LEVEL 2: Neural Networks and Deep Learning

Day 3

Course Outline

- Machine Learning Limitations
- Neural Networks and Deep Learning
- Vectorization
- Activation functions
- Backpropagation
- Weights Initialization
- Optimization
- Learning rate

Learning Objectives

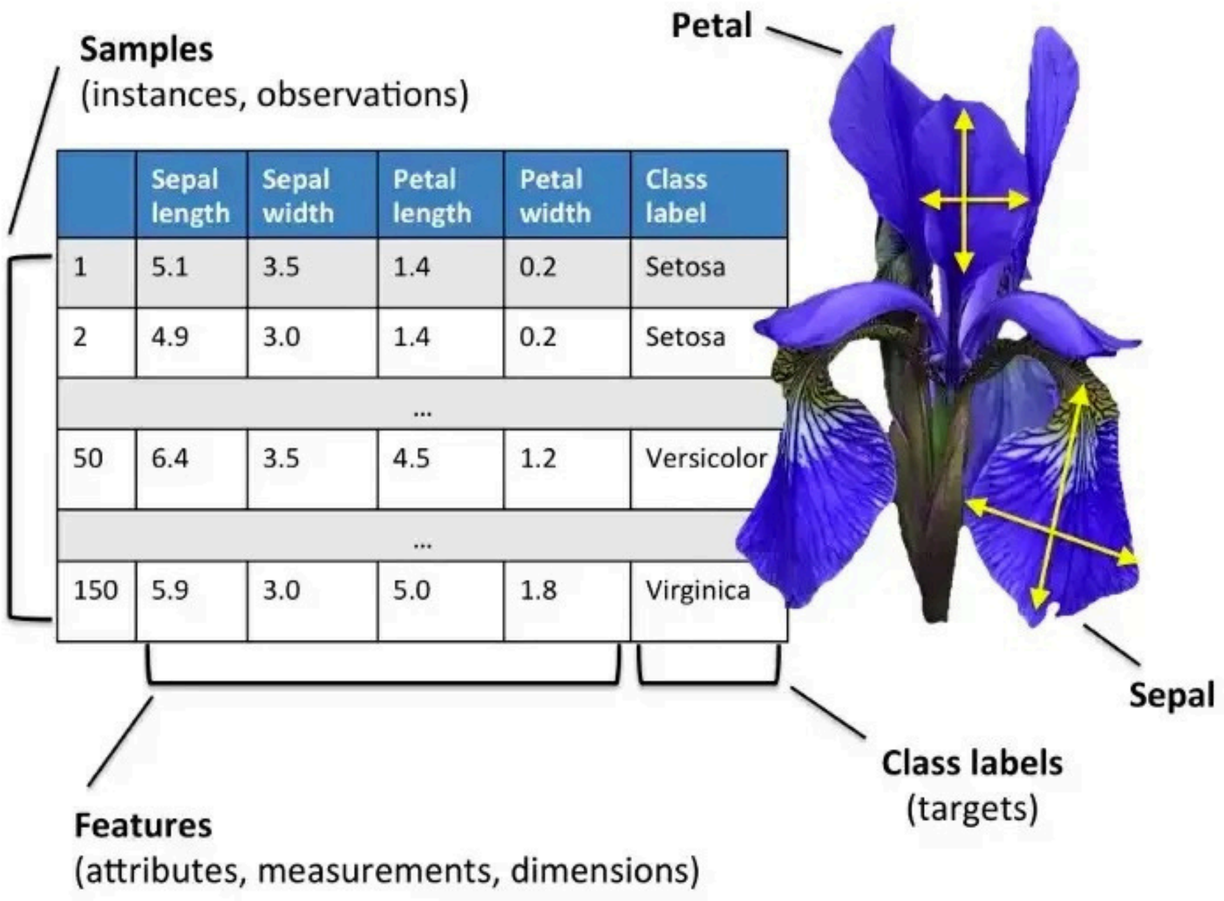
- Understand the key limitations of traditional Machine Learning models
- Explain how Neural Networks and Deep Learning address these limitations
- Apply vectorization to efficiently process multiple inputs
- Describe the role of activation functions in introducing non-linearity
- Understand the core idea of backpropagation for training neural networks
- Recognize the importance of proper weight initialization
- Identify common optimization methods used in deep learning
- Explain how the learning rate affects convergence and training stability

Machine Learning Limitations

Machine Learning Limitations

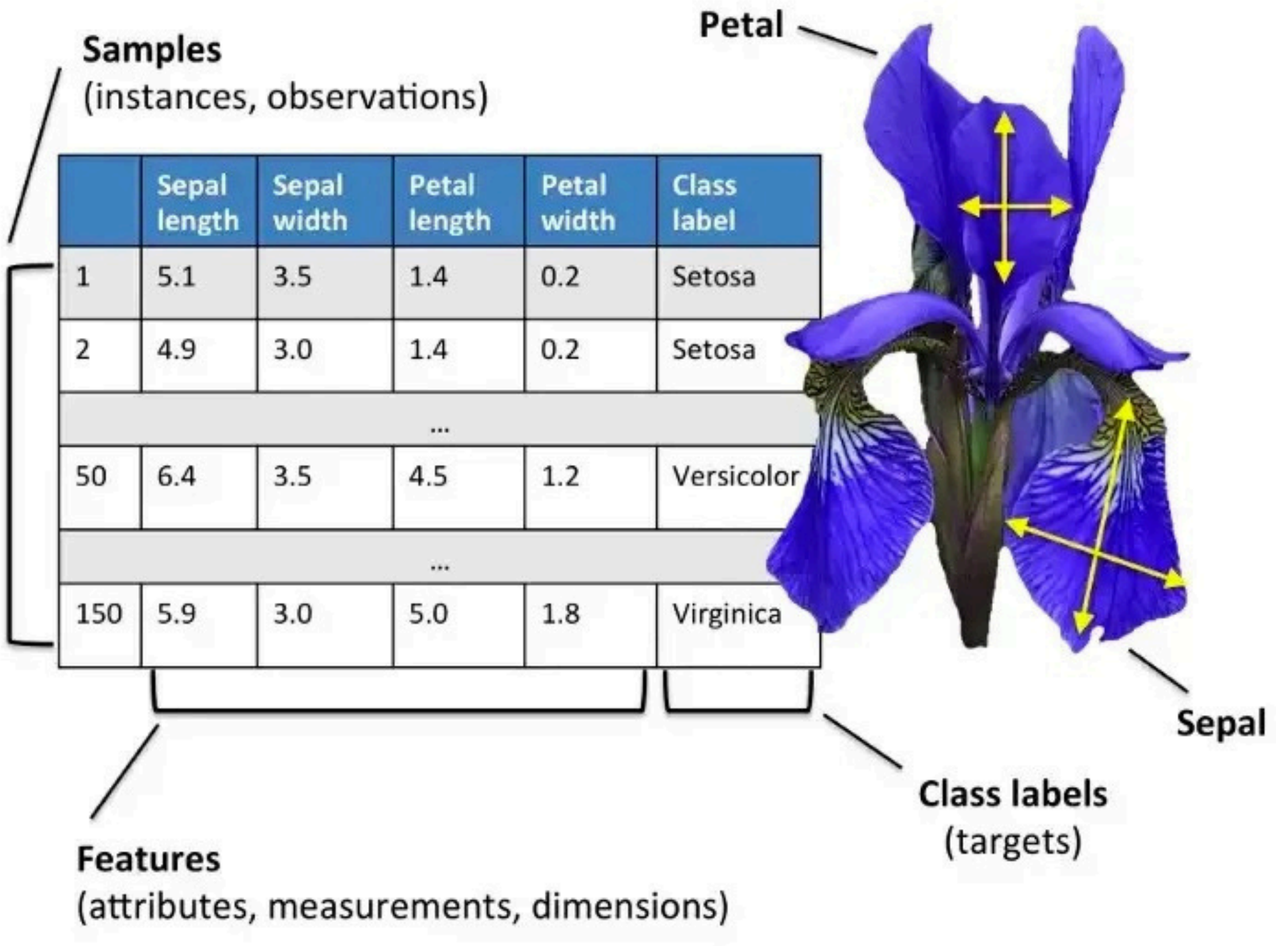
- How to classify these flowers?

Machine Learning Limitations



	Sepal length	Sepal width	Petal length	Petal width	Class label
1	5.1	3.5	1.4	0.2	Setosa
2	4.9	3.0	1.4	0.2	Setosa
...					
50	6.4	3.5	4.5	1.2	Versicolor
...					
150	5.9	3.0	5.0	1.8	Virginica

Machine Learning Limitations



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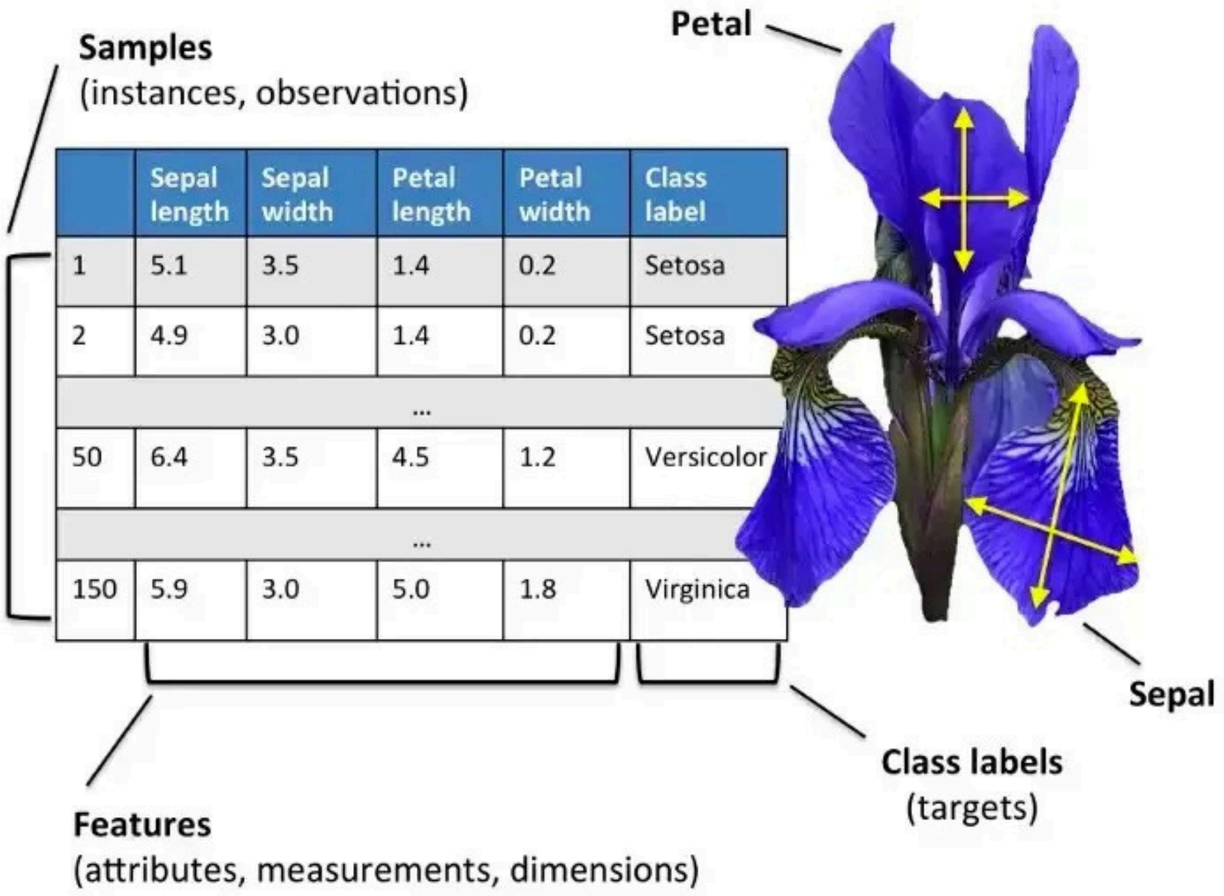
What's the problem?

Machine Learning Limitations



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- Human-designed features capture only simple, linear patterns.

Machine Learning Limitations

512 × 512 pixels

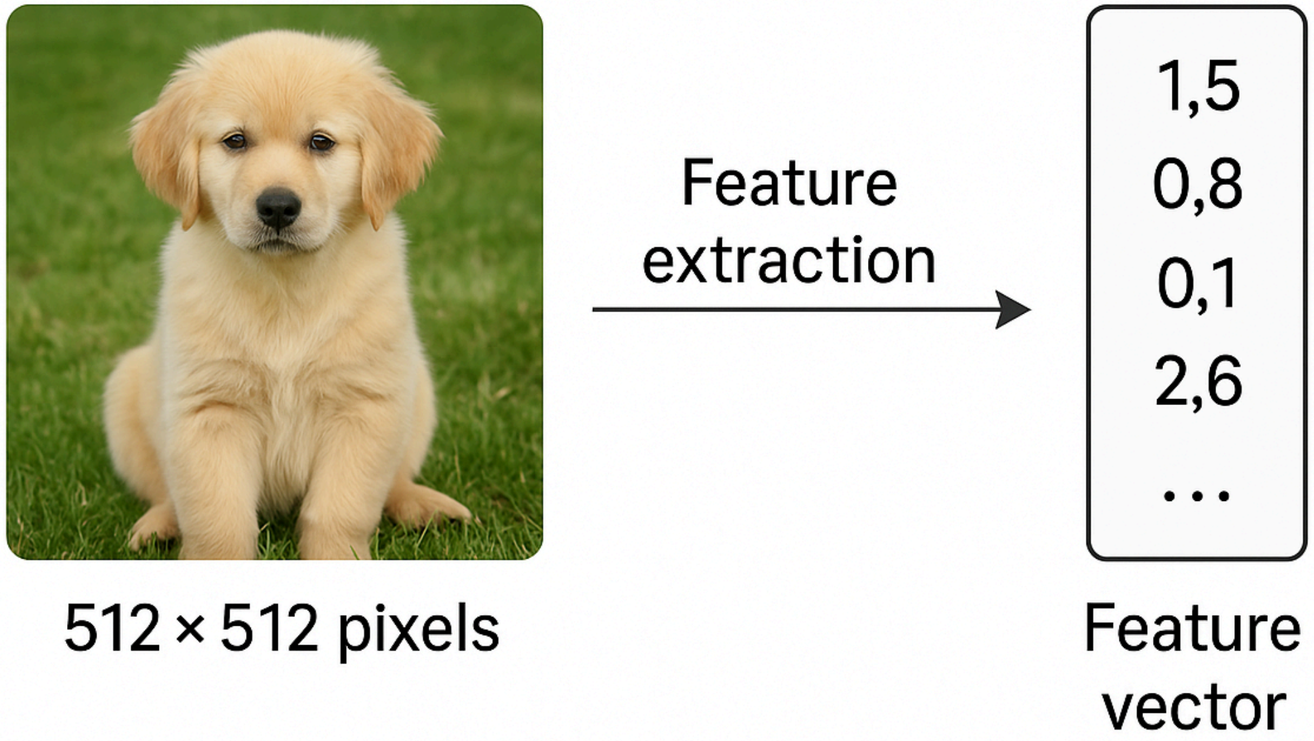
Feature extraction

1,5
0,8
0,1
2,6
...

Feature vector

What's the problem here?

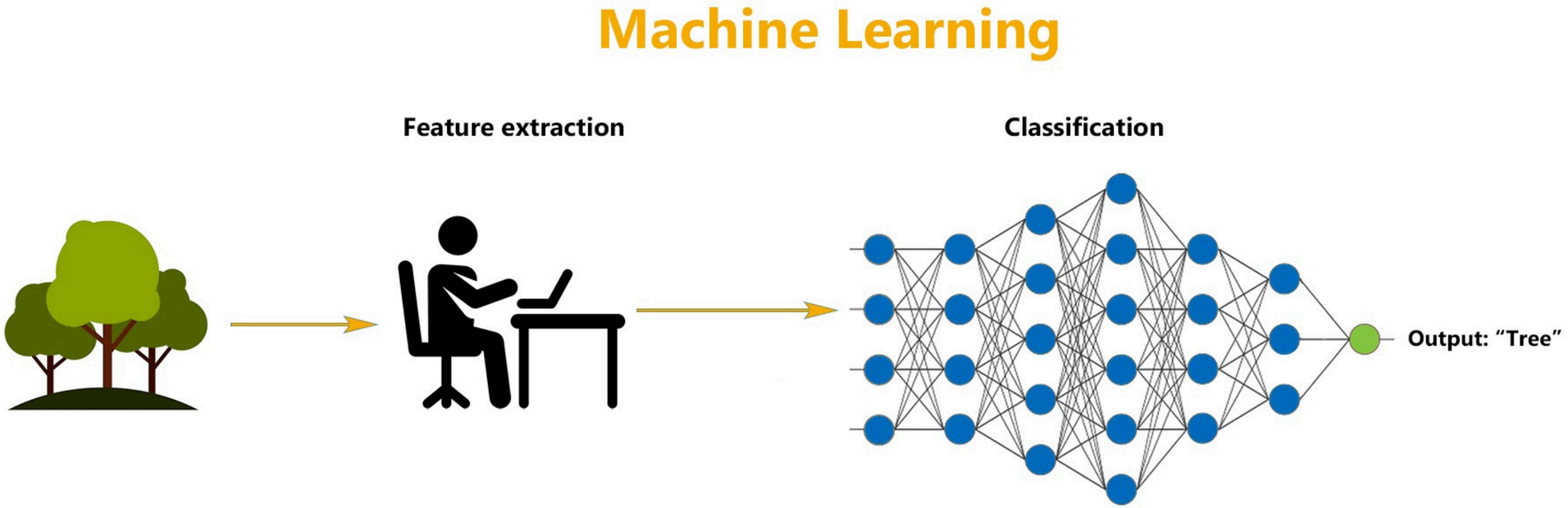
Machine Learning Limitations



- Most raw data information is lost when reduced to a few features.

Machine Learning Limitations

- Traditional ML needs manually extracted features, while deep learning learns features directly from images automatically.



