▷ Stammfunktionen

1.) $f(x) = 4x^5 - 6x^3 + 8x^2 - 3x + 5$

$F(x) = 4\frac{x^6}{6} - 6\frac{x^4}{4} + 8\frac{x^3}{3} - 3\frac{x^2}{2} + 5x + C$

$= \frac{2}{3}x^6 - \frac{3}{2}x^4 + \frac{8}{3}x^3 - \frac{2}{2}x^2 + 5x + C.$

2.) $f(t) = 2e^t - \frac{5}{t} + 1 = 2e^t - 5\cdot\frac{1}{t} + 1$

$F(t) = 2e^t - 5\ln(|t|) + t + C.$

3.) $f(x) = \frac{-2}{\sqrt{1-x^2}} - \frac{1}{\cos^2(x)}$ → Siehe Formelsammlung!

$F(x) = -2\arcsin(x) - \tan(x) + C.$

4.) $f(x) = \frac{1-2x^2-4x^3}{2x} + 3 = \frac{1}{2}\cdot\frac{1}{x} - x - 2x^2 + 3$

$F(x) = \frac{1}{2}\ln(|x|) - \frac{1}{2}x^2 - \frac{2}{3}x^3 + 3x + C.$

▷ Unbestimmte Integrale Formelsammlung: $\sin(2x) = 2\sin(x)\cos(x)$

1.) $\int \frac{\tan(x)}{\sin(2x)}\,dx = \int \underbrace{\frac{\sin(x)}{\cos(x)}}_{\tan(x)} \cdot \frac{1}{2\sin(x)\cdot\cos(x)}\,dx = \frac{1}{2}\int\frac{1}{\cos^2(x)}\,dx$

$\overset{\text{Formels.}}{=} \frac{1}{2}\tan(x) + C.$

2.) $\int\sqrt{x\sqrt{x}}\,dx = \int(\underbrace{x\cdot x^{1/2}}_{x^{3/2}})^{1/2}\,dx = \int x^{\frac{3}{2}\cdot\frac{1}{2}}\,dx = \int x^{\frac{3}{4}}\,dx = \frac{x^{3/4+1}}{3/4+1} + C$

$= \frac{x^{7/4}}{7/4} + C = \frac{4}{7}x^{7/4} + C.$

3.) $\int \frac{10}{\cosh^2(x)} - 3a^x - b\cdot\sin(x)\,dx = 10\tanh(x) - 3\frac{e^{x\cdot\ln(a)}}{\ln(a)} + b\cdot\cos(x) + C =$

↑ Formels. $a^x = (e^{\ln(a)})^x = e^{x\cdot\ln(a)}$

$= 10\tanh(x) - \frac{3}{\ln(a)}a^x + b\cos(x) + C.$

4.) $\int\frac{\sqrt[3]{x^5}}{\sqrt[5]{x^4}}\,dx = \int\frac{x^{5/3}}{x^{4/5}}\,dx = \int x^{5/3-4/5}\,dx = \int x^{13/15}\,dx =$

$\frac{5}{3} - \frac{4}{5} = \frac{25-12}{15} = \frac{13}{15}$

$= \frac{x^{\frac{13}{15}+1}}{\frac{13}{15}+1} + C = \frac{x^{\frac{28}{15}}}{\frac{28}{15}} + C = \frac{15}{28}x^{\frac{28}{15}} + C.$

▷ <u>Biegegleichung</u>

$$y''(x) = -\frac{F}{2EI}\left(\ell x - x^2\right)$$

$$y'(x) = -\frac{F}{2EI}\left(\ell \frac{x^2}{2} - \frac{x^3}{3} + C\right)$$

$$y'(\ell/2) = -\frac{F}{2EI}\left(\underbrace{\ell \frac{\ell^2}{8} - \frac{\ell^3}{3\cdot 8} + C}\right) \overset{!}{=} 0$$

$$\frac{\ell^3}{8}\left(1 - \frac{1}{3}\right) + C \overset{!}{=} 0$$

$$\frac{\ell^3}{4\cancel{8}} \cdot \frac{\cancel{2}}{3} + C = 0 \Rightarrow C = -\frac{\ell^3}{12}$$

$$y'(x) = -\frac{F}{2EI}\left(\ell \frac{x^2}{2} - \frac{x^3}{3} - \frac{\ell^3}{12}\right)$$

