ADA University, School of IT and Engineering (SITE)

Syllabus of the course MATH 3501 Linear Algebra (*6 ECTS,* SPRING 2024)

Syllabus follows the format of Curriculum Guidelines for Undergraduate Degree Programs in Computer Science, ACM and IEEE.

Course Information

Instructors: Dr. Fuad Hajiyev**,** Dr. Elchin Hasanalizada

Format of the course: **Hybrid, on-campus and on-line** (Blackboard Platform) modes

Teaching Hours:

Joint Lectures: (Dr. Fuad Hajiyev)

Mon, 10:00 – 11:15, **Hybrid, on-campus and on-line** modes. Venue: ____ (for on-campus mode),
Guest Link (not for the registered students): ______________________________

Wed: 10:00 – 11:15, **Hybrid, on-campus and on-line** modes. Venue: _____ (for on-campus mode),
Guest Link (not for the registered students): _______________________________

Problem solving sessions* (PSS)*:

Dr. Fuad Hajiyev

CRN 20397 ________________________________, Class will be available for online students as well

Guest Link (not for the registered students) ____

CRN 20400 _________________________________, Class will be available for online students as well

Guest Link (not for the registered students)

Dr. Elchin Hasanalizada

CRN 20398 _________________________________, Class will be available for online students as well

Guest Link (not for the registered students) _______________________________________

CRN 20399 _________________________________, Class will be available for online students as well

Guest Link (not for the registered students) _____________________________________

All course materials will be available on **Blackboard**:

1. All students, regardless of which CRN they are registered with, have equal access to all **PSS** and can attend classes with the same discussion topic.
2. Textbooks are placed in the folder Course Content/Textbooks
3. Lecture Notes will be available in the folder Course Content/Lecture Notes
4. Homeworks will be placed in the folder Course Content/Homeworks
5. Midterms and Final tasks will be placed in the folder Course Content/Midterms and Finals
6. Syllabus is available in the folder Course Content/Syllabus

Introduction.

Linear Algebra (LA) is a mandatory component of all IT-oriented Bachelor programs of the School of IT and Engineering at ADA University.

LA is an essential part of the mathematical background required of mathematicians, economists, engineers, computer scientists, physicists, statisticians and other scientists. This requirement reflects the importance and wide applications of the subject matter.

This course is usually delivered at the 1st year of the undergraduate education as a basic mathematical course that includes simple and at the same time very important tools for the next Math and Computer Science, Computer Engineering courses.

Knowledge Area (KA) and Body of Knowledge

Body of Knowledge coverage

KA	Knowledge Unit	Topic Covered	Acad. Hours
LA	Vectors in R^n and operations	Introduction. Vectors in R^n. Vector addition and scalar multiplication and their basic properties. Dot product. Norm and distance in R^n. Located Vectors, Hyperplanes, Lines and Curves in R^n. Vectors in R^3 (Spatial Vectors), **ijk** Notation. Cross Product. Linear combination of vectors. Dependent and independent vectors	150 min
LA	Algebra of Matrices	Matrices. Matrix Addition and Scalar Multiplication. Matrix Multiplication. Square Matrices. Diagonal and Trace. Identity Matrix, Scalar Matrices. Powers of Matrices, Polynomials in Matrices. Invertible (Nonsingular) Matrices. Special Types of Square Matrices: Diagonal and Triangular Matrices. Special Real Square Matrices: Symmetric, Orthogonal matrices. Linear Combinations of Rows (row-vectors). Elementary Row Operations. Echelon Matrices, Row Canonical Form, Row Equivalence. Rank of a Matrix. Matrix equivalence as a particular case of equivalence relation. Uniqueness of row canonical form of a matrix. Elementary matrices. Applications of Elementary Matrices. Application to Finding the Inverse of an n x n Matrix.	225 min

