# *ONLINE LESSION*

## *-CS-UNIT-3-*

## *Introduction to Boolean Algebra.*

## *Part 4-KMAP*

## 1. Boolean Laws and Postulates.

- **Commutative Law** 2
  - ✓ $A + B = B + A$
  - ✓ $A . B = B . A$
- **Identity Law** 1
  - ✓ $A + A = A$
  - ✓ $A . A = A$
- **Associate Law**
  - ✓ $(A + B) + C = A + (B + C)$
  - ✓ $(A B) C = A (B C)$
- **Redundancy Law**
  - ✓ $A + AB = A$
  - ✓ $A (A + B) = A$
- **Distributive Law**
  - ✓ $A (B + C) = A B + A C$
  - ✓ $A + (BC) = (A + B) (A + C)$
- **Demorgan's Theorem** 3
  - ✓ $\overline{A + B} = \bar{A} . \bar{B}$
  - ✓ $\overline{A . B} = \bar{A} + \bar{B}$

## 2. Other useful postulates.

✓ $A . \bar{B} + A . B = A$
$(A + \bar{B})(A + B) = A$

✓ $1 + A = 1$
4 $1 . A = A$

✓ $A + \bar{A}.B = A + B$
$A (\bar{A} + B) = AB$

✓ $A + 0 = A$
$A . 0 = 0$

✓ $A + \bar{A} = 1$
5 $A . \bar{A} = 0$

✓ $\bar{A}.\bar{B} + A.B = \overline{A \oplus B}$
$\bar{A}.B + \bar{B}.A = A \oplus B$

## 3. Observe following truth table,

6

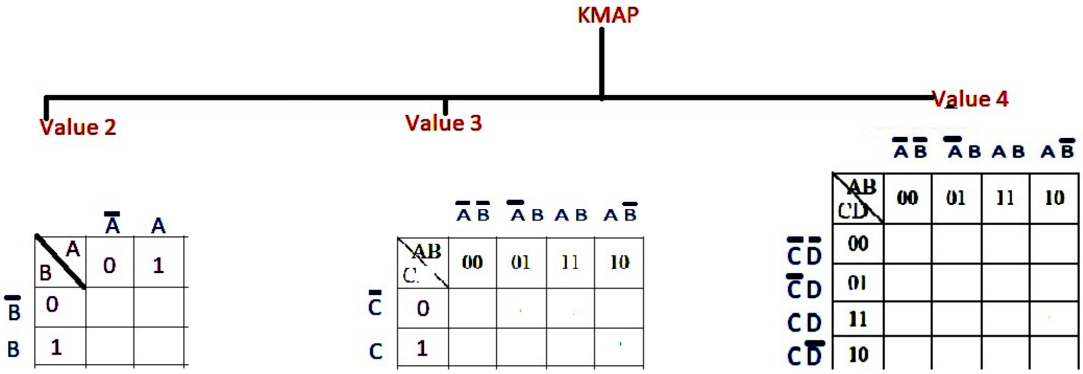
$$\bar{A}.B + \bar{B}.A = A \oplus B$$

| A | B | $\bar{A}$ | $\bar{B}$ | $\bar{A}.B$ | $\bar{B}.A$ | $\bar{A}.B + \bar{B}.A$ | $A \oplus B$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 * 0 = 0 | 1 * 0 = 0 | 0 + 0 = 0 | $0 \oplus 0 = 0$ |
| 0 | 1 | 1 | 0 | 1 * 1 = 1 | 0 * 0 = 0 | 1 + 0 = 1 | $0 \oplus 1 = 1$ |
| 1 | 0 | 0 | 1 | 0 * 0 = 0 | 1 * 1 = 1 | 0 + 1 = 1 | $1 \oplus 0 = 1$ |
| 1 | 1 | 0 | 0 | 0 * 1 = 0 | 0 * 1 = 0 | 0 + 0 = 0 | $1 \oplus 1 = 0$ |

