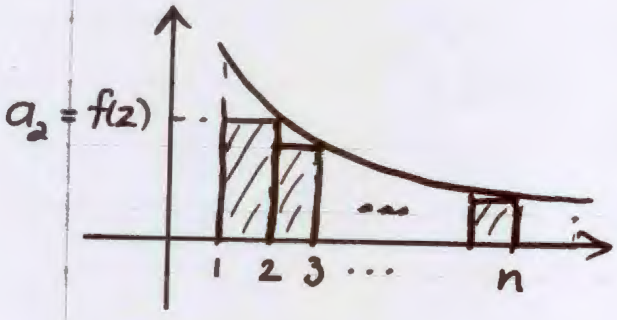
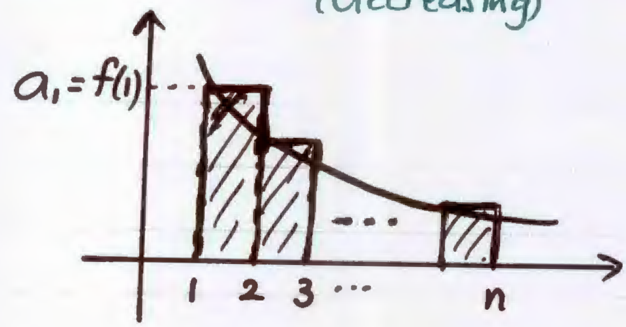
Integraaltoets: Skets van bewys/proof

Gestel f is kontinu, positief en dalend op $[1,\infty)$.
(decreasing)





$$\left.\begin{array}{ll}(1) & a_2 + a_3 + \cdots + a_n \leq \int_1^n f(x)\,dx \\ (2) & a_1 + a_2 + \cdots + a_{n-1} \geq \int_1^n f(x)\,dx\end{array}\right\}$$ omdat f dalend is (decreasing)

$\Rightarrow$ Gestel $\int_1^\infty f(x)\,dx < \infty$. Dan volg uit (1):

$$S_n = a_1 + \sum_{i=2}^n a_i \leq a_1 + \int_1^n f(x)\,dx \leq a_1 + \int_1^\infty f(x)\,dx$$

$\{S_n\}$ is ook stygend.
(increasing)

Dus konvergeer $\{S_n\}$ (uit MRS), sodat $\sum_{n=1}^\infty a_n < \infty$.
(Monotone Sequence Thm)

$\Leftarrow$ Gestel $\sum_{n=1}^\infty a_n < \infty$.

As $b_n = \int_1^n f(x)\,dx$, dan volg uit (2) dat

$$b_n \leq \sum_{i=1}^{n-1} a_i \leq \sum_{n=1}^\infty a_n .$$

$\{b_n\}$ is ook stygend/increasing.

Dus konvergeer $\left\{\int_1^n f(x)\,dx\right\}$ (uit MRS). (MST)

Dus $\int_1^\infty f(x)\,dx < \infty$ (uit Tut. 2 nr. 2(d)).