MATHEMATICS

Homework 1. Sets, Relations and Functions

In the questions 1, 2, 3 and 4 you have to choose one correct answer from the list, in the questions 5 and 6 you have to give a solution.

Question 1. At a university, each of 100 students chose at least one course in Mathematics or Computer Science. It is known that 70 students chose courses in Computer Science and 25 students chose courses both in Mathematics and Computer Science. How many students chose courses in Mathematics?

A 35 B 40 C 50 D 55 E 70

Question 2. For $A = \{1, 2, 3\}$, $B = \{-1, 0, 1\}$ and the universal set $U = \{-2, -1, 0, 1, 2, 3\}$, determine which of the following sets has cardinality 3:

A $A \cap B$ B $\overline{B}$ C $A \oplus B$ D $A - B$ E $A \times B$

Question 3. A relation R is defined on $\mathbb{N}$ by $a\ R\ b$ if $a^2 + b^3$ is even. Choose one true proposition.

A R is a finite set;

B R isn't transitive;

C the class $[2023]$ is the set of all odd natural numbers;

D $2023\ R\ 2024$;

E there are exactly four equivalence classes for R.

Question 4. Determine which of the following functions is a bijection from $\mathbb{R}$ to $\mathbb{R}$.

A $f(x) = \tan x$

B $f(x) = x(x - 4) + 4$

C $f(x) = x^3 - x + 1$

D $f(x) = 2x^3 + 1$

E $f(x) = 3^x$

Question 5. For the universal set $U = \{1, 2, ..., 10\}$, let $A = \{1, 3, 6\}$, $B = \{2, 3, 5, 7\}$ and $C = \{2, 4, 7, 8\}$. Determine the following sets with explanation:

A $A \oplus B$

B $B - C$

C $\overline{A} \cap B$

D $A \times (B - C)$

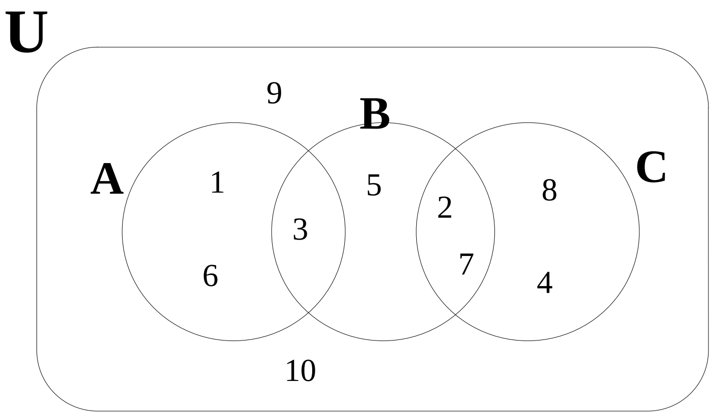
E $|A - \overline{(B \cup C)}|$

Question 6. Prove that the function $f : \mathbb{R} - \{5\} \to \mathbb{R} - \{1\}$ defined by

$$f(x) = \frac{x + 1}{x - 5}$$

is bijective.

Question 5 Answer:



A = {1, 3, 6}
B = {2, 3, 5, 7}
C = {2, 4, 7, 8}
U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

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a: $\mathbf{A \oplus B}$

$A \oplus B = (A - B) \cup (B - A)$

$(A - B) = \{1, 6\}$

$(B - A) = \{2, 5, 7\}$

$A \oplus B = (A - B) \cup (B - A) =$ **{1, 2, 5, 6, 7}**

b: $\mathbf{B - C}$

$B - C =$ **{3, 5}**

c: $\mathbf{\overline{A} \cap B}$

$\overline{A} = \{2, 4, 5, 7, 8, 9, 10\}$

$\overline{A} \cap B = B - A =$ **{2, 5, 7}**

d: $\mathbf{A \times (B - C)}$

B – C = {3, 5}

A × (B − C) = {{1, 3}, {1, 5}, {3, 3}, {3, 5}, {6, 3}, {6, 5}}

e: $\mathbf{|A - \overline{(B \cup C)}|}$

$B \cup C = \{2, 3, 4, 5, 7, 8\}$

$\overline{(B \cup C)} = \{1, 6, 9, 10\}$

$A - \overline{(B \cup C)} = \{3\}$

$|A - \overline{(B \cup C)}| =$ **1**

Question 6 Answer:

To show that the function is bijective, we need to prove both injectivity and surjectivity.

1. prove Injectivity :

Assume f(x) = f(y)
(x + 1) / (x - 5) = (y + 1) / (y – 5)
(x + 1)(y - 5) = (y + 1)(x – 5)
xy - 5x + y - 5 = xy - 5y + x – 5
-5x + y = -5y + x
-5x – x = -5y – y
-6x = -6y
y = x

This implies x = y which means the function is **injective**.

2. prove Surjectivity:

Consider any y in R - {1} . We need to find an x in R – {5} such that f(x) = y

(x + 1) / (x - 5) = y
x + 1 = y(x – 5)
x + 1 = xy – 5y
x - xy = -5y – 1
x(1 - y) = -5y – 1
x = (-5y - 1) / (1 – y)

This expression gives a valid x for any y in R - {1}. Therefore, f is **surjective**.

Since we've demonstrated that f is both **injective** and **surjective**, it is **bijective**.