Discrete-Time Fourier Series

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A. Discrete-time Fourier series

Given the signal $x[n]$ as following:

$$x[n] = 4\cos(2.4\pi n) + 2\sin(3.2\pi n)$$

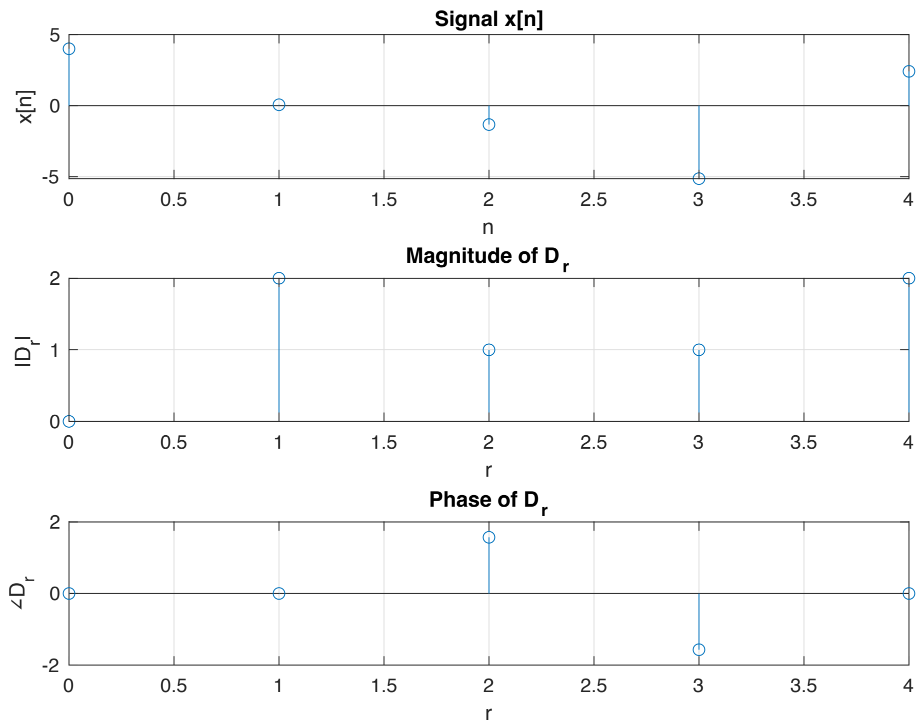
$$\frac{m}{N_1} = \frac{\Omega_1}{2\pi} = \frac{2.4}{2\pi} = \frac{6}{5} \rightarrow N_1 = 5$$

$$\frac{n}{N_2} = \frac{\Omega_2}{2\pi} = \frac{3.2}{2\pi} = \frac{8}{5} \rightarrow N_2 = 5$$

The fundamental period of $x[n]$ is: $N_0 = LCM(5,5) = 5$

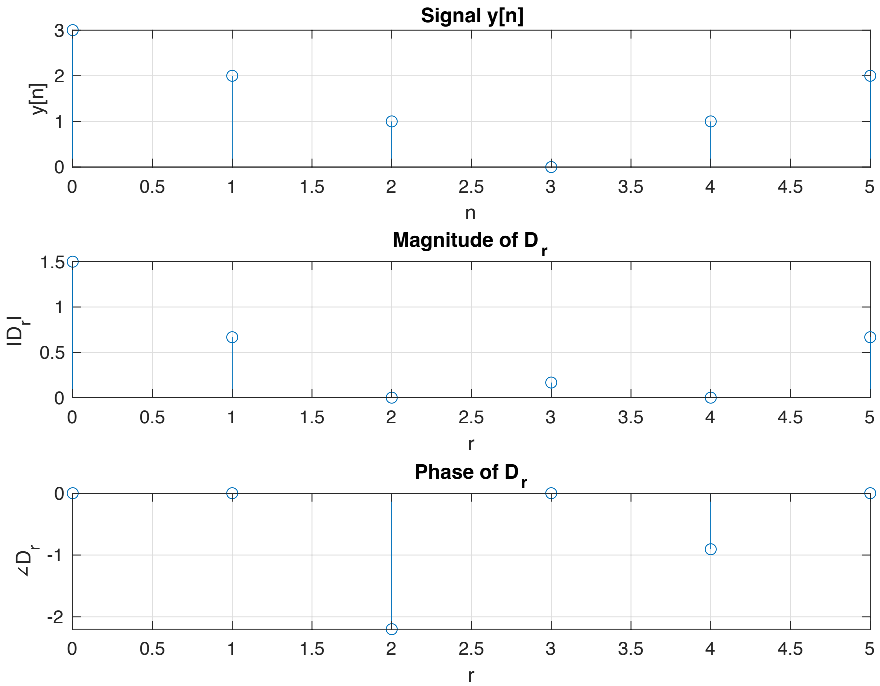
The fundamental frequency of $x[n]$ is: $\Omega_0 = \frac{2\pi}{N_0} = \frac{2\pi}{5}$

```
% A.2

N_0 = 5; %fundamental period
omega_0 = 2*pi/5; % fundamental frequency

n = 0:N_0-1;

% Signal x[n] can be rewritten as following:
x = @(n) (4.*cos(0.4*pi*n) + 2.*sin(1.2*pi*n));

for r = 0:N_0-1
    D_r(r+1) = (1/N_0).*sum(x(n).*exp(-j*r*omega_0*n));
end

r = n;

figure;
subplot(3,1,1);
stem(n,x(n));
grid;
title("Signal x[n]");
xlabel('n');
ylabel('x[n]');

subplot(3,1,2);
stem(r,abs(D_r));
grid;
title("Magnitude of D_r");
xlabel('r');
ylabel('|D_r|');

subplot(3,1,3);
stem(r,angle(D_r));
grid;
title("Phase of D_r");
xlabel('r');
ylabel('\angleD_r');
```



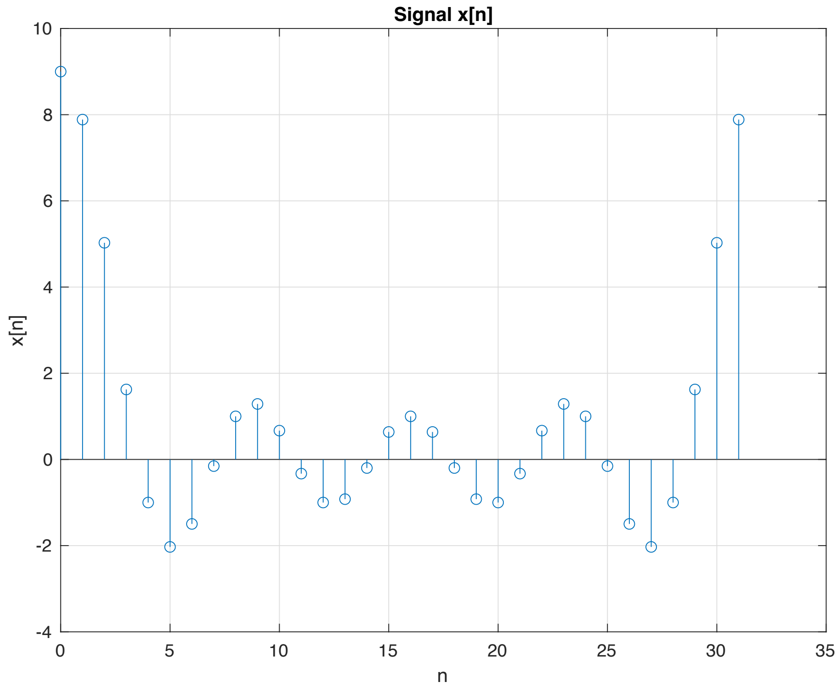

```
% A.3

N_0 = 6; %fundamental period
omega_0 = 2*pi/6; % fundamental frequency

n = 0:N_0-1;

% Signal y[n] can be represented as following:
y_n = [3 2 1 0 1 2];

for r = 0:N_0-1
    D_r(r+1) = (1/N_0).*sum(y_n.*exp(-j*r*omega_0*n));
end

r = n;

figure;
subplot(3,1,1);
stem(n,y_n);
grid;
title("Signal y[n]");
xlabel('n');
ylabel('y[n]');

subplot(3,1,2);
stem(r,abs(D_r));
grid;
title("Magnitude of D_r");
xlabel('r');
ylabel('|D_r|');

subplot(3,1,3);
```

```
stem(r,angle(D_r));
grid;
title("Phase of D_r");
xlabel('r');
ylabel('\angleD_r');
```

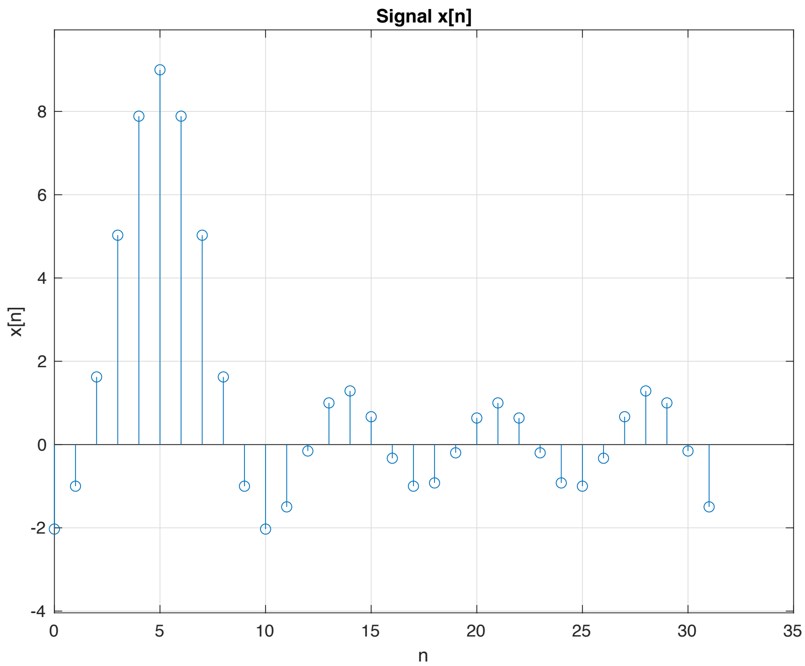


B. Inverse DTFS and time shifting property

```
% B.1
N_0 = 32; %fundamental period
omega_0 = 2*pi/32; % fundamental frequency

r = 0:N_0-1;

% Rewrite Xr in the interval of [0,N_0-1]
X_r = [ones(1,5) zeros(1,23) ones(1,4)];

for n = 0:N_0-1
    x_n(n+1) = sum(X_r.*exp(-j*r*omega_0*n));
end

n = r;

figure;
stem(n,x_n);
grid;
title("Signal x[n]");
xlabel('n');
ylabel('x[n]');
```



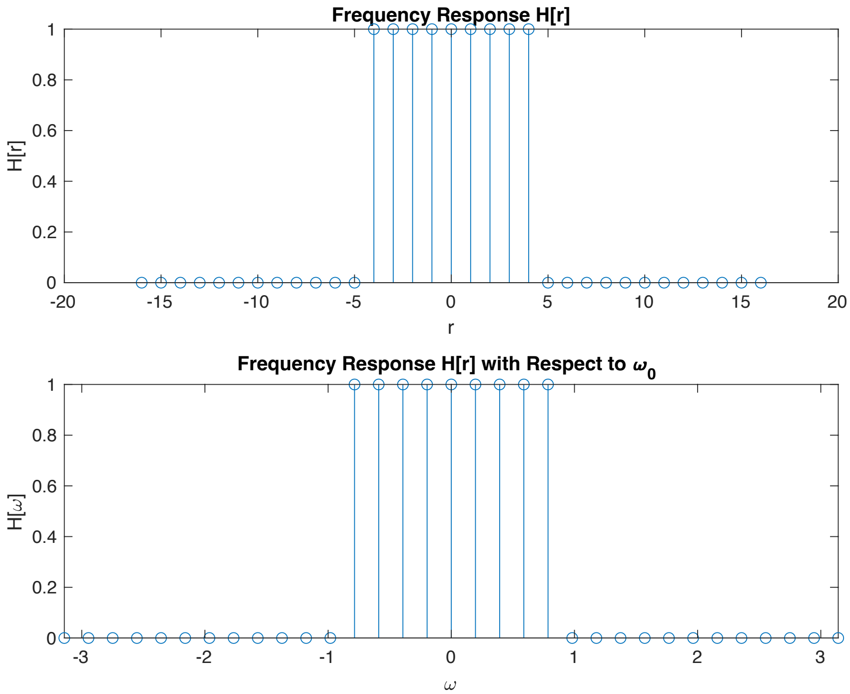

```
% B.2
N_0 = 32; %fundamental period
omega_0 = 2*pi/32; % fundamental frequency

r = 0:N_0-1;

% Rewrite Xr in the interval of [0,N_0-1]
X_r = [ones(1,5) zeros(1,23) ones(1,4)].*exp(-j*5*omega_0*r);

for n = 0:N_0-1
    x_n(n+1) = sum(X_r.*exp(j*r*omega_0*n));
end

n = r;

figure;
stem(n,x_n);
grid;
title("Signal x[n]");
xlabel('n');
ylabel('x[n]');
```




```
% Signal x[n] obtained in this part is shifted version of signal from the
% previous part.
```

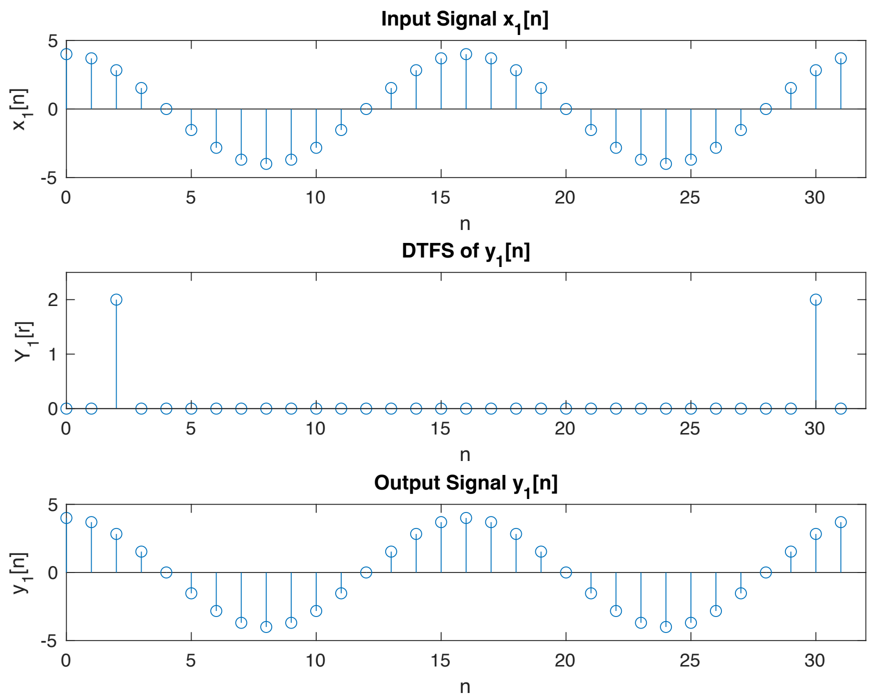
C. System response

```
% C.1
N_0 = 32; %fundamental period
omega_0 = 2*pi/N_0; % fundamental frequency

u = @(r) (r >= 0) * 1.0 .* (mod(r,1)==0);
H = @(r) u(r+4)-u(r-5);

r = (-16:16);

omega = r.*omega_0;

figure;
subplot(2, 1, 1);
stem(r,H(r));
xlabel('r');
ylabel('H[r]');
title('Frequency Response H[r]');

subplot(2, 1, 2);
stem(omega,H(r));
xlabel('\omega');
ylabel('H[\omega]');
axis([-pi pi 0 1]);
title ('Frequency Response H[r] with Respect to \omega_0 ');
```



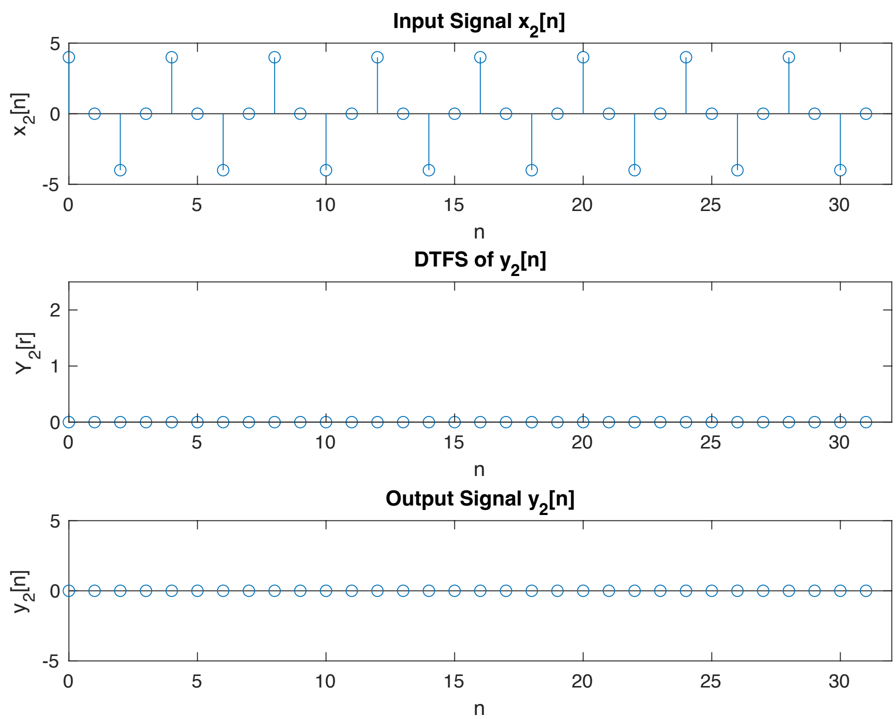

```
% C.2

N_0 = 32; %fundamental period
omega_0 = 2*pi/N_0;% fundamental frequency

n = 0:N_0-1;

% Rewrite H[r] in the interval of [0,N_0-1]
H = [ones(1,5) zeros(1,23) ones(1,4)];
x1 = @(n) 4.0*cos(pi*n/8); % Input Signal x1[n]

for r = 0:N_0-1
    X1(r+1) = (1/N_0).*sum(x1(n).*exp(-j.*r.*(omega_0).*n));
end

r = n;

Y1 = H.*X1; % Calculate DTFS of y2[n]

for n = 0:N_0-1
    y1(n+1) = sum(Y1.*exp(-j*r*(omega_0)*n));
end

n = r;

figure;
subplot(3,1,1);
stem(n,x1(n));
title('Input Signal x_1[n]');
axis([0 32 -5 5]);
xlabel('n');
ylabel('x_1[n]');
```

```
subplot(3,1,2);
stem(r,Y1);
title('DTFS of y_1[n]');
axis([0 32 0 2.5]);
xlabel('n');
ylabel('Y_1[r]');

subplot(3,1,3);
stem(n,y1);
title('Output Signal y_1[n]');
axis([0 32 -5 5]);
xlabel('n');
ylabel('y_1[n]');
```




```
% C.3
N_0 = 32; %fundamental period
omega_0 = 2*pi/N_0; % fundamental frequency

n = 0:N_0-1;

% Rewrite H[r] in the interval of [0,N_0-1]
H = [ones(1,5) zeros(1,23) ones(1,4)];
x2 = @(n) 4.0*cos(pi*n/2); % Input signal

for r = 0:N_0-1
    X2(r+1) = (1/N_0).*sum(x2(n).*exp(-j.*r.*(omega_0).*n));
end

r = n;

Y2 = H.*X2; % Calculate DTFS of y2[n]

for n = 0:N_0-1
    y2(n+1) = sum(Y2.*exp(-j*r*(omega_0)*n));
```

```
end

n = r;

figure;
subplot(3,1,1);
stem(n,x2(n));
title('Input Signal x_2[n]');
axis([0 32 -5 5]);
xlabel('n');
ylabel('x_2[n]');

subplot(3,1,2);
stem(r,Y2);
title('DTFS of y_2[n]');
axis([0 32 0 2.5]);
xlabel('n');
ylabel('Y_2[r]');

subplot(3,1,3);
stem(n,y2);
title('Output Signal y_2[n]');
axis([0 32 -5 5]);
xlabel('n');
ylabel('y_2[n]');
```




```
% C.4

% x1[n] passes through the filter unaltered and x2[n] is completely
% rejected by the filter. The reason is frequecies presented in x1[n]
% lie inside the passband of the filter (where filter allows to pass)
% while frequecies presented in x2[n] lie inside the reject-band of
% the filter (where filter does not allows to pass).
```