

Grenoble, the 14th of October 2013  
Thesis from the University of Grenoble

# Macroscopic Analysis of Large-scale Systems

Robin Lamarche-Perrin

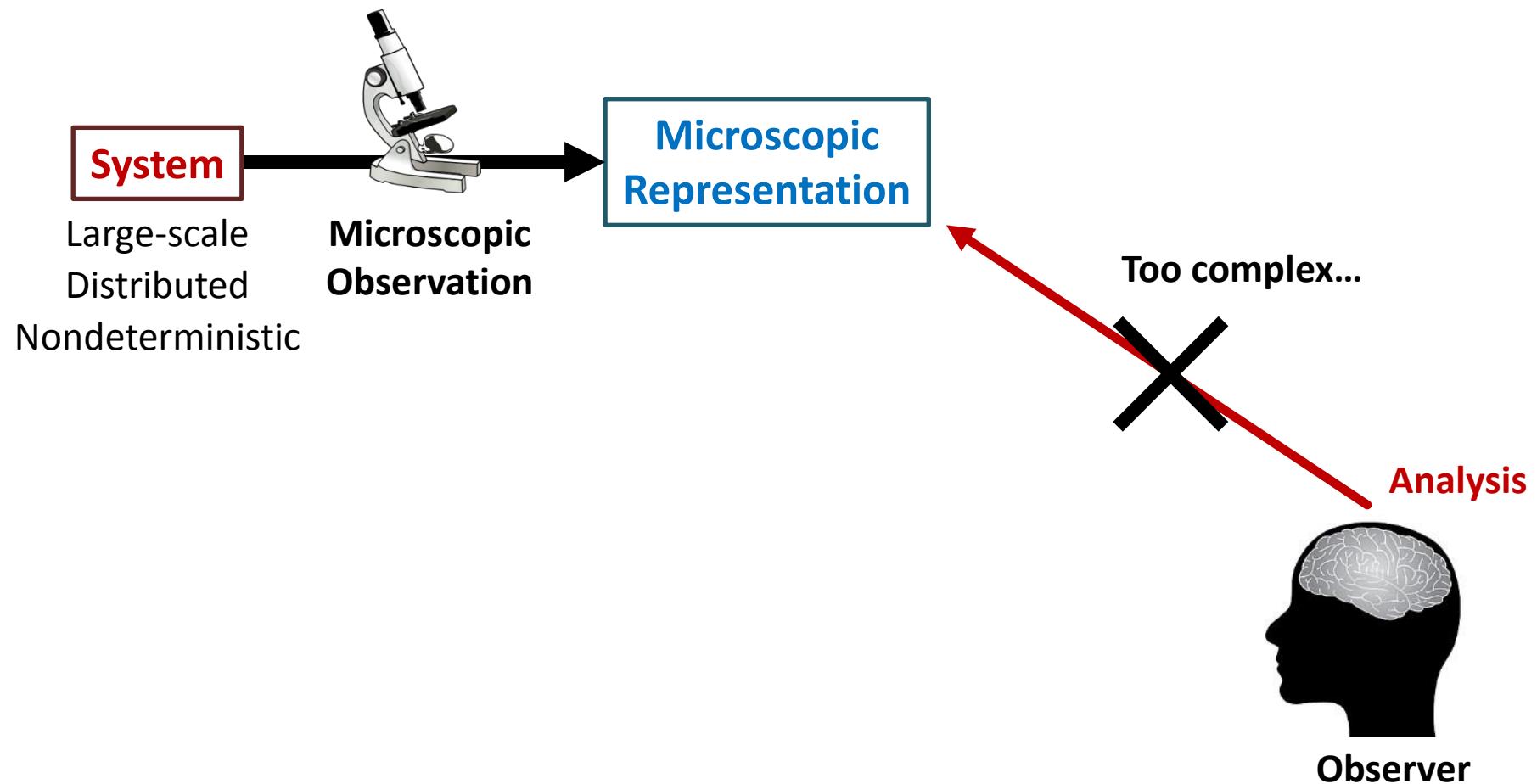
**Reviewers:** Bernard Moulin and Éric Fleury

**Examiners:** Salima Hassas and Brigitte Plateau

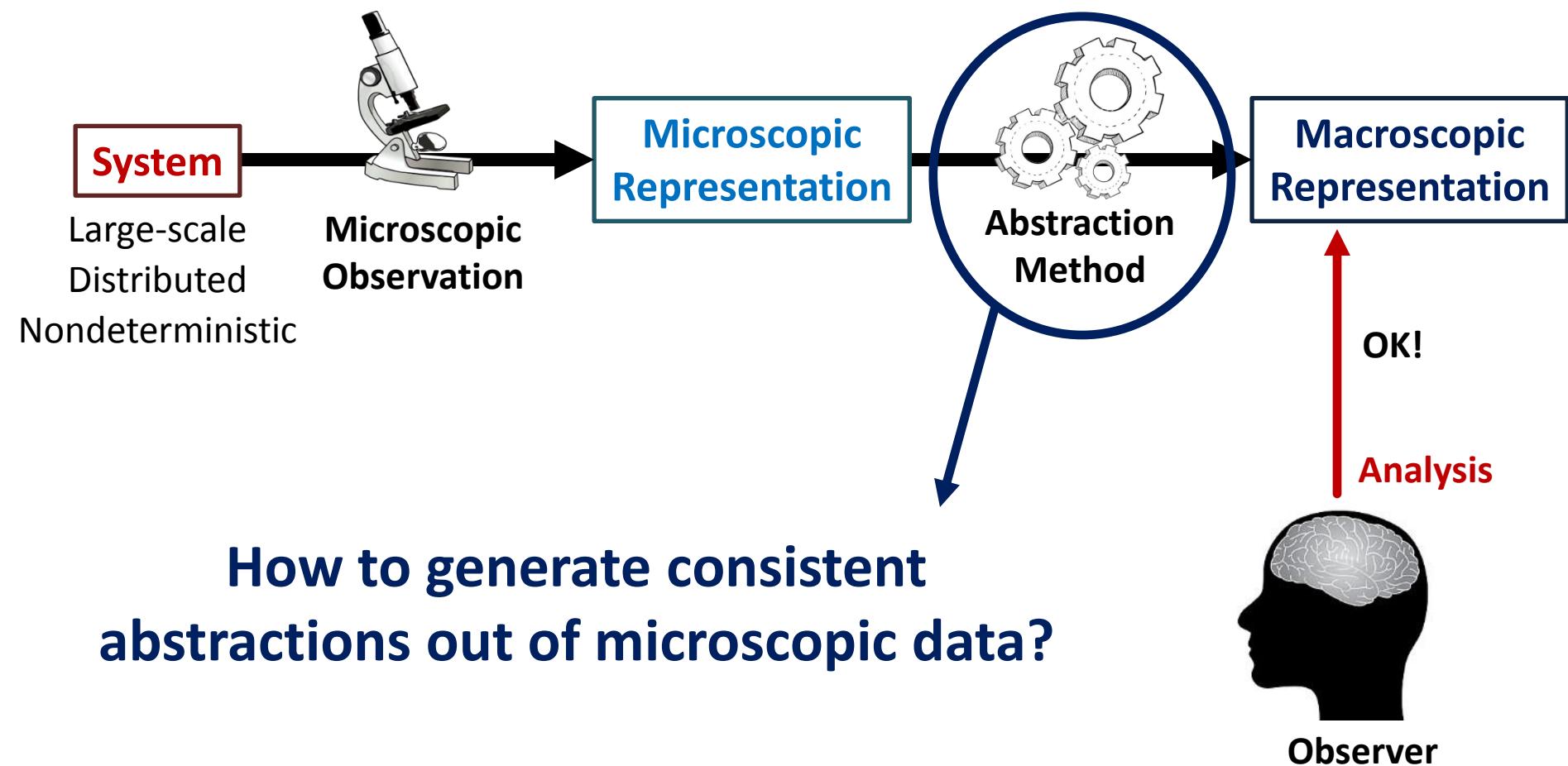
**Supervisors:** Yves Demazeau and Jean-Marc Vincent



# The Analysis of Large-scale Systems



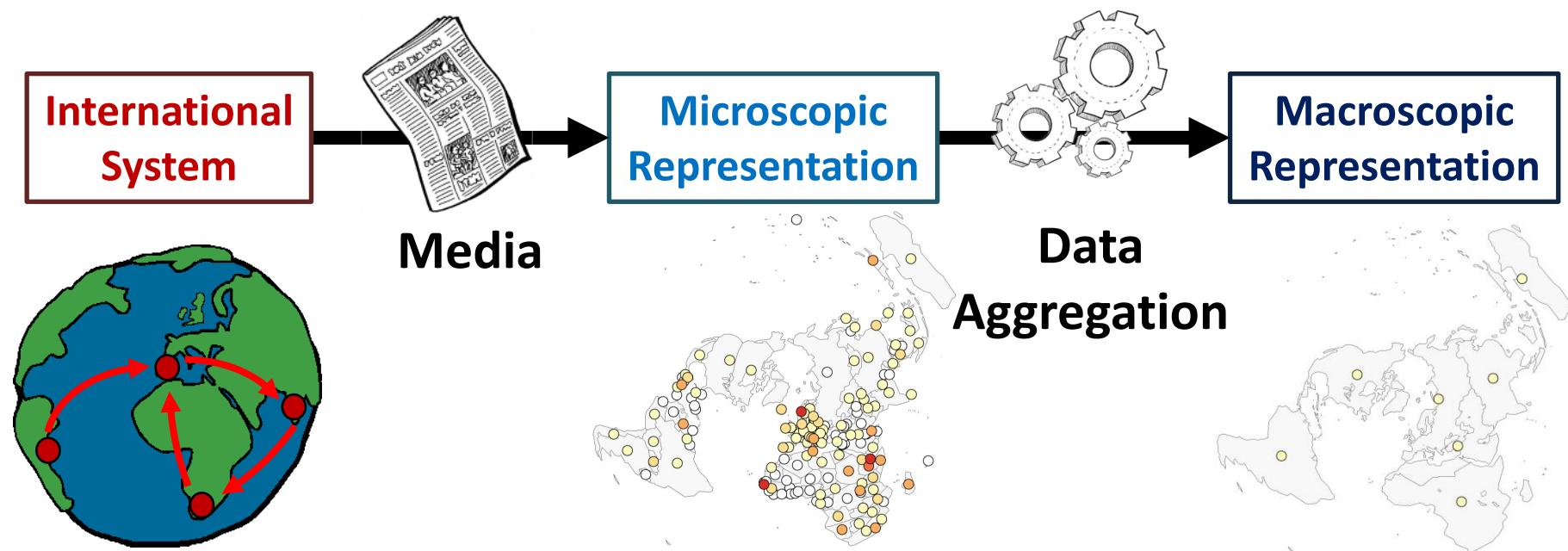
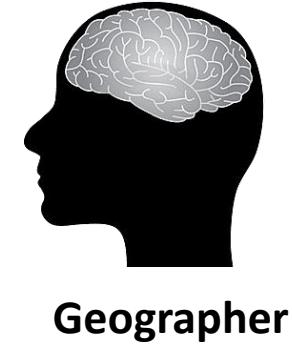
# The Analysis of Large-scale Systems



# Analysis of International Relations

**Hypothesis:** media constitute an adequate instrument to observe the national level

[Grasland *et al.*, 2011]



# Data from Print Media

THE GUARDIAN



“Japan”

THE TIMES OF INDIA



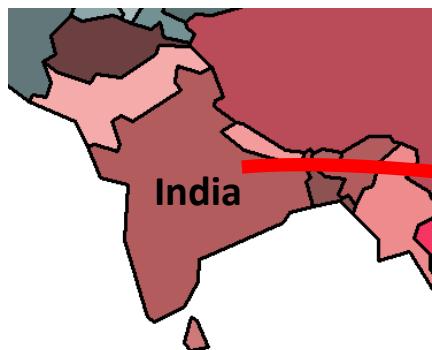
“Madrid”



“French”



“Spain”



The GEOMEDIA Database  
(ANR CORPUS GUI-AAP-04)

150 newspapers

1,944,000 papers

GEOGRAPHIC INFORMATION  
193 countries (UN members)

# Data from Print Media

THE GUARDIAN

Paper 1



“Japan”

30<sup>th</sup> May 2011

THE TIMES OF INDIA

Paper 2



“Madrid”

30<sup>th</sup> May 2011

Paper 3



“French”

19<sup>th</sup> July 2012

“Spain”

The GEOMEDIA Database  
(ANR CORPUS GUI-AAP-04)

150 newspapers

1,944,000 papers

**GEOGRAPHIC INFORMATION**  
193 countries (UN members)

**TEMPORAL INFORMATION**

889 days / 127 weeks  
(from the 3<sup>rd</sup> May 2011 to today)

# Microscopic Representation of the International System

Newspaper LE MONDE

$\pi$

Space

	USA	Libya	Syria	France	Israel	...	Total
2 May	25	12	11	10	4	...	142
9 May	14	6	12	12	5	...	108
16 May	20	11	12	6	9	...	142
23 May	15	9	6	13	5	...	120
30 May	10	16	17	9	4	...	137
6 June	14	16	16	9	4	...	114
13 June	15	14	17	9	6	...	119
20 June	17	13	12	12	7	...	123
27 June	7	6	7	20	2	...	103
4 July	12	13	8	10	6	...	129
11 July	21	10	10	14	3	...	107
18 July	7	3	8	4	5	...	61
25 July	16	7	6	13	4	...	128
1 Aug.	21	1	9	7	4	...	88
...	...	...	...	...	...	...	...
Total	423	308	260	248	153	...	3520

$t$

Time

Number of observed citations

$$v(\pi, t)$$

..

