

Introduction to Unmanned Vehicle Systems

Fall 2019

Dynamical Systems Modeling and Simulation Homework

A. Transfer Function

5378 Homework

Dynamic Systems Modeling and Simulation

Given Differential Equation:

$$a\ddot{y} + b\dot{y} + cy + dy = 4u$$

where $a=2$, $b=5$, $c=3$, $d=5$

a) obtain transfer function $H(s)$ of system.

→ solution:

Assuming all initial conditions equal to zero.

By applying Laplace transform to equation we get,

$$as^3Y(s) + bs^2Y(s) + csY(s) + dY(s) = 4U(s)$$

$$\Rightarrow (as^3 + bs^2 + cs + d)Y(s) = 4U(s)$$

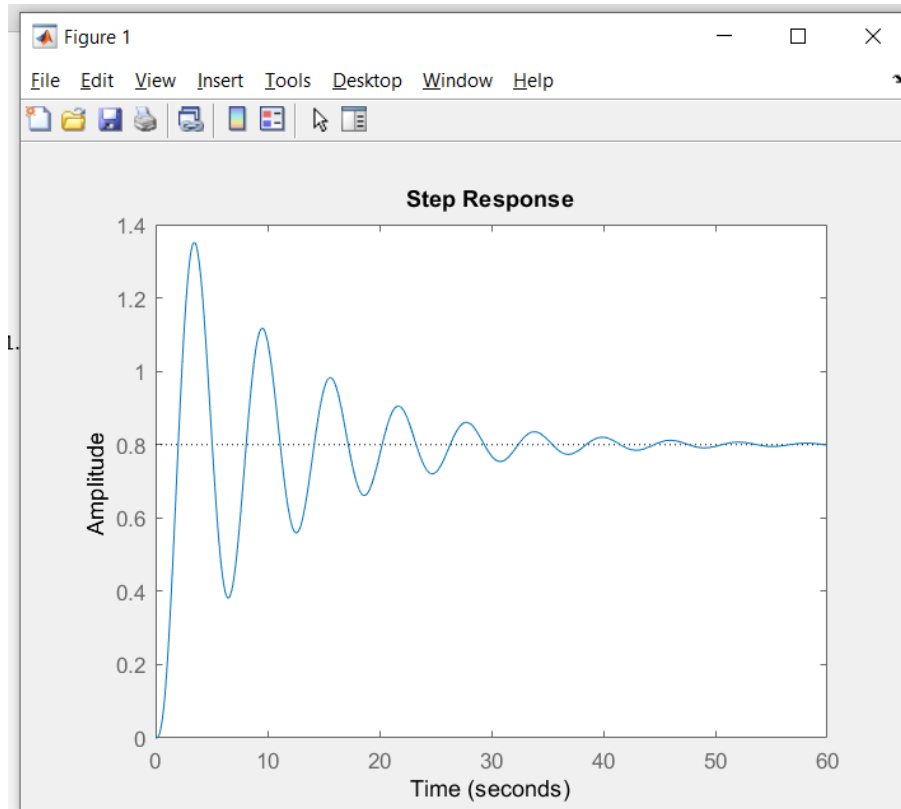
$$H(s) = \frac{Y(s)}{U(s)} = \frac{4}{as^3 + bs^2 + cs + d}$$

$H(s)$ represents the transfer function.

- **Transfer function simulation in Matlab:**

```
Editor - C:\Users\jalit\Documents\MS CS\Sem2\5383\matlabHW\transfuncHW.m
transfuncHW.m  statespaceHW.m  ssdyn.m  MATLAB Function*  +
1  % Transfer Function of Dynamical System Homework
2  % H(s) = 4/(as^3 + bs^2 + cs + d)
3
4  a = 2;
5  b = 5;
6  c = 3;
7  d = 5;
8
9  num=4;
10 den=[a b c d];
11
12 tfsystem = tf(num, den);
13
14 stepplot(tfsystem)
15
```

