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Welcome
to
PHYS 212: Physics

Chapter#27

Circuits

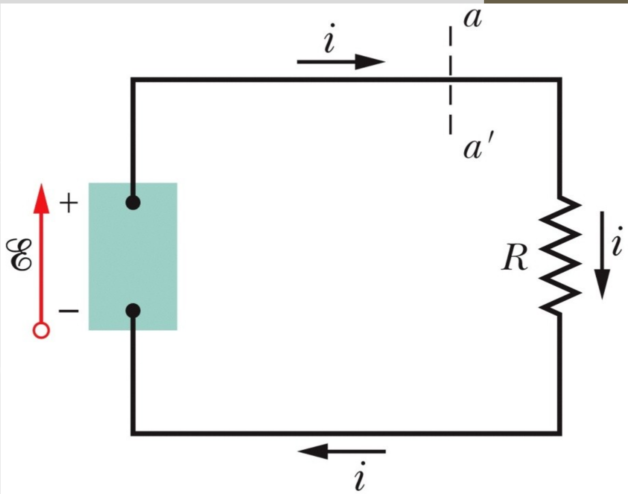
Topics

- Electromotive Force (e.m.f).
- Resistance in Series.
- Resistance in Parallel.
- Multiloop Circuits (Kirchhoff's Rules).

27-1 Electromotive Force (e.m.f)

❖ To produce a steady flow of charge, you need a "**charge pump**," a device that—by doing work on the charge carriers—maintains a potential difference between a pair of terminals.

❖ We call such a device an ***emf* device**, which means that it does work on charge carriers.



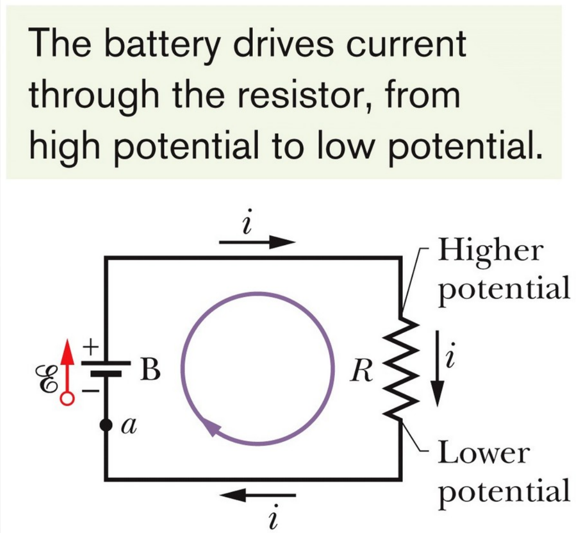
❖ If ***dW*** is the work the device does to force positive charge ***dq*** from the negative to the positive terminal, then the ***emf*** (**work per unit charge**) of the device is

$$\mathscr{E} = \frac{dW}{dq} \quad \text{(definition of } \mathscr{E}\text{).}$$

➢ **The emf of a battery is the maximum possible voltage that the battery can provide between its terminals.**

Single-Loop Circuits

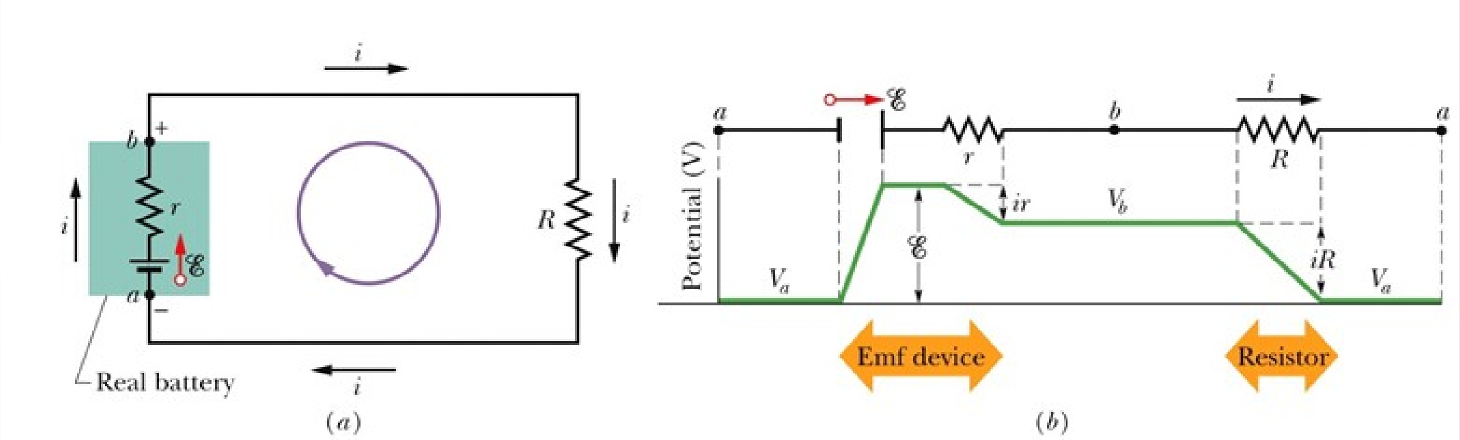
❖ An **ideal *emf* device** is one that lacks any internal resistance. The potential difference between its terminals is equal to the *emf*.



