D Ableiten

1.) $f(x) = 4\sqrt[3]{x^5} - 4e^x + \sin(x)$

$f'(x) = 4(x^{5/3})' - 4(e^x)' + \cos(x) =$

$= 4 \cdot \frac{5}{3} x^{\frac{5}{3}-1} - 4e^x + \cos(x) =$

$= \underline{\frac{20}{3}\sqrt[3]{x^2} - 4e^x + \cos(x)}.$

2.) $f(x) = 2x^2 \cdot \ln(x)$ → Produktregel

$f'(x) = (2x^2)' \cdot \ln(x) + 2x^2 \cdot (\ln(x))' =$

$= 4x \cdot \ln(x) + 2x^2 \cdot \frac{1}{x} =$

$= \underline{4x\ln(x) + 2x}.$

3.) $f(x) = \frac{10x}{x^2+1}$ → Quotientenregel

$f'(x) = \frac{(10x)' \cdot (x^2+1) - 10x \cdot (x^2+1)'}{(x^2+1)^2} =$

$= \frac{10 \cdot (x^2+1) - 10x \cdot 2x}{(x^2+1)^2} = \frac{10x^2+10-20x^2}{(x^2+1)^2} = \frac{10-10x^2}{(x^2+1)^2} =$

$= \underline{\frac{10(1-x^2)}{(x^2+1)^2}}.$

4.) $f(x) = 3e^{-4x}$ → Kettenregel

$f'(x) = 3 \cdot e^{-4x} \cdot (-4) = \underline{-12e^{-4x}}.$

5.) $f(x) = \sin^2(2x-4) = [\sin(2x-4)]^2$ → Kettenregel zwei mal

$f'(x) = 2 \cdot \sin(2x-4) \cdot \cos(2x-4) \cdot 2 =$

$= \underline{4 \cdot \sin(2x-4) \cdot \cos(2x-4)}.$

6.) $f(t) = \sin(\omega t)$ → Kettenregel

$f'(t) = \cos(\omega t) \cdot \omega$

$f''(t) = -\sin(\omega t) \cdot \omega \cdot \omega = \underline{-\omega^2 \cdot \sin(\omega t)} = -\omega^2 \cdot f(t).$

▷ Differenzieren

1.) $y(x) = \frac{x^2}{1+x^2} \rightarrow$ Quotientenregel

$$y'(x) = \frac{(x^2)'\cdot(1+x^2) - x^2\cdot(1+x^2)'}{(1+x^2)^2} =$$

$$= \frac{2x\cdot(1+x^2) - x^2\cdot 2x}{(1+x^2)^2} = \frac{2x+2x^3-2x^3}{(1+x^2)^2} =$$

$$= \frac{2x}{(1+x^2)^2}.$$

$$y''(x) = \frac{(2x)'\cdot(1+x^2)^2 - 2x\cdot[(1+x^2)^2]'}{(1+x^2)^4} =$$

$$= \frac{2(1+x^2)^{\cancel{2}\,1} - 2x\,[2\,\cancel{(1+x^2)}\cdot 2x]}{(1+x^2)^{\cancel{4}\,3}} =$$

$$= \frac{2+2x^2-8x^2}{(1+x^2)^3} = \frac{2-6x^2}{(1+x^2)^3}.$$

2.) $y(x) = 4^{x\cdot\sin(x)} = [e^{\ln(4)}]^{x\cdot\sin(x)} = e^{\ln(4)\cdot x\cdot\sin(x)}$

$$y'(x) = e^{\ln(4)\cdot x\cdot\sin(x)}\cdot[\ln(4)\cdot x\cdot\sin(x)]' =$$

$$= 4^{x\cdot\sin(x)}\cdot\ln(4)\,[x'\cdot\sin(x) + x\cdot(\sin(x))'] =$$

$$= 4^{x\cdot\sin(x)}\cdot\ln(4)\,(\sin(x) + x\cdot\cos(x)).$$

3.) $y(x) = x\cdot\ln(x) \rightarrow$ Produktregel

$$y'(x) = (x)'\cdot\ln(x) + x\cdot(\ln(x))' =$$

$$= 1\cdot\ln(x) + x\cdot\frac{1}{x} = \ln(x)+1.$$

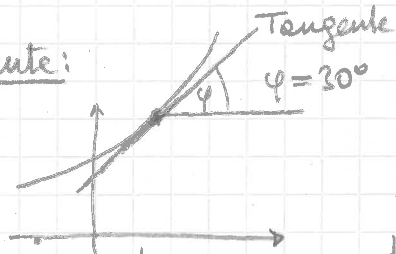
$$y''(x) = \frac{1}{x}.$$

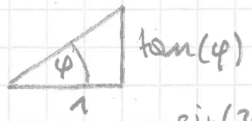
$$y'''(x) = \left(\frac{1}{x}\right)' = (x^{-1})' = -1\,x^{-1-1} = -x^{-2} = -\frac{1}{x^2}.$$

$$y'''(1) = -\frac{1}{1^2} = -1.$$

▷ **Differenzieren, Tangente:**

1.) $y(t) = 2e^{3t}$





$$\tan(30°) = \frac{\sin(30°)}{\cos(30°)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

$$y'(t) = 2e^{3t}\cdot 3 = 6\cdot e^{3t} \overset{!}{=} \tan(\varphi)$$

$$6\cdot e^{3t} = \frac{1}{\sqrt{3}}$$

$$e^{3t} = \frac{1}{6\cdot\sqrt{3}} \quad |\ln(\ )$$

$$\ln(e^{3t}) = \ln\left(\frac{1}{6\cdot\sqrt{3}}\right)$$

$$3t = \underbrace{\ln(1)}_{=0} - \ln(6\cdot\sqrt{3}) = -\ln(6\sqrt{3})$$

$$t = -\frac{\ln(6\sqrt{3})}{3} = \underline{-0{,}78036.}$$

$$y(-0{,}78036) = 2e^{3\cdot(-0{,}78036)} = \underline{0{,}19245.}$$

$$\underline{P = (-0{,}78036;\ 0{,}19245).}$$

2.) $y(x) = \frac{1}{3}x^3 - x$ Gerade $y = \frac{1}{4}x - 2$ hat Steigung $\frac{1}{4}$.