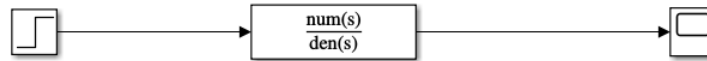
Transfer Function Matlab Code



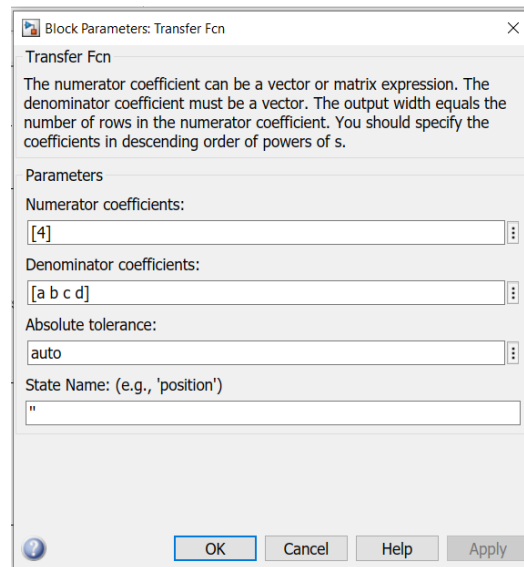
Matlab Transfer Function Resulting Plot

- **Transfer Function Simulation in Simulink**

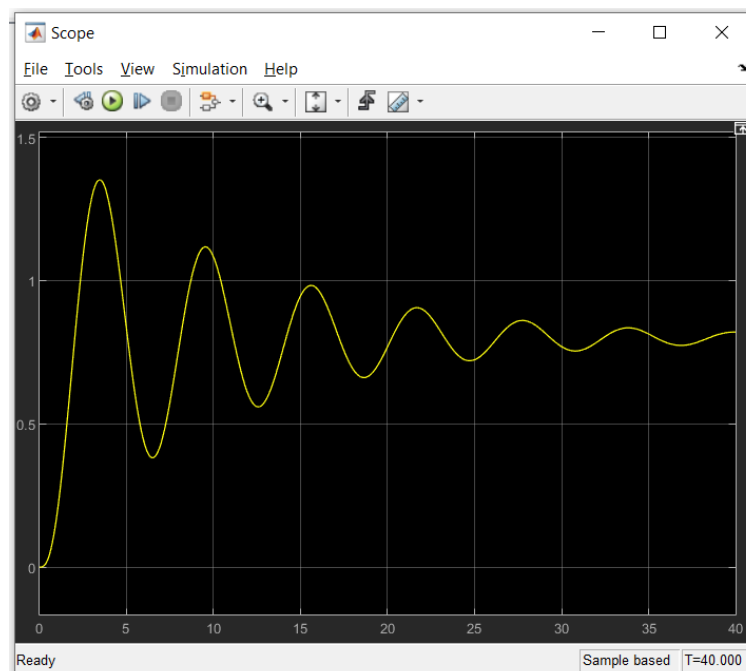
Homework: SIMULATION OF TRANSFER FUNCTION OF DYNAMICAL SYSTEM



Snapshot: Simulink block for Transfer Function of system.



Block parameters for Transfer Function



Simulink Resulting Transfer Function Plot

B. State Space Representation:

(b) State Space Representation of System

Differential Equation:

$$a\ddot{y} + b\dot{y} + cy + dy = 4fu \quad \text{where,}$$
$$a=2, b=5, c=3, d=5$$

This is a third order equation.

Therefore it has 3 states.

The states are $y(t)$, \dot{y} and \ddot{y}

Rename the states of the system as

$$y_1 = y$$

$$y_2 = \dot{y}$$

$$y_3 = \ddot{y}$$

The state space model of system is second order differential equation.

The derivative of our state variables are,

$$\dot{y}_1 = \dot{y} = y_2$$

$$\dot{y}_2 = \ddot{y} = y_3$$

$$\dot{y}_3 = \ddot{\ddot{y}}$$

$$\dot{y}_3 = -\frac{b}{a}\ddot{y} - \frac{c}{a}\dot{y} - \frac{d}{a}y + \frac{4f}{a}u$$

We have the derivative of our state variables as

$$\dot{y}_1 = \dot{y} = y_2$$

$$\dot{y}_2 = \ddot{y} = y_3$$

$$\dot{y}_3 = \ddot{\ddot{y}} = -\frac{b}{a}\ddot{y} - \frac{c}{a}\dot{y} - \frac{d}{a}y + \frac{4}{a}Fu$$

$$\dot{y}_3 = \ddot{\ddot{y}} = -\frac{b}{a}y_3 - \frac{c}{a}y_2 - \frac{d}{a}y_1 + \frac{4}{a}Fu$$

We can express this model in matrix form as

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{d}{a} & -\frac{c}{a} & -\frac{b}{a} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{4}{a} \end{bmatrix} Fu$$

