DUAL NATURE OF RADIATION AND MATTER

Some Important Discoveries of the late 19th Century

- X-Rays were discovered by Roentgen
- Electron was discovered by J.J. Thomson
- Cathode rays were discovered by William Crookes and were confirmed by J.J. Thompson
- J.J. Thomson calculated the e/m ratio of cathode ray particles
- Charge on an electron was discovered by R.A. Millikan by his famous Oil-drop experiment. This established that charge on an electron is quantized.

Cathode Rays

If a gas is filled in a glass tube at extremely low pressure and very high voltage is applied then, a discharge takes place between the two electrodes on applying electric field. A fluorescent glow appeared on the glass opposite to the cathode, the color of which depends on the glass. This glow was attributed to the radiation coming from the cathode which was named as cathode rays.
It was found out that the cathode rays consisted of a stream of negatively charged particles.

e/m Ratio

By applying mutually perpendicular electric and magnetic fields across the discharge tube, J.J. Thomson was able to calculate the specific charge (charge to mass ratio) for the cathode rays

- The value of e/m ratio was found to be independent of nature of material/metal used as the cathode and the gas in the discharge tube
- The value of e/m ratio of cathode rays (electrons) is 1.76×10^{11} C/kg

Electron Emission

We know that metals have free electrons but these free electrons cannot escape the metal because when an electron escapes the metal, the metal surface becomes positively charged and it pulls back the electron. So, an electron can only escape a metal if it has a minimum amount of energy.

Work function (ϕ_o) – The minimum energy required by an electron to escape from the metal surface is called the work function of the metal. It is generally measured in eV (electron volt). The work function of the metal depends on its properties and the nature of its surface.

The energy required for electron emission can be supplied by-

1. **Thermionic Emission-** By suitably heating, sufficient energy can be supplied to the electrons to enable them to come out of the metal
2. **Field emission-** By applying very strong electric field to a metal, electrons can be pulled out of the metal, as in a spark plug.
3. **Photo-electric emission-** When light of suitable frequency illuminates a metal surface, electrons are emitted from the metal surface. The photo (light) generated electrons are called photo electrons.

Photoelectric Effect

Hertz' Observations

The phenomenon of photoelectric emission was discovered by Heinrich Hertz.
Light shining on the metal surface somehow facilitated the escape of free, charged particles which we now know as electrons. When light falls on a metal surface, some electrons near the surface absorb enough energy from the incident radiation to overcome the attraction of the positive ions in the material of the surface. After gaining sufficient energy from the incident light, the electrons escape from the surface of the metal into the surrounding space.

Hallwachs' and Lenard's Observations

- Lenard observed that when ultraviolet radiations were allowed to fall on the emitter plate of an evacuated glass tube enclosing two electrodes (metal plates), current flows in the circuit.
- As soon as the ultraviolet radiations were stopped, the current flow also stopped.

These observations indicate that when ultraviolet radiations fall on the emitter plate C, electrons are ejected from it which are attracted towards the positive, collector plate A by the electric field. The electrons flow through the evacuated glass tube, resulting in the current flow. Thus, light falling on the surface of the emitter causes current in the external circuit.

Hallwachs, undertook the study further and connected a negatively charged zinc plate to an electroscope. He observed that-

- the zinc plate lost its charge when it was illuminated by ultraviolet light.
- Further, the uncharged zinc plate became positively charged when it was irradiated by ultraviolet light.
- Positive charge on a positively charged zinc plate was found to be further enhanced when it was illuminated by ultraviolet light.

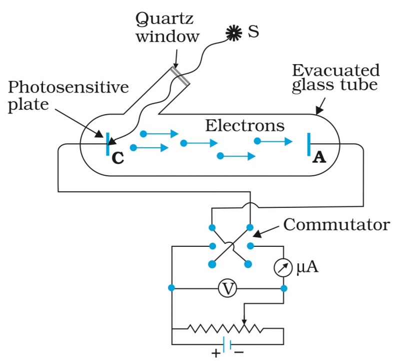
From these observations he concluded that negatively charged particles were emitted from the zinc plate under the action of ultraviolet light.

Experimental Study of Photoelectric effect

The Figure depicts a schematic view of the arrangement used for the experimental study of the photoelectric effect.

