Informele Definisie / Informal Definition
$\lim_{x\to\infty} f(x) = L$ if we can make $f(x)$ as close to L as we wish by taking x large enough.

So $\lim_{x\to\infty} f(x) = L$ means that we can make the distance between $f(x)$ and L, i.e. $|f(x) - L|$, as small as we like, provided that x is large enough.

"we can make $|f(x) - L|$ as small as we like" means that, given an arbitrary positive number ϵ (possibly very small), we can make $|f(x) - L|$ so small that it is less than ϵ;

"x is large enough" means that there is some number $N \in \mathbb{R}$ (possibly very large), so that x is so large that $x \geq N$.

So $\lim_{x\to\infty} f(x) = L$ means that, given any $\epsilon > 0$, then $|f(x) - L| < \epsilon$, provided that $x \geq N$ for some (well-chosen) $N \in \mathbb{R}$.

Formele Definisie / Formal Definition
$\lim_{x\to\infty} f(x) = L$ if, given any $\epsilon > 0$, there exists an $N \in \mathbb{R}$ such that if $x \geq N$, then $|f(x) - L| < \epsilon$.