$\mathscr{E} - iR = 0.$ → $\mathscr{E} = iR.$ → $i = \dfrac{\mathscr{E}}{R}.$

Single-Loop Circuits

❖ A **real *emf* device** has internal resistance. The potential difference between its terminals is equal to the *emf* only if there is no current through the device.



$$\mathscr{E} - ir - iR = 0. \quad \Rightarrow \quad i = \frac{\mathscr{E}}{R + r}.$$

LOOP RULE: The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

Potential Difference Between Two Points

- We often want to find the potential difference between two points in a circuit. For example, in Fig. 27-6, what is the potential difference $V_b - V_a$ between points a and b?

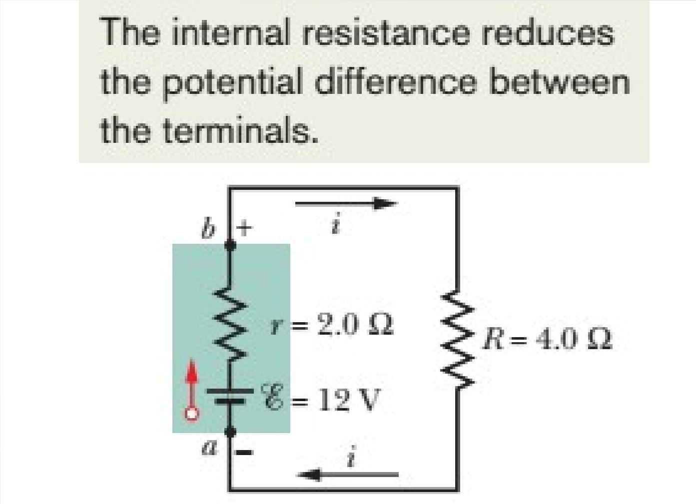


Figure 27-6 Points *a* and *b*, which are at the terminals of a real battery, differ in potential.

- To find out, let's start at point a (at potential V_a) and move through the battery to point b (at potential V_b) while keeping track of the potential changes we encounter.

Potential Difference Between Two Points

- To find out, let's start at point a and move through the battery to point b.

- When we pass through the battery's emf, the potential increases. When we pass through the battery's internal resistance r the potential decreases by ir. We are then at the potential of point b and we have

$$V_a + \mathscr{E} - ir = V_b,$$

$$\text{or} \quad V_b - V_a = \mathscr{E} - ir.$$

$$\text{or} \quad \Delta V = \mathscr{E} - ir.$$

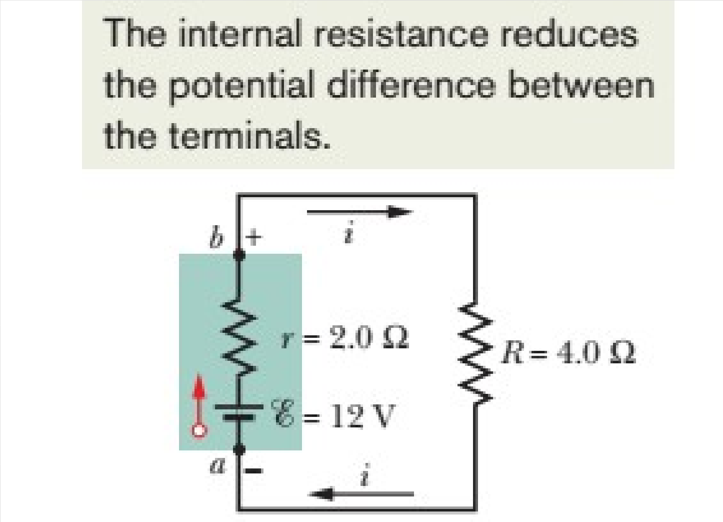


Figure 27-6 Points a and b, which are at the terminals of a real battery, differ in potential.

