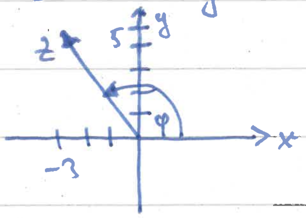
Darstellungsformen

1.) $z = -3 + 5j \rightarrow z = r \cdot e^{j\varphi}$, $r = \sqrt{(-3)^2 + 5^2} = \sqrt{34} \approxeq 5{,}83$



$\arctan\left(\frac{5}{-3}\right) \approxeq -59{,}04°$

$\varphi = \arctan\left(\frac{5}{-3}\right) + 180° \approxeq 120{,}96°$

Somit ist $z \approxeq 5{,}83 \cdot e^{j \cdot 120{,}96°}$.

$\bar{z} = -3 - 5j \approxeq 5{,}83\ e^{-j \cdot 120{,}96°} = 5{,}83 \cdot e^{j\,239{,}04°}$

$-120{,}96° + 360° = 239{,}04°$

2.) $z = 3 \cdot e^{j\,30°} = 3 \cdot (\cos(30°) + j \cdot \sin(30°))$

$= 3\cos(30°) + j \cdot 3 \cdot \sin(30°)$

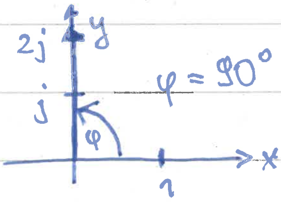
$\approxeq 2{,}60 + j \cdot 1{,}50 = 2{,}60 + 1{,}50j$

$\bar{z} = z^* = 2{,}60 - 1{,}50j = 3 \cdot e^{-j \cdot 30°}$.

Polarform

1.) $z = \frac{6+8j}{4-3j} = \frac{(6+8j)\cdot(4+3j)}{(4-3j)\cdot(4+3j)} = \frac{24 + 18j + 32j + 24j^2}{4^2 - (3j)^2} =$

$= \frac{24 + 50j - 24}{16 + 9} = \frac{50j}{25} = 2j = \underline{2\,e^{j\frac{\pi}{2}}}$.

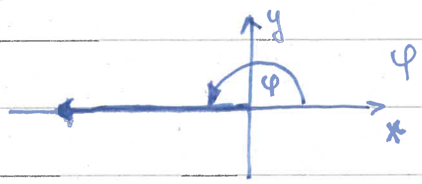


$\varphi = 90° = \frac{\pi}{2}$, $r = 2$

2.) $z = \frac{3 + \sqrt{2}j}{-\sqrt{2} - \frac{2}{3}j} = \frac{(3+\sqrt{2}j)(-\sqrt{2} + \frac{2}{3}j)}{(-\sqrt{2} - \frac{2}{3}j)(-\sqrt{2} + \frac{2}{3}j)} = \frac{-3\sqrt{2} + 2j - 2j + \sqrt{2}\frac{2}{3}j^2}{(-\sqrt{2})^2 + (\frac{2}{3})^2}$

$= \frac{\sqrt{2}(-3 - \frac{2}{3})}{2 + \frac{4}{9}} = -\sqrt{2}\ \frac{\frac{11}{3}}{\frac{22}{9}} = -\sqrt{2}\ \frac{11}{3} \cdot \frac{9}{22} =$

$= -\sqrt{2}\ \frac{3}{2} = \underline{\sqrt{2}\ \frac{3}{2}\ e^{j\pi}}$.



$\varphi = 180° = \pi$

▷ Kartesische Form

1.) $z = \sqrt{8}\left(\cos\left(\frac{\pi}{4}\right) + j\cdot\sin\left(\frac{\pi}{4}\right)\right) =$

$= \sqrt{8}\left(\frac{1}{\sqrt{2}} + j\cdot\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{8}}{\sqrt{2}} + j\,\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}} + j\cdot\sqrt{\frac{8}{2}} =$

$= \sqrt{4} + j\sqrt{4} = 2 + j2 = \underline{2+2j}.$

2.) $z = \sqrt{50}\left(\cos\left(\frac{3\pi}{4}\right) + j\sin\left(\frac{3\pi}{4}\right)\right) =$

$= \sqrt{50}\left(-\frac{1}{\sqrt{2}} + j\cdot\frac{1}{\sqrt{2}}\right) = -\frac{\sqrt{50}}{\sqrt{2}} + j\cdot\frac{\sqrt{50}}{\sqrt{2}} = -\sqrt{\frac{50}{2}} + j\sqrt{\frac{50}{2}} =$

$= -\sqrt{25} + j\sqrt{25} = -5 + j5 = \underline{-5+5j}.$

▷ Kartesisch Rechnen $\quad z_1 = -4j,\ z_2 = 3-2j,\ z_3 = -1+j$

1.) $\frac{z_1^* \cdot z_2}{z_3} = \frac{4j\cdot(3-2j)}{-1+j} = \frac{12j - 8j^2}{-1+j} = \frac{8+12j}{-1+j} =$