Machine Learning Limitations

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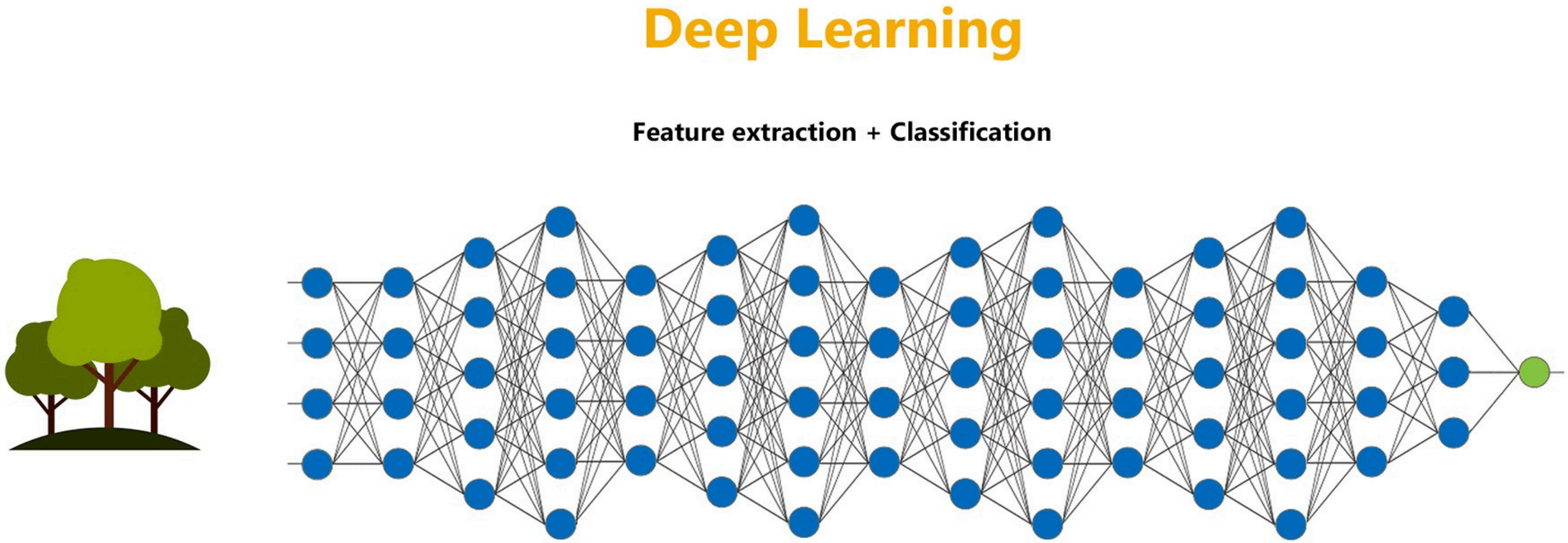
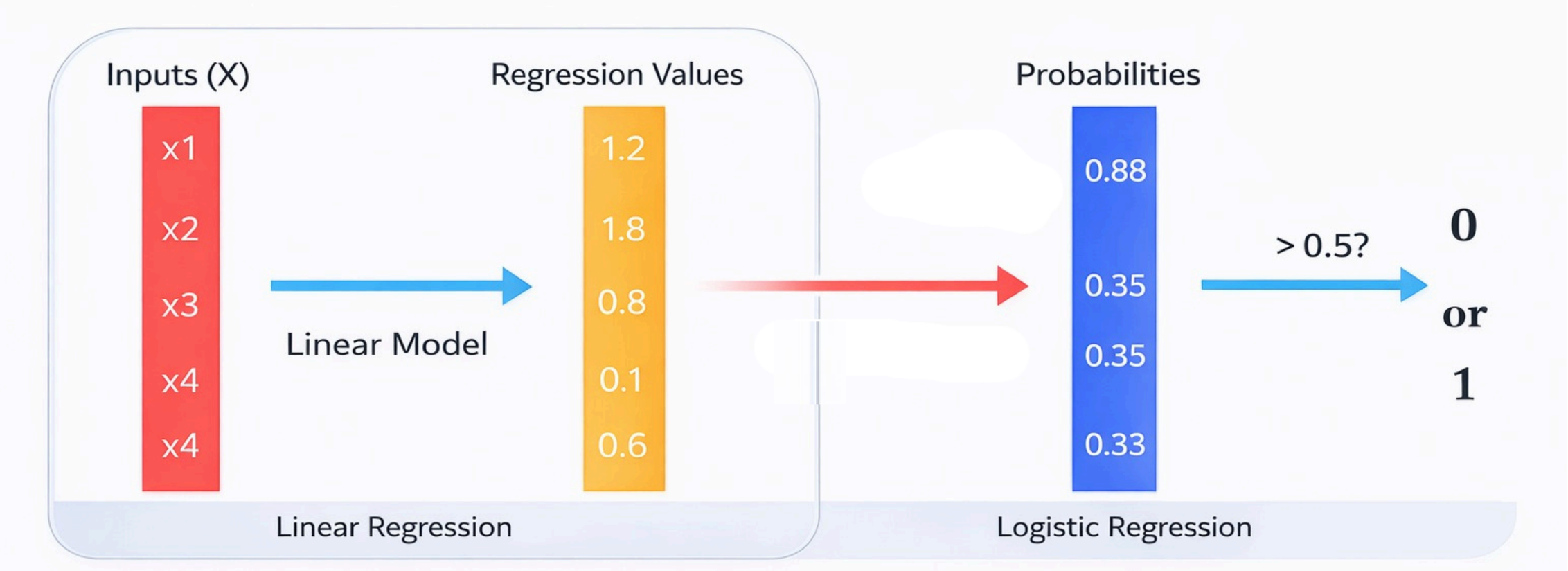


Neural Networks and Deep Learning

Neural Networks

Logistic Regression



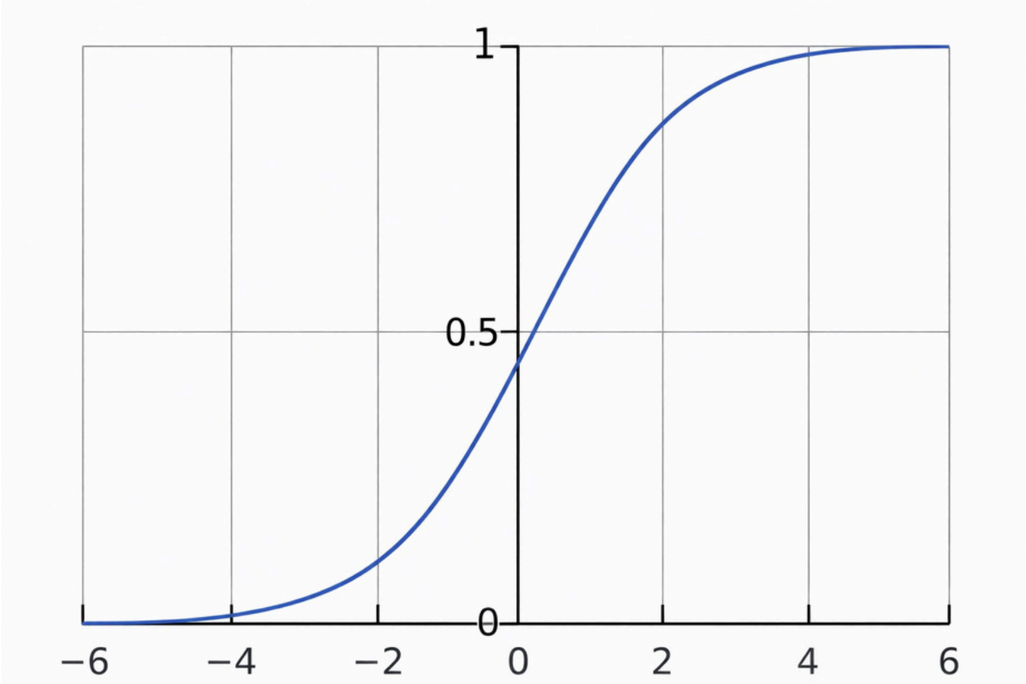
Neural Networks

- Logistic Regression
  - Sigmoid maps real values to probabilities

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

  - Suitable for gradient descent

$$\sigma'(z) = \sigma(z)\,(1 - \sigma(z))$$



$$\lim_{z \to -\infty} \sigma(z) = 0 \qquad \lim_{z \to +\infty} \sigma(z) = 1$$

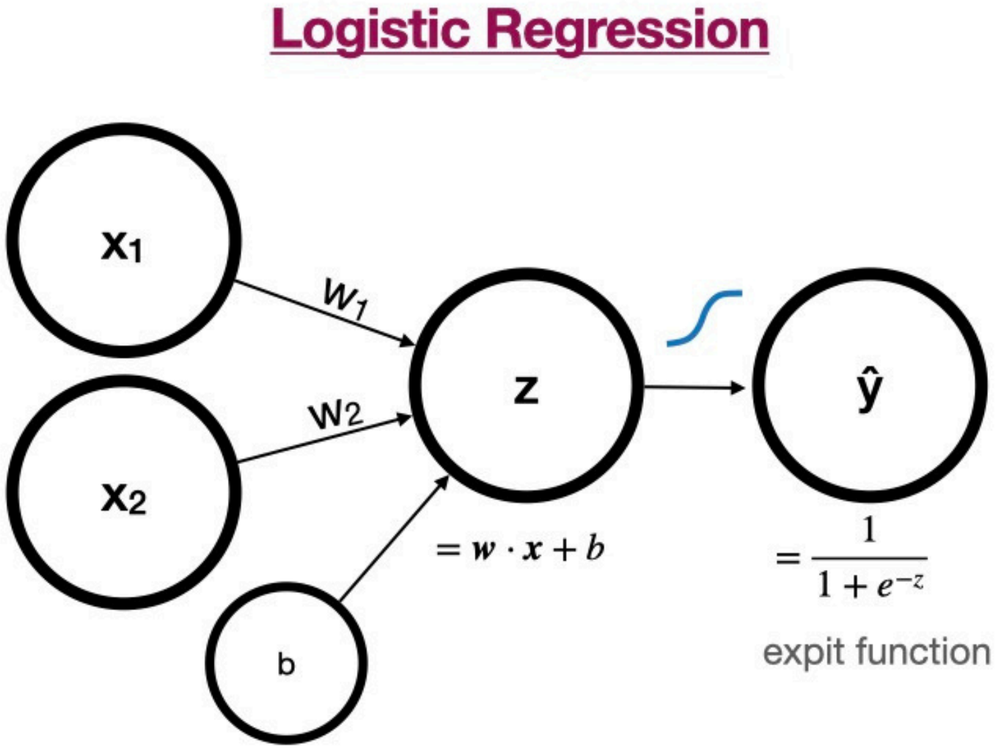
Neural Networks

- Logistic Regression
  - Logistic Regression computes:

$$y = \sigma(w^T x + b)$$

$$z = w^T x + b$$

$$y = \sigma(z)$$



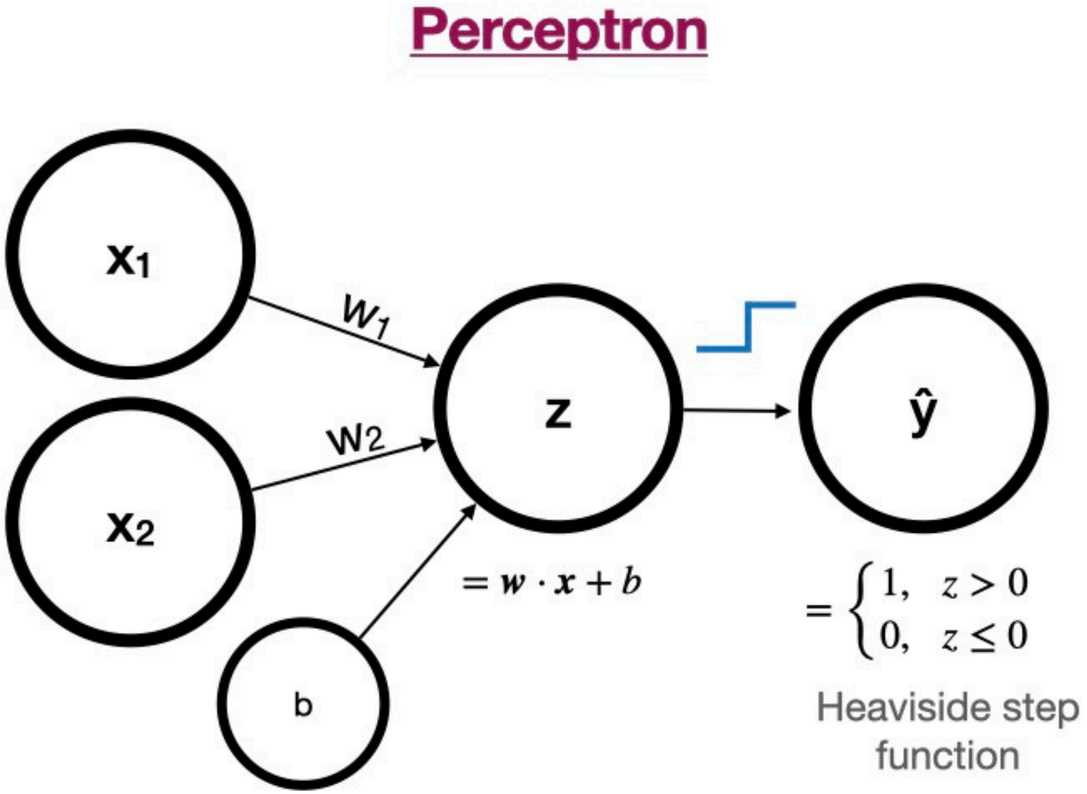
Neural Networks

Perceptron

- A perceptron is a generalized neuron where the activation function is not restricted to a sigmoid.

