MATHEMATICS

Homework 4. Combinatorics

In the questions 1, 2 and 3 you have to choose **two** correct answers from the list, in the questions 4 and 5 you have to give a **solution**.

Question 1 (2 answers)**.** How many distinct 10-letter words can be formed from the letters of EQUALITIES?

A $10!$

B $\frac{10!}{2}$

C $\frac{10!}{4}$

D $\frac{8!}{2}$ words starting with EQ.

E $\frac{8!}{4}$ words starting with EQ.

Question 2 (2 answers)**.** In an organic market, 8 types of cakes are being sold (cakes of the same type are identical).

A There are $\binom{27}{7}$ ways to buy 20 cakes.

B There are $\binom{8}{6}$ ways to buy 6 cakes.

C There are 20^8 ways to buy 20 cakes.

D There are $\binom{20}{6}$ ways to buy 20 cakes to taste at least six types.

E There are $\binom{19}{7}$ ways to buy 20 cakes to taste all eight types.

Question 3 (2 answers)**.** There are 20 students in the class. They are going to form two studying groups of 5 and 15 people. In how many ways can they do it?

A If no student can be in more than one group, then $\frac{20!}{5!} \cdot 15$ ways.

B If no student can be in more than one group, then $\binom{20}{5}$ ways.

C If any student can be in any number of groups, then $\frac{20!}{5!15!}$ ways.

D If any student can be in any number of groups, then $\frac{(20!)^2}{5!15!}$ ways.

E If any student can be in any number of groups, then $\binom{20}{5}^2$ ways.

Question 4. Find with explanation the number of ways that one can choose 8 cards from a standard deck of 52 cards in such a way that all four suits are present.

Question 5. A man buys 7 donuts, each of which is a plain donut, a powdered donut or a glazed donut. How many possible selections are there if he buys at least one donut of each kind?

A Answer the question above by determining the coefficient of x^7 in a product of polynomials and/or power series.

B Answer the question above by computing $\binom{s+t-1}{s}$ for an appropriate choice of s and t.

Question 4 – Answer.

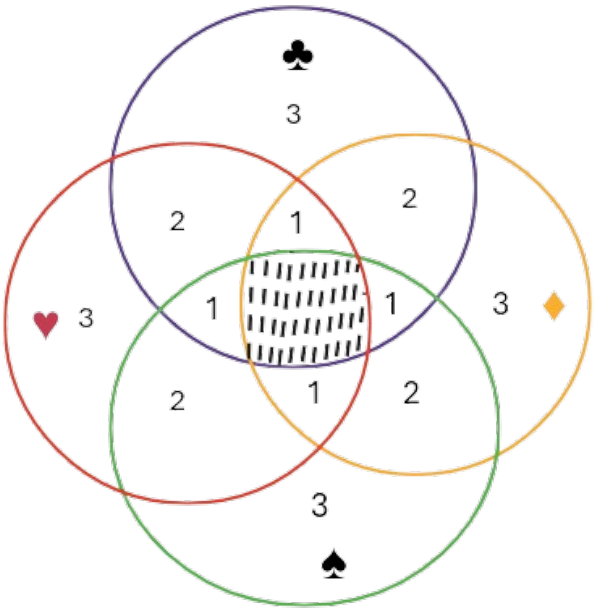
- the number of ways to choose 8 cards without any restrictions = **C(52, 8)**

- the number of ways to choose 8 cards with one suit missing = **C(39, 8)**

- the number of ways to choose 8 cards with two suits missing = **C(26, 8)**

- the number of ways to choose 8 cards with three suits missing = **C(18, 8)**

- the number of ways to choose 8 cards with four suits missing = **C(0, 8)**



By using the principle of **inclusion-exclusion**, the answer will be:

$$C(52,8)-(4\times C(39,8))+(6\times C(26,8))-(4\times C(13,8))$$

Question 5 – Answer.

A:

$$(x+x^2+x^3+x^4+x^5)\times(x+x^2+x^3+x^4+x^5)\times(x+x^2+x^3+x^4+x^5) \;=\; (x+x^2+x^3+x^4+x^5)^3$$

Let the first term corresponds to the plain donut, the second to the powdered donut, the third to the glazed donut.

Then possible selections are there if he buys at least one donut of each kind is equal to coefficient in front of x^7

$$(x+x^2+x^3+x^4+x^5)^3 \;=$$

$$x^{15}+3x^{14}+6x^{13}+10x^{12}+15x^{11}+18x^{10}+19x^9+18x^8+15x^7+10x^6+6x^5+3x^4+x^3$$

The answer is equal to 15

B:

if the total number of donuts the man buys is n , and the number of dount kind is equal to **r**,

by Balls and Walls Counting Idea the answer will be $\binom{n-1}{r-1}$