Oneindige Reekse / Infinite Series §11.2

Definisie 2
Beskou die *(oneindige) reeks*

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \cdots + a_n + \cdots.$$

Laat

$$s_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n$$

die n-de *parsiële som / partial sum* van die reeks wees.

As die ry $\{s_n\}$ konvergeer, sê $\lim_{n\to\infty} s_n = s \in \mathbb{R}$, dan sê ons die reeks $\sum_{n=1}^{\infty} a_n$ is *konvergent* en skryf

$$\sum_{n=1}^{\infty} a_n = s \quad \text{of} \quad a_1 + a_2 + \cdots = s.$$

Die getal s word die *som / sum* van die reeks genoem.

Indien $\{s_n\}$ divergeer, dan sê ons die reeks is *divergent*.

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} \sum_{i=1}^{n} a_i.$$

Stelling 4

Die meetkundige reeks / geometric series

$$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \cdots$$

konvergeer as $|r| < 1$, met som

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}.$$

As $|r| \geq 1$, dan is die meetkundige reeks divergent.

Huiswerk

Ex. 11.2 nr. 1, 2, 16, 17, 25, 53, 61, 67

Stelling 8

As $\sum_{n=1}^{\infty} a_n$ en $\sum_{n=1}^{\infty} b_n$ konvergente reekse is, en c is 'n konstante, dan is die reekse $\sum_{n=1}^{\infty} ca_n$, $\sum_{n=1}^{\infty} (a_n + b_n)$ en $\sum_{n=1}^{\infty} (a_n - b_n)$ konvergent, en

- $\sum_{n=1}^{\infty} ca_n = c \sum_{n=1}^{\infty} a_n$

- $\sum_{n=1}^{\infty} (a_n + b_n) = \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$

- $\sum_{n=1}^{\infty} (a_n - b_n) = \sum_{n=1}^{\infty} a_n - \sum_{n=1}^{\infty} b_n$

Huiswerk

Ex. 11.2 nr. 29, 31, 39, 43, 45, 81, 82, 85, 87, 89(a)