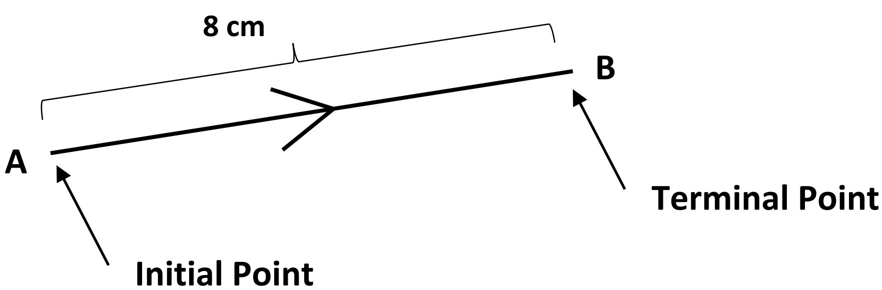
Vectors

1. Scalars vs Vectors

	Scalars		Vectors
	1. Have a **Magnitude**		1. Have a **Magnitude**
			2. Have a **Direction**
Ex:	Distance	Ex:	Displacement
	Speed		Velocity, Acceleration
	Mass		Weight, Force

2. Vector Notation

"A vector is represented as a **Directed Line Segment** in Geometry". A directed line segment has both magnitude (distance between the two end points) and direction.



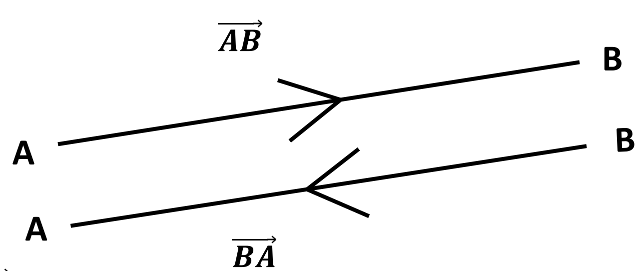
Here, AB directed line segment is a vector and it is denoted as $\overrightarrow{\boldsymbol{AB}}$ (not as $\cancel{\overleftarrow{\boldsymbol{BA}}}$).

As $\overrightarrow{\boldsymbol{AB}}$ is a vector, it has a magnitude and a direction.

- **Magnitude** is denoted as $\left|\overrightarrow{\boldsymbol{AB}}\right|$ and $\left|\overrightarrow{\boldsymbol{AB}}\right| = \mathbf{8}(The\ length\ of\ the\ directed\ line\ segment)$
- **Direction** is denoted by the arrow head and it is from **A to B**

3. Special Vectors

- **Negative of a Vector**



If $\overrightarrow{\boldsymbol{AB}}$ is a vector then it's **negative** is $-\left(\overrightarrow{\boldsymbol{AB}}\right) = \overrightarrow{\boldsymbol{BA}}$

Here, $\left|\overrightarrow{\boldsymbol{AB}}\right| = \left|\overrightarrow{\boldsymbol{BA}}\right| \ \rightarrow \mathbf{1}$

But, $Direction\ of\ \overrightarrow{\boldsymbol{AB}} = Opposite\ Direction\ of\ \overrightarrow{\boldsymbol{BA}}$

$Direction\ of\ \overrightarrow{\boldsymbol{AB}} = Direction\ of\ -\left(\overrightarrow{\boldsymbol{BA}}\right) \rightarrow \mathbf{2}$

$\underline{From\ \mathbf{1}\ and\ \mathbf{2},\quad \overrightarrow{\boldsymbol{AB}} = -\left(\overrightarrow{\boldsymbol{BA}}\right)}$

- **Null/Zero Vector** ($\underline{\odot}$)

When a magnitude of a vector is equal to zero, it is called as a Null or Zero vector. When the magnitude is equal to zero it is represented as a point in geometry, such that there is no direction.