Construction-

It consists of an evacuated glass/quartz tube having a photosensitive plate C and another metal plate A.



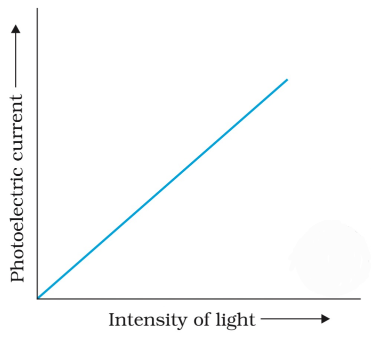
Working-

- Monochromatic light from the source S of sufficiently short wavelength passes through the window W and falls on the photosensitive plate C (emitter).
- A transparent quartz window is sealed on to the glass tube, which permits ultraviolet radiation to pass through it and irradiate the photosensitive plate C.
- The electrons are emitted by the plate C and are collected by the plate A (collector), by the electric field created by the battery.
- The battery maintains the potential difference between the plates C and A, that can be varied.
- The polarity of the plates C and A can be reversed by a commutator. Thus, the plate A can be maintained at a desired positive or negative potential with respect to emitter C.
- When the collector plate A is positive with respect to the emitter plate C, the electrons are attracted to it. The emission of electrons causes flow of electric current in the circuit.

Effect of Intensity of light on photocurrent

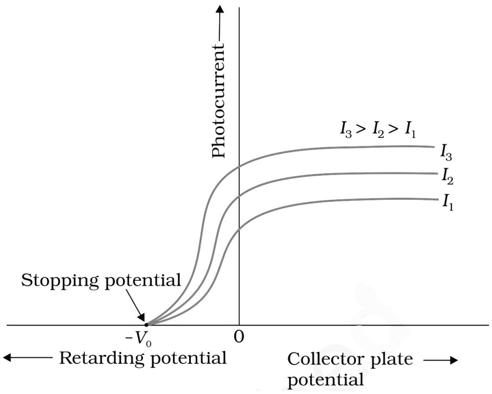
It is found that the photocurrent increases linearly with intensity of incident light as shown graphically. The photocurrent is directly proportional to the number of photoelectrons emitted per second. This implies that the number of photoelectrons emitted per second is directly proportional to the intensity of incident radiation.

$$\boxed{I \propto i_p}$$



Effect of potential on photoelectric current

The photoelectric current increases with increase in accelerating (positive) potential. At some stage, for a certain positive potential of plate A, all the emitted electrons are collected by the plate A and the photoelectric current becomes maximum or saturates. If we increase the accelerating potential of plate A further, the photocurrent does not increase. This maximum value of the photoelectric current is called saturation current. Saturation current corresponds to the case when all the photoelectrons emitted by the emitter plate C reach the collector plate A.



When we apply a negative (retarding) potential to the plate A with respect to the plate C and make it increasingly negative gradually. When the polarity is reversed, the electrons are repelled and only the most energetic electrons are able to reach the collector A. The photocurrent is found to decrease rapidly until it drops to zero at a certain sharply defined, critical value of the negative potential V_0 on the plate A. For a particular frequency of incident radiation, the minimum negative (retarding) potential V_0 given to the plate A for which the photocurrent stops or becomes zero is called the **cut-off** or **stopping potential (V_0)**

Interpretation-

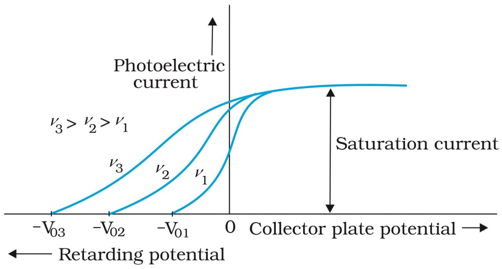
All the photoelectrons emitted from the metal do not have the same energy. Photoelectric current is zero when the stopping potential is sufficient to repel even the most energetic photoelectrons, with the maximum kinetic energy K_{max}, such that-

$$\boxed{K_{max} = eV_0}$$

Also, if the intensity of the incident light is increased, we find that the current still saturates but at a higher value. This indicates that more electrons are emitted per second (as photocurrent is directly proportional to intensity) but still, the value of stopping potential remains the same (given the frequency of incident radiation remains the same). Thus, for a given frequency of incident radiation, the stopping potential is independent of its intensity. This also means that the maximum kinetic energy of the photoelectrons depends on the frequency and not intensity of incident light.