LA	Systems of Linear Equations (SLE)	Introduction. Equivalent Systems and Elementary Row Operations. Linear Combinations of Rows. Systems in Triangular and Echelon Forms. Gaussian Elimination Method. Examples SLE and rank of coefficient and augmented matrix of SLE: Existence and Uniqueness Theorem. Matrix Equation of a SLE. Matrix Equation of a Square SLE. Homogeneous SLE (HSLE). Nonhomogeneous and Associated Homogeneous Systems.	225 min
LA	Determinants	Introduction. Permutations. Determinants of arbitrary order (definition). Properties of Determinants. Major Properties of Determinants. Minors and cofactors. Laplace Expansion Formula. Evaluation of Determinants: Algorithm (Reduction of the order of a determinant). Applications to Linear Equations, Cramer’s Rule. Block Matrices and Determinants. Determinants and Volume. Multilinearity and Determinants.	150 min
LA	Vector spaces	Introduction. Vector spaces. Examples of vector spaces. Solution Space of Homogeneous Systems. Linear Combinations and Spanning Sets. Subspaces. Intersection of subspaces. Linear spans, Row Space of a Matrix. Linear Dependence and Linear Combinations. Linear Dependence and Echelon Matrices. Basis and Dimension Example of Bases. Dimension and Subspaces. Application to Matrices, Rank of a Matrix. Basis-Finding Problems. Theorems on Bases. Application to HSLE. Sums and Direct Sums.	225 min
LA	Linear mappings	Definition and examples. Matrices as Linear Mappings. Kernel and image. Kernel and Image of Matrix Mappings. The Main Dimension Formula. Singular and nonsingular mappings, Isomorphisms. Linear mappings and systems of linear equations. Operations on a set of linear mappings. Algebra of linear operators. Determinant of a linear operator.	225 min
LA	Linear Mappings and Matrices	Matrix Representation of a Linear Operator. Matrix Mappings and Their Matrix Representation. Algorithm for Finding Matrix Representations. Properties of Matrix Representations. Change of Basis. Applications. Examples. Similarity. Functions and Similar Matrices.	225 min
LA	Diagonalization: Eigenvalues and Eigenvectors	Introduction. Matrix Point of View. Linear Operator Point of View. Equivalence of the Two Points of View. Polynomials of Matrices. Matrices and Linear Operators. Characteristic Polynomial, Cayley–Hamilton Theorem. Characteristic Polynomial of a Linear Operator. Diagonalization, Eigenvalues and Eigenvectors. Properties of Eigenvalues and Eigenvectors. Diagonalization of Linear Operators. Computing Eigenvalues and Eigenvectors, Diagonalizing Matrices. Minimal Polynomial. Minimal Polynomial of a Linear Operator.	300 min
LA	Canonical Forms	Introduction. Triangular Form. Invariance. Invariant Direct-Sum Decompositions.	75 min
LA	Linear Functionals and the Dual Space	Introduction. Linear Functionals and the Dual Space. Second Dual Space. Annihilators. Transpose of a Linear Mapping.	150 min
LA	Linear Optimization	Introduction. Transportation Problem of Linear Programming. Simplex Method.	300 min

Where does the course fit in your curriculum?

This course covers all the required Linear Algebra for Computer and Information Technologies majors, in particular, Computer Science, Information Technology, Computer Engineering etc. This knowledge area is a prerequisite for almost any mathematics in Science and Engineering.

Course Objectives.

Linear Algebra is a course designed to meet the requirements of Computer and Information Science degree programs. Furthermore, this course is designed to meet the following program objectives. Upon successful completion of this course, students will be able to:

- Demonstrate critical thinking, analytical reasoning, and problem solving skills
- Apply appropriate mathematical concepts and operations to interpret data and to solve problems
- Identify a problem and analyze it in terms of its significant parts and the information needed to solve it.
- Formulate and evaluate possible solutions to problems, and select and defend the chosen solutions.