Ex: If $\overrightarrow{\boldsymbol{AB}}$ is a null vector, then $\overrightarrow{\boldsymbol{AB}} = \underline{\odot}$

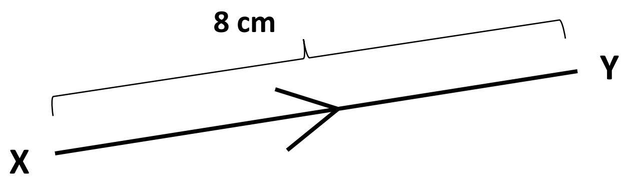
- **Unit Vector of a point**

When a magnitude of a vector is equal to one it is called a **Unit Vector**.

Ex: If $\overrightarrow{\boldsymbol{AB}}$ is a unit vector, then $\left|\overrightarrow{\boldsymbol{AB}}\right| = \mathbf{1}$

❖ **Unit Vector in the direction of a given vector**

The unit vector in the direction of a given vector can be found by, **dividing the vector by its magnitude**.



Unit Vector in the direction of $\overrightarrow{\boldsymbol{XY}} = \dfrac{\overrightarrow{\boldsymbol{XY}}}{\left|\overrightarrow{\boldsymbol{XY}}\right|} = \dfrac{\mathbf{8}\ (\boldsymbol{X} \rightarrow \boldsymbol{Y})}{\mathbf{8}} = \mathbf{1}\ (\boldsymbol{X} \rightarrow \boldsymbol{Y})$

- **Base Vectors**

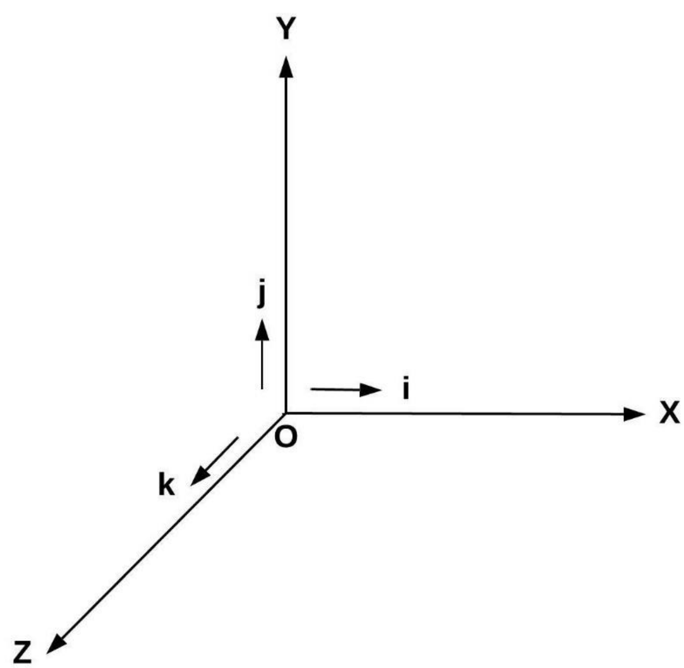
Unit Vectors, in the direction of 3 axes (X, Y, Z) in the 3D space are called **Base Vectors ($\underline{\boldsymbol{i}}$, $\underline{\boldsymbol{j}}$, $\underline{\boldsymbol{k}}$)**.
Here, Base Vectors ($\underline{\boldsymbol{i}}$, $\underline{\boldsymbol{j}}$, $\underline{\boldsymbol{k}}$) are in the positive directions of X, Y, Z respectively.

Unit Vector in the direction of $\overrightarrow{\boldsymbol{OX}} = \underline{\boldsymbol{i}}$

Unit Vector in the direction of $\overrightarrow{\boldsymbol{OY}} = \underline{\boldsymbol{j}}$

Unit Vector in the direction of $\overrightarrow{\boldsymbol{OZ}} = \underline{\boldsymbol{k}}$

$$|\underline{\boldsymbol{i}}| = |\underline{\boldsymbol{j}}| = |\underline{\boldsymbol{k}}| = \mathbf{1}$$