Example 28.1 Terminal Voltage of a Battery

A battery has an emf of 12.0 V and an internal resistance of 0.05 Ω. Its terminals are connected to a load resistance of 3.00 Ω.

(A) Find the current in the circuit and the terminal voltage of the battery.

Solution Equation 28.3 gives us the current:

$$I = \frac{\mathcal{E}}{R + r} = \frac{12.0\ \text{V}}{3.05\ \Omega} = \boxed{3.93\ \text{A}}$$

and from Equation 28.1, we find the terminal voltage:

$$\Delta V = \mathcal{E} - Ir = 12.0\ \text{V} - (3.93\ \text{A})(0.05\ \Omega) = \boxed{11.8\ \text{V}}$$

To check this result, we can calculate the voltage across th load resistance R:

$$\Delta V = IR = (3.93\ \text{A})(3.00\ \Omega) = 11.8\ \text{V}$$

(B) Calculate the power delivered to the load resistor, the power delivered to the internal resistance of the battery, and the power delivered by the battery.

Solution The power delivered to the load resistor is

$$\mathcal{P}_R = I^2R = (3.93\ \text{A})^2\ (3.00\ \Omega) = \boxed{46.3\ \text{W}}$$

The power delivered to the internal resistance is

$$\mathcal{P}_r = I^2r = (3.93\ \text{A})^2\ (0.05\ \Omega) = \boxed{0.772\ \text{W}}$$

Hence, the power delivered by the battery is the sum of these quantities, or 47.1 W. You should check this result, using the expression $\mathcal{P} = I\mathcal{E}$.

27-1 Resistance in Series

❖ **Figure (a)** shows three resistances connected in series to an ideal battery with ***emf***. The potential difference is maintained across **a** and **b** by the battery.

❖ To find total resistance R_{eq} in **Fig. (b)**, we apply the **loop rule** to both circuits. For Fig. (a), starting at **a** and going clockwise around the circuit, we find

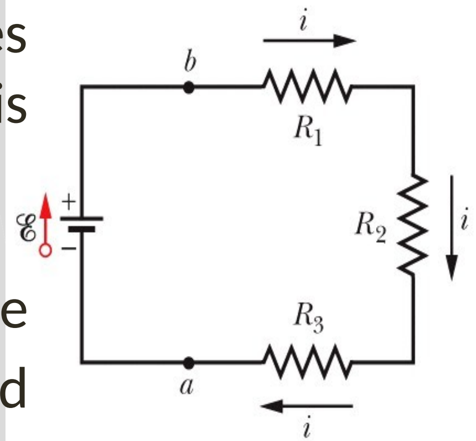
$$\mathscr{E} - iR_1 - iR_2 - iR_3 = 0, \quad \Rightarrow \quad i = \frac{\mathscr{E}}{R_1 + R_2 + R_3}.$$

❖ **For Fig. (b)**, with the three resistances replaced with a single equivalent resistance R_{eq}, we find

$$\mathscr{E} - iR_{eq} = 0, \quad \Rightarrow \quad i = \frac{\mathscr{E}}{R_{eq}}.$$

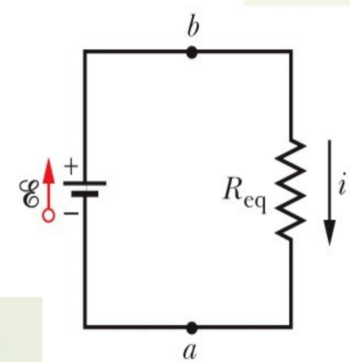
❖ Equating them, we get,

$$R_{eq} = R_1 + R_2 + R_3. \quad \Rightarrow \quad R_{eq} = \sum_{j=1}^{n} R_j \quad (n \text{ resistances in series}).$$



(a)

Series resistors and their equivalent have the same current ("ser-i").



