Feature Detection and Matching

CSE 6367: Computer Vision

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Introduction

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- What kinds of *features* should be *detected* and then *matched* in a given image?

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- These kinds of localized features are called **keypoint features** or **interest points** (or even **corners**) and are described by the appearance of patches of pixels surrounding the point location

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- Edges can be grouped into longer **curves** and **straight line segments**, which can be directly matched or analyzed to find **vanishing points** and thus internal and external camera parameters

Introduction

(a)

(b)

(c)

(d)

- A variety of feature detectors and descriptors can be used to analyze, describe, and match images

Point Features

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- Such correspondences can also be used to align different images, e.g. when stitching image mosaics or performing video stabilization

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- A key advantage of keypoints is that they permit matching even in the presence of clutter (occlusion) and large-scale and orientation changes
- Feature-based correspondence techniques have been used since the early days of stereo matching and have gained popularity for image-stitching applications as well as fully automated 3D modeling

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- The first approach is more suitable when images are taken from nearby viewpoints or in rapid succession (e.g. video images)
- The second approach is more suitable when a large number of motion or appearance change is expected (e.g. object recognition)

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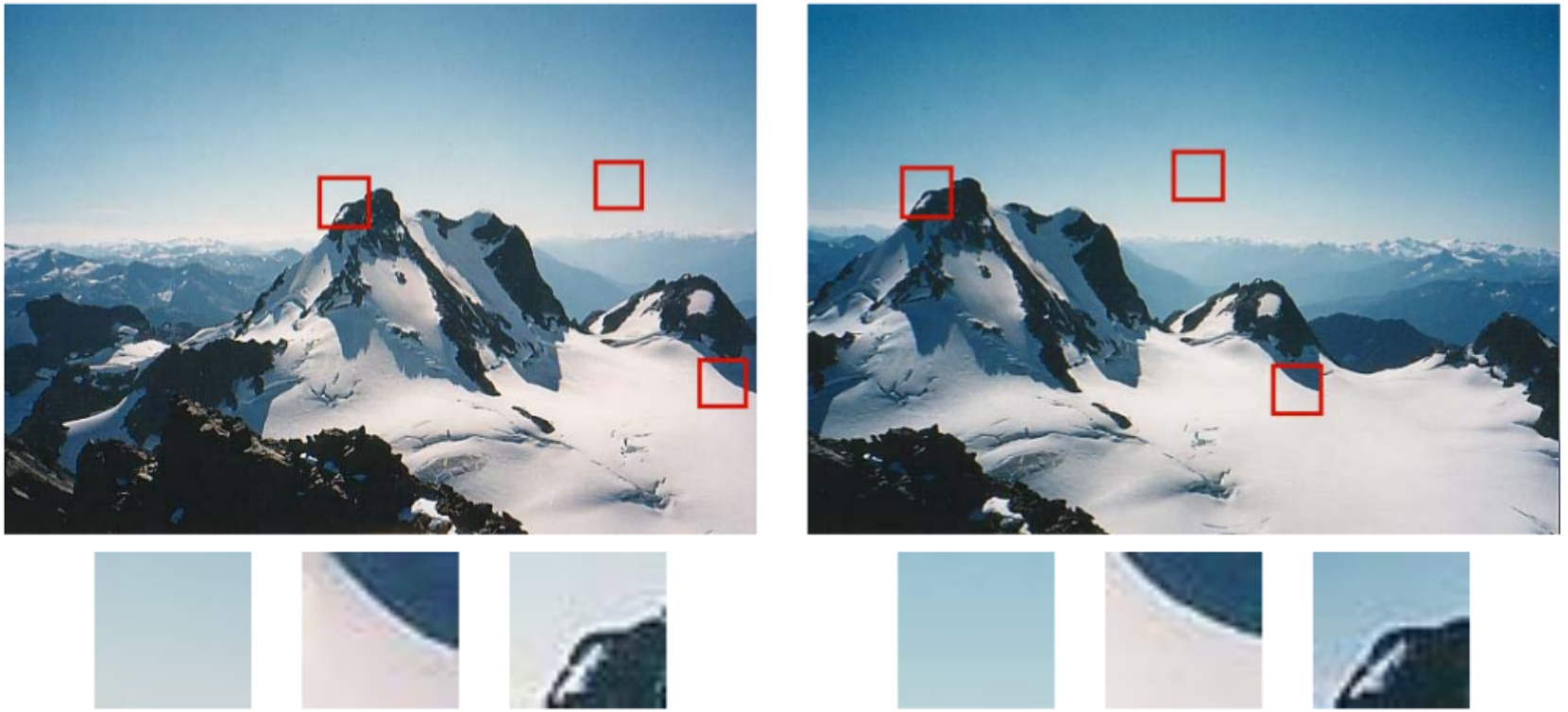
  - **Feature matching** stage - efficiently searches for likely matching candidates in other images
  - **Feature tracking** stage - is an alternative to the third stage that only searches a small neighborhood around each detected feature and is therefore more suitable for video processing

Feature Detectors



- How can we find image locations where we can reliably find correspondences with other images, i.e. what are good features to track?

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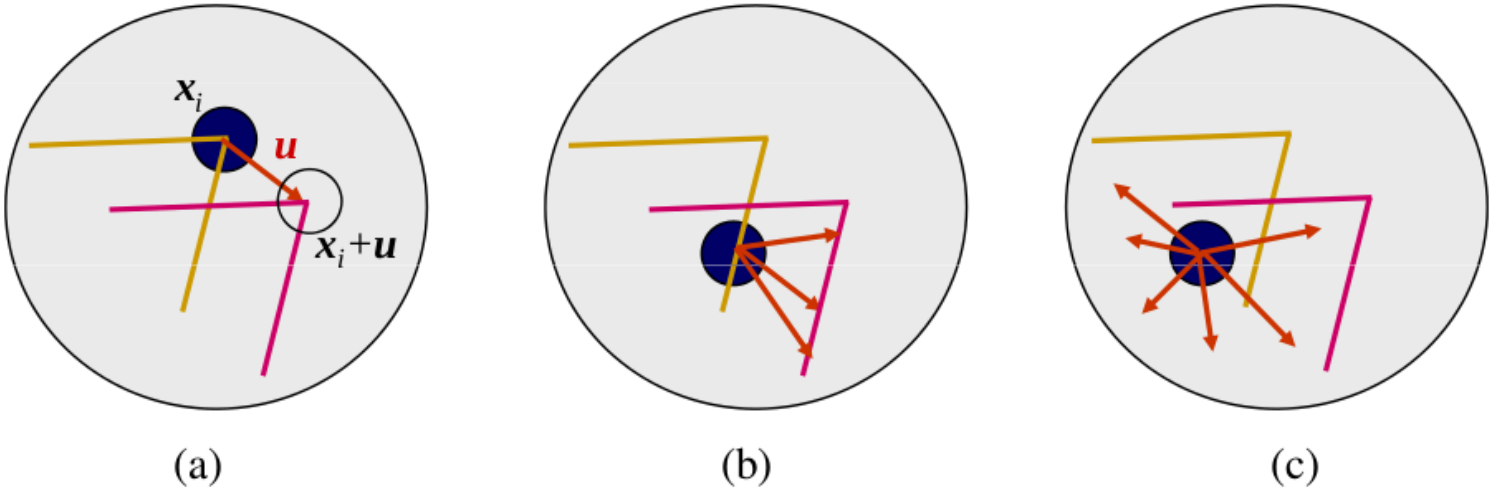
- Patches with gradients in at least two (significantly) different orientations are the easiest to localize

Feature Detectors



- Aperture problems for different image patches: (a) stable ("corner-like") flow; (b) classic aperture problem (barber-pole illusion); (c) textureless region

Feature Detectors

- The simplest possible matching criterion for comparing two image patches is their (weighted) summed square difference

$$E_{\mathrm{WSSD}}(\mathbf{u}) = \sum_i w(\mathbf{x}_i)[I_1(\mathbf{x}_i + \mathbf{u}) - I_0(\mathbf{x}_i)]^2$$

where I_0 and I_1 are the two images being compared, $\mathbf{u} = [u, v]$ is the *displacement* vector, $w(\mathbf{x})$ is a spatially varying weighting (or window) function, and i is over all pixels in the patch

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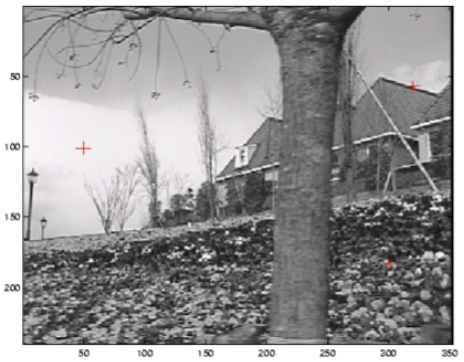
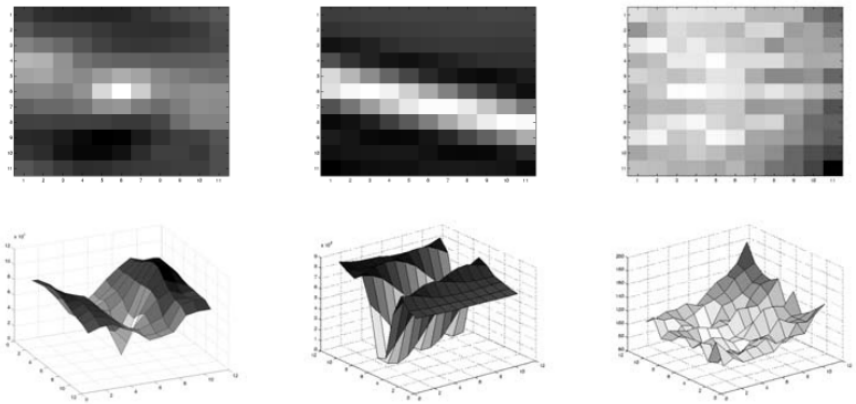
- Therefore, we can only compute how stable this metric is w.r.t small variations in position $\Delta\mathbf{u}$ by comparing an image patch against itself which is known as an **auto-correlation surface** function

$$E_{\mathrm{AC}}(\Delta\mathbf{u}) = \sum_i w(\mathbf{x}_i)[I_0(\mathbf{x}_i + \Delta\mathbf{u}) - I_0(\mathbf{x}_i)]^2$$

Feature Detectors



(a)



(b) (c) (d)

- Three auto-correlation surfaces: (a) original image; (b) flower bed patch (good unique minimum); (c) roof edge patch (1D aperture problem); (d) cloud patch (no good peak)

Feature Detectors

- Using a Taylor series expansion of the image function $I_0(\mathbf{x}_i + \Delta\mathbf{u}) \approx I_0(\mathbf{x}_i) + \nabla I_0(\mathbf{x}_i) \cdot \Delta\mathbf{u}$, we can approximate the auto-correlation surface as

$$\begin{aligned} E_{\mathrm{AC}}(\Delta\mathbf{u}) &= \sum_i w(\mathbf{x}_i)[I_0(\mathbf{x}_i + \Delta\mathbf{u}) - I_0(\mathbf{x}_i)]^2 \\ &\approx \sum_i w(\mathbf{x}_i)[I_0(\mathbf{x}_i) + \nabla I_0(\mathbf{x}_i) \cdot \Delta\mathbf{u} - I_0(\mathbf{x}_i)]^2 \\ &= \sum_i w(\mathbf{x}_i)[\nabla I_0(\mathbf{x}_i) \cdot \Delta\mathbf{u}]^2 \\ &= \Delta\mathbf{u}^T A \Delta\mathbf{u} \end{aligned}$$

where $\nabla I_0(\mathbf{x}_i) = (\frac{\partial I_0}{\partial x}, \frac{\partial I_0}{\partial y})(\mathbf{x}_i)$ is the **image gradient** at $\mathbf{x}_i$

Feature Detectors

- The auto-correlation matrix A can be written as

$$A = w * \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

where the weighted summations have been replaced with discrete convolutions using the weighting kernel w

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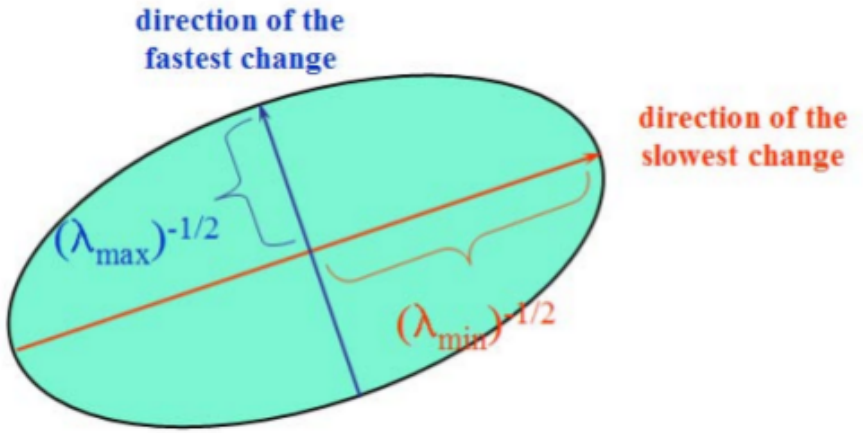
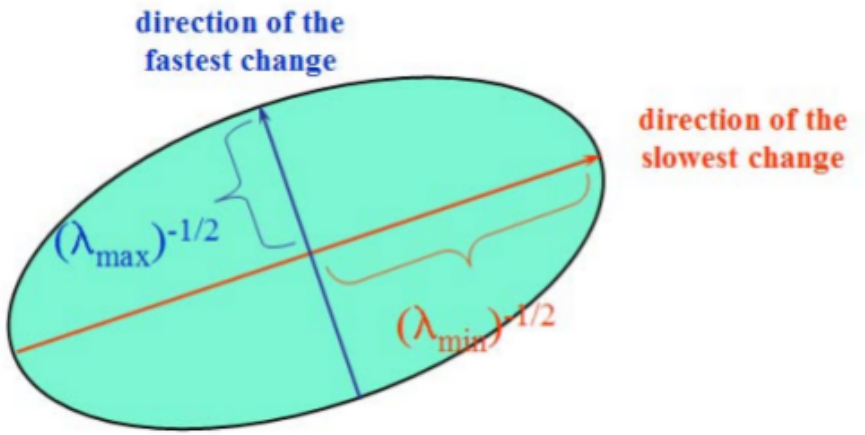
- The inverse of A provides a lower bound on the uncertainty in the location of a matching pixel and thus is a useful indicator of which patches can be reliably matched

Feature Detectors



- Uncertainty ellipse corresponding to an eigenvalue analysis of the auto-correlation matrix A

Feature Detectors



- Uncertainty ellipse corresponding to an eigenvalue analysis of the auto-correlation matrix A
- Since the larger uncertainty depends on the smaller eigenvalue, it makes sense to find maxima in the smaller eigenvalue to locate good features to track

Feature Detectors

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- The minimum eigenvalue λ_1 is not the only quantity that can be used to find these keypoints

Harris Corner Detector

- The **Harris corner detector** makes use of a simpler quantity

$$\det(A) - \alpha \text{trace}(A)^2 = \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

with $\alpha = 0.06$

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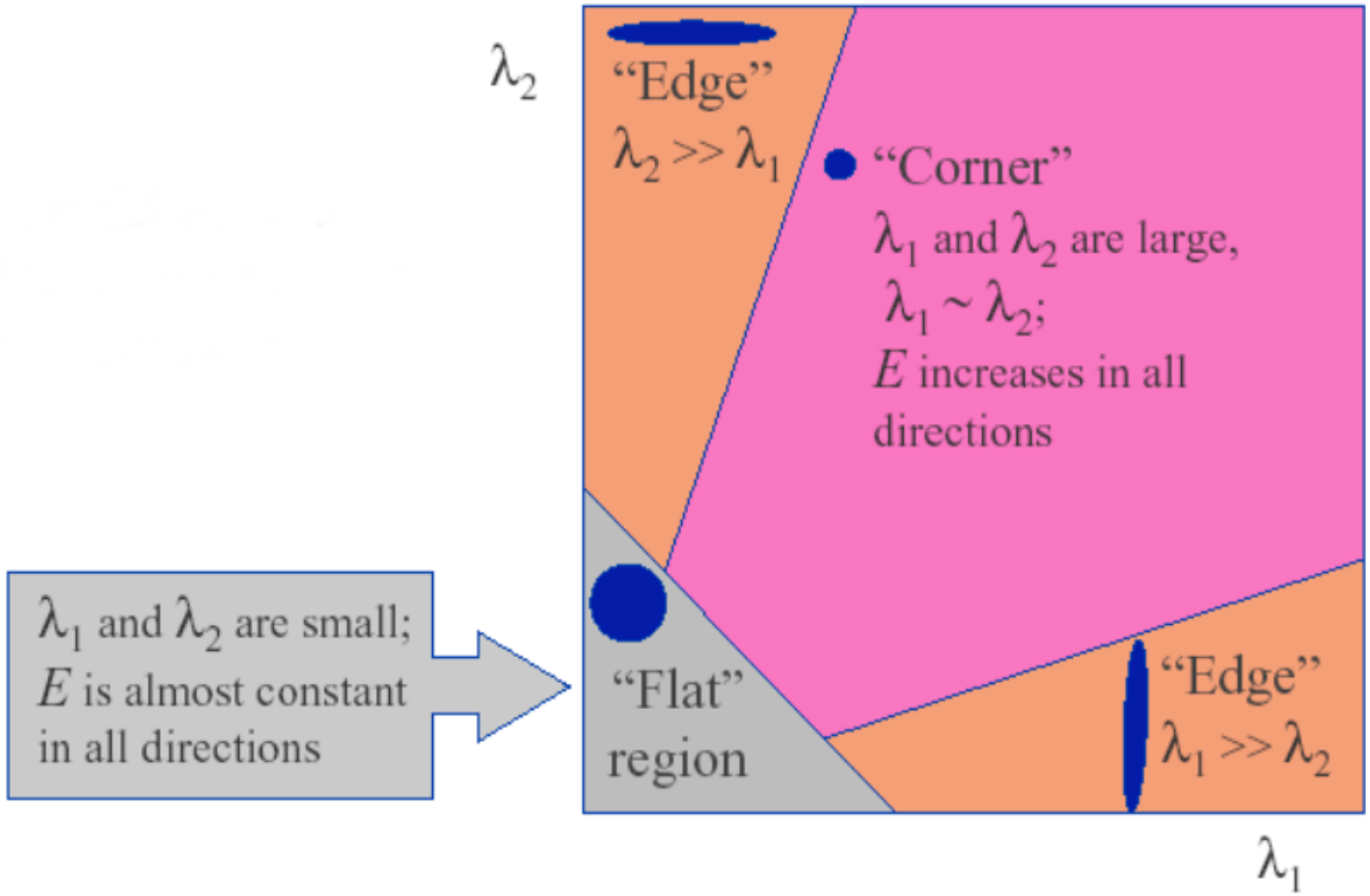
- It also downweights edge-like features where $\lambda_1 \gg \lambda_2$ or $\lambda_2 \gg \lambda_1$

Harris Corner Detector



Harris Corner Detector Algorithm

1. Compute the x and y derivatives of the image

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3. For each pixel (x, y) in the window w define the matrix

$$A(x,y) = \begin{bmatrix} \sum_{(x,y)\in w} I_x^2 & \sum_{(x,y)\in w} I_x I_y \\ \sum_{(x,y)\in w} I_x I_y & \sum_{(x,y)\in w} I_y^2 \end{bmatrix}$$

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4. Compute the response of the detector at each pixel

$$R = \det(A) - \alpha \text{trace}(A)^2$$

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5. Threshold on the value of R and apply non-maximal suppression

Example: Detecting Corners using MATLAB

- Detect corners using the Harris-Stephens algorithm:

```
I = checkerboard;
corners = detectHarrisFeatures(I);
imshow(I); hold on;
plot(corners.selectStrongest(50));
```

Interest Operator Responses

(a) (b) (c)

- (a) sample image; (b) Harris response; (c) DoG response

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- (a) sample image; (b) Harris response; (c) DoG response
- The circle sizes and colors indicate the scale at which each interest point was detected
- Note how the two detectors tend to respond at complementary locations