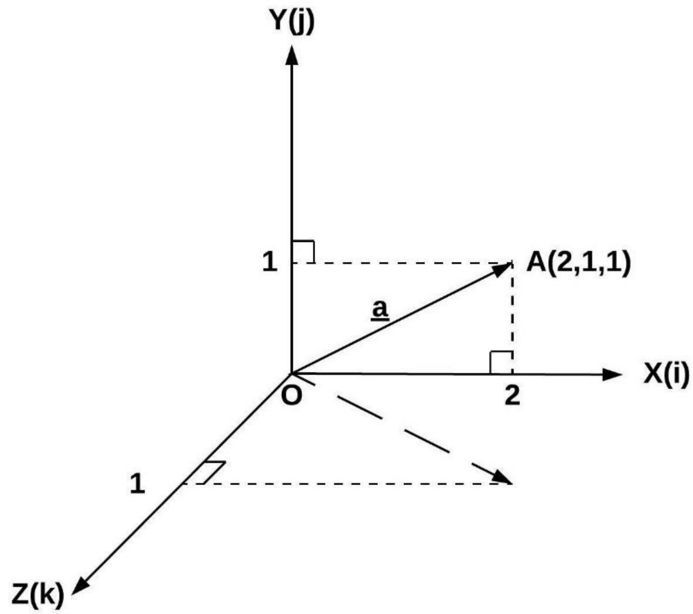
- **Position Vector of a point in the 3D space**

Position vector of a given point is represented by the directed line segment **from the origin to that point**. It is uniquely denoted using the **underlined simple letter** of the point.

Position Vector of the point A = $\overrightarrow{OA}$ = $\underline{a}$

➢ **An expression for the position vector, using base vectors and coordinates**

$$\overrightarrow{OA} = \underline{a} = x\underline{i} + y\underline{j} + z\underline{k}$$
$$= 2\underline{i} + 1\underline{j} + 1\underline{k}$$
$$= 2\underline{i} + \underline{j} + \underline{k}$$



➢ **Magnitude of the position vector**

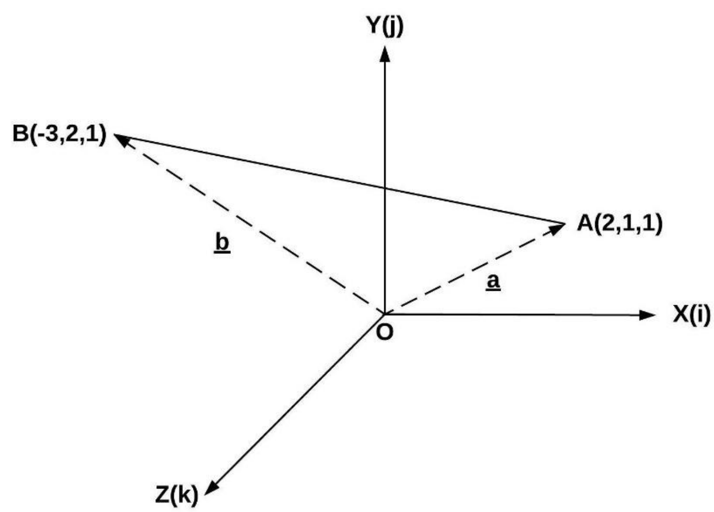
$$|\overrightarrow{OA}| = |\underline{a}| = +\sqrt{x^2 + y^2 + z^2}$$
$$= +\sqrt{2^2 + 1^2 + 1^2}$$
$$= +\sqrt{6}$$

➢ **Displacement between two points**

Displacement between A and B $\Big\}$ = $|\overrightarrow{AB}| = |\overrightarrow{BA}|$

$\overrightarrow{AB}$ = **(Position Vector of the Terminal Point) - (Position Vector of the Initial Point)**

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \underline{b} - \underline{a}$$
$$= \left(-3\underline{i} + 2\underline{j} + 1\underline{k}\right) - \left(2\underline{i} + 1\underline{j} + 1\underline{k}\right)$$
$$= \left((-3-2)\underline{i} + (2-1)\underline{j} + (1-1)\underline{k}\right)$$
$$= \left(-5\underline{i} + 1\underline{j} + 0\underline{k}\right)$$
$$= -5\underline{i} + \underline{j}$$
$$|\overrightarrow{AB}| = +\sqrt{(-5)^2 + (1)^2} = \sqrt{26}$$



