computer Science & IT
Department of CS
Term 2191

Welcome to PHYS 212: Physics

Chapter#3

Vectors

Topics

- Quantity
- Magnitude
- Resolving Vectors
- Unit Vectors
- Scalar Product
- Vector Product

What is a quantity?

- A Quantity is a property that can exist in numbers or an amount of a property.

Example:

- A mass is a quantity that tells us how heavy something is and it is measured in kg, g, pounds, etc.
- Temperature is a quantity that tells us how hot something is.
- Distance/displacement tells us how far something has gone.

- **Physical Quantities** → **Scalar Quantities** / **Vector Quantities**

SCALARS

- **A scalars** quantity is the quantity that has a magnitude but no direction.
- Temperature, pressure, energy, mass, speed and time, for example, do not “point” anywhere . We call such quantities **scalars.**

VECTORS

A vector quantity is a quantity that has both a magnitude and a direction and thus can be represented with a vector ($\longrightarrow$).

- Because direction is an important characteristic of a vector, ***arrows*** are used to represent them.

Some physical quantities that are vector quantities are displacement, velocity, Force and acceleration

What is a magnitude?

The magnitude of the physical quantity is given as:

(Number + Unit)

Example:

- A car is moving to the north at 45 km/hr.

So, *North* is the direction and *45 km/hr* is the *magnitude* of how fast the car is going.

The magnitude of a vector in a *scaled* and is always a positive number.

