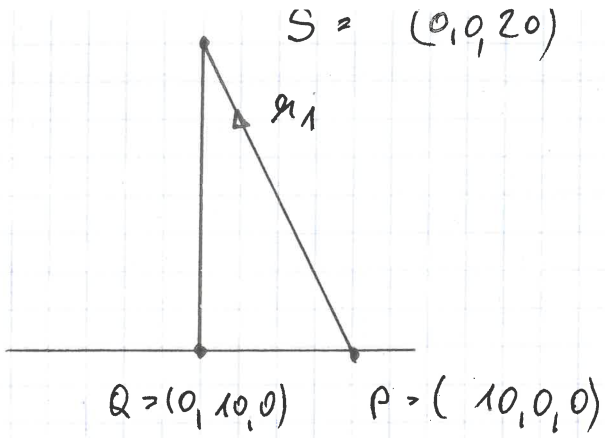
1.)



a.) $\mathcal{E}_1$: $\vec{r}_1 = \begin{pmatrix} -10 \\ 0 \\ 20 \end{pmatrix} \sim \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$

$\vec{r}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$$\vec{n}_1 = \vec{r}_1 \times \vec{r}_2 = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix}}}$$

$$\mathcal{E}_1: \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 20 \end{pmatrix}\right) \cdot \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} = 0$$

$$-2x - z = -20$$

$$\underline{\underline{2x + z = 20}}$$

b.) $\mathcal{E}_2$: $\vec{r}_1 = \begin{pmatrix} 0 \\ -10 \\ 20 \end{pmatrix} \sim \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix}$ $\quad r_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$\vec{n}_2 = \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

$$\mathcal{E}_2: \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 20 \end{pmatrix}\right) \cdot \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} = 0 \qquad \underline{\underline{2y + z = 20}}$$

c) Schnittwinkel:

$$\vec{n_1} \cdot \vec{n_2} = |\vec{n_1}| \cdot |\vec{n_2}| \cdot \cos(\varphi)$$

$$\cos\varphi = \frac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}| \cdot |\vec{n_2}|} = \frac{\begin{pmatrix}2\\0\\1\end{pmatrix} \cdot \begin{pmatrix}0\\2\\1\end{pmatrix}}{\sqrt{5} \cdot \sqrt{5}} = \underline{\underline{\frac{1}{5}}}$$

$$\underline{\underline{\varphi = 78{,}46^\circ}}$$

2a.)

$$z^2 - 5z + iz + 8 = i$$

$$z^2 - z\cdot(5-i) + 8 - i = 0$$

$$z_{1,2} = \frac{5}{2} - \frac{i}{2} \pm \sqrt{\frac{1}{4}\left((5-i)^2 - 32 + 4i\right)} = \frac{5}{2} - \frac{i}{2} \pm \frac{1}{2}\cdot\sqrt{25 - 10i - 1 - 32 + 4i}$$

$$z_{1,2} = \frac{5}{2} - \frac{i}{2} \pm \frac{1}{2}\cdot\sqrt{-8-6i}$$

$$\sqrt{-8-6i} = \pm\left(\sqrt{\frac{1}{2}(10-8)} - i\cdot\sqrt{\frac{1}{2}(10+8)}\right) = \pm(1-3i)$$

$$z_{1,2} = \frac{5}{2} - \frac{i}{2} \pm \left(\frac{1}{2} - \frac{3i}{2}\right)$$

$$z_1 = \frac{5}{2} - \frac{i}{2} + \frac{1}{2} - \frac{3i}{2} = \underline{\underline{3-2i}}$$

$$z_2 = \frac{5}{2} - \frac{i}{2} - \frac{1}{2} + \frac{3i}{2} = \underline{\underline{2+i}}$$

2.b.)

$$z^5 = 3 - 4i$$

$$z = \sqrt[5]{3-4i} \qquad |z| = \sqrt{9+16} = \sqrt{25} = 5.$$

$$k = 0, 1, 2, 3, 4.$$

$$\sqrt[5]{z} = \sqrt[5]{|z|} \cdot \left(\cos\left(\frac{\varphi + 2k\pi}{5}\right) + i\sin\left(\frac{\varphi + 2k\pi}{5}\right)\right) \qquad \varphi = \arctan\left(\frac{-4}{3}\right) = \underline{\underline{306{,}87^\circ}}$$