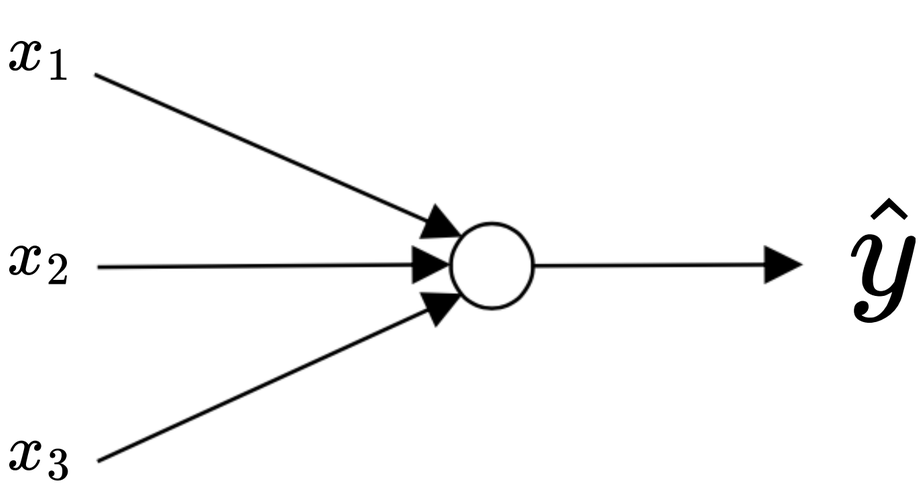
$$z = w^T x + b$$

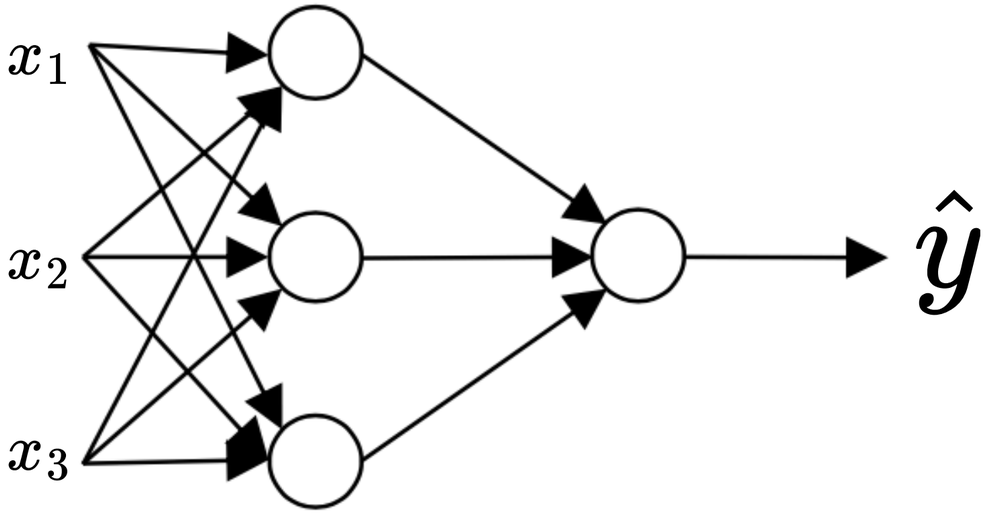
$$y = f(z)$$



Neural Networks

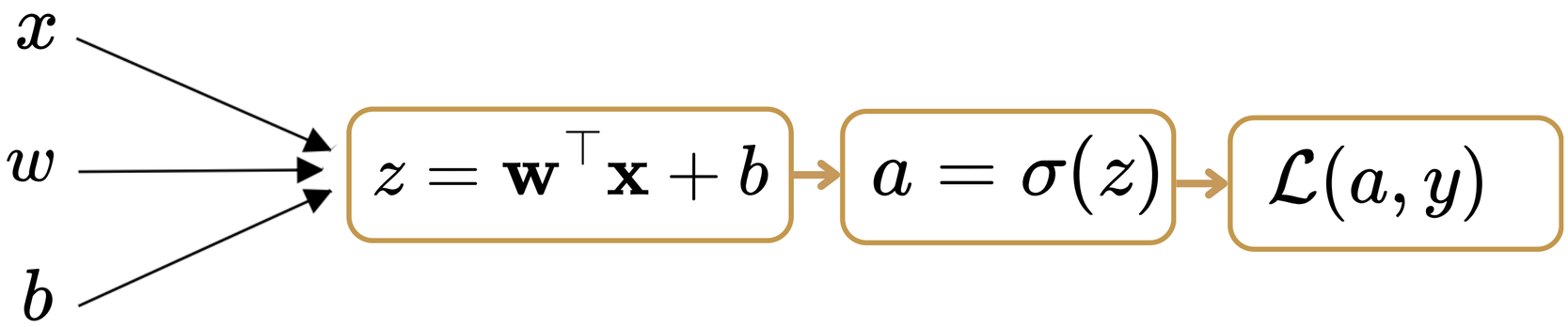
- Perceptron

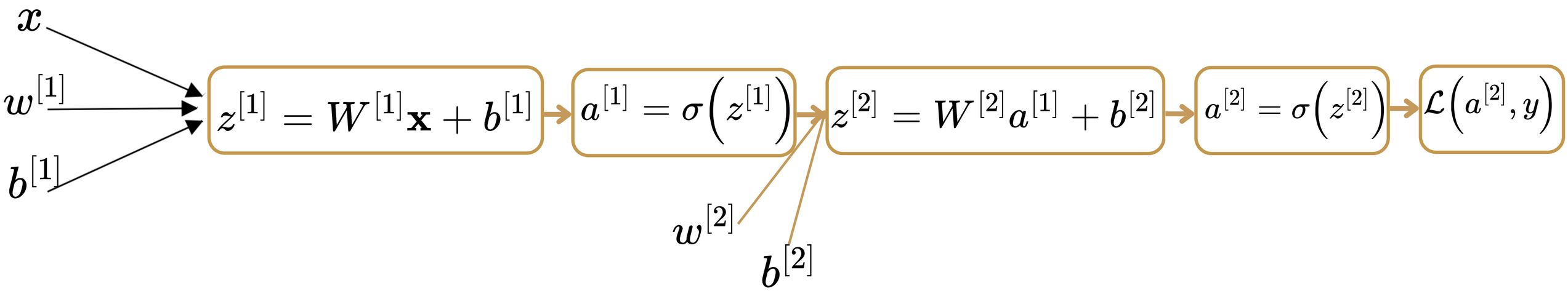




One neuron $\longrightarrow$ Many neurons

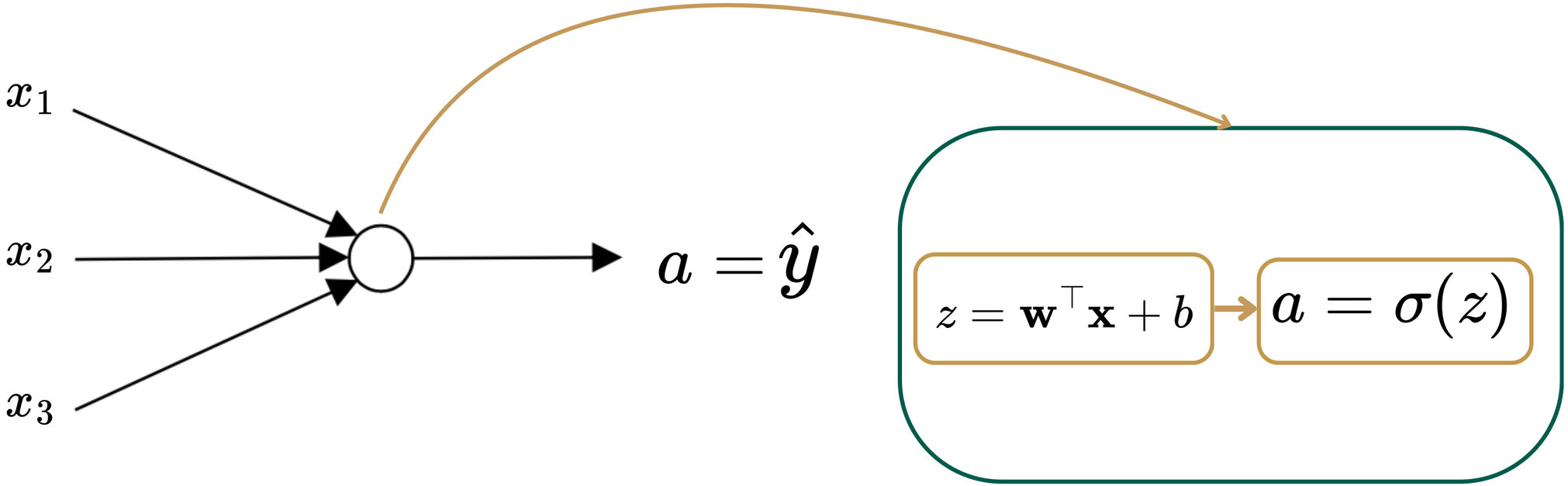
Neural Networks





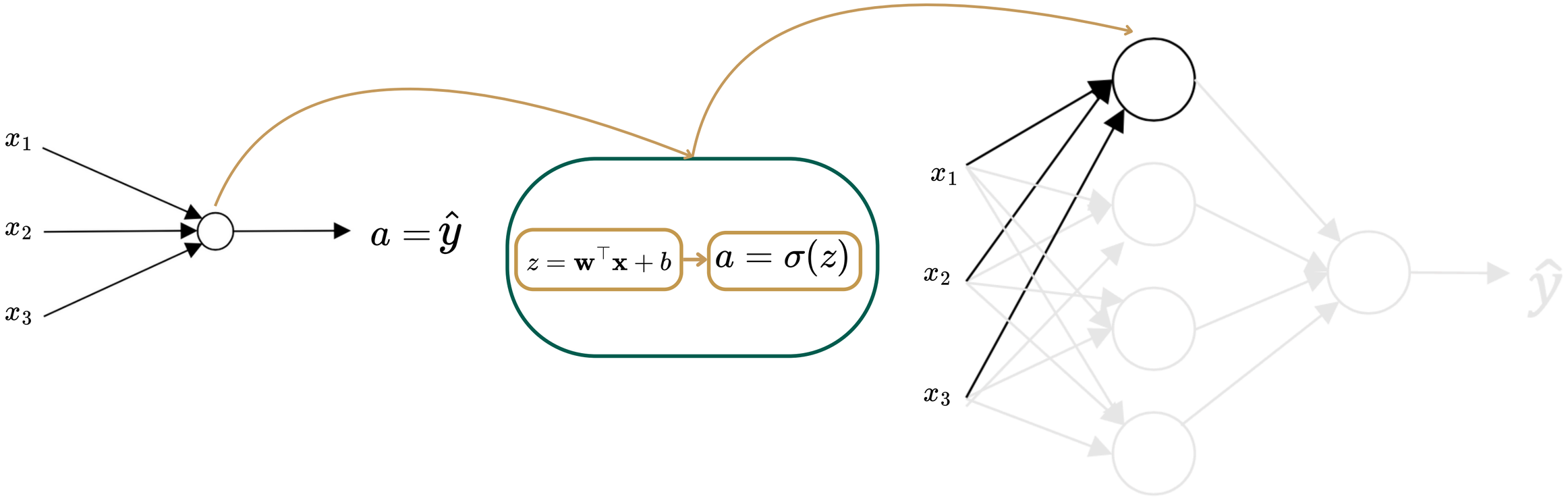
Neural Networks

Neuron Computation



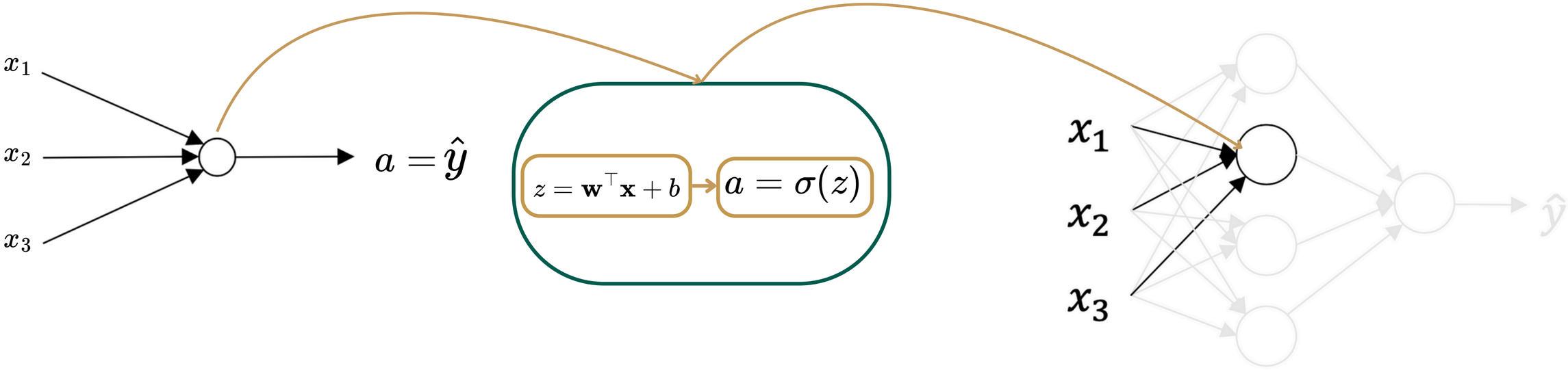
Neural Networks

Neuron Computation



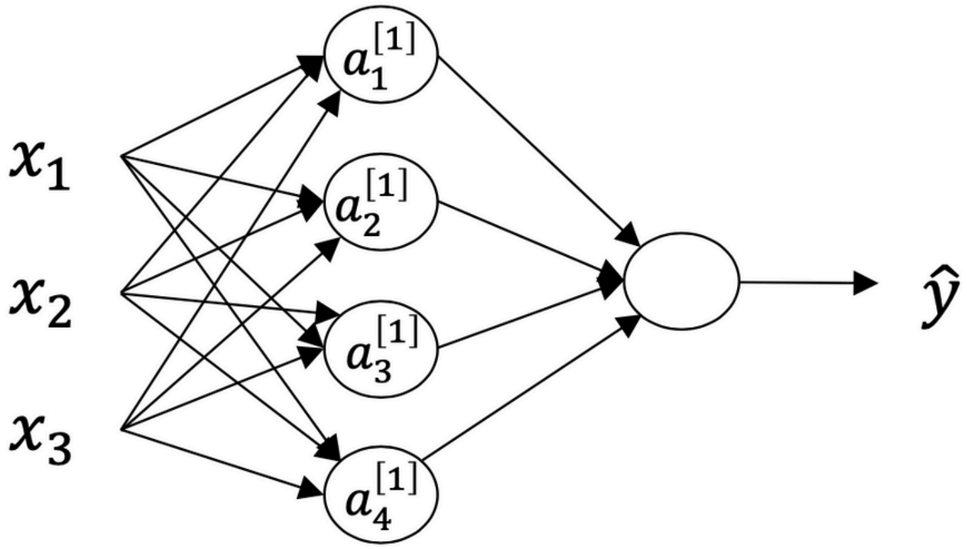
Neural Networks

Neuron Computation



Neural Networks

Neuron Computation



$$z_1^{[1]} = \mathbf{w}_1^{[1]T}\mathbf{x} + b_1^{[1]}, \qquad a_1^{[1]} = \sigma\left(z_1^{[1]}\right)$$

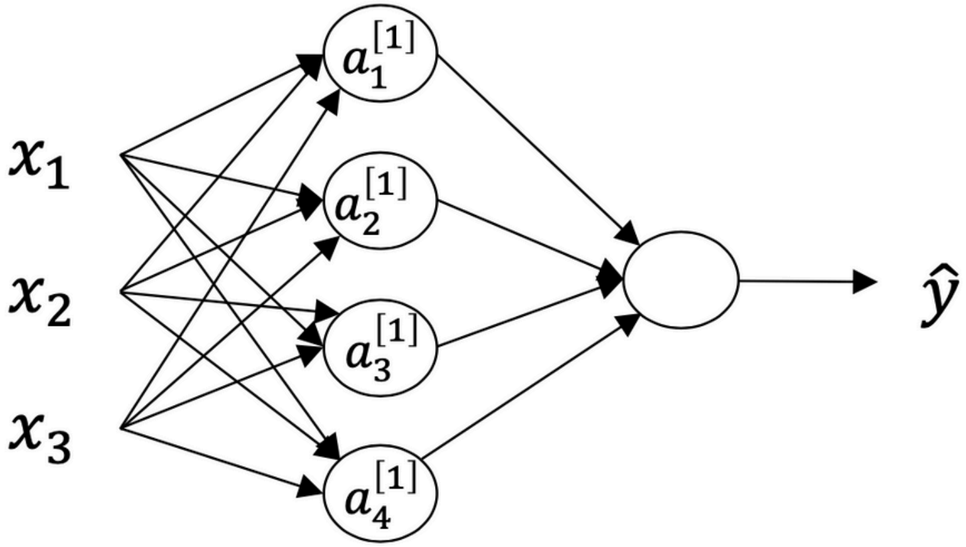
$$z_2^{[1]} = \mathbf{w}_2^{[1]T}\mathbf{x} + b_2^{[1]}, \qquad a_2^{[1]} = \sigma\left(z_2^{[1]}\right)$$

$$z_3^{[1]} = \mathbf{w}_3^{[1]T}\mathbf{x} + b_3^{[1]}, \qquad a_3^{[1]} = \sigma\left(z_3^{[1]}\right)$$

$$z_4^{[1]} = \mathbf{w}_4^{[1]T}\mathbf{x} + b_4^{[1]}, \qquad a_4^{[1]} = \sigma\left(z_4^{[1]}\right)$$

Neural Networks

Neuron Computation



Given input $\mathbf{x}$:

$$z^{[1]} = W^{[1]}\mathbf{x} + b^{[1]}$$

$$a^{[1]} = \sigma\left(z^{[1]}\right)$$

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = \sigma\left(z^{[2]}\right)$$

Neural Networks

Vectorization

Process many examples at once instead of looping.

Before Vectorization

$$z^{[1](i)} = W^{[1]} x^{(i)} + b^{[1]}$$

$$a^{[1](i)} = \sigma\left(z^{[1](i)}\right)$$

Vectorization

$$X = \begin{bmatrix} x^{(1)} & x^{(2)} & \cdots & x^{(m)} \end{bmatrix}$$

$$A^{[1]} = \sigma\left(Z^{[1]}\right)$$

$$Z^{[2]} = W^{[2]} A^{[1]} + b^{[2]}$$

$$A^{[2]} = \sigma\left(Z^{[2]}\right)$$

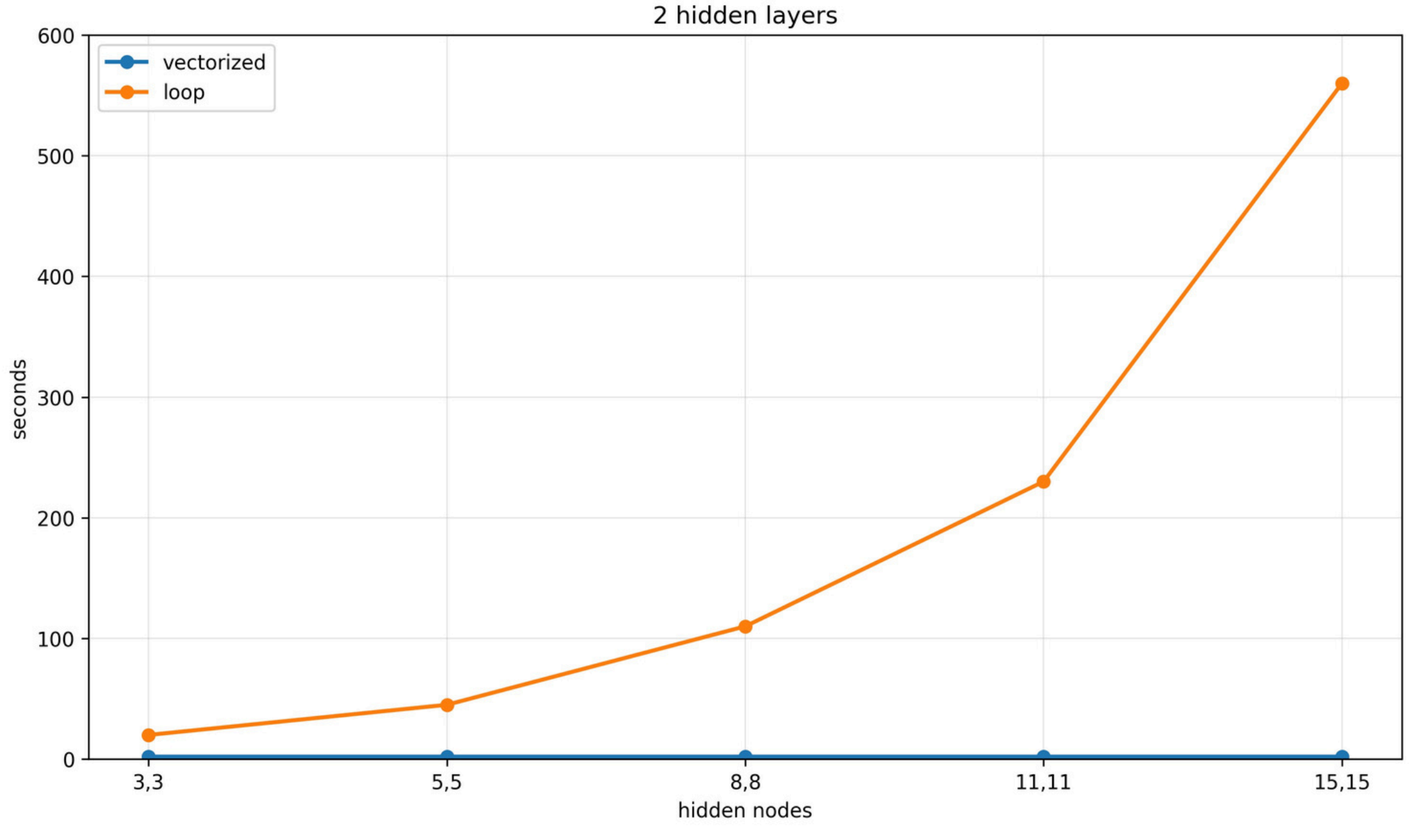
One matrix operation replaces a loop over examples.