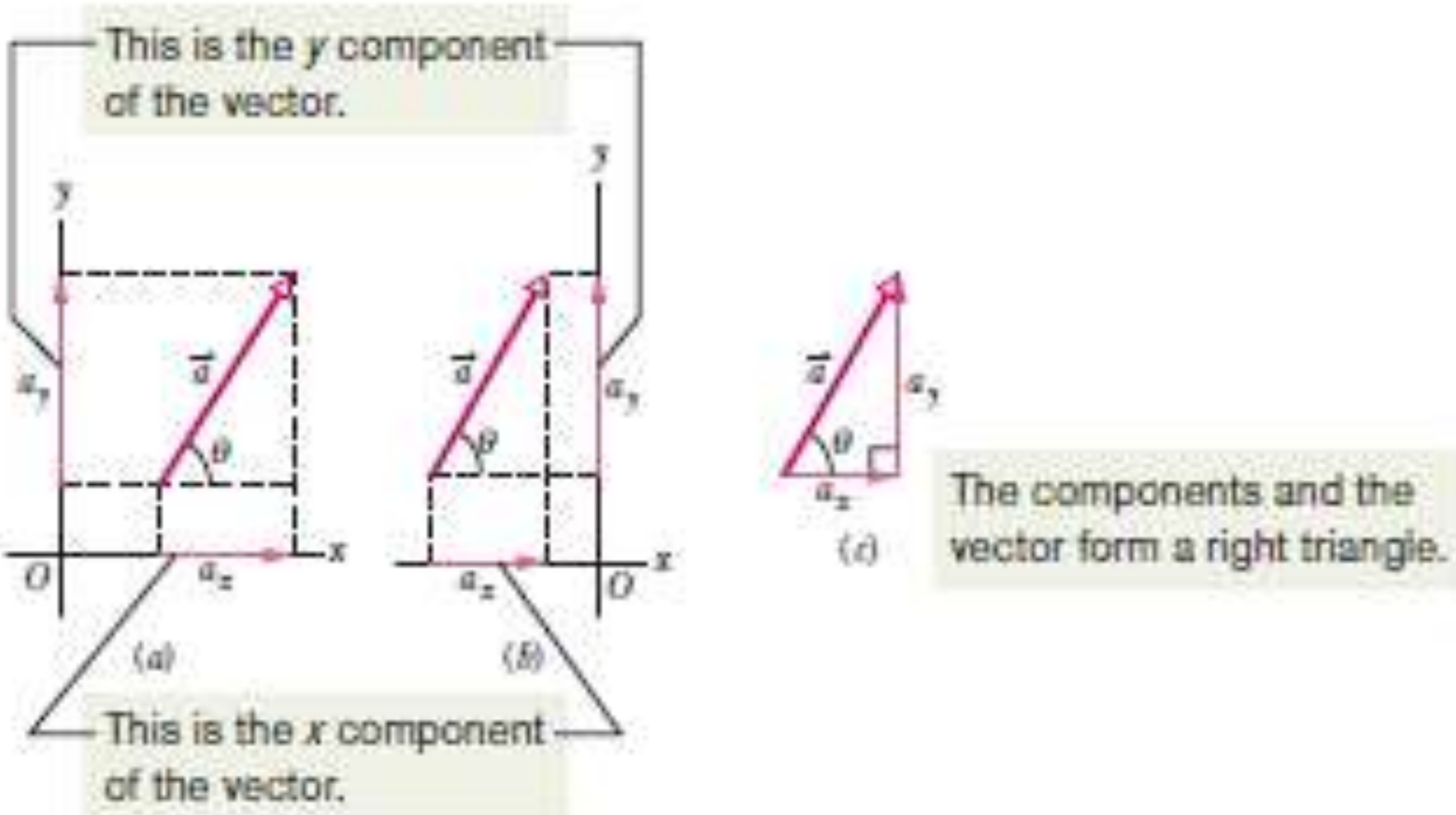


Fig. 3-8 (a) The components a_x and a_y of vector $\vec{a}$. (b) The components are unchanged if the vector is shifted, as long as the magnitude and orientation are maintained. (c) The components form the legs of a right triangle whose hypotenuse is the magnitude of the vector.

Resolving vectors

- $\vec{a}$ = Vector that is going to be separated into a_x and a_y.

  $a_x = acos\theta$

  $a_y = asin\theta$

Where a is the magnitude of vector $\vec{a}$

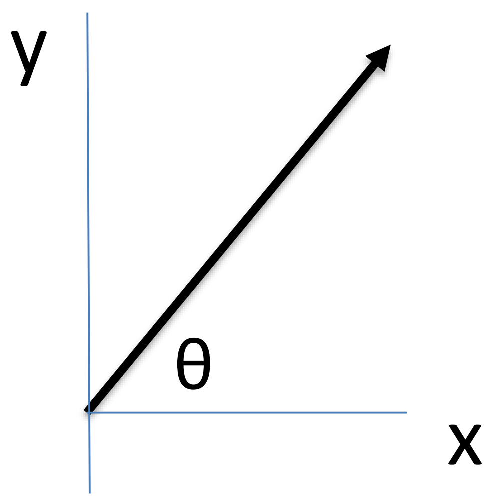
- a_x and a_y are scalars and they are called the components of vector $\vec{a}$.

Fig. 3-8a is given (completely determined) by a and θ. It can also be given by its components a_x and a_y. Both pairs of values contain the same information. If we know a vector in *component notation* (a_x and a_y) and want it in *magnitude-angle notation* (a and θ), we can use the equations

$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \tan\theta = \frac{a_y}{a_x} \tag{3-6}$$

Problem 3.02 Finding components, airplane flight

A small airplane leaves an airport on an overcast day and is later sighted 215 km away, in a direction making an angle of 22° east of due north. This means that the direction is not due north (directly toward the north) but is rotated 22° toward the east from due north. How far east and north is the airplane from the airport when sighted?