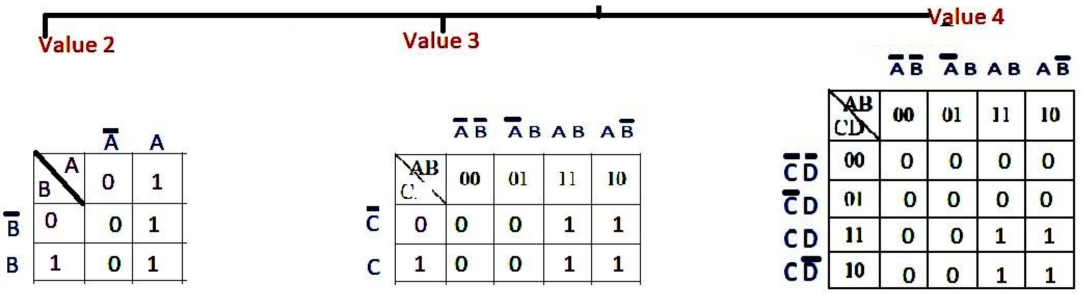
*Both outputs are same,*

# Introduction to KMAP.

1. Draw KMAP.




2. Mark the values.

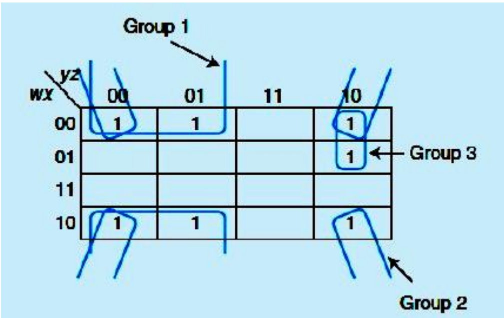



$A\bar{B}+AB$

$AB\bar{C}+A\bar{B}\bar{C}+ABC+A\bar{B}C$

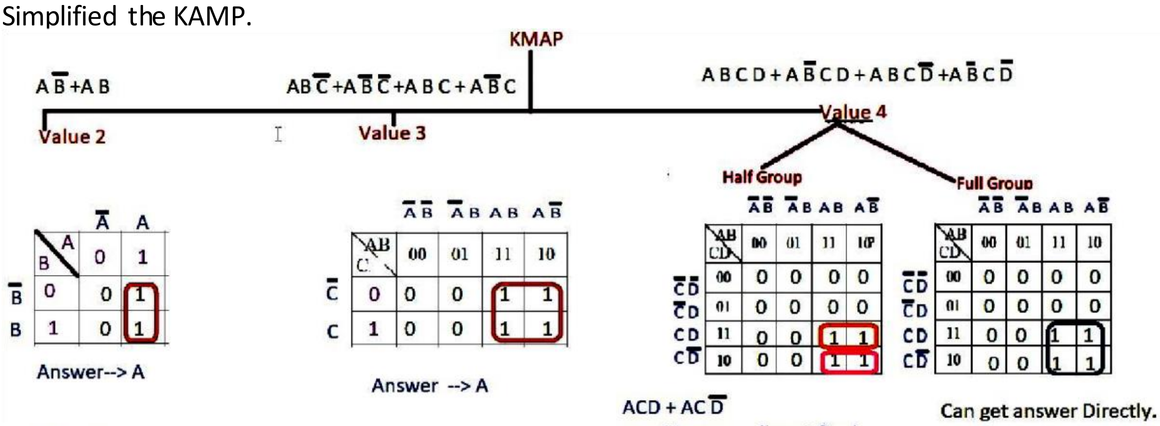
$ABCD+A\bar{B}CD+ABC\bar{D}+A\bar{B}C\bar{D}$

3. Grouping Techniques.




*1. The groups can only contain 1s; no 0s.*
*2. Only 1s in adjacent cells can be grouped*
*3. Diagonal grouping is not allowed.*
*4. The number of 1s in a group must be a power of 2.*
*5. The groups must be as large as possible while still following all rules.*
*6. All 1s must belong to a group, even if it is a group of one.*