Effect of frequency of incident radiation on stopping potential

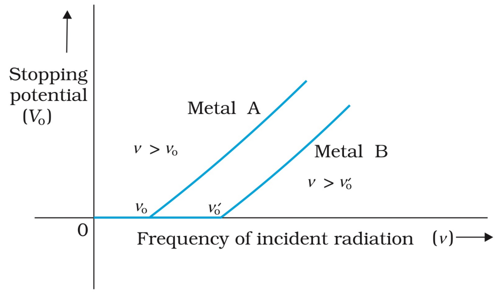
If light of same intensity but different frequency is incident on a metal surface, we observe that we get the same value of saturation current but different values of stopping potential. This is because the energy of the emitted electrons depends on the frequency of the incident radiations. The stopping potential is more negative for higher frequencies of incident radiation. This implies that greater the frequency of incident light, greater is the maximum kinetic energy of the photoelectrons. Consequently, we need greater retarding potential to stop them completely.



If we draw a graph between the stopping potential and frequency of incident radiation, we get a straight line.

From the graph we can say-

1. The stopping potential V_0 varies linearly with the frequency of incident radiation for a given photosensitive material.
2. There exists a certain minimum cut-off frequency V_0 for which the stopping potential is zero.



Implications-

1. The maximum kinetic energy of the photoelectrons varies linearly with the frequency of incident radiation, but is independent of its intensity.
2. For a frequency ν of incident radiation, lower than the cut-off frequency V_0, no photoelectric emission is possible even if the intensity is large. This minimum, cut-off frequency V_0, is called the **threshold frequency**. It is different for different metals.

Note: If frequency of the incident radiation exceeds the threshold frequency, the photoelectric emission starts instantaneously without any apparent time lag, even if the incident radiation is very dim. It is now known that emission starts in a time of the order of 10– 9 s or less.

Photoelectric effect and Wave theory of light

The wave picture of light cannot explain the observations on photoelectric effect because-

- According to the wave picture of light, the free electrons at the surface of the metal (over which the beam of radiation falls) absorb the radiant energy continuously. The greater the intensity of radiation, the greater are the amplitude of electric and magnetic fields.
- Consequently, the greater the intensity, the greater should be the energy absorbed by each electron. In this picture, the maximum kinetic energy of the photoelectrons on the surface is then expected to increase with increase in intensity.
- Also, no matter what the frequency of radiation is, a sufficiently intense beam of radiation (over sufficient time) should be able to impart enough energy to the electrons, so that they exceed the minimum energy needed to escape from the metal surface. A threshold frequency, therefore, should not exist.
- In the wave picture, the absorption of energy by electron takes place continuously over the entire wavefront of the radiation. Since a large number of electrons absorb energy, the energy absorbed per electron per unit time turns out to be small. Explicit calculations estimate that it can take hours or more for a single electron to pick up sufficient energy to overcome the work function and come out of the metal.

But these points contradict the observations we saw in the previous part, therefore we cannot explain photoelectric effect using the wave model of light.

Einstein's Photoelectric equation: Energy Quantum of Radiation

Albert Einstein proposed a radically new picture of electromagnetic radiation to explain photoelectric effect. In this picture-

- photoelectric emission does not take place by continuous absorption of energy from radiation. Radiation energy is built up of discrete units called quanta of energy of radiation
- Each quantum of radiant energy has energy hν, where h is Planck's constant and ν the frequency of light.
- In photoelectric effect, an electron absorbs a quantum of energy (hν) of radiation. If this quantum of energy absorbed exceeds the minimum energy needed for the electron to escape from the metal surface (work function ϕ_o), the electron is emitted with maximum kinetic energy-

$$\boxed{K_{max} = h\nu - \phi_0}$$

This equation is known as **Einstein's Photoelectric equation**

Inferences-

1. K_{max} depends linearly on ν, and is independent of intensity of radiation. This is because photoelectric effect arises from the absorption of a single quantum of radiation by a single electron and intensity of light doesn't affect this process

2. Since K_{max} must be non-negative, this implies that photoelectric emission is possible only if-

$$h\nu > \phi_0$$
$$\text{or} \quad \nu > \nu_0, \text{where}$$
$$\boxed{\nu_0 = \frac{\phi_0}{h}}$$

This shows that the greater the work function ϕ_o, the higher the minimum or threshold frequency ν_o needed to emit photoelectrons. Thus, there exists a threshold frequency ν_o (= ϕ_o/h) for the metal surface, below which no photoelectric emission is possible, no matter how intense the incident radiation may be or how long it falls on the surface.