Adaptive Non-Maximal Suppression Detector

- While most feature detectors simply look for local maxima in the interest function, this can lead to an uneven distribution of feature points across the image (e.g. points will be denser in regions of higher contrast)

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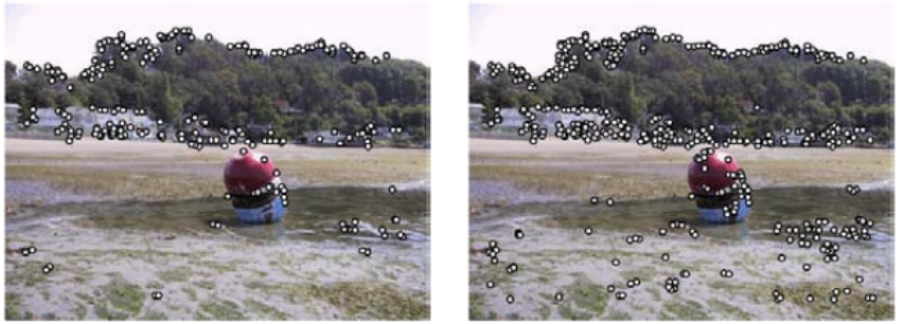
- While most feature detectors simply look for local maxima in the interest function, this can lead to an uneven distribution of feature points across the image (e.g. points will be denser in regions of higher contrast)
- To mitigate this problem the **adaptive non-maximal suppression** (ANMS) detector only detects features that are both local maxima and whose response value is significantly (10%) greater than its neighbors within a radius r

Adaptive Non-Maximal Suppression Detector

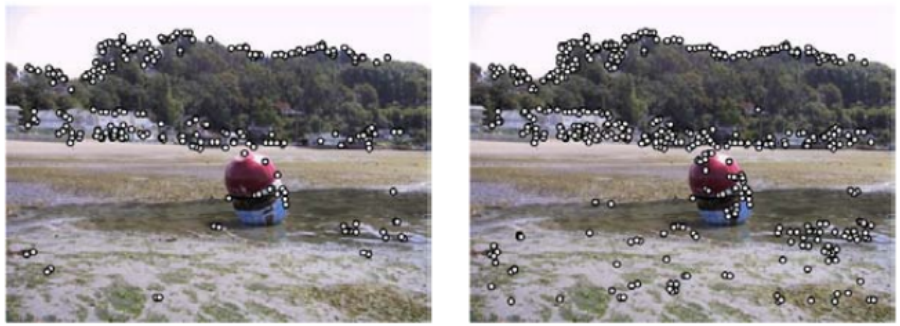


(a) Strongest 250 (b) Strongest 500

(c) ANMS 250, $r = 24$ (d) ANMS 500, $r = 16$

- The upper two images show the strongest 250 and 500 interest points

Adaptive Non-Maximal Suppression Detector



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- The upper two images show the strongest 250 and 500 interest points
- The lower two images show the interest points selected with ANMS along with the suppression radius *r*

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4. Compute a scalar interest measure using one of the discussed formulas
5. Find local maxima above a certain threshold and report them as detected feature point locations

Measuring Repeatability

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- The **repeatability** of a feature detector is defined as the frequency in which keypoints detected in one image are found within ϵ (say $\epsilon = 1.5$) pixels of the corresponding location in a transformed image
- The **information content** available at each detected feature point can be defined as the entropy of a set of rotationally invariant local grayscale descriptors

Scale Invariance

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- In many situations, detecting features at the finest scale possible may not be appropriate (e.g. when matching images with little high frequency detail, fine-scale features may not exist
- One solution is to extract features at a variety of scales, e.g. by performing the same operations at multiple resolutions in a pyramid and then matching features at the same level

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- However, for most object recognition applications, the scale of the object in the image is unknown
- Thus, instead of extracting features at many different scales and then matching all of them, it is more efficient to extract features that are stable in both location *and* scale

Rotational Invariance and Orientation Estimation

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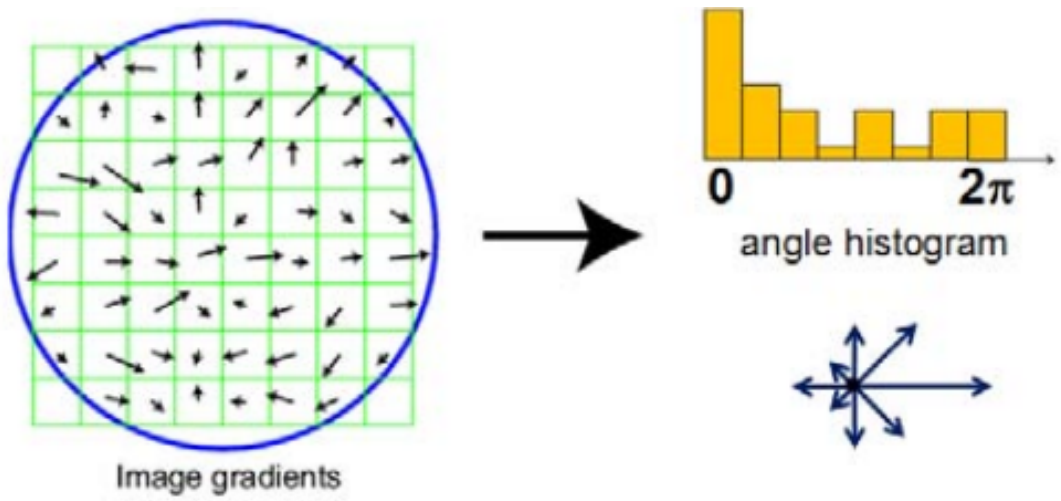
- One way to address this problem is to design descriptors that are rotationally invariant, however such descriptors have poor discriminability (i.e. they map different looking patches to the same descriptor)

Rotational Invariance and Orientation Estimation

- In addition to dealing with scale changes, most image matching and object recognition algorithms need to deal with in-plane image rotation
- One way to address this problem is to design descriptors that are rotationally invariant, however such descriptors have poor discriminability (i.e. they map different looking patches to the same descriptor)
- A better method is to estimate a **dominate orientation** at each detected keypoint, then a scaled and oriented patch around the detected point can be extracted and used to form a feature descriptor

Dominate Orientation Estimate



- A dominate orientation estimate can be computed by creating a histogram of all the gradient orientations (weighted by their magnitude or after thresholding out small gradients) and then finding the significant peaks in this distribution

Affine Invariance

- Affine invariant detectors not only respond at consistent locations after scale and orientation changes, they also respond consistently across affine deformations such as (local) perspective foreshortening

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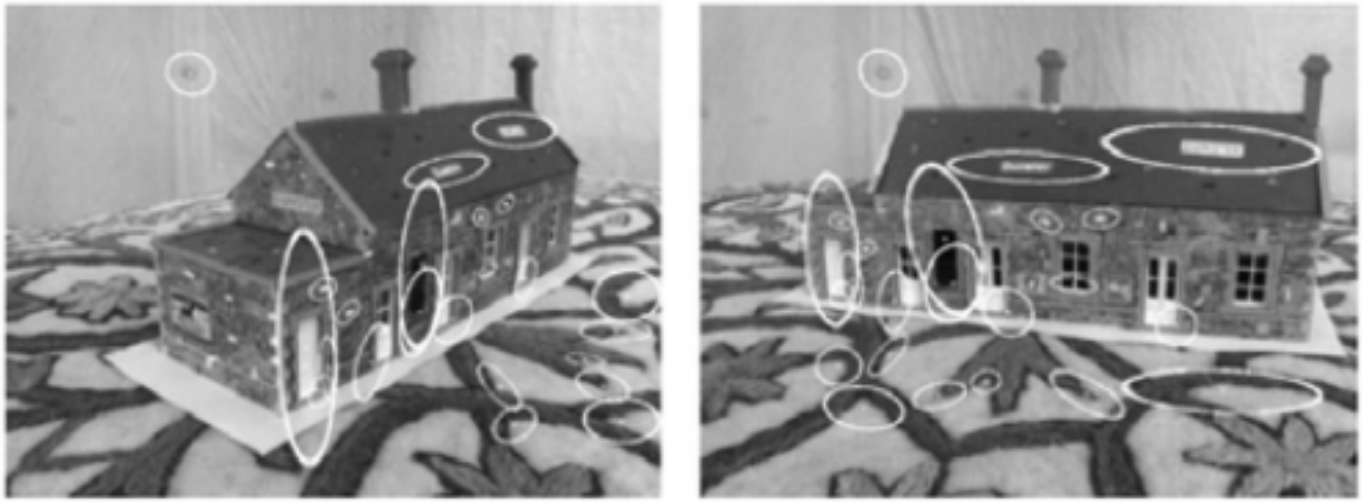
- Affine invariant detectors not only respond at consistent locations after scale and orientation changes, they also respond consistently across affine deformations such as (local) perspective foreshortening
- Affine invariance may be introduced by fitting an ellipse to the auto-correlation or Hessian matrix (using eigenvalue analysis) and then using the principle axes and ratios of this fit as the affine coordinate frame

Affine Region Detectors



- Affine region detectors used to match two images taken from dramatically different viewpoints

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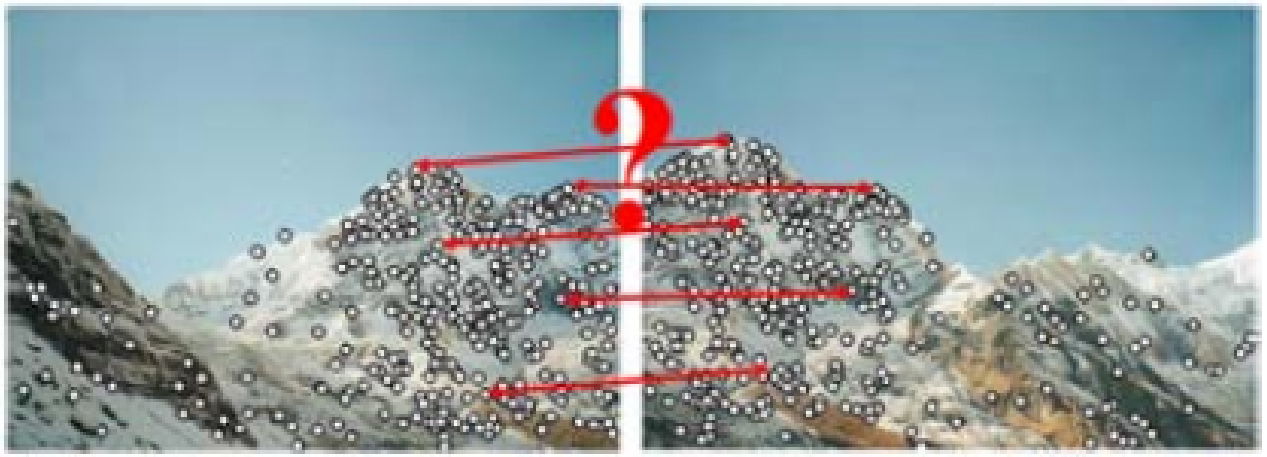
- The local appearance of features will change in orientation and scale (sometimes even undergo affine deformations)
- Therefore, we are interested in extracting a local scale, orientation, or affine frame estimate and then using this to resample the patch before forming the feature descriptor

Feature Descriptors



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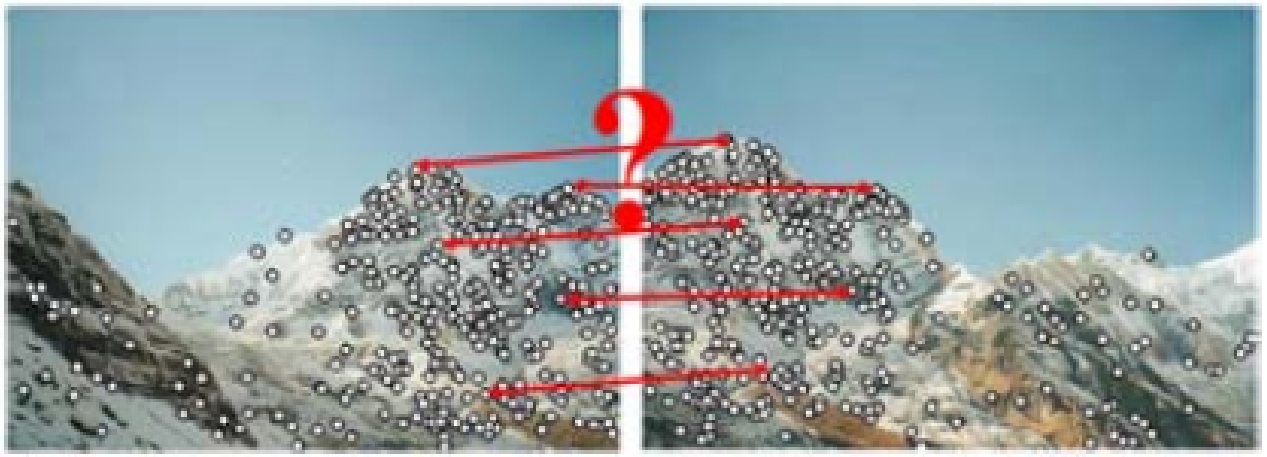

- Even after compensating for these changes, the local appearance of image patches will usually vary from image to image
- How can we extract local descriptors that are invariant to inter-image variations yet still discriminative enough to establish correct correspondences?

Bias and Gain Normalization (MOPS)

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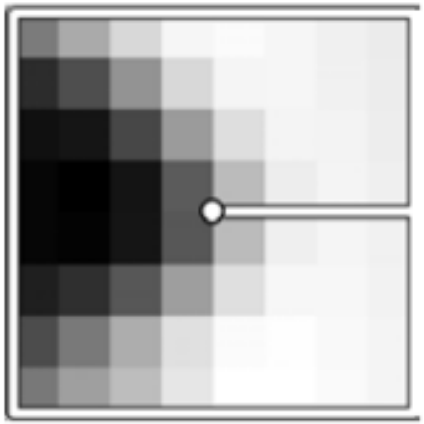
- For tasks that do not exhibit large amounts of foreshortening (e.g. image stitching) simple normalized intensity patches perform reasonably well and are simple to implement
- To compensate for slight inaccuracies in the feature point detector (location, orientation, and scale), these **multi-scale oriented patches** (MOPS) are sampled at a spacing of five pixels relative to the detection scale

Bias and Gain Normalization (MOPS)



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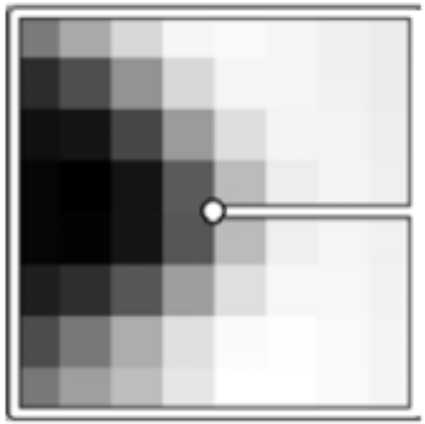
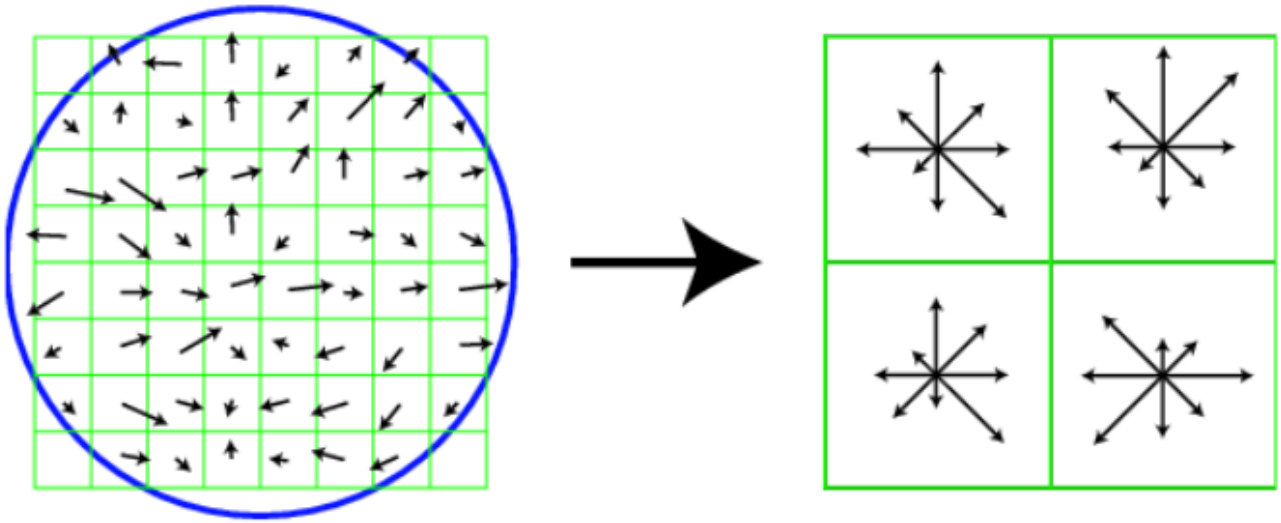


- MOPS descriptors are formed using an 8×8 sampling of bias and gain normalized intensity values
- This low frequency sampling gives the features some robustness to interest point location error and is achieved by sampling at a higher pyramid level than the detection scale

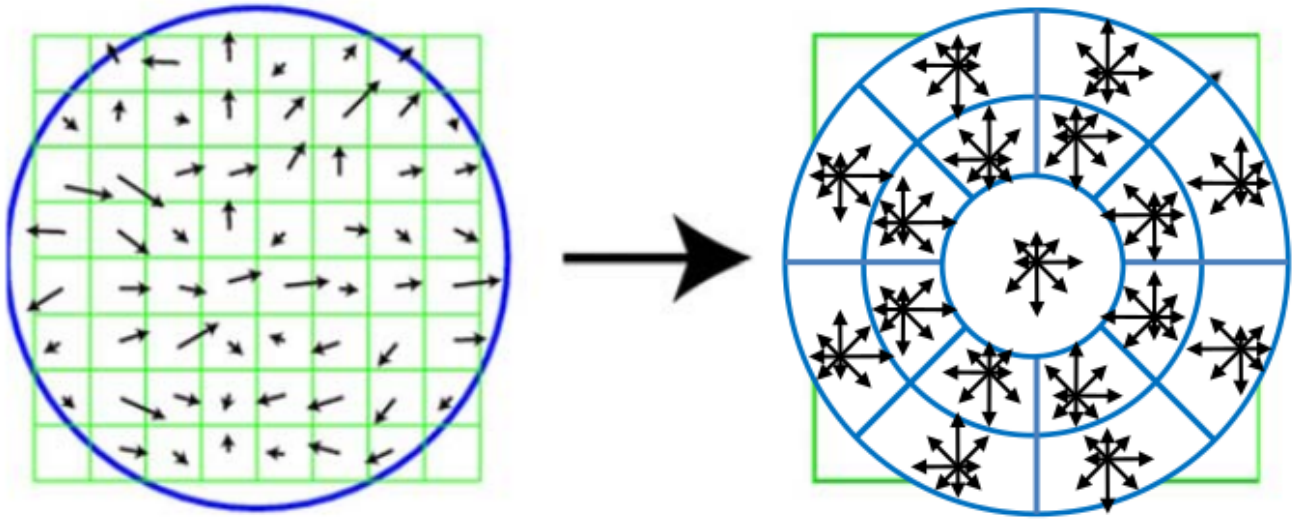
Scale Invariant Feature Transform (SIFT)



(a) image gradients (b) keypoint descriptor

- A schematic representation of **SIFT**: (a) gradient orientations and magnitudes are computed at each pixel and weighted by a Gaussian fall-off function (blue circle); (b) a weighted gradient orientation histogram is then computed in each subregion using trilinear interpolation

Gradient Location-Orientation Histogram (GLOH)