$$X \in \mathbb{R}^{n_x \times m}, \quad Z^{[l]}, A^{[l]} \in \mathbb{R}^{n_l \times m}$$

Neural Networks

Vectorization

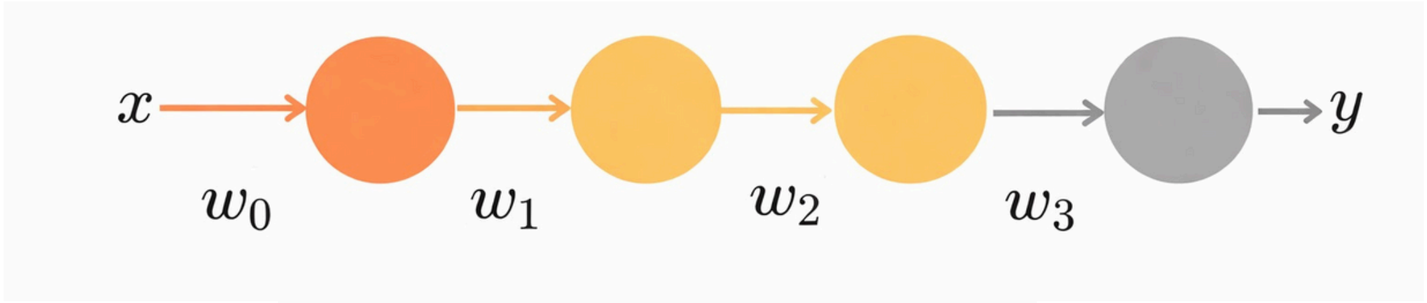
- Replacing Python loops with matrix operations dramatically improves performance

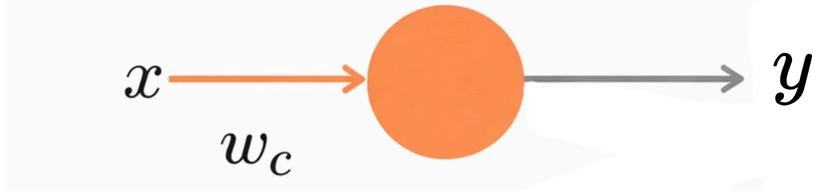


Activation Functions

Activation Functions

- Activation functions



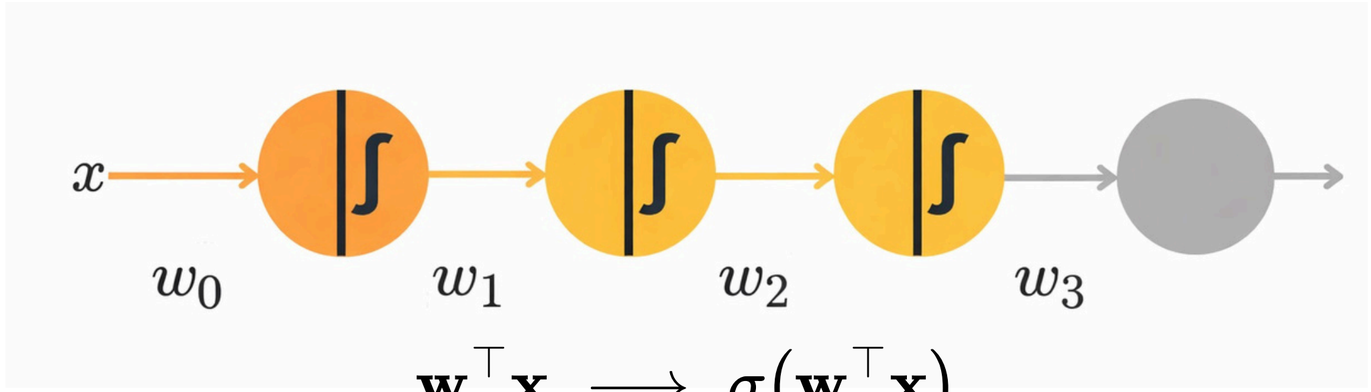


$$y = x \cdot w_0 w_1 w_2 w_3 = x \cdot w_c$$

- Without non-linear activation functions, a deep neural network reduces to a single linear model.

Activation Functions

- Activation functions

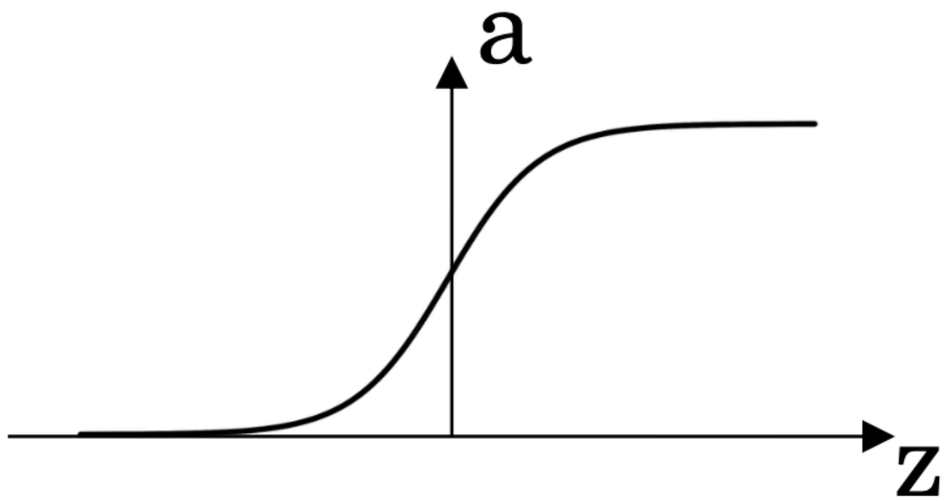


- Non-linear activations prevent layer collapse.

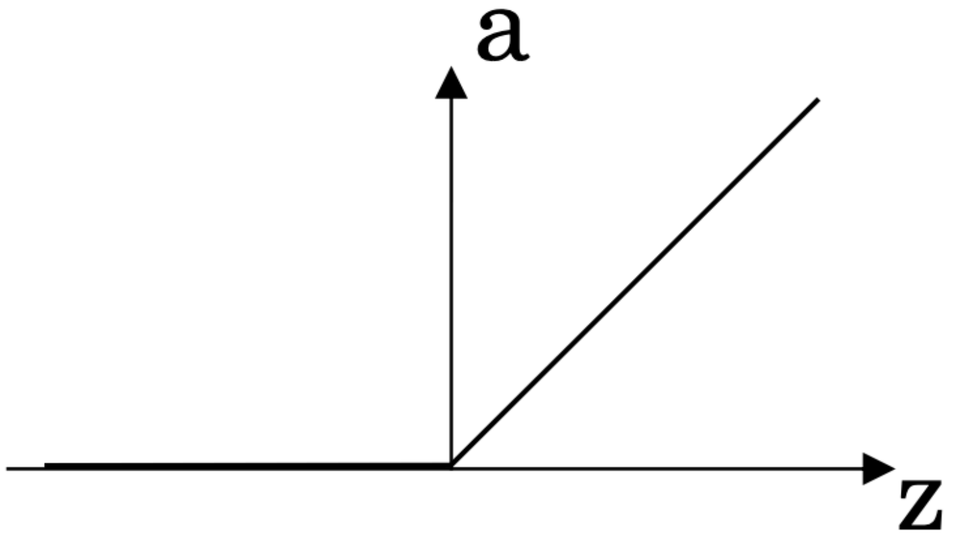
$$y = \sigma\Big(w_3\,\sigma\big(w_2\,\sigma\big(w_1\,\sigma(w_0 x)\big)\big)\Big)$$

Activation Functions

- Activation functions



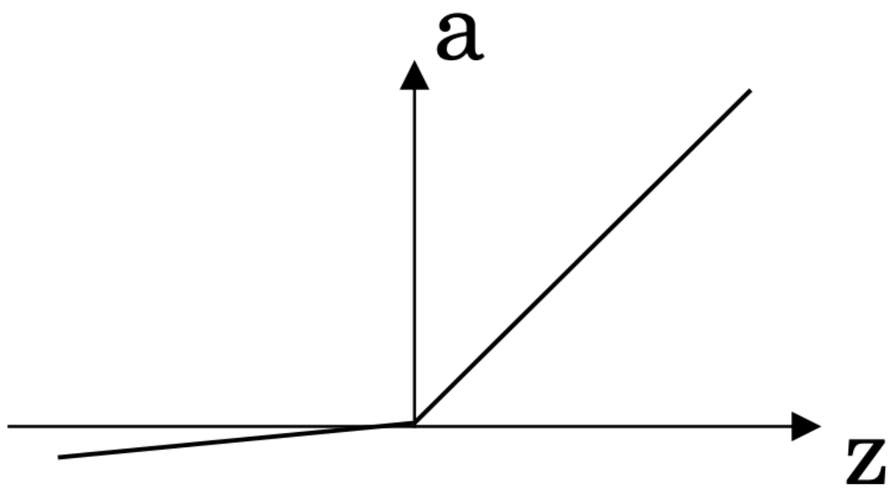
$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

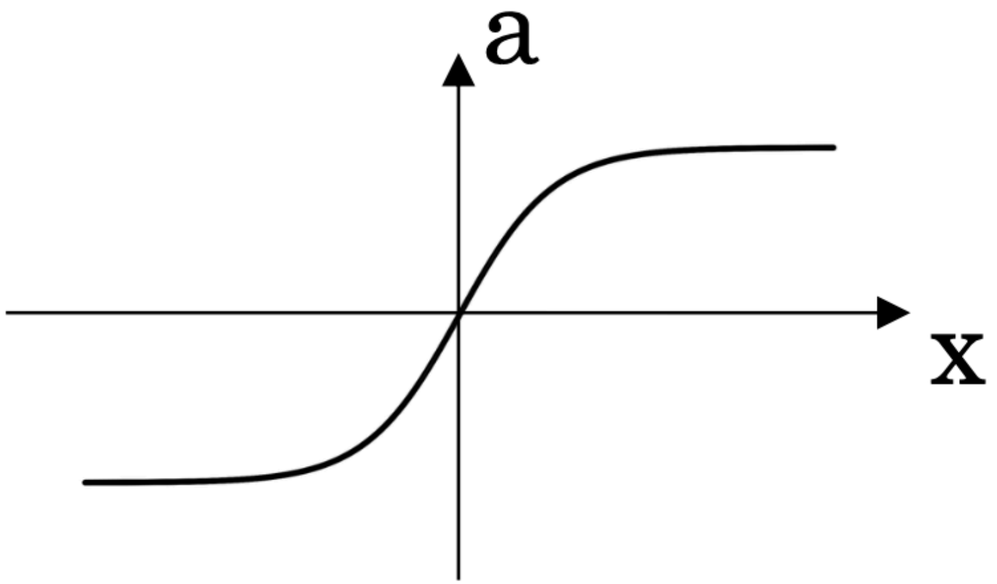


$$\mathrm{ReLU}(z) = \max(0, z)$$

Activation Functions

- Activation functions





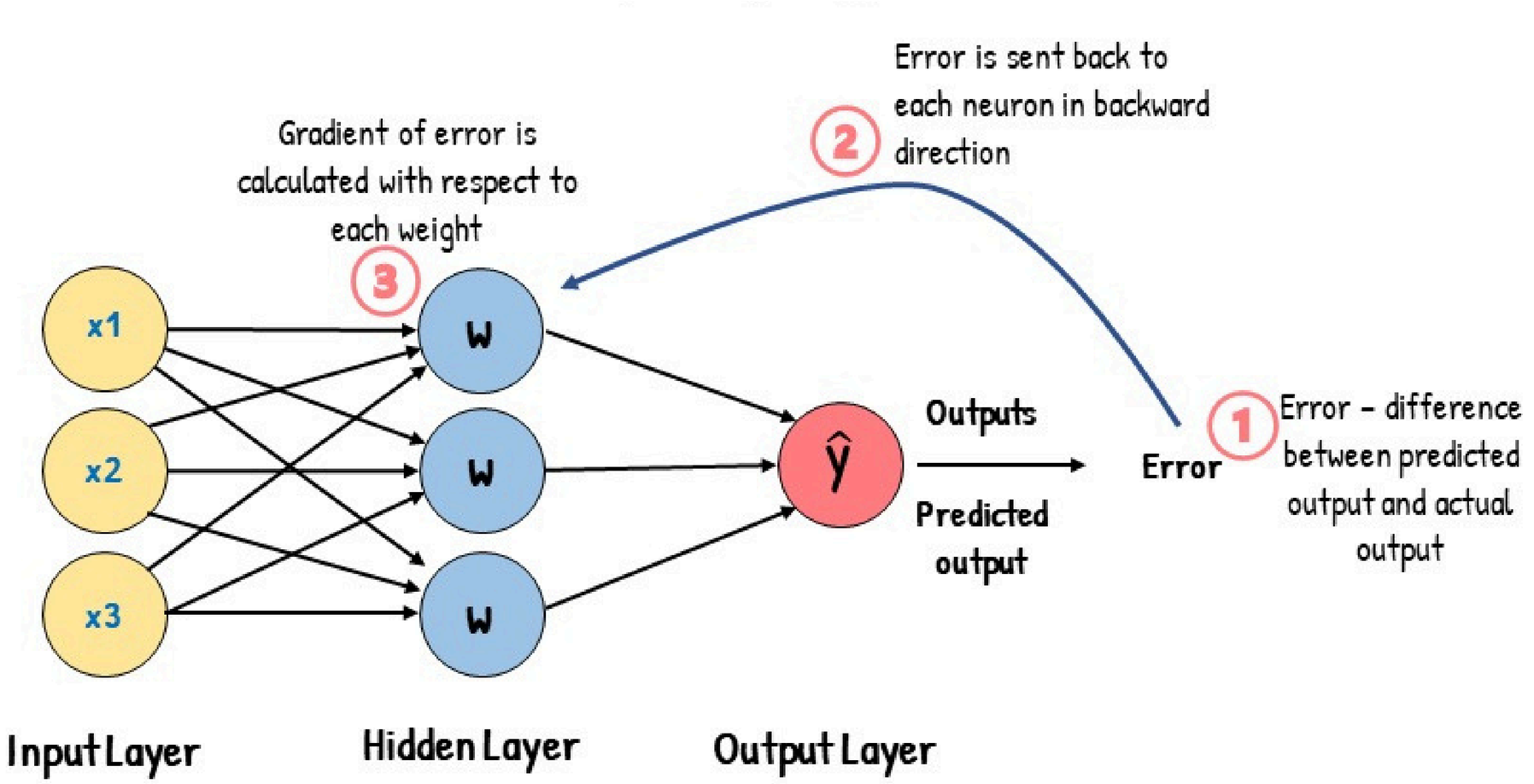
$$\text{LeakyReLU}(z) = \max(\alpha z, z)$$

$$\tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

Backpropagation

Activation Functions

- Backpropagation



Backpropagation

- The error is first calculated at the output layer
- Then, it propagated backward through the network to update weights and biases in each layer.

$$dz^{[2]} = a^{[2]} - y$$

$$dW^{[2]} = dz^{[2]}\, {a^{[1]}}^{T}$$

$$db^{[2]} = dz^{[2]}$$

$$dz^{[1]} = {W^{[2]}}^{T} dz^{[2]} \odot {g^{[1]}}'\left(z^{[1]}\right)$$

$$dW^{[1]} = dz^{[1]}\, x^{T}$$

$$db^{[1]} = dz^{[1]}$$

Backpropagation

- The vectorized version of backpropagation allows gradients to be computed efficiently over all training examples at once by averaging across the batch.