(b)

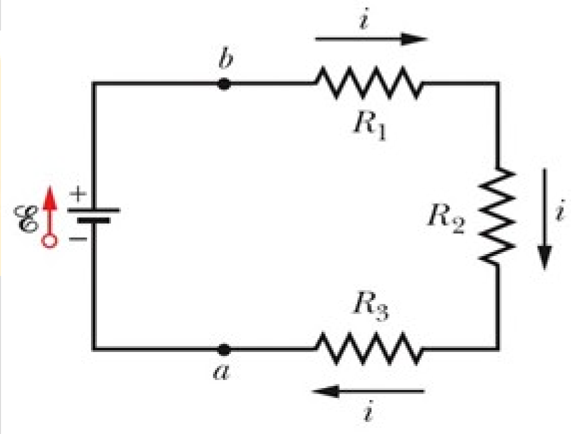
Resistance in Series

When a potential difference V is applied across resistances connected in series, the resistances have identical currents i. The sum of the potential differences across the resistances is equal to the applied potential difference V.

Resistances connected in series can be replaced with an equivalent resistance R_{eq} that has the same current i and the same *total* potential difference V as the actual resistances.

Checkpoint 2

In Fig. *a*, if $R_1 > R_2 > R_3$, rank the three resistances according to (a) the current through them and (b) the potential difference across them, greatest first.



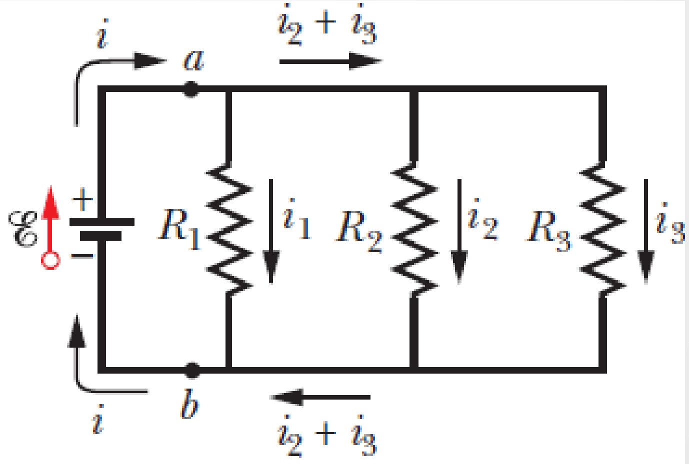
(a)

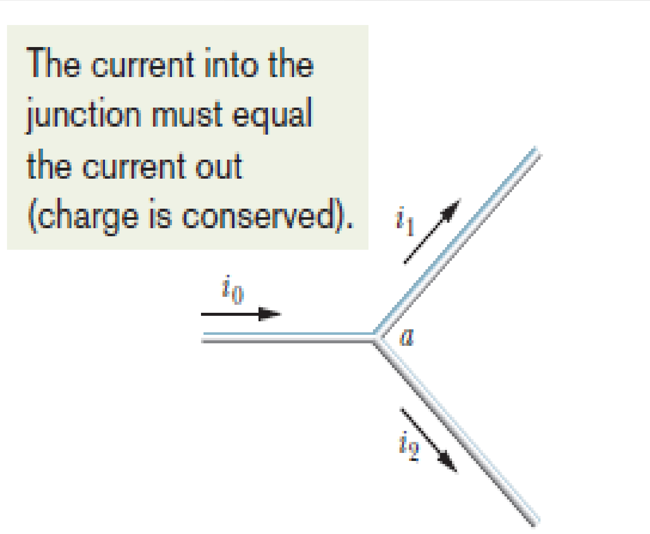
Answer: (a) current is same for all resistors in series.

(b) V_1, V_2, and V_3

27-2 Resistance in Parallel

When a potential difference V is applied across resistances connected in parallel, the resistances all have that same potential difference V.