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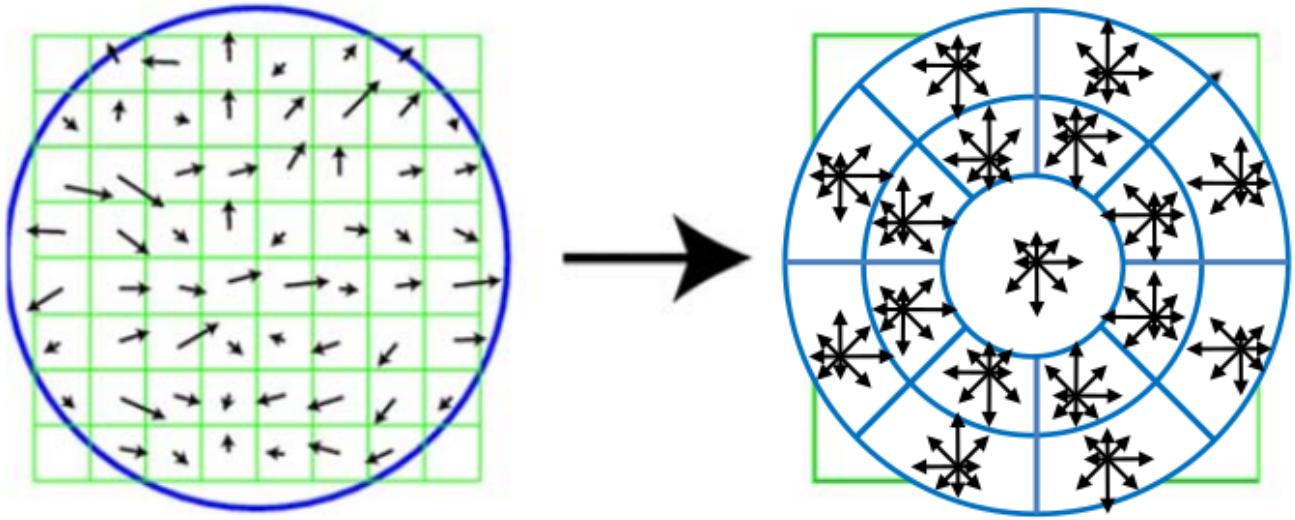

(a) image gradients (b) keypoint descriptor

- The **GLOH** descriptor is a variant on SIFT that uses a log-polar binning structure instead of square bins to compute orientation histograms
- The 272-dimensional descriptor is then projected onto a 128-dimensional descriptor using PCA trained on a large database

Performance of Local Descriptors

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- In addition to optimizing for repeatability across *all* object classes, it is also possible to develop class- or instance-specific feature detectors that maximize *discriminability* from other classes

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  - Select a **matching strategy** to determine which correspondences are passed on to the next stage for further processing
  - Devise efficient **algorithms** and **data structures** to perform this matching as quickly as possible

Application: Recognizing Objects in a Cluttered Scene

- Two of the training images in the database are shown on the left

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- The affine warp of each recognized database image is shown as a larger parallelogram

Matching Strategy and Error Rates

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- To begin, we assume that the feature descriptors have been designed so that Euclidean (vector magnitude) distances in feature space can be directly used for ranking potential matches

Matching Strategy and Error Rates

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  - TN - true negatives (non-matches that were correctly rejected)

Matching Strategy and Error Rates

	True matches	True non-matches		
Predicted matches	TP = 18	FP = 4	P' = 22	PPV = 0.82
Predicted non-matches	FN = 2	TN = 76	N' = 78	
	P = 20	N = 80	Total = 100	
	TPR = 0.90	FPR = 0.05		ACC = 0.94

- The number of matches correctly and incorrectly estimated by a feature matching algorithm can be tabulated in a **confusion matrix** (contingency table)

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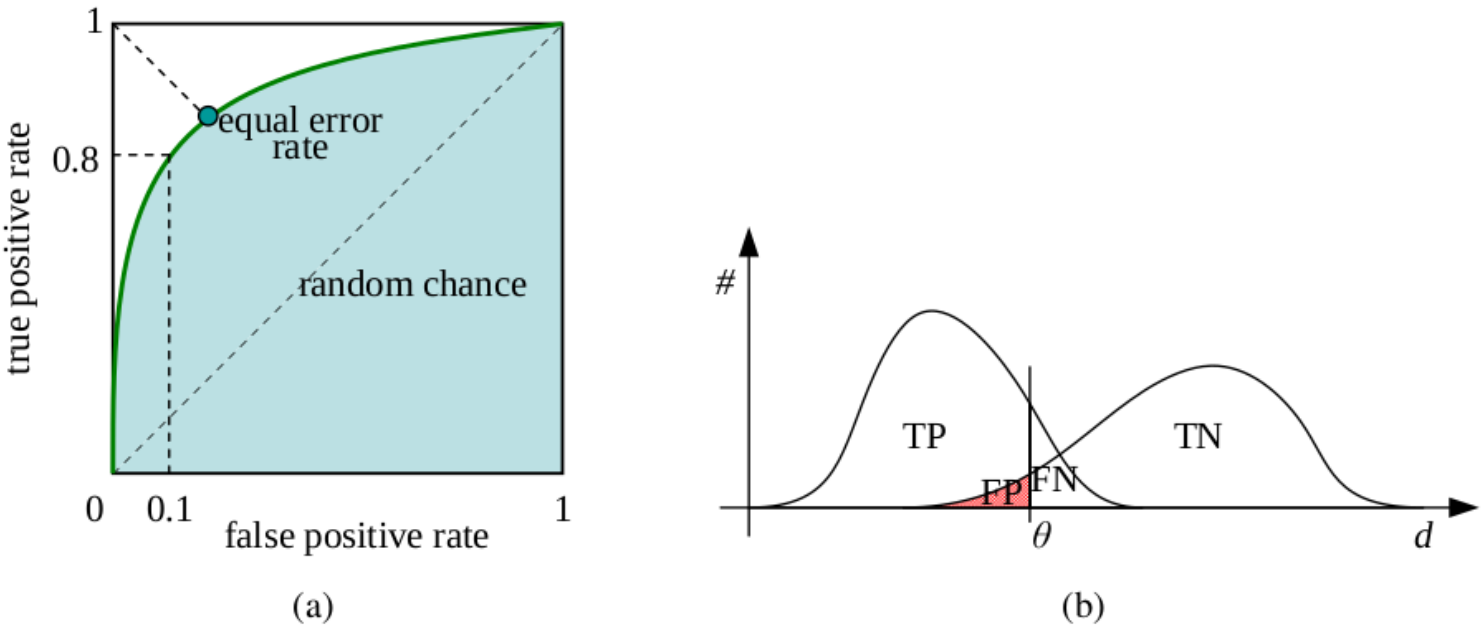
- Ideally, the true positive rate will be close to 1 and the false positive rate close to 0
- As we vary the matching threshold, we obtain a family of such points which are collectively known as the **receiver operating characteristic** (ROC) curve

Matching Strategy and Error Rates



- (a) the ROC curve plots the true positive rate against the false positive rate for a particular combination of feature extraction and matching algorithms

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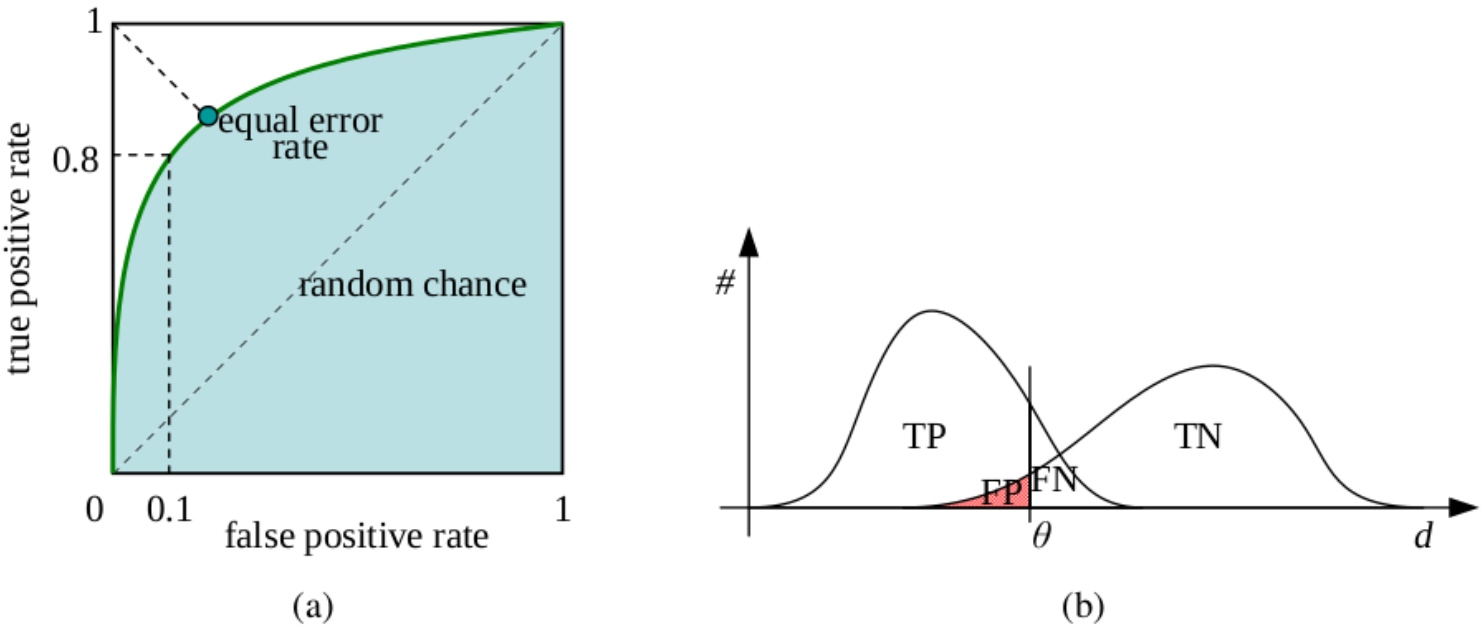

- (a) the ROC curve plots the true positive rate against the false positive rate for a particular combination of feature extraction and matching algorithms
- (b) the distribution of positives (matches) and negatives (non-matches) as a function of inter-feature distance d

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- This can be done by devising an **indexing structure**, such as a multi-dimensional search tree or a hash table, to rapidly search for features near a given feature

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- An indexing structure can be built for each image independently (useful if we want to only consider certain potential matches, e.g. searching for a particular object)
- Alternatively, an indexing structure can be built globally for all the images in a given database, which can potentially be faster since it removes the need to iterate over each image

Feature Tracking

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- This kind of *detect then track* approach is more widely used for video tracking applications where the amount of motion and appearance deformation between adjacent frames is expected to be small

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- In practice, regions containing high gradients in both directions, i.e. which have high eigenvalues in the auto-correlation matrix, provide stable locations at which to find correspondences

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- The latter strategy runs the risk of the feature drifting from its original location to some other location in the image

Kanada-Lucas-Tomasi (KLT) Tracker

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- A preferable solution to the tracking problem is to compare the original patch to later image locations using an *affine* motion model
- The **Kanada-Lucas-Tomasi** (KLT) tracker first compares patches in neighboring frames using a translational model
- The tracker then uses the location estimates produced by this model to initialize an affine registration between the patch in the current frame and the base frame where a feature was first detected

Edge Features

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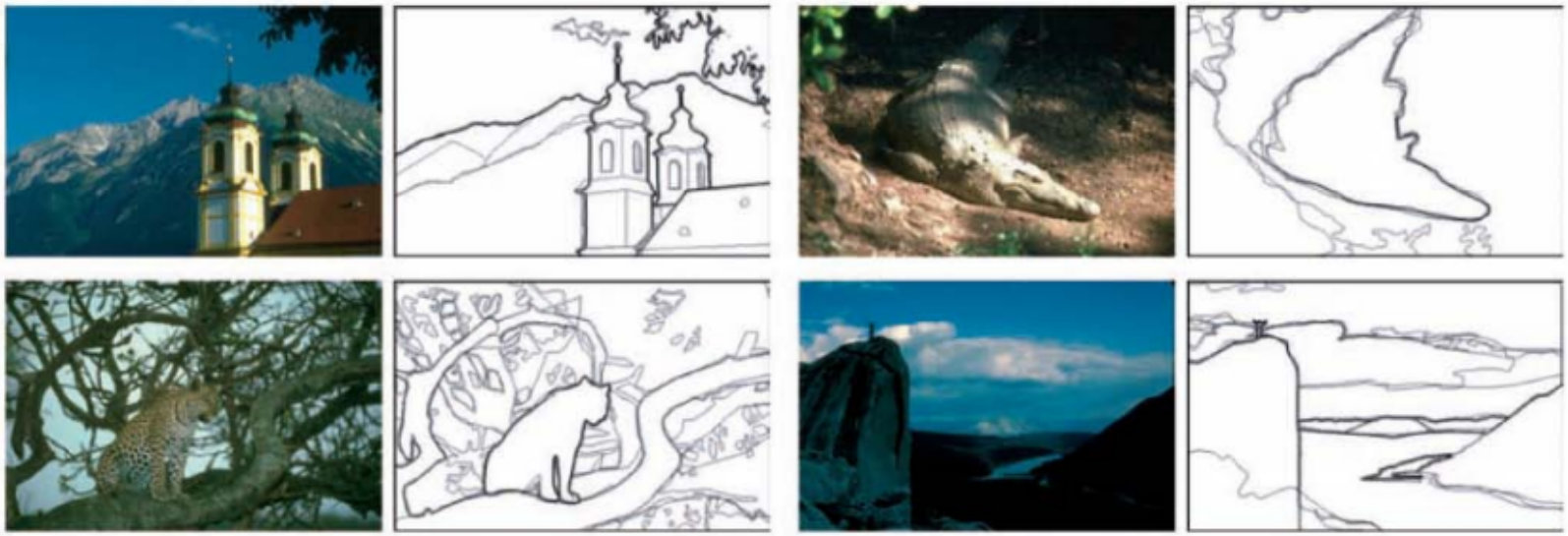
- For example, the boundaries of objects which also correspond to occlusion events in 3D are usually delineated by visible contours
- Edges and lines are more compact than pixels and are used in object recognition, image matching (e.g., stereo, mosaics), robotics, and much more

Edge Detection



- Given an image, how can we find the most "salient" or "strongest" edges or the object boundaries?

Edge Detection

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- Unfortunately, segmenting an image into coherent regions is a difficult task
- Often, it is preferable to detect edges using only purely local information

Edge Detection

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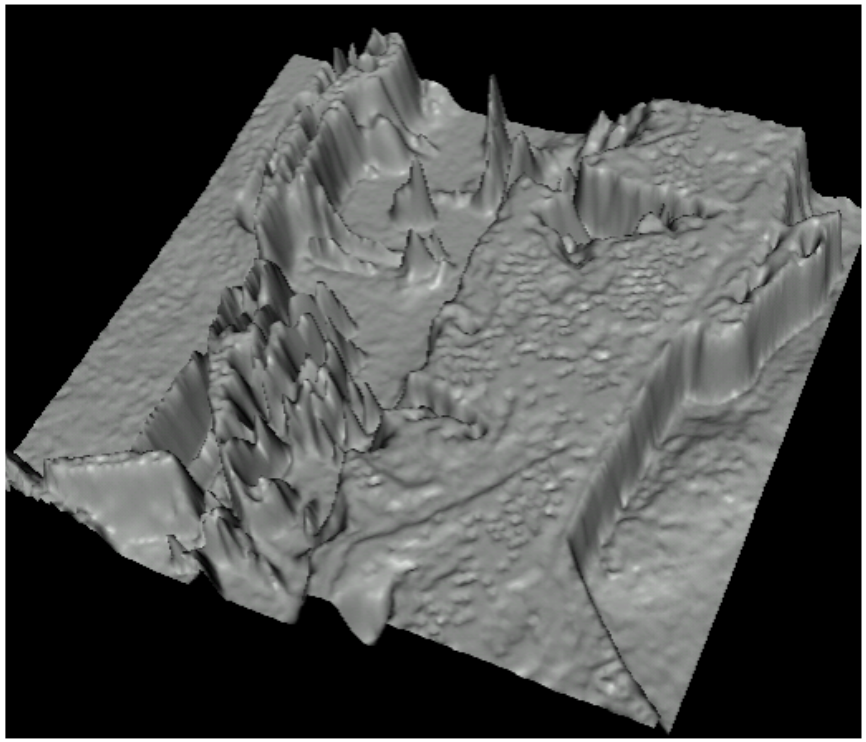
Edge Detection

- Under such conditions, a reasonable approach is to define an edge as the location of **rapid intensity variation**
- If we think of an image as a height field, then on such a surface, edges occur at locations of **steep slopes**, or equivalently, in regions of closely packed contour lines (on a topographic map)

Edge Detection



- Edges look like steep slopes (i.e. cliffs)

Image Gradients

- Mathematically, we define the slope and direction of a surface through its gradient

$$\mathbf{J}(\mathbf{x}) = \nabla I(\mathbf{x}) = (\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y})(\mathbf{x})$$

where the local gradient vector **J** points in the direction of **steepest ascent** in the intensity function

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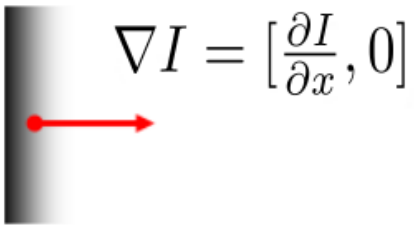
  where the local gradient vector **J** points in the direction of **steepest ascent** in the intensity function

- Its magnitude is an indication of the slope or strength of the variation, while its orientation points in a direction *perpendicular* to the local contour

Image Gradients



$$\nabla I = [\frac{\partial I}{\partial x}, 0]$$

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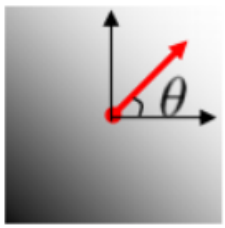

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- The edge strength is given by the **gradient magnitude**

$$||\nabla I|| = \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2}$$

Discrete Image Gradients

y

x

62	79	23	119	120	105	4	0
10	10	9	62	12	78	34	0
10	58	197	46	46	0	0	48
176	135	5	188	191	68	0	49
2	1	1	29	26	37	0	77
0	89	144	147	187	102	62	208
255	252	0	166	123	62	0	31
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- How can we differentiate a digital image $I(x, y)$?
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- Better approximations of the derivative exist

Sobel Operator

- The **Sobel** operator is commonly used to approximate image derivatives

$$G_x = \frac{1}{8}\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \qquad G_y = \frac{1}{8}\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

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  - It does not make a difference for edge detection
  - However, the term is needed to get the correct gradient value

Other Gradient Operators

- The **Roberts Cross** operator

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- The **Prewitt** operator

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Example: Edge detection using MATLAB

- Read an image into the workspace and find edges using the Sobel and Canny methods:

```
I = imread('circuit.tif');
imshow(I)
E1 = edge(I, 'Sobel');
E2 = edge(I, 'Canny');
imshowpair(E1, E2, 'montage')
```

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- Therefore, it is good practice to smooth the image with a low-pass filter prior to computing the gradient
- Because we would like the response of our edge detector to be independent of orientation, a circularly symmetric smoothing filter is desirable

Image Smoothing

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- Since differentiation is a linear operation, it commutes with other linear filtering operations
- The gradient of the smoothed image can therefore be written as

$$\mathbf{J}_\sigma(\mathbf{x}) = \nabla[G_\sigma(\mathbf{x}) * I(\mathbf{x})] = [\nabla G_\sigma](\mathbf{x}) * I(\mathbf{x})$$

i.e. we can convolve the image with the horizontal and vertical derivatives of the Gaussian kernel function

$$\nabla G_\sigma(\mathbf{x}) = (\frac{\partial G_\sigma}{\partial x}\frac{\partial G_\sigma}{\partial y})(\mathbf{x}) = [-x-y]\frac{1}{\sigma^3}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

where σ indicates the width of the Gaussian

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Laplician of Gaussian

- For many applications, we want to thin such a continuous gradient image to only return isolated edges, i.e. as single pixels at discrete locations along the edge contours
- This can be achieved by looking for *maxima* in the edge strength (gradient magnitude) in a direction *perpendicular* to the edge orientation, i.e. along the gradient direction

Laplician of Gaussian

- Finding this maximum corresponds to taking a directional derivative of the strength field in the direction of the gradient and then looking for zero crossings

