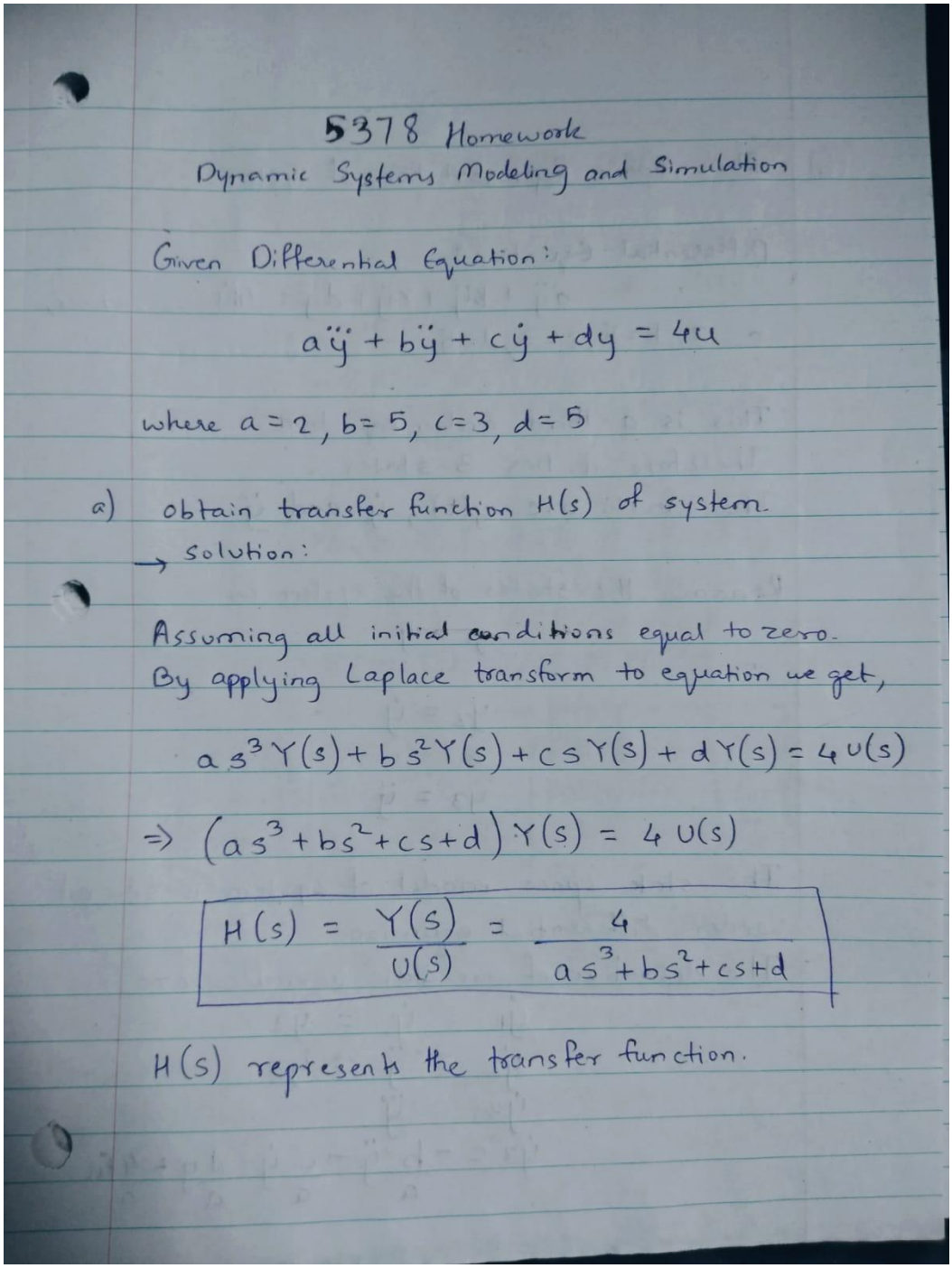
Introduction to Unmanned Vehicle Systems

Fall 2019

Dynamical Systems Modeling and Simulation Homework

A. Transfer Function



5378 Homework

Dynamic Systems Modeling and Simulation

Given Differential Equation:

$$a\dddot{y} + b\ddot{y} + c\dot{y} + dy = 4u$$

where $a = 2, b = 5, c = 3, d = 5$

a) obtain transfer function $H(s)$ of system.