4. Simplified the KAMP.



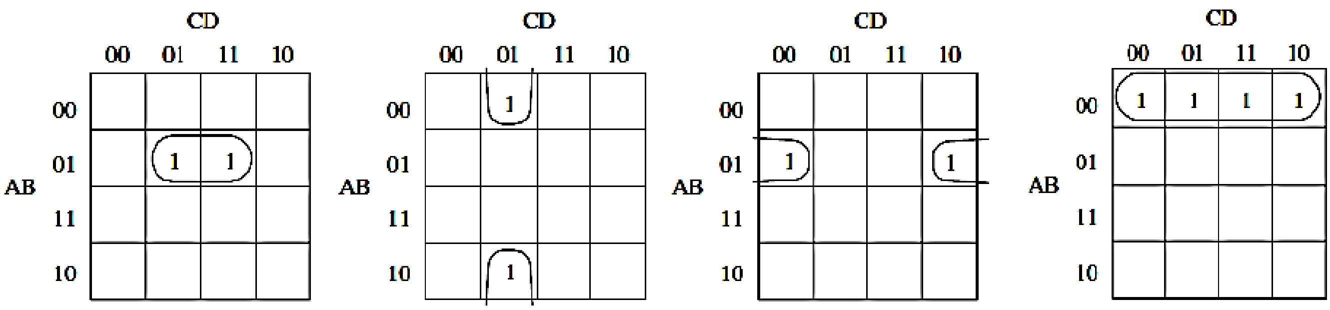

Get X and Y axis common variables.

$ACD + AC\bar{D}$
want to manually get final answer.
$AC[D+\bar{D}]=AC$
Answer -->AC

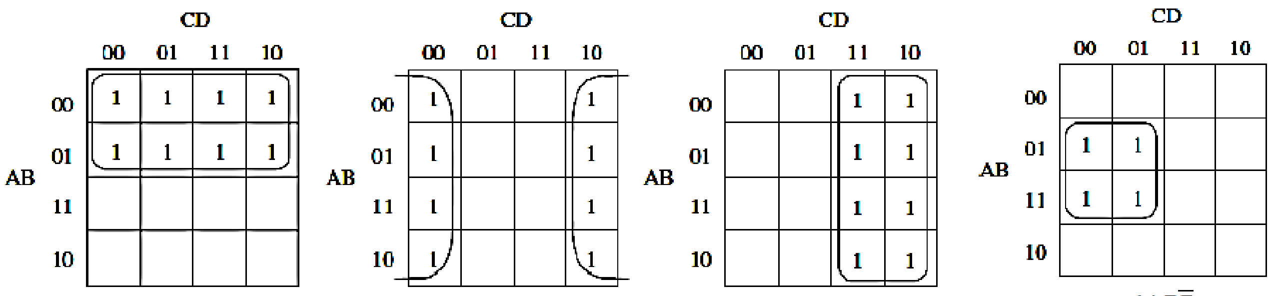
Can get answer Directly.

ANSWER ->AC

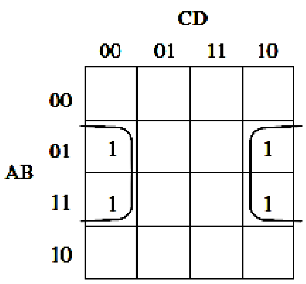
# KMAP-Grouping methods.



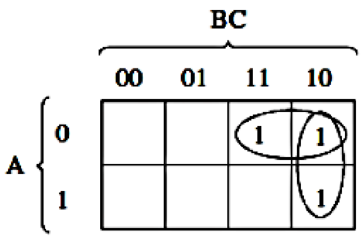
(a) $\overline{A}BD$ (b) $\overline{B}\,\overline{C}D$ (b) $\overline{A}B\overline{D}$ (d) $\overline{A}\,\overline{B}$



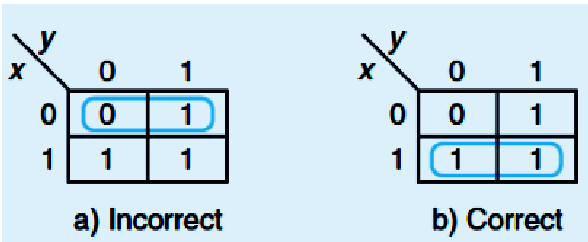
(g) $\overline{A}$ (h) $\overline{D}$ (i) C (e) $B\overline{C}$