$$v(\pi, .)$$

$$v(., .)$$

Number of expected citations

$$v^*(\pi, t) = \frac{v(\pi, .) v(., t)}{v(., .)}$$

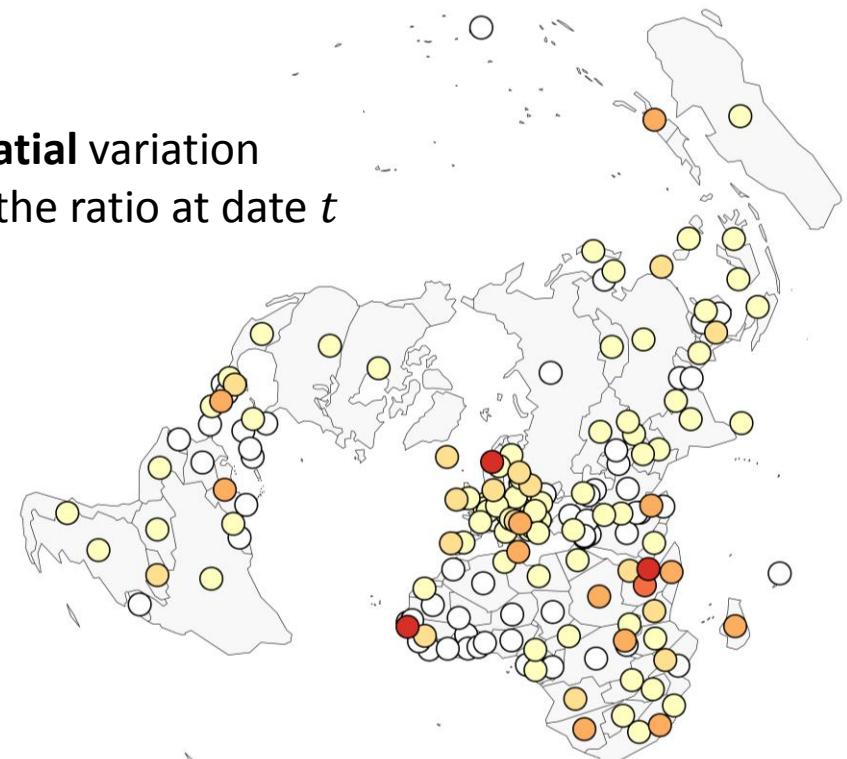
# Geographical Representation

**Space**

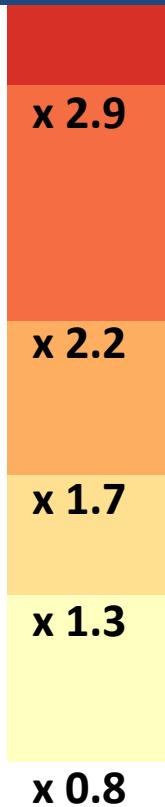
	USA	Libya	Syria	France	Israel	...	Total
2 May	25	12	11	10	4	...	142
9 May	14	6	12	12	5	...	108
16 May	20	11	12	6	9	...	142
23 May	15	9	6	13	5	...	120
30 May	10	16	17	9	4	...	137
6 June	14	16	16	9	4	...	114
13 June	15	14	17	9	6	...	119
20 June	17	13	12	12	7	...	123
27 June	7	6	7	20	2	...	103
4 July	12	13	8	10	6	...	129
11 July	21	10	10	14	3	...	107
18 July	7	3	8	4	5	...	61
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...	...	...	...	...	...	...	...
Total	423	308	260	248	153	...	3520

Observed-to-expected  
ratio of citation number

$$\rho(\pi, t) = \frac{v(\pi, t)}{v^*(\pi, t)} = \frac{v(\pi, t) v(., .)}{v(\pi, .) v(., t)}$$