$= \frac{(8+12j)(-1-j)}{(-1+j)(-1-j)} = \frac{-8-8j-12j-12j^2}{(-1)^2+1^2} =$

$= \frac{4-20j}{2} = \frac{4}{2} - \frac{20}{2}j = \underline{2-10j}.$

2.) $\frac{2j}{3-4j} + 2e^{j(-30°)} + 3\left[\cos\left(\frac{\pi}{4}\right) + j\sin\left(\frac{\pi}{4}\right)\right] =$

$= \frac{2j(3+4j)}{(3-4j)(3+4j)} + 2\left[\cos(-30°) + j\cdot\sin(-30°)\right] + \frac{3}{\sqrt{2}} + j\cdot\frac{3}{\sqrt{2}} =$

$\cong \frac{6j+8j^2}{(3)^2+(-4)^2} + 1{,}732 - j + 2{,}121 + 2{,}121j =$

$= \frac{-8+6j}{25} + 3{,}853 + 1{,}121j =$

$= -0{,}32 + 0{,}24j + 3{,}853 + 1{,}121j = \underline{3{,}533 + 1{,}361j}.$

▷ Real- und Imaginärteil von $z = \frac{(1+j)^2}{3+2j} = \frac{1^2+2\cdot1\cdot j + j^2}{3+2j} =$

$= \frac{2j}{3+2j} = \frac{2j(3-2j)}{(3+2j)(3-2j)} = \frac{6j-4j^2}{3^2+2^2} = \frac{4+6j}{13} =$

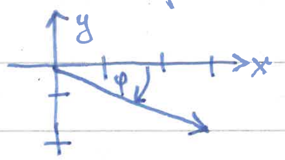
$= \frac{4}{13} + \frac{6}{13}j.\quad \underline{Re(z) = \frac{4}{13}},\ \underline{Im(z) = \frac{6}{13}}.$

▷ Kartesisch Rechnen und Potenzieren

1.) $\frac{4(3-j)^*}{(1+j)(-1+j)} = \frac{4(3+j)}{-1+j-j+j^2} = \frac{12+4i}{-2} = \underline{-6-2j}.$

2.) $(\underbrace{3-\sqrt{3}j}_{\text{in Polarform}})^4$

in Polarform: $r = \sqrt{3^2+\sqrt{3}^2} = \sqrt{9+3} = \sqrt{12}$

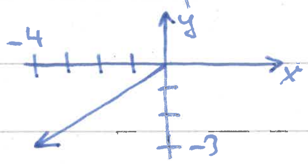


$\varphi = \arctan\left(\frac{-\sqrt{3}}{3}\right) = \arctan\left(-\frac{1}{\sqrt{3}}\right)$
$= -30° = -30° + 360° = 330°.$

$(3-\sqrt{3}j)^4 = \left(\sqrt{12}\cdot e^{-30°j}\right)^4 = \sqrt{12}^4 \cdot e^{-30°j\cdot 4} = 12^2\cdot e^{-120°j}$
$= 144\left(\cos(-120°) + j\cdot\sin(-120°)\right) =$
$= 144\cdot\left(-\frac{1}{2}\right) + 144\cdot\left(-\frac{\sqrt{3}}{2}\right)\cdot j =$
$= \underline{-72 - 124{,}71j.}$

3.) $(\underbrace{-4-3j}_{\text{Polarform}})^3$

Polarform: $r = \sqrt{(-4)^2+(-3)^2} = \sqrt{25} = 5$



$\varphi = \arctan\left(\frac{-3}{-4}\right) + 180° = 216{,}87°$

$(-4-3j)^3 = \left(5\cdot e^{j\cdot 216{,}87°}\right)^3 = 5^3\cdot\left(e^{j\cdot 216{,}87°}\right)^3 = 125\cdot e^{j\cdot 216{,}87°\cdot 3} =$
$= 125\, e^{j650{,}61°} = 125\, e^{j290{,}61°} = \underline{44{,}00 - 117{,}00j.}$

▷ Kartesisch Rechnen und Potenzieren:

1.) $z = (2-4j)^2 + \frac{|1-\sqrt{3}j|}{j} = 4 - 16j + 16j^2 + \frac{\sqrt{1^2+\sqrt{3}^2}\cdot(-j)}{j\cdot(-j)} =$

$= 4 - 16j - 16 + \frac{-2j}{1} = -12 - 18j.$ $\underline{Re(z) = -12}$, $\underline{Im(z) = -18}.$

2.) $(3e^{j\pi})^5 = 3^5\cdot(e^{j\pi})^5 = 243\, e^{5\pi j} = \underline{243\, e^{j\pi}} = \underline{-243}.$