Probleem 1: Beskou die volgende aanvangswaardeprobleem, wat 'n vrye ongedempte veer-massa stelsel beskryf:

Problem 1: Consider the following initial value problem, that describes a free undamped spring-mass system:

$$x'' + 9x = 0, \quad x(0) = 1, \quad x'(0) = 3\sqrt{3}.$$

Los die probleem op, skryf die oplossing in amplitude-fase vorm, en skets x as 'n funksie van t. Dui ook die amplitude, periode, frekwensie en fase-verskuiwing van die beweging aan.

Solve the problem, write the solution in amplitude-phase form, and sketch x as a function of t. Also indicate the amplitude, period, frequency and phase shift of the motion.

Probleem 2: Die twee probleme hieronder beskryf elkeen 'n vrye veer-massa stelsel met demping:

Problem 2: The two problems below each describes a free spring-mass system with damping:

$$\textbf{(a)} \quad x'' + 2x' + 5x = 0, \quad x(0) = 1, \quad x'(0) = -3$$

$$\textbf{(b)} \quad x'' + 3x' + 2x = 0, \quad x(0) = -1, \quad x'(0) = 4$$

Bepaal in elke geval of die demping swaar, kritiek of lig is. Los dan elke probleem op. Skryf die oplossing van (a) in amplitude-fase vorm en skets $x(t)$.

Determine whether each is over-, critically or underdamped. Then solve each problem. Write the solution of (a) in amplitude-phase form and sketch $x(t)$.

Probleem 3: Beskou die volgende aanvangswaardeprobleem wat 'n veer-massa stelsel beskryf:

Problem 3: Consider the following initial value problem, which describes a spring-mass system:

$$\frac{d^2x}{dt^2} + 2x = \sin(t), \quad x(0) = 0, \quad x'(0) = 0.$$

(a) Is die veer-massa stelsel: lineêr? gedemp? aangedrewe? Wat is die natuurlike frekwensie van die stelsel? Wat is die drywingsfrekwensie van die stelsel?

(b) Los die aanvangswaardeprobleem.

(a) Is the spring-mass system: linear? damped? driven? What is the natural frequency of the system? What is the driving frequency of the system?

(b) Solve the initial value problem.

Probleem 4: Beskou die volgende veer-massa stelsel:

Problem 4: Consider the following spring-mass system:

$$x'' + 4x = 4.1\cos(2.1t), \quad x(0) = 0, \quad x'(0) = 0.$$

(a) Hoewel die voorwerp aan die veer aanvanklik in die ewewigsposisie hang, in rus, sal dit begin beweeg. Wat veroorsaak hierdie beweging?

(b) Bevestig dat $x(t) = 10\big[\cos(2t) - \cos(2.1t)\big]$ wel die DV en aanvangswaardes hierbo bevredig.

(c) Skryf die oplossing wat in deel (b) gegee word in die vorm $x(t) = A\sin(\alpha t)\sin(\beta t)$, waar jy A, α en β moet bepaal.

(d) Gebruik die resultaat in deel (c) om 'n grafiek van x teenoor t te skets vir $0 \le t \le 125$. Jou skets moet die gedrag van die oplossing duidelik toon.

(a) Even though the object hanging from the spring is initially in the equilibrium position, in rest, it will start moving. What causes this motion?

(b) Verify that $x(t) = 10\big[\cos(2t) - \cos(2.1t)\big]$ does indeed satisfy the above DE and initial conditions.

(c) Write the solution given in part (b) in the form $x(t) = A\sin(\alpha t)\sin(\beta t)$, where you need to determine A, α and β.

(d) Use the result in part (c) to sketch a graph of x against t for $0 \le t \le 125$. Your sketch must show the behaviour of the solution clearly.