Brief Introduction to PID Controllers

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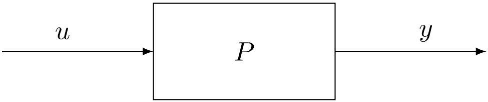
Open-loop System

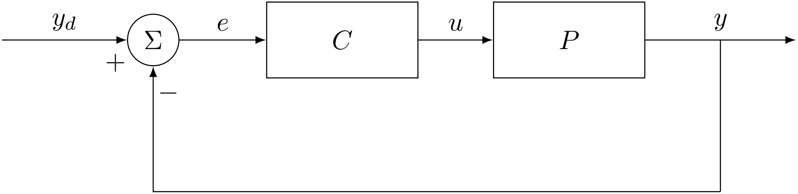
The plant (P) is the dynamical system to be controlled.



If we provide an arbitrary control input $u(t)$, we observe a corresponding output (y).

Closed-loop Systems

A **feedback controller** measures the state of a system and applies the appropriate input to correct the deviation of the output with respect to a desired value



Closed-loop Systems

- y_d represents the desired trajectory for the output of the system.
- The error signal $e(t)$ is defined as

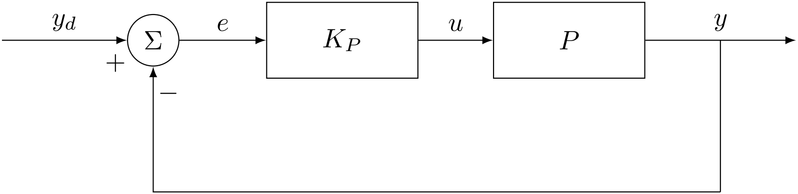
$$e(t) = y_d(t) - y(t)$$

- Many control methods with different characteristics exist in the literature. The PID controller is one of the most used in the industry.

Proportional Control

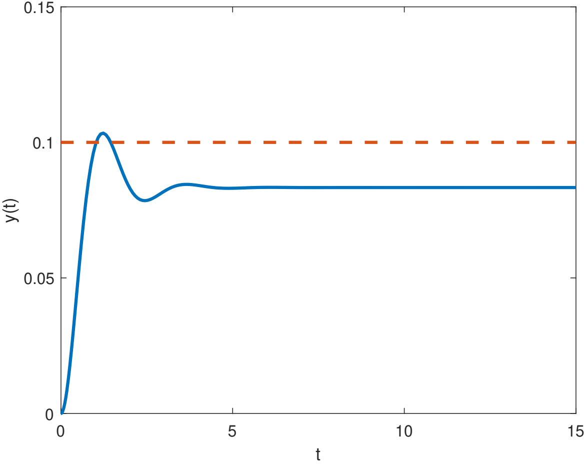
Let the control input be the error signal amplified by a constant gain

$$u(t) = K_P e(t)$$



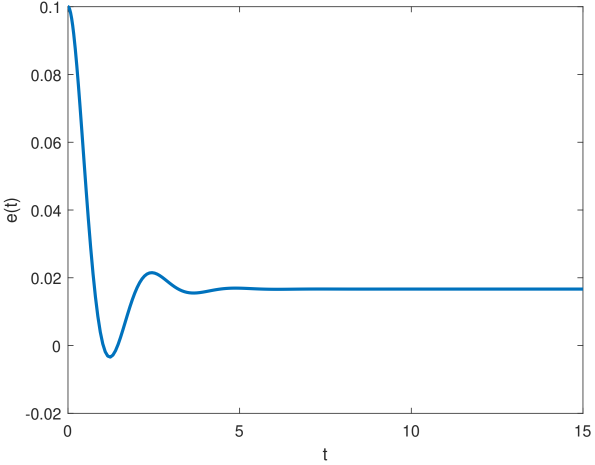
Example: Proportional Control

For a given system, use $K_P = 20$.