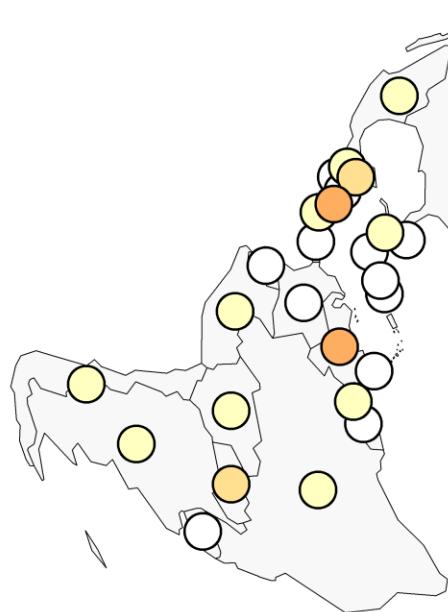


# Detection of Media Events



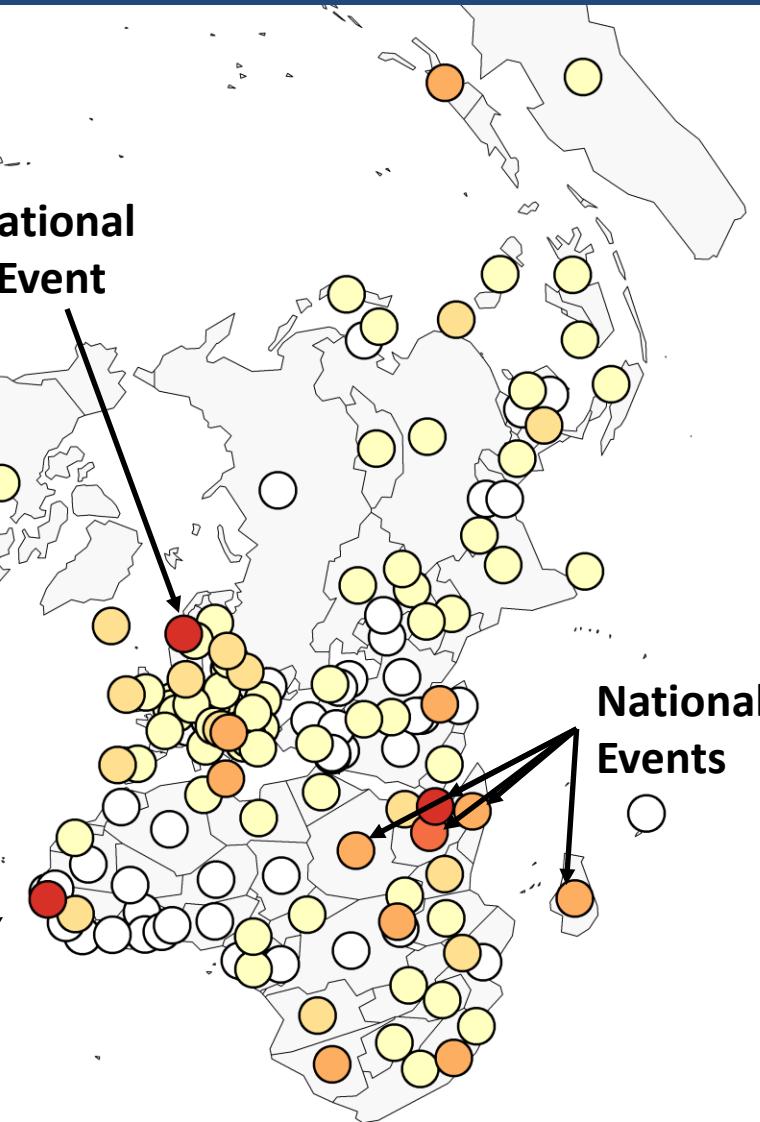
Newspaper LE MONDE in July 2011

MICROSCOPIC  
REPRESENTATION



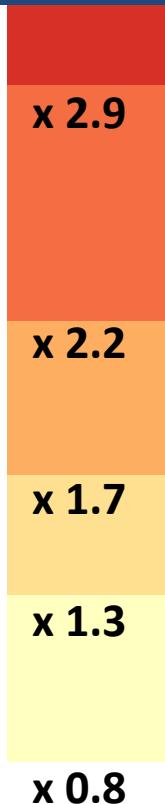
National  
Event

National  
Event



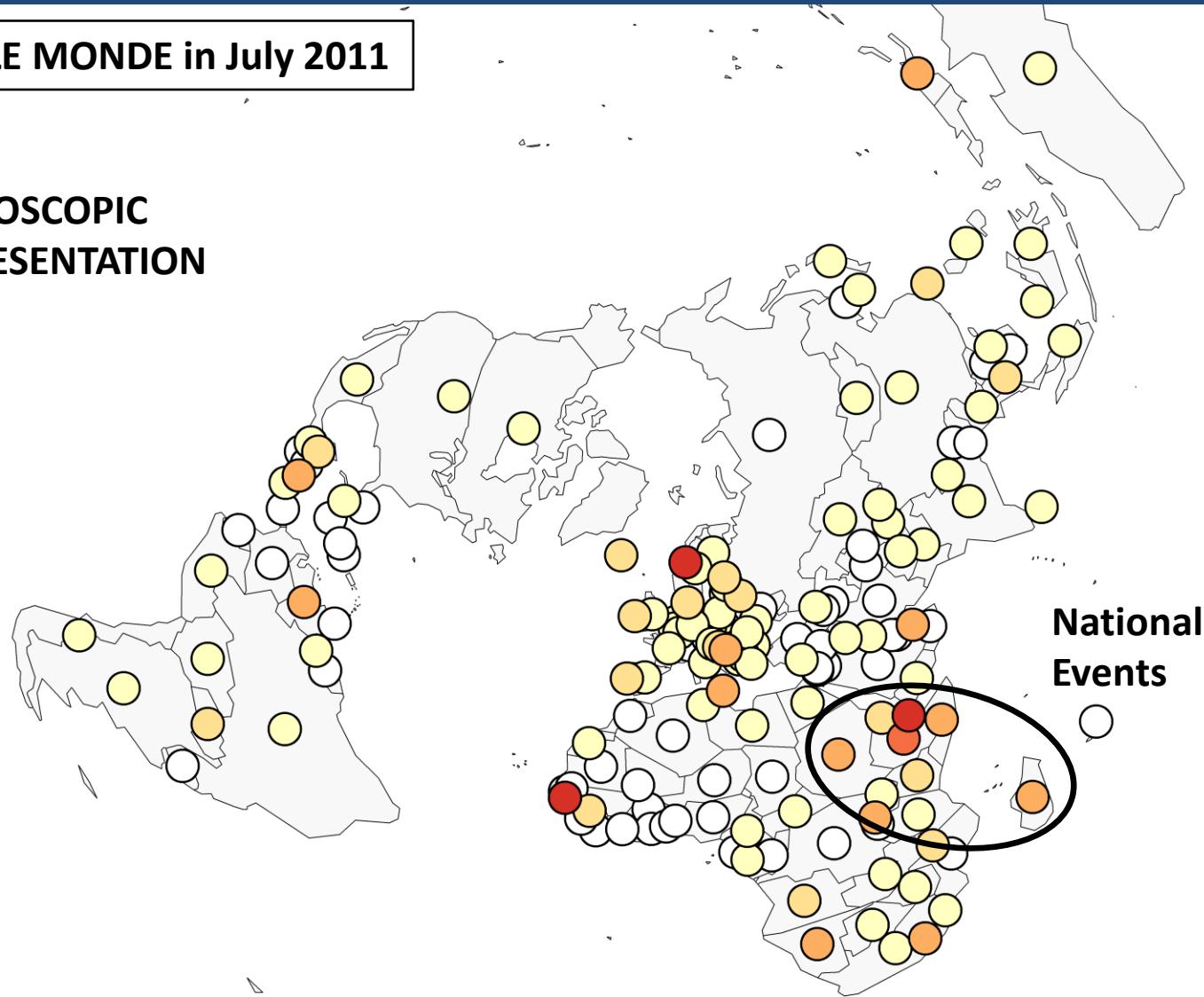
National  
Events

# Detection of Media Events

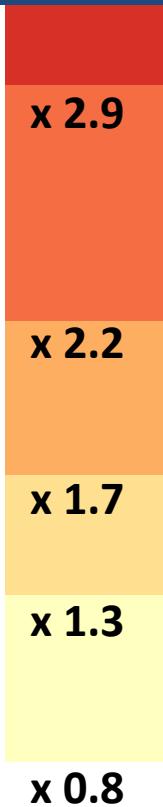


Newspaper LE MONDE in July 2011

MICROSCOPIC  
REPRESENTATION

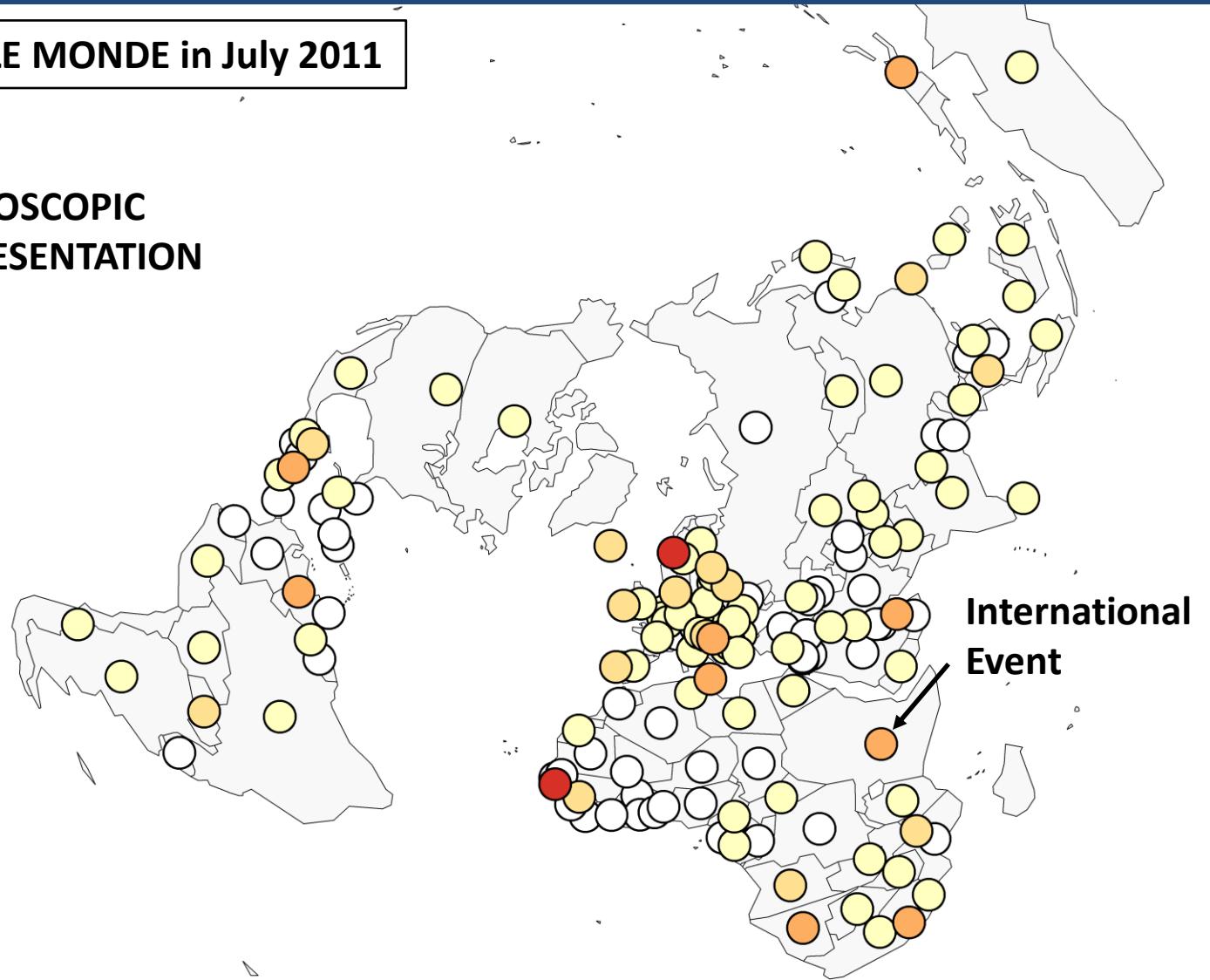


# Detection of Media Events



Newspaper LE MONDE in July 2011

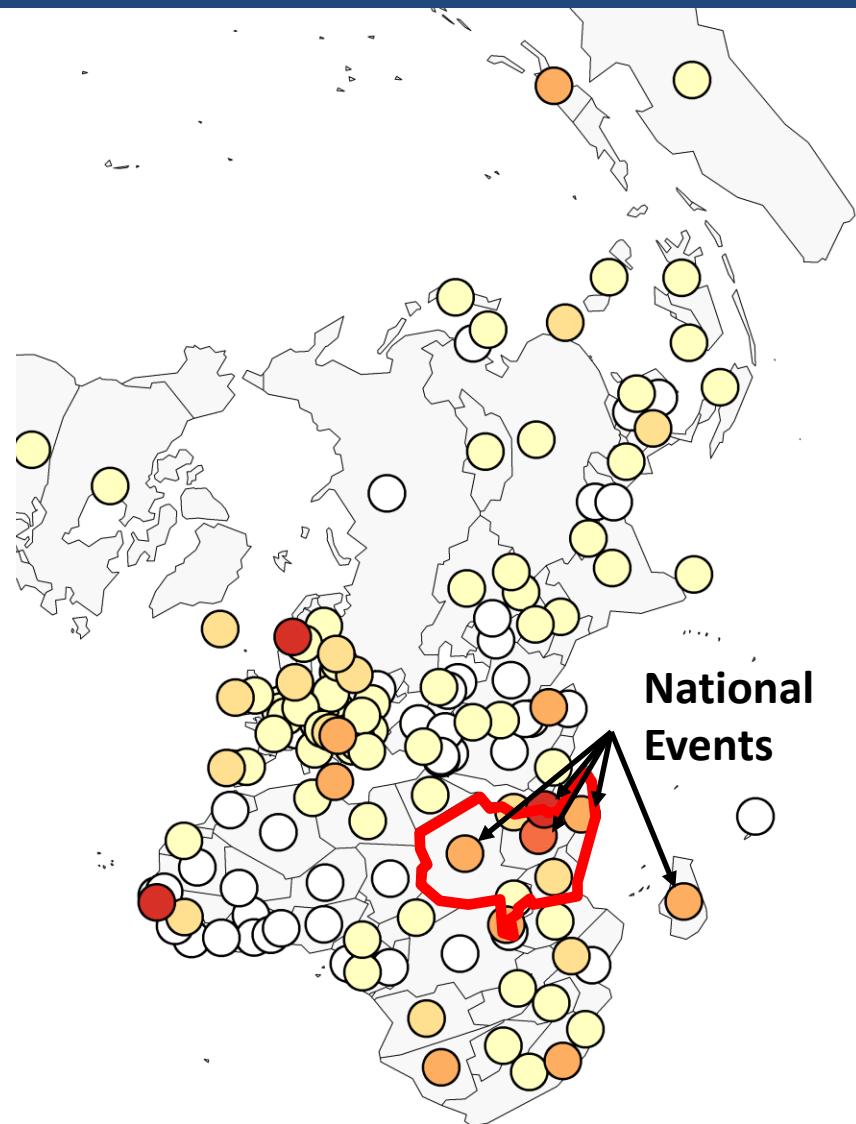
MICROSCOPIC  
REPRESENTATION



# Data Aggregation

**Space**

	USA	Libya	Syria	France	Israel	...	Total
2 May	25	12	11	10	4	...	142
9 May	14	6	12	12	5	...	108
16 May	20	11	12	6	9	...	142
23 May	15	9	6	13	5	...	120
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...	...	...	...	...	...	...	...
Total	423	308	260	248	153	...	3520

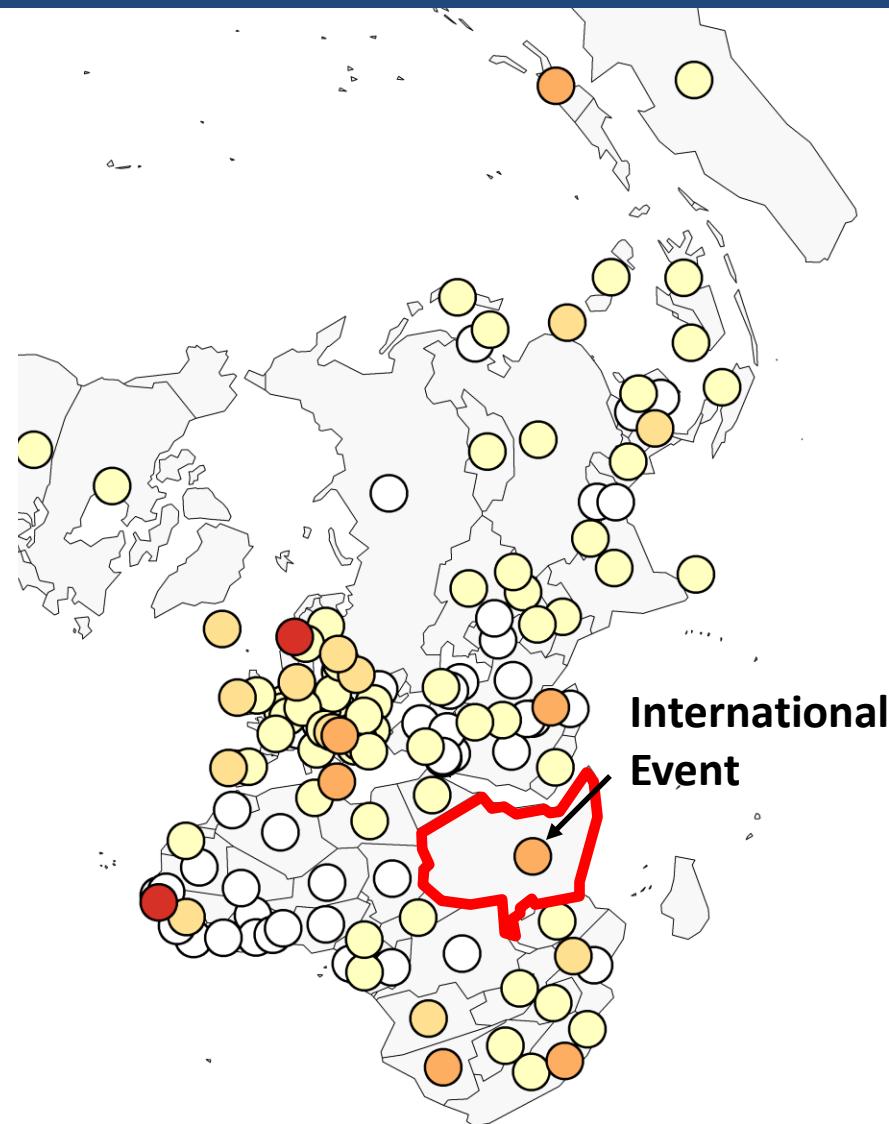


# Data Aggregation

**Space**

	USA	Aggregate	Israel	...	Total
2 May	25	13+11+10	4	...	142
9 May	14	6+12+12	5	...	108
16 May	20	11+12+6	9	...	142
23 May	15	9+6+13	5	...	120
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...	...	...	...	...	...
<b>Total</b>	<b>423</b>	<b>308+260+248</b>	<b>153</b>	...	<b>3520</b>

Time ↓



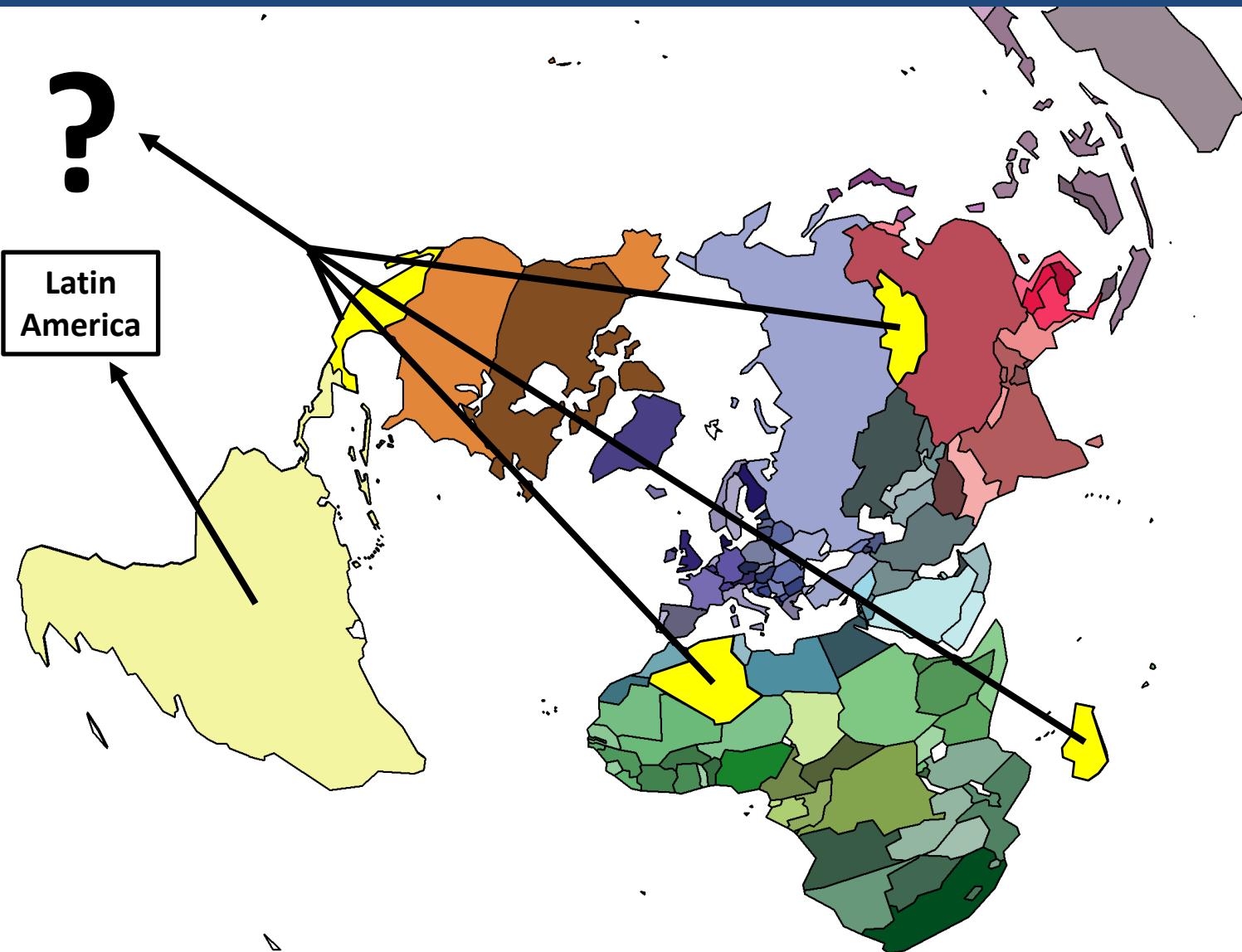
# P1: The Semantics of Geographical Aggregates



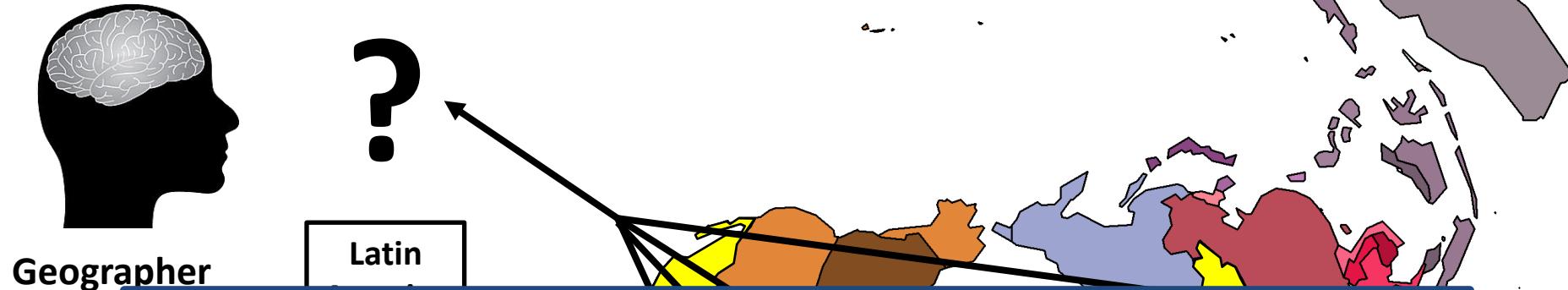
Geographer

?