Laplician of Gaussian

- Finding this maximum corresponds to taking a directional derivative of the strength field in the direction of the gradient and then looking for zero crossings
- The desired directional derivative is equivalent to the dot product between a second gradient operator and the results of the first

$$S_\sigma(\mathbf{x}) = \nabla \cdot \mathbf{J}_\sigma(\mathbf{x}) = [\nabla^2 G_\sigma](\mathbf{x}) * I(\mathbf{x})]$$

Laplician of Gaussian

- The gradient operator dot product with the gradient is called the **Laplacian**

Laplician of Gaussian

- The gradient operator dot product with the gradient is called the **Laplacian**
- The convolution kernel

$$\nabla^2 G_\sigma(\mathbf{x}) = \frac{1}{\sigma^3}\left(2 - \frac{x^2+y^2}{2\sigma^2}\right)\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

is therefore called the **Laplacian of Gaussian** (LoG) kernel

Laplician of Gaussian

- This kernel can be split into two separable parts

$$\nabla^2 G_\sigma(\mathbf{x}) = \frac{1}{\sigma^3}\left(1-\frac{x^2}{2\sigma^2}\right) G_\sigma(x) G_\sigma(y) + \frac{1}{\sigma^3}\left(1-\frac{y^2}{2\sigma^2}\right) G_\sigma(x) G_\sigma(y)$$

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- In practice, it is quite common to replace the LoG convolution with a DoG computation since the kernel shapes are qualitatively similar

Edge Elements

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- The sub-pixel location of this crossing can be obtained by computing the "x-intercept" of the "line" connecting $S(\mathbf{x}_i)$ and $S(\mathbf{x}_j)$

$$\mathbf{x}_z = \frac{\mathbf{x}_i S(\mathbf{x}_j) - \mathbf{x}_j S(\mathbf{x}_i)}{S(\mathbf{x}_j) - S(\mathbf{x}_i)}$$

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- If edges have been detected using zero crossings of some function, then linking them up is straightforward since adjacent edgels share common endpoints
- Linking edgels into chains involves sorting them and then picking an unlinked edgel and following its neighbors in both directions

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- Once the edgels have been linked into chains, we can apply an optional thresholding with **hysteresis** to remove low-strength contour segments
- The basic idea of hysteresis is to set two different thresholds and allow a curve being tracked above the higher threshold to dip in strength down to the lower threshold

Edge Linking

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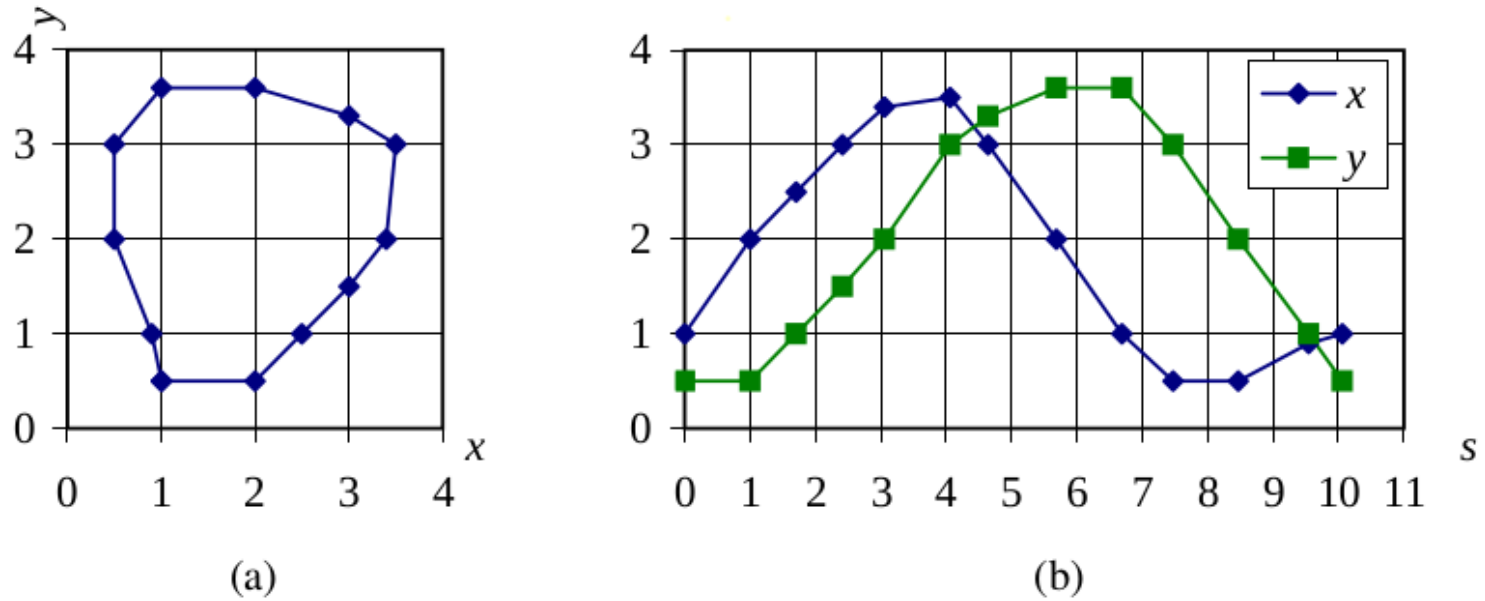
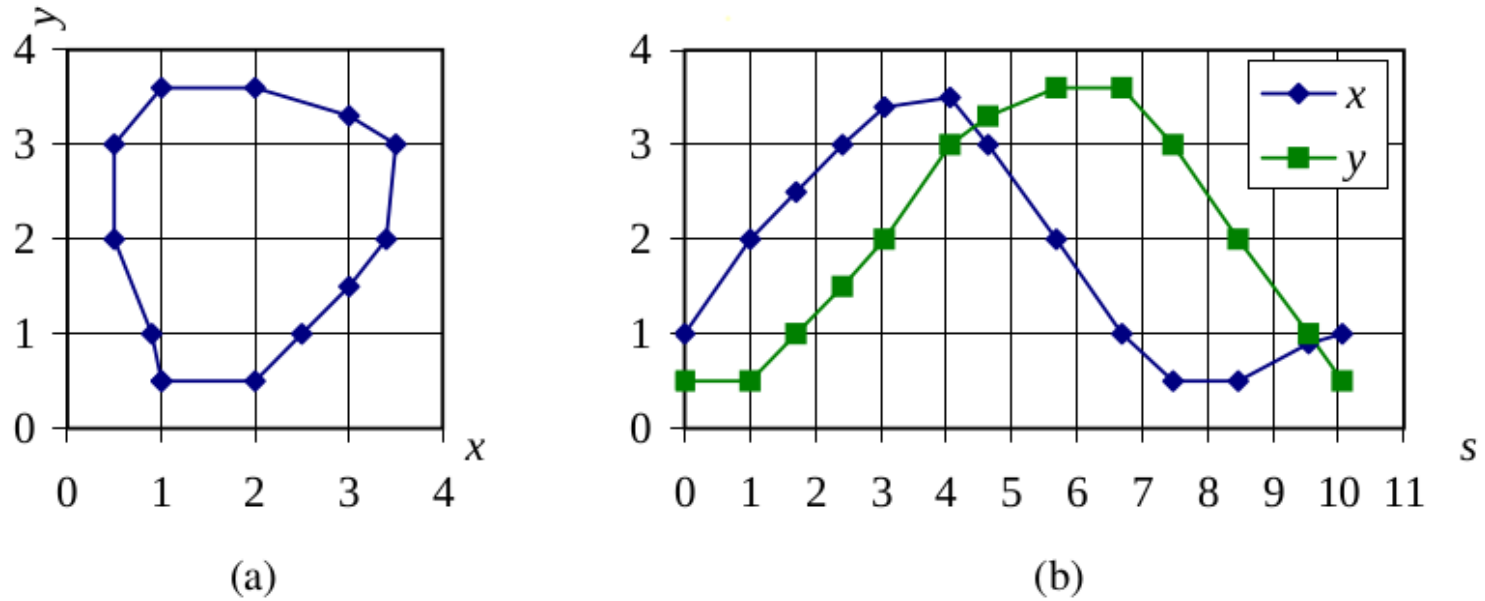
- Linked edgel lists can be encoded more compactly using a variety of alternative representations such as the **arc length parameterization** of a contour, $\mathbf{x}_s$, where s denotes the arc length along a curve
- The advantage of the arc-length parameterization is that it makes matching and processing (e.g. smoothing) operations much easier

Edge Linking



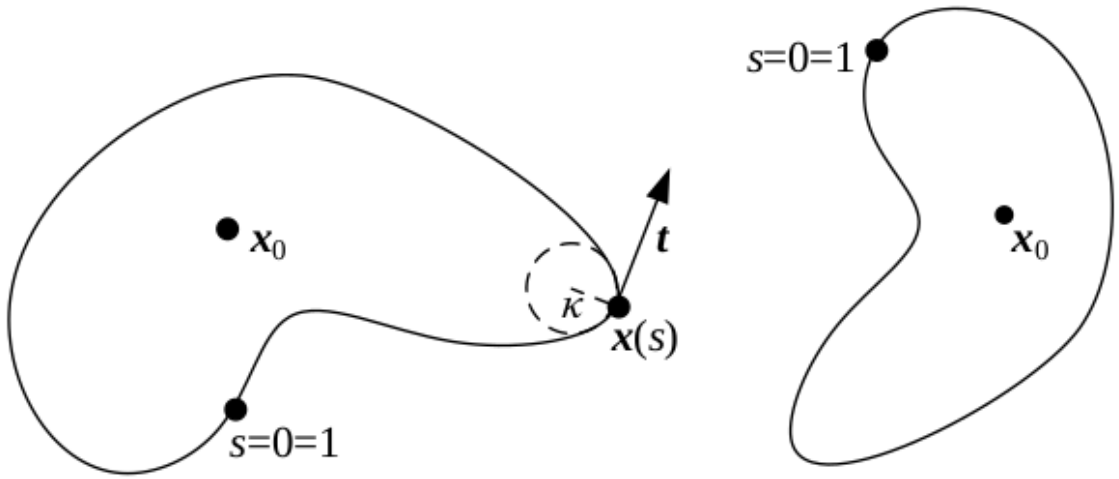
- Arc-length parameterization of a contour: (a) discrete points along the contour are first transcribed as (b) (x, y) pairs along the arc length s

Edge Linking



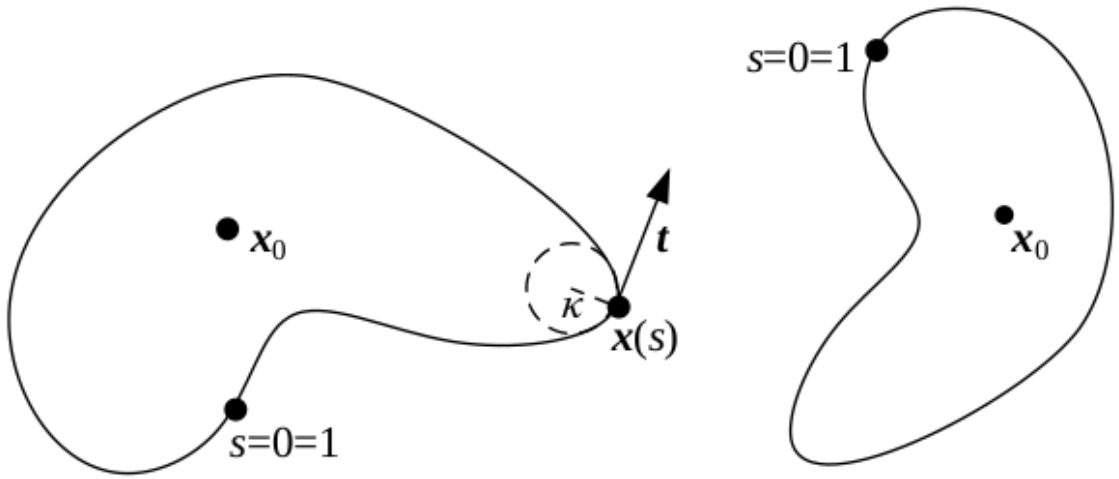
- Arc-length parameterization of a contour: (a) discrete points along the contour are first transcribed as (b) (x, y) pairs along the arc length s
- This curve can then be regularly re-sampled or converted into alternative (e.g. Fourier) representations

Edge Linking



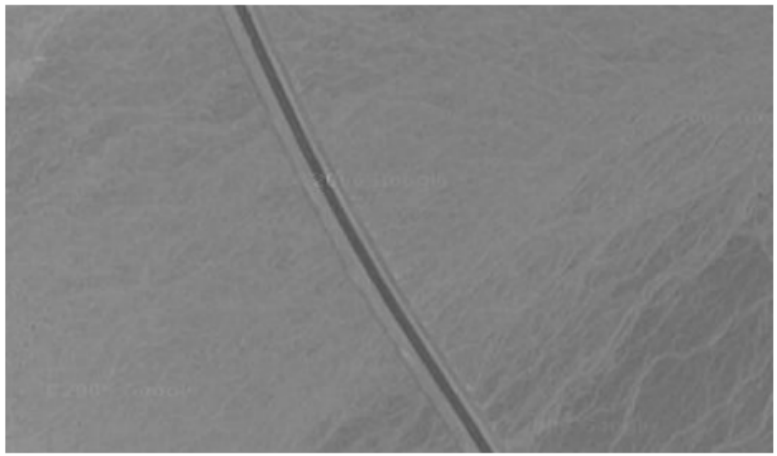
- Matching two contours using their arc-length parameterization

Edge Linking



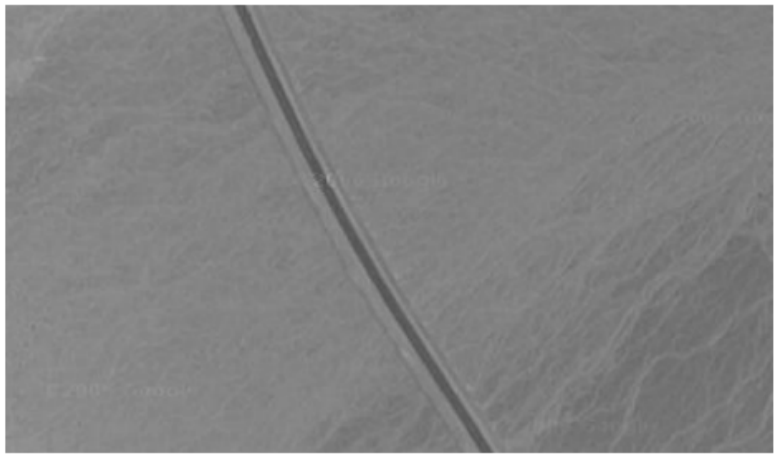
- Matching two contours using their arc-length parameterization
- If both curves are normalized to unit length, $s \in [0, 1]$ and centered around their centroid $\mathbf{x}_0$, they will have the same descriptor up to an overall "temporal" shift (due to different starting points for $s = 0$) and a phase (x-y) shift (due to rotation)

Detecting and Matching Lines




- Although edges and general curves are suitable for describing the contours of natural objects, our world is full of straight lines

Detecting and Matching Lines




- Although edges and general curves are suitable for describing the contours of natural objects, our world is full of straight lines
- Detecting and matching these lines can be useful in a variety of computer vision applications

What is a Straight Line?

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- It is the shortest path that connects two points

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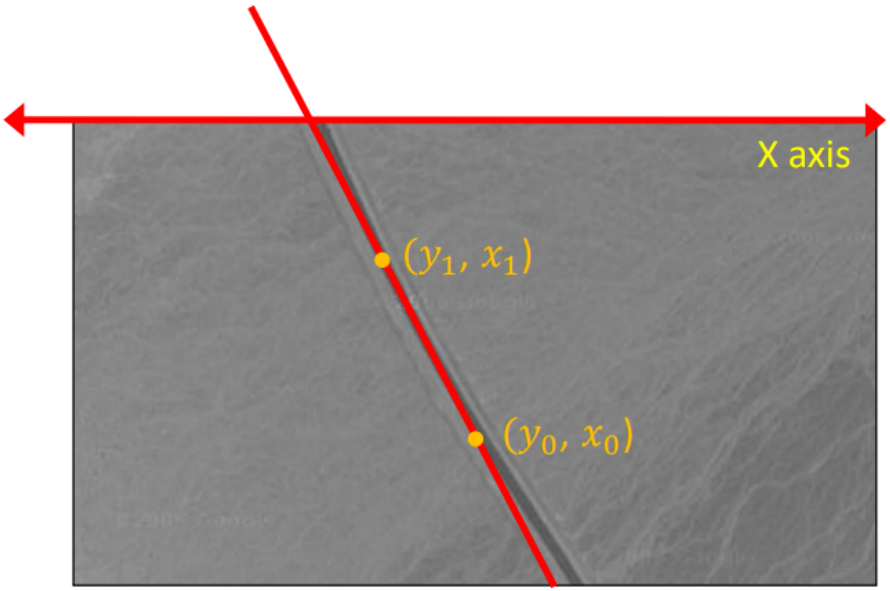
- How many parameters do we need to define a line in an image?

Defining Straight Lines



- One option for defining a line is to use four numbers, i.e. points the (y_0, x_0) and (y_1, x_1) (slope-intercept)

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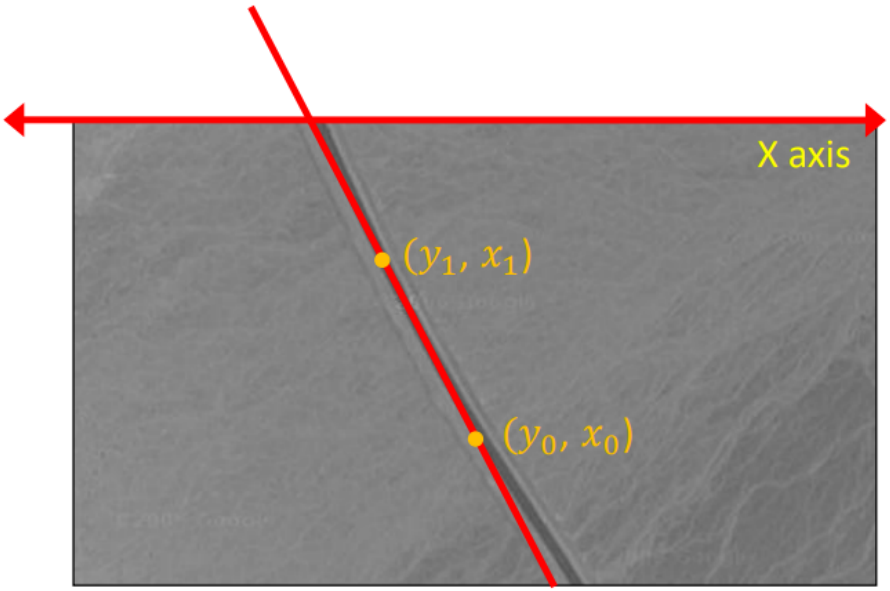

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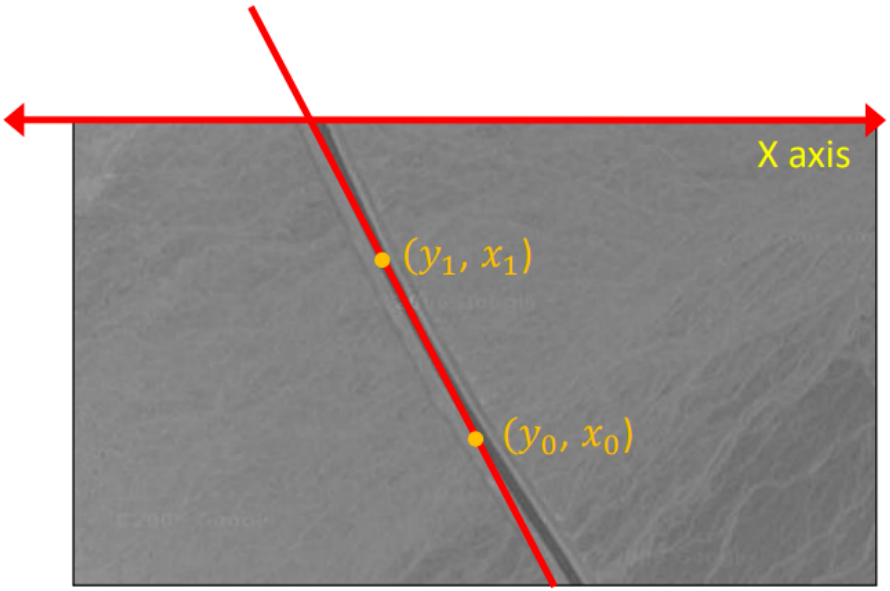