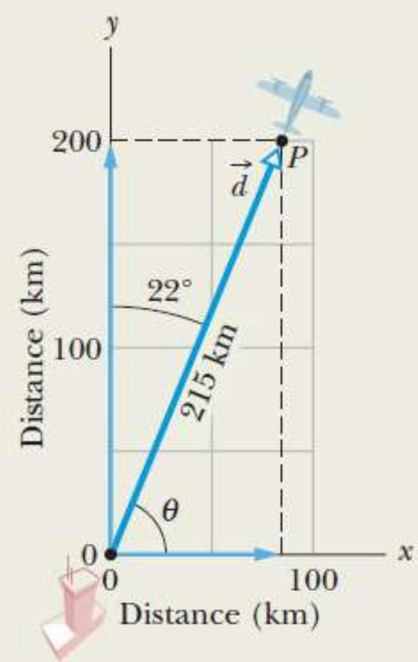


Figure 3-10 A plane takes off from an airport at the origin and is later sighted at P.

KEY IDEA

We are given the magnitude (215 km) and the angle (22° east of due north) of a vector and need to find the components of the vector.

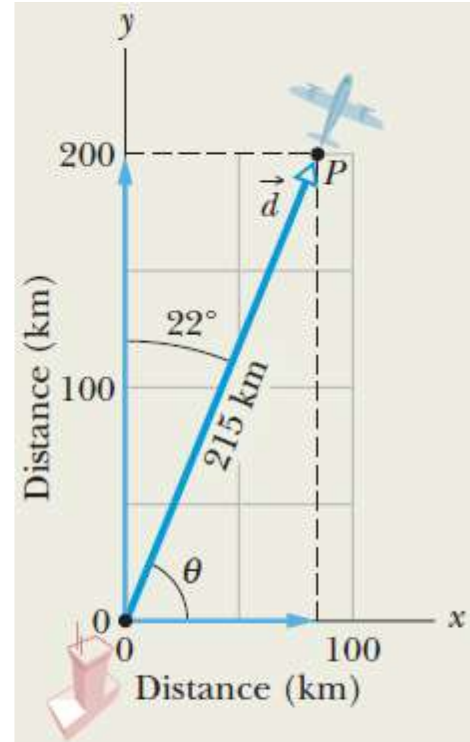
Calculations: We draw an xy coordinate system with the positive direction of x due east and that of y due north (Fig. 3-10). For convenience, the origin is placed at the airport. (We don't have to do this. We could shift and misalign the coordinate system but, given a choice, why make the problem more difficult?) The airplane's displacement $\vec{d}$ points from the origin to where the airplane is sighted.

To find the components of $\vec{d}$, we use Eq. 3-5 with $\theta = 68°$ $(= 90° - 22°)$:

$$d_x = d\cos\theta = (215\text{ km})(\cos 68°)$$
$$= 81\text{ km} \qquad \text{(Answer)}$$

$$d_y = d\sin\theta = (215\text{ km})(\sin 68°)$$
$$= 199\text{ km} \approx 2.0 \times 10^2\text{ km.} \qquad \text{(Answer)}$$

Thus, the airplane is 81 km east and 2.0×10^2 km north of the airport.



Unit Vectors

- A unit vector is a vector that has a magnitude of 1 exactly.
- It always points in a particular direction.
- It is always written in the form of $\hat{\imath}, \hat{\jmath}\ or\ \hat{k}$ depending on the direction of the vector.
- $\hat{\imath}$ always comes with the x- component of the vector , $\hat{\jmath}$ with the y-component and k with the z-component.

The unit vectors point along axes.