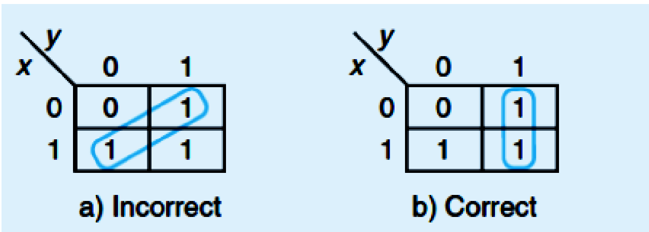
(f) $B\overline{D}$



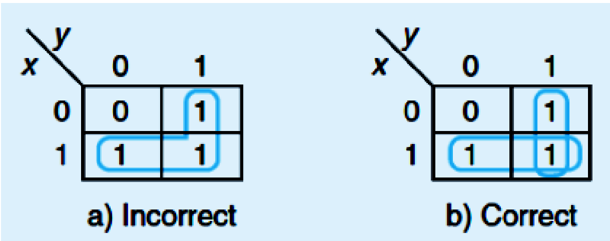
(a) $F = \overline{A}B + B\overline{C}$



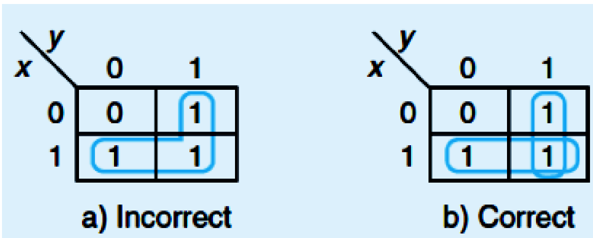

Groups Contain Only 1s




Groups Cannot Be Diagonal




Groups Must Be Powers of 2




Groups Must Be as Large as Possible

2010-12 2012-18 2011-14 **2013-13 2014-17 2015-16**

12) Consider the following Karnaugh map?

| CD \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | 1 | 1 |
| 01 | 1 | 0 | 0 | 1 |
| 11 | 0 | 0 | 0 | 0 |
| 10 | 0 | 1 | 1 | 0 |

Also consider the following compact Boolean forms.

(i) $\overline{B}.\overline{C}+B.\overline{D}$

(ii) $\overline{C}.\overline{D}+\overline{C}.D.\overline{B}+B.C.\overline{D}$

(iii) $\overline{C.D}+\overline{C}.B+\overline{D}.B$

(iv) $A.B.C+B.\overline{D}$

(v) $\overline{B}.\overline{C}+\overline{B}.\overline{D}$

Which of the above is the most compact form of a Boolean expression which represents the given Karnaugh map?

| | | |
|---|---|---|
| (a) Only (i) | (b) Only (i) and (ii) | (c) Only (ii) and (iii) |
| (d) Only (iii) | (e) Only (iv) and (v) | |

***2010 -12***

| CD \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | 1 | 1 |
| 01 | 1 | 0 | 0 | 1 |
| 11 | 0 | 0 | 0 | 0 |
| 10 | 0 | 1 | 1 | 0 |

***Group 1***

***X common --> $\overline{D}$***

***y common --> B***

***Group 2***

***X common --> $\overline{C}$***

***y common --> $\overline{B}$***

***Answer $B\overline{D}+\overline{B}\,\overline{C}$***

2012-18

18) Consider the following Karnaugh map.

| CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | 1 | 1 |
| 01 | 1 | 0 | 0 | 1 |
| 11 | 0 | 0 | 0 | 0 |
| 10 | 1 | 0 | 0 | 1 |

Which of the following is the most compact form of a Boolean function representing the above Karnaugh map?