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- Note that any pair of distinct points on the line can be used

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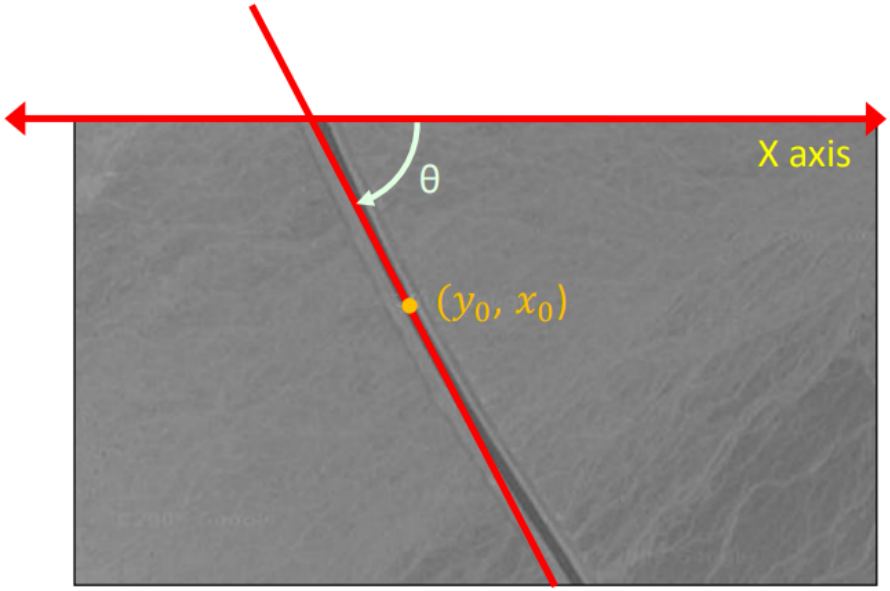

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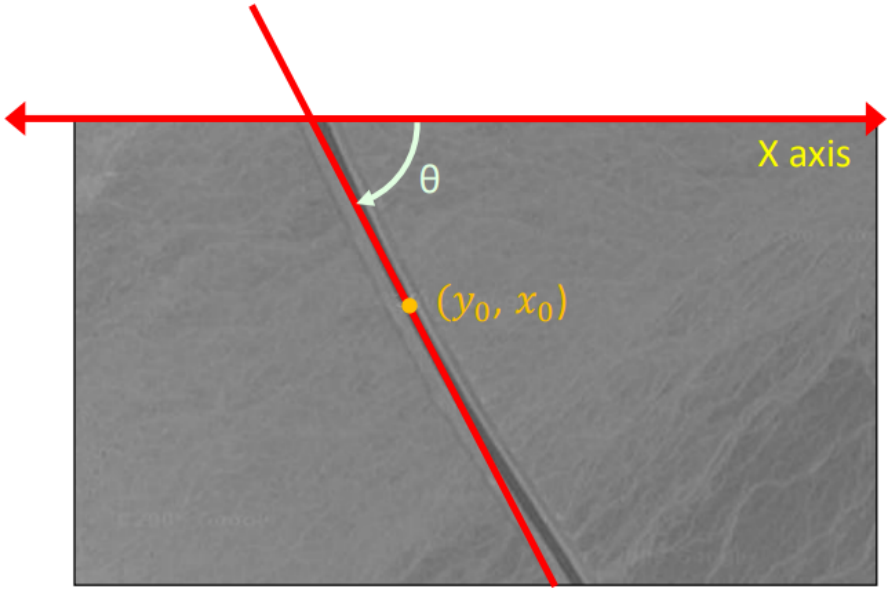

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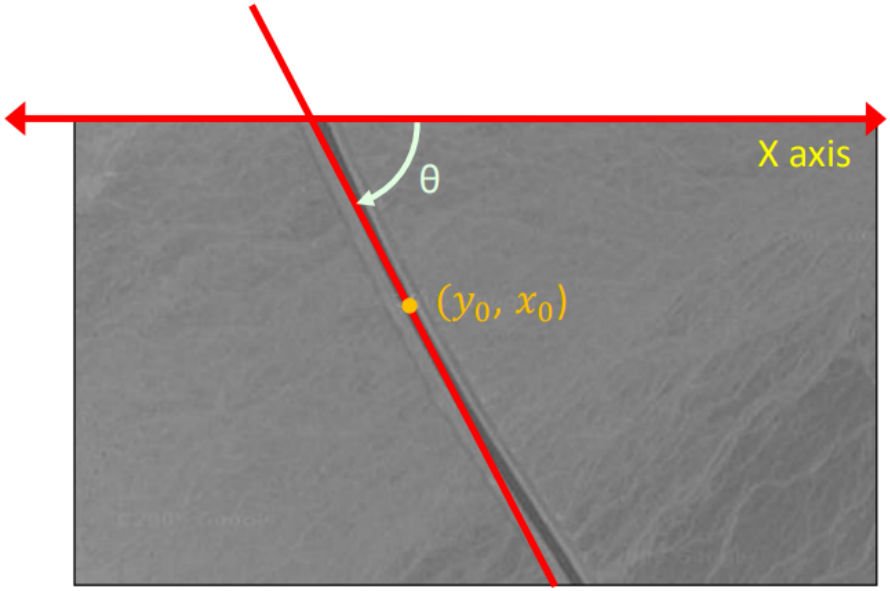

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- Then, the line is $\{(y, x) \,|\, y = y_0 + \tan(\theta) \cdot (x - x_0)\}$
- Note that $\theta \in [0, 180]$ and any point (y_0, x_0) on the line can be used

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- Thus, we have an infinite number of choices to define a line
  - We prefer to have a single choice

Defining Vertical Lines

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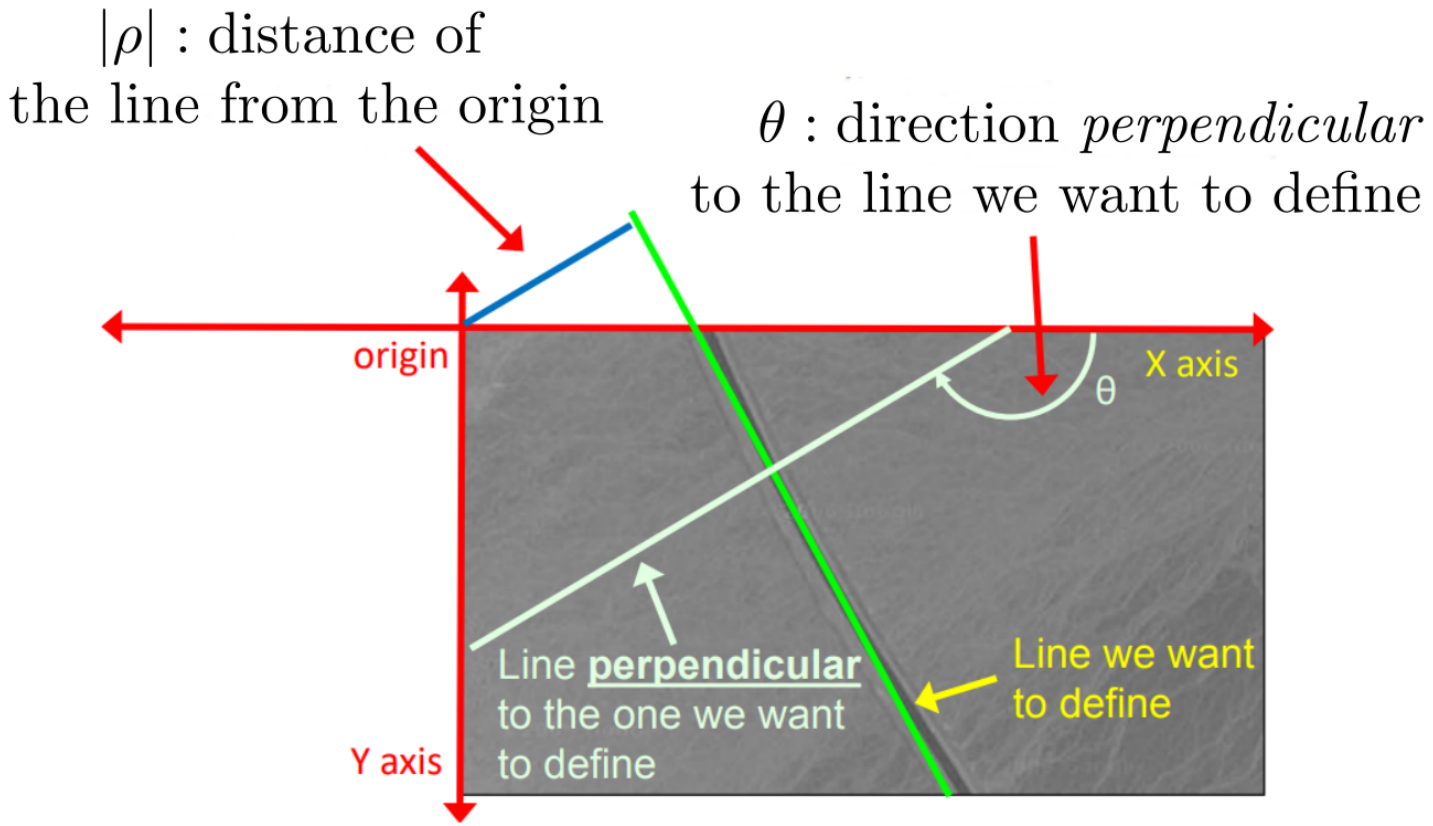
  - $\{(y, x) \,|\, y = y_0 + \underbrace{\frac{y_1 - y_0}{x_1 - x_0}}_{\text{division by 0}} \cdot (x - x_0)\}$

Defining Vertical Lines

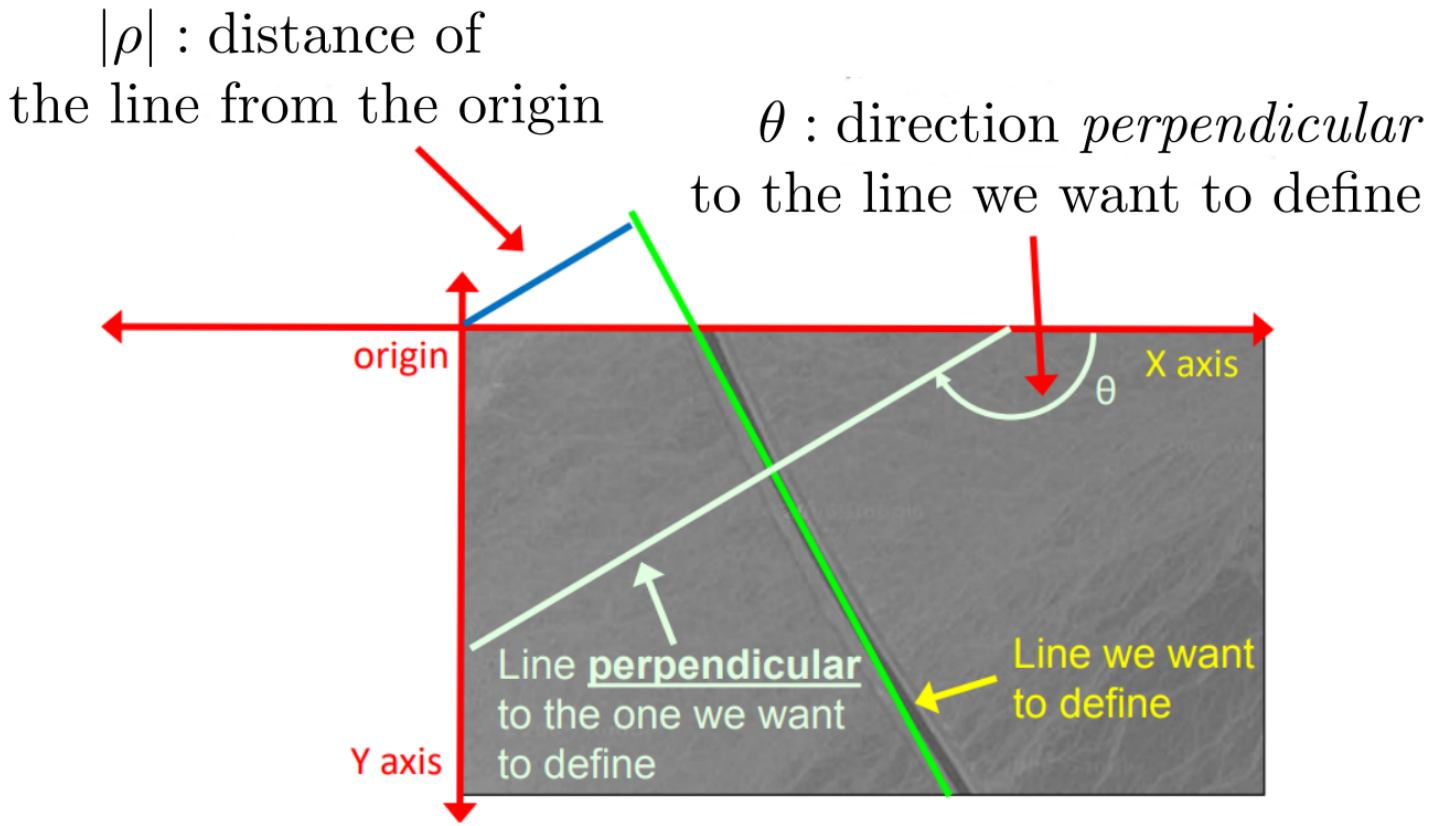
- Additionally, none of these options are appropriate for defining a *vertical* line
  - $\{(y, x) \,|\, y = y_0 + \underbrace{\frac{y_1 - y_0}{x_1 - x_0}}_{\text{division by 0}} \cdot (x - x_0)\}$
  - $\{(y, x) \,|\, y = y_0 + \underbrace{\tan(\theta)}_{\tan(90°)=\infty} \cdot (x - x_0)\}$

Defining Straight Lines in Polar Coordinates



- We define the line in polar coordinates using ρ and θ

Defining Straight Lines in Polar Coordinates



- We define the line in polar coordinates using ρ and θ
- Then, the line is $\{(y, x) \,|\, \rho = x \cdot \cos(\theta) + y \cdot \sin(\theta)\}$

Defining Straight Lines in Polar Coordinates

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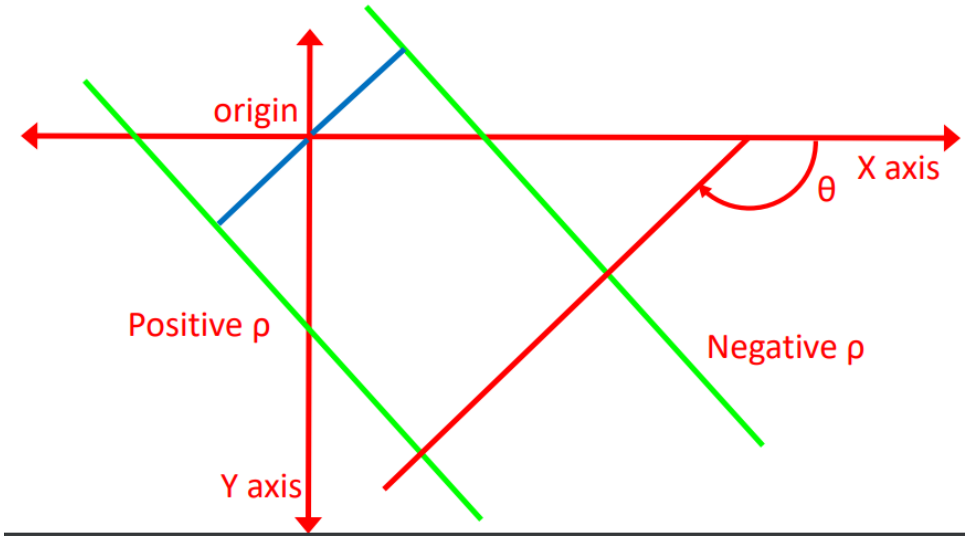
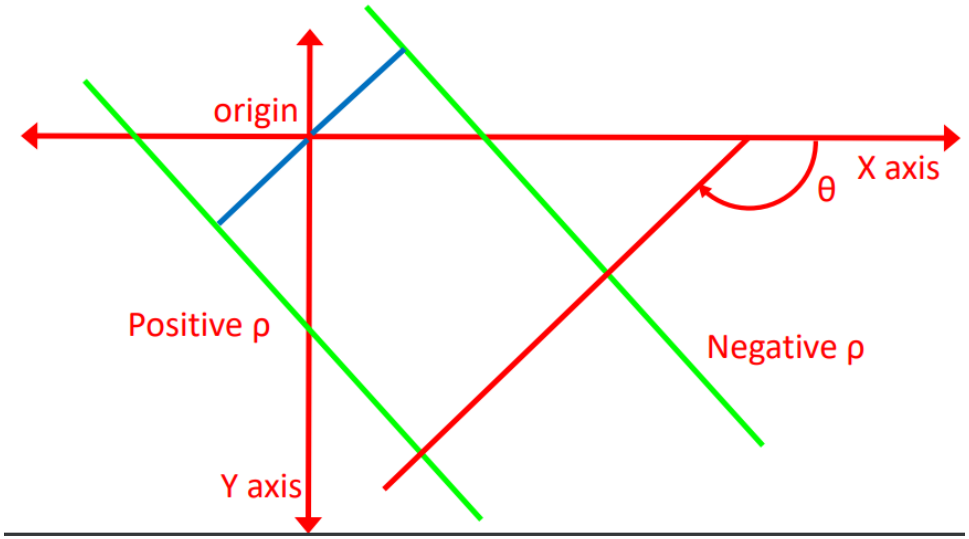
- Note the following important characteristics of this definition
  - Non-redundant - a single line corresponds to a single ρ and a single θ as long as $0^\circ \leq \theta < 180^\circ$
  - Robust - any line (including vertical lines) can be defined using this form

Positive and Negative ρ



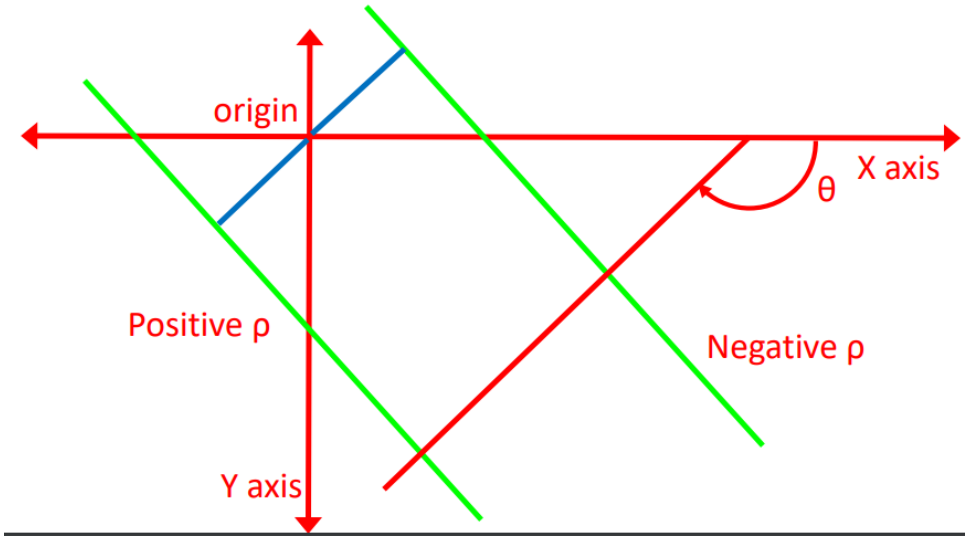
- Note that ρ can be positive or negative

Positive and Negative ρ



- Note that ρ can be positive or negative
- Consider the two green lines

Positive and Negative ρ



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  - They have the same θ and same distance to the origin