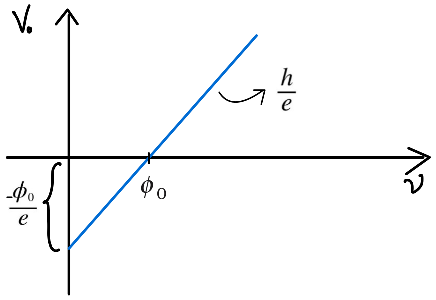
3. Intensity of radiation is proportional to the number of energy quanta per unit area per unit time. The greater the number of energy quanta available, the greater is the number of electrons absorbing the energy quanta and greater, therefore, is the number of electrons coming out of the metal (for $\nu > \nu_o$). This explains why, for $\nu > \nu_o$, photoelectric current is proportional to intensity.

4. The basic elementary process involved in photoelectric effect is the absorption of a light quantum by an electron. This process is instantaneous. Thus, whatever may be the intensity i.e., the number of quanta of radiation per unit area per unit time, photoelectric emission is instantaneous.

5. Relation with stopping potential-

$$e V_0 = h\nu - \phi_0; \text{ for } \nu \geq \nu_0$$

or $$\boxed{V_0 = \left(\frac{h}{e}\right)\nu - \frac{\phi_0}{e}}$$



This predicts that the V_0 versus ν curve is a straight line with slope = (h/e), independent of the nature of the material.

Important PYQs 

Ques: Photons of energy 1eV and 2eV are successively incident on a metal surface of work function 0.5 eV. The ratio of kinetic energy of the most energetic photon in both cases will be? **(PYQ 2020) [1M]**

a) 1:2
b) 1:1
c) 1:3
d) 1:4

Ans: We know-

$$K_{max} = h\nu - \phi_0$$

Therefore-

$$K_1 = 1 - 0\cdot 5$$
$$= 0.5\, eV$$

$$K_2 = 2 - 0\cdot 5$$
$$= 1\cdot 5\, eV$$

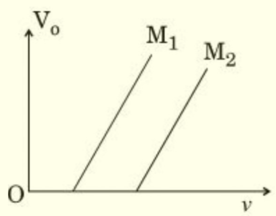
So,

$$\frac{k_1}{k_2} = \frac{0.5}{1.5} = 1{:}3$$

Ques: The variation of stopping potential with frequency of incident light is on two different photosensitive materials M_1 and M_2 is shown in the graph below. Identify the surface with greater value of work function. **(PYQ 2020) [1M]**

Ans: We know that x-intercept of this graph gives the value of the work function. Therefore, the graph with bigger x intercept will have greater work function. I.e-

$$M_2 > M_1$$



Ques: Find the frequency of light which ejects electrons from a metal surface, fully stopped by a retarding potential of 3.3 V. If photo electric emission begins in this metal at a frequency of 8 x 10^{14} Hz, calculate the work function (in eV) for this metal. **(PYQ 2018) [2M]**

Ans:

$$\phi_0 = h\nu_0$$
$$\phi_0 = 6.62 \times 10^{-34} \times 8 \times 10^{14}$$
$$\phi_0 = 3.3\ \ eV$$

We know,

$$eV_0 = h\nu - \phi_0$$

$$\nu = \frac{eV_0 + \phi_0}{h}$$

$$\nu = 1 \cdot 3 \times 10^{15}\text{Hz}$$

Ques: Monochromatic light of frequency 6.0 x 10^{14} Hz is produced by a laser. The power emitted is 2.0 x 10^{-3} W. Calculate the (i) energy of a photon in the light beam and (ii) number of photons emitted on an average by the source. **(PYQ 2018) [2M]**

