Oneindige Reekse / Infinite Series §11.2

Definition 2

Consider the *(infinite) series*

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \cdots + a_n + \cdots.$$

Let

$$s_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n$$

be the nth *parsiële som / partial sum* of the series.

If the sequence $\{s_n\}$ converges, say $\lim_{n\to\infty} s_n = s \in \mathbb{R}$, then we say that the series $\sum_{n=1}^{\infty} a_n$ is *convergent* and write

$$\sum_{n=1}^{\infty} a_n = s \qquad \text{or} \qquad a_1 + a_2 + \cdots = s.$$

The number s is called the *som / sum* of the series.

If $\{s_n\}$ diverges, then we say that the series is *divergent*.

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} \sum_{i=1}^{n} a_i.$$

Stelling 4

Die meetkundige reeks / The geometric series

$$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \cdots$$

converges if $|r| < 1$, with sum

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}.$$

If $|r| \geq 1$, then the geometric series is divergent.

Homework

Ex. 11.2 nr. 1, 2, 16, 17, 25, 53, 61, 67

Theorem 8

If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent series, and c is a constant, then the series $\sum_{n=1}^{\infty} ca_n$, $\sum_{n=1}^{\infty} (a_n + b_n)$ and $\sum_{n=1}^{\infty} (a_n - b_n)$ are convergent, and

- $\sum_{n=1}^{\infty} ca_n = c \sum_{n=1}^{\infty} a_n$
- $\sum_{n=1}^{\infty} (a_n + b_n) = \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$
- $\sum_{n=1}^{\infty} (a_n - b_n) = \sum_{n=1}^{\infty} a_n - \sum_{n=1}^{\infty} b_n$

Homework

Ex. 11.2 nr. 29, 31, 39, 43, 45, 81, 82, 85, 87, 89(a)