R5. $(k+l).\vec{v} = k.\vec{v} + l.\vec{v}$?

Try: $\vec{v} = \vec{b}$, $k = 1$, $l = 1$.

$LHS = (1+1).\vec{b}$ $\quad$ $RHS = 1.\vec{b} + 1.\vec{b}$

$= 2.\vec{b}$ $\quad$ $= \vec{a} + \vec{a}$

$= \vec{a}$ $\quad$ $= \vec{a}$

<u>Proof it is true in general:</u>

Let $k, l \in \mathbb{R}$.

<u>Case: $\vec{v} = \vec{a}$</u>

$LHS = (k+l).\vec{a}$ $\quad$ $RHS = k.a + l.\vec{a}$

$= \vec{a}$ $\quad$ $= \vec{a} + \vec{a}$

$= \vec{a}$

So $LHS = RHS$.

Case: $\vec{v} = \vec{b}$

$$LHS = (k+l).\vec{b}$$
$$= \vec{a}$$

$$RHS = k.\vec{b} + l.\vec{b}$$
$$= \vec{a} + \vec{a}$$
$$= \vec{a}$$

so $LHS = RHS$.

$\therefore$ R5 is true.

<u>But</u> :

R7 : Take $\vec{v} = \vec{b}$.

LHS $= 1 . \vec{b} = \vec{a}$

but

RHS $= \vec{b}$

$\therefore$ LHS $\neq$ RHS for R7.

$\therefore$ R7 not satisfied.

$\therefore$ V is not a vector space.

<u>Example</u> The set

such that

$$B := \{ (b_1, b_2, b_3) \in \mathbb{R}^3 \text{ s.t. } b_1 - b_2 + b_3 = 0 \}$$

equipped with the following data, is a vector space.

D1. <u>addition</u>

$$(b_1, b_2, b_3) + (c_1, c_2, c_3) := (b_1 + c_1, b_2 + c_2, b_3 + c_3)$$

D2. <u>zero vector</u>

$$\vec{0} := (0, 0, 0)$$

D3. <u>scalar mult</u>: $k.(b_1, b_2, b_3) := (kb_1, kb_2, kb_3)$

Check:

R1 Let $\vec{b} = (b_1, b_2, b_3)$
$\vec{c} = (c_1, c_2, c_3)$.

Then:

$\vec{b} + \vec{c} = (b_1, b_2, b_3) + (c_1, c_2, c_3)$

$= (b_1 + c_1, b_2 + c_2, b_3 + c_3)$

[by defn of addition in B]

$= (c_1 + b_1, c_2 + b_2, c_3 + b_3)$

[since $x + y = y + x$ for real numbers]

$= (c_1, c_2, c_3) + (b_1, b_2, b_3)$

[by def. of + in B]

$= \vec{c} + \vec{b}$

<u>Example</u> The zero vector space.

$$Z := \{\vec{z}\}$$

D1. $\vec{z} + \vec{z} := \vec{z}$

D2. $\vec{0} := \vec{z}$

D3. $k.\vec{z} := \vec{z}$

<u>Exercise</u> Check this satisfies R1 to R8.

<u>R5</u> $\text{LHS} = (k+\ell).\vec{z}$
$= \vec{z}$

$\text{RHS} = k.\vec{z} + \ell.\vec{z}$
$= \vec{z} + \vec{z}$
$= \vec{z}$ so LHS = RHS.

Example ($\mathbb{R}^n$)

$$\mathbb{R}^n := \left\{ (x_1, \cdots, x_n) \; : \; x_i \in \mathbb{R} \;\; \forall\, i = 1 \ldots n \right\}$$

D1. $(x_1, \cdots, x_n) + (y_1, \cdots, y_n)$

$$:= (x_1 + y_1, \cdots, x_n + y_n)$$

D2. $\vec{0} := (0, \cdots, 0)$

D3. $k.(x_1, \cdots, x_n) := (kx_1, \cdots, kx_n)$

Example ($\mathbb{R}^\infty$)

$$\mathbb{R}^\infty := \left\{ (x_1, x_2, x_3, \cdots), \, x_i \in \mathbb{R}, \; i = 1 \ldots \infty \right\}.$$

D1. $(x_1, x_2, \cdots) + (y_1, y_2, \cdots)$

$$:= (x_1 + y_1, x_2 + y_2, \cdots)$$

D2, D3 similar.

<u>Example</u> Let X be any set.

$$\mathrm{Fun}(X) := \{\text{functions } f : X \longrightarrow \mathbb{R}\}$$

eg. $X = \{a, b, c\}$

$f(a) = 7$	$g(a) = 2$
$f(b) = 1.2$	$g(b) = 0$
$f(c) = 6$	$g(c) = -5.9$
$f \in \mathrm{Fun}(X)$	$g \in \mathrm{Fun}(X)$

$\mathrm{Fun}(X) =$

D1. $(f+g)(x) := f(x) + g(x)$

$x \in X$

D2. $\vec{0} := z$

where $z(x) = 0$

for all $\rightarrow \forall x \in X$

D3. $(k.f)(x) := \underbrace{k}_{\text{number}}\,\underbrace{f(x)}_{\text{number}}$

number function