$$y(x) = -\frac{F}{2EI}\left(\ell \frac{x^3}{6} - \frac{x^4}{12} - \frac{\ell^3}{12} x + \tilde{C}\right)$$

$$y(0) = -\frac{F}{2EI} \cdot \tilde{C} \overset{!}{=} 0 \Rightarrow \tilde{C} = 0$$

$$\underline{y(x) = -\frac{F}{24EI}\left(2\ell x^3 - x^4 - \ell^3 x\right).}$$

▷ <u>Integrationskonstante</u> $y(x)$ geht durch $P=(0|2)$

$$y'(x) = \sin(x) + 3e^x - \frac{1}{3}x^2 + \frac{4}{1+x^2}$$

$$y(x) = -\cos(x) + 3e^x - \frac{1}{3}\frac{x^3}{3} + 4\arctan(x) + C$$

$$y(0) = -\cos(0) + 3\cdot e^0 - \frac{1}{9}0^3 + 4\arctan(0) + C \overset{!}{=} 2$$

$$= -1 + 3\cdot 1 - 0 + 4\cdot 0 + C = 2$$

$$2 + C = 2$$

$$\underline{C = 0}$$

$$\underline{y(x) = -\cos(x) + 3e^x - \frac{1}{9}x^3 + 4\cdot\arctan(x).}$$

▷ Substitution

1.) $\int \sin(3x)\,dx \underset{\substack{\uparrow\\ \text{Schnellformel}}}{=} \frac{-\cos(3x)}{3} + C = -\frac{1}{3}\cos(3x) + C.$

Probe: $\left[-\frac{1}{3}\cos(3x) + C\right]' = -\frac{1}{3}(-\sin(3x))\cdot 3 + 0 = \sin(3x)$ ✓

2.) $\int \sin(x)\cdot e^{\cos(x)}\,dx$. Setze $u := \cos(x)$

$du = -\sin(x)\,dx$

$\sin(x) = -\frac{du}{dx}$

$= \int -\frac{du}{\cancel{dx}}\cdot e^{u}\,\cancel{dx} = -\int e^{u}\,du = -e^{u} + C =$

$= -e^{\cos(x)} + C$. Probe: $\left[-e^{\cos(x)} + C\right]' = -e^{\cos(x)}\cdot(-\sin(x)) + 0$

$= e^{\cos(x)}\cdot\sin(x)$ ✓

3.) $\int \frac{3x}{1+x^2}\,dx$. Setze $u := 1+x^2$

$du = 2x\,dx$

$x\,dx = \frac{1}{2}\,du$

$= \int \frac{3}{u}\,\frac{1}{2}\,du = \frac{3}{2}\int \frac{1}{u}\,du = \frac{3}{2}\ln(|u|) + C =$

$= \frac{3}{2}\ln(|\underbrace{1+x^2}_{>0}|) + C = \frac{3}{2}\ln(1+x^2) + C.$

Probe: $\left[\frac{3}{2}\ln(1+x^2) + C\right]' = \frac{3}{2}\,\frac{1}{1+x^2}\cdot 2x + 0 =$

$= \frac{3x}{1+x^2}$. ✓

▷ Integrationsmethoden

1.) $\int_0^{\pi} \cos^3(x) \cdot \sin(x)\, dx$ Substitution $\cos(x) = u$, Grenzen: $\cos(0) = 1$

$-\sin(x) = \frac{du}{dx}$, $\cos(\pi) = -1$

$\sin(x)\,dx = -du.$

$$= \int_1^{-1} u^3\,(-du) = -\int_1^{-1} u^3\,du = -\frac{1}{4}u^4\Big|_1^{-1} = -\frac{1}{4}\left((-1)^4 - 1^4\right) = \underline{0.}$$

2.) $\int \underbrace{x \cdot \cos(x)}_{f \cdot g'}\, dx = \underbrace{x \cdot \sin(x)}_{f \cdot g} - \underbrace{\int 1 \cdot \sin(x)\, dx}_{\int f' \cdot g} =$

$= \underline{x \cdot \sin(x) + \cos(x) + C.}$

3.) $\int \ln(x)\, dx = \int \underbrace{1 \cdot \ln(x)}_{f' \cdot g}\, dx = \underbrace{x \cdot \ln(x)}_{f \cdot g} - \underbrace{\int x \cdot \frac{1}{x}\, dx}_{\int f \cdot g'} =$

$= \underline{x \cdot \ln(x) - x + C.}$