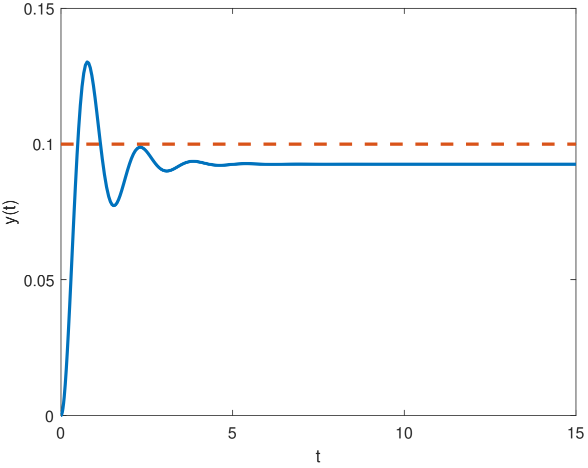
Example: Proportional Control

The error of the system is:



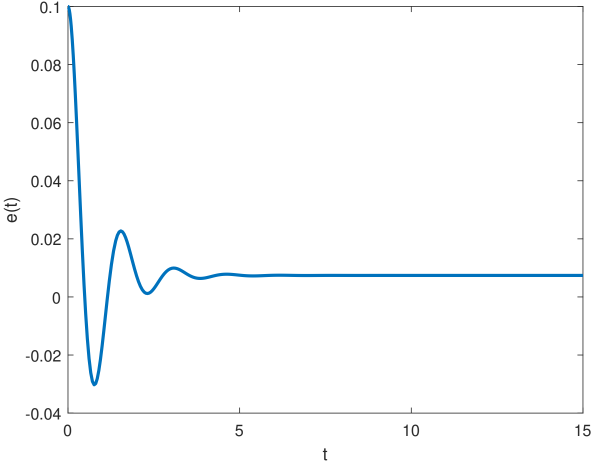
Example: Proportional Control 2

Now use $K_P = 50$.



Example: Proportional Control 2

The error signal is smaller. In many applications, we will not be able to eliminate the error with a proportional control.



Proportional-Integral (PI) Control

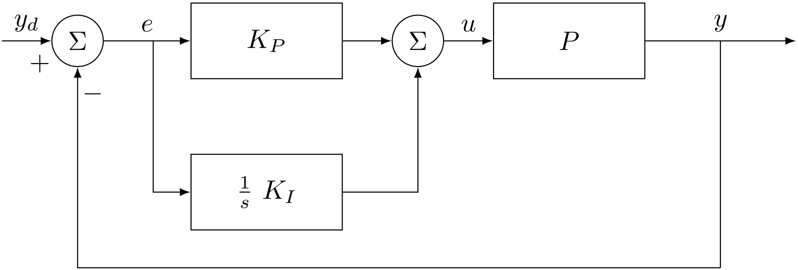
An integral control action is defined by

$$u_I(t) = \int_0^t e(\tau) d\tau$$

Add this integral term to the system controller to obtain

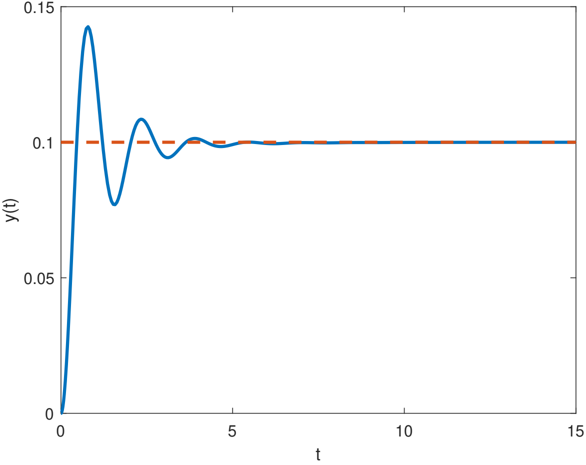
$$u(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau$$

PI Control



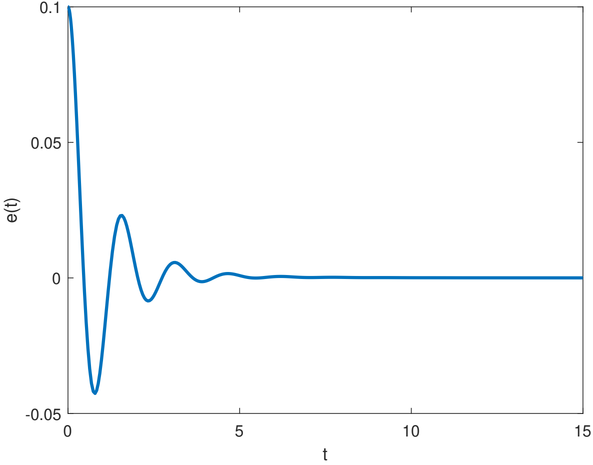
Example: PI Control

The PI controller eliminates the error in steady state



Example: PI Control

There is a large overshoot, and the error oscillates a lot.