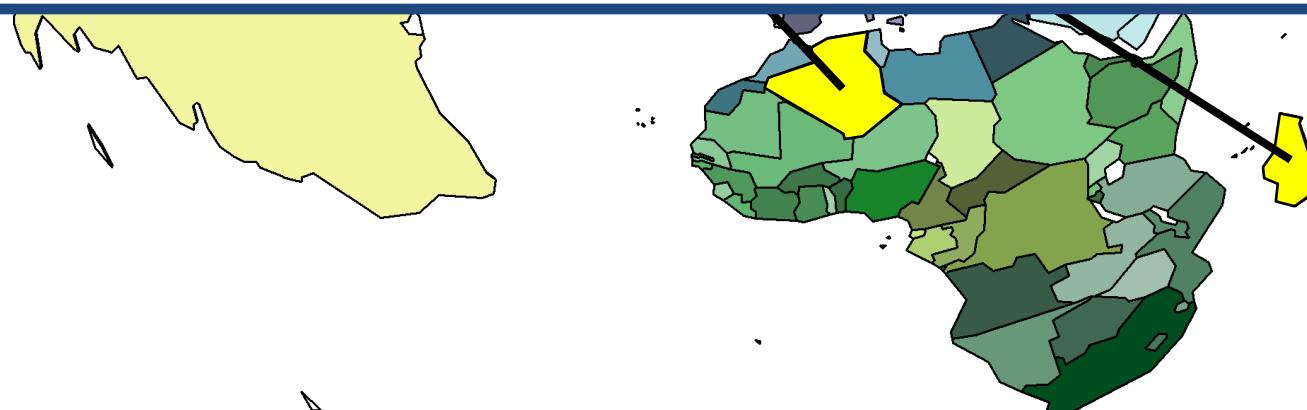
Latin  
America



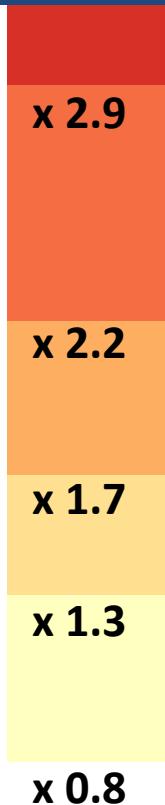
# P1: The Semantics of Geographical Aggregates