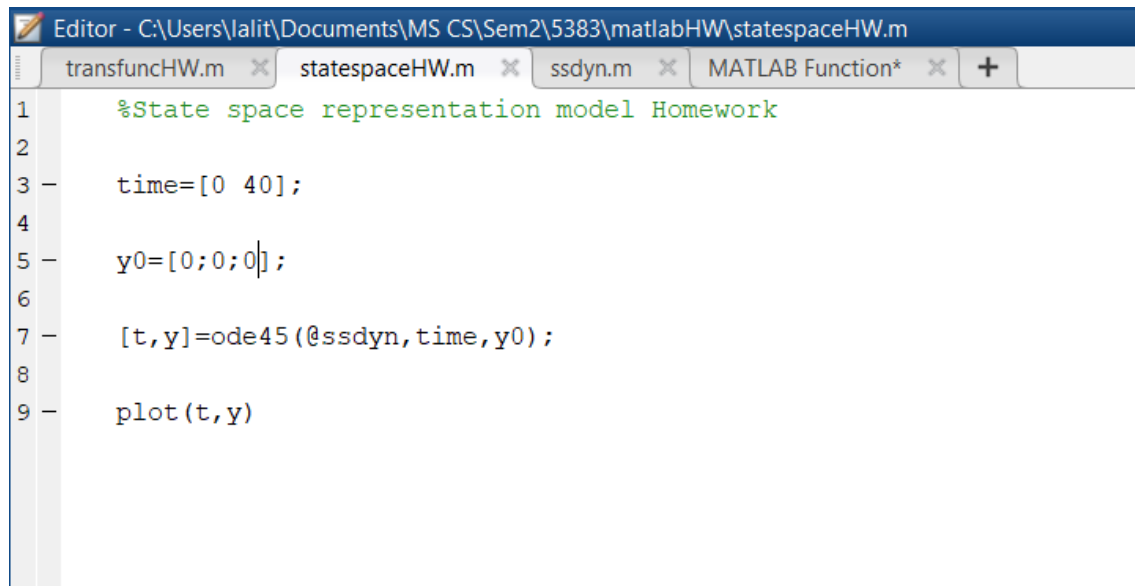
A linear system can be expressed as

$$\dot{y} = Ay + Bu$$

$$\text{or } \dot{y} = f(y, u)$$

$$\text{or } \dot{y} = f(y) + g(y)u$$

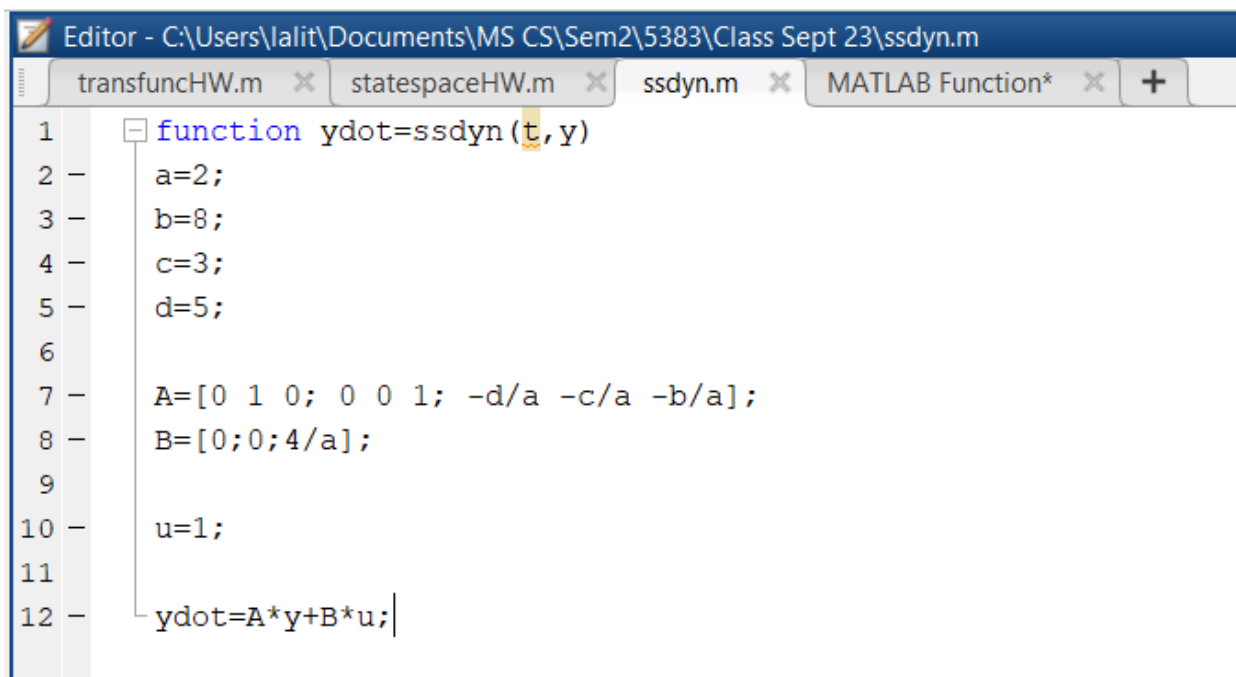
- **State Space Simulation in Matlab**



The image shows a MATLAB Editor window with the title bar "Editor - C:\Users\lalit\Documents\MS CS\Sem2\5383\matlabHW\statespaceHW.m". The window contains four tabs: "transfuncHW.m", "statespaceHW.m", "ssdyn.m", and "MATLAB Function*". The "statespaceHW.m" tab is active, displaying the following code:

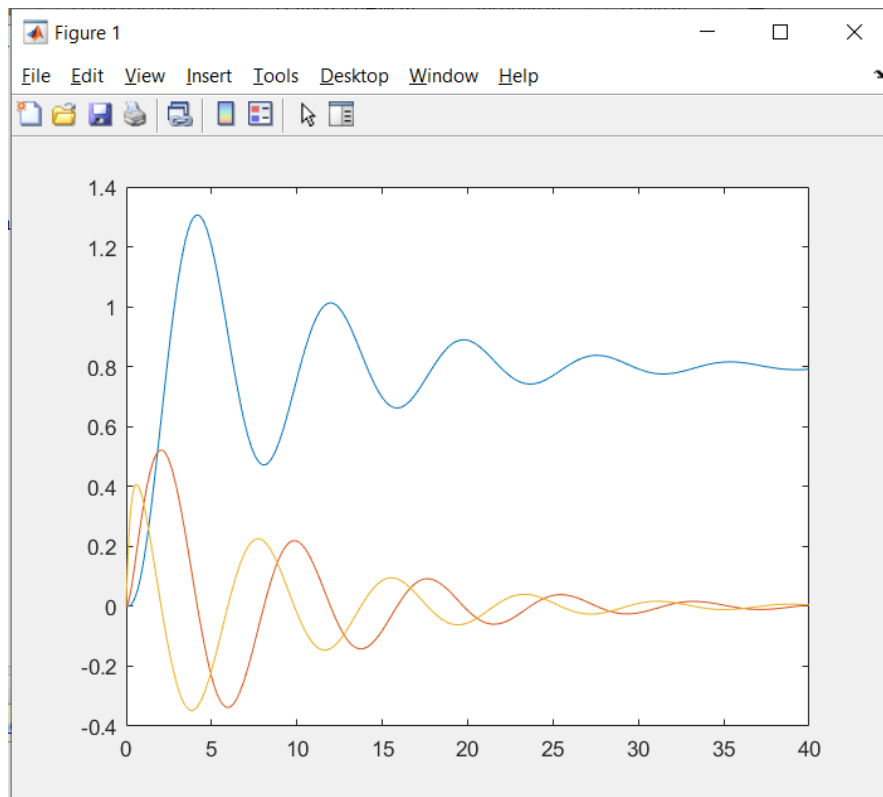
```
1 %State space representation model Homework
2
3 time=[0 40];
4
5 y0=[0;0;0];
6
7 [t,y]=ode45(@ssdyn,time,y0);
8
9 plot(t,y)
```

State Space Modeling Matlab Code.



The image shows a MATLAB Editor window with the title bar "Editor - C:\Users\lalit\Documents\MS CS\Sem2\5383\Class Sept 23\ssdyn.m". The window contains four tabs: "transfuncHW.m", "statespaceHW.m", "ssdyn.m", and "MATLAB Function*". The "ssdyn.m" tab is active, displaying the following function definition:

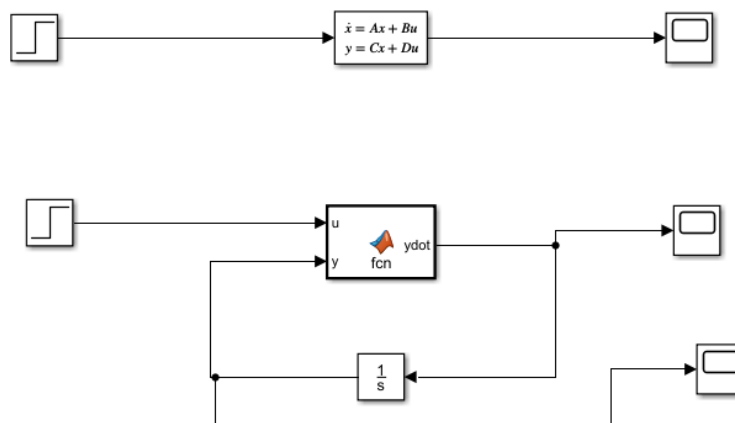
```
1 function ydot=ssdyn(t,y)
2     a=2;
3     b=8;
4     c=3;
5     d=5;
6
7     A=[0 1 0; 0 0 1; -d/a -c/a -b/a];
8     B=[0;0;4/a];
9
10    u=1;
11
12    ydot=A*y+B*u;
```



State Space Matlab Resulting Plot

- State Space Simulation in Simulink

Homework: SIMULATION OF STATE SPACE OF DYNAMICAL SYSTEM



Simulink State Space Block

- All initial conditions equal to zero.