→ solution:

Assuming all initial conditions equal to zero.

By applying Laplace transform to equation we get,

$$as^3Y(s) + bs^2Y(s) + csY(s) + dY(s) = 4U(s)$$

$$\Rightarrow (as^3 + bs^2 + cs + d)Y(s) = 4U(s)$$

$$\boxed{H(s) = \frac{Y(s)}{U(s)} = \frac{4}{as^3 + bs^2 + cs + d}}$$

$H(s)$ represents the transfer function.

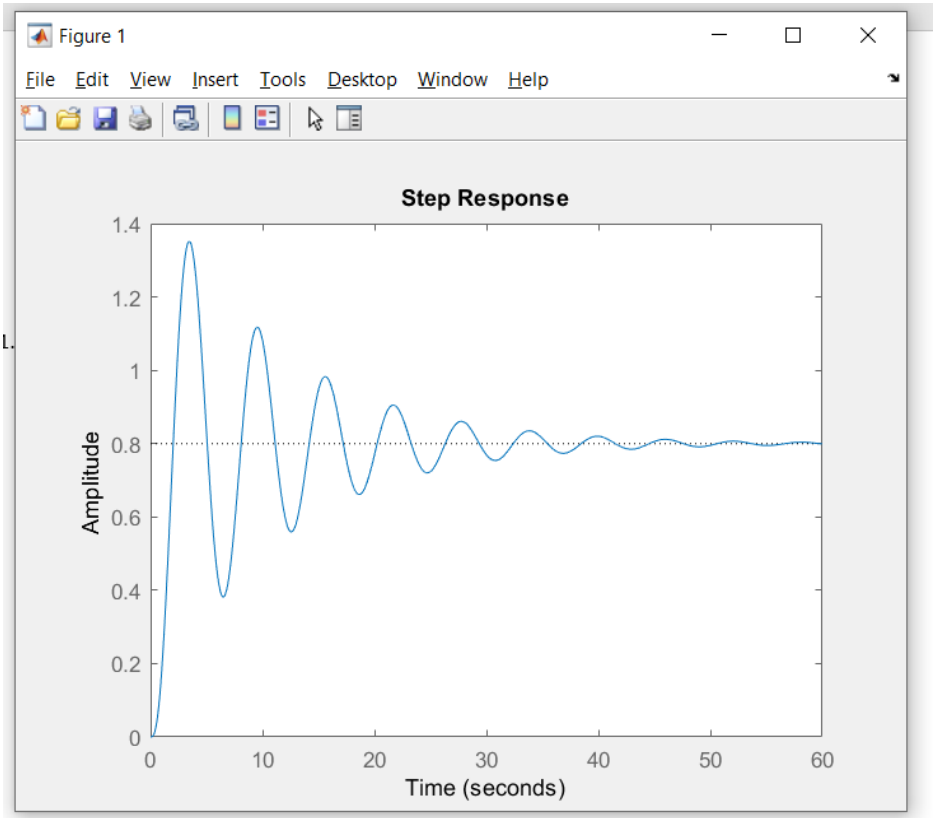
- **Transfer function simulation in Matlab:**

Editor - C:\Users\lalit\Documents\MS CS\Sem2\5383\matlabHW\transfuncHW.m

transfuncHW.m | statespaceHW.m | ssdyn.m | MATLAB Function*

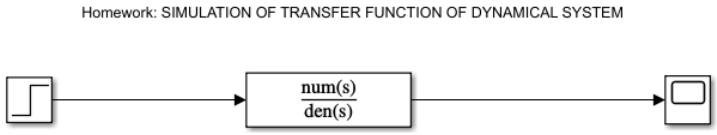
```
1   % Transfer Function of Dynamical System Homework
2   % H(s) = 4/(as^3 + bs^2 + cs + d)
3
4   a = 2;
5   b = 5;
6   c = 3;
7   d = 5;
8
9   num=4;
10  den=[a b c d];
11
12  tfsystm = tf(num, den);
13
14  stepplot(tfsystm)
15
```