$$dZ^{[2]} = A^{[2]} - Y$$

$$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]T}$$

$$db^{[2]} = \frac{1}{m} \sum_{i=1}^{m} dZ^{[2]}_{:,i}$$

$$dZ^{[1]} = W^{[2]T} dZ^{[2]} \odot g^{[1]'}(Z^{[1]})$$

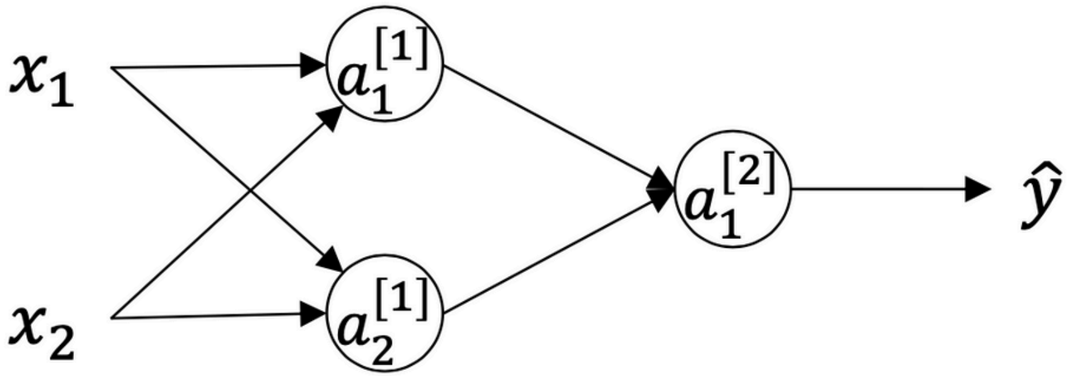
$$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^{T}$$

$$db^{[1]} = \frac{1}{m} \sum_{i=1}^{m} dZ^{[1]}_{:,i}$$

Weights Initialization

Weights Initialization

- Initialize weights to zero



$$W^{[1]} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \qquad W^{[2]} = (0 \quad 0)$$

$$b^{[2]} = 0$$

$$b^{[1]} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

All neurons start identical when weights are initialized to zero

Weights Initialization

- Initialize weights to zero

$$W^{[1]} = 0,\ b^{[1]} = 0$$

$$z^{[1]} = W^{[1]}x + b^{[1]} = 0$$

$$a^{[1]} = g(z^{[1]}) = g(0)$$

$$dz^{[1]} = W^{[2]T}dz^{[2]} \odot g'(z^{[1]}) = W^{[2]T}dz^{[2]} \odot g'(0)$$

$$dW^{[1]} = dz^{[1]}x^T$$

$$dW_1^{[1]} = dW_2^{[1]} = \cdots$$

$$W_1^{[1]} = W_2^{[1]} = \cdots \quad \forall \text{ iterations}$$

Weights Initialization

- Initialize weights to zero

$$W^{[1]} = 0,\ b^{[1]} = 0$$

$$z^{[1]} = W^{[1]}x + b^{[1]} = 0$$

$$a^{[1]} = g(z^{[1]}) = g(0)$$

$$dz^{[1]} = W^{[2]T}dz^{[2]} \odot g'(z^{[1]}) = W^{[2]T}dz^{[2]} \odot g'(0)$$

$$dW^{[1]} = dz^{[1]}x^T$$

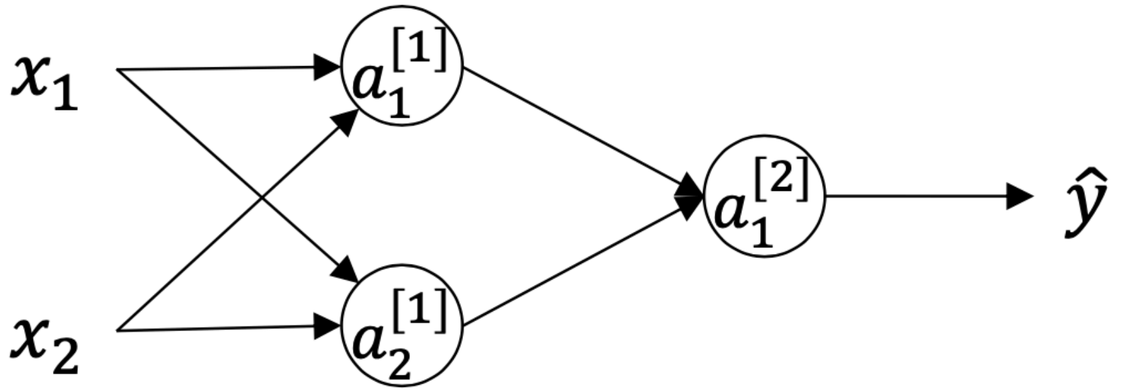
$$dW_1^{[1]} = dW_2^{[1]} = \cdots$$

$$W_1^{[1]} = W_2^{[1]} = \cdots \quad \forall \text{ iterations}$$

Zero initialization $\Rightarrow$ no symmetry breaking

Weights Initialization

- Initialize weights randomly



$$W^{[1]} = \begin{pmatrix} \epsilon_1 & \epsilon_2 \\ \epsilon_3 & \epsilon_4 \end{pmatrix} \qquad W^{[2]} = \begin{pmatrix} \epsilon_5 & \epsilon_6 \end{pmatrix}$$

$$b^{[1]} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad b^{[2]} = 0$$

ϵ are small random numbers

Weights Initialization

- Initialize weights randomly

$$W^{[1]} \sim \mathcal{N}(0, \epsilon), \quad b^{[1]} = 0$$

$$z^{[1]} = W^{[1]}x + b^{[1]} \neq 0$$

$$a^{[1]} = g(z^{[1]}) \quad \text{(different for each neuron)}$$

Weights Initialization

- Initialize weights randomly

Backpropagation effect

$$dz^{[1]} = W^{[2]T} dz^{[2]} \odot g'(z^{[1]})$$

$$dW^{[1]} = dz^{[1]} x^T$$

$$dW_1^{[1]} \neq dW_2^{[1]} \neq \cdots$$

$$W_1^{[1]} \neq W_2^{[1]} \neq \cdots \quad \forall \text{ iterations}$$

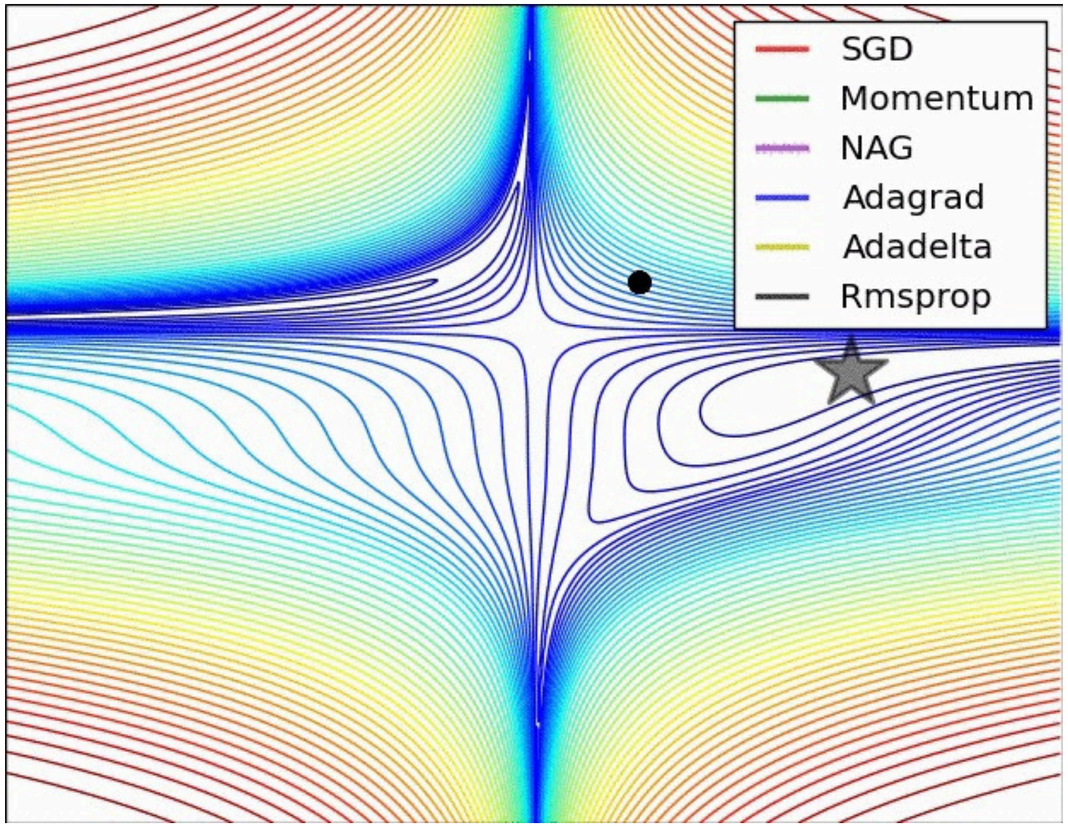
Random initialization $\Rightarrow$ symmetry breaking

Optimization

Optimization

- Optimizers are algorithms that update a model's parameters in order to minimize the loss function during training.

  - Optimizers are a core component of deep learning models.
  - Different optimizers lead to different convergence behaviors and performance.

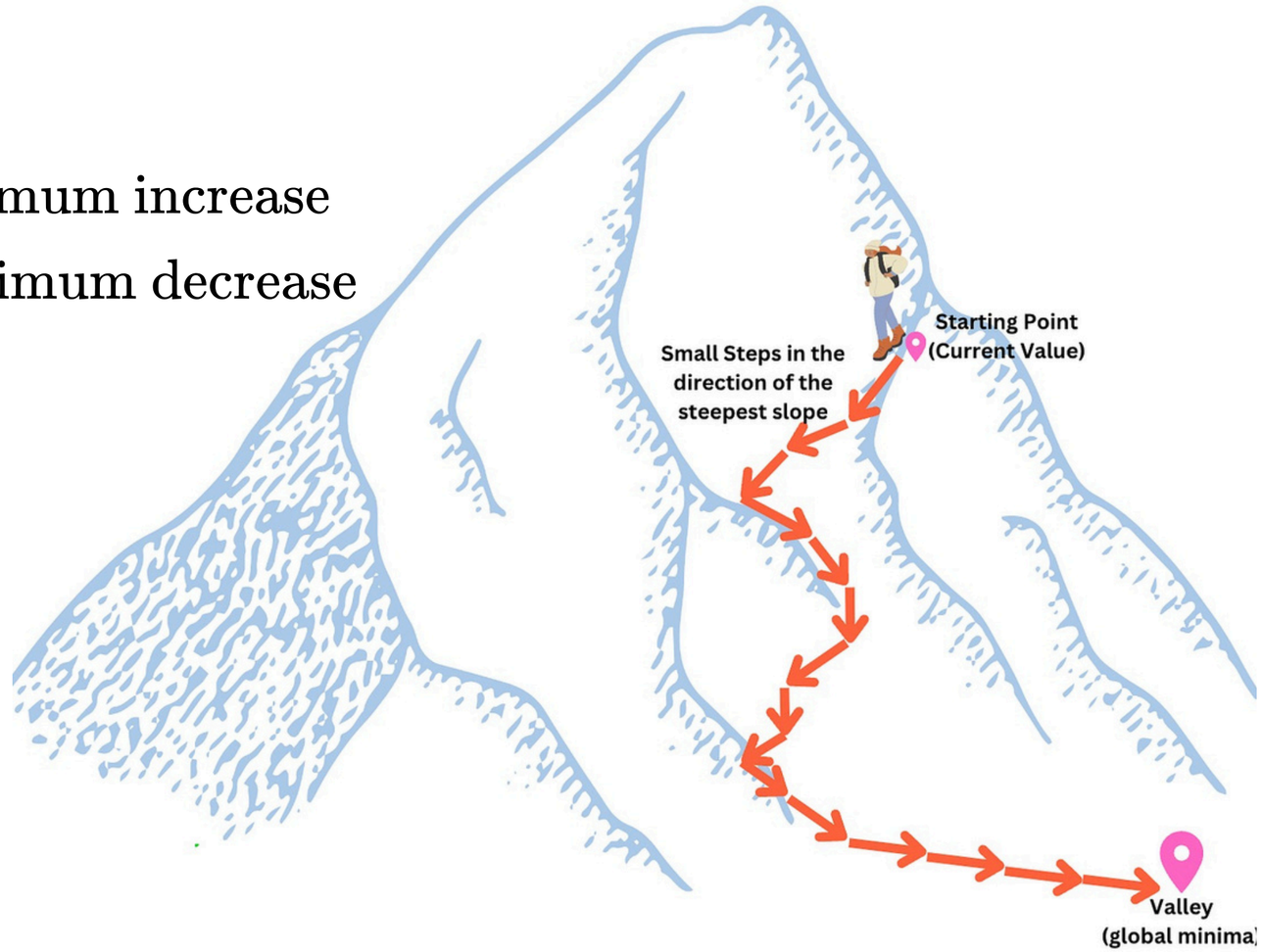


Optimization

- Direction of Maximum Increase and Decrease

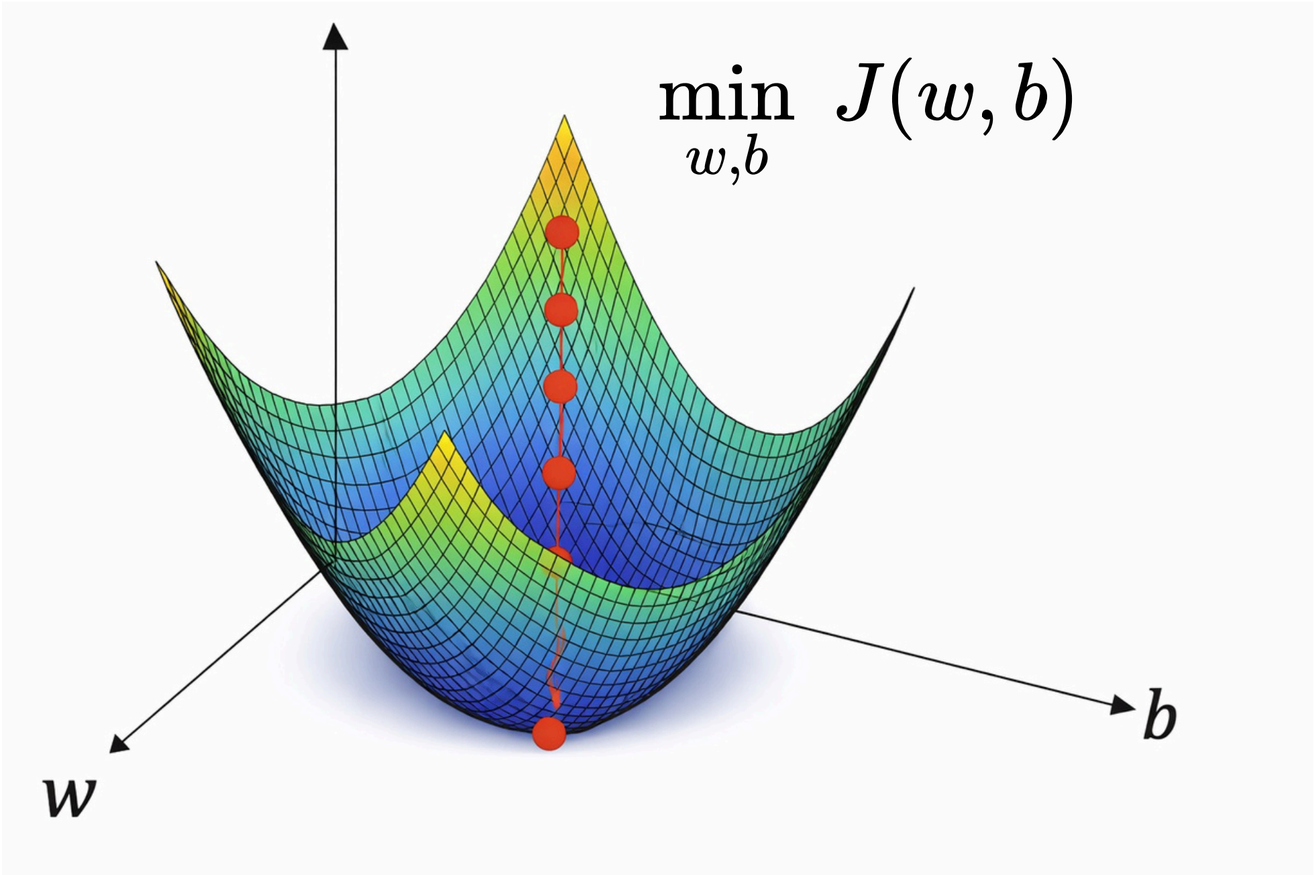
$\nabla f(x)$: direction of maximum increase