**Problem 1:** How to generate abstractions  
that are **meaningful** for the observer?

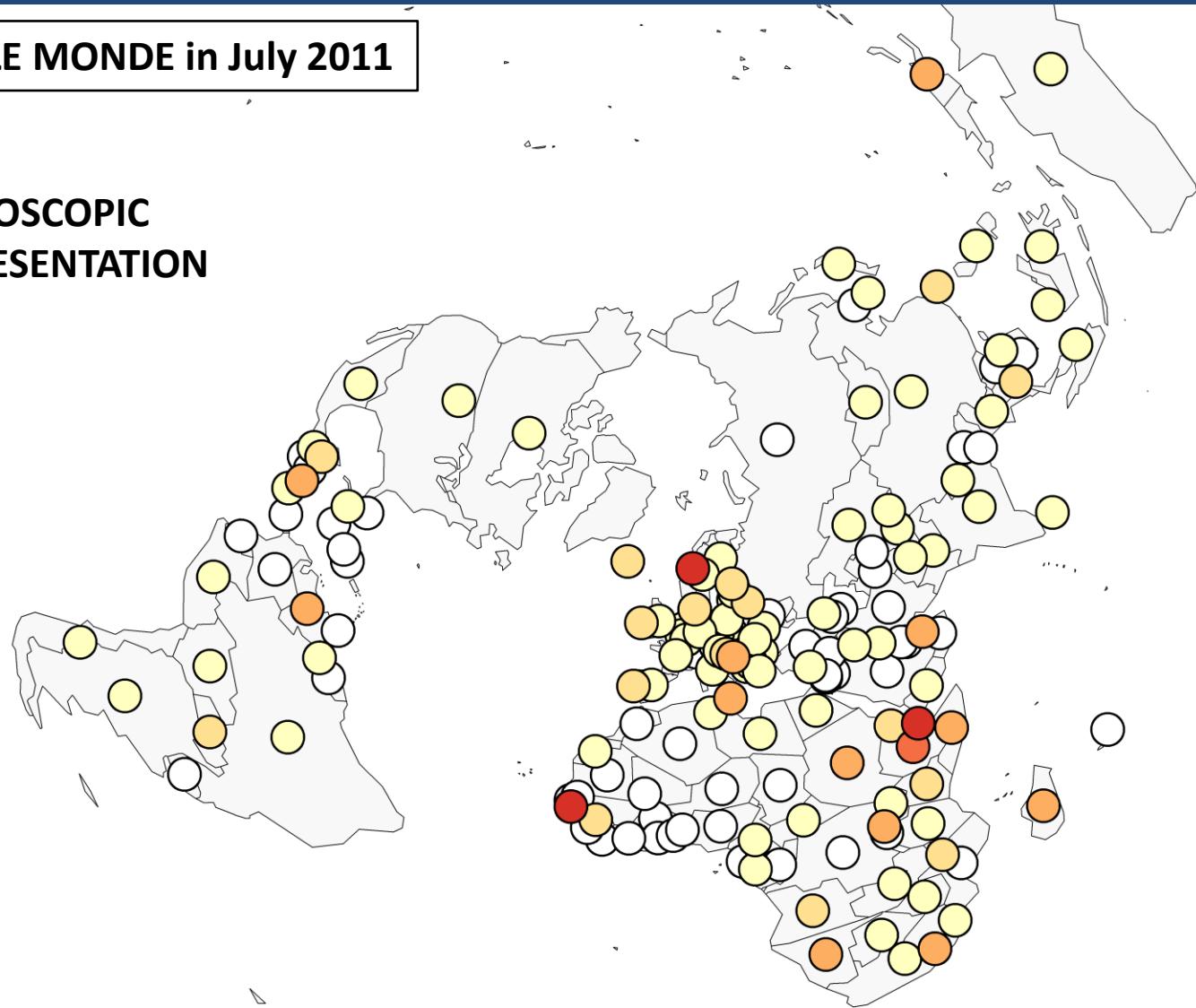


# P2: The Levels of Representation

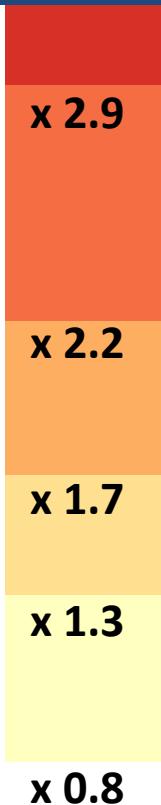


Newspaper LE MONDE in July 2011

MICROSCOPIC  
REPRESENTATION

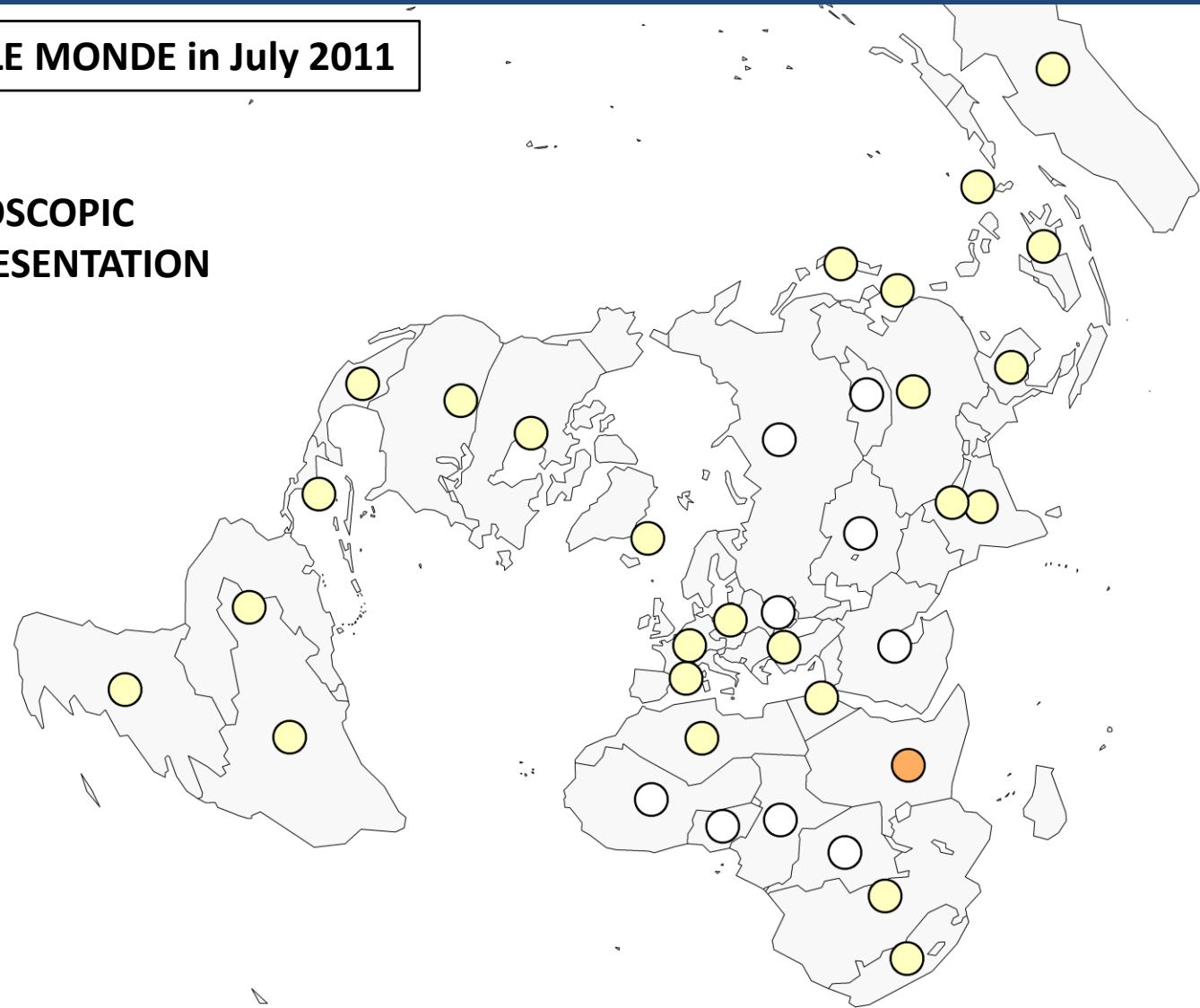


# P2: The Levels of Representation

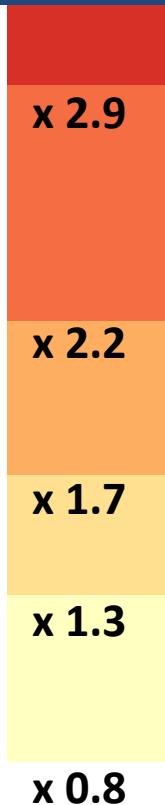


Newspaper LE MONDE in July 2011

MESOSCOPIC  
REPRESENTATION

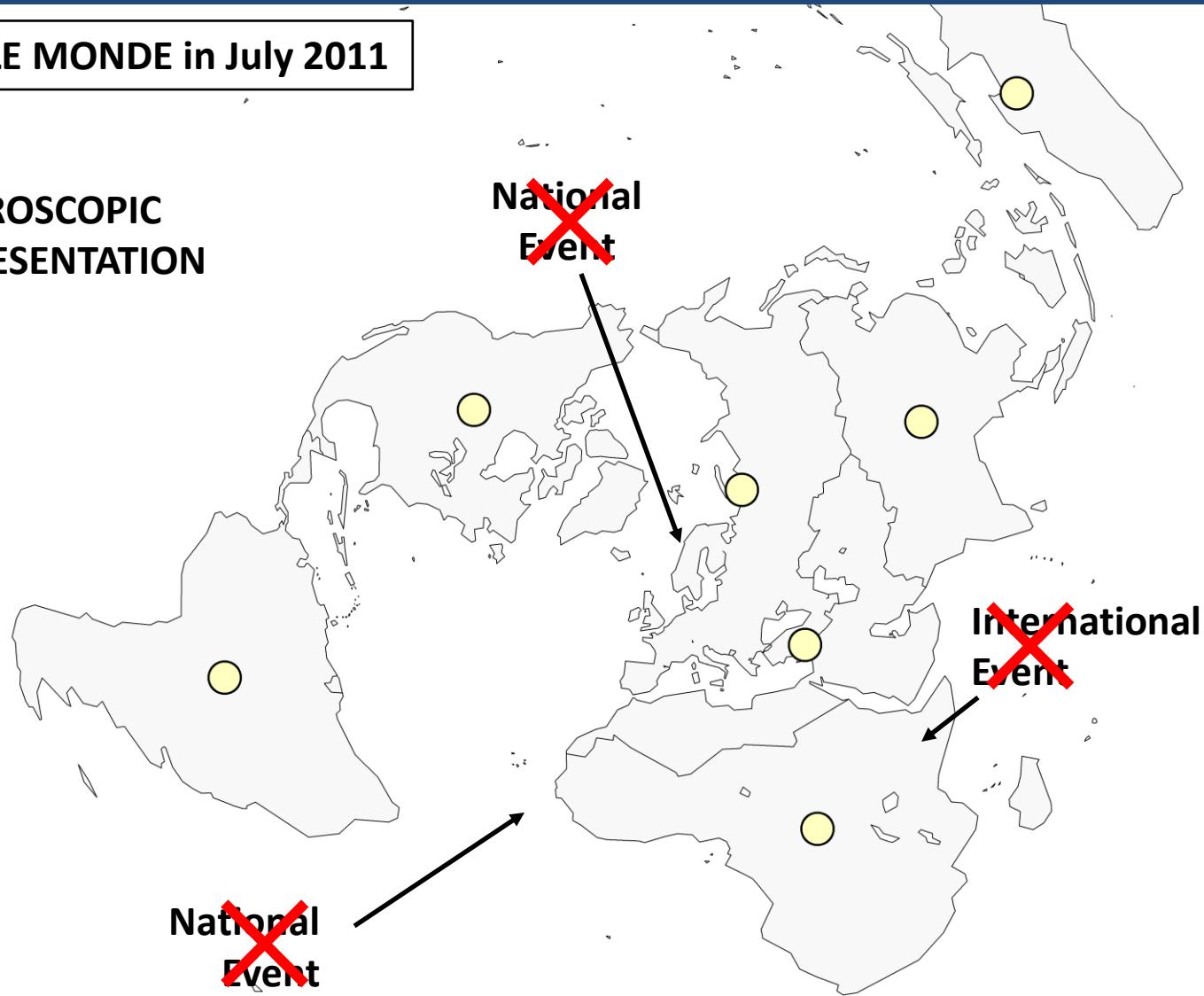


# P2: The Levels of Representation

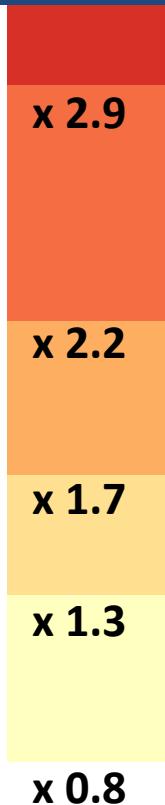


Newspaper LE MONDE in July 2011

MACROSCOPIC  
REPRESENTATION

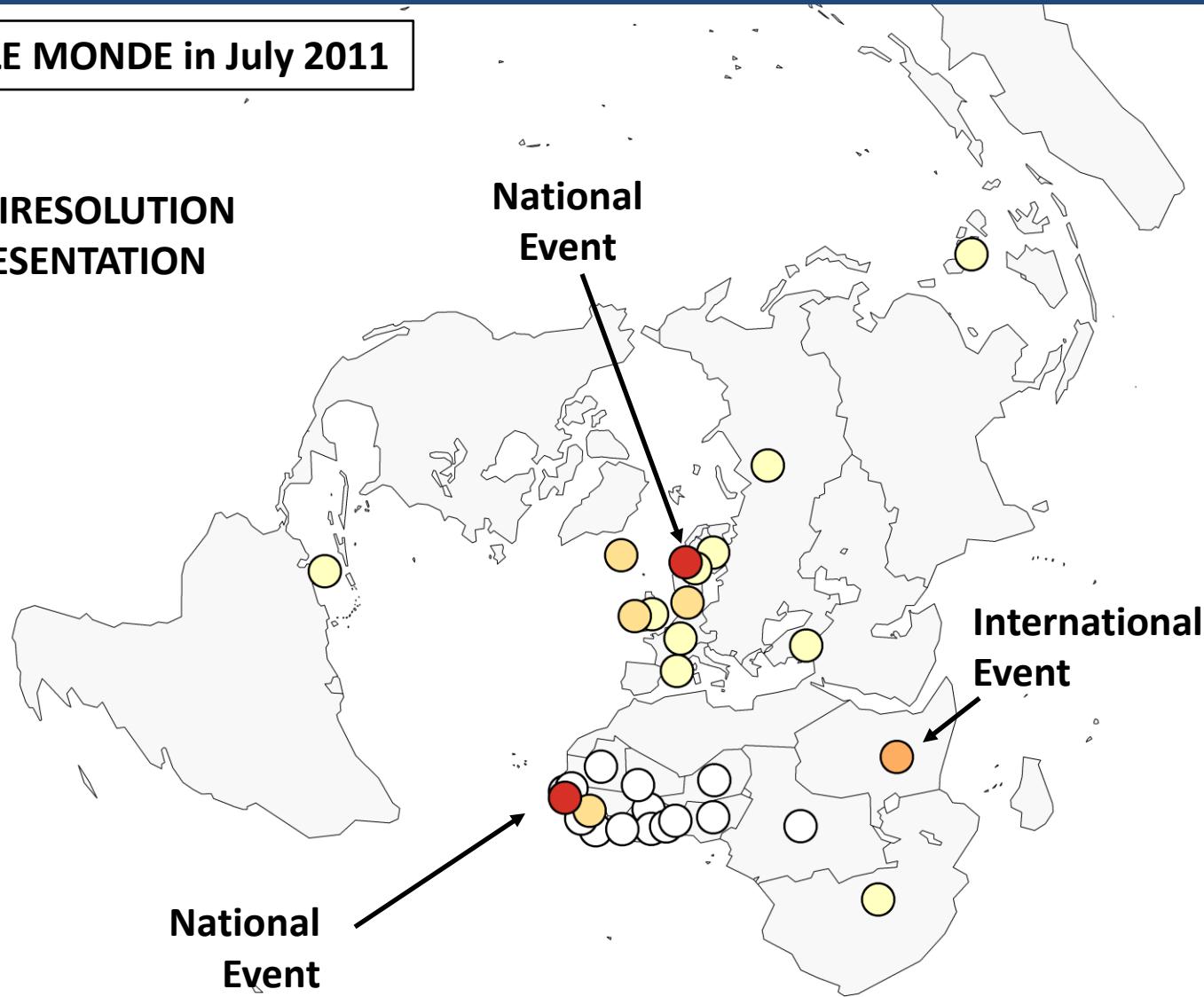


# P2: The Levels of Representation



Newspaper LE MONDE in July 2011

MULTIRESOLUTION  
REPRESENTATION



# P2: The Levels of Representation



Newspaper LE MONDE in July 2011

MULTIRESOLUTION  
REPRESENTATION

National  
Event

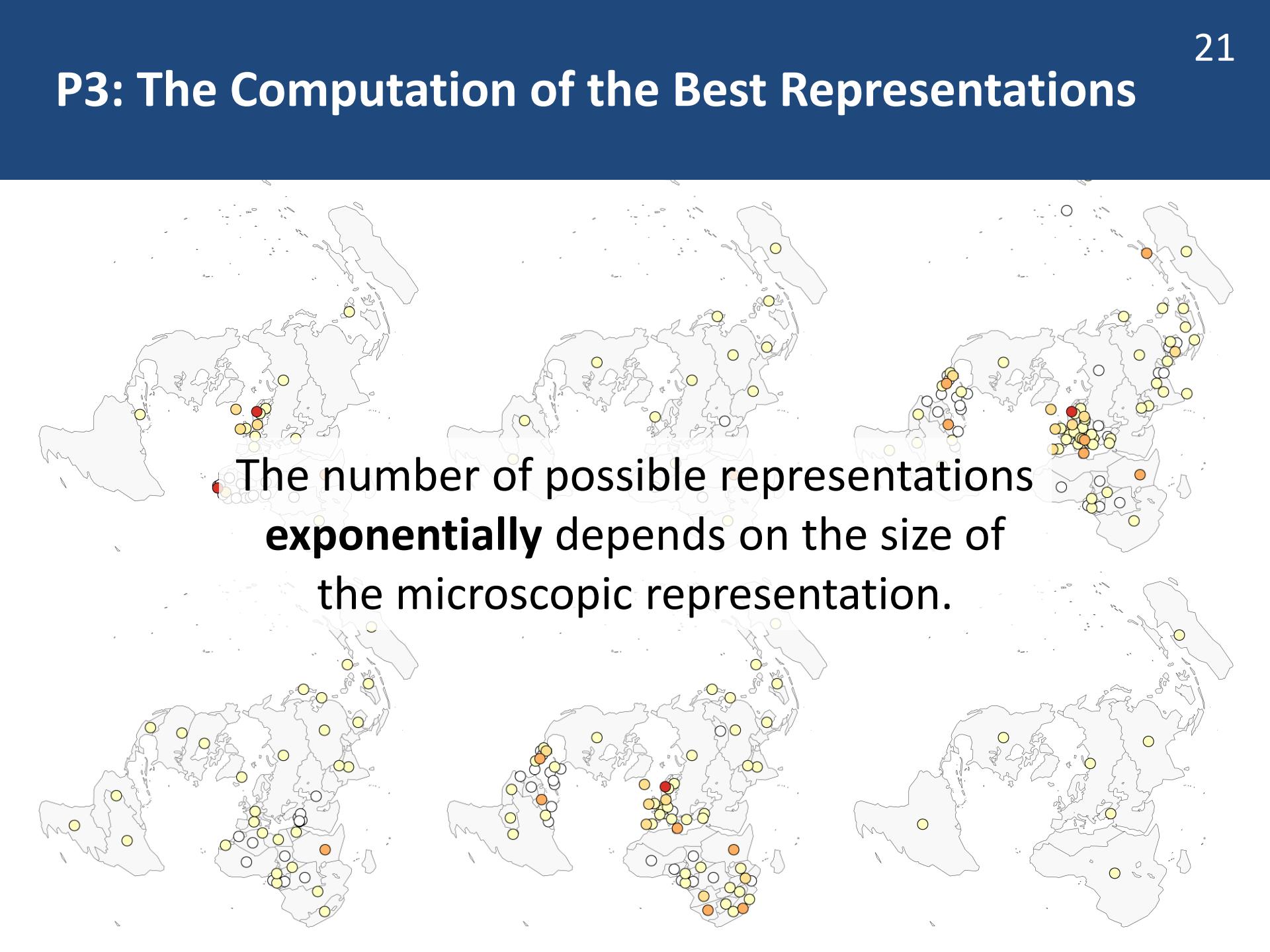
**Problem 2:** How to find the representation levels that are **relevant** for the analysis?

National  
Event

Event

onal

## P3: The Computation of the Best Representations

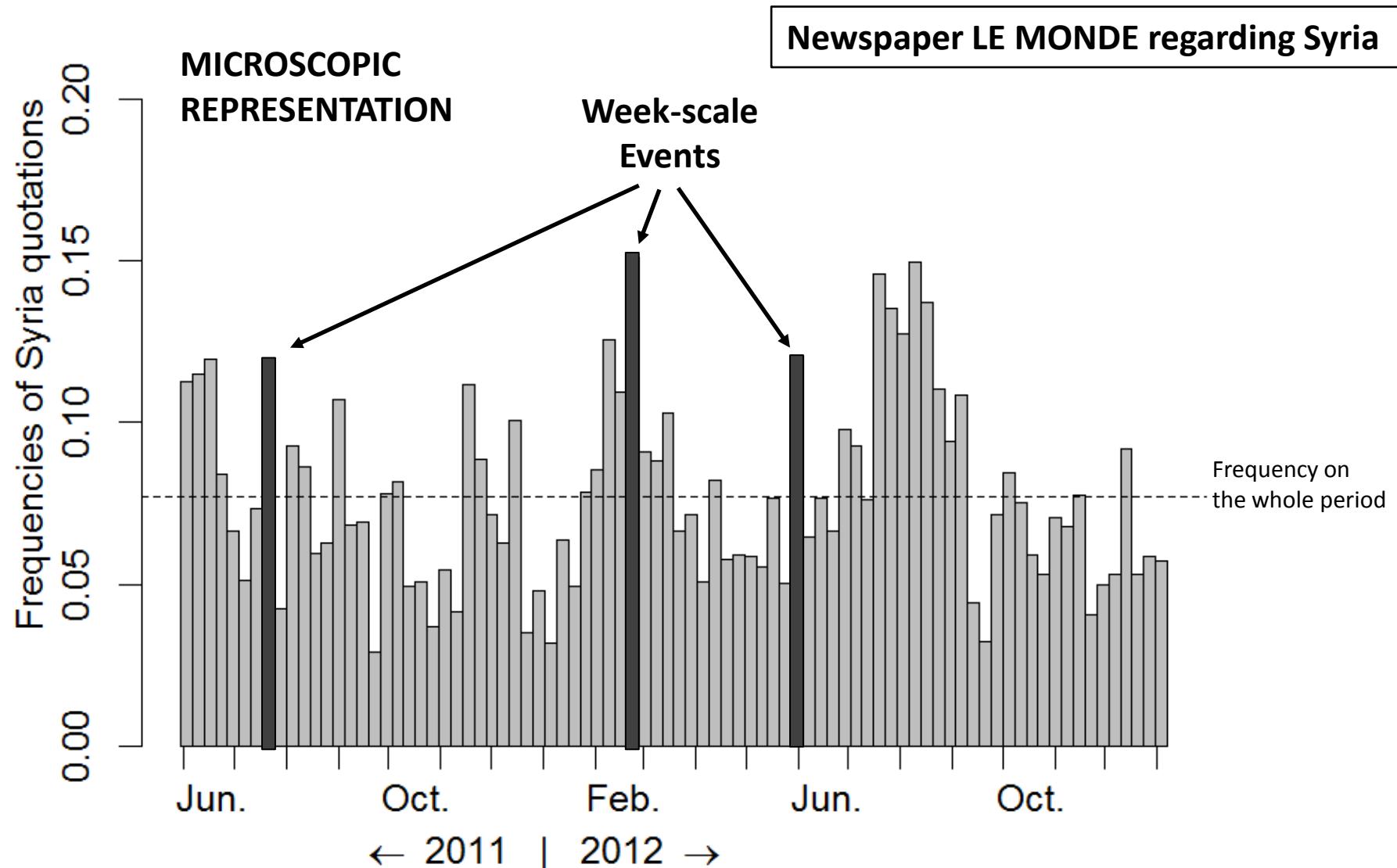


The number of possible representations **exponentially** depends on the size of the microscopic representation.

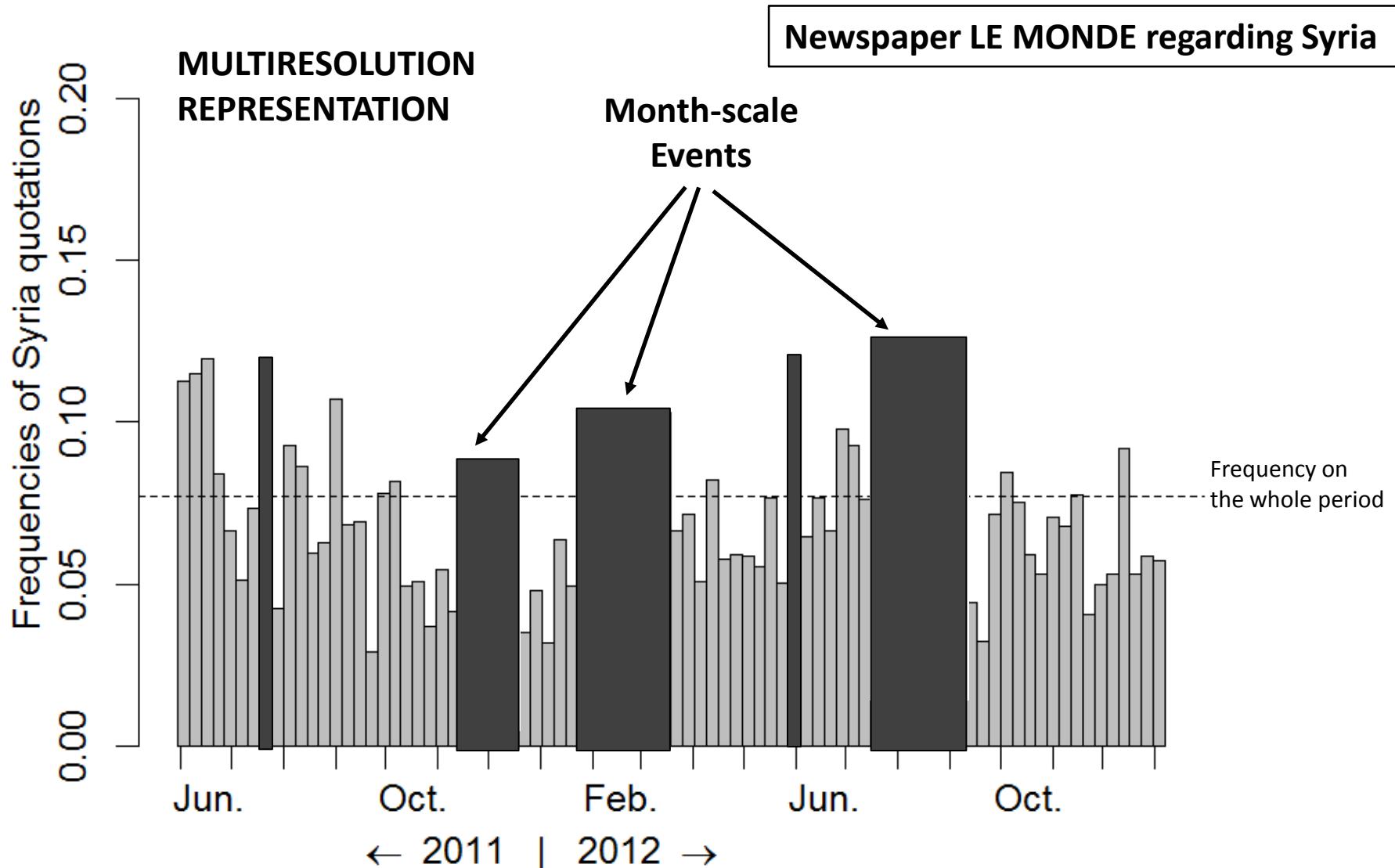
# P3: The Computation of the Best Representations