(i) $\overline{BC}+\overline{B}\,\overline{D}+\overline{C}\,\overline{D}$

(ii) $\overline{CD}+\overline{BC}+\overline{A}\,\overline{BD}+A\overline{BD}$

(iii) $\overline{CD}+\overline{BD}+\overline{A}\,\overline{BC}+A\overline{BC}$

(iv) $B.C+B.\overline{D}+\overline{CD}$

(v) $\overline{B}.\overline{C}+\overline{B}.\overline{D}+C\overline{D}$

| | | |
|---|---|---|
| (a) Only (i) | (b) Only (i) and (ii) | (c) Only (ii) and (iii) |
| (d) Only (iii) | (e) Only (iv) and (v) | |

**2012-18**

| CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | 1 | 1 |
| 01 | 1 | 0 | 0 | 1 |
| 11 | 0 | 0 | 0 | 0 |
| 10 | 1 | 0 | 0 | 1 |

***Group 1***

***X common*** --> $\overline{C}\,\overline{D}$

***y common*** -->

***Group 2***

***X common*** --> $\overline{C}$

***y common*** --> $\overline{B}$

Group 3

X common--> $\overline{D}$

y common--> $\overline{B}$

Answers $\overline{C}\,\overline{D}+\overline{B}\,\overline{C}+\overline{B}\,\overline{D}$

2011-14

14) Which of the following K-Maps represent(s) the Boolean expression $Q = A + C \oplus B$ ?

(a)

| C \ BA | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

(b)

| C \ BA | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 0 |

(c)

| C \ BA | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |

(d)

| C \ BA | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 |

(e)

| C \ BA | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 |

$A + C \oplus B = A + \overline{C}B + \overline{B}C$

Answer B

**2013-13**

2013-13

| CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 0 | 0 |
| 01 | 0 | 1 | 1 | 0 |
| 11 | 0 | 1 | 1 | 0 |
| 10 | 1 | 0 | 0 | 1 |

***Group 1***

***X common*** --> D

***y common*** --> B

***Group 2***

***X common*** --> $C\overline{D}$

***y common*** --> $\overline{B}$

Group 3

X common--> $\overline{D}$

y common--> $\overline{A}\,\overline{B}$

Answers $B D + \overline{B} C \overline{D} + \overline{A}\,\overline{B}\,\overline{D}$

**2014-17**

2014-17

| AB / CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | 1 | 1 |
| 01 | 1 | 0 | 0 | 1 |
| 11 | 0 | 0 | 0 | 0 |
| 10 | 1 | 0 | 0 | 1 |

***Group 1***

***X common*** $\to \overline{C}\,\overline{D}$

***y common*** $\to$

***Group 2***

***X common*** $\to \overline{D}$

***y common*** $\to \overline{B}$

Group 3

X common--> $\overline{C}$

y common--> $\overline{B}$

Answers $\overline{C}\,\overline{D} + \overline{B}\,\overline{D} + \overline{B}\,\overline{C}$

# Boolean function simplified using KMAP.

2010-13 **2013-18**

13) Consider the following logic function

$$F = A.B.C + A.B.\overline{C} + A.\overline{B}.C + \overline{A}.\overline{B}.\overline{C} + \overline{A}.B.\overline{C}$$

Also consider the following compact Boolean forms.

(i) $A.B + A.C + \overline{A}.B$

(ii) $A.B + A.\overline{C} + \overline{A}.\overline{C}$

(iii) $A.B + A.C + \overline{A}.\overline{C}$

(iv) $A.C + \overline{B}.\overline{C} + \overline{A}.\overline{C}$

(v) $A.C + B.\overline{C} + \overline{A}.\overline{C}$

Which of the above would the results be if the given logic function were to be simplified using Karnaugh map?

| | | |
|---|---|---|
| (a) Only (i) and (ii) | (b) Only (i) and (iii) | (c) Only (ii) and (iv) |
| (d) Only (iii) and (v) | (e) Only (iv) and (v) | |

2010-13

| AB \ C | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 |

**Group 1**

**X common ->**

**y common ->** A B

**Group 2**

**X common ->** $\overline{C}$

**y common ->** $\overline{A}$

Group 3

X common--> C

y common--> A

| AB \ C | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 |

**Group 1**

**X common ->** $\overline{C}$

**y common ->** B

Answers -->A B + $\overline{A}\,\overline{C}$ + A C

--> B $\overline{C}$ +A C + $\overline{A}\,\overline{C}$

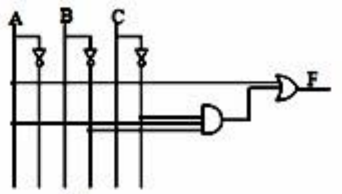
## Logic circuit diagram me simplication.