Learning outcomes:

1. Know operations on vectors in R^n, understand the geometrical interpretations of them. [Usage].
2. Explain linear dependence of vectors [Usage].
3. Perform the operations over Matrices. Know how to find inverse matrix [Usage].
4. Evaluate the rank of a matrix [Usage].
5. Understand equivalence of matrices in terms of row operations and understand uniqueness of row canonical form for a matrix [Usage]
6. Apply elimination method to solve systems of linear equations. [Usage]
7. Calculate determinants of a matrix [Usage]
8. Explain relationship between: a) existence of inverse matrix and determinant of the given matrix, b) the rank of a square matrix and his rank.
9. State and explain definition of vector spaces and subspaces. Identify the dimension of a space and subspace [Usage].
10. Explain a linear mapping concept between two vector spaces. Know what the kernel and image of a linear mapping is. Know the dimension formula generated by a linear mapping [Usage].
11. Know how to find a matrix representation of a linear mapping.
12. Explain how the matrix of a linear mapping is changed if basis of the domain will change.
13. Understand the concept of similarity of matrices.
14. Explain the Eigenvalue and Eigenvector and the diagonalization of a matrix.
15. Know canonical forms of linear operators and explain how a linear operator can be reduced to canonical forms.
16. Explain the concept of a Dual space as a space of linear functionals on a given vector space.
17. Understand the concept of the second Dual Space.
18. Know how to solve linear optimization problem by simplex method
19. Know the algorithm of Transportation Problem and how to apply this algorithm.

What is the format of the course?

Lectures: There are TWO Joint Lectures a week, each 75 minutes.

Problem Solving Session: 75 min in a week.

Course materials:

Textbooks and web resources:

Main:

[1]. Seymour Lipschutz, Marc Lipson: Linear Algebra. (Shaum's Outline), 4th Ed, 2009, McGraw-Hill.

[2]. David Lay, Linear Algebra and its Applications. 4th Ed, 2012, Addison Wessley, ISBN-13: 978-0-321-38517-8; ISBN-10: 0-321-38517-9;

[3]. https://people.richland.edu/james/ictcm/2006/simplex.html

[4]. https://people.richland.edu/james/ictcm/2006/table.html

[5]. https://study.com/academy/lesson/the-transportation-problem-features-types-solutions.html

[6]. https://ocw.ehu.eus/pluginfile.php/8170/mod_resource/content/1/5_Transportation_Slides.pdf

Supplementary:

[7]. W.Keith Nicholson, Linear Algebra with Applications, 6th edition, McGraw-Hill, 2009; ISBN-13: 978-0-07-0985-10-0; ISBN-10: 0-07-0985-10-3

[8]. http://www.phpsimplex.com/en/simplex_method_example.htm

[9]. https://study.com/academy/lesson/the-transportation-problem-features-types-solutions.html

Homework Assignments: There will be five homework assignments. The details of the assignments will be announced on the Blackboard.

Exercises: Students are strongly advised to study the textbook, especially the solved examples and solve as many exercises. Exercises, mainly selected from the textbook, will be announced on regular basis, however solutions of the exercises should not be submitted. Students are encouraged expected to work with other students and attend the office hours for further help from the instructor.

Important: As you are the central figure in the teaching-learning process your effort will be decisive in achieving the goals of this course. A lecturer is one who only helps you to achieve these goals. Reading the textbooks and lecture notes are of utmost importance. Without this type of work it is very unlikely that you will succeed in this course.

Course Assessment.

The assessment of objectives is achieved through homework assignments, and examinations

Grading Policy

Grade components are:

- Homework Assignments–15 % (5 Assignments by 3 point each)
- Midterm Exam 1 – 20%
- Midterm Exam 2 – 20%
- Midterm Exam 3 – 20%
- Final exam –25%

Exam Rules

- A missed examination with no excuse counts as a zero.
- Instructor may upload sample exams and worksheets from previous years

Academic Policy

ADA University standard academic policy is used.

- Cheating, duplication, falsification of data, plagiarism, and crib are not permitted under any circumstances!
- Attendance is mandatory.
- Mobile phones must always be switched off in class.
- Students should always show tolerance, consideration and mutual support towards other students.

Disclaimer

Instructor could modify schedule of the classes and content of the syllabus as necessary.