Ans: 1) $E = h\nu$

$$E = 6 \cdot 62 \times 10^{-34} \times 6 \times 10^{14}$$

$$E = 2 \cdot 5 \quad eV$$

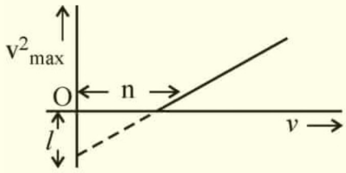
2) $P = nE$ (Where n is number of photons emitted per second)

$$n = \frac{P}{E}$$

$$n = \frac{2 \times 10^{-3}}{6.62 \times 10^{-34} \times 6 \times 10^{14}}$$

$$n = 5 \times 10^{15}$$

Ques: Light of frequency ν is incident on a photosensitive surface. A graph of the square of the maximum speed of the electrons (v^2_{max}) vs. ν is obtained as shown in the figure. Using Einstein's photoelectric equation, obtain expressions for (i) Planck's constant (ii) work function of the given photosensitive material in terms of parameters *l*, n and mass of the electron m. **(PYQ 2018) [2M]**



Ans: We know-

$$\frac{1}{2}mv^2 = h\nu - \phi_0$$

$$v^2 = \left(\frac{2h}{m}\right)\nu - \frac{2\phi_0}{m}$$

Comparing equation to graph -

I) $\frac{2\phi_0}{m} = l$

$$\phi_0 = \frac{\text{ml}}{2}$$

ii) when ν is 0-

$$0 = \left(\frac{2h}{m}\right)n - \frac{2\phi_0}{m}$$

$$h = \frac{\phi_0}{n}$$

$$h = \frac{\text{ml}}{2n}$$

Ques: The maximum kinetic energy of the photoelectrons gets doubled when the wavelength of light incident on the surface changes from λ_1 to λ_2. Derive the expressions for the threshold wavelength λ_0, and work function for the metal surface. **(PYQ 2015) [3M]**

Ans: $K_1 = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_0}$

$$K_2 = \frac{hc}{\lambda_2} - \frac{hc}{\lambda_0}$$

ATQ-

$$K_2 = 2K_1$$

$$\frac{2hc}{\lambda_1} - \frac{2hc}{\lambda_0} = \frac{hc}{\lambda_2} - \frac{hc}{\lambda_0}$$

$$\frac{2}{\lambda_1} - \frac{1}{\lambda_2} = \frac{2}{\lambda_0} - \frac{1}{\lambda_0}$$

$$\lambda_0 = \frac{\lambda_1\lambda_2}{2\lambda_2 - \lambda_1}$$

$$\phi_0 = \frac{hc}{\lambda_0}$$

$$\phi_0 = \frac{hc(2\lambda_2 - \lambda_1)}{\lambda_2\lambda_1}$$

Particle Nature of Light: The Photon

In the particle picture of light-

1. In interaction of radiation with matter, radiation behaves as if it is made up of particles called photons.

2. Each photon has energy E (=hν) and momentum p (= h ν/c), and speed c, the speed of light.

3. All photons of light of a particular frequency ν, or wavelength λ, have the same energy E (=hν = hc/λ) and momentum p (= hν/c = h/λ), whatever the intensity of radiation may be. By increasing the intensity of light of given wavelength, there is only an increase in the number of photons per second crossing a given area, with each photon having the same energy. Thus, photon energy is independent of intensity of radiation.

4. Photons are electrically neutral and are not deflected by electric and magnetic fields.

5. In a photon-particle collision (such as photon-electron collision), the total energy and total momentum are conserved. However, the number of photons may not be conserved in a collision. The photon may be absorbed or a new photon may be created.

Wave nature of matter

Louis Victor de Broglie put forward the bold hypothesis that moving particles of matter should display wave-like properties under suitable conditions. He proposed that the wave length λ associated with a particle of momentum p is given as-

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Where m is the mass of the particle and v is its speed. This equation is called the de Broglie relation and the wavelength of matter waves is called de Broglie wavelength.
This relation is also satisfied by a photon. For a photon-

$$p = \frac{h\nu}{c}$$

$$\lambda = \frac{h}{p} = \frac{c}{\nu}$$

That is, the de Broglie wavelength of a photon equals the wavelength of electromagnetic radiation of which the photon is a quantum of energy and momentum.
Note: The wavelength for everyday objects for e.g., a ball is extremely small. Therefore, the macroscopic objects around us do not show wave nature.