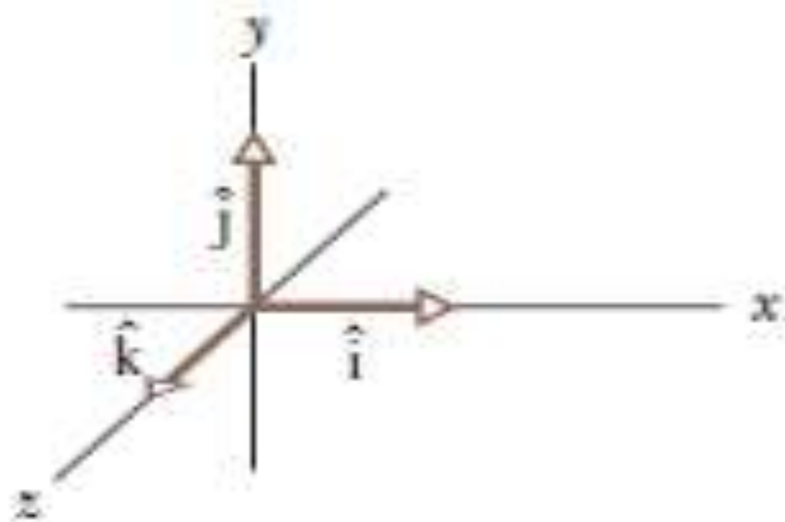
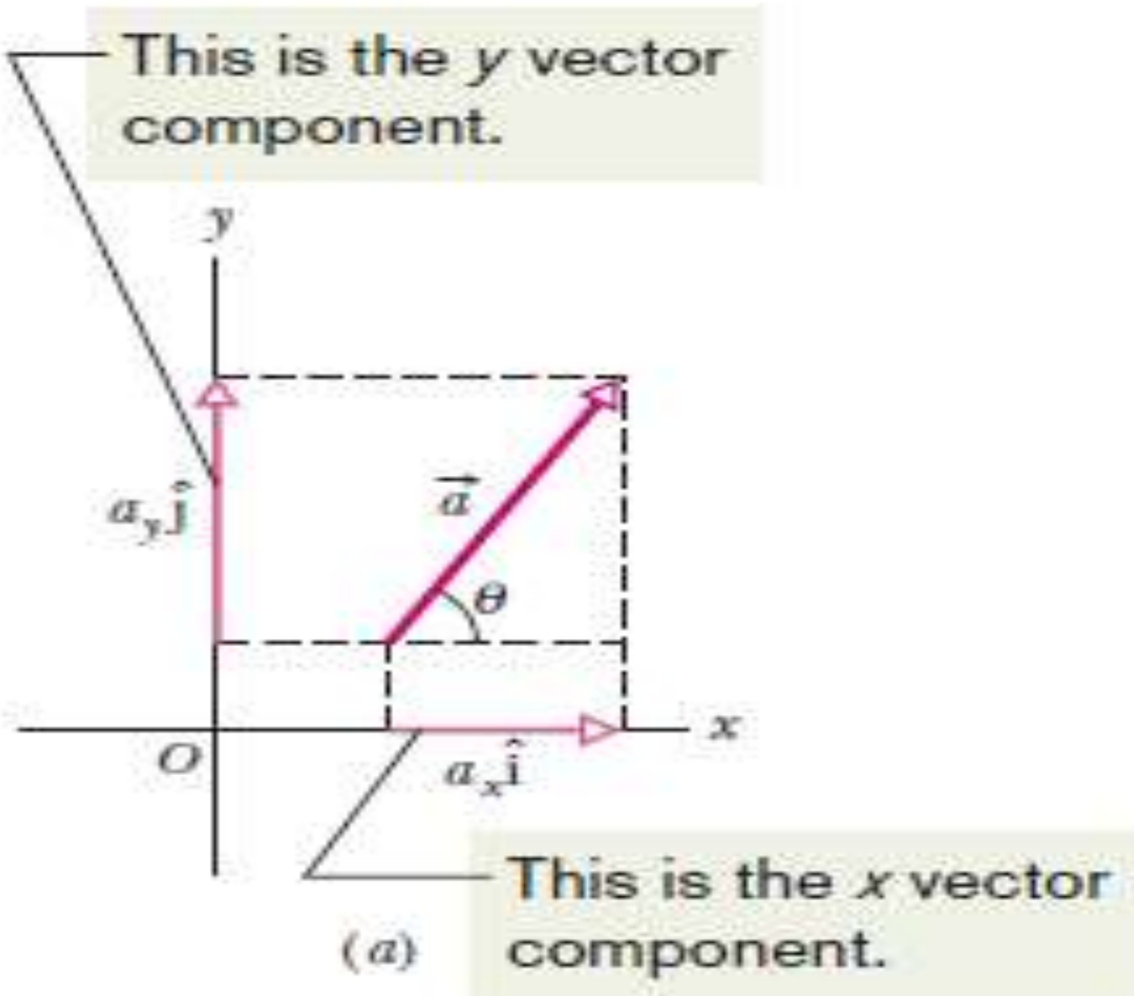