2010-14 2012-16 **2013-14** **2015-17**

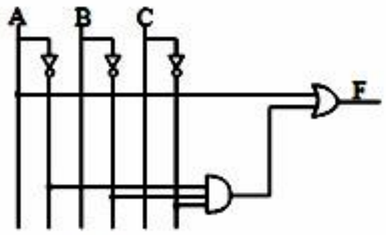
14) Consider the following logic function

$$F = A.B.C + \overline{A}.\overline{B}.\overline{C} + A.\overline{B}.\overline{C} + A.B.\overline{C} + A.\overline{B}.C$$

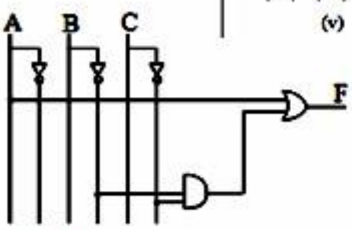
Also consider the following logic circuit diagrams.



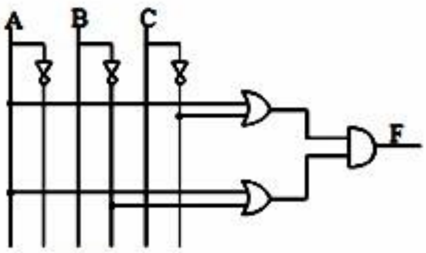
(v)



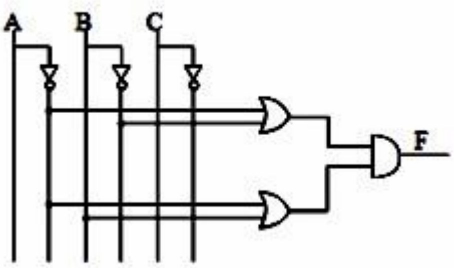
(i)



(ii)



(iii)



(iv)

[2010-14] Answers (b) //

$$ABC + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + AB\bar{C} + A\bar{B}C = A + \bar{B}\bar{C}$$

Simplified the equation using KMAP.

| C \ AB | $\bar{A}\bar{B}$ | $\bar{A}B$ | $AB$ | $A\bar{B}$ |
|---|---|---|---|---|
| $\bar{C}$ | 1 | 0 | 1 | 1 |
| $C$ | 0 | 0 | 1 | 1 |

$\overrightarrow{x}$ $y\downarrow$ $A$

$\overrightarrow{x}$ $y\downarrow$ $\bar{B}\bar{C}$

Answer $A + \bar{B}\bar{C}$

(a) $A + \bar{A}\bar{B}\bar{C}$ want to Draw T Table

✓(b) $A + \bar{B}\bar{C}$ ✓

✓(c) $(A+\bar{C}).(A+\bar{B})$

(d) $(\bar{A}+\bar{B}).(\bar{A}+B)$ want to Draw T Table.

(e) $A + A\bar{B}\bar{C}$ ← want to Draw T Table.

(c) $(A+\bar{C})(A+\bar{B})$

$A(A+\bar{B}) + \bar{C}(A+\bar{B})$

$\underset{\sim}{AA} + A\bar{B} + A\bar{C} + \bar{B}\bar{C}$

$A + A\bar{B} + A\bar{C} + \bar{B}\bar{C}$

$A[\underbrace{1+\bar{B}+\bar{C}}_{1}] + \bar{B}\bar{C}$

$A + \bar{B}\bar{C}$ //

(d) $(\bar{A}+\bar{B})(\bar{A}+B)$

$\bar{A}(\bar{A}+B) + \bar{B}(\bar{A}+B)$

$\underset{\sim}{\bar{A}\bar{A}} + \bar{A}B + \bar{A}\bar{B} + \underset{0}{B\bar{B}}$