Transfer Function Matlab Code

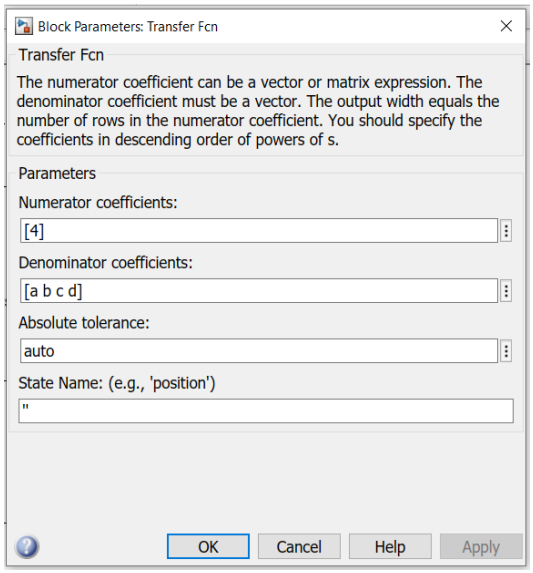


Matlab Transfer Function Resulting Plot

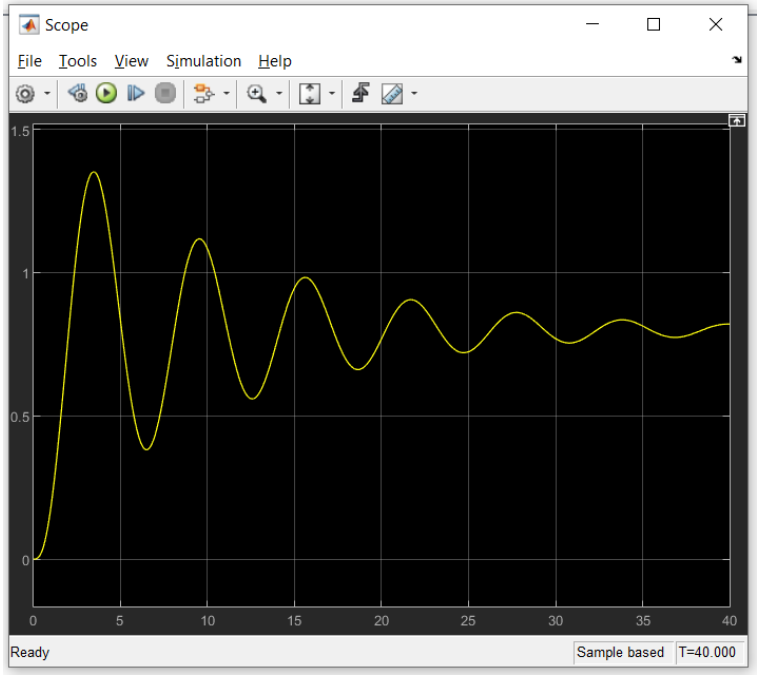
- **Transfer Function Simulation in Simulink**



Snapshot: Simulink block for Transfer Function of system.