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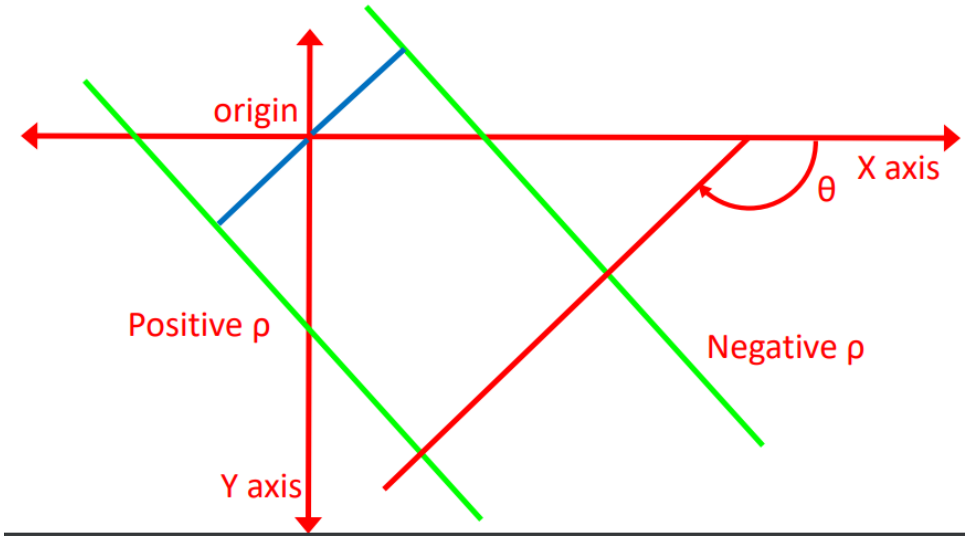

- Note that ρ can be positive or negative
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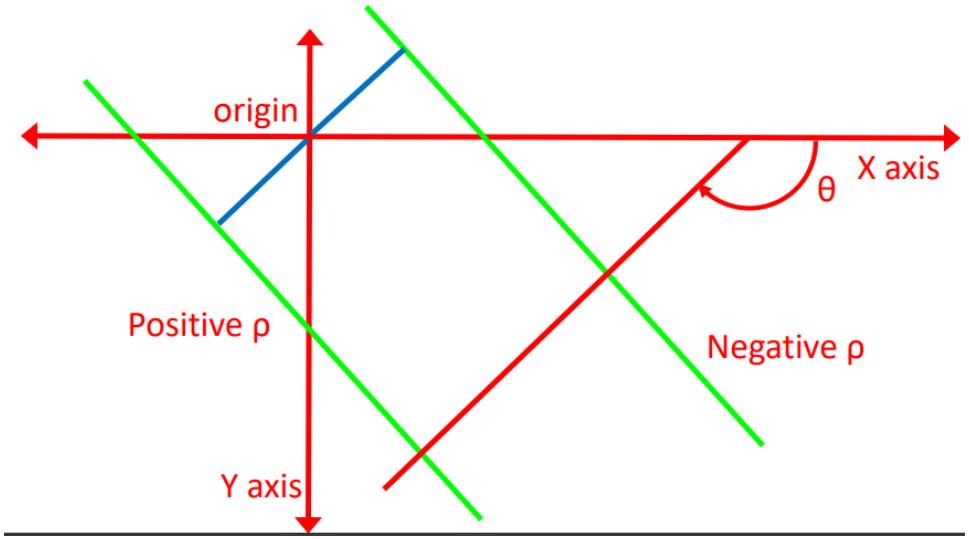

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- Consider the two green lines
  - They have the same θ and same distance to the origin
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  - For both lines, $|\rho|$ is their distance from the origin

Hough Transform

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- This is done by using the local information at each edge pixel to "vote" for all the lines that it is a part of
- Each detected line will be specified by its ρ and θ parameters

Voting

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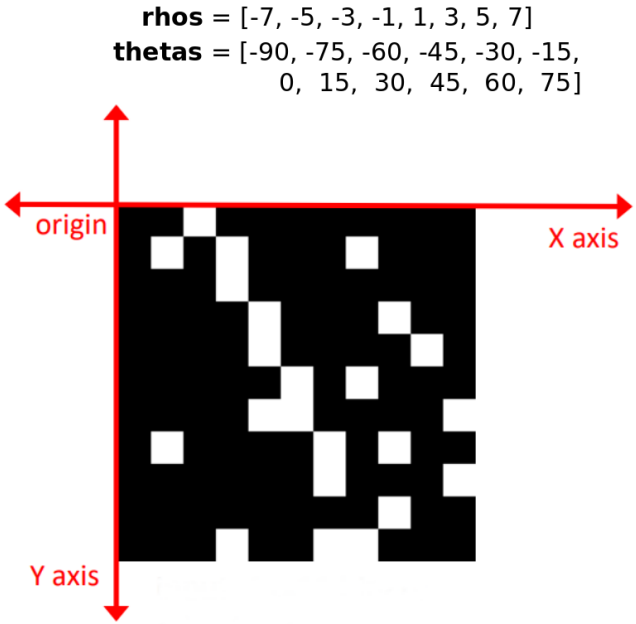
  - Solution: we discretize the set of lines to a finite number, i.e. we choose the numbers of ρ and θ
- Thus, the voting matrix (accumulator) is of size # of $\rho \times$ # of θ

Hough Transform Algorithm

```
define rhos array
define thetas array
accumulator = zeros(size of rhos, size of thetas)
for every pixel (i,j):
 if (i,j) not an edge pixel, then continue
 for every θ in thetas:
   ρ = find_rho(i,j,θ)
   Add 1 to accumulator for line with parameters (ρ,θ)
```

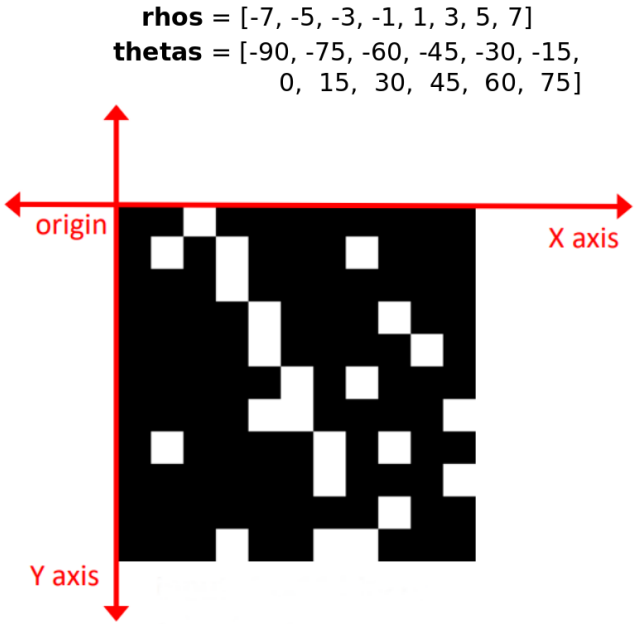
Example: Voting on an 11×11 Binary Edge Image

- Do we have any other options for the size of **rhos** and **thetas**?



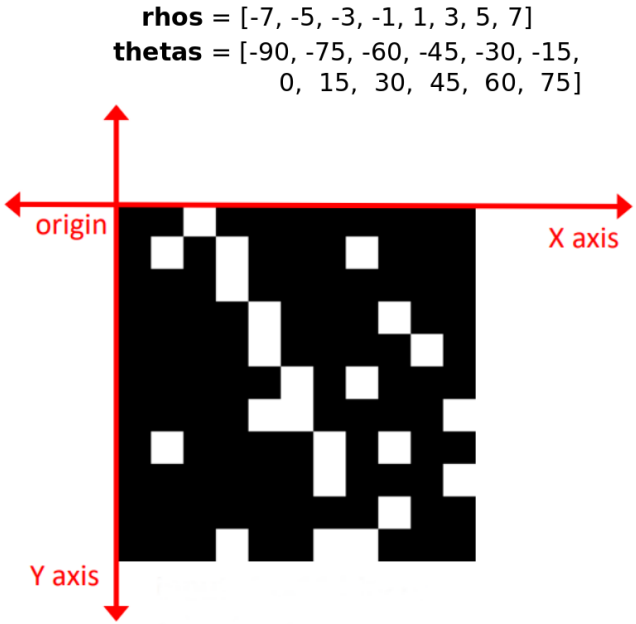
Example: Voting on an 11×11 Binary Edge Image

- Do we have any other options for the size of **rhos** and **thetas**?
- What is the size of the accumulator?



Example: Voting on an 11×11 Binary Edge Image

- Size of the accumulator: 8×12



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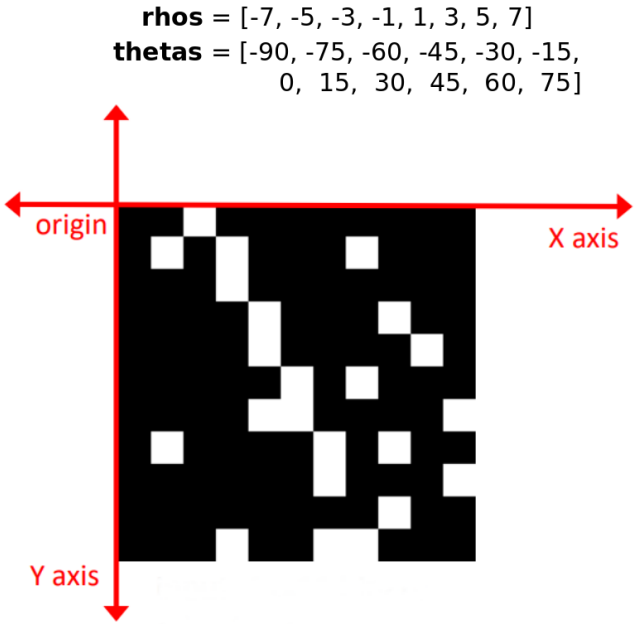

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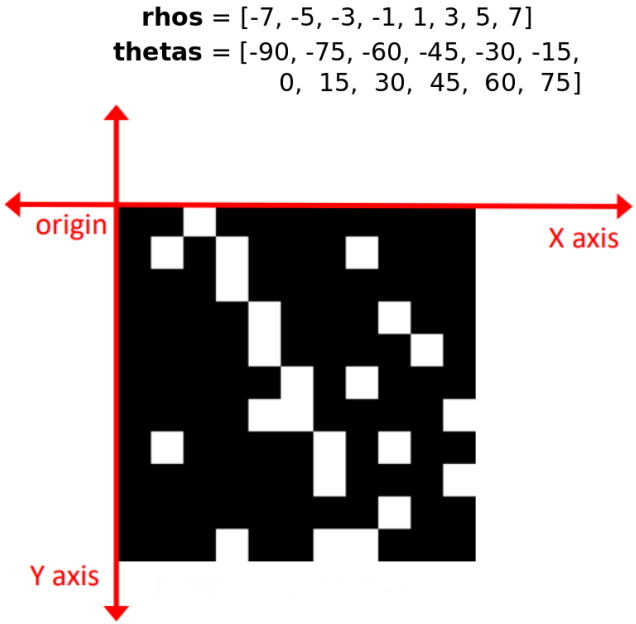
- For each pixel:



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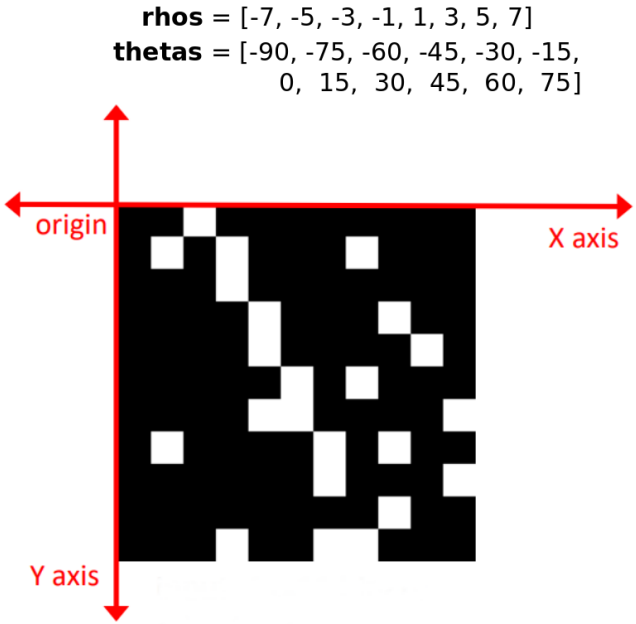

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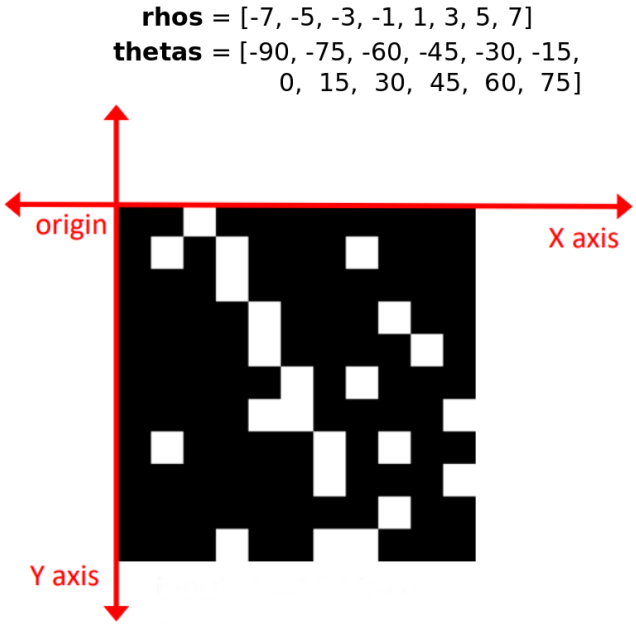
- For each pixel:
  - If it is an edge pixel:
    - For each θ in **thetas**:

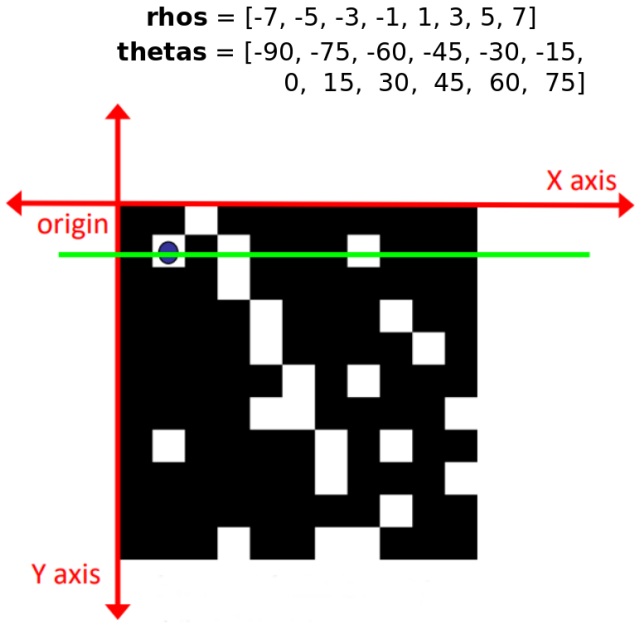
      - Find the corresponding ρ

      - Add a vote to the position in the accumulator for that ρ and θ



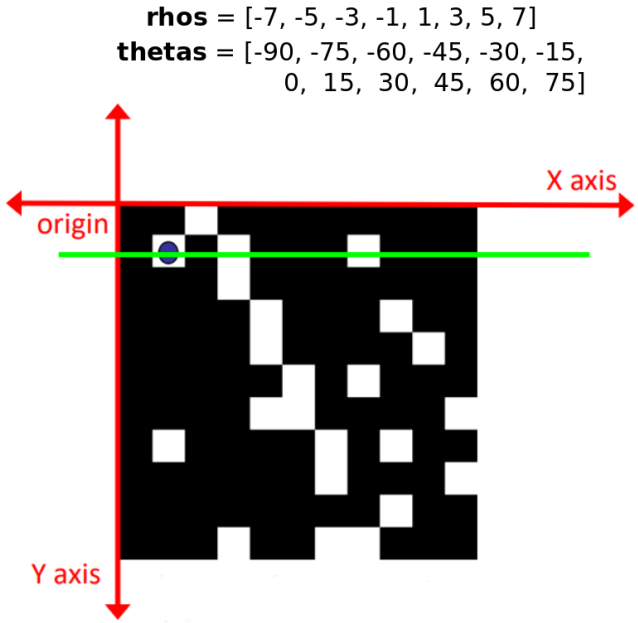
Example: Voting on an 11×11 Binary Edge Image

- Suppose we are at pixel (2,2)



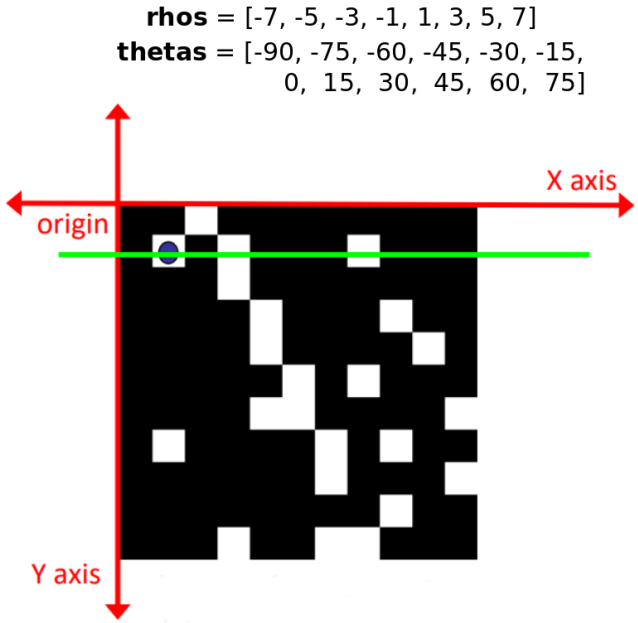
Example: Voting on an 11×11 Binary Edge Image

- Suppose we are at pixel (2,2)
- For each θ in **thetas**:
  - What do we do?