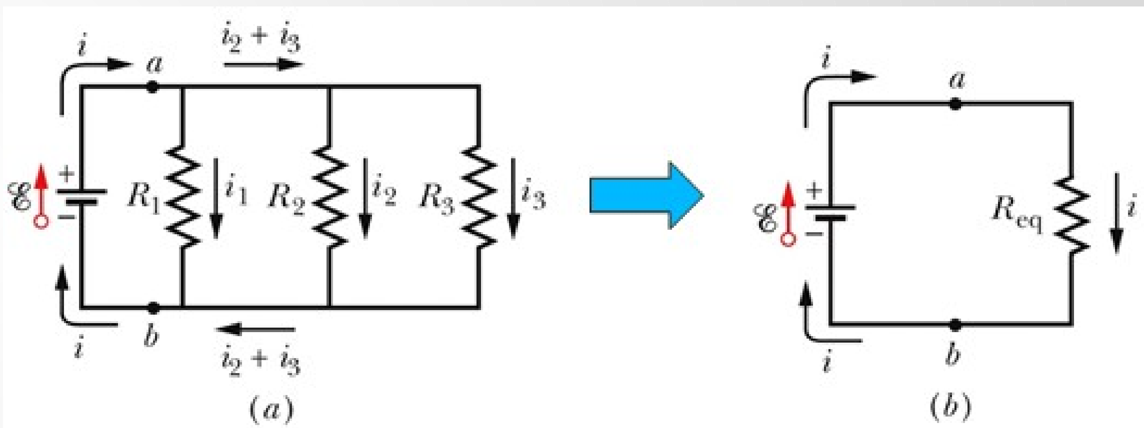


JUNCTION RULE: The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.

Resistance in Parallel

❖ **Figure (a)** shows three resistances connected in parallel to an ideal battery of *emf*.

❖The applied potential difference **V** is maintained by the battery.



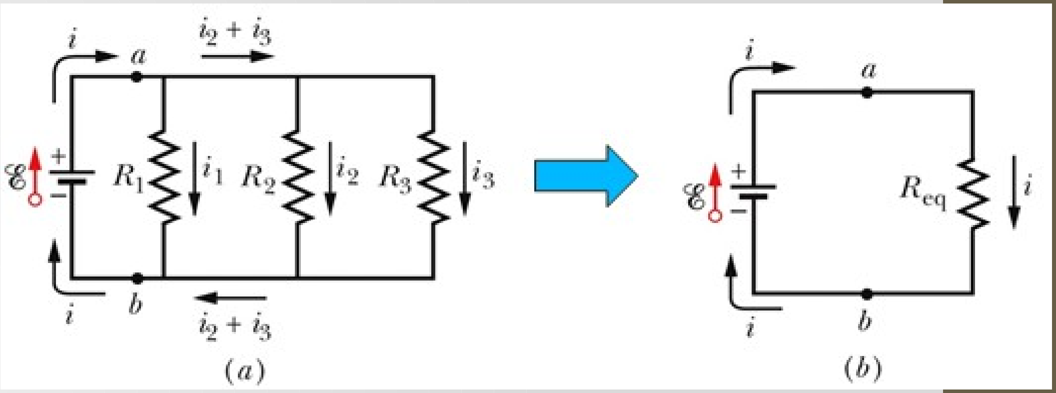
❖To derive an expression for $\boldsymbol{R_{eq}}$ in Fig. (b), we first write the current in each actual resistance in Fig. (a) as

$$i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2}, \quad \text{and} \quad i_3 = \frac{V}{R_3},$$

Resistance in Parallel

❖ If we apply the **junction rule** at point **a** in Fig. (a) and then substitute these values, we find

$$i = i_1 + i_2 + i_3 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right).$$



❖ If we replaced the parallel combination with the equivalent resistance R_{eq} (Fig. b), we would have

$$i = \frac{V}{R_{eq}}.$$

❖ and thus substituting the value of ***i*** from above equation we get,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}. \quad \Rightarrow \quad \frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j} \quad (n \text{ resistances in parallel}).$$

Resistance and capacitors

Table 27-1 Series and Parallel Resistors and Capacitors

Series	Parallel	Series	Parallel
Resistors		Capacitors	
$R_{eq} = \sum_{j=1}^{n} R_j$ Eq. 27-7	$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$ Eq. 27-24	$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$ Eq. 25-20	$C_{eq} = \sum_{j=1}^{n} C_j$ Eq. 25-19
Same current through all resistors	Same potential difference across all resistors	Same charge on all capacitors	Same potential difference across all capacitors

Checkpoint 4

A battery, with potential V across it, is connected to a combination of two identical resistors and then has current i through it. What are the potential difference across and the current through either resistor if the resistors are (a) in series and (b) in parallel?