Consider an electron (mass m, charge e) accelerated from rest through a potential V. The kinetic energy K of the electron equals the work done (eV) on it by the electric field:

$$K = eV$$

Also-

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

Therefore-

$$p = \sqrt{2mk} = \sqrt{2meV}$$

Therefore, the de Broglie wavelength is-

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mk}} = \frac{h}{\sqrt{2meV}}$$

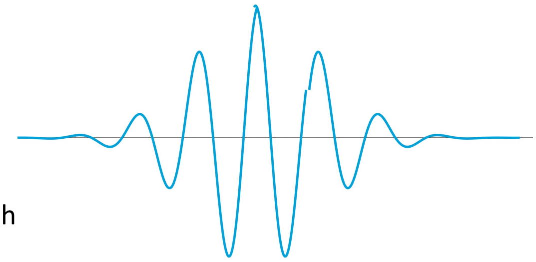
Substituting values of h, m and e

$$\lambda = \frac{1.227}{\sqrt{V}} nm$$

Heisenberg's Uncertainty Principle

According to the principle, it is not possible to measure both the position and momentum of an electron (or any other particle) at the same time exactly. Let uncertainty in measuring position be Δx and that in measuring momentum be Δp. Then-

$$\boxed{\Delta x \Delta p \approx \hbar} \qquad \left[\hbar = \frac{h}{2\times\pi}\right]$$



Uncertainty principle and de Broglie's hypothesis-

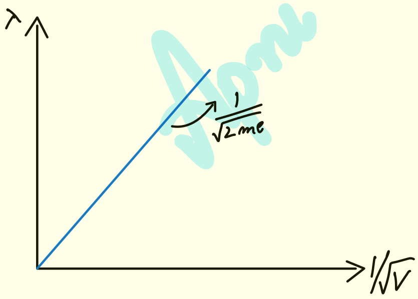
If an electron has a definite momentum p, (i.e., Δp = 0), by the de Broglie relation, it has a definite wavelength λ. A wave of definite (single) wavelength extends all over space. By Born's probability interpretation this means that the electron is not localised in any finite region of space. That is, its position uncertainty is infinite (Δx → ∞), which is consistent with the uncertainty principle.

Note: In general, the matter wave associated with the electron is not extended all over space. It is a wave packet extending over some finite region of space. In that case Δx is not infinite but has some finite value depending on the extension of the wave packet.

Important PYQs

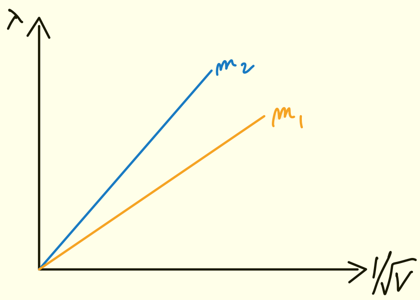
Ques: Plot a graph showing the variation of de Broglie wavelength (λ) of a charged particle of mass m, versus 1/√V, where V is the potential difference through which the particle is accelerated. How does this graph give us information about the magnitude of the charge on the particle? **(PYQ 2019) [2M]**

Ans: $\lambda = \frac{h}{\sqrt{2meV}}$ The greater the slope, the less is the charge on the particle



Ques: Plot a graph showing variation of de-Broglie wavelength λ versus 1/√V where V is accelerating potential for two particles A and B carrying same charge but of masses m_1, m_2, ($m_1 > m_2$). Which one of the two represents a particle of smaller mass and why? **(PYQ 2016) [2M]**

Ans: $\lambda = \frac{h}{\sqrt{2meV}}$ The greater the slope, the less is the mass of the particle



Ques: A proton and an a-particle have the same de-Broglie wavelength. Determine the ratio of their accelerating potentials and (ii) their speeds. **(PYQ 2015) [2M]**