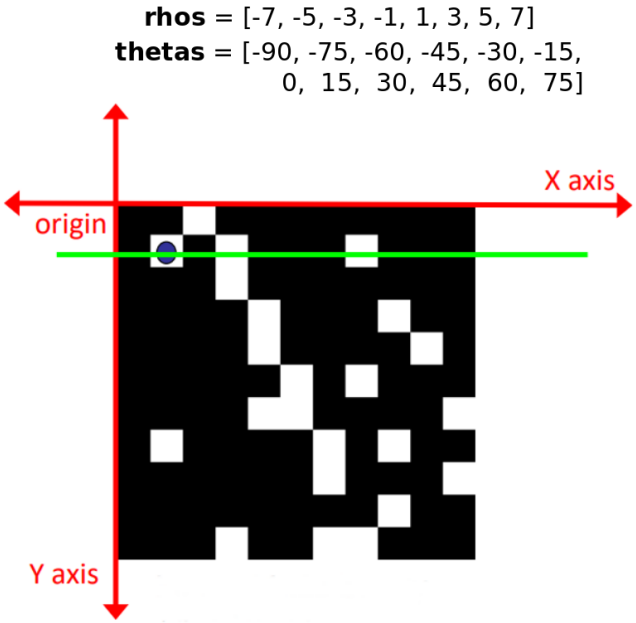
Example: Voting on an 11×11 Binary Edge Image

- We start with $\theta = -90°$



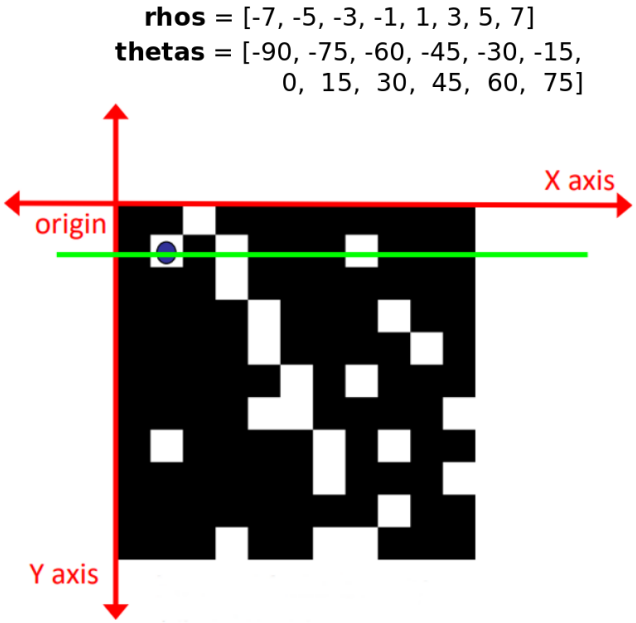
Example: Voting on an 11×11 Binary Edge Image

- We start with $\theta = -90°$
- Remember θ is the direction perpendicular to the line, thus the line itself has orientation of $0°$



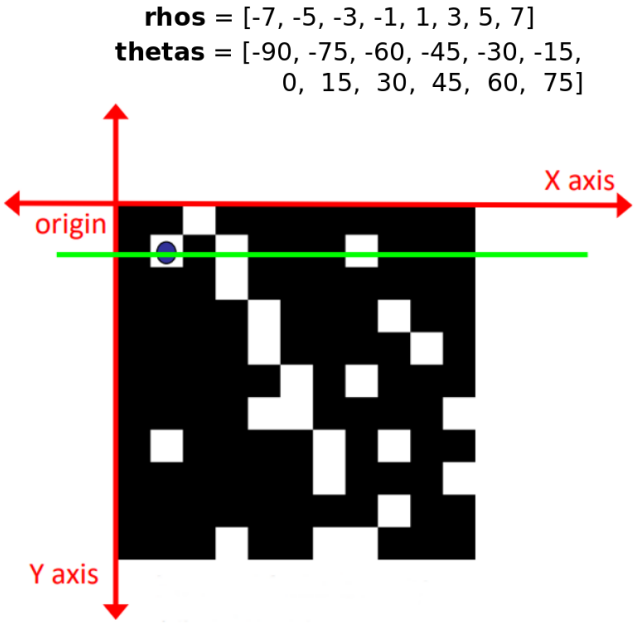
Example: Voting on an 11×11 Binary Edge Image

- We start with $\theta = -90°$
- Remember θ is the direction perpendicular to the line, thus the line itself has orientation of $0°$
- How do we compute ρ?



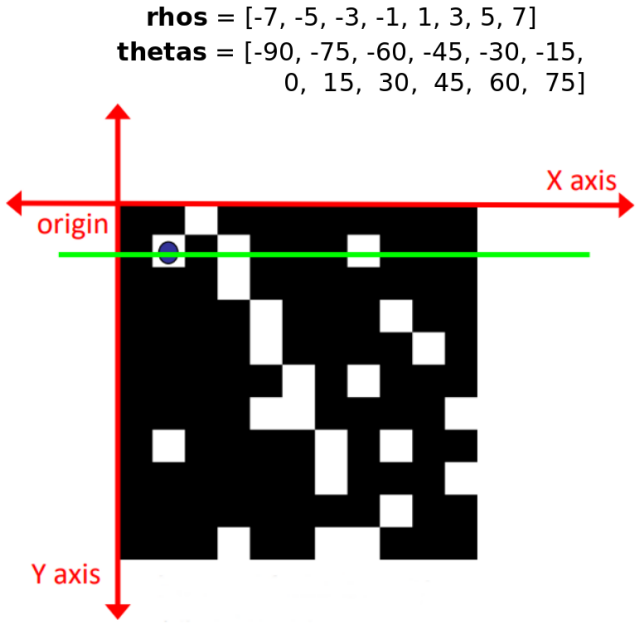
Example: Voting on an 11×11 Binary Edge Image

- We start with $\theta = -90°$
- Remember θ is the direction perpendicular to the line, thus the line itself has orientation of $0°$
- How do we compute ρ?
  - $\rho = x \cdot \cos(\theta) + y \cdot \sin(\theta) = 2 \cdot 0 + 2 \cdot -1 = -2$



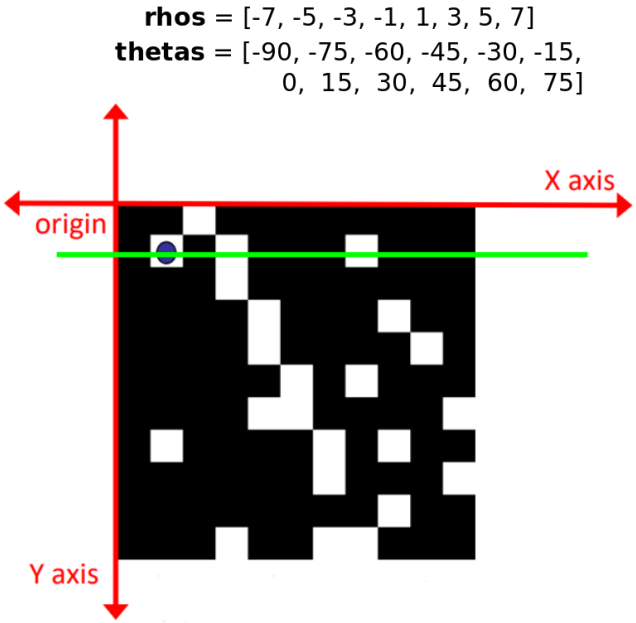
Example: Voting on an 11×11 Binary Edge Image

- What do we vote for?



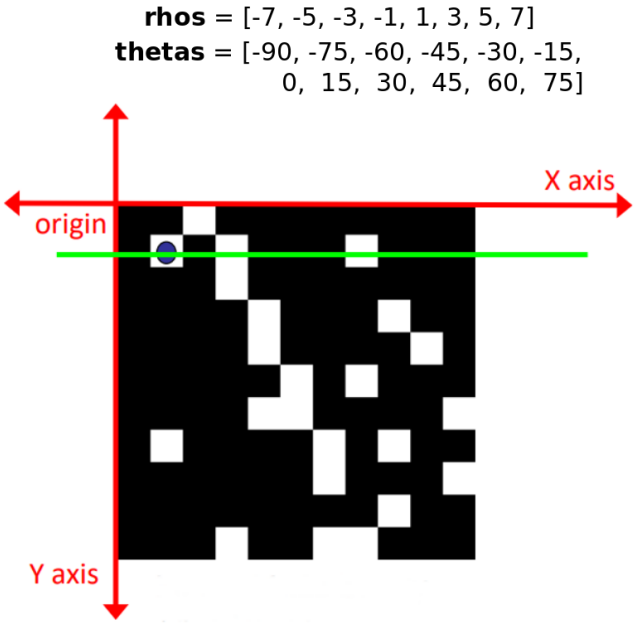
Example: Voting on an 11 × 11 Binary Edge Image

- What do we vote for?
- The closest values to -2 in **rhos** are -3 and -1 (-2 is halfway between)



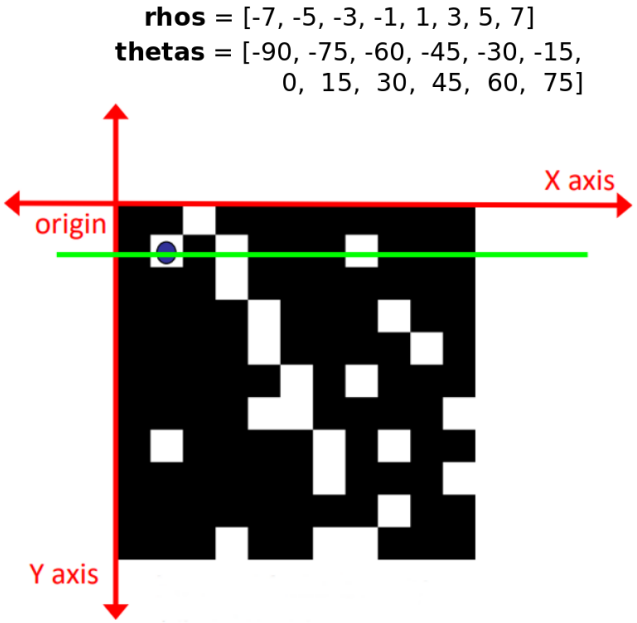
Example: Voting on an 11 × 11 Binary Edge Image

- What do we vote for?
- The closest values to -2 in **rhos** are -3 and -1 (-2 is halfway between)
  - Ties like this are rare, but they can happen



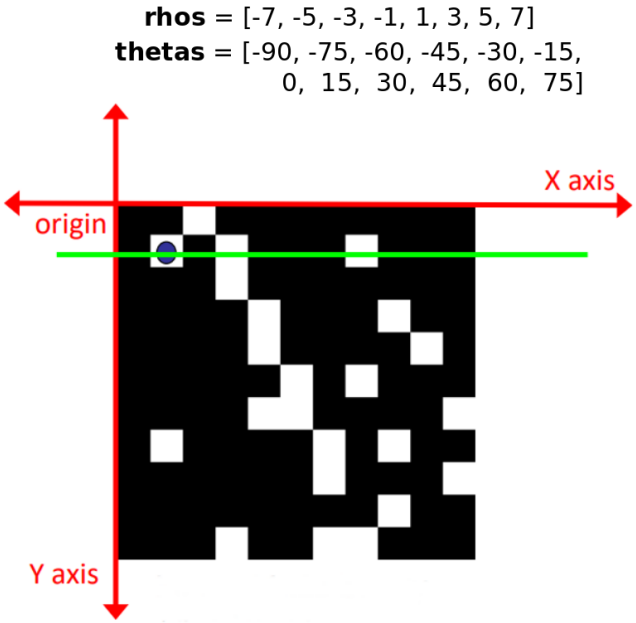
Example: Voting on an 11×11 Binary Edge Image

- What do we vote for?
- The closest values to -2 in **rhos** are -3 and -1 (-2 is halfway between)
  - Ties like this are rare, but they can happen
  - Thus, we split the vote between $(\rho = -3, \theta = -90)$ and $(\rho = -1, \theta = -90)$ by giving a 0.5 vote to each



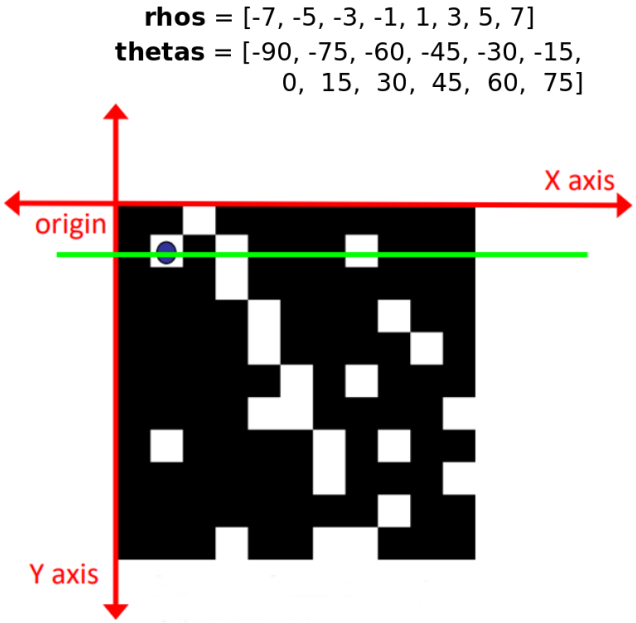
Example: Voting on an 11×11 Binary Edge Image

- Which position in the accumulator corresponds to $(\rho = -3, \theta = -90)$?



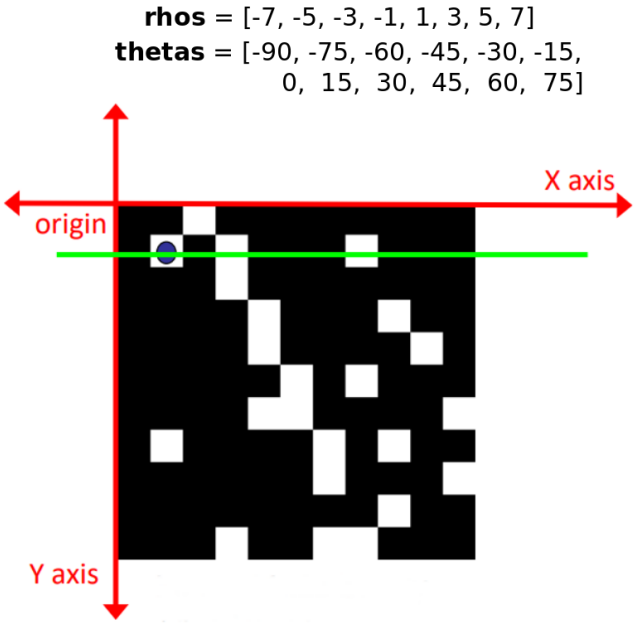
Example: Voting on an 11 × 11 Binary Edge Image

- Which position in the accumulator corresponds to $(\rho = -3, \theta = -90)$?
  - -3 is in the 3rd position of **rhos**



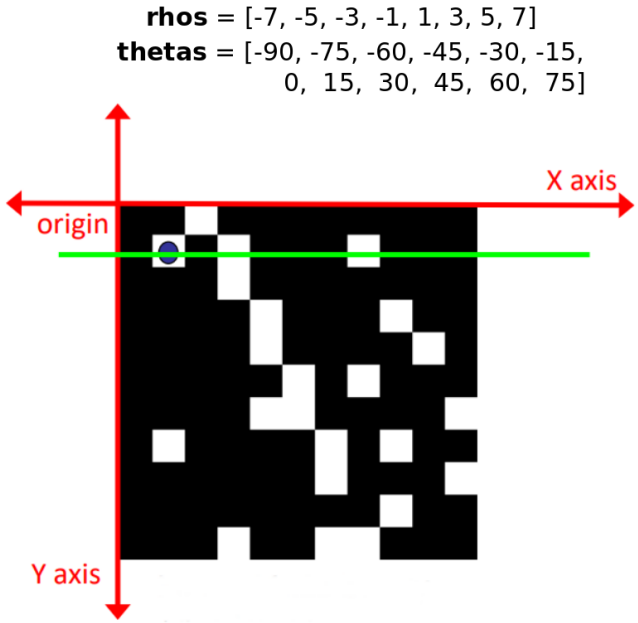
Example: Voting on an 11×11 Binary Edge Image

- Which position in the accumulator corresponds to $(\rho = -3, \theta = -90)$?
  - -3 is in the 3rd position of **rhos**
  - -90 is in the 1st position of **thetas**



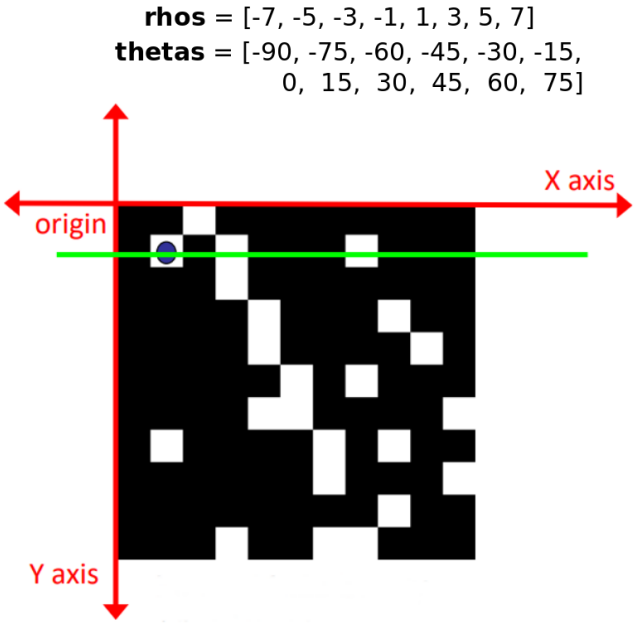
Example: Voting on an 11×11 Binary Edge Image

- Which position in the accumulator corresponds to $(\rho = -3, \theta = -90)$?
  - -3 is in the 3rd position of **rhos**
  - -90 is in the 1st position of **thetas**
  - Therefore, we add 0.5 to accumulator(3,1)



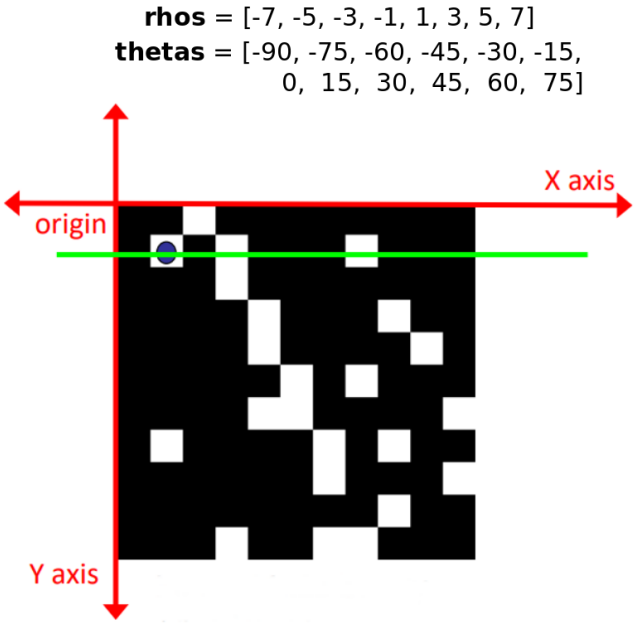
Example: Voting on an 11×11 Binary Edge Image

- Which position in the accumulator corresponds to $(\rho = -1, \theta = -90)$?