**Problem 3 : How to compute the optimal representation in an efficient way?**

# And for Other Dimensions?

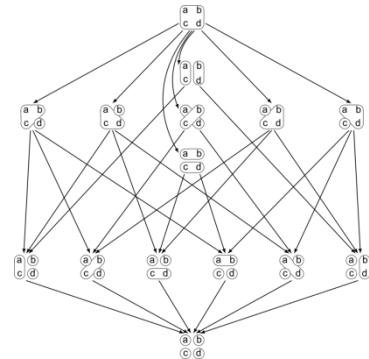


# And for Other Dimensions?

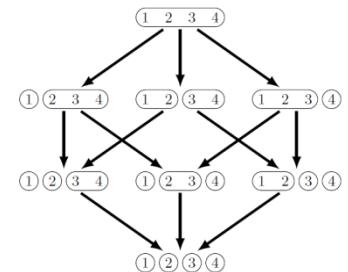


# My Approach

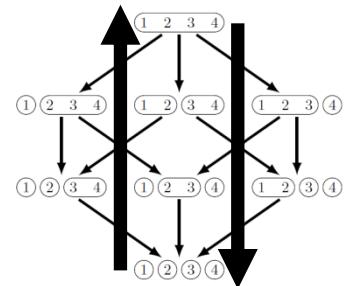
**P0** To characterize the aggregation process  
→ The algebra of possible partitions



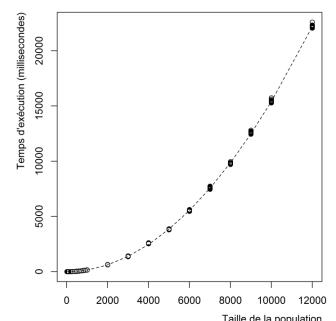
**P1** To preserve the system's semantics  
→ A constrained partitioning method  
To aggregate according to several dimension  
→ Some constraints expressing the system's topology



**P2** To evaluate and compare the representations  
→ Some measures of complexity and information  
To offer several granularity levels  
→ The optimization of a compromise



**P3** To compute the best representations  
→ A generic algorithm of constrained optimization



# My Approach

P0

To characterize the aggregation process  
→ The algebra of possible partitions

P1

To preserve the system's semantics  
→ A constrained partitioning method  
To aggregate according to several dimension  
→ Some constraints expressing the system's topology

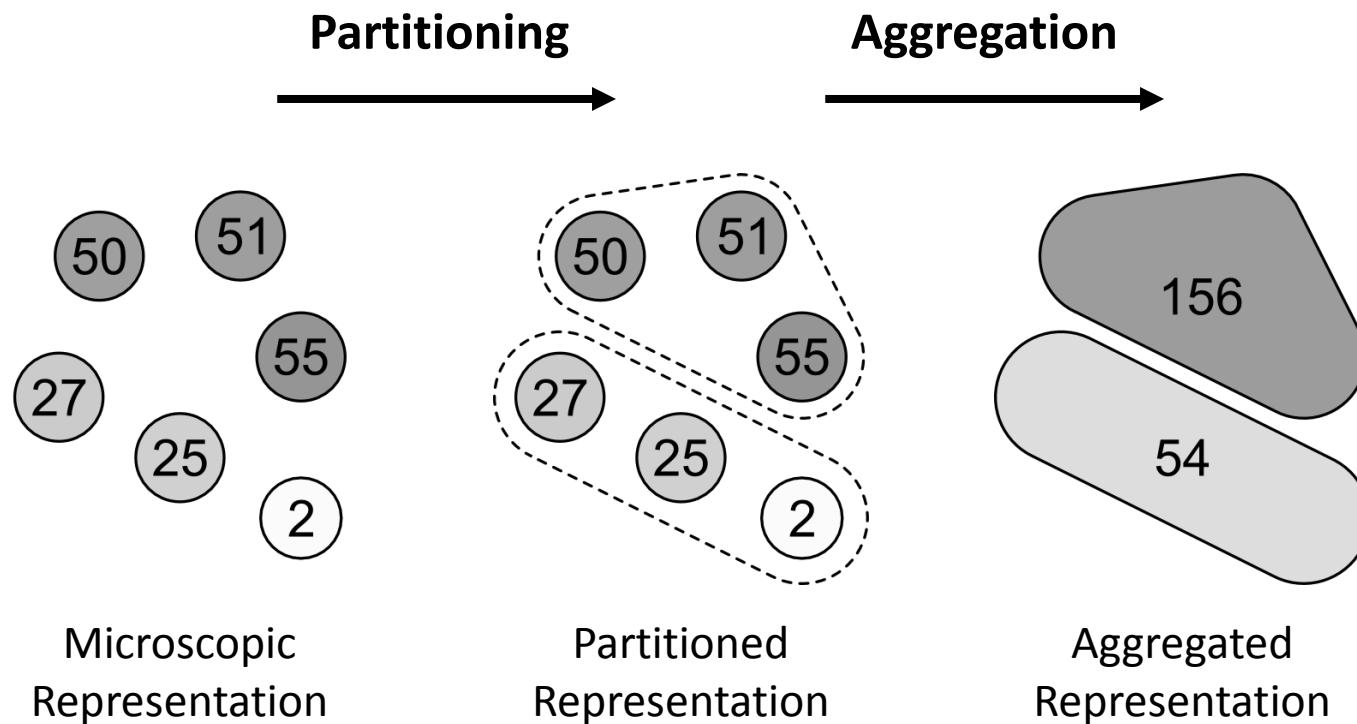
P2

To evaluate and compare the representations  
→ Some measures of complexity and information  
To offer several granularity levels  
→ The optimization of a compromise

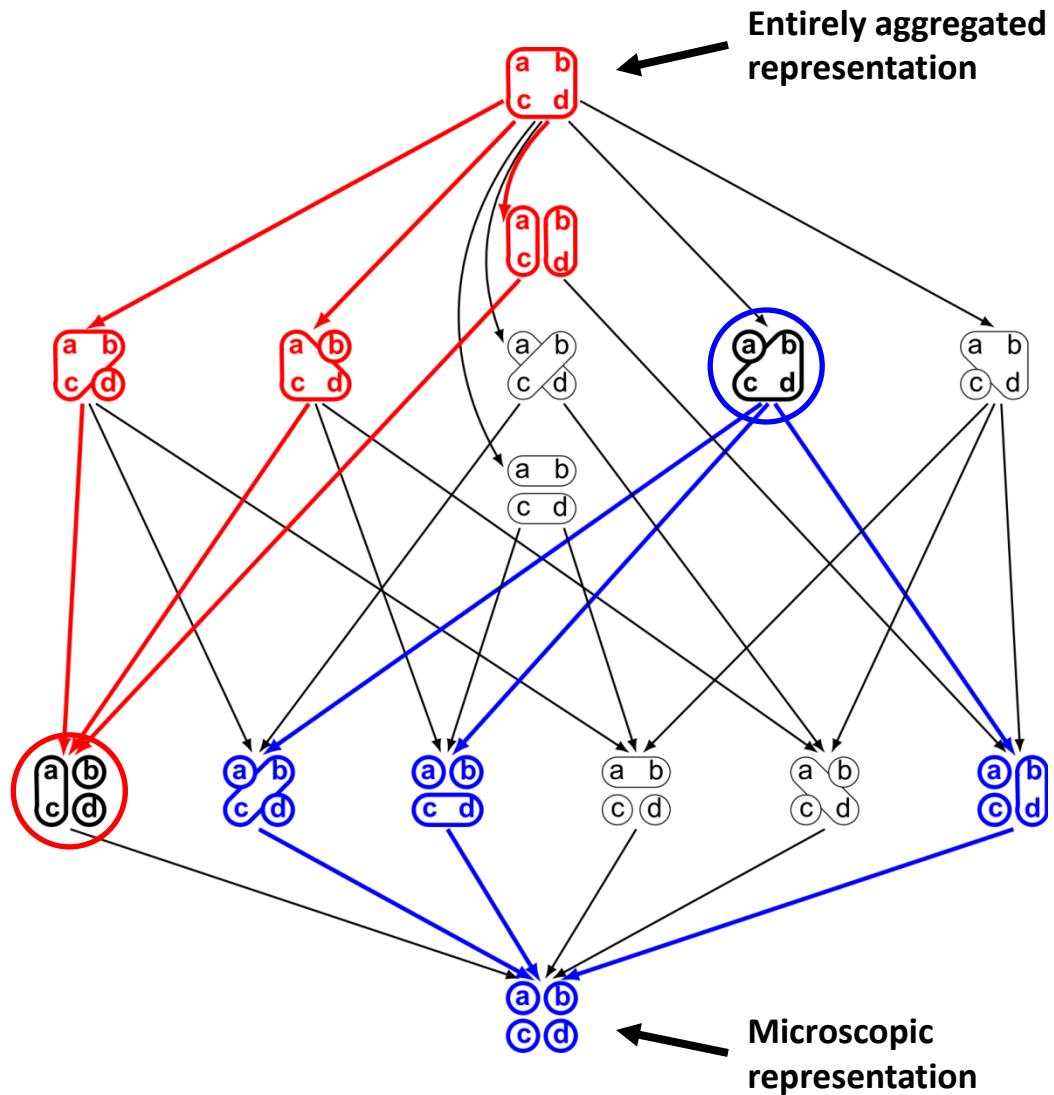
P3

To compute the best representations  
→ A generic algorithm of constrained optimization

# The Aggregation Process



# Set of Possible Partitions

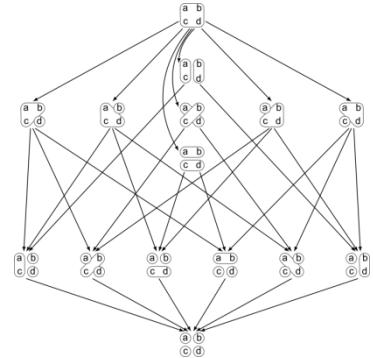


## Algebraic Structure

A partial order on the set of possible partitions

→ the refinement relation

# My Approach



**P0** To characterize the aggregation process  
→ The algebra of possible partitions

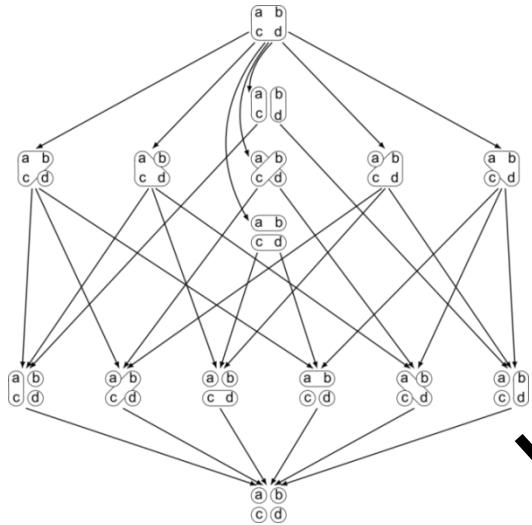
**P1** To preserve the system's semantics  
→ A constrained partitioning method  
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To offer several granularity levels  
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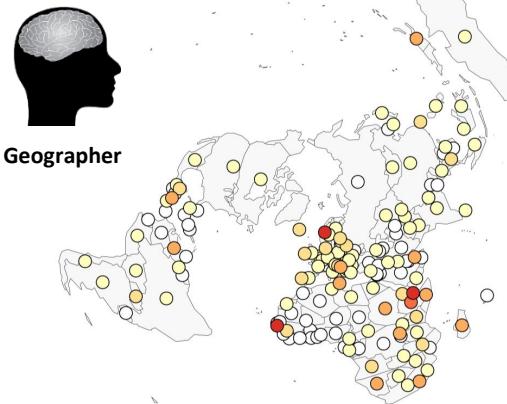
**P3** To compute the best representations  
→ A generic algorithm of constrained optimization

# Problems and Objectives

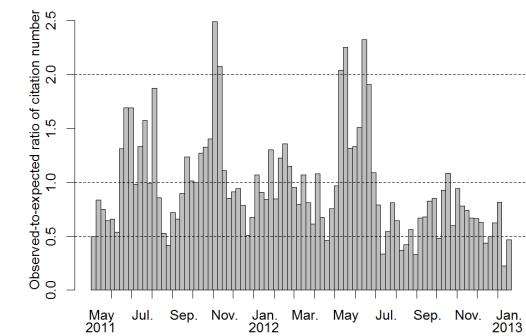
**Set of possible partitions**



**Geographical Semantics**

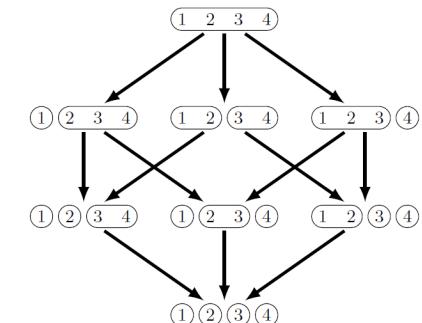
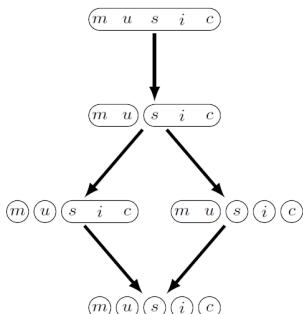


**Temporal Semantics**



**Constraints**

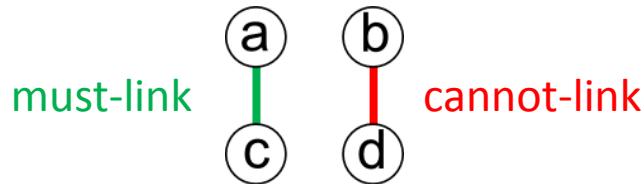
**Set of admissible partitions**



# Constrained Partitioning

**At the instances level**

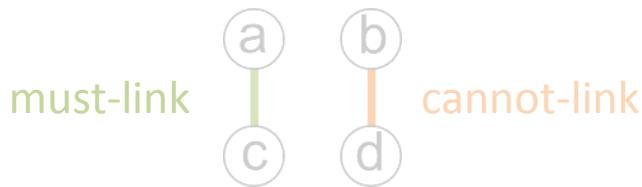
[Davidson and Basu, 2007]



# Constrained Partitioning

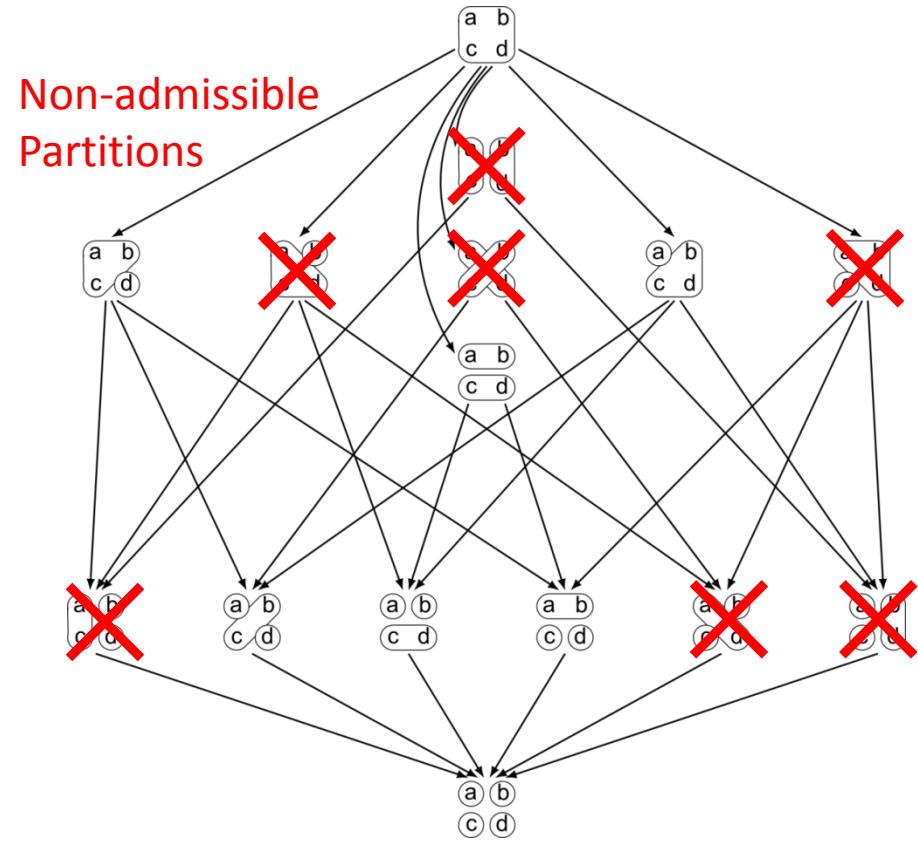
## At the instances level

[Davidson and Basu, 2007]



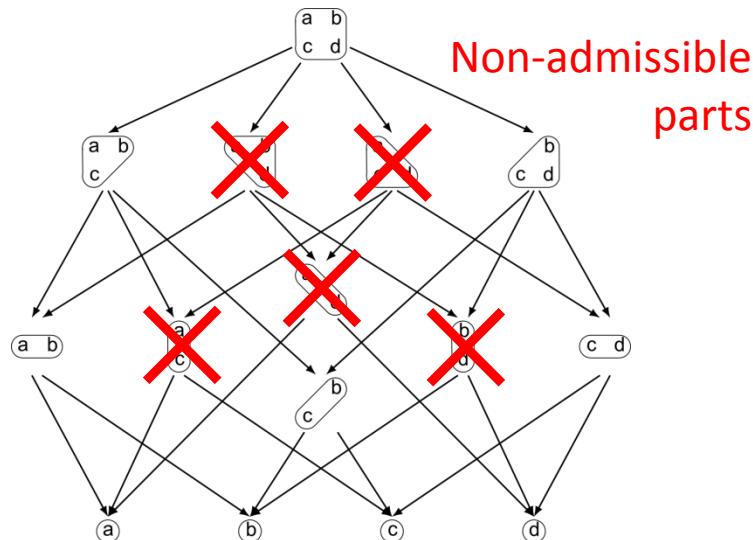
## At the partitions level

[Lamarche-Perrin *et al.*, IAT 2013]



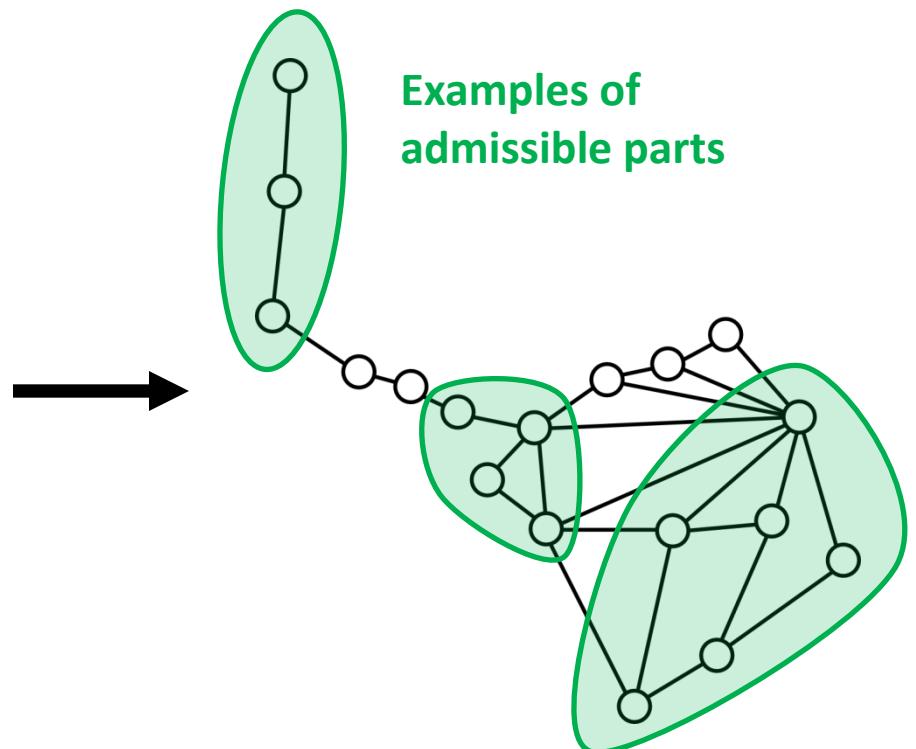
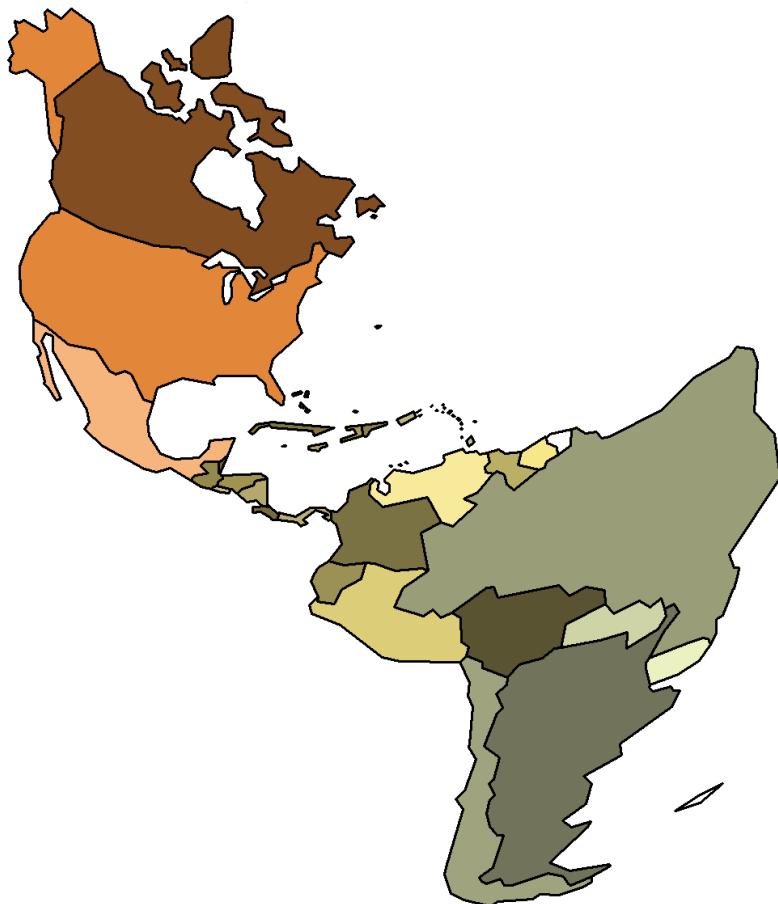
## At the parts level

[Lamarche-Perrin *et al.*, IAT 2013]



# Preserving the Neighborhood Relation

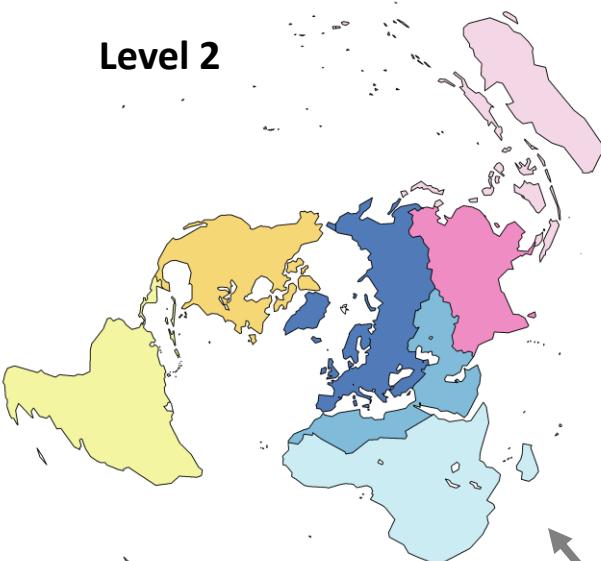
**Admissible Parts:** set of connected countries regarding the adjacency graph



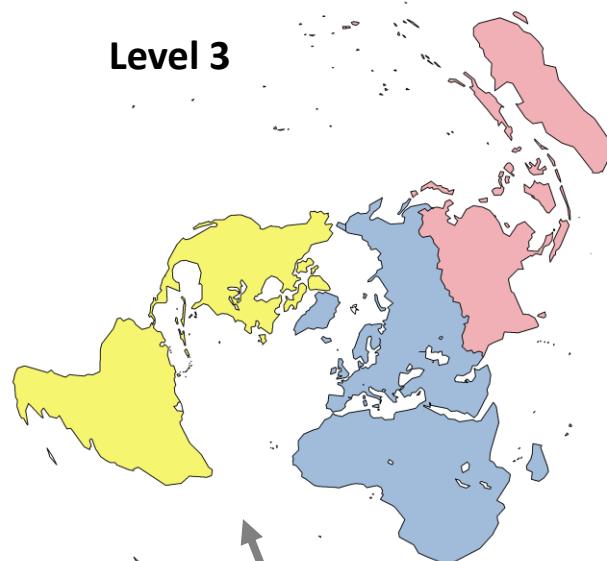
# The WUTS Hierarchy

[Grasland and Didelon, 2007]

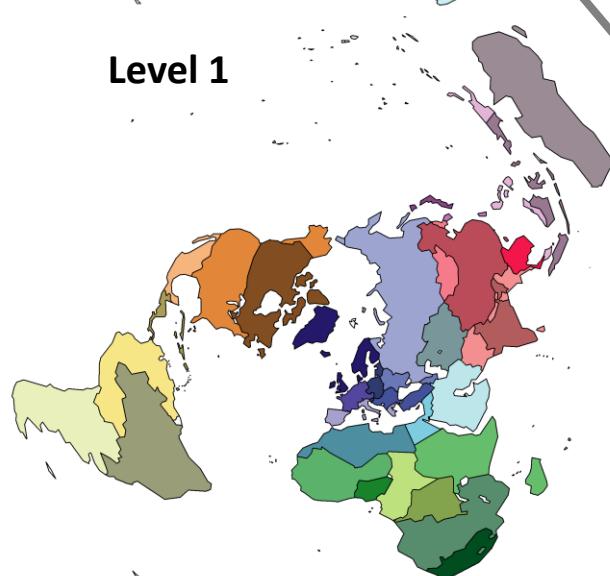
Level 2



Level 3

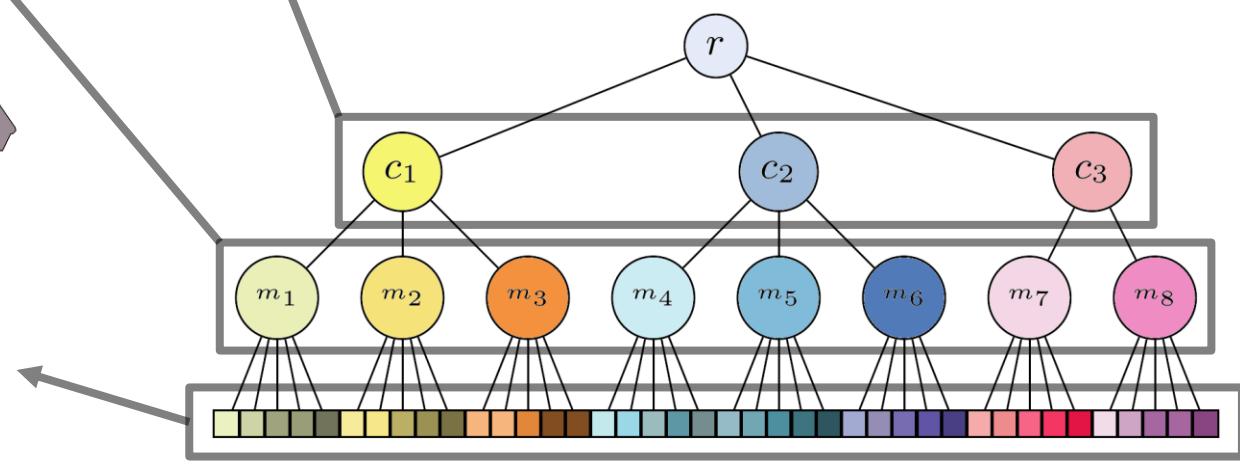


Level 1



## Admissible Parts:

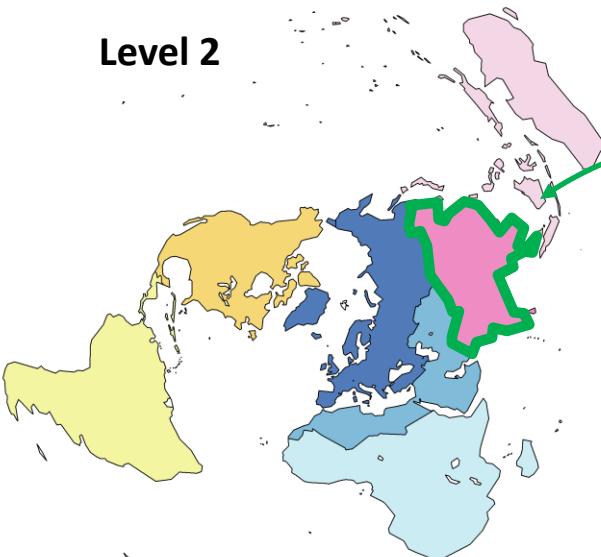
set of countries that are politically, culturally and economically consistent



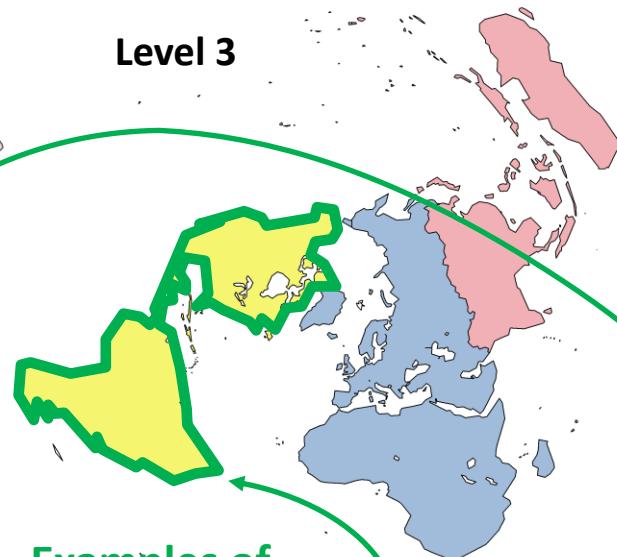
# The WUTS Hierarchy

[Grasland and Didelon, 2007]

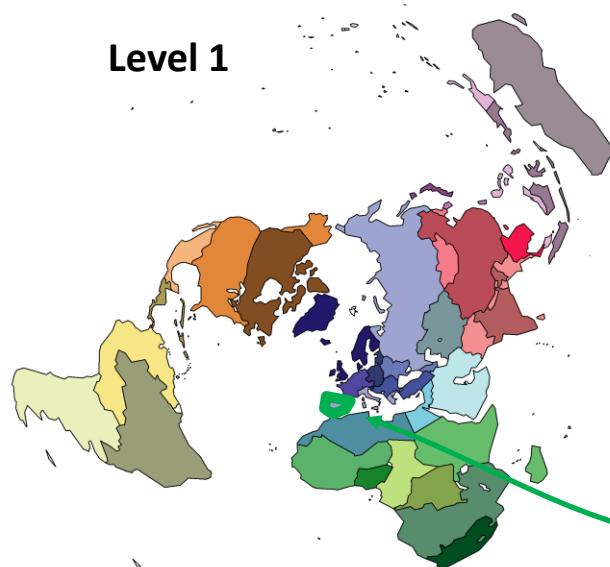
Level 2



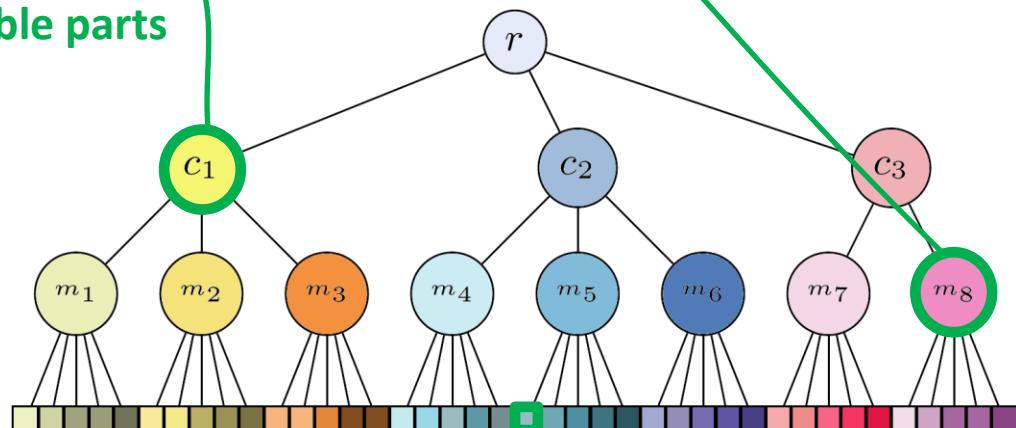
Level 3



Level 1



Examples of  
admissible parts

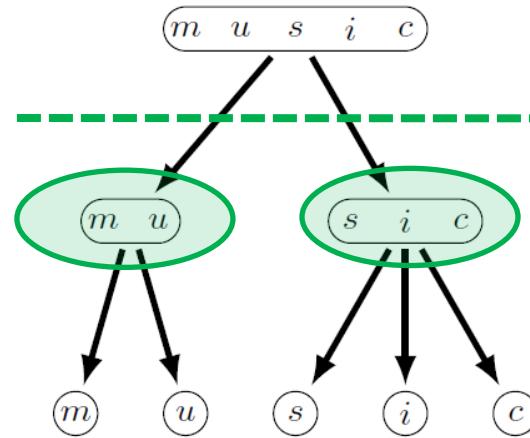


## Admissible Parts:

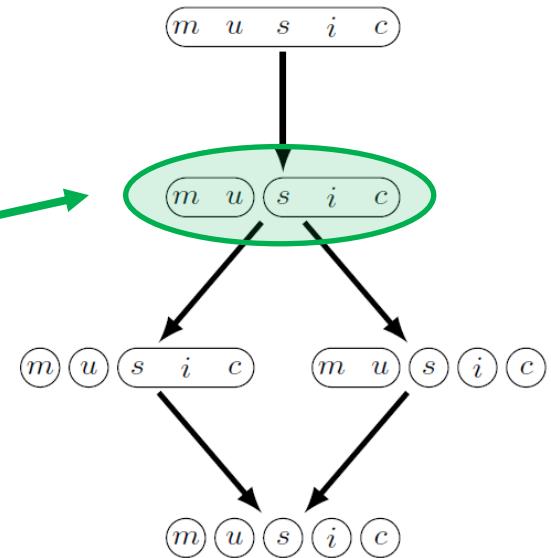
set of countries that are politically, culturally and economically consistent

# Aggregating according to a Hierarchy

**Admissible Parts**  
(nodes of the hierarchy)

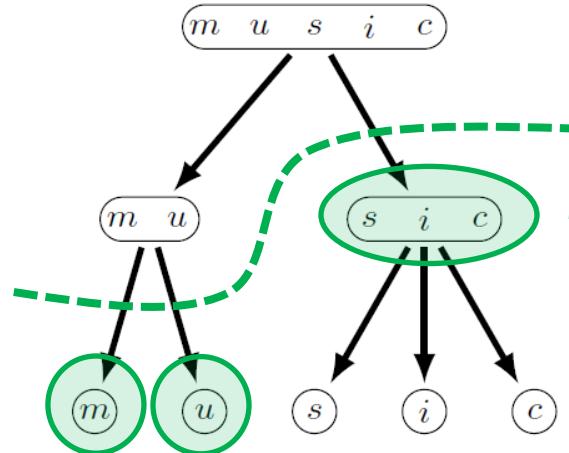


**Admissible Partitions**  
(cuts of the hierarchy)

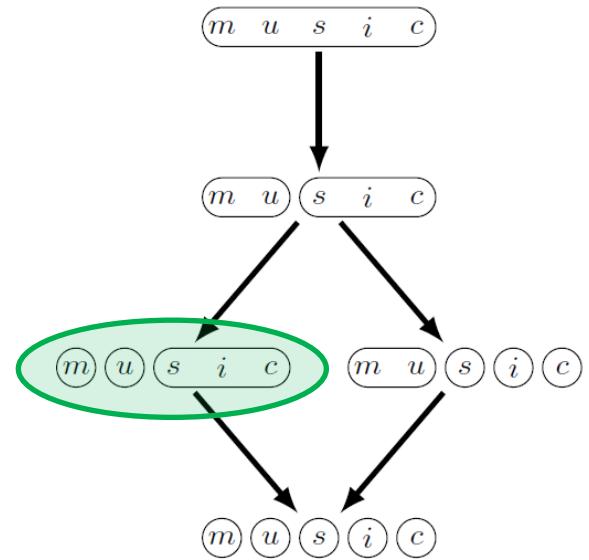


# Aggregating according to a Hierarchy

**Admissible Parts**  
(nodes of the hierarchy)



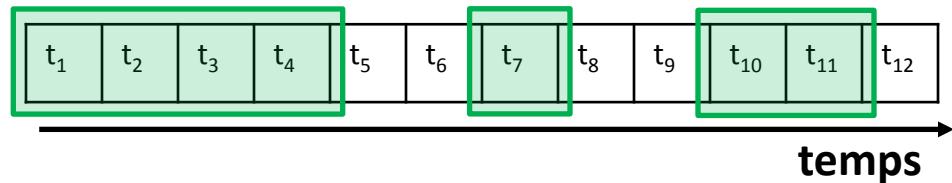
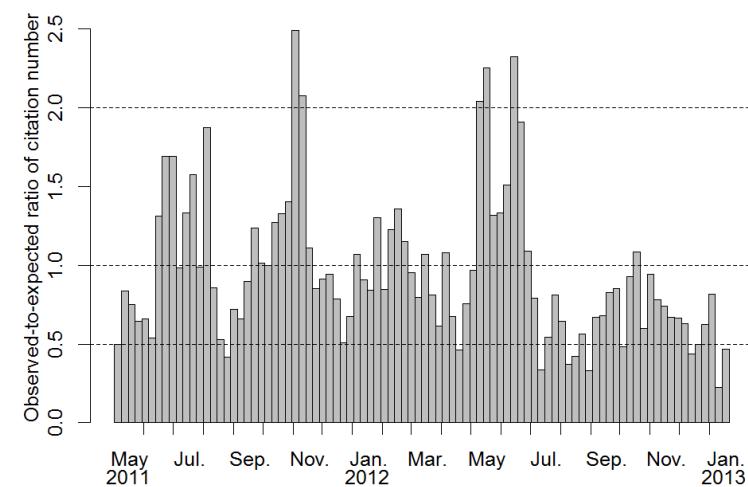
**Admissible Partitions**  
(cuts of the hierarchy)



# Preserving the Order of Time

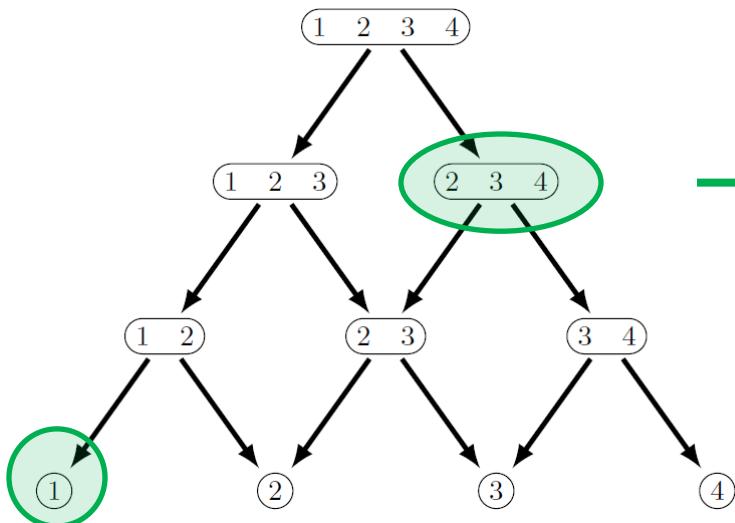
**Admissible Parts:**  
time intervals

Examples of admissible parts

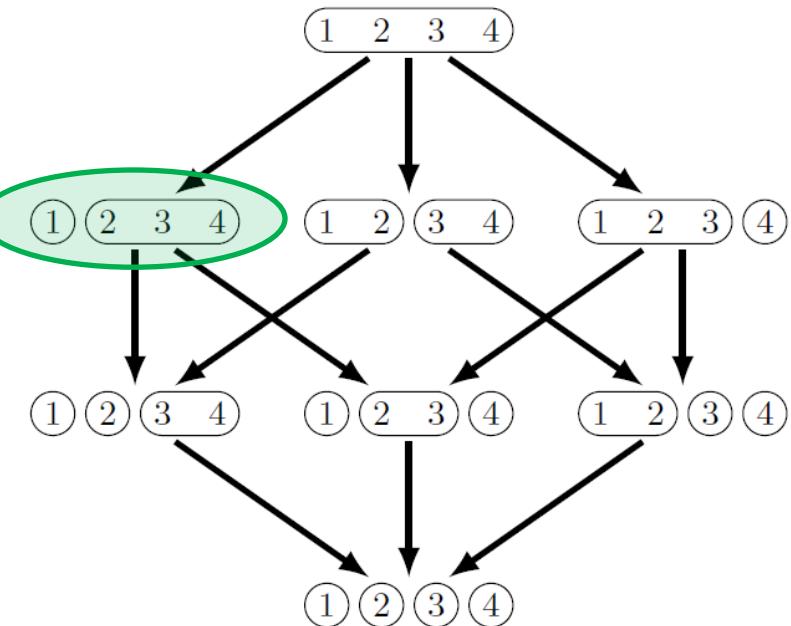


# Aggregating according to a Total Order

**Admissible Parts**  
(time intervals)

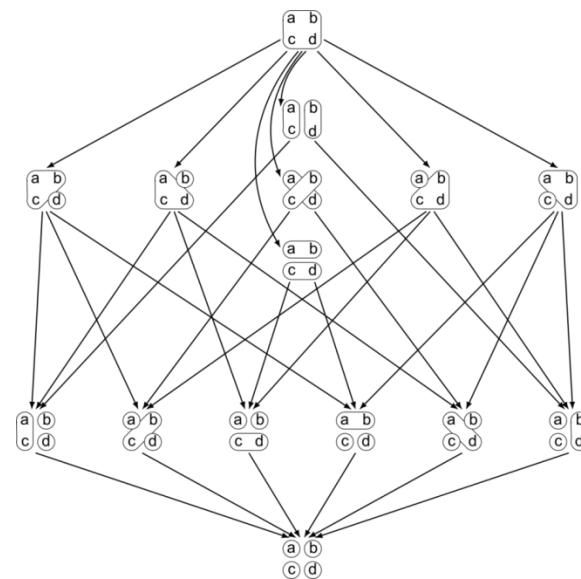


**Admissible Partitions**  
(interval sequences)

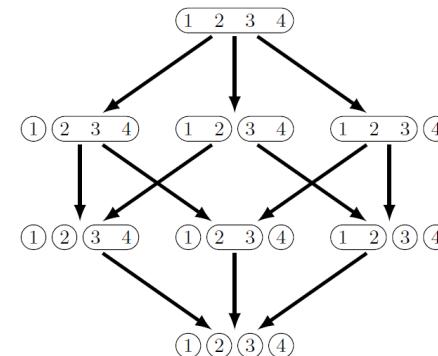


# Complexity of Algebraic Structures