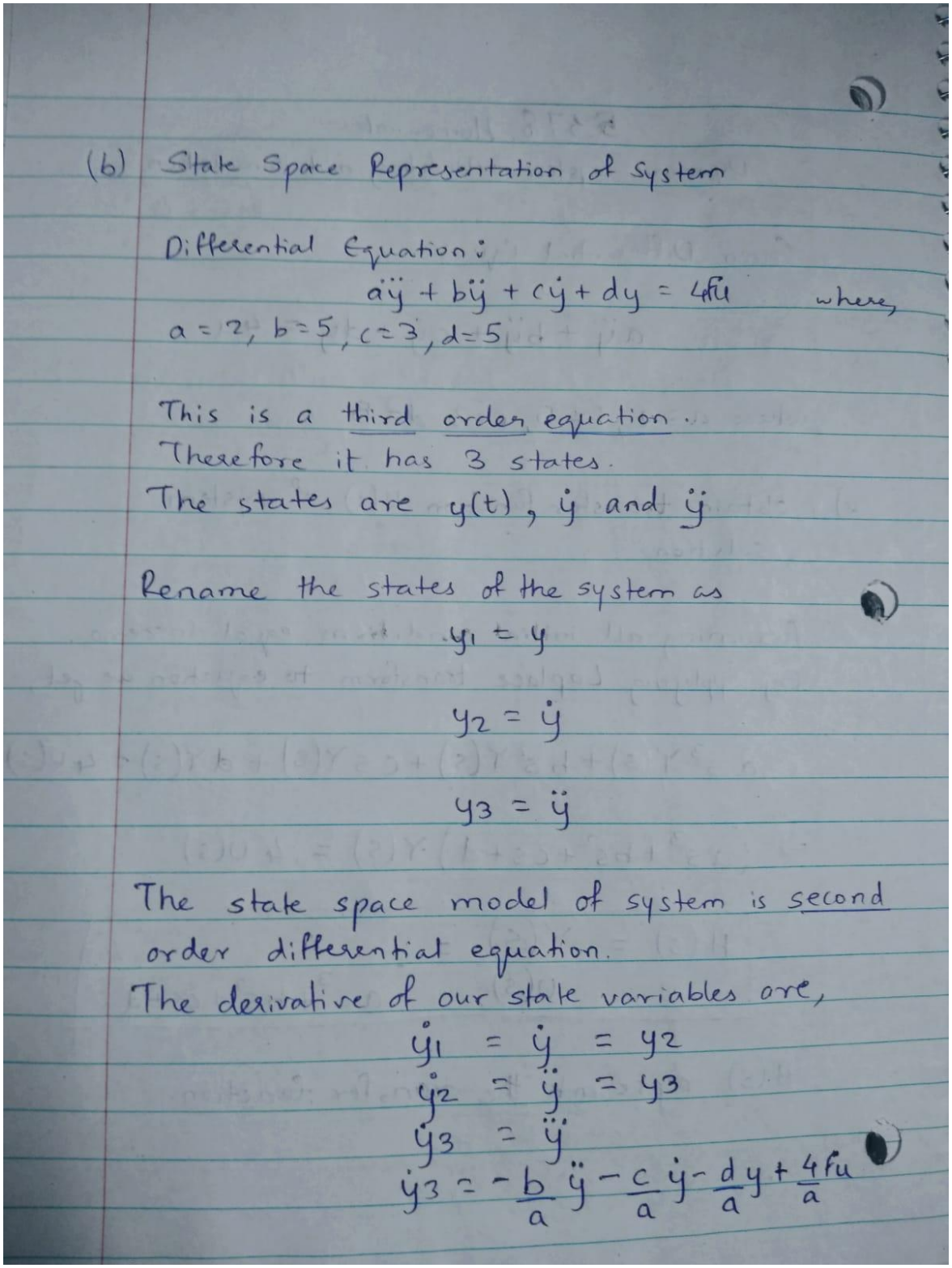


Block parameters for Transfer Function



Simulink Resulting Transfer Function Plot

B. State Space Representation:



(b) State Space Representation of System

Differential Equation:

$$a\dddot{y} + b\ddot{y} + c\dot{y} + dy = 4fu \quad \text{where,}$$

$a = 2,\ b = 5,\ c = 3,\ d = 5$

This is a third order equation.
Therefore it has 3 states.
The states are $y(t)$, $\dot{y}$ and $\ddot{y}$

Rename the states of the system as

$$y_1 = y$$

$$y_2 = \dot{y}$$

$$y_3 = \ddot{y}$$

The state space model of system is second order differential equation.
The derivative of our state variables are,

$$\dot{y}_1 = \dot{y} = y_2$$

$$\dot{y}_2 = \ddot{y} = y_3$$

$$\dot{y}_3 = \dddot{y}$$

$$\dot{y}_3 = -\frac{b}{a}\ddot{y} - \frac{c}{a}\dot{y} - \frac{d}{a}y + \frac{4fu}{a}$$

We have the derivative of our state variables as

$$\boxed{\dot{y}_1 = \dot{y} = y_2}$$

$$\boxed{\dot{y}_2 = \ddot{y} = y_3}$$

$$\dot{y}_3 = \dddot{y} = -\frac{b}{a}\ddot{y} - \frac{c}{a}\dot{y} - \frac{d}{a}y + \frac{4}{a}Fu$$

$$\boxed{\dot{y}_3 = \dddot{y} = -\frac{b}{a}y_3 - \frac{c}{a}y_2 - \frac{d}{a}y_1 + \frac{4}{a}Fu}$$

We can express this model in matrix form as

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{d}{a} & -\frac{c}{a} & -\frac{b}{a} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{4}{a} \end{bmatrix} Fu$$