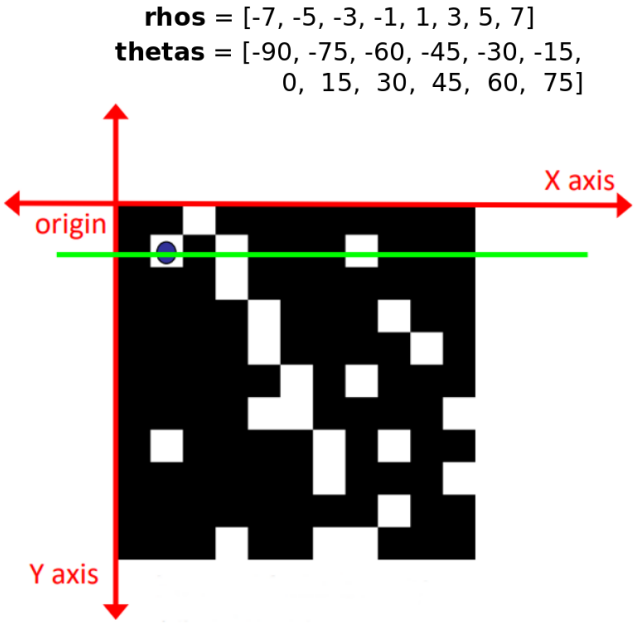
Example: Voting on an 11×11 Binary Edge Image

- Which position in the accumulator corresponds to $(\rho = -1, \theta = -90)$?
  - -1 is in the 4th position of **rhos**



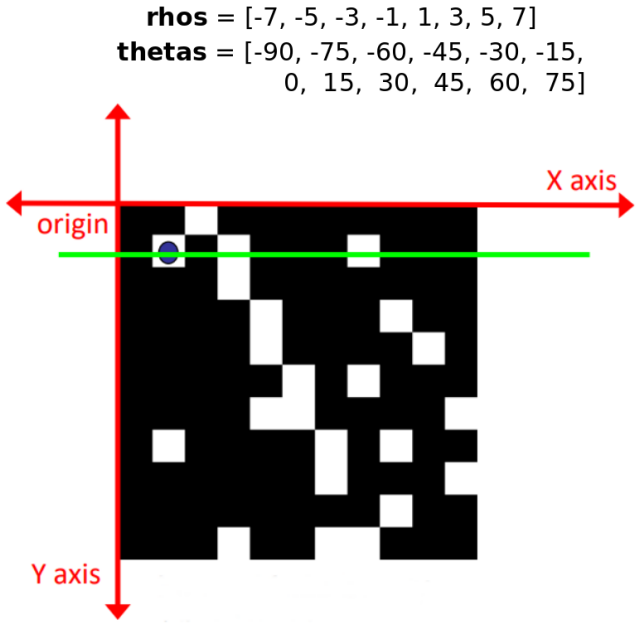
Example: Voting on an 11×11 Binary Edge Image

- Which position in the accumulator corresponds to $(\rho = -1, \theta = -90)$?
  - -1 is in the 4th position of **rhos**
  - -90 is in the 1st position of **thetas**



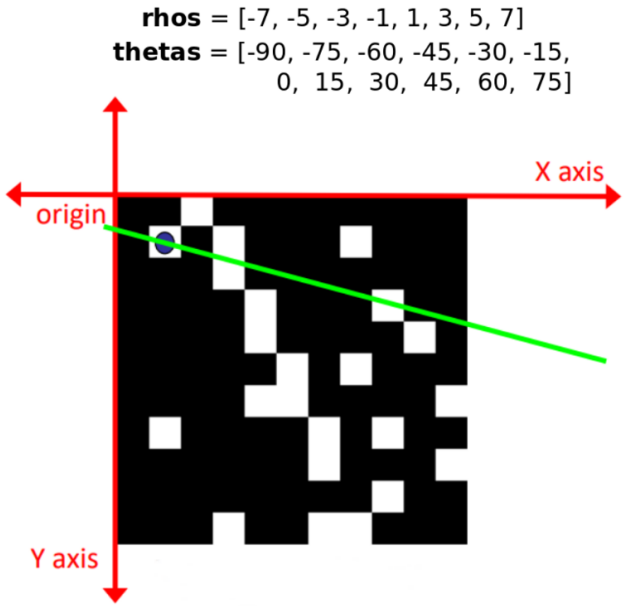
Example: Voting on an 11 × 11 Binary Edge Image

- Which position in the accumulator corresponds to $(\rho = -1, \theta = -90)$?
  - -1 is in the 4th position of **rhos**
  - -90 is in the 1st position of **thetas**
  - Therefore, we add 0.5 to accumulator(4,1)



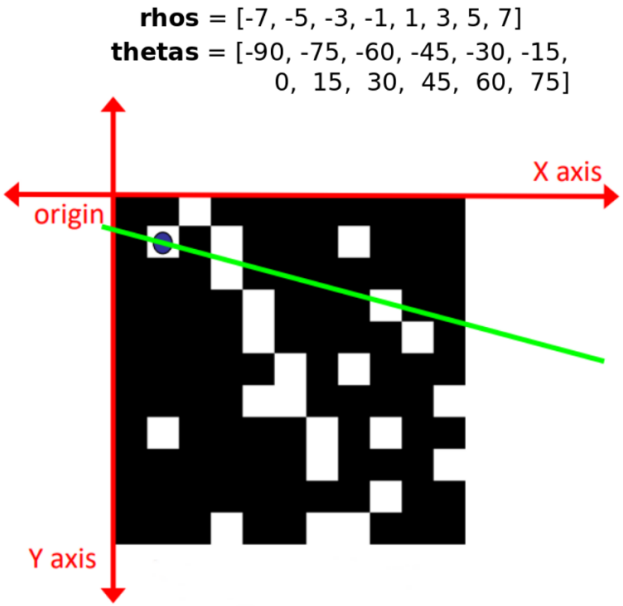
Example: Voting on an 11×11 Binary Edge Image

- Next, we process $\theta = -75°$



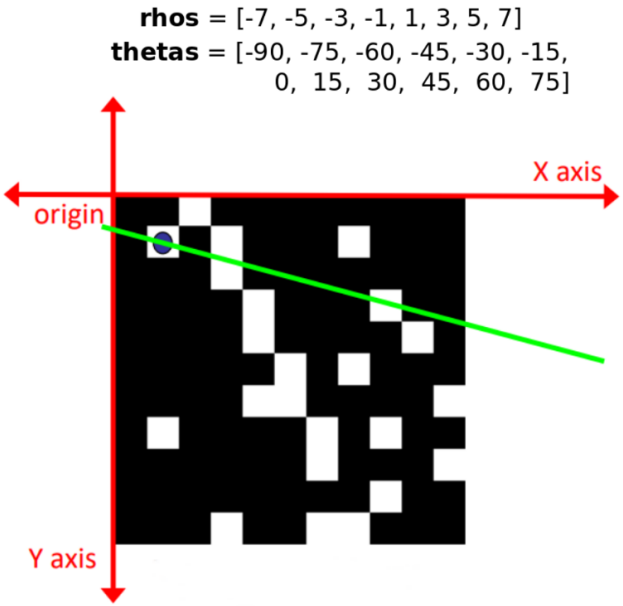
Example: Voting on an 11×11 Binary Edge Image

- Next, we process $\theta = -75°$
- Remember θ is the direction perpendicular to the line, thus the line itself has orientation of $15°$



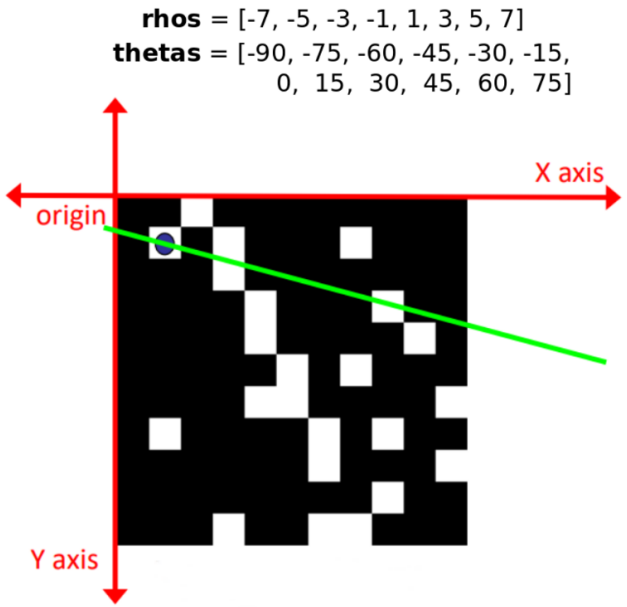
Example: Voting on an 11×11 Binary Edge Image

- Next, we process $\theta = -75°$
- Remember θ is the direction perpendicular to the line, thus the line itself has orientation of $15°$
- $\rho = x \cdot \cos(\theta) + y \cdot \sin(\theta) =$
  $2 \cdot \cos(-75) + 2 \cdot \sin(-75) = -1.414$



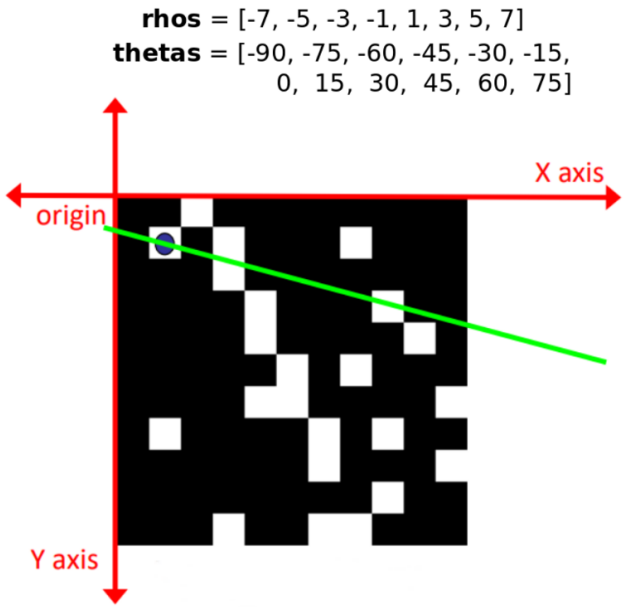
Example: Voting on an 11×11 Binary Edge Image

- What do we vote for?



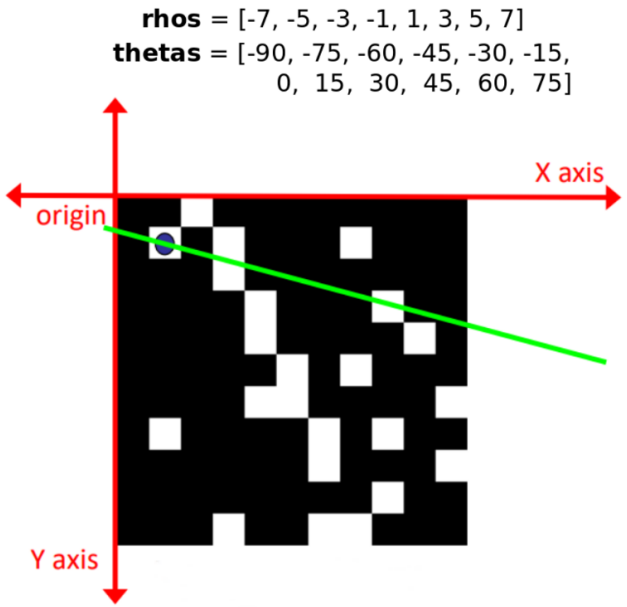
Example: Voting on an 11×11 Binary Edge Image

- What do we vote for?
- The closest value to -1.414 in **rhos** is -1



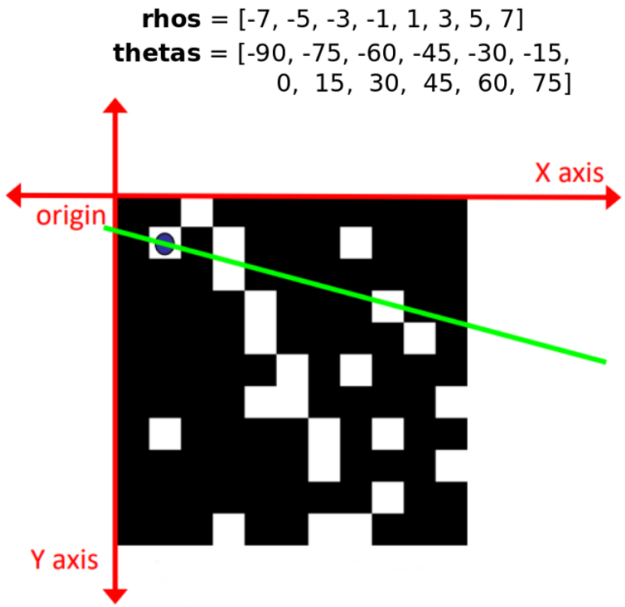
Example: Voting on an 11×11 Binary Edge Image

- What do we vote for?
- The closest value to -1.414 in **rhos** is -1
  - Thus, we vote for $\rho = -1, \theta = -75$



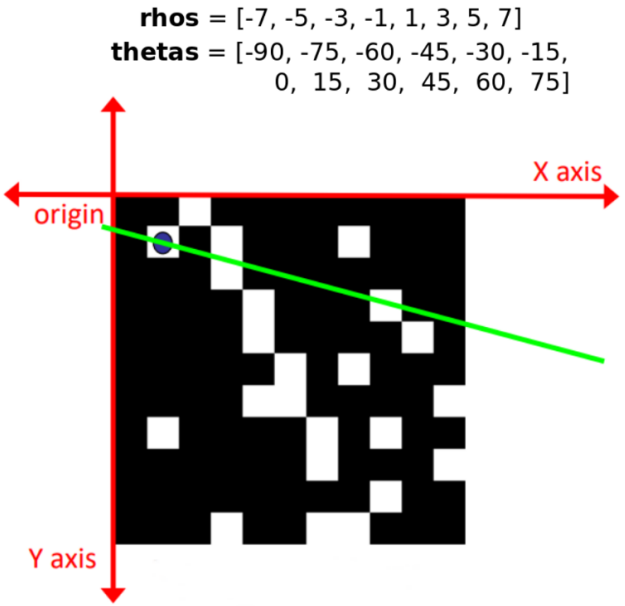
Example: Voting on an 11×11 Binary Edge Image

- Which position in the accumulator corresponds to $(\rho = -1, \theta = -75)$?



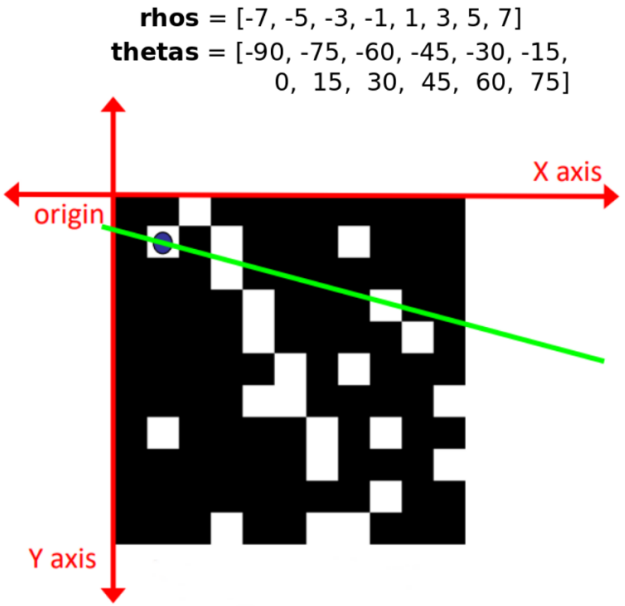
Example: Voting on an 11×11 Binary Edge Image

- Which position in the accumulator corresponds to $(\rho = -1, \theta = -75)$?
  - -1 is in the 4th position of **rhos**



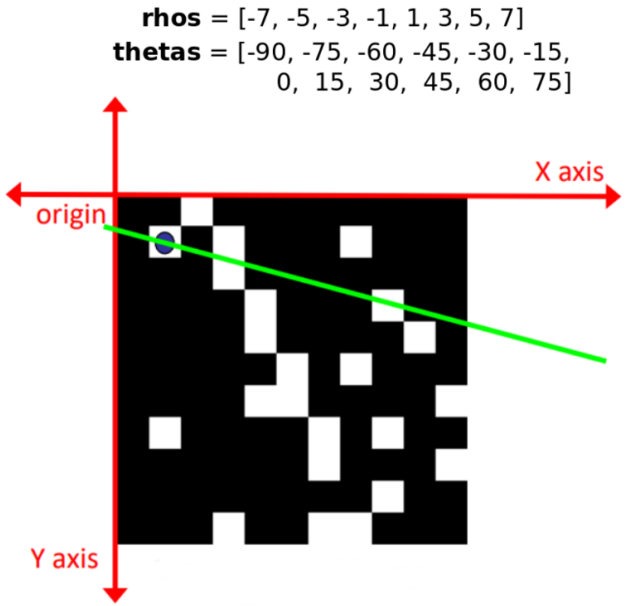
Example: Voting on an 11×11 Binary Edge Image

- Which position in the accumulator corresponds to $(\rho = -1, \theta = -75)$?
  - -1 is in the 4th position of **rhos**
  - -90 is in the 2nd position of **thetas**



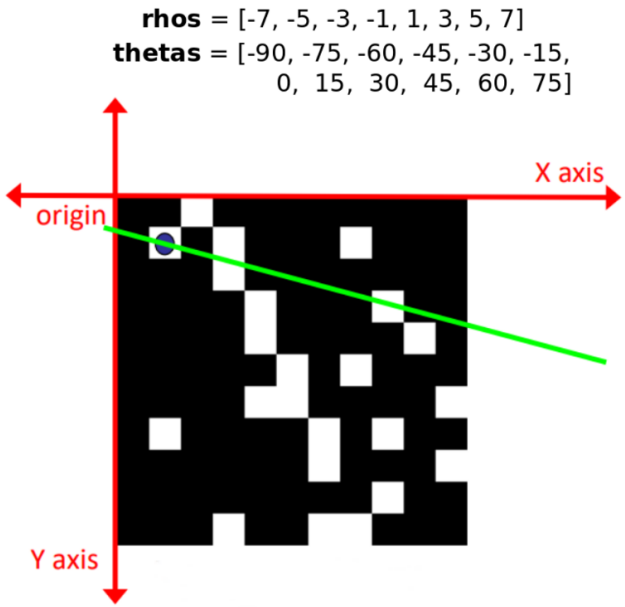
Example: Voting on an 11×11 Binary Edge Image

- Which position in the accumulator corresponds to $(\rho = -1, \theta = -75)$?
  - -1 is in the 4th position of **rhos**
  - -90 is in the 2nd position of **thetas**
  - Therefore, we add 1 to accumulator(4,2)



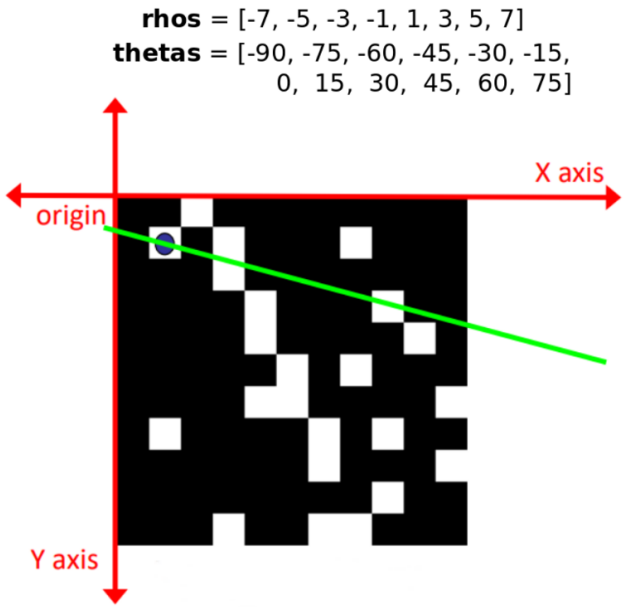
Example: Voting on an 11×11 Binary Edge Image

- To finish processing pixel (2,2), we proceed with the remaining values in **thetas**



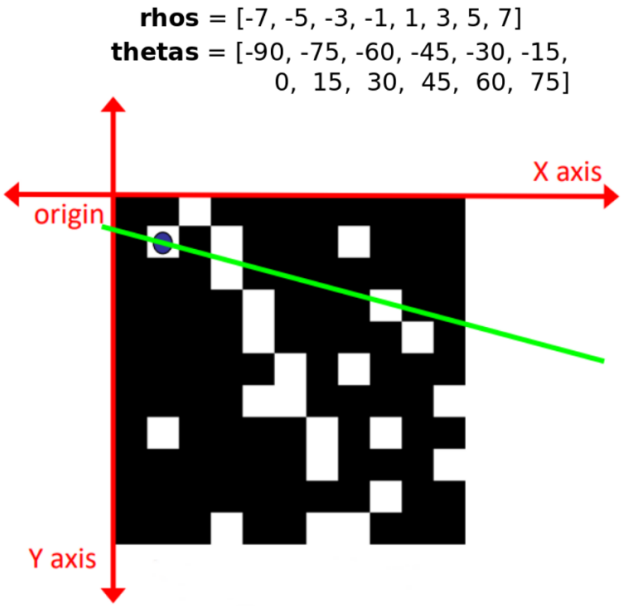
Example: Voting on an 11×11 Binary Edge Image

- To finish processing pixel (2,2), we proceed with the remaining values in **thetas**
  - For each θ, we find the corresponding ρ and we add one vote for the line with parameters (ρ, θ)



Example: Voting on an 11 × 11 Binary Edge Image

- To finish processing pixel (2,2), we proceed with the remaining values in **thetas**
  - For each θ, we find the corresponding ρ and we add one vote for the line with parameters (ρ, θ)
- After we are done with pixel (2,2), we continue processing the rest of the edge pixels the same way



Hough Transform for Lines

- The Hough transform for lines simply computes and accumulates the votes for all lines

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- There are also more complicated Hough transforms for ellipses and other shapes

Complexity of the Hough Transform

- How long does it take to build the accumulator?

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- How long does it take to build the accumulator?
  - Loose estimate: $O(\#\text{ of pixels} \cdot \#\text{ of } \theta)$
  - Tighter estimate: $O(\#\text{ of pixels} + \#\text{ of edge pixels} \cdot \#\text{ of } \theta)$

Hough Transform Accumulator Dimensions

- The number of bins to use along each axis of the accumulator depends on the accuracy of the position and orientation estimate available at each edge pixel and the expected line density

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- This parameter is best set experimentally with some test runs on sample imagery

General Hough Transforms

- In theory, we can use Hough transforms to detect more complicated shapes

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- In theory, we can use Hough transforms to detect more complicated shapes
- In practice, this requires memory and time *exponential* to the number of parameters and is usually not practical

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- Detecting and matching features is a significant part of many computer vision systems
- We've explored some practical approaches to detecting features and also discussed how feature correspondences can be established across different images
- We've seen how to implement basic edge and line detectors