$-\nabla f(x)$: direction of maximum decrease



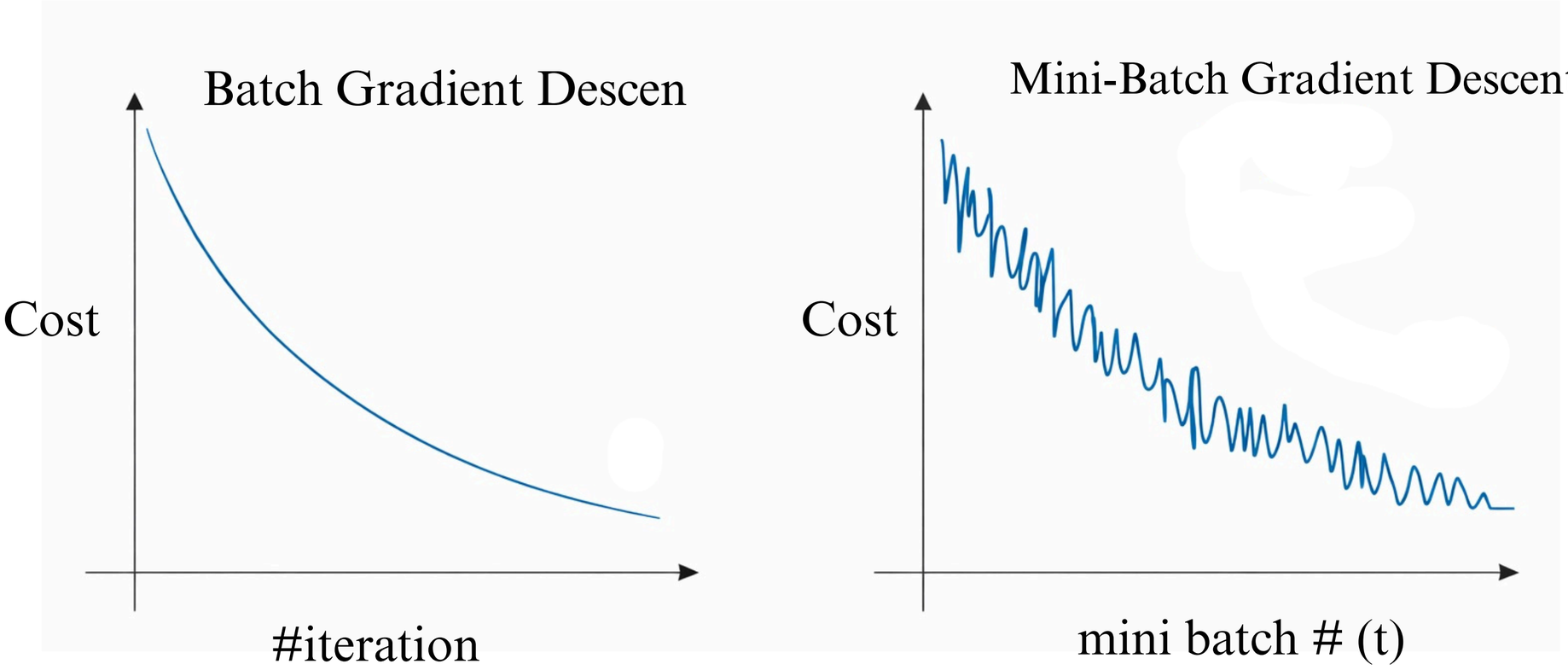
Optimization

- Gradient Descent
  - Moving in the direction of steepest loss decrease



Optimization

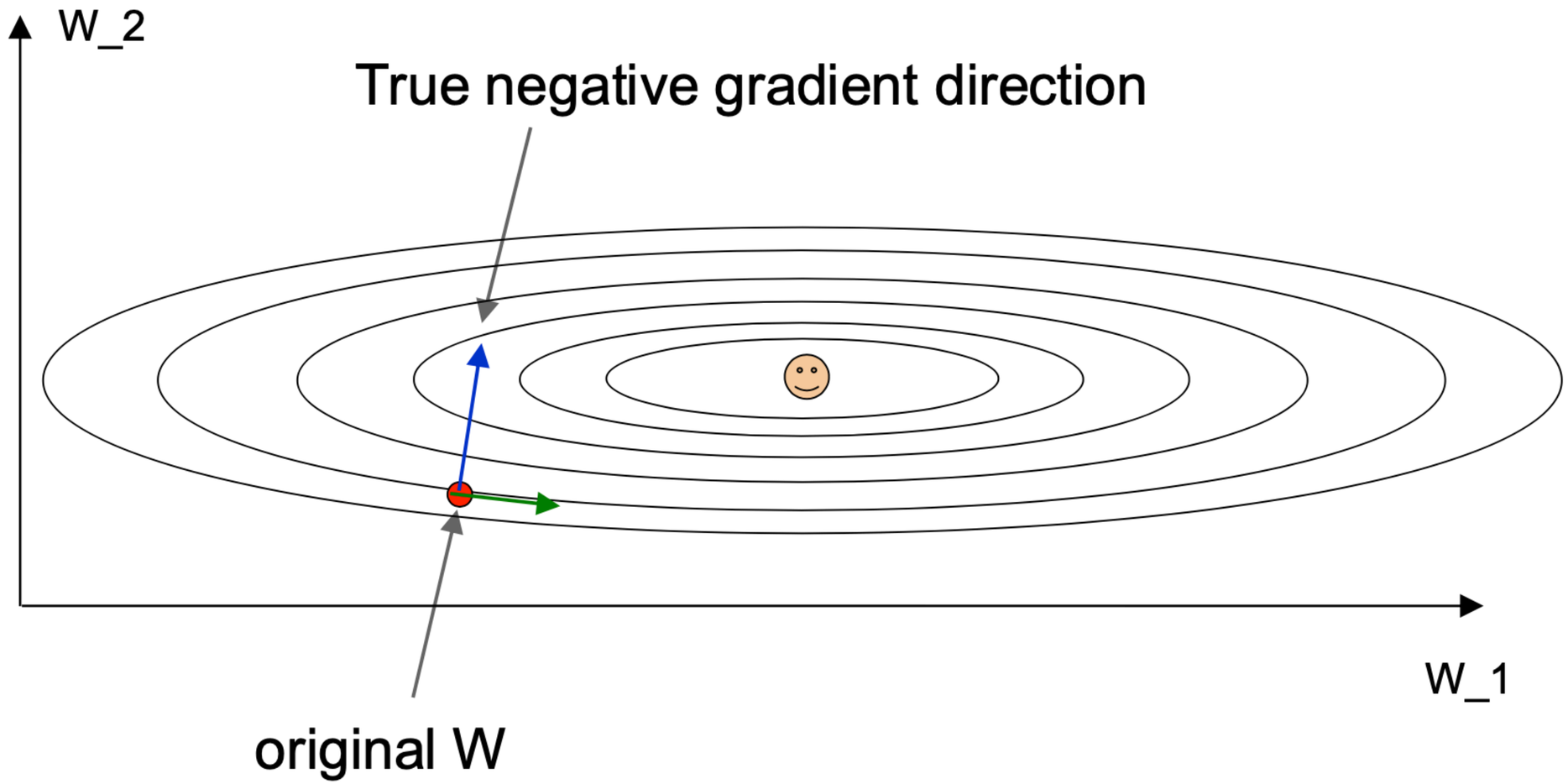
- Mini-Batch Gradient Descent
  - Instead of using all training examples or just one, we update parameters using a small batch of samples.



Common mini-batch sizes are 32/64/128 examples

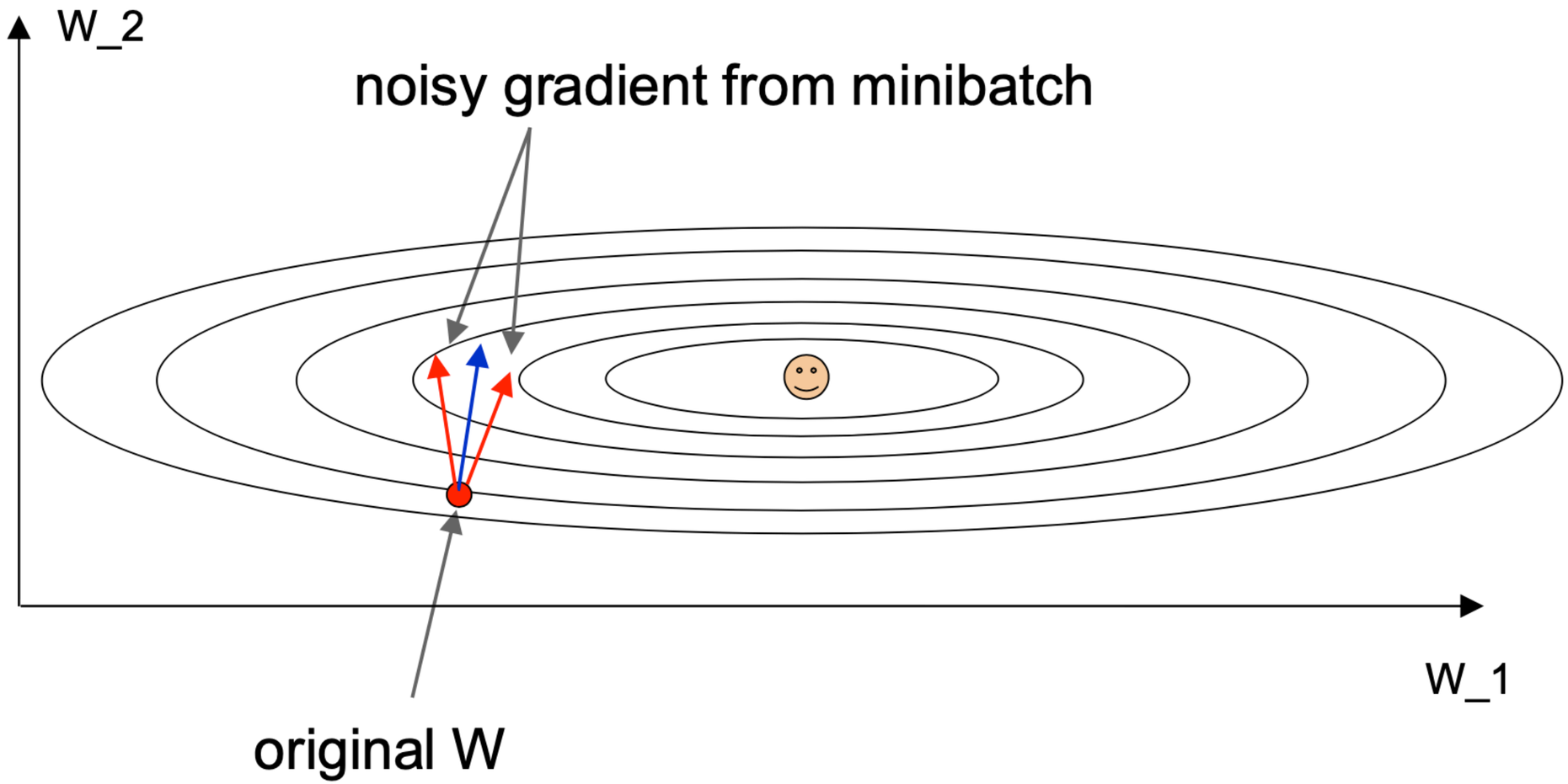
Optimization

- Minibatch updates



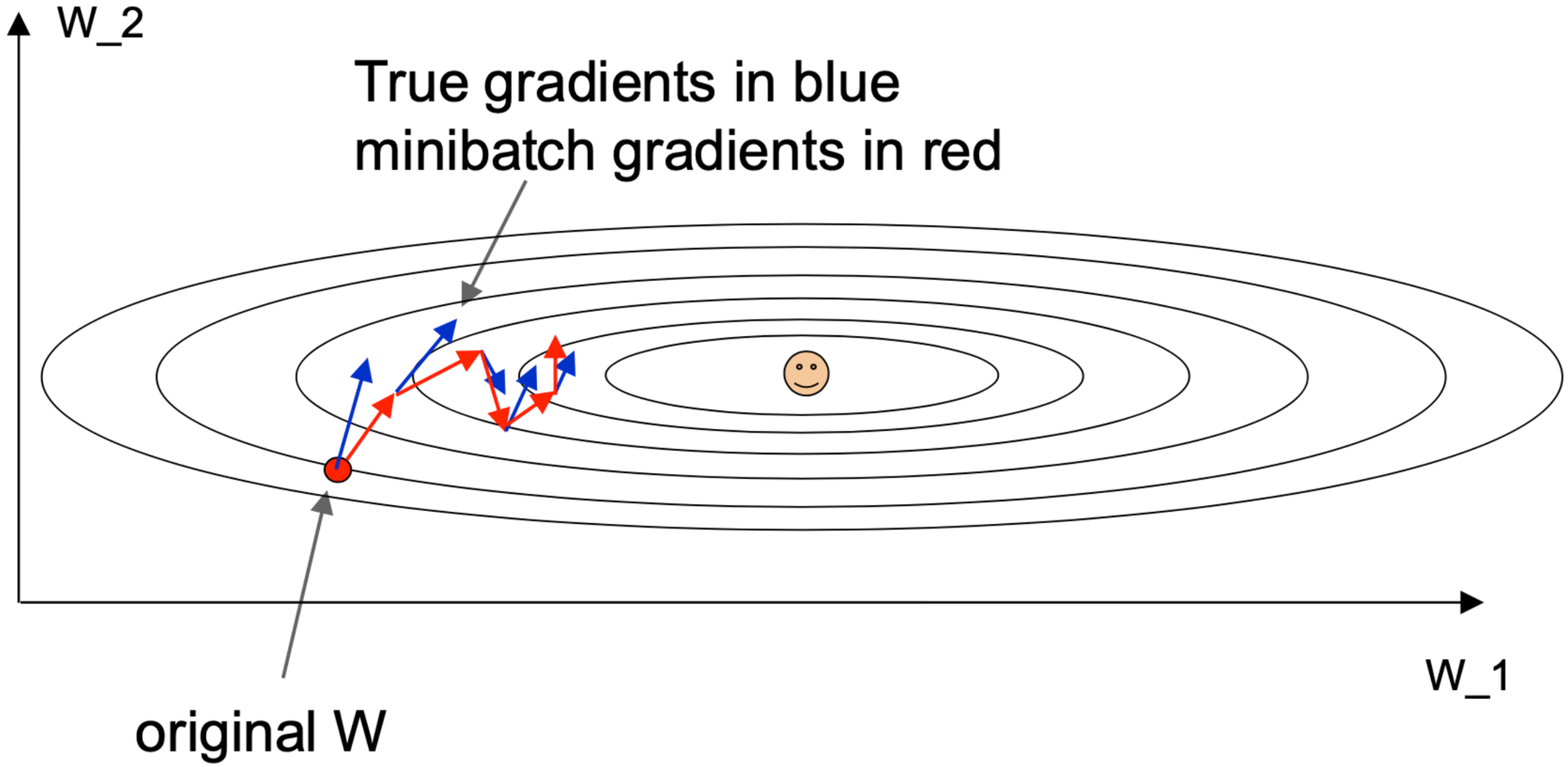
Optimization

- Minibatch updates



Optimization

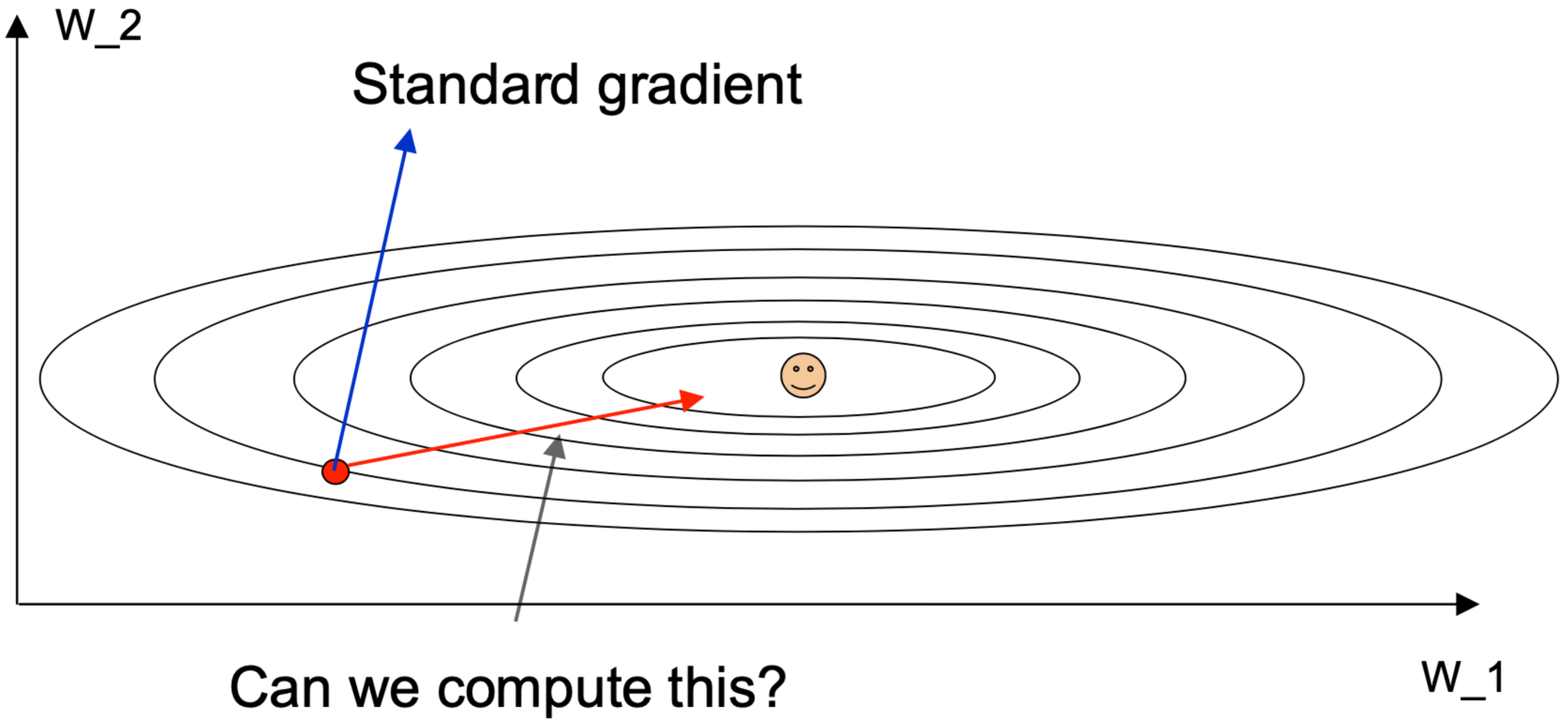
- Minibatch updates



Gradients are noisy but still make good progress on average

Optimization

- Minibatch updates



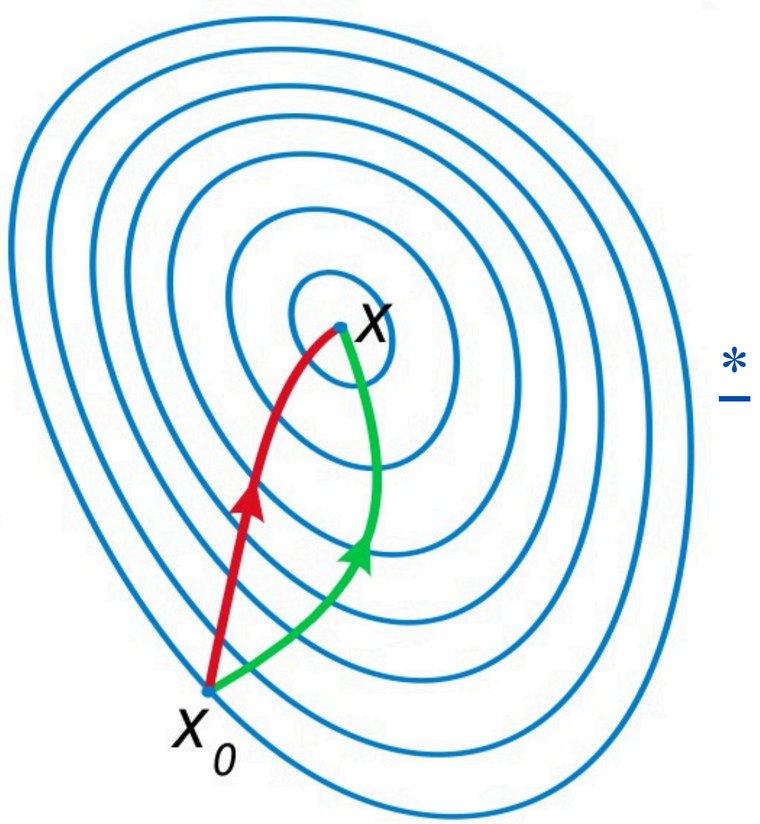
Optimization

- Newton's Method for Optimization

$$x_{n+1} = x_n - H_f(x_n)^{-1} \nabla f(x_n)$$

$\nabla f(x_n)$: gradient of f at x_n

$H_f(x_n) = \nabla^2 f(x_n)$: Hessian matrix (second-order derivatives)

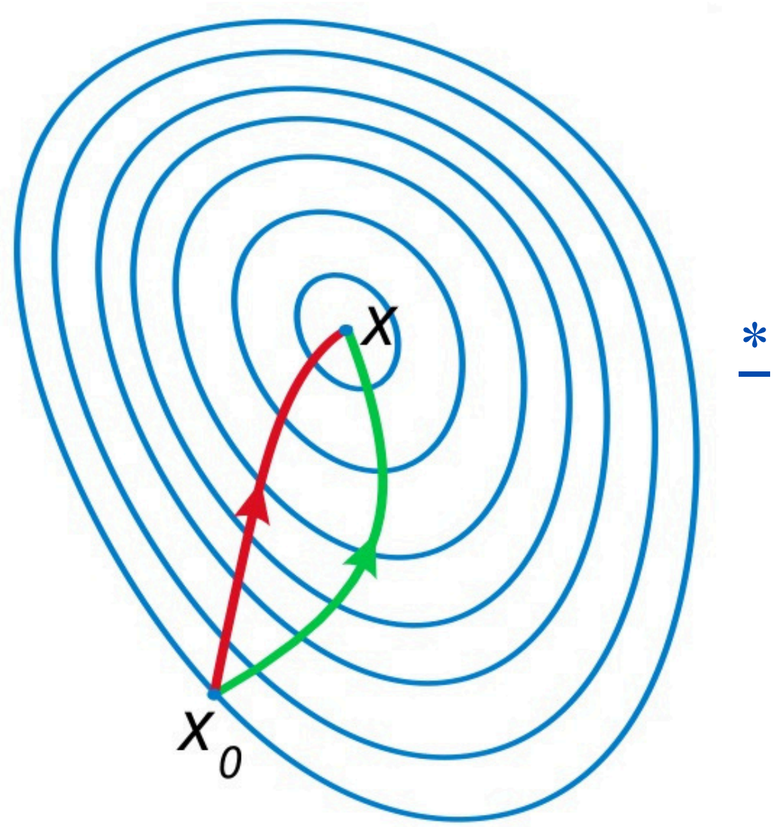


A comparison of gradient descent(green) and Newton's method (red) for minimizing a function

Optimization

Newton's Method for Optimization

- Uses second-order curvature information
- Quadratic convergence near the optimum
- No learning rate required



Why Rarely Used in Deep Learning?