$$y'(x) = x^2 - 1 \overset{!}{=} \frac{1}{4}$$

$$x^2 = \frac{1}{4} + 1 = \frac{5}{4}$$

$$x_{1,2} = \pm\frac{\sqrt{5}}{2}.$$

$$\underline{P_1 = (1{,}118;\ -0{,}652),\quad P_2 = (-1{,}118;\ 0{,}652).}$$

▷ **Tangentengleichung:** $y(x) = \sqrt{25 - x^2}$, $x_0 = 4$.

$$y(x_0) = y(4) = \sqrt{25 - 4^2} = \sqrt{9} = 3.$$

$$y'(x) = \left[(25 - x^2)^{\frac{1}{2}}\right]' = \frac{1}{2}(25 - x^2)^{\frac{1}{2}-1}\cdot(-2x) =$$

$$= -x(25 - x^2)^{-\frac{1}{2}} = -\frac{x}{\sqrt{25 - x^2}}.$$

$$y'(x_0) = y'(4) = -\frac{4}{\sqrt{25 - 16}} = -\frac{4}{3}.$$

Tangentengleichung $y = kx + d$ mit $k = -\frac{4}{3}$

$y = -\frac{4}{3}x + d$ Einsetzen des Punktes $(4, 3)$

$$3 = -\frac{4}{3}\cdot 4 + d = -\frac{16}{3} + d$$

$$d = 3 + \frac{16}{3} = \frac{9 + 16}{3} = \frac{25}{3}$$

$$\underline{y = -\frac{4}{3}x + \frac{25}{3}.}$$

D <u>Kurvendiskussion:</u>

$$y(x) = \frac{x^2+1}{x-3}$$

maximale Definitionsmenge $= \mathbb{R}\setminus\{3\}$, weil bei $x=3$ der Nenner Null ist. Bei $x=3$ ist auch eine Polstelle, weil der Nenner dort Null, der Zähler aber nicht Null ist.

Nullstellen: Zähler = Null setzen liefert $x^2+1=0$

$x^2=-1$ hat

keine Lösung in $\mathbb{R}$ $\Rightarrow$ keine reellen Nullstellen

Minima und Maxima: $y'(x) = \frac{2x(x-3)-(x^2+1)\cdot 1}{(x-3)^2} =$

$$= \frac{2x^2-6x-x^2-1}{(x-3)^2} =$$

$$= \frac{x^2-6x-1}{(x-3)^2}.$$

$y'(x)=0$: $x^2-6x-1=0$

$$x_{1,2} = 3 \pm \sqrt{9+1} = 3 \pm \sqrt{10} \quad \begin{array}{l} \nearrow \underline{x_1 \cong 6{,}162}. \\ \searrow \underline{x_2 \cong -0{,}162}. \end{array}$$

$$y''(x) = \frac{(2x-6)(x-3)^{\cancel{2}\,1}-(x^2-6x-1)2\cancel{(x-3)}}{(x-3)^{\cancel{4}\,3}} =$$

$$= \frac{(2x-6)(x-3)-2x^2+12x+2}{(x-3)^3} = \frac{2x^2-6x-6x+18-2x^2+12x+2}{(x-3)^3} =$$

$$= \underline{\frac{20}{(x-3)^3}}.$$

$y''(x_1) \cong \frac{20}{\sqrt{10}^3} > 0 \Rightarrow$ <u>x_1 ist ein lokales Minimum</u>

$y''(x_2) \cong \frac{20}{-\sqrt{10}^3} < 0 \Rightarrow$ <u>x_2 ist ein lokales Maximum.</u>

D Extremwerte

$$f(x) = (x-1)\,e^{-2x}.$$

$$f'(x) = 1\cdot e^{-2x} + (x-1)\,e^{-2x}\cdot(-2) =$$

$$= e^{-2x} - 2(x-1)\,e^{-2x} = e^{-2x}\left(1-2(x-1)\right) =$$

$$= e^{-2x}(1-2x+2) = e^{-2x}(3-2x).$$

$$f'(x)=0: \quad \underbrace{e^{-2x}}_{>0}\,\underbrace{(3-2x)}_{3-2x=0} = 0$$

$$2x = 3, \quad x = \frac{3}{2} = 1{,}5.$$

$$f''(x) = e^{-2x}(-2)\cdot(3-2x) + e^{-2x}\cdot(-2)$$

$$= -2e^{-2x}\cdot(3-2x) - 2\cdot e^{-2x}$$

$$f''\left(\tfrac{3}{2}\right) = \underbrace{-2\cdot e^{-3}\cdot\underbrace{\left(3-2\cdot\tfrac{3}{2}\right)}_{=0}}_{=0} \underbrace{-\,2\cdot\underbrace{e^{-3}}_{>0}}_{<0} < 0 \Rightarrow \text{lokales Maximum bei } x = 1{,}5.$$