☐ **2015-10** ☐ **2019-10** ☐ **Assign-01** ☐ **2010-17** ☐ **Assign-02** ☐ **2017-17** ☐ **2014-15**
☐ **2008-15**

- **2015-10**

10) If $\underline{x} = 2a\underline{i} + 3a\underline{j} - \sqrt{3}a\underline{k}$ is a unit vector, then the value of a could be

(a) $\frac{1}{4}$.	(b) $-\frac{1}{4}$.	(c) $\frac{1}{2}$.
(d) $-\frac{1}{2}$.	(e) $\frac{1}{8}$.	

- **2019-10**

10) If $\underline{a} = 3\underline{i} + p\underline{j} + q\underline{k}$ and $|\underline{a}| = \sqrt{19}$, where p and q are integers, then possible (p, q) values are,

(a) (2,5).	(b) (1,3).	(c) (-3,1).	(d) (1,1).	(e) (3,3).

- **Assignment-01**

If A(5, -10, 3) and B(-1, 2, 12) are two arbitrary points in a three-dimensional space, then the vector **AB** is represented in the usual notation by

Select one or more:

- ☐ A. - 6i + 12**j** + 9**k**.
- ☐ B. 6i - 12 **j** - 9**k**.
- ☐ C. – 6**i** + **j** – **k**.
- ☐ D. – 5**i** + 13**j** + 9**k**.
- ☐ E. 5i + 12j – 8**k**.

- **2010-17**

17) If $\overrightarrow{OA} = (\underline{i} + 2\underline{j} + 3\underline{k})$ and $\overrightarrow{OB} = (2\underline{i} + 2\underline{j} + 4\underline{k})$, then the unit vector in the direction of $\overrightarrow{AB}$ is

(a) $\frac{1}{\sqrt{2}}(\underline{i} + \underline{j})$	(b) $\frac{1}{\sqrt{2}}(\underline{j} + \underline{k})$	(c) $\frac{1}{\sqrt{2}}(\underline{i} + \underline{k})$
(d) $\frac{1}{\sqrt{2}}(\underline{i} - \underline{k})$	(e) $\frac{1}{\sqrt{2}}(\underline{j} - \underline{k})$	

- **Assignment-02**

A particle moves in space. It starts at the point A(1, -1, 3) and moves first to the point B(3, 1, 2) and then to the point C(9, 11, 12). Which of the following is (are) correct?

Select one or more:

- A. The distance covered by the particle moving from A to B equals 4.
- B. The position vector of the particle located at the point A is i – j +3k.
- C. The total displacement of the particle is represented by 8i + 12j + 9k.
- D. The magnitude of the total displacement of the particle is 17.
- E. The displacement of the particle from B to C is represented by 6i +10j – 9k.

- **2017-17**

17) The shortest distance between the fixed position $i + j$ and a variable position $i - 2tj + k$, where t is time, is equal to

(a) 1	(b) 9	(c)) 10
(d) 0	(e) $\sqrt{10}$	

- **2014-15**

15) If points A and B have position vectors $(2t+1)\underline{i}+(t+1)\underline{j}+3\underline{k}$ and $(t+1)\underline{i}+5\underline{j}+2\underline{k}$ respectively, then the minimum value of $\left|\overrightarrow{AB}\right|$ is

(a) 3	(b) 5	(c) 9
(d) 11	(e) $\sqrt{11}$	

- **2008-15**

15) If $\underline{a}=\alpha\underline{i}+\underline{j}$, $\underline{b}=\beta\underline{j}+\underline{k}$, and $\underline{c}=\gamma\underline{k}+\underline{i}$ are the position vectors of the vertices of an equilateral triangle, then which of the following is true?

(a) $\alpha=\beta=\gamma=t$	(b) $\alpha=-\beta=\gamma=t$
(c) $\alpha=\beta=-\gamma=t$	(d) $-\alpha=\beta=\gamma=t$
(e) There are no possible real values for α, β and γ.	