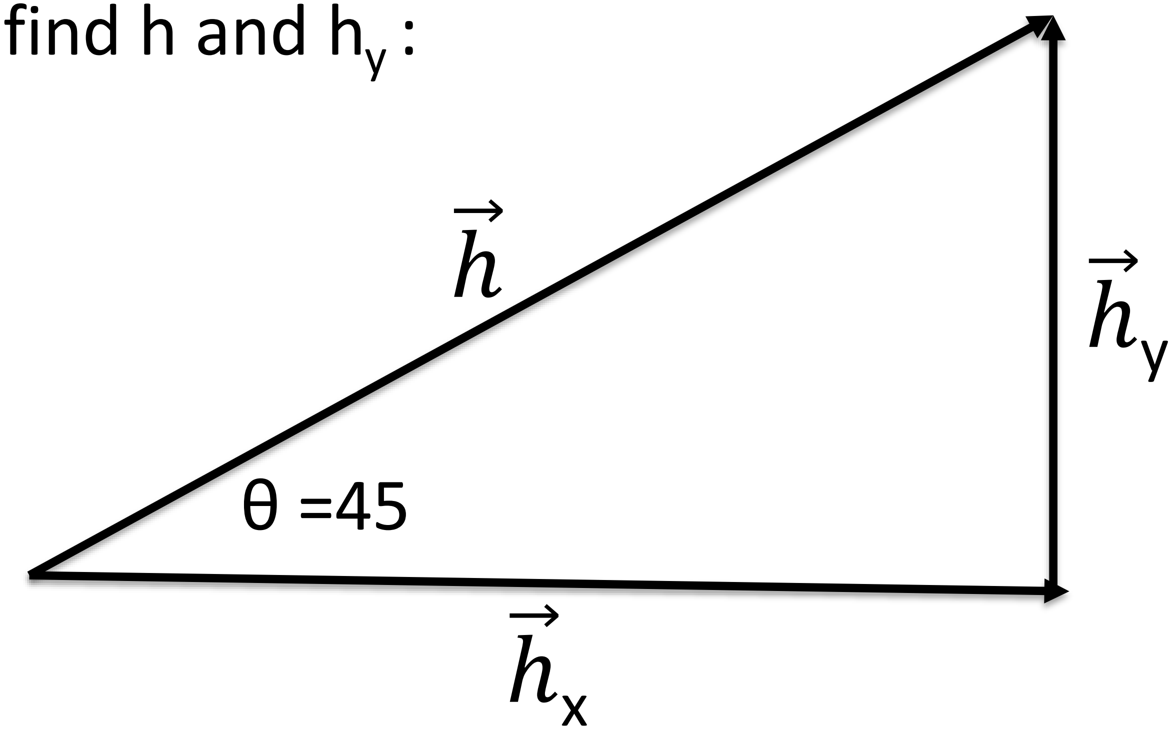