A linear system can be expressed as

$$\dot{y} = Ay + Bu$$

or $\dot{y} = f(y,u)$

or $\dot{y} = f(y) + g(y)u$

- **State Space Simulation in Matlab**

Editor - C:\Users\lalit\Documents\MS CS\Sem2\5383\matlabHW\statespaceHW.m

transfuncHW.m | statespaceHW.m | ssdyn.m | MATLAB Function*

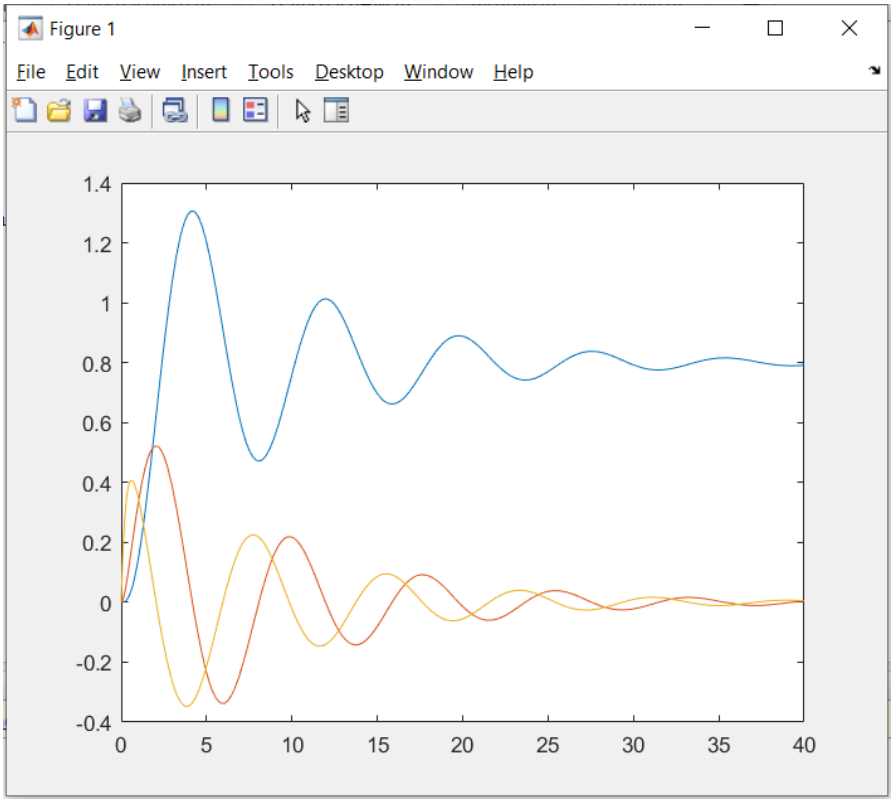
```
1    %State space representation model Homework
2
3 -  time=[0 40];
4
5 -  y0=[0;0;0];
6
7 -  [t,y]=ode45(@ssdyn,time,y0);
8
9 -  plot(t,y)
```

State Space Modeling Matlab Code.

Editor - C:\Users\lalit\Documents\MS CS\Sem2\5383\Class Sept 23\ssdyn.m

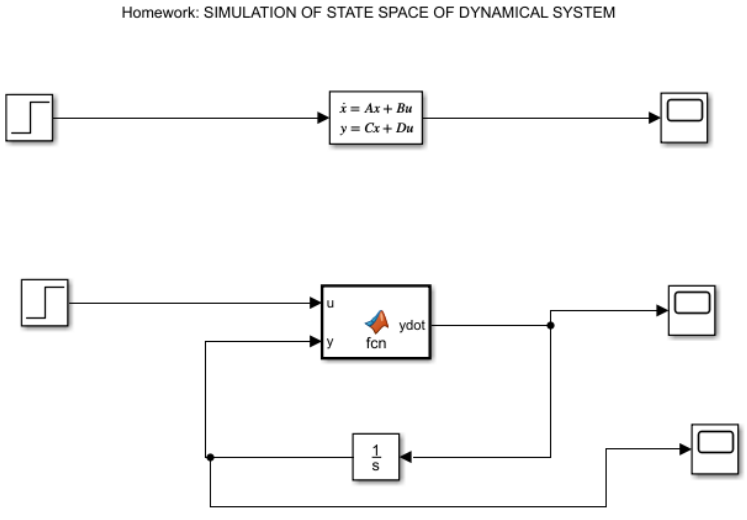
transfuncHW.m | statespaceHW.m | ssdyn.m | MATLAB Function*

```
1    function ydot=ssdyn(t,y)
2 -  a=2;
3 -  b=8;
4 -  c=3;
5 -  d=5;
6
7 -  A=[0 1 0; 0 0 1; -d/a -c/a -b/a];
8 -  B=[0;0;4/a];
9
10 - u=1;
11
12 - ydot=A*y+B*u;
```



