

AMATH 581: Homework 4

November 18, 2018

Part (a)

$$\begin{aligned}\omega_t + [\psi, \omega] &= \nu \nabla^2 \omega \\ \Rightarrow \omega_t &= -\psi_x \omega_y + \psi_y \omega_x + \nu (\partial_x^2 + \partial_y^2) \omega \\ &= -B\psi \cdot C\omega + C\psi \cdot B\omega + \nu A\omega\end{aligned}$$

Use this as the rhs equation in ode45, where A, B, C are calculated as in HW 3.

Part (a) and (b)

`ode45(rhs_equation, timespan, ω_0)`

Inside of the rhs equation, we need to solve for ψ using:

- backslash: $\psi = A \backslash \omega$
- LU decomposition: do LU decomposition once in the main program: $[L, U, P] = \text{lu}(A)$, and pass L, U, P to the rhs equation:
 $y = L \backslash (P * \omega); \psi = U \backslash y;$
- FFT:

$$\tilde{\psi} = -\frac{\tilde{\omega}}{k_x^2 + k_y^2}$$

Part(b)

FFT:

k rescaled to 2π domain:

$$L = 20; k_x = k_y = (2 * \pi / L) * [0 : (n/2 - 1) \quad (-n/2) : -1];$$

To avoid dividing by zero:

$$k_x(1) = 10^{-6}; k_y(1) = 10^{-6};$$