Proportional-Integral-Derivative (PID) Control

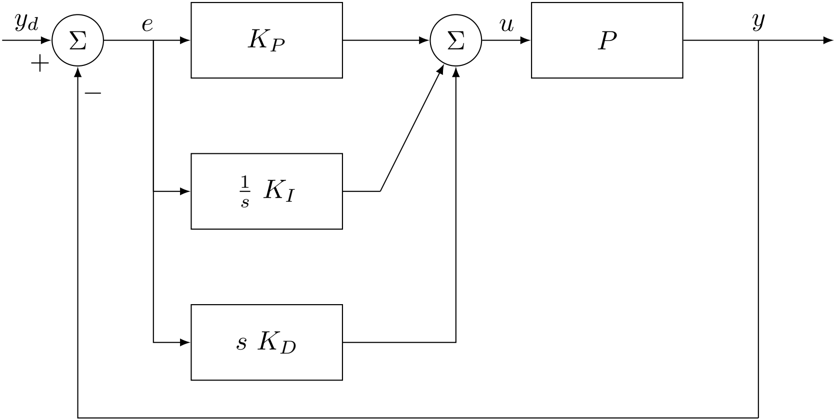
The derivative control action is given by

$$u_D(t) = \frac{de(t)}{dt}$$

This is the third term of our controller,

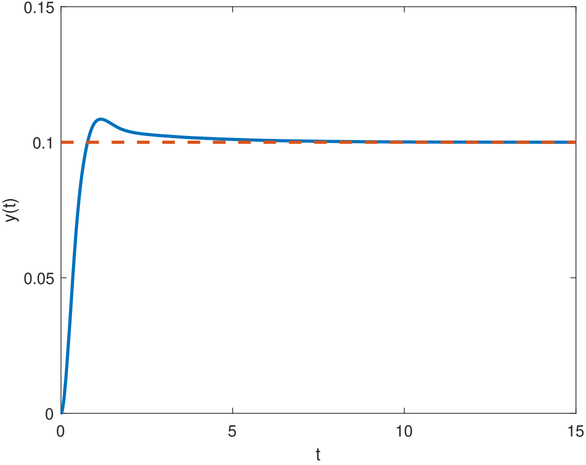
$$u(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau + K_D \frac{de(t)}{dt}$$

PID Control



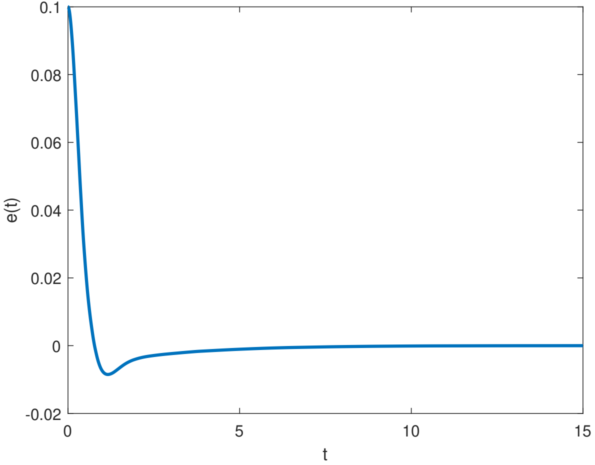
Example: PID Control

The performance of the controller is now satisfactory



Example: PID Control

Error signal



PID Control Characteristics

- The integral term guarantees a steady state error of zero by limiting the area below the error curve.
- The derivative term responds to the velocity of change of $e(t)$ and applies a corrective action before it becomes too large.
- We only need to select the gains K_P, K_I and K_D to obtain the desired performance of the closed-loop system.