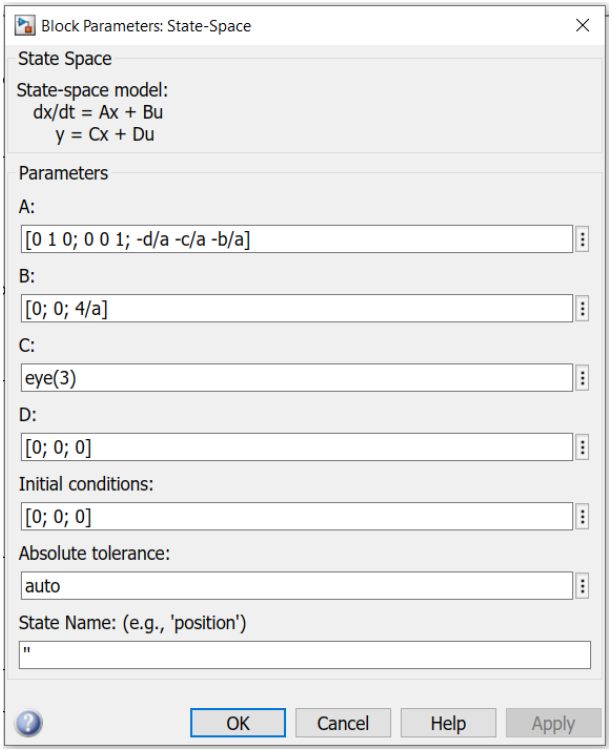
State Space Matlab Resulting Plot

- **State Space Simulation in Simulink**

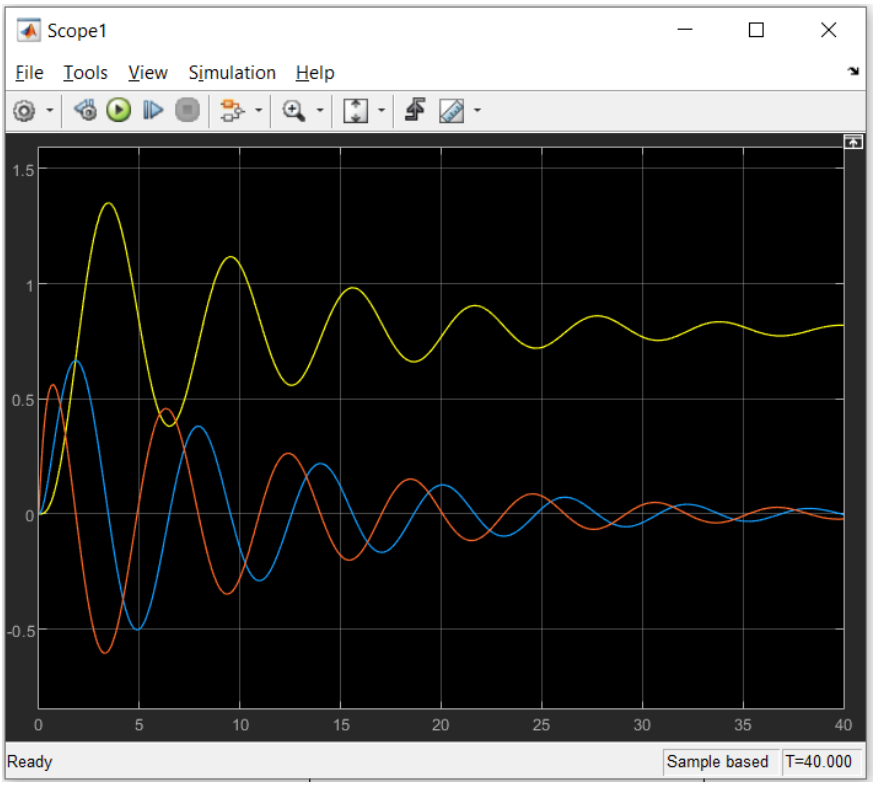


Simulink State Space Block

- **All initial conditions equal to zero.**



State Space Block parameters at 0 initials



Simulink State Space Resulting Plot at 0

C. State Space Representation:

- **State Space Simulation in Matlab**

Editor - C:\Users\lalit\Documents\MS CS\Sem2\5383\matlabHW\statespaceHW.m

transfuncHW.m | statespaceHW.m | ssdyn.m | MATLAB Function*

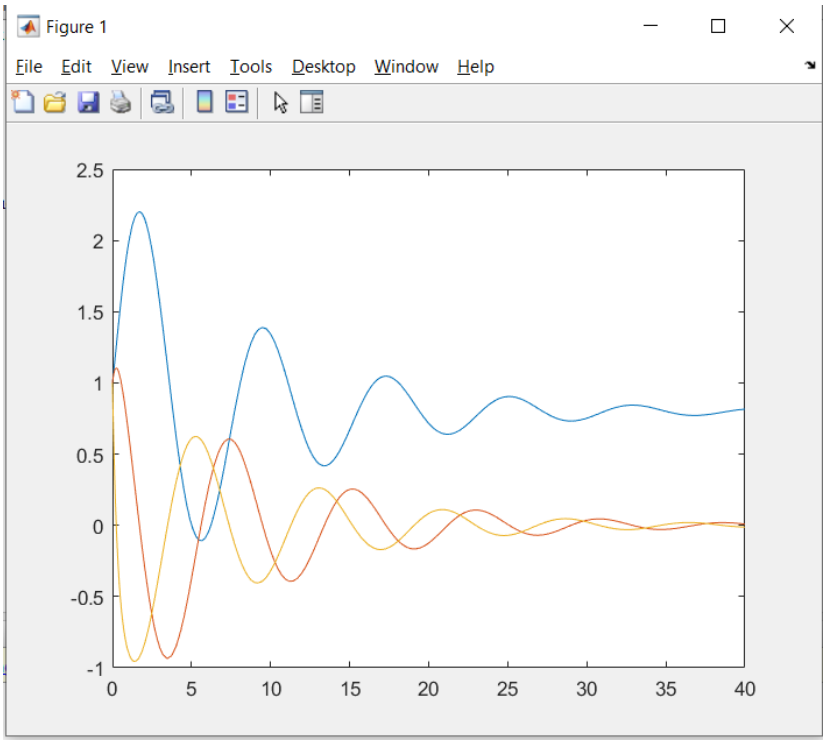
