Bewys van Wisselreekstoets

Beskou die ewe (even) parsiële somme:

$S_2 = b_1 - b_2 \geq 0$, want $b_2 \leq b_1$.

$S_4 = b_1 - b_2 + b_3 - b_4 = S_2 + b_3 - b_4 \geq S_2$,

want $b_4 \leq b_3$.

$S_6 = b_1 - b_2 + b_3 - b_4 + b_5 - b_6 = S_4 + b_5 - b_6 \geq S_4$,

want $b_6 \leq b_5$, ens.

Dus is $0 \leq S_2 \leq S_4 \leq S_6 \leq \cdots$, sodat $\{S_{2n}\}$ stygend (increasing) is.

Ook is $0 \leq S_{2n} = b_1 - b_2 + b_3 - b_4 + \cdots - b_{2n-2} + b_{2n-1} - b_{2n}$

$$= b_1 - (b_2 - b_3) - (b_4 - b_5) - \cdots - (b_{2n-2} - b_{2n-1}) - b_{2n}$$

$$\leq b_1,$$

sodat $\{S_{2n}\}$ begrens (bounded) is.

Dus konvergeer $\{S_{2n}\}$ (MRS) (MST), sê $\lim_{n \to \infty} S_{2n} = S$.

Dit volg dat $\lim_{n\to\infty} S_{2n+1} = \lim_{n\to\infty} (S_{2n} + b_{2n+1})$

$$= \lim_{n\to\infty} S_{2n} + \lim_{n\to\infty} b_{2n+1}$$

$$= S + 0 = S.$$

Dus is $\lim_{n\to\infty} S_n = S$, sodat $\sum_{n=1}^{\infty} (-1)^{n-1} b_n < \infty$ (Ex. 11.1 nr 92(a)).