W214 Linear Algebra

Lecturer: Dr Bartlett

1.1. Abstract vector spaces

"such that"

real numbers

$$A := \{ (a_1, a_2) : a_1, a_2 \in \mathbb{R} \}$$

$:=$ "is defined to be"

"element of"

"the set of all"

"pairs a-one, comma a-two"

$$B := \{ (b_1, b_2, b_3) : b_1, b_2, b_3 \in \mathbb{R}, \; b_1 - b_2 + b_3 = 0 \}$$

"3-tuple"

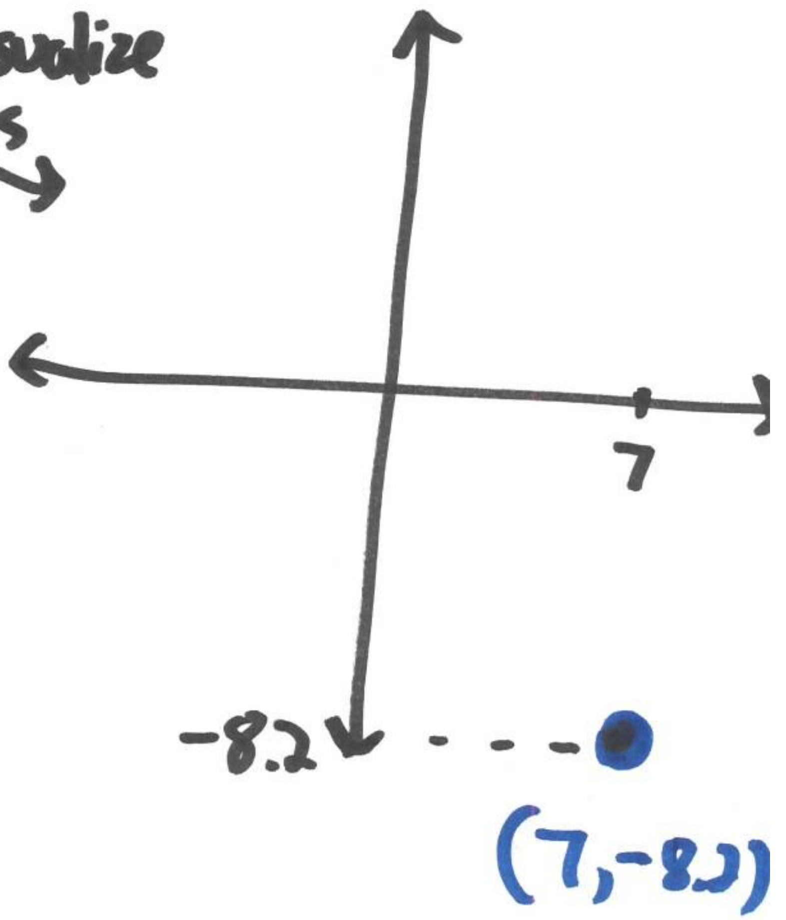
$$C := \{ \text{all polynomials of degree} \leq 3 \}$$

Set A

eg. $(7, -8.2) \in A$

$(\pi, e^{\pi}) \in A$

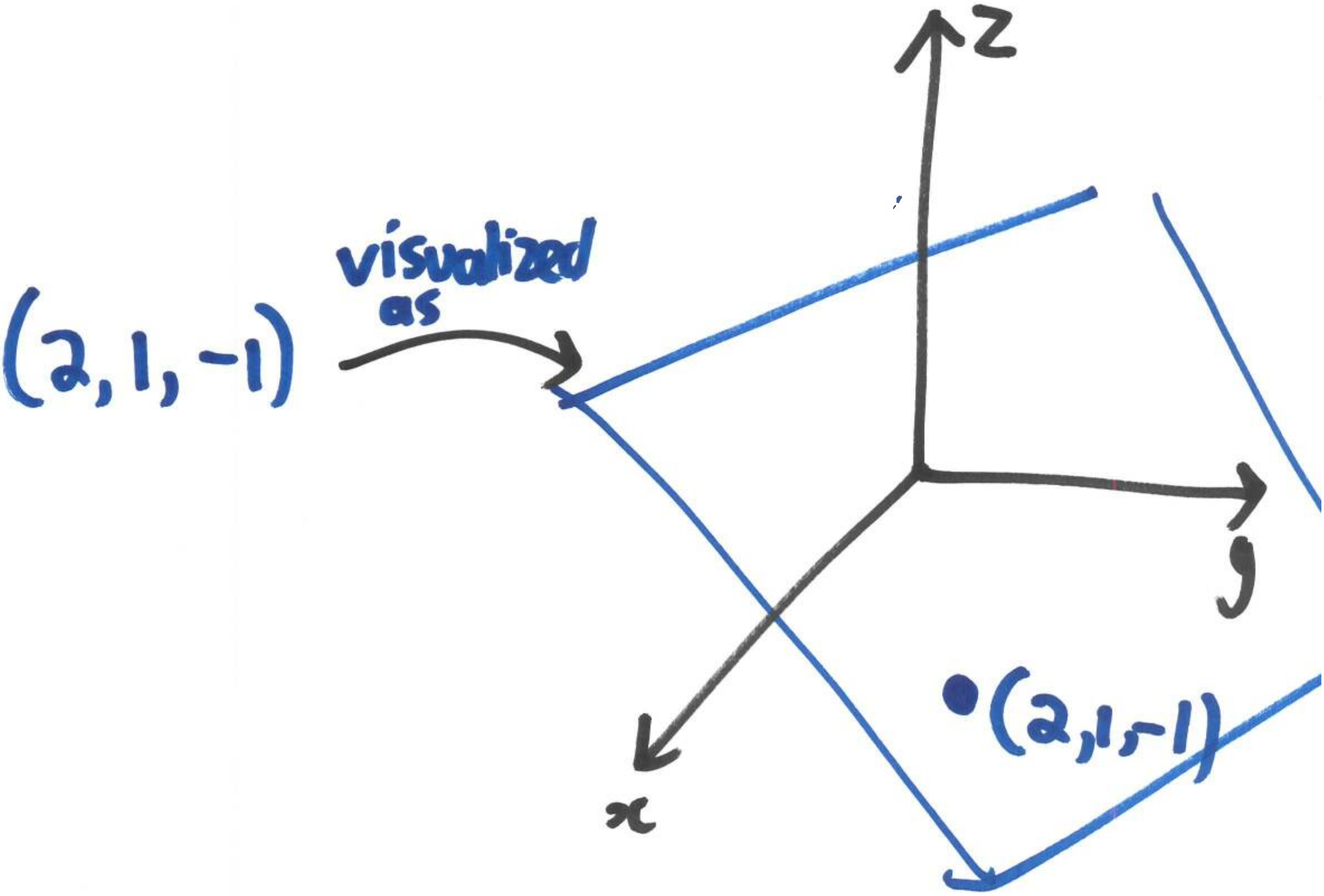
$\boxed{(7, -8.2)} \in A$ $\xrightarrow{\text{visualize as}}$



Set B

eg. $(2, 1, -1) \in B$

$(1, 1, 1) \notin B$



Set C

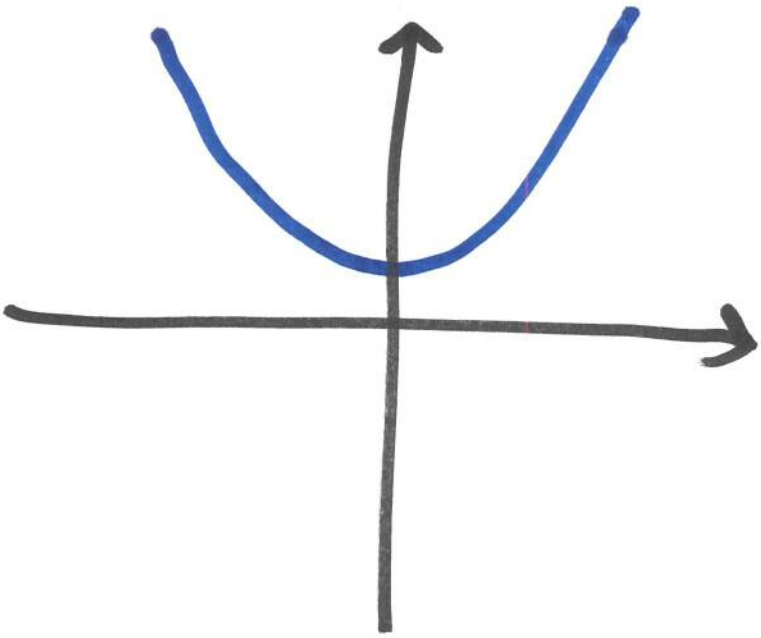
eg. $f = 2x^3 - x + 7 \quad \in C$

$g = -1.1x^2 + 2x \quad \in C$

$h = 7x^4 - 9 \quad \notin C$

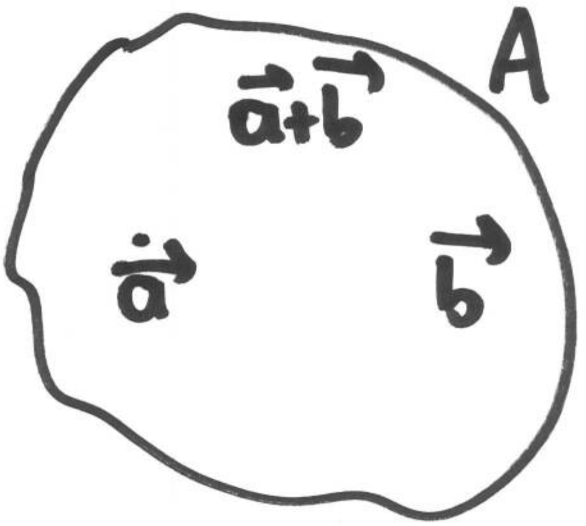
$k = x^2 + 1 \quad \in C$

$\boxed{k} \xrightarrow{\text{visualize as}}$



Features in common

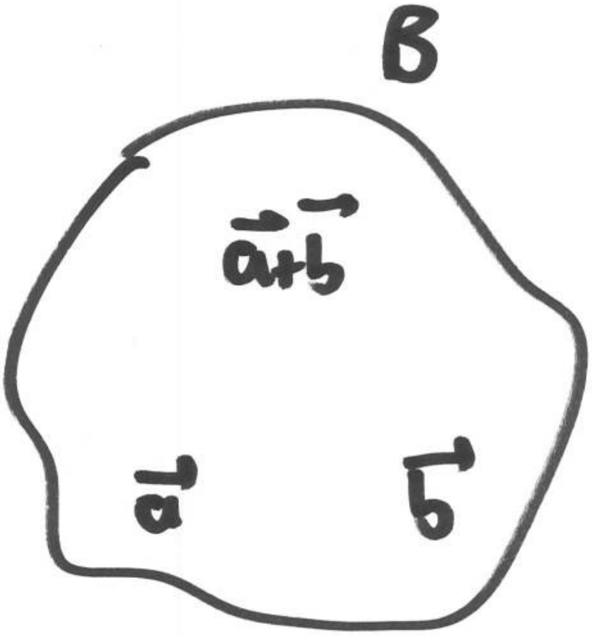
- **Addition operation:**



eg. $\vec{a} = (2,3)$

$\vec{b} = (7,-4)$

$\vec{a}+\vec{b} = (9,-1)$



$$\vec{a} = (2, 1, -1) \in B$$

$$\vec{b} = (3, 4, 1) \in B$$

$$\vec{a}+\vec{b} := (5, 5, 0) \in B$$

Definition In B, we define

$$(b_1, b_2, b_3) + (b_1', b_2', b_3')$$

$$:= (b_1 + b_1', b_2 + b_2', b_3 + b_3').$$

Lemma If $(b_1, b_2, b_3) \in B$ and

$(b_1', b_2', b_3') \in B$,

then $(b_1 + b_1', b_2 + b_2', b_3 + b_3') \in B$

<u>Proof</u> We must show that

$$(b_1 + b_1', b_2 + b_2', b_3 + b_3') \in B$$

Check:

$$(b_1 + b_1') - (b_2 + b_2') + (b_3 + b_3')$$

$$= \underbrace{(b_1 - b_2 + b_3)}_{=0} + \underbrace{(b_1' - b_2' + b_3')}_{=0}$$

$$= 0 + 0$$

$$= 0$$

□