$k=0$: $z_1 = 1{,}38 \cdot \left(\cos\left(\frac{306{,}87}{5}\right) + i\sin\left(\frac{306{,}87}{5}\right)\right) = 1{,}38 \cdot (\cos(61{,}37) + i\sin(61{,}37)) = \underline{\underline{0{,}661 + i\,1{,}21}}$

$k=1$: $z_2 = 1{,}38 \cdot \left(\cos\left(\frac{306{,}87 + 360}{5}\right) + i\sin\left(\frac{306{,}87 + 360}{5}\right)\right) = 1{,}38 \cdot (\cos(133{,}37) + i\sin(133{,}37)) = \underline{\underline{-0{,}95 + i \cdot 0{,}73}}$

$k=2$: $z_3 = 1{,}38 \cdot \left(\cos\left(\frac{306{,}87 + 720}{5}\right) + i\sin\left(\frac{306{,}87 + 720}{5}\right)\right) = 1{,}38 \cdot (\cos(205{,}37) + i\sin(205{,}37)) = \underline{\underline{-1{,}25 - i \cdot 0{,}43}}$

$k=3$: $z_4 = 1{,}38 \cdot \left(\cos\left(\frac{306{,}87 + 1080}{5}\right) + i\sin\left(\frac{306{,}87 + 1080}{5}\right)\right) = 1{,}38 \cdot (\cos(277{,}37) + i\sin(277{,}37)) = \underline{\underline{0{,}18 - i\,0{,}98}}$

$k=4$: $z_5 = 1{,}38 \cdot \left(\cos\left(\frac{306{,}87 + 1440}{5}\right) + i\sin\left(\frac{306{,}87 + 1440}{5}\right)\right) = 1{,}38 \cdot (\cos(349{,}37) + i\sin(349{,}37)) = \underline{\underline{1{,}36 - i \cdot 0{,}25}}$

3.)

$$\lim_{x\to 0} \frac{\sqrt{1+x\sin(x)} - \cos(x)}{\sin^2\left(\frac{x}{2}\right)} = \frac{0}{0} = \lim_{x\to 0} \frac{\frac{1}{2}\cdot\frac{1}{\sqrt{}}\cdot\left(\sin(x) + x\cos(x)\right) + \sin(x)}{\cancel{2}\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\cdot\frac{1}{\cancel{4}}}$$

$$= \lim_{x\to 0} \frac{\cancel{\frac{1}{2}}\,\frac{\sin(x) + x\cos(x)}{\sqrt{1+x\sin(x)}} + 2\sin(x)}{\cancel{\frac{1}{2}}\sin(x)} = \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{\frac{\left(2\cos(x) - x\sin(x)\right)\cdot\sqrt{} - \left(\sin(x) + x\cos(x)\right)\cdot\frac{1}{2}\frac{1}{\sqrt{}}\cdot\left(\sin(x) + x\cos(x)\right)}{\left(1+x\sin(x)\right)} + 2\cos(x)}{\cos(x)} = \frac{4}{1} = 4$$

4.)

$$f(x) = 1 - \frac{x^3}{x^3+1}$$

$$A = x - \frac{x^4}{x^3+1}$$

$$\frac{dA}{dx} = 1 - \frac{4x^3 \cdot (x^3+1) - x^4 \cdot 3x^2}{(x^3+1)^2} = 0$$

$$(x^3+1)^2 - 4x^6 - 4x^3 + 3x^6 = 0$$

$$\cancel{x^6} + 2x^3 + 1 - \cancel{4x^6} - 4x^3 + \cancel{3x^6} = 0$$

$$2x^3 = 1$$

$$x^3 = \frac{1}{2}$$

$$\underline{\underline{x = \frac{1}{\sqrt[3]{2}}}} = \underline{\underline{0,7937}}$$

$$A = \frac{1}{\sqrt[3]{2}} - \frac{\left(\frac{1}{2}\right)^{\frac{4}{3}}}{\frac{3}{2}} = \underline{\underline{0,5291}}$$

5.) a. $\int_0^{\frac{\pi}{4}} \sqrt{x}\cdot\sin(\sqrt{x})\,dx$ Subst: $u = \sqrt{x}$