Tentative Schedule and Body of Knowledge coverage

Wks	Lecture #	Topic and Content	Readings & Home tasks
1	1	**Vectors in R^n and operations.** Introduction. Vectors in R^n. Vector addition and scalar multiplication and their basic properties. Linear combination of vectors in $\mathbf{R^n}$. Dependent and independent vectors in $\mathbf{R^2}$ (with extension to the case $\mathbf{R^n}$). Dot product. Norm and distance in R^n.	[1]. Ch. 1, pp 1-6. Problem Set: pp. 13-26
1	2	**Vectors in R^n and operations. (continuation).** **ijk** Notation. Cross Product of vectors in $\mathbf{R^3}$. Hyperplanes, Lines and Curves in $\mathbf{R^n}$.	[1]. Ch. 1, pp 7-13. Problem Set: pp. 13-26
2	3	**Algebra of Matrices.**	[1]. Ch. 2, pp 27-41.

		Matrices. Matrix Addition and Scalar Multiplication. Matrix Multiplication. Square Matrices. Diagonal and Trace. Identity Matrix, Scalar Matrices. Powers of Matrices, Polynomials in Matrices. Invertible (Nonsingular) Matrices. Special Types of Square Matrices: Diagonal and Triangular Matrices. Special Real Square Matrices: Symmetric, Orthogonal, Normal Matrices.	Problem Set: pp. 41-56
2	4	**Algebra of Matrices (continuation).** Linear Combinations of Rows (row-vectors). Elementary Row Operations. Echelon Matrices, Row Canonical Form, Row Equivalence. Rank of a Matrix. Matrix equivalence as a particular case of equivalence relation. Uniqueness of row canonical form of a matrix.	[1]. Ch.3, pp. 61, 70-73, *Problem Set:* pp. 89-111
3	5	**Algebra of Matrices (continuation).** Elementary matrices. Applications of Elementary Matrices. Application to Finding the Inverse	[1]. Ch.3, pp. 84-86 *Problem Set:* pp. 89-111
3	6	**Systems of Linear Equations (SLE).** Basic Definitions, Solutions. Equivalent Systems and Elementary Row Operations. Linear Combinations of Rows. Systems in Triangular and Echelon Forms. Gaussian Elimination Method. Examples	[1]. Ch. 3, pp 57-60, 62-69, 73-75. Problem Set: pp. 89-111
4	7	**Systems of Linear Equations (continuation).** SLE and rank of coefficient and augmented matrix of SLE: Existence and Uniqueness Theorem. Matrix Equation of a SLE. Matrix Equation of a Square SLE. Homogeneous SLE. Nonhomogeneous and Associated Homogeneous Systems.	[1]. Ch. 3, pp 75-83. Problem Set: pp. 89-111
4	8	**Systems of Linear Equations (continuation).** Overview.	[1]. Ch. 3, pp 57-88. Problem Set: pp. 89-111
5	9	**Determinants.** Introduction. Permutations. Determinants of arbitrary order (definition). Properties of Determinants. Major Properties of Determinants. Minors and cofactors. Laplace Expansion Formula. Evaluation of Determinants: Algorithm (Reduction of the order of a determinant).	[1]. Ch. 8, pp 264-271. Problem Set: pp. 276-291
5	10	**Determinants (Continuation).** Applications to Linear Equations, Cramer’s Rule. Block Matrices and Determinants. Determinants and Volume. Multilinearity and Determinants.	[1]. Ch. 8, pp 272-276. Problem Set: pp. 276-291
6	11	**Vector spaces.** Introduction. Vector spaces. Examples of vector spaces. Solution Space of Homogeneous Systems. Linear Combinations and Spanning Sets.	[1]. Ch. 4, pp 112-117. Problem Set: pp. 133-163

