

Homework 2. Logic

In the questions 1, 2 and 3 you have to choose one or several correct answers from the list, in the questions 4 and 5 you have to give a solution.

Question 1. Which of the following sentences are logical statements?

- ☐ A Find the square root of 4.
- ☒ B The number π is irrational.
- ☐ C Is the number 91 prime?
- ☐ D The lecture about combinatorics was fascinating!
- ☒ E The number 28 is a multiple of 7.
- ☒ F $e = 2,71828$.

Question 2. Which of the following statements is false?

- ☐ A If the angle of a regular triangle is acute then the number 97 is prime.
- ☒ B If the number 97 is prime then the elephants can play a piano.
- ☒ C If the elephants can play a piano then $\pi < 3$.
- ☐ D If $\pi < 3$ then the angle of a regular triangle is acute.

Question 3. Let x and y be integers. Which of the following statements are true?

- ☐ A $\forall x \exists y (x^2 = y^4)$;
- ☐ B $\forall x \exists y (x^4 = y^2)$;
- ☒ C $\forall x \exists y (xy \text{ is a perfect square})$;
- ☐ D $\forall x \exists y (xy = 1)$;
- ☒ E $\forall x \exists y (xy = 0)$.

Question 4. For a real number x , consider the open sentences $P(x) : x \geq 3$ and $Q(x) : x^2 > 9$. Give an example of x such that

- ☐ A the statement $P(x) \Rightarrow Q(x)$ is true; x = 4 , if 4 >= 3, then 16 > 9
- ☐ B the statement $P(x) \Rightarrow Q(x)$ is false. x = 3, if 3 >= 3, then 9 > 9

Question 5. Construct the truth tables and check if it is always truth that:

- ☐ A $(\neg A \wedge B) \vee \neg B$;
- ☐ B $(A \vee B) \wedge \neg A$;

Question 5 Answer.

A: $(\neg A \wedge B) \vee \neg B$

A	B	$\neg A$	$\neg B$	$\neg A \wedge B$	$(\neg A \wedge B) \vee \neg B$
T	T	F	F	F	F
T	F	F	T	F	T
F	T	T	F	T	T
F	F	T	T	F	T

B: $(A \vee B) \wedge \neg A$;

A	B	$\neg A$	$A \vee B$	$(A \vee B) \wedge \neg A$
T	T	F	T	F
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F