FINAL REPORT

Course: Applied Calculus for IT

Course Code: 501031

I. REGULATIONS

This report is a replacement for the final written exam in the first semester of the academic year 2025-2026. Students need to adhere to the following regulations:

• Each report is conducted by a group of **one or two students**.

• For each question, students are required to present **detailed calculation steps**, and the final results.

• The report must be submitted in **PDF format**, and the content must be written based on the report/essay format of the Faculty of Information Technology. **In case students do not follow the Faculty's format, they will receive 0 points for the Report.**

• Each group of students must complete the **self-evaluation form** using the following template:

   o   https://docs.google.com/spreadsheets/d/1OIvardCzSXRSCwiDs199qLlnFO-JCs1A/edit?usp=sharing&ouid=104602653034506388459&rtpof=true&sd=true
   o   Download the form as an Excel file (.xlsx extension)

• The report must include the following contents:

   a. Chapter 1: Solutions

        Write **detailed calculation steps** and the final results of each question (in the section III). In each calculation step, insert the related Background knowledge (theorems, laws, rules, tests…) in case you apply it, for ex.,:

- By Squeeze Theorem, $\lim_{x \to a} f(x)\sin(g(x)) = 0$.
- Based on/ According to the Intermediate Value Theorem, there exists a number $c \in (0,2)$ such that $f(c) = 0$.
- By the First Derivative Test, we found one local maximum at $x = 0$, and two local minimum at $x = -1$ and $x = 2$.
- …

• Unusual cases (such as identical patterns, signs of plagiarism, etc.) will be scheduled for online discussions with the instructor.

II. Submission guideline

- Filenames of the report and self-evaluation files must be the **Student IDs**, for ex.,

III. Questions

1. Tell whether the following functions are even, odd, or neither. Give reasons for your answer. (1.0 point)
   - $f(x) = x^2 + x$
   - $f(x) = x^3 + x$
   - $f(x) = \frac{4}{x^4 - 4}$
   - $f(x) = \frac{x^3}{x^4 - 4}$

2. Find the following limit $\lim \frac{555}{x^2 - 25}$ as: (1.0 point)
   - $x \to 5^+$
   - $x \to 5^-$
   - $x \to -5^+$
   - $x \to -5^-$

3. Find the derivatives $\frac{dy}{dx}$ of the following functions: (1.0 point)
   - $y = \frac{\sqrt{x} - 4}{\sqrt{x} + 4}$
   - $y = \left(\frac{\sqrt{x}}{10} - 1\right)^{-10}$

4. Find an equation of the tangent line to the graph of $y = 1 + 2e^x$ at the point where $x = 0$. (1.0 point)

5. Given the derivative $f'(x) = (\sin x + \cos x)(\sin x - \cos x)$, $0 \le x \le 2\pi$. (1 point)
   - What are the critical numbers of ?
   - On what open intervals is f increasing or decreasing?
   - At what points, if any, does f assume local maximum and minimum values?

6. Find all curves through a point where x = 1 whose arc length is the following L value: (1.0 point)

$$L = \int_1^5 \sqrt{1 + \frac{1}{x^2}}\, dx$$

7. Given that $a_1, a_2, a_3, \dots, a_n, \dots$ are real numbers fulfilling the following conditions: (1 point)

- $a_n > 0, n \in \mathbb{Z}^+$
- $a_1 \geq a_2 \geq a_3 \geq \cdots \geq a_n \geq \cdots$
- The series $a_2 + a_4 + a_8 + a_{16} + \cdots + a_{2^n} + \cdots$ diverges.

Determine the convergence or divergence of the following series. Explain in details.

$$a_1 + \frac{a_2}{2} + \frac{a_3}{3} + \frac{a_4}{4} + \cdots + \frac{a_n}{n} + \cdots$$

8. Find all values of x such that the following series is absolutely convergent: (1 point)

$$\sum_{n=1}^{\infty} \frac{nx^n}{(n+1)(2x+1)^n}$$

9. One thousand earphones sell for \$55 each, resulting in a revenue of $(1000)(\$55) =$ \$55,000. For each \$5 increase in the price, 20 fewer earphones are sold. For ex., if the price of each earphone is \$60, there will be 980 $(1000 - 20)$ earphones sold; if the price of each earphone is \$65, there will be 960 $(1000 - 20 - 20)$ earphones sold; so on. Find the revenue in case the price of each earphone is \$255 (1 point).