Mathematics

Homework 5. Number Theory

In the questions 1, 2, 3 and 4 you have to choose one correct answer from the list, in the questions 5, 6 and 7 you have to give an answer.

Question 1. Find the residue of the number $4444^{7777} + 7777^{4444}$ modulo 5.

A 0

B 1

C 2

D 3

E 4

Question 2. Find the last digit of the number $1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + ... + 2021 \cdot 2023$.

A 1

B 3

C 5

D 7

E 9

Question 3. Find the number of positive integers which are smaller than 96000 and are relatively prime to it.

A 9600;

B 12800;

C 19200;

D 25600;

E 30000.

Question 4. Find the residue of the number 19^{2023} modulo 12.

A 1

B 3

C 5

D 7

E 9

$\boxed{\text{F}}$ 11

Question 5. Find the smallest natural x such that $29x \equiv 35 \pmod{53}$.

Question 6. Find the smallest natural number which gives remainder 2 modulo 3, remainder 1 modulo 5, remainder 3 modulo 7 and remainder 7 modulo 11.

Question 7. Find the smallest natural n such that the number $n^2 + 3n + 2$ gives remainder 30 modulo 45.

Question 5 – Answer.

29x ≡ 35 (mod 53).
29x = 53 k + 35

29y = 53m + 1
*by applying the **Euclid's algorithm**:*

$53=29\times1+24$	$24=53\times1-29\times1$	$1=29\times5-(53\times1-29\times1)\times6=29\times\mathbf{11}-53\times\mathbf{6}$
$29=24\times1+5$	$5=29\times1-24\times1$	$1=(29\times1-24\times1)\times5-24\times1=29\times5-24\times6$
$24=5\times4+4$	$4=24\times1-5\times4$	$1=5\times1-(24\times1-5\times4)\times1=5\times5-24\times1$
$5=4\times1+1$	$1=5\times1-4\times1$	$1=5\times1-4\times1$

$$29\times\mathbf{11}-53\times\mathbf{6}=1$$

Thus **y = 11, m = 6**, *and by multiply these numbers by* **35**, *we get*
y = 385 , m = 210
by taking y mod 53
y = 14

***Thus** the smallest natural* **x is 14.**

Question 6– Answer.

$X \equiv 2 (mod 3)$

$X \equiv 1 (mod 5)$

$X \equiv 3 (mod 7)$

$X \equiv 7 (mod 11)$

By using ***CRT:***

$X = (a M_1 M_1^{-1} + b M_2 M_2^{-1} + c M_3 M_3^{-1} + d M_4 M_4^{-1}) mod M$

$M = m_1 \times m_2 \times m_3 \times m_4 = 1155$

$a=2$	$m_1=3$	$M_1 = M/m_1 = 385$	$M_1 M_1^{-1} = 1 mod m_1$	$385 M_1^{-1} = 1 mod 3$	$M_1^{-1} = 1$
$b=1$	$m_2=5$	$M_2 = M/m_2 = 231$	$M_2 M_2^{-1} = 1 mod m_2$	$231 M_2^{-1} = 1 mod 5$	$M_2^{-1} = 1$
$c=3$	$m_3=7$	$M_3 = M/m_3 = 165$	$M_3 M_3^{-1} = 1 mod m_3$	$165 M_3^{-1} = 1 mod 7$	$M_3^{-1} = 2$
$d=7$	$m_4=11$	$M_4 = M/m_4 = 105$	$M_4 M_4^{-1} = 1 mod m_4$	$105 M_4^{-1} = 1 mod 11$	$M_4^{-1} = 2$

$X = (a M_1 M_1^{-1} + b M_2 M_2^{-1} + c M_3 M_3^{-1} + d M_4 M_4^{-1}) mod M$

$X = (2 \times 385 \times 1 + 1 \times 231 \times 1 + 3 \times 165 \times 2 + 7 \times 105 \times 2) mod 1155$

$X = (770 + 231 + 990 + 1470) mod 1155$

$X = 3461 mod 1155$

$X = \mathbf{1151}$

Question 7– Answer.

$$n^2+3n+2\equiv 30\,(mod\ 45)$$

Since the congruence is equivalent to: $n^2+3n+2-30\equiv 0\,(mod\ 45)$
then $n^2+3n+2=30$ *, and by solving this equation, n will be the smallest natural number such that the number* n^2+3n+2 *gives remainder 30 modulo 45.*

$$n^2+3n+2=30$$

$$(n+1)(n+2)=30$$

$$n=4$$