Fig. 3-13 Unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$ define the directions of a right-handed coordinate system.


This is the y vector component.
y
$a_y\hat{j}$
$\vec{a}$
θ
O
x
$a_x\hat{i}$
This is the x vector component.
(a)

Example: If $\vec{h}_x$ has a magnitude of 20 km/h, let us find h and h_y :



h_x= hcos θ
20=hcos 45
20=h 0.707
h= 28.28 km/h

h_y=hsin θ
h_y =28.28*sin45
h_y =28.28*0.707
h_y =20 km/h

Rules of multiplying unit vectors

Let $\hat{\mathrm{i}}$, $\hat{\mathrm{j}}$, and $\hat{\mathrm{k}}$ be unit vectors in the x, y, and z directions. Then

$$\hat{\mathrm{i}} \cdot \hat{\mathrm{i}} = \hat{\mathrm{j}} \cdot \hat{\mathrm{j}} = \hat{\mathrm{k}} \cdot \hat{\mathrm{k}} = 1, \quad \hat{\mathrm{i}} \cdot \hat{\mathrm{j}} = \hat{\mathrm{j}} \cdot \hat{\mathrm{k}} = \hat{\mathrm{k}} \cdot \hat{\mathrm{i}} = 0,$$

Basis vectors:

•Any vector with itself gives zero

$$\hat{\mathrm{i}} \times \hat{\mathrm{i}} = \hat{\mathrm{j}} \times \hat{\mathrm{j}} = \hat{\mathrm{k}} \times \hat{\mathrm{k}} = 0,$$

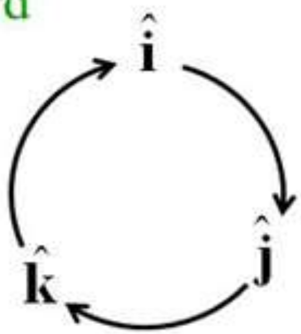
•Think of ijk as a circle: any two <u>in order</u> gives the third

•Any two in <u>reverse order</u> gives minus the third

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \qquad \hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}}$$

$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}} \qquad \hat{\mathbf{k}} \times \hat{\mathbf{j}} = -\hat{\mathbf{i}}$$

$$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \qquad \hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}$$



Multiplying Vectors

- There are two ways of multiplying vectors:

The Scalar Product

The **scalar product** of the vectors $\vec{a}$ and $\vec{b}$ in Fig. 3-18*a* is written as $\vec{a} \cdot \vec{b}$ and defined to be

$$\vec{a} \cdot \vec{b} = ab \cos \phi, \tag{3-20}$$

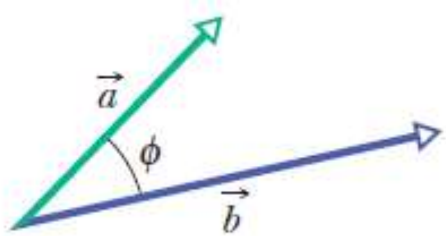


Figure 3-18 (*a*) Two vectors $\vec{a}$ and $\vec{b}$, with an angle ϕ between them.

Scalar Product

When two vectors are in unit-vector notation, we write their dot product as

$$\vec{a} \cdot \vec{b} = (a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}}) \cdot (b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}} + b_z\hat{\mathrm{k}}), \tag{3-22}$$

which we can expand according to the distributive law: Each vector component of the first vector is to be dotted with each vector component of the second vector. By doing so, we can show that

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z. \tag{3-23}$$

CHECKPOINT 4

Vectors $\vec{C}$ and $\vec{D}$ have magnitudes of 3 units and 4 units, respectively. What is the angle between the directions of $\vec{C}$ and $\vec{D}$ if $\vec{C} \cdot \vec{D}$ equals (a) zero, (b) 12 units, and (c) -12 units?

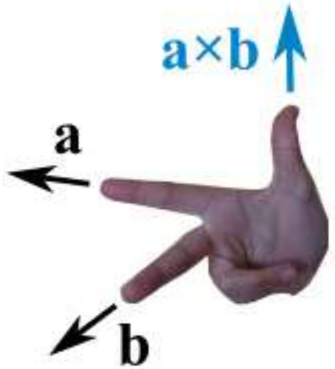
Vector Product

- The second way of multiplying methods is called Vector or Cross Product.