```
1    %State space representation model Homework
2
3 -  time=[0 40];
4
5 -  y0=[1;1;1];
6
7 -  [t,y]=ode45(@ssdyn,time,y0);
8
9 -  plot(t,y)
```

State Space Matlab Code for all initial condition at 1

Editor - C:\Users\lalit\Documents\MS CS\Sem2\5383\Class Sept 23\ssdyn.m

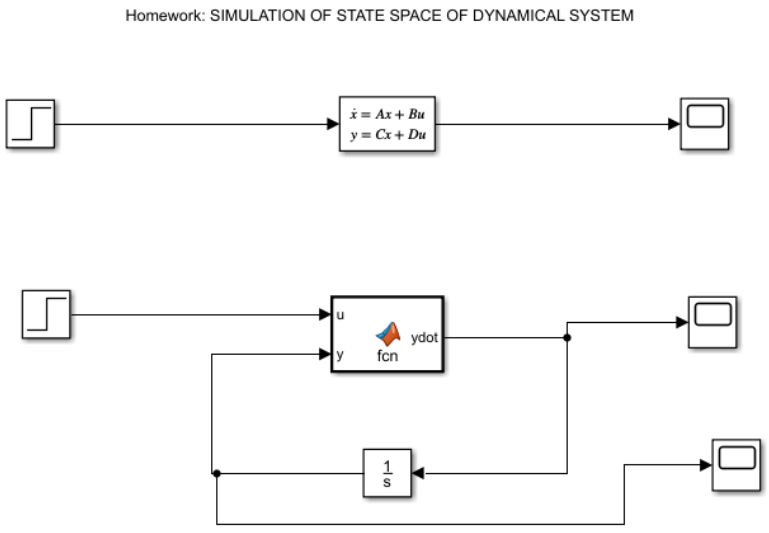
transfuncHW.m | statespaceHW.m | ssdyn.m | MATLAB Function*

```
1    function ydot=ssdyn(t,y)
2 -  a=2;
3 -  b=8;
4 -  c=3;
5 -  d=5;
6
7 -  A=[0 1 0; 0 0 1; -d/a -c/a -b/a];
8 -  B=[0;0;4/a];
9
10 - u=1;
11
12 - ydot=A*y+B*u;
```