Block Parameters: State-Space

State Space

State-space model:
 $\dot{x}/dt = Ax + Bu$
 $y = Cx + Du$

Parameters

A:

B:

C:

D:

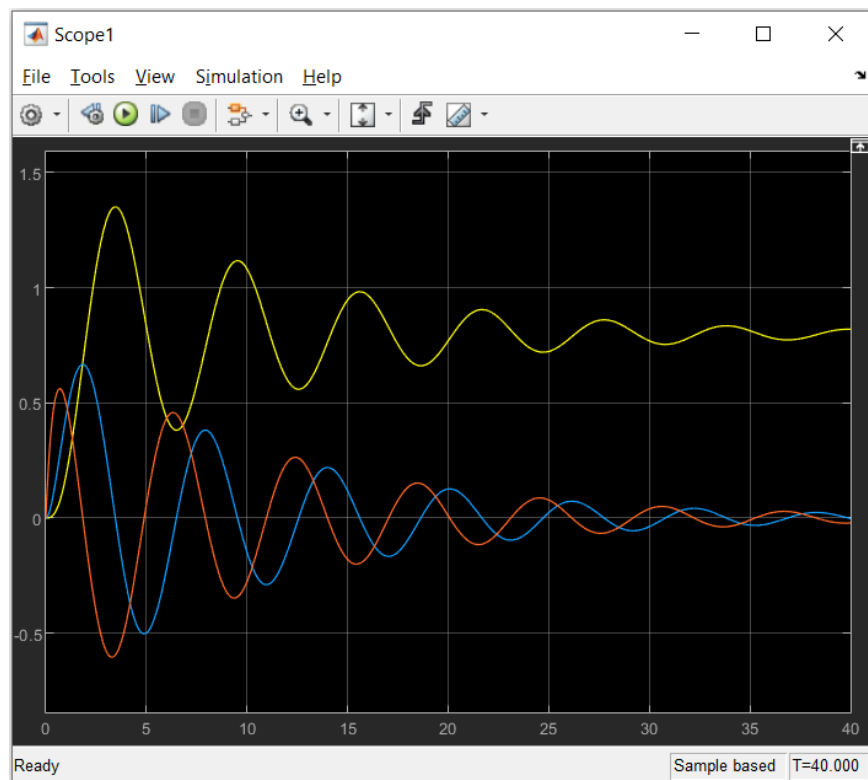
Initial conditions:

Absolute tolerance:

State Name: (e.g., 'position')

OK Cancel Help Apply

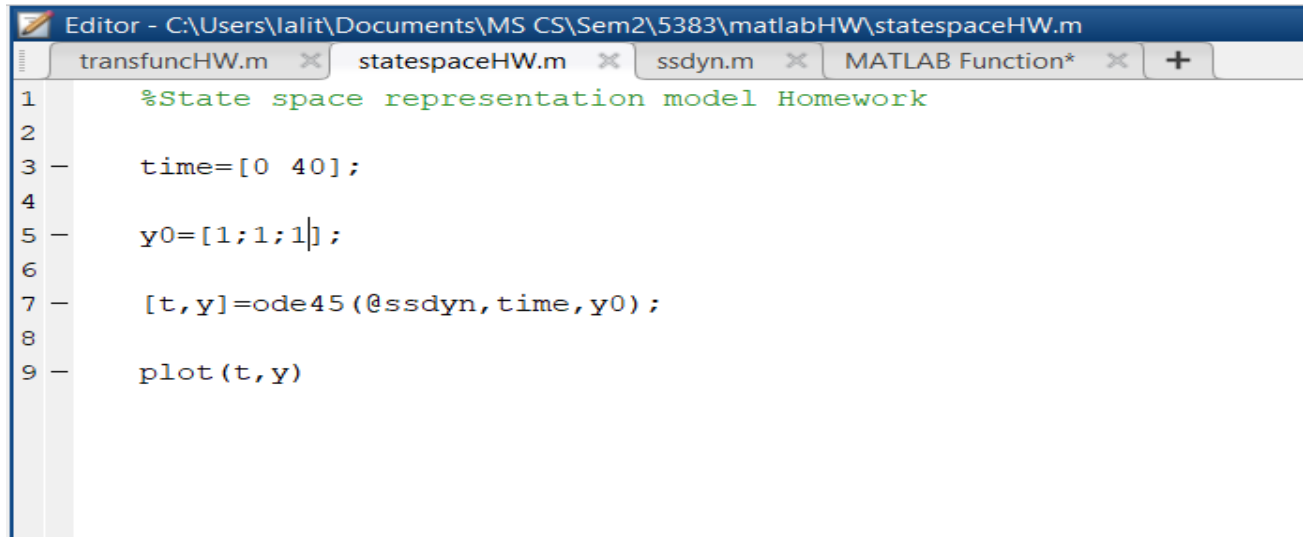
State Space Block parameters at 0 initials