Optimization

- Newton's Method for Optimization

Why Rarely Used in Deep Learning?

- Hessian is $O(d^2)$ to store
- Matrix inversion is $O(d^3)$
- Unstable for non-convex loss landscapes
- Not scalable to millions of parameters

Deep Learning trades fast convergence for scalability.

Optimization

- Momentum Gradient Descent

$$v_{t+1} = \beta v_t + \nabla J(\theta_t)$$

Momentum Update

$$v_{t+1} = \beta v_t + \nabla J(\theta_t)$$

$$\theta_{t+1} = \theta_t - \alpha v_{t+1}$$

α : learning rate

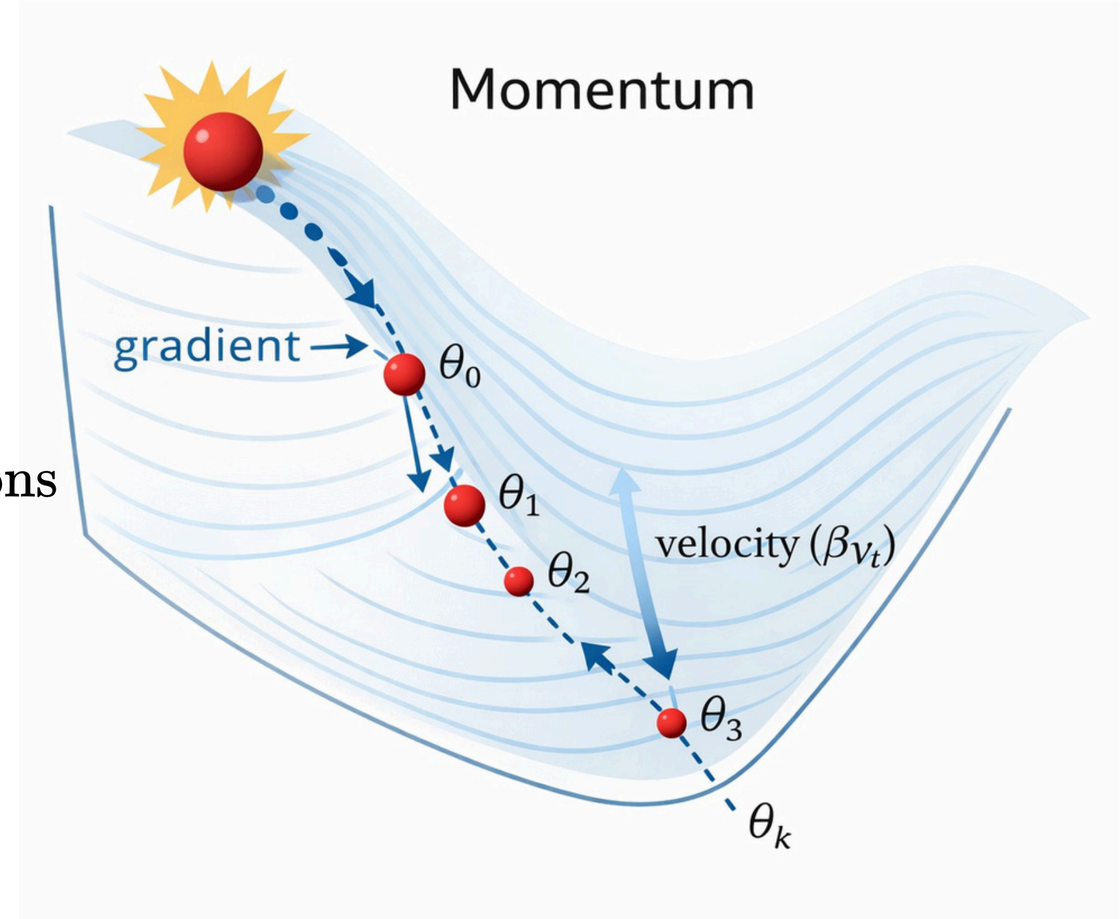
$\beta \in [0, 1)$: momentum coefficient (typically 0.9)

v_t : velocity (exponentially averaged gradient)

Optimization

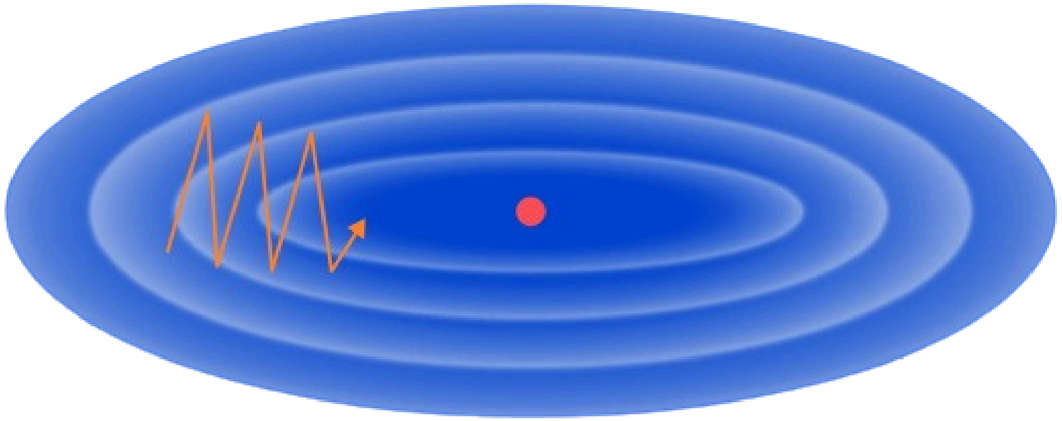
Momentum Gradient Descent

- Accumulates gradients over time
- Dampens oscillations in steep directions
- Accelerates convergence along consistent directions

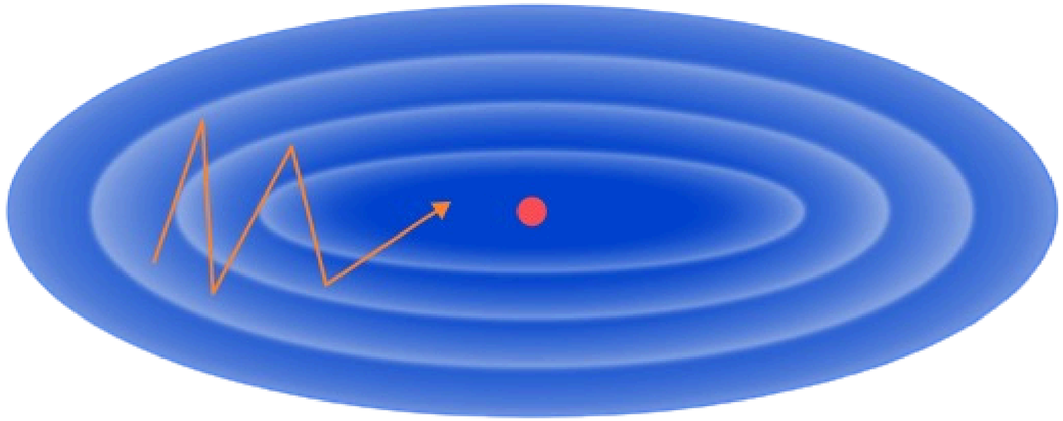


Optimization

- SGD without Momentum



- SGD with Momentum



Optimization

- AdaGrad update

$$g_t = \nabla_\theta J(\theta_t)$$

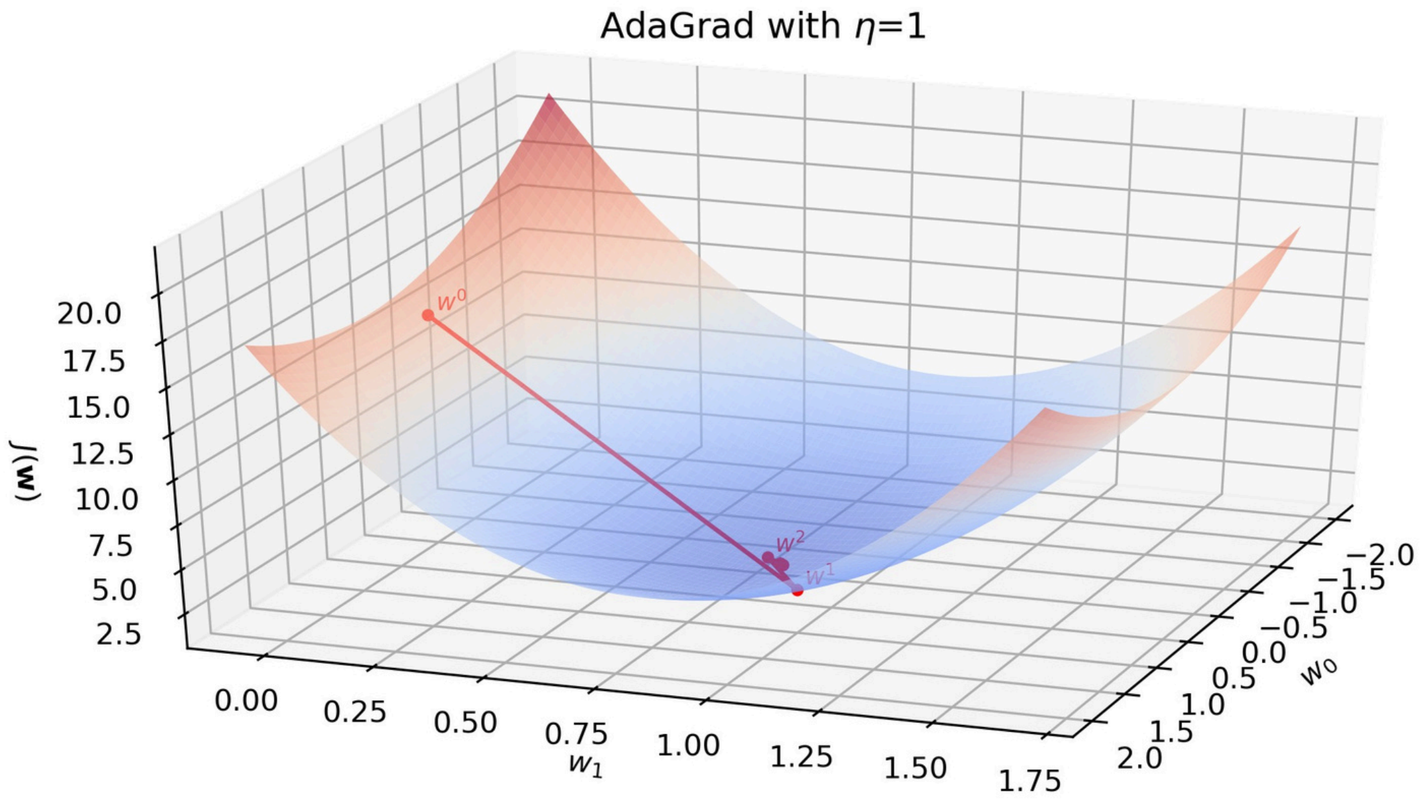
$$G_t = G_{t-1} + g_t \odot g_t$$

$$\theta_{t+1} = \theta_t - \frac{\alpha}{\sqrt{G_t} + \varepsilon} \odot g_t$$

Scalar (per-parameter i) form:

$$G_{t,i} = G_{t-1,i} + g_{t,i}^2$$

$$\theta_{t+1,i} = \theta_{t,i} - \frac{\alpha}{\sqrt{G_{t,i}} + \varepsilon} g_{t,i}$$



Optimization

RMSProp

$$g_t = \nabla_\theta J(\theta_t)$$

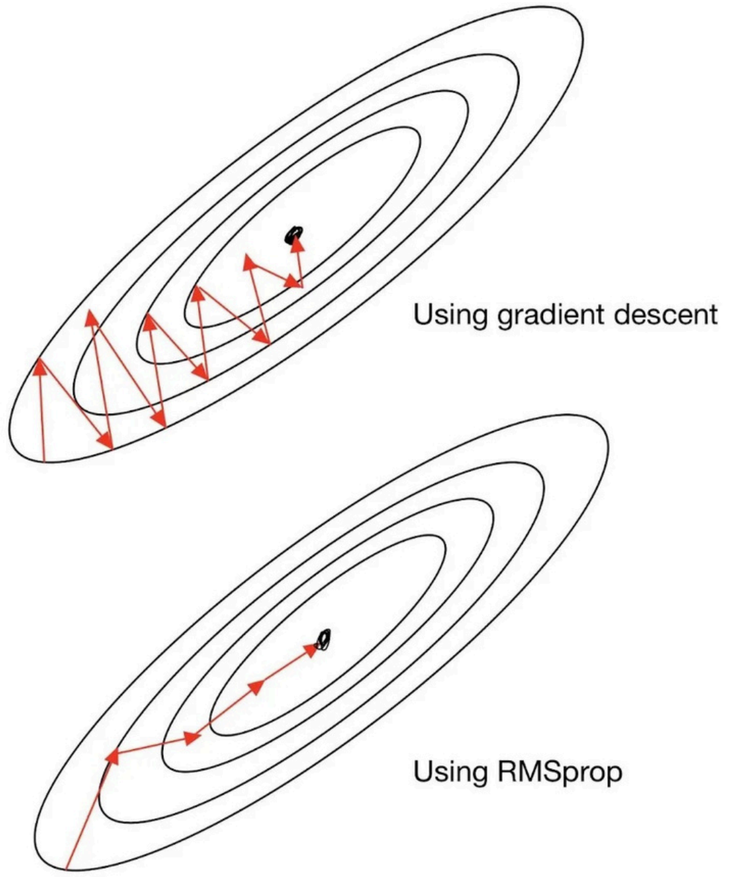
$$E[g^2]_t = \rho\, E[g^2]_{t-1} + (1-\rho)\, g_t \odot g_t$$

$$\theta_{t+1} = \theta_t - \frac{\alpha}{\sqrt{E[g^2]_t + \varepsilon}} \odot g_t$$

α : learning rate

$\rho \in [0, 1)$: decay rate (typically 0.9)

ε : small constant for numerical stability



RMSProp uses an exponentially decaying average of squared gradients.