$\bar{A} + \bar{A}B + \bar{A}\bar{B}$

$\bar{A}[\underbrace{B+\bar{B}}_{1}]$

$\bar{A}$ //

Draw the Truth Table. [verified answers (a) (d) (e)]

Total Results, $2^3 \rightarrow 8$ //

| | A | B | C | $A+\bar{B}\bar{C}$ | (a) ✓ $A+\bar{A}\bar{B}\bar{C} \rightarrow$ | (d) × $\bar{A} \rightarrow$ | (e) × $A+A\bar{B}\bar{C} \rightarrow$ |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | $0+1\cdot1 \rightarrow 1$ | $0+1\cdot1\cdot1 \rightarrow 1$ | 1 | $0+0\cdot1\cdot1 \rightarrow 0$ |
| 2 | 0 | 0 | 1 | $0+1\cdot0 \rightarrow 0$ | $0+1\cdot1\cdot0 \rightarrow 0$ | 1 | $0+0\cdot1\cdot0 \rightarrow 0$ |
| 3 | 0 | 1 | 0 | $0+0\cdot1 \rightarrow 0$ | $0+1\cdot0\cdot1 \rightarrow 0$ | 1 | $0+0\cdot0\cdot1 \rightarrow 0$ |
| 4 | 0 | 1 | 1 | $0+0\cdot0 \rightarrow 0$ | $0+1\cdot0\cdot0 \rightarrow 0$ | 1 | $0+0\cdot0\cdot0 \rightarrow 0$ |
| 5 | 1 | 0 | 0 | $1+1\cdot1 \rightarrow 1$ | $1+0\cdot1\cdot1 \rightarrow 1$ | 0 | $1+1\cdot1\cdot1 \rightarrow 1$ |
| 6 | 1 | 0 | 1 | $1+1\cdot0 \rightarrow 1$ | $1+0\cdot1\cdot0 \rightarrow 1$ | 0 | $1+1\cdot1\cdot0 \rightarrow 1$ |
| 7 | 1 | 1 | 0 | $1+0\cdot1 \rightarrow 1$ | $1+0\cdot0\cdot1 \rightarrow 1$ | 0 | $1+1\cdot0\cdot1 \rightarrow 1$ |
| 8 | 1 | 1 | 1 | $1+0\cdot0 \rightarrow 1$ | $1+0\cdot0\cdot0 \rightarrow 1$ | 0 | $1+1\cdot0\cdot0 \rightarrow 1$ |

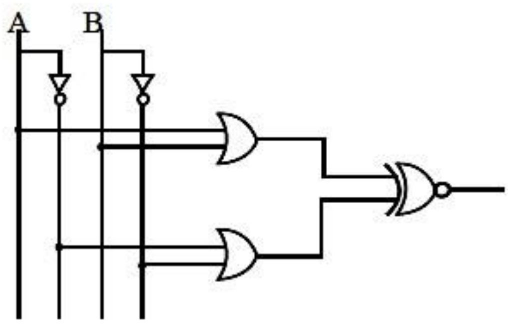
2015-17

17) Consider the following logic function

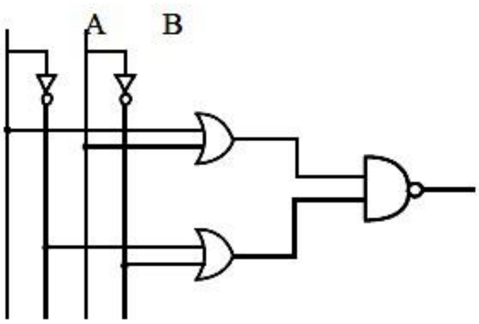
$$F(AB) = \overline{A}\overline{B} + AB$$

Which of the following logic circuit diagrams provide a similar output to the above logic function F(AB)?