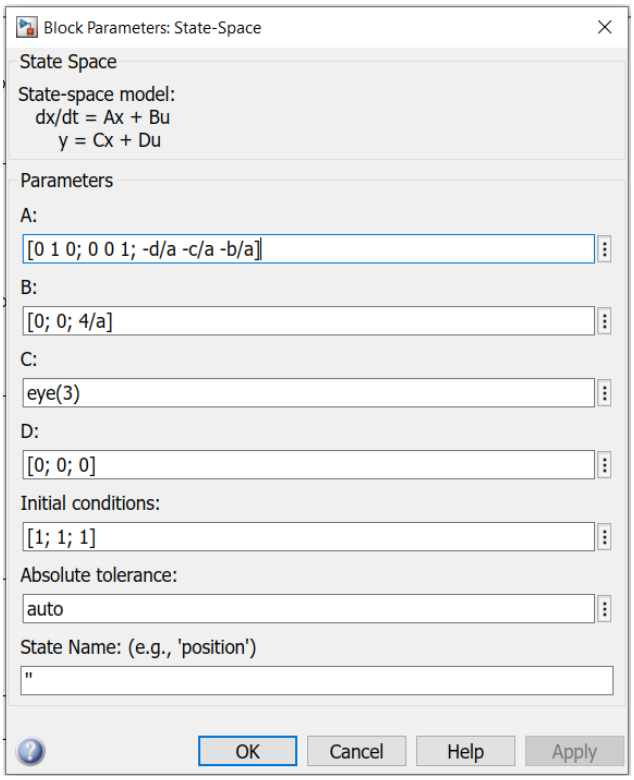
Matlab State Space Resulting Plot at 1

- **State Space Simulation in Simulink**

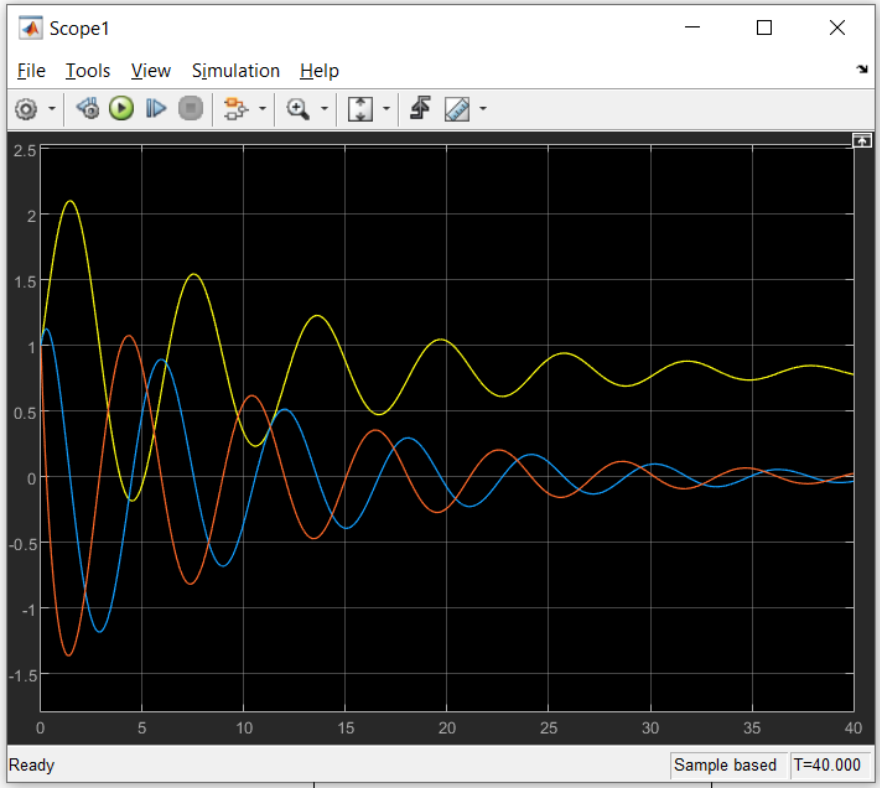


Simulink State Space Block

- **All initial conditions equal to one.**



State Space Block parameters at 1 initials



Simulink State Space Resulting Plot at 1

- **Simulink State Space Modeling using User Defined Block**

Matlab Function Code for user defined State Space block: