

Discrete-Time Fourier Series

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A. Discrete-time Fourier series

Given the signal $x[n]$ as following:

$$x[n] = 4 \cos(2.4\pi n) + 2 \sin(3.2\pi n)$$

$$\frac{m}{N_1} = \frac{\Omega_1}{2\pi} = \frac{2.4}{2\pi} = \frac{6}{5} \rightarrow N_1 = 5$$

$$\frac{n}{N_2} = \frac{\Omega_2}{2\pi} = \frac{3.2}{2\pi} = \frac{8}{5} \rightarrow N_2 = 5$$

The fundamental period of $x[n]$ is: $N_0 = LCM(5,5) = 5$

The fundamental frequency of $x[n]$ is: $\Omega_0 = \frac{2\pi}{N_0} = \frac{2\pi}{5}$

% A.2

```
N_0 = 5; %fundamental period
omega_0 = 2*pi/5; % fundamental frequency

n = 0:N_0-1;

% Signal x[n] can be rewritten as following:
x = @(n) (4.*cos(0.4*pi*n) + 2.*sin(1.2*pi*n));

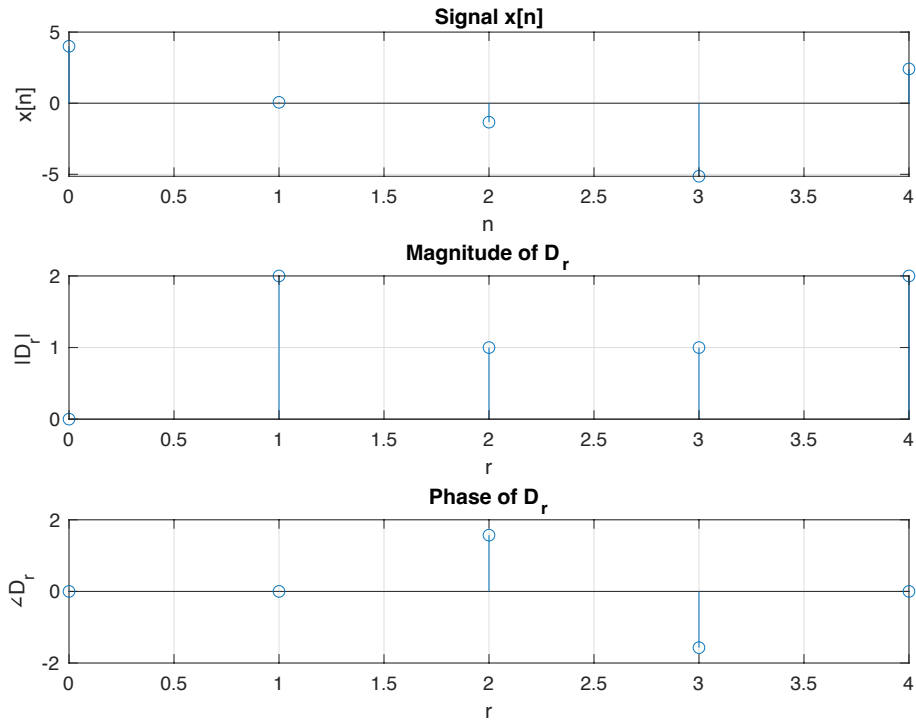
for r = 0:N_0-1
    D_r(r+1) = (1/N_0).*sum(x(n).*exp(-j*r*omega_0*n));
end

r = n;

figure;
subplot(3,1,1);
stem(n,x(n));
grid;
title("Signal x[n]");
xlabel('n');
ylabel('x[n]');

subplot(3,1,2);
stem(r,abs(D_r));
grid;
title("Magnitude of D_r");
xlabel('r');
ylabel('|D_r|');

subplot(3,1,3);
stem(r,angle(D_r));
grid;
title("Phase of D_r");
xlabel('r');
ylabel('\angle D_r');
```



`% A.3`

```

N_0 = 6; %fundamental period
omega_0 = 2*pi/6; % fundamental frequency

n = 0:N_0-1;

% Signal  $y[n]$  can be represented as following:
y_n = [3 2 1 0 1 2];

for r = 0:N_0-1
    D_r(r+1) = (1/N_0).*sum(y_n.*exp(-j*r*omega_0*n));
end

r = n;

figure;
subplot(3,1,1);
stem(n,y_n);
grid;
title("Signal  $y[n]$ ");
xlabel('n');
ylabel('y[n]');

subplot(3,1,2);
stem(r,abs(D_r));
grid;
title("Magnitude of  $D_r$ ");
xlabel('r');
ylabel('|D_r|');

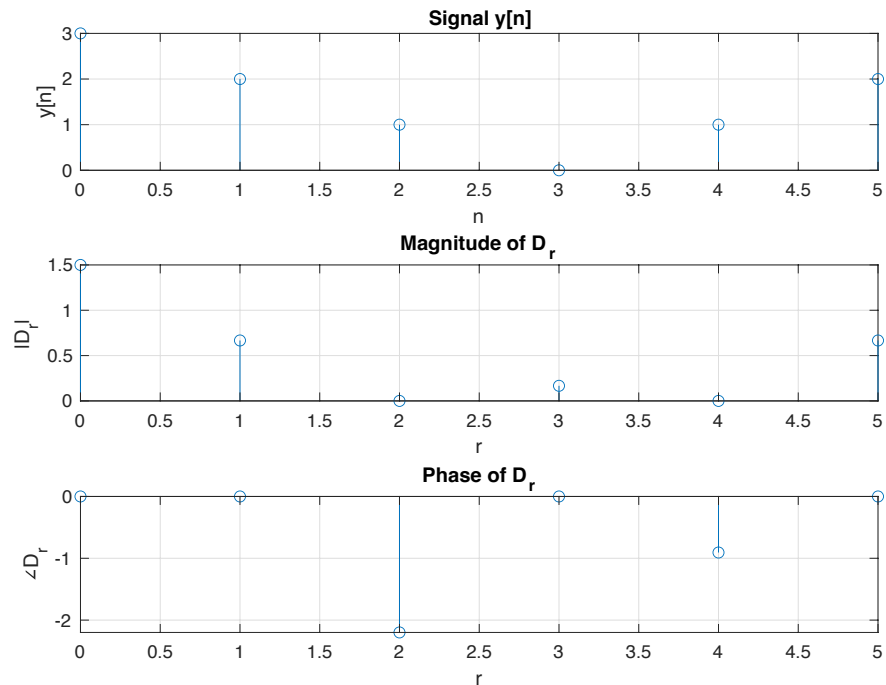
subplot(3,1,3);

```

```

stem(r,angle(D_r));
grid;
title("Phase of D_r");
xlabel('r');
ylabel('\angled_r');

```



B. Inverse DTFS and time shifting property

```

% B.1
N_0 = 32; %fundamental period
omega_0 = 2*pi/32; % fundamental frequency

r = 0:N_0-1;

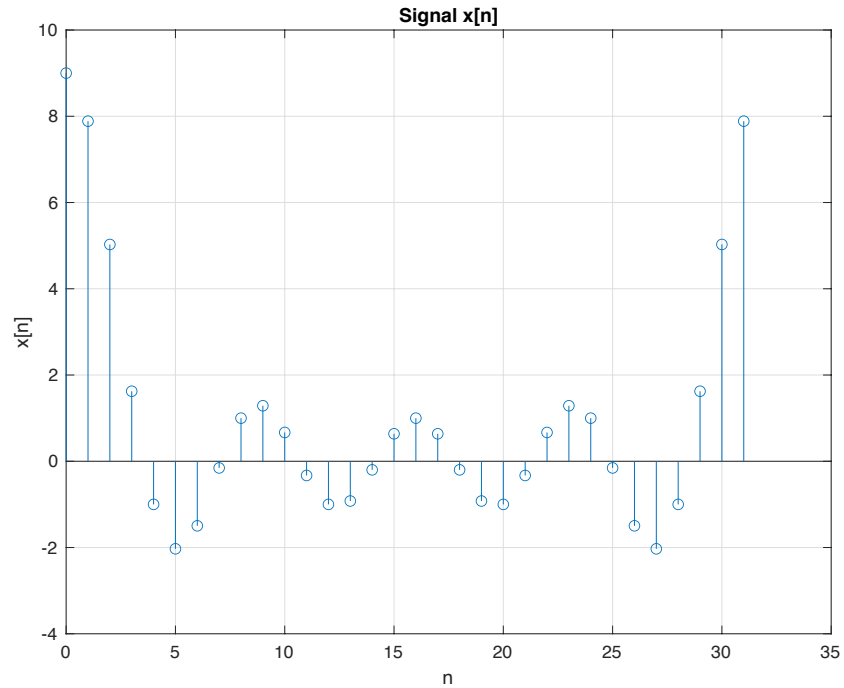
% Rewrite X_r in the interval of [0,N_0-1]
X_r = [ones(1,5) zeros(1,23) ones(1,4)];

for n = 0:N_0-1
    x_n(n+1) = sum(X_r.*exp(-j*r*omega_0*n));
end

n = r;

figure;
stem(n,x_n);
grid;
title("Signal x[n]");
xlabel('n');
ylabel('x[n]');

```



```
% B.2
N_0 = 32; %fundamental period
omega_0 = 2*pi/32; % fundamental frequency

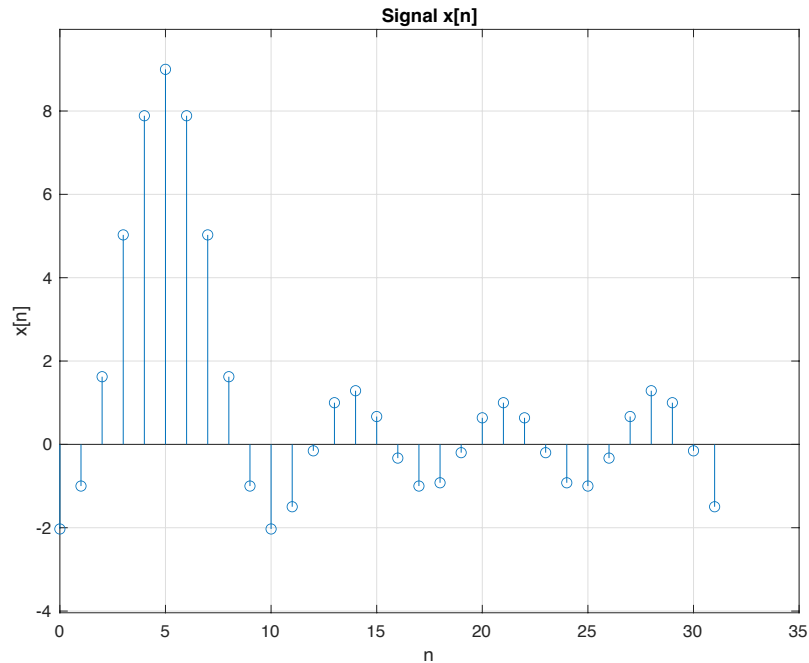
r = 0:N_0-1;

% Rewrite Xr in the interval of [0,N_0-1]
X_r = [ones(1,5) zeros(1,23) ones(1,4)].*exp(-j*5*omega_0*r);

for n = 0:N_0-1
    x_n(n+1) = sum(X_r.*exp(j*r*omega_0*n));
end

n = r;

figure;
stem(n,x_n);
grid;
title("Signal x[n]");
xlabel('n');
ylabel('x[n]');
```



% Signal $x[n]$ obtained in this part is shifted version of signal from the
% previous part.

C. System response

```
% C.1
N_0 = 32; %fundamental period
omega_0 = 2*pi/N_0; % fundamental frequency

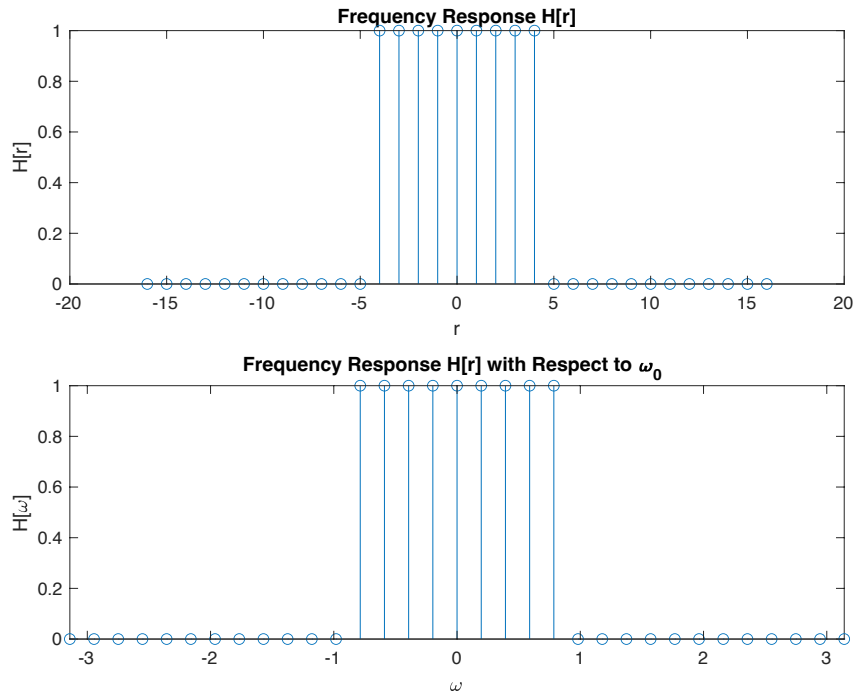
u = @(r) (r >= 0) * 1.0 .* (mod(r,1)==0);
H = @(r) u(r+4)-u(r-5);

r = (-16:16);

omega = r.*omega_0;

figure;
subplot(2, 1, 1);
stem(r,H(r));
xlabel('r');
ylabel('H[r]');
title('Frequency Response H[r]');

subplot(2, 1, 2);
stem(omega,H(r));
xlabel('\omega');
ylabel('H[\omega]');
axis([-pi pi 0 1]);
title ('Frequency Response H[r] with Respect to \omega_0 ');
```



% C.2

```

N_0 = 32; %fundamental period
omega_0 = 2*pi/N_0;% fundamental frequency

n = 0:N_0-1;

% Rewrite H[r] in the interval of [0,N_0-1]
H = [ones(1,5) zeros(1,23) ones(1,4)];
x1 = @(n) 4.0*cos(pi*n/8); % Input Signal x1[n]

for r = 0:N_0-1
    X1(r+1) = (1/N_0).*sum(x1(n).*exp(-j.*r.*(omega_0).*n));
end

r = n;

Y1 = H.*X1; % Calculate DTFS of y2[n]

for n = 0:N_0-1
    y1(n+1) = sum(Y1.*exp(-j*r*(omega_0)*n));
end

n = r;

figure;
subplot(3,1,1);
stem(n,x1(n));
title('Input Signal x_1[n]');
axis([0 32 -5 5]);
xlabel('n');
ylabel('x_1[n]');

```

The figure consists of three vertically stacked plots, all sharing a common horizontal axis labeled n ranging from 0 to 32.

- Top Plot: Input Signal $x_1[n]$**
The vertical axis is labeled $x_1[n]$ and ranges from -5 to 5. The signal is a periodic sequence of impulses with a period of 8 samples. The values of the impulses are approximately: $x_1[0] \approx 4.5$, $x_1[1] \approx 4.0$, $x_1[2] \approx 3.5$, $x_1[3] \approx 2.0$, $x_1[4] = 0$, $x_1[5] \approx -1.5$, $x_1[6] \approx -2.5$, $x_1[7] \approx -3.5$, $x_1[8] \approx -3.0$, $x_1[9] \approx -1.5$, $x_1[10] \approx -2.5$, $x_1[11] \approx -1.0$, $x_1[12] = 0$, $x_1[13] \approx 2.0$, $x_1[14] \approx 3.5$, $x_1[15] \approx 4.5$, $x_1[16] \approx 4.0$, $x_1[17] \approx 3.5$, $x_1[18] \approx 2.0$, $x_1[19] \approx 1.0$, $x_1[20] = 0$, $x_1[21] \approx -1.5$, $x_1[22] \approx -2.5$, $x_1[23] \approx -3.5$, $x_1[24] \approx -3.0$, $x_1[25] \approx -1.5$, $x_1[26] \approx -2.5$, $x_1[27] \approx -1.0$, $x_1[28] = 0$, $x_1[29] \approx 2.0$, $x_1[30] \approx 3.5$, $x_1[31] \approx 4.5$.
- Middle Plot: DTFS of $y_1[r]$**
The vertical axis is labeled $y_1[r]$ and ranges from 0 to 2. The signal is a periodic sequence of impulses with a period of 8 samples. The values of the impulses are approximately: $y_1[0] = 0$, $y_1[1] = 0$, $y_1[2] = 2.0$, $y_1[3] = 0$, $y_1[4] = 0$, $y_1[5] = 0$, $y_1[6] = 0$, $y_1[7] = 0$, $y_1[8] = 0$, $y_1[9] = 0$, $y_1[10] = 0$, $y_1[11] = 0$, $y_1[12] = 0$, $y_1[13] = 0$, $y_1[14] = 0$, $y_1[15] = 0$, $y_1[16] = 0$, $y_1[17] = 0$, $y_1[18] = 0$, $y_1[19] = 0$, $y_1[20] = 0$, $y_1[21] = 0$, $y_1[22] = 0$, $y_1[23] = 0$, $y_1[24] = 0$, $y_1[25] = 0$, $y_1[26] = 0$, $y_1[27] = 0$, $y_1[28] = 0$, $y_1[29] = 0$, $y_1[30] = 2.0$, $y_1[31] = 0$.
- Bottom Plot: Output Signal $y_1[n]$**
The vertical axis is labeled $y_1[n]$ and ranges from -5 to 5. The signal is a periodic sequence of impulses with a period of 8 samples. The values of the impulses are approximately: $y_1[0] \approx 4.5$, $y_1[1] \approx 4.0$, $y_1[2] \approx 3.5$, $y_1[3] \approx 2.0$, $y_1[4] = 0$, $y_1[5] \approx -1.5$, $y_1[6] \approx -2.5$, $y_1[7] \approx -3.5$, $y_1[8] \approx -3.0$, $y_1[9] \approx -1.5$, $y_1[10] \approx -2.5$, $y_1[11] \approx -1.0$, $y_1[12] = 0$, $y_1[13] \approx 2.0$, $y_1[14] \approx 3.5$, $y_1[15] \approx 4.5$, $y_1[16] \approx 4.0$, $y_1[17] \approx 3.5$, $y_1[18] \approx 2.0$, $y_1[19] \approx 1.0$, $y_1[20] = 0$, $y_1[21] \approx -1.5$, $y_1[22] \approx -2.5$, $y_1[23] \approx -3.5$, $y_1[24] \approx -3.0$, $y_1[25] \approx -1.5$, $y_1[26] \approx -2.5$, $y_1[27] \approx -1.0$, $y_1[28] = 0$, $y_1[29] \approx 2.0$, $y_1[30] \approx 3.5$, $y_1[31] \approx 4.5$.

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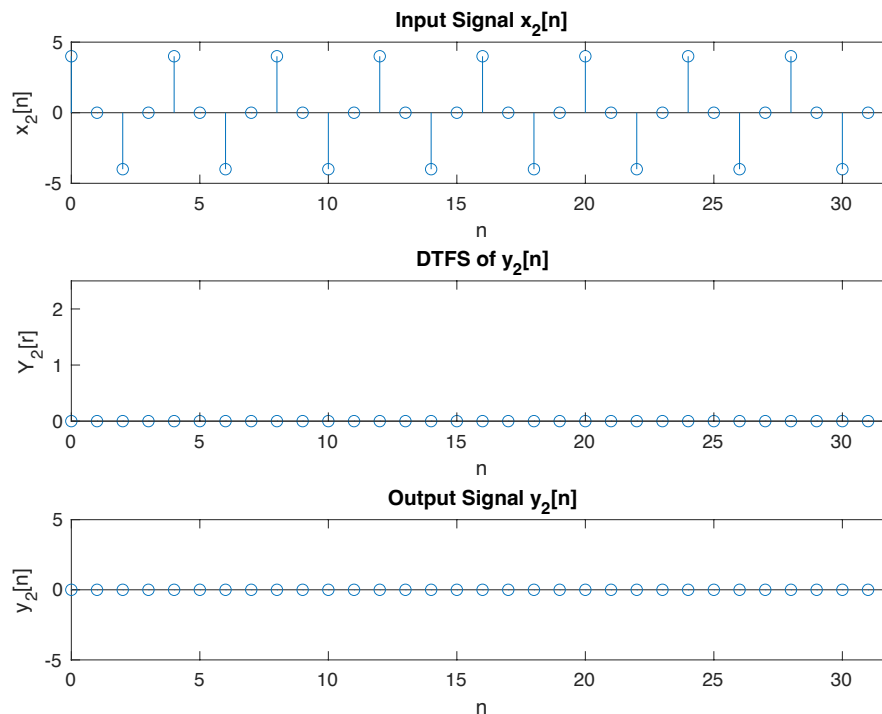

```
end
```

```
n = r;
```

```
figure;  
subplot(3,1,1);  
stem(n,x2(n));  
title('Input Signal x_2[n]');  
axis([0 32 -5 5]);  
xlabel('n');  
ylabel('x_2[n]');
```

```
subplot(3,1,2);  
stem(r,Y2);  
title('DTFS of y_2[n]');  
axis([0 32 0 2.5]);  
xlabel('n');  
ylabel('Y_2[r]');
```

```
subplot(3,1,3);  
stem(n,y2);  
title('Output Signal y_2[n]');  
axis([0 32 -5 5]);  
xlabel('n');  
ylabel('y_2[n]');
```



```
% C.4
```

```
% x1[n] passes through the filter unaltered and x2[n] is completely  
% rejected by the filter. The reason is frequencies presented in x1[n]  
% lie inside the passband of the filter (where filter allows to pass)  
% while frequencies presented in x2[n] lie inside the reject-band of  
% the filter (where filter does not allow to pass).
```