Optimization

Adam

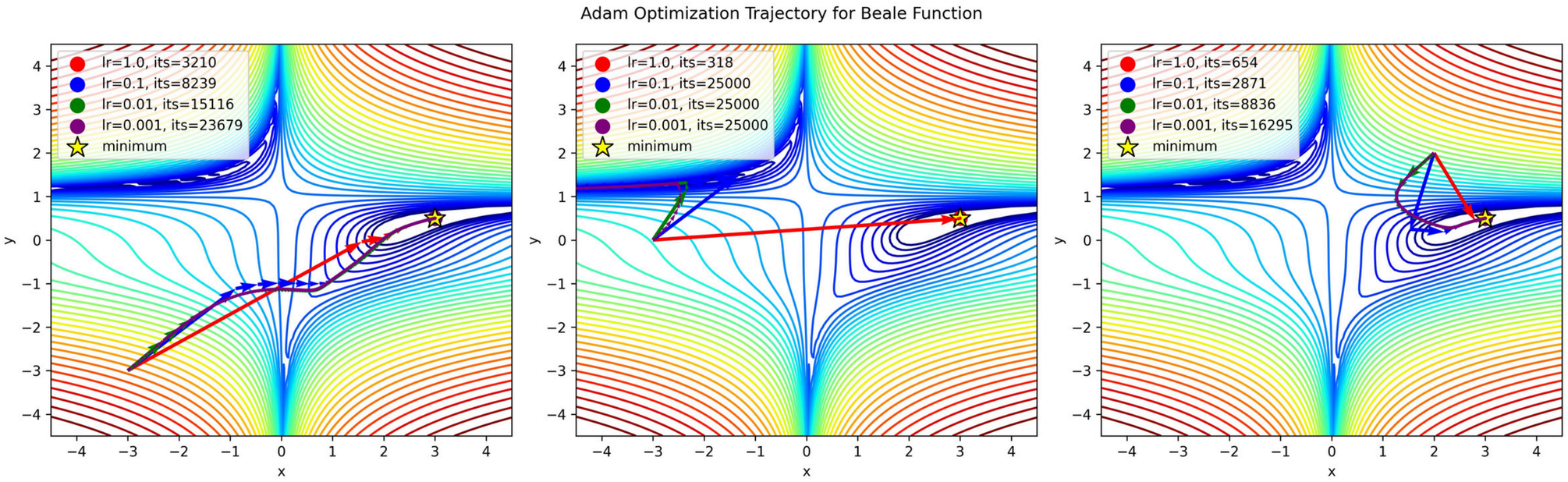
$$g_t = \nabla_\theta J(\theta_t)$$

$$m_t = \beta_1 m_{t-1} + (1-\beta_1) g_t$$

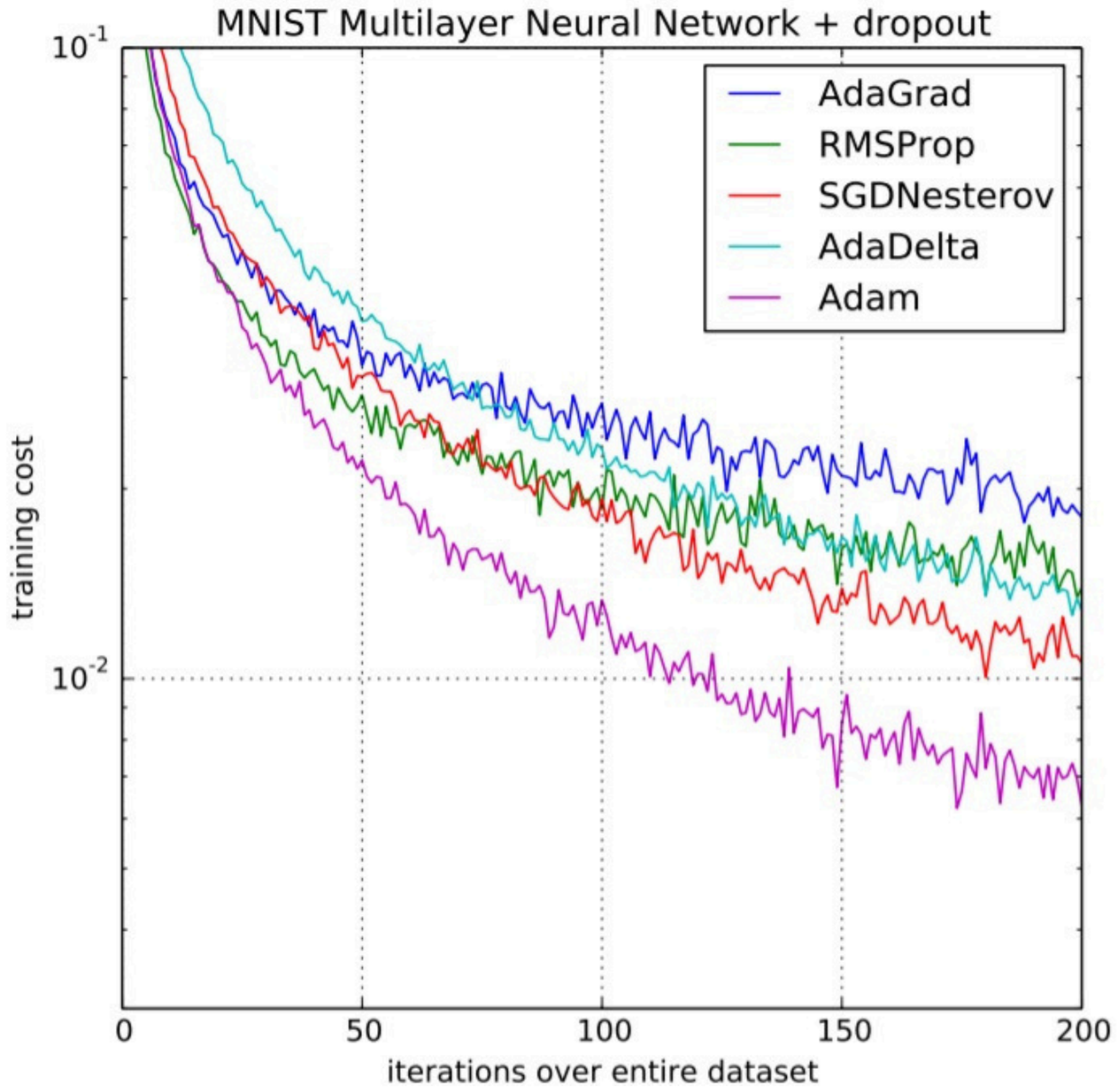
$$v_t = \beta_2 v_{t-1} + (1-\beta_2) g_t^2$$

$$\hat{m}_t = \frac{m_t}{1-\beta_1^t}, \qquad \hat{v}_t = \frac{v_t}{1-\beta_2^t}$$

$$\text{Adam} = \text{Momentum} + \text{RMSProp}$$



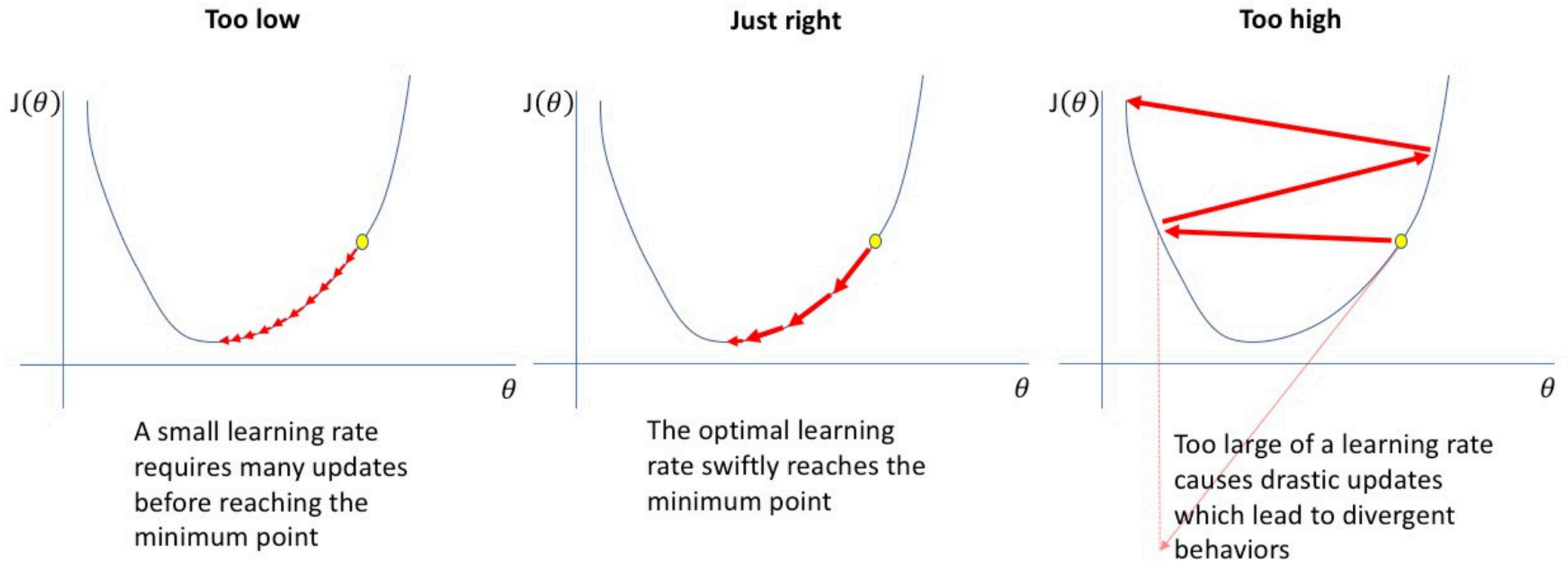
Optimization



Learning Rate

Learning Rate

- SGD, SGD with Momentum, Adagrad, RMSProp, Adam all have learning rate as a hyperparameter.



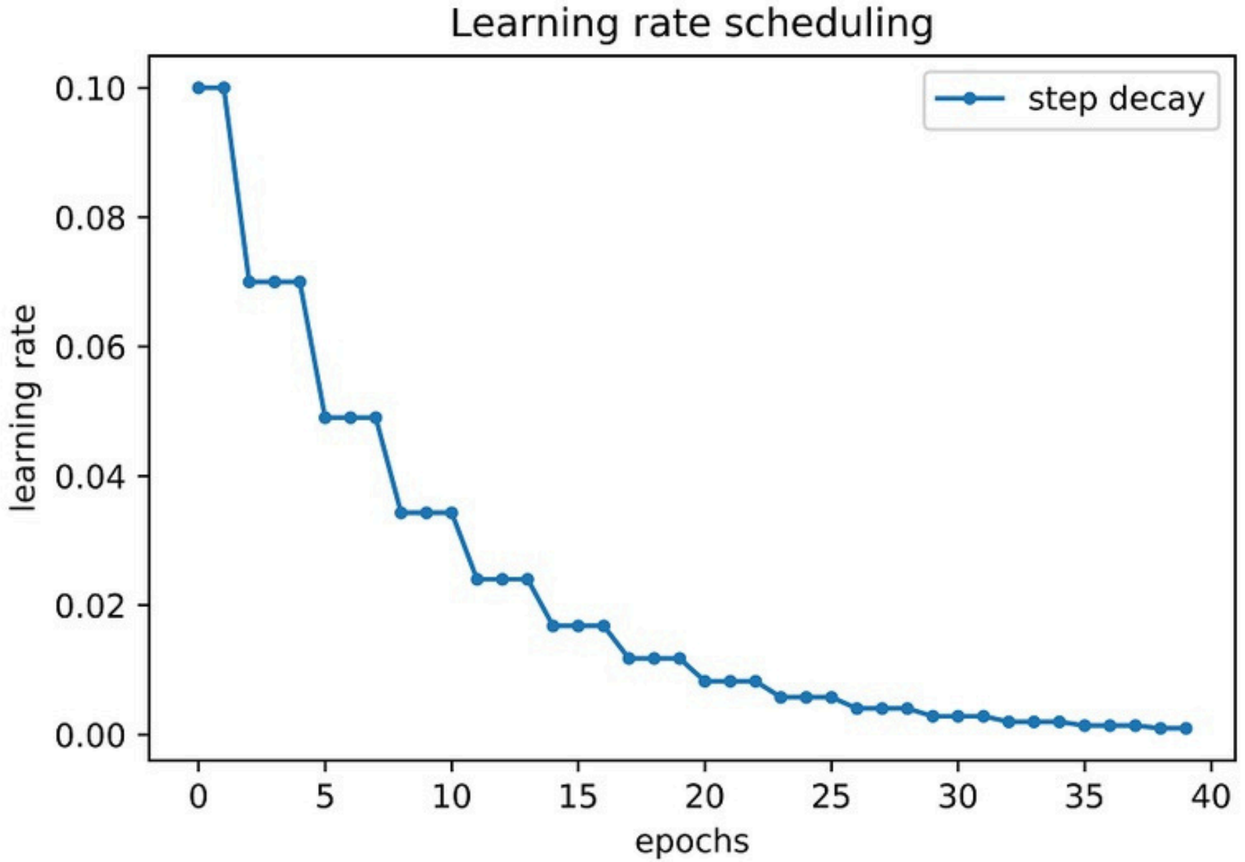
Learning Rate

Learning Rate Schedulers

Step Decay

$$\alpha_t = \alpha_0 \cdot \gamma^{\lfloor t/k \rfloor}$$

Drop learning rate every k epochs.



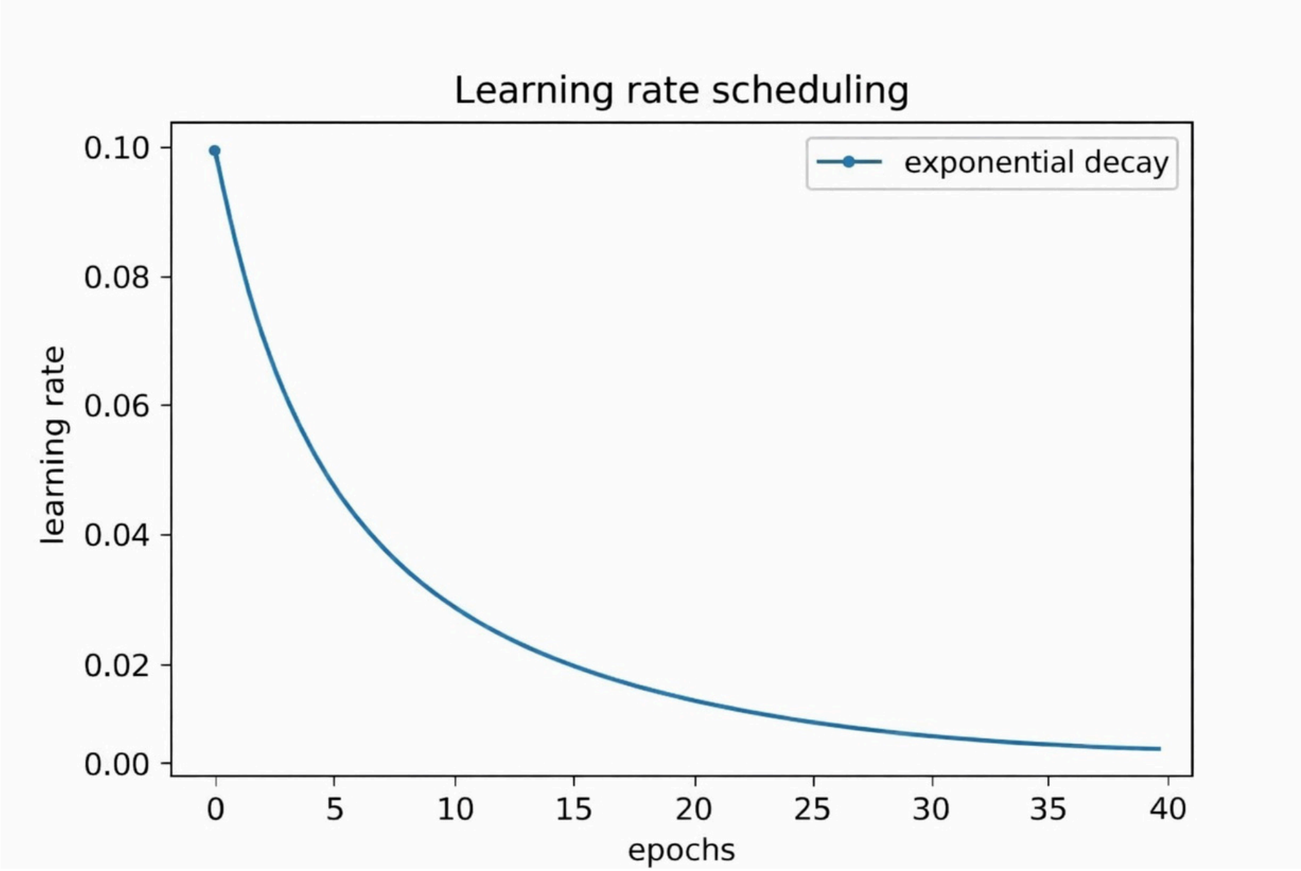
Learning Rate

Learning Rate Schedulers

Exponential Decay

$$\alpha_t = \alpha_0 e^{-kt}$$

Smooth continuous decay.

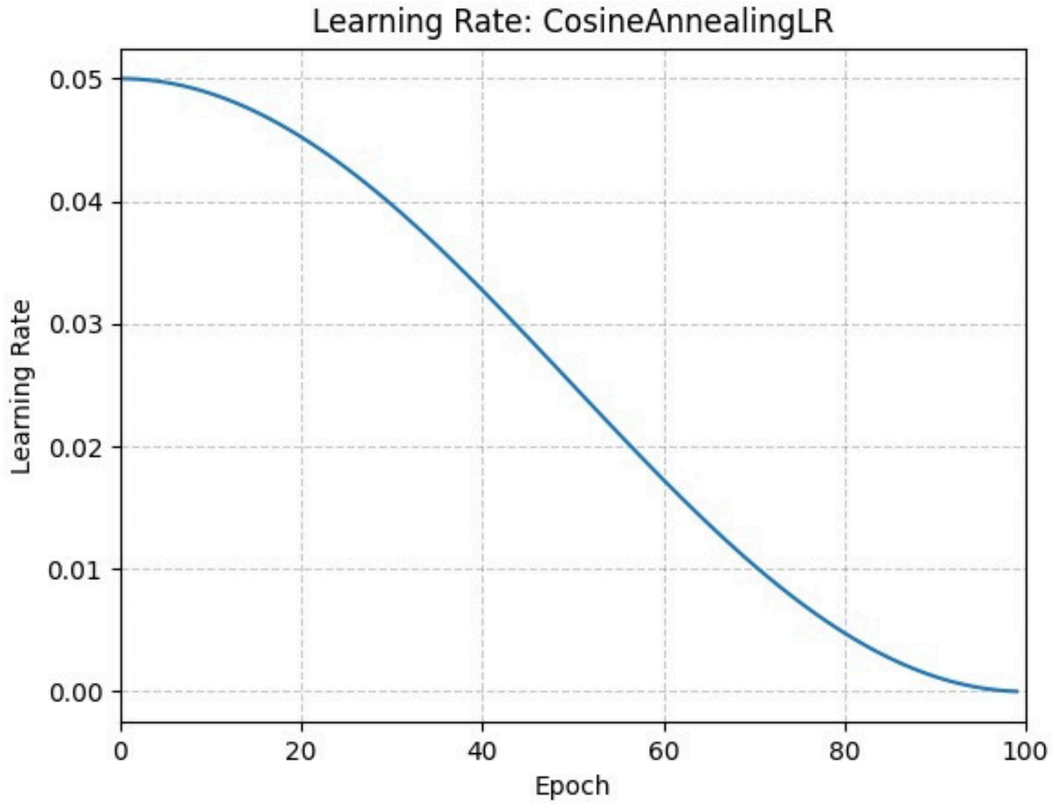


Learning Rate Schedulers

Cosine Annealing

$$\eta_t = \eta_{\min} + \frac{1}{2}(\eta_{\max} - \eta_{\min})\left(1 + \cos\left(\frac{T_{\text{cur}}}{T_{\max}}\pi\right)\right)$$

Most popular in modern deep learning.




Q
A

Thank You