$x = u^2$

$dx = 2u\,du$

$$= \int_{u=0}^{\frac{\pi}{2}} u\cdot\sin(u)\cdot 2u\,du = 2\cdot\int_0^{\pi/2} \underset{g}{u^2}\,\underset{h'}{\sin(u)}\,du$$

$$= 2\cdot\left(\underbrace{u^2\cdot(-\cos(u))}_{0}\Big|_0^{\pi/2} + \int_0^{\pi/2} 2u\cdot\cos(u)\,du\right) = 4\cdot\int_0^{\pi/2} \underset{g}{u}\cdot\underset{h'}{\cos(u)}\,du$$

$$= 4\cdot\left(u\cdot\sin(u)\Big|_0^{\pi/2} - \int_0^{\pi/2} 1\cdot\sin(u)\,du\right)$$

$$= 4\cdot\left(\frac{\pi}{2} + \cos(u)\Big|_0^{\pi/2}\right) = \underline{\underline{4\cdot\left(\frac{\pi}{2}-1\right)}}$$

5.6.)

$$\int \frac{x^3+x}{x^3+x^2-5x+3}\,dx$$

$$\overbrace{(x-1)^2\cdot(x+3)}$$

$$(x^3+x):(x^3+x^2-5x+3)=1$$

$$\underline{-x^3 \pm x^2 \pm 5x \pm 3}$$

$$-x^2+6x-3 \quad \text{Rest.}$$

$$\int 1-\frac{x^2-6x+3}{(x-1)^2\cdot(x+3)}\,dx = x - \int \frac{x^2-6x+3}{(x-1)^2\cdot(x+3)}\,dx$$

Achtung.

PBZ:

$$\frac{x^2-6x+3}{(x-1)^2\cdot(x+3)} = \frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+3}$$

$$x^2-6x+3 = A\cdot(x-1)\cdot(x+3)+B(x+3)+C\cdot(x-1)^2$$

$x=1$: $\quad -2 = B\cdot 4 \qquad \underline{\underline{B=-\frac{1}{2}}}$

$x=-3$: $\quad 30 = C\cdot 16 \qquad C=\frac{30}{16}=\underline{\underline{\frac{15}{8}}}$

$x=0$: $\quad 3 = -3A-\frac{3}{2}+\frac{15}{8}$

$$3A = -3-\frac{3}{2}+\frac{15}{8} = \frac{-24-12+15}{8} = -\frac{21}{8} \qquad \underline{\underline{A=-\frac{7}{8}}}$$

$$\int ..\,dx = x+\frac{7}{8}\int\frac{dx}{x-1}+\frac{1}{2}\int\frac{dx}{(x-1)^2}-\frac{15}{8}\int\frac{1}{x+3}\,dx =$$

$$= x+\frac{7}{8}\ln|x-1|-\frac{1}{2}\frac{1}{x-1}-\frac{15}{8}\ln|x+3|+c$$

6.)

$f(x) = \sin(x) + \cos(x) \qquad \rho(x) = x \qquad x \in [0, \frac{\pi}{2}]$

$$M = \pi \cdot \int_0^{\pi/2} (f(x))^2 \cdot \rho(x) \cdot dx = \pi \cdot \int_0^{\pi/2} x \cdot \left(\sin^2(x) + 2\sin(x)\cos(x) + \cos^2(x) \right) \cdot dx$$

$$M = \pi \cdot \int_0^{\pi/2} x \cdot (1 + \sin(2x))\, dx = \pi \cdot \left(\underbrace{\int_0^{\pi/2} x\, dx}_{\mathrm{I}} + \underbrace{\int_0^{\pi/2} x \cdot \sin(2x)\, dx}_{\mathrm{II}} \right)$$

$$\mathrm{I}: \int_0^{\pi/2} x \cdot dx = \frac{x^2}{2} \Big|_0^{\pi/2} = \underline{\underline{\frac{\pi^2}{8}}}$$

$$\mathrm{II}: \int_0^{\pi/2} \underset{g}{x} \cdot \underset{h'}{\sin(2x)}\, dx = x \cdot -\frac{1}{2}(\cos(2x)) \Big|_0^{\pi/2} - \int_0^{\pi/2} -\frac{1}{2}\cos(2x)\, dx$$