Answer: (a) Potential difference across each resistor: *V/2*

Current through each resistor: *i*

(b) Potential difference across each resistor: *V*

Current through each resistor: *i/2*

Problem-1:

•24 In Fig. 27-36, $R_1 = R_2 = 4.00\ \Omega$ and $R_3 = 2.50\ \Omega$. Find the equivalent resistance between points D and E. (*Hint:* Imagine that a battery is connected across those points.)

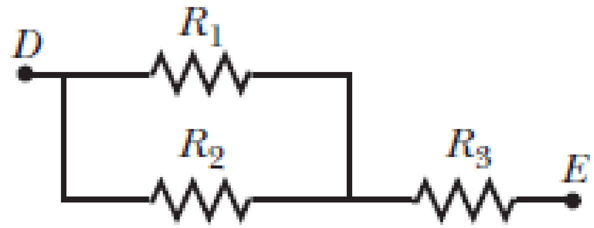


Figure 27-36 Problem 24.

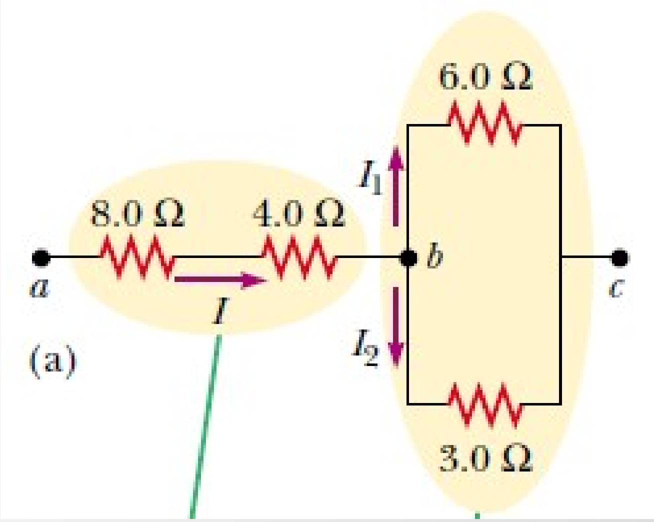
Answer:

$R_{eq} = 4.50\ \Omega$

Problem-2:

Four resistors are connected as shown in Figure 28.9a.

(A) Find the equivalent resistance between points *a* and *c*.



(B) What is the current in each resistor if a potential difference of 42 V is maintained between *a* and *c*?

Answer:

(A) R_{eq} = 14 Ω

(B) I = 3 A

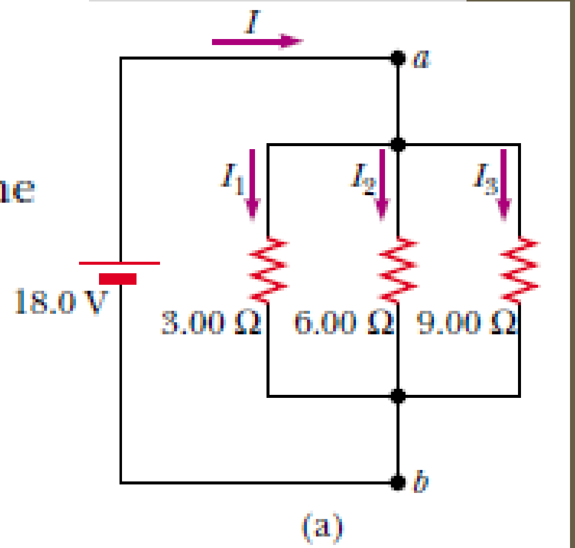
Problem-3:

Three resistors are connected in parallel as shown in Figure 28.11a. A potential difference of 18.0 V is maintained between points *a* and *b*.

(A) Find the current in each resistor.

(B) Calculate the power delivered to each resistor and the total power delivered to the combination of resistors.

(C) Calculate the equivalent resistance of the circuit.