6	12	**Vector spaces (Continuation).** Subspaces. Intersection of subspaces. Linear spans, Row Space of a Matrix. Linear Dependence and Linear Combinations. Linear Dependence and Echelon Matrices.	[1]. Ch. 4, pp 117-124. Problem Set: pp. 133-163
7	13	**Vector spaces (Continuation).** Basis and Dimension Example of Bases. Dimension and Subspaces. Application to Matrices, Rank of a Matrix. Basis-Finding Problems. Theorems on Bases. Application to Homogeneous Systems of Linear Equations. Sums and Direct Sums.	[1]. Ch. 4, pp 124-133. Problem Set: pp. 133-163
7	14	**Linear mappings.** Definition and examples. Matrices as Linear Mappings. Vector Spaces Isomorphism.	[1]. Ch. 5, pp 164-169. Problem Set: pp. 176-194
8	15	**Linear mappings. (Continuation).** Kernel and image of a Linear Mapping. Kernel and Image of Matrix Mappings. Dimension Formula for a Linear Mapping. Application to Systems of Linear Equations.	[1]. Ch. 5, pp 169-172. Problem Set: pp. 176-194
8	16	**Linear mappings (Continuation).** Singular and nonsingular mappings, Isomorphisms. Operations with linear maps. Algebra of linear operators. Invertible operator. Determinant of a linear operator.	[1]. Ch. 5, pp 173-176. Ch.8, p. 275 Problem Set: pp. 176-194
9	17	**Linear Mappings and Matrices.** Matrix Representation of a Linear Operator. Matrix Mappings and Their Matrix Representation. Algorithm to find a Matrix Representation. Properties of Matrix Representations.	[1]. Ch. 6, pp 195-199. Problem Set: pp. 205-225
9	18	**Linear Mappings and Matrices (Continuation).** Change of Basis. Applications of Change-of-Basis Matrix. Examples.	[1]. Ch. 6, pp 199-203. Problem Set: pp. 205-225
10	19	**Linear Mappings and Matrices (Continuation).** Similarity. Functions and Similar Matrices. Matrices and General Linear Mappings.	[1]. Ch. 6, pp 203-205. Problem Set: pp. 205-225
10	20	**Diagonalization: Eigenvalues and Eigenvectors.** Introduction. Matrix Point of View. Linear Operator Point of View. Equivalence of the Two Points of View. Polynomials of Matrices. Matrices and Linear Operators. Characteristic Polynomial, Cayley–Hamilton Theorem. Characteristic Polynomial of a Linear Operator.	[1]. Ch. 9, pp 292-295. Problem Set: pp. 306-324
11	21	**Diagonalization: Eigenvalues and Eigenvectors (Continuation).**	[1]. Ch. 9, pp 295-299. Problem Set: pp. 306-324

		Diagonalization, Eigenvalues and Eigenvectors. Properties of Eigenvalues and Eigenvectors. Diagonalization of Linear Operators.	
11	22	**Diagonalization: Eigenvalues and Eigenvectors (Continuation).** Computing Eigenvalues and Eigenvectors, Diagonalizing Matrices.	[1]. Ch. 9, pp 299-303. Problem Set: pp. 306-324
12	23	**Diagonalization: Eigenvalues and Eigenvectors (Continuation).** Minimal Polynomial. Minimal Polynomial of a Linear Operator. Characteristic and Minimal Polynomials of Block Matrices. Minimal Polynomial and Block Diagonal Matrices.	[1]. Ch. 9, pp 304-306. Problem Set: pp. 306-324
12	24	**Canonical Forms.** Introduction. Triangular Form. Invariance. Invariant Direct-Sum Decompositions.	[1]. Ch. 10, pp 325-328. Problem Set: pp. 332-348
13	25	**Linear Functionals and the Dual Space.** Introduction. Linear Functionals and the Dual Space. Second Dual Space.	[1]. Ch. 11, pp 349-351. Problem Set: pp. 352-358
13	26	**Linear Functionals and the Dual Space (Continuation).** Annihilators. Transpose of a Linear Mapping.	[1]. Ch. 11, pp 351-352. Problem Set: pp. 352-358
14	27	**Linear Optimization.** Introduction. Linear Programming. Transportation Problem Algorithm	https://ocw.ehu.eus/pluginfile.php/8170/mod_resource/content/1/5_Transportation_Slides.pdf
14	28	**Linear Optimization.** Linear Programming. Applications of Transportation Problem Algorithm	https://ocw.ehu.eus/pluginfile.php/8170/mod_resource/content/1/5_Transportation_Slides.pdf
15	29	**Linear Optimization.** Simplex Method. Geometric interpretation and examples.	https://people.richland.edu/james/ictcm/2006/simplex.html
15	30	**Linear Optimization.** Simplex Method. General Algorithm and applications.	https://people.richland.edu/james/ictcm/2006/table.html

Good luck!