The Vector Product

The **vector product** of $\vec{a}$ and $\vec{b}$, written $\vec{a} \times \vec{b}$, produces a third vector $\vec{c}$ whose magnitude is

$$c = ab \sin \phi, \qquad (3\text{-}27)$$



- **Cross product of vectors is Vector Product**

In unit-vector notation, we write

$$\vec{a} \times \vec{b} = (a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}}) \times (b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}} + b_z\hat{\mathrm{k}}), \qquad (3\text{-}29)$$

Problem:

If $\vec{a} = 3\hat{i} - 4\hat{j}$ and $\vec{b} = -2\hat{i} + 3\hat{k}$, what is $\vec{c} = \vec{a} \times \vec{b}$?

KEY IDEA

When two vectors are in unit-vector notation, we can find their cross product by using the distributive law.

Calculations: Here we write

$$\vec{c} = (3\hat{i} - 4\hat{j}) \times (-2\hat{i} + 3\hat{k})$$
$$= 3\hat{i} \times (-2\hat{i}) + 3\hat{i} \times 3\hat{k} + (-4\hat{j}) \times (-2\hat{i}) + (-4\hat{j}) \times 3\hat{k}.$$

We next evaluate each term with Eq. 3-24, finding the direction with the right-hand rule. For the first term here, the angle ϕ between the two vectors being crossed is 0. For the other terms, ϕ is 90°. We find

$$\vec{c} = -6(0) + 9(-\hat{j}) + 8(-\hat{k}) - 12\hat{i}$$
$$= -12\hat{i} - 9\hat{j} - 8\hat{k}. \qquad \text{(Answer)}$$

This vector $\vec{c}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, a fact you can check by showing that $\vec{c}\cdot\vec{a} = 0$ and $\vec{c}\cdot\vec{b} = 0$; that is, there is no component of $\vec{c}$ along the direction of either $\vec{a}$ or $\vec{b}$.

In general: A cross product gives a perpendicular vector, two perpendicular vectors have a zero dot product, and two vectors along the same axis have a zero cross product.

Second way for vector multiplication: Matrices way

$$\vec{a}\, x\, \vec{b} = \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ -2 & 0 & 3 \end{vmatrix}$$

$$=\hat{i}(-1)^{(1+1)}(-12-0)+$$

$$\hat{j}(-1)^{(1+2)}(9-0)+$$

$$\hat{k}(-1)^{(1+3)}(0-8)$$

$$=\hat{i}\,(-12)-\hat{j}\,(9)+\hat{k}\,(-8) = -12\hat{i}-9\hat{j}-8\hat{k}$$

$$\vec{c} = \vec{c}_x + \vec{c}_y + \vec{c}_z$$

Problems

•2 A displacement vector $\vec{r}$ in the xy plane is 15 m long and directed at angle $\theta = 30°$ in Fig. 3-26. Determine (a) the x component and (b) the y component of the vector.

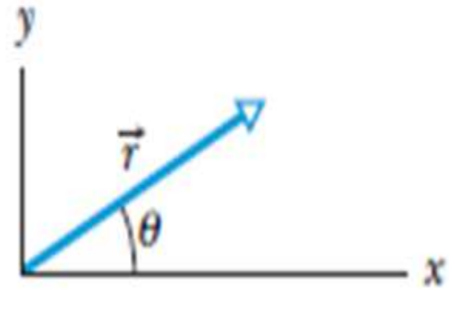


Fig. 3-26

•11 SSM (a) In unit-vector notation, what is the sum $\vec{a} + \vec{b}$ if $\vec{a} = (4.0\text{ m})\hat{i} + (3.0\text{ m})\hat{j}$ and $\vec{b} = (-13.0\text{ m})\hat{i} + (7.0\text{ m})\hat{j}$? What are the (b) magnitude and (c) direction of $\vec{a} + \vec{b}$?

Problems

•34 Two vectors are presented as $\vec{a} = 3.0\hat{i} + 5.0\hat{j}$ and $\vec{b} = 2.0\hat{i} + 4.0\hat{j}$. Find (a) $\vec{a} \times \vec{b}$, (b) $\vec{a} \cdot \vec{b}$, (c) $(\vec{a} + \vec{b}) \cdot \vec{b}$