Roll No. ....

H

TMC–104

M. C. A. (FIRST SEMESTER)
END SEMESTER EXAMINATION, 2019
DISCRETE STRUCTURE AND COMBINATORICS
Time : Three Hours
Maximum Marks : 100

Note : (i) This question paper contains five questions.

(ii) All questions are compulsory.

(iii) Instructions on how to attempt a question are mentioned against it.

(iv) Total marks assigned to each question are twenty.

1. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) Define Partial order relation and equivalence relation with suitable examples.

(b) Let A = {1, 2, 3, 4} and consider R = {(1, 1), (1, 2), (1, 3), (2, 2), (3, 2) (3, 3), (4, 2), (4, 3), (4, 4)}.

Is R an equivalence relation ? Is R a partial order relation ? If so, draw its Hasse Diagram.

(c) Let A = {1, 2, 3} and let R and S be the relation on A such that :

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, M_S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Find $M_{R^{-1}}, M_{R'}, M_{R \cap S}, M_{R \circ S}, M_{R^2}$.

2. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) (i) If a function $f : \mathbf{R} \to \mathbf{R}$ defined by :

$$f(x) = \begin{cases} 3x - 4, & x > 0 \\ -3x + 2, & x \leq 0 \end{cases}$$

Determine (1) $f^{-1}(2)$ and $f^{-1}(7)$. (2) Is f a bijective function ?

(ii) Write short notes on the following :

(a) Connectives

(b) Predicates and Quantifiers

(b) (i) Let *a, b* be positive integer. Suppose Q is defined recursively as :

$$Q(a,b) = \begin{cases} 0 & , \text{ if } a < b \\ Q(a-b,b)+1, & \text{if } b \leq a \end{cases}$$

Find (1) Q (2, 5) and (2) Q (12, 5).

(ii) Use mathematical induction to show that $n^2 \geq 2n + 1$ for $n \geq 3$.

(c) (i) Prove that $p \wedge q \to (p \to q)$ is a tautology and $p \wedge (q \wedge \sim p)$ is a contradiction.

(ii) (1) Write the converse, inverse and contra-positive of "If $x + 4 = 10$, then $x = 6$."

(2) Obtain principal disjunctive normal form of $\sim p \vee q$.

3. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

(a) If ${}^nC_x = 56$ and ${}^nP_x = 336$, find *n* and *x*.

(b) (i) What is the coefficient of x^2y^4 in $(x+y)^6$?

(ii) In how many ways can a committee of 5 teachers and 4 students be chosen from 9 teachers and 15 students ?

(c) (i) Solve $a_n = a_{n-1} + 2a_{n-2}, n \geq 2$ with initial conditions $a_0 = 0, a_1 = 1$.

(ii) Determine Generating function of the following sequence :

$$a_r = \begin{cases} 2^r & \text{if } r \text{ is even} \\ -2^r & \text{if } r \text{ is odd} \end{cases}$$

4. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

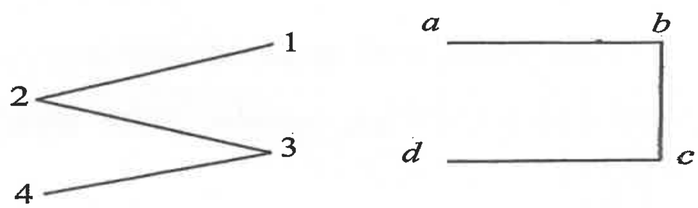
   (a) Show that the set G = {1, 2, 3, 4, 5, 6} forms an abelian group with respect to multiplication modulo 7. Is G a cyclic group ?

   (b) Define subgroups of a group. Prove that the intersection of two subgroup of a group G is again a subgroup of G.

   (c) Define Integral domain and Field. Give an example of integral domain which is not a field.

5. Attempt any *two* questions of choice from (a), (b) and (c). (2×10=20 Marks)

   (a) Define Isomorphic Graphs. Show that the given pairs of graphs are isomorphic :



   (b) Define Eulerian and Hamiltonian Graph with suitable examples.

   (c) Explain vertex set and edge set of a graph. Define union and intersection of two graphs with suitable examples.

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