Simulink State Space Resulting Plot at 0

C. State Space Representation:

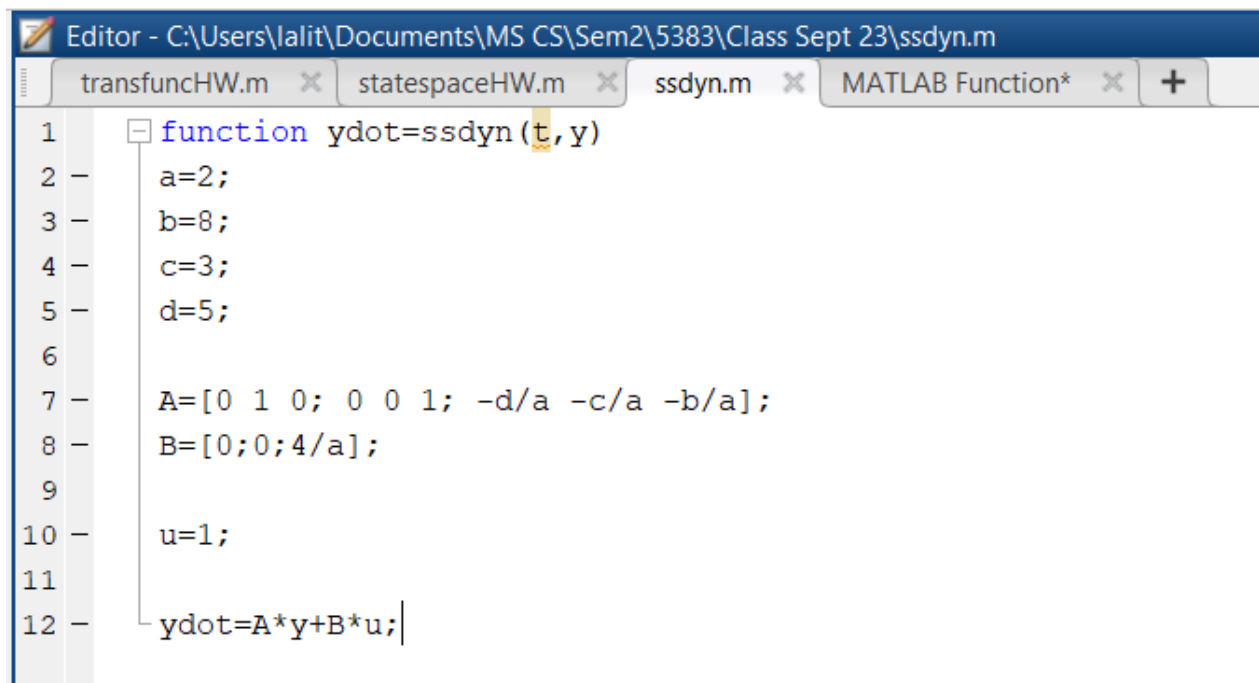
- State Space Simulation in Matlab



The image shows a MATLAB Editor window with the title bar "Editor - C:\Users\lalit\Documents\MS CS\Sem2\5383\matlabHW\statespaceHW.m". The window contains a script named statespaceHW.m with the following code:

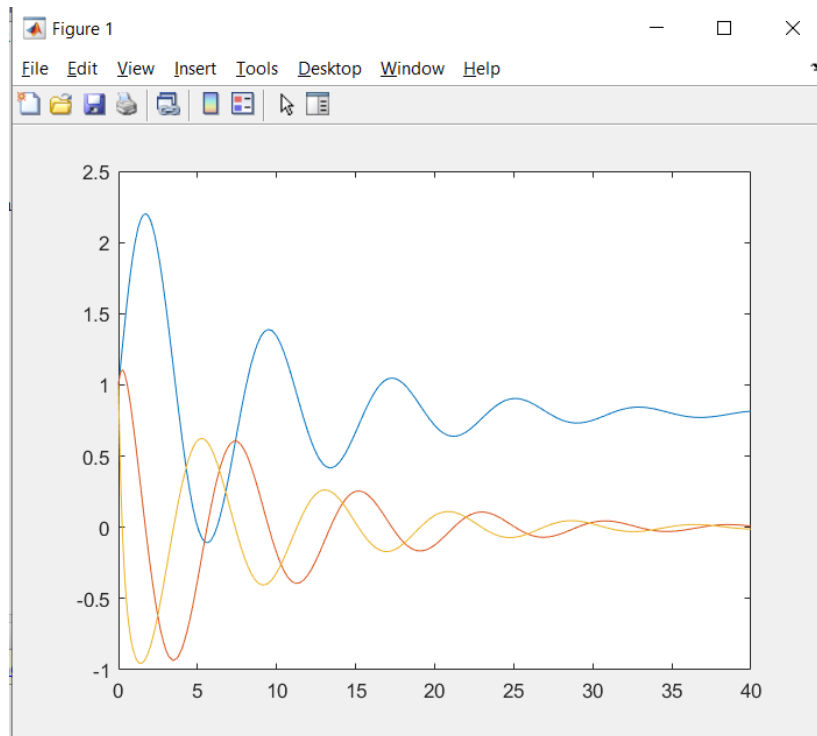
```
1 %State space representation model Homework
2
3 time=[0 40];
4
5 y0=[1;1;1];
6
7 [t,y]=ode45(@ssdyn,time,y0);
8
9 plot(t,y)
```

State Space Matlab Code for all initial condition at 1



The image shows a MATLAB Editor window with the title bar "Editor - C:\Users\lalit\Documents\MS CS\Sem2\5383\Class Sept 23\ssdyn.m". The window contains a function named ssdyn.m with the following code:

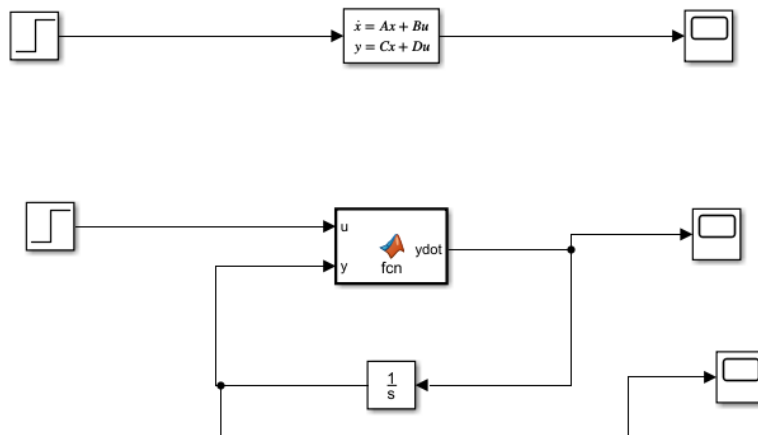
```
1 function ydot=ssdyn(t,y)
2     a=2;
3     b=8;
4     c=3;
5     d=5;
6
7     A=[0 1 0; 0 0 1; -d/a -c/a -b/a];
8     B=[0;0;4/a];
9
10    u=1;
11
12    ydot=A*y+B*u;
```



Matlab State Space Resulting Plot at 1

- **State Space Simulation in Simulink**

Homework: SIMULATION OF STATE SPACE OF DYNAMICAL SYSTEM



Simulink State Space Block

- All initial conditions equal to one.

Block Parameters: State-Space

State Space

State-space model:
 $\dot{x}/dt = Ax + Bu$
 $y = Cx + Du$

Parameters

A:

B:

C:

D:

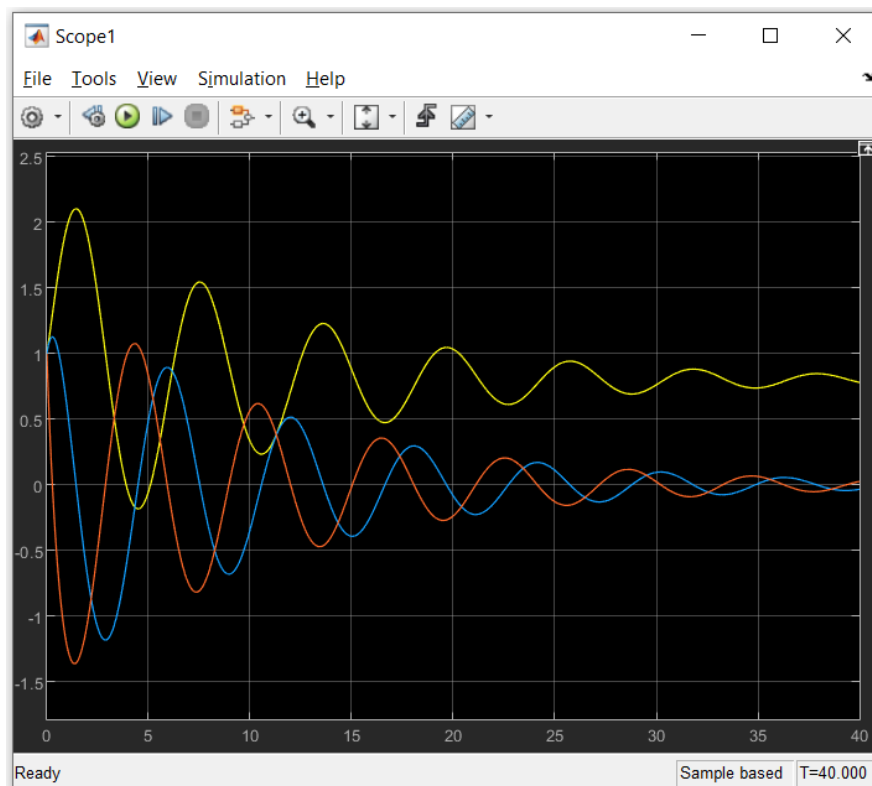
Initial conditions:

Absolute tolerance:

State Name: (e.g., 'position')

OK Cancel Help Apply

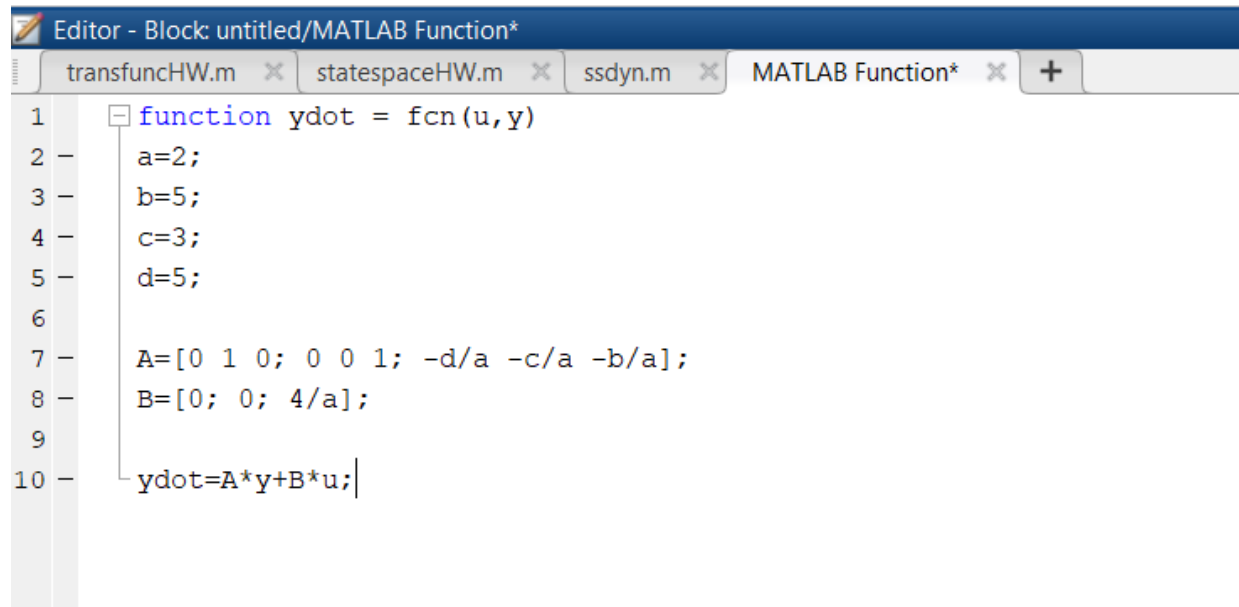
State Space Block parameters at 1 initials



Simulink State Space Resulting Plot at 1

- **Simulink State Space Modeling using User Defined Block**

Matlab Function Code for user defined State Space block:



The image shows a MATLAB Editor window titled "Editor - Block: untitled/MATLAB Function*". The window has several tabs open: "transfuncHW.m", "statespaceHW.m", "ssdyn.m", and "MATLAB Function*". The "MATLAB Function*" tab is active, displaying the following code:

```
1 function ydot = fcn(u,y)
2     a=2;
3     b=5;
4     c=3;
5     d=5;
6
7     A=[0 1 0; 0 0 1; -d/a -c/a -b/a];
8     B=[0; 0; 4/a];
9
10    ydot=A*y+B*u;
```