(a)

Answer:

$I_1 = \boxed{6.00\ \mathrm{A}}$

$I_2 = \boxed{3.00\ \mathrm{A}}$

$I_3 = \boxed{2.00\ \mathrm{A}}$

$\mathcal{P}_1 = \boxed{108\ \mathrm{W}}$

$\mathcal{P}_2 = \boxed{54.0\ \mathrm{W}}$

$\mathcal{P}_3 = \boxed{36.0\ \mathrm{W}}$

$R_{eq} = \boxed{1.64\ \Omega}$

Problem-4:

•19 A total resistance of 3.00 Ω is to be produced by connecting an unknown resistance to a 12.0 Ω resistance. (a) What must be the value of the unknown resistance, and (b) should it be connected in series or in parallel?

Answer:

(a) $R_x = 4.00\ \Omega$.

(b) the resistors must be connected in parallel.

Problem-5:

•20 When resistors 1 and 2 are connected in series, the equivalent resistance is 16.0 Ω. When they are connected in parallel, the equivalent resistance is 3.0 Ω. What are (a) the smaller resistance and (b) the larger resistance of these two resistors?

Answer:

(a) The smaller resistance is $R_1 = 4.0\ \Omega$.

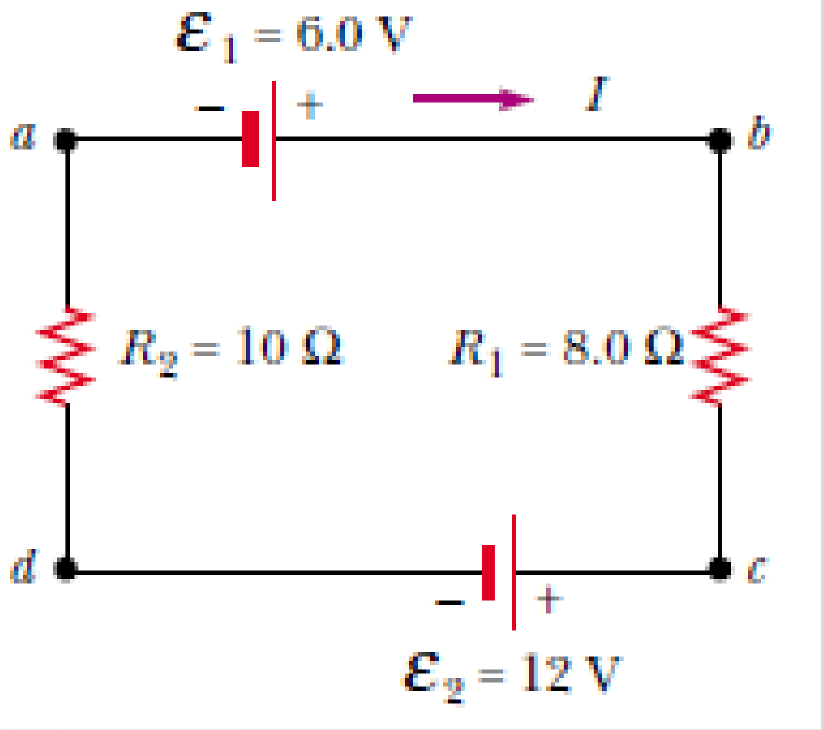
(b) The larger resistance is $R_2 = 12\ \Omega$.

Problem-6:

A single-loop circuit contains two resistors and two batteries, as shown in Figure 28.16. (Neglect the internal resistances of the batteries.)

(A) Find the current in the circuit.

(B) What power is delivered to each resistor? What power is delivered by the 12-V battery?



Answer:

$$I = -0.33\ \text{A}$$

$$\mathcal{P}_1 = 0.87\ \text{W}$$

$$\mathcal{P}_2 = 1.1\ \text{W}$$

27-2 Multiloop Circuits (Kirchhoff's Rules)

❖ As we saw in the preceding section, simple circuits can be analyzed using the expression **$\Delta V = IR$** and the rules for series and parallel combinations of resistors. Very often, it is not possible to reduce a circuit to a single loop.

❖ The procedure for analyzing more complex circuits is greatly simplified if we use two principles called ***Kirchhoff's rules:***

❑ **Junction rule** (or ***Kirchhoff's current law***):

The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction.

$$\sum I_{in} = \sum I_{out}$$

❑ **Loop rule** (or ***Kirchhoff's voltage law***):

The sum of the potential differences across all elements around any closed circuit loop must be zero.

$$\sum_{\substack{Closed\\loop}} \Delta V = 0$$