MATHEMATICS

Homework 3. Sequences

In the questions 1, 2 and 3 you have to choose one correct answer from the list, in the questions 4 and 5 you have to give a solution.

Question 1. Choose one true proposition.

A All terms of a sequence must be positive.

B All terms of a sequence must be integer.

C A sequence can have both an explicit and a recursive definition.

D A sequence can have at most two recursive relations.

E Initial values are not always necessary to define a sequence recursively.

Question 2. Choose a closed-form expression for the nth term of the sequence $\{a_n\}$, where $n \in \mathbb{N}$, whose first four terms are $7, 19, 39, 67, \ldots$.

A $a_n = 7n$.

B $a_n = 2 \cdot 3^n + 1$.

C $a_n = 4n^2 + 3$.

D $a_n = 2n^3 + 1$.

E $a_n = 2^{2n} + 5$.

Question 3. Let s_n be the number of n-digit numbers such that every digit is 0, 1, 2 or 3 and the sequence "00" never appears (note that a number cannot start with 0). Choose one true proposition.

A $s_n = 3s_{n-1} + 3$ for all $n > 2$.

B $s_n = 3s_{n-1} + s_{n-2}$ for all $n > 2$.

C $s_n = s_{n-1} + 3s_{n-2}$ for all $n > 2$.

D $s_n = 3s_{n-1} + 3s_{n-2}$ for all $n > 2$.

E All the four previous relations are false.

Question 4. Prove that the Fibonacci sequence satisfies the following relation for every $n \geq 1$:

$$F_{n+7} = F_{n+5} + 4F_{n+3} + F_n.$$

Question 5. Solve the recurrence relation

$$2a_n = 7a_{n-2} - 3a_{n-1}$$

with $a_1 = 1$ and $a_2 = 3$.