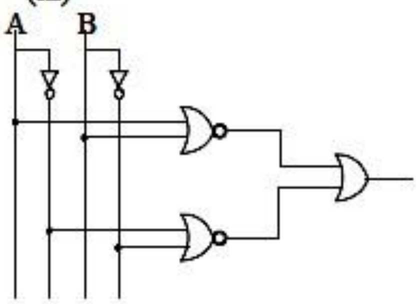
(i)



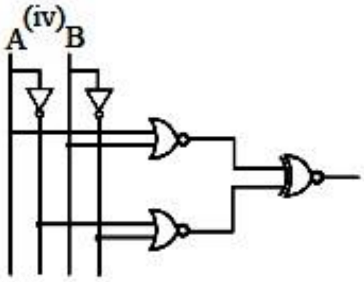

(ii)




(iii)




(iv)




(a) Only (i) and (ii)
(b) Only (ii) and (iii)
(c) Only (i)and (iii)
(d) Only (i),(ii) and (iii)
(e) Only (ii),(iii) and (iv)

[2015-17] function $\bar{A}\bar{B}+AB$

× (i) $\overline{(A+B) \oplus (\bar{A}+\bar{B})}$

✓ (ii) $\overline{(A+B) \cdot (\bar{A}+\bar{B})}$

$\overline{(A+B) \cdot (\bar{A}+\bar{B})}$ Apply Demorgen Law.

$\overline{(A+B)} + \overline{(\bar{A}+\bar{B})}$ Apply Demorgen Law

$\bar{A}\cdot\bar{B} + \bar{\bar{A}}\cdot\bar{\bar{B}}$

$\bar{A}\bar{B} + AB$ //=

✓ (iii) $\overline{(A+B)} + \overline{(\bar{A}+\bar{B})}$

$\overline{(A+B)} + \overline{(\bar{A}+\bar{B})}$ Apply Demorgen

$\bar{A}\bar{B} + (\bar{\bar{A}} \bullet \bar{\bar{B}})$ // $\rightarrow$ $\bar{A}\bar{B} + AB$ //=

× (Iv) $\overline{\overline{(A+B)} \oplus \overline{(\bar{A}+\bar{B})}}$

Apply Demorgen.

$\overline{\bar{A}\cdot\bar{B} \oplus \bar{\bar{A}}\cdot\bar{\bar{B}}}$ $\rightarrow$ $\overline{\bar{A}\cdot\bar{B} \oplus AB}$ //=

Draw the Truth table for verify Answer (i) and IV//

| A | B | $\bar{A}\bar{B}+AB$ | $\overline{(A+B)\oplus(\bar{A}+\bar{B})}$ | $\overline{\bar{A}\bar{B}\oplus AB}$ |
|---|---|---|---|---|
| 0 | 0 | $1\cdot1+0\cdot0\rightarrow1$ | $(0+0)\rightarrow\overline{0\oplus1}\leftarrow(1+1)$ $\rightarrow 0$ | $(1\cdot1)\rightarrow\overline{1\oplus0}\leftarrow(0\cdot0)$ $\rightarrow 0$ |
| 0 | 1 | $1\cdot0+0\cdot1\rightarrow0$ | $(0+1)\rightarrow\overline{1\oplus1}\leftarrow(1+0)$ $\rightarrow 1$ | $(1\cdot0)\rightarrow\overline{0\oplus0}\leftarrow(0\cdot1)$ $\rightarrow 1$ |
| 1 | 0 | $0\cdot1+1\cdot0\rightarrow0$ | $(1+0)\rightarrow\overline{1\oplus1}\leftarrow(0+1)$ $\rightarrow 1$ | $(0\cdot1)\rightarrow\overline{0\oplus0}\leftarrow(1\cdot0)$ $\rightarrow 1$ |
| 1 | 1 | $0\cdot0+1\cdot1\rightarrow1$ | $(1+1)\rightarrow\overline{1\oplus0}\leftarrow(0+0)$ $\rightarrow 0$ | $(0\cdot0)\rightarrow\overline{0\oplus1}\leftarrow(1\cdot1)$ $\rightarrow 0$ |

Answers,

(ii) And (iii) $\longrightarrow$ Answer (b)