Ans: I) $\lambda_p = \dfrac{h}{\sqrt{2meV_p}}$

$$\lambda_\alpha = \frac{h}{\sqrt{2\times 4m\times 2\times eV_\alpha}}$$

$$\frac{V_p}{V_\alpha} = 2\times\sqrt{2}$$

ii) $\lambda_p = \dfrac{h}{\sqrt{2m\left(\frac{1}{2}mv_p^2\right)}}$

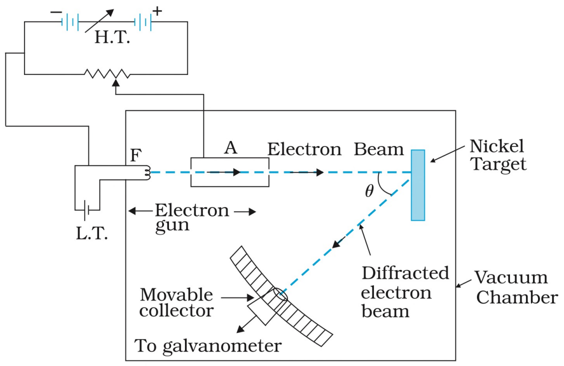
$$\lambda_\alpha = \frac{h}{\sqrt{2\times(4m)\times\left(\frac{1}{2}\times(4m)v_\alpha^2\right)}}$$

$$\frac{v_p}{v_\alpha} = 2$$

Davisson and Germer Experiment

Aim

This experiment verified the wave nature of electrons.



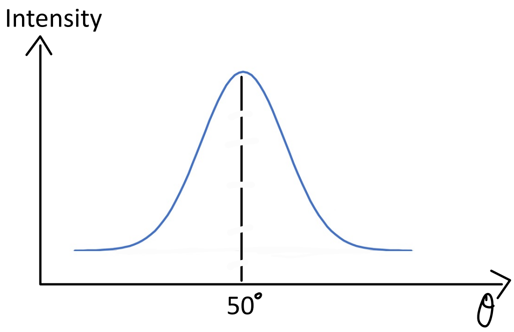
Apparatus and working

1. An electron gun which comprises of a tungsten filament F, coated with barium oxide is heated by a low voltage power supply and emits electrons which are accelerated by using high voltage

2. They are made to pass through a cylinder with fine holes along its axis, producing a fine beam. The beam is made to fall on the surface of a nickel crystal. The electrons are scattered in all directions by the atoms of the crystal.
3. The intensity of the electron beam, scattered in a given direction, is measured by the electron detector (collector).
4. The detector can be moved on a circular scale and is connected to a sensitive galvanometer, which records the current. The deflection of the galvanometer is proportional to the intensity of the electron beam entering the collector

Observations

The variation of the intensity (I) of the scattered electrons with the angle of scattering θ is obtained for different accelerating voltages. It was noticed that a strong peak appeared in the intensity (I) of the scattered electron for an accelerating voltage of 54V at a scattering angle θ = 50°



Calculations

From the electron diffraction measurements, the wavelength of matter waves was found to be 0.165 nm.

Form de Broglie's relation-

$$\lambda = \frac{12\cdot 27}{\sqrt{V}}A^\circ$$

$\lambda = 1.66A^\circ$ (For v= 54 V)

Result

There is an excellent agreement between the theoretical value and the experimentally obtained value of de Broglie wavelength. Davisson Germer experiment thus strikingly confirms the wave nature of electrons and the de Broglie relation

Note: The appearance of the peak in a particular direction is due to the constructive interference of electrons scattered from different layers of the regularly spaced atoms of the crystals.

Definitions and Derivations asked as PYQs 

Ques: Define the term 'threshold frequency' in terms of photoelectric emission? **(PYQ 2019) [1M]**

Ques: Define the term 'Intensity' in photon picture of electromagnetic radiation **(PYQ 2019) [1M]**

Ques: Why is wave theory of electromagnetic radiation not able to explain photoelectric effect? How does photon picture resolve this problem? **(PYQ 2019) [2M]**

Ques: State Einstein's photoelectric equations **(PYQ 2018) [1M]**

Ques: Using photon picture of light, show how Einstein's photoelectric equation can be established. Write two features of photoelectric effect which cannot be explained by wave theory. **(PYQ 2017, 2016) [3M]**

Ques: Write Einstein's Photoelectric equation and mention which important features in photoelectric effect can be explained with the help of this equation. **(PYQ 2015, 2012) [2M]**

Ques: Why photoelectric effect can not be explained on the basis of wave nature of light? Give reasons **(PYQ 2013) [2M]**