$$= -\frac{\pi}{2}\left(-\frac{1}{2}\right) + \frac{1}{2}\int_0^{\pi/2} \cos(2x)\, dx = \frac{\pi}{4} + \frac{1}{4}\sin(2x) \Big|_0^{\pi/2} = \underline{\underline{\frac{\pi}{4}}}$$

$$M = \pi \cdot \left(\frac{\pi^2}{8} + \frac{\pi}{4} \right) = \frac{\pi}{4} \cdot \left(\frac{\pi}{2} + 1 \right) \pi \approx \underline{\underline{2{,}019099}} \cdot \pi = \underline{\underline{\underline{6{,}3431}}}$$

7.)

$$y' + \frac{y}{x^2} = \frac{1}{x^2} \qquad y(1) = 0$$

a.) homogene:

$$y' = -\frac{y}{x^2}$$

$$\frac{dy}{y} = -\frac{dx}{x^2}$$

$$\ln|y| = \frac{1}{x} + \ln(c)$$

$$y_h = C e^{\frac{1}{x}}$$

b.) partikulär:

$$y_p = C(x)\, e^{\frac{1}{x}}$$

$$y_p' = C' e^{1/x} + C \cdot e^{1/x} \cdot \left(-\frac{1}{x^2}\right)$$

$$C' e^{\frac{1}{x}} - \cancel{\frac{C}{x^2} e^{\frac{1}{x}}} + \cancel{\frac{C}{x^2} e^{1/x}} = \frac{1}{x^2}$$

$$C' = \frac{1}{x^2} e^{-\frac{1}{x}}$$

$$C = + e^{-\frac{1}{x}}$$

$$y_p = 1$$

$$y(x) = C e^{\frac{1}{x}} + 1$$

$$y(1) = 0 = C \cdot e^{1} + 1 \qquad C = -\frac{1}{e}$$

$$y(x) = -e^{\frac{1}{x} - 1} + 1$$

8.)

$$xy' = y - x - xe^{-\frac{y}{x}} \qquad y(1) = 0 \qquad y(x) = x \cdot \ln\left(\frac{2}{x} - 1\right)$$

$$y' = \frac{y}{x} - 1 - e^{-\frac{y}{x}} \qquad u = \frac{y}{x} \qquad y = u \cdot x \qquad y' = u'x + u$$

$$u'x + \cancel{u} = \cancel{u} - 1 - e^{-u}$$

$$u'x = -(1 + e^{-u})$$

$$\frac{du}{1 + e^{-u}} = -\frac{dx}{x}$$

$$\frac{e^{u}\,du}{e^{u}+1} = -\frac{dx}{x}$$

$$e^{u} = z$$

$$e^{u}\,du = dz$$

$$\frac{dz}{z+1} = -\frac{dx}{x}$$

$$\ln(z+1) = -\ln(x) + \ln(c)$$

$$z + 1 = \frac{c}{x}$$

$$e^{u} = \frac{c}{x} - 1$$

$$u = \ln\left(\frac{c}{x} - 1\right)$$

$$\underline{\underline{y = x \cdot \ln\left(\frac{c}{x} - 1\right)}}$$

$$y(1) = 0 \qquad 0 = 1 \cdot \ln\left(\frac{c}{1} - 1\right) \Rightarrow \underline{\underline{c = 2}}$$

$$\underline{\underline{y(x) = x \cdot \ln\left(\frac{2}{x} - 1\right)}}$$

9.) a) • streng monoton steigend $\Rightarrow$ injektiv

• e^x hat Bild $(0,\infty)$ $\Rightarrow$ (x-Verschiebung und y-Skalierung um pos. Faktor) $2\cdot e^{x-1}$ hat auch Bild $(0,\infty)$ $\Rightarrow$ surjektiv

Insgesamt bijektiv $\Leftrightarrow$ umkehrbar.

b)
$$y = 2e^{x-1}$$
$$\frac{y}{2} = e^{x-1} \quad | \ln()$$
$$\ln(y/2) = x-1$$
$$x = \ln(y/2)+1$$
$$f^{-1}: (0,\infty) \to \mathbb{R}: y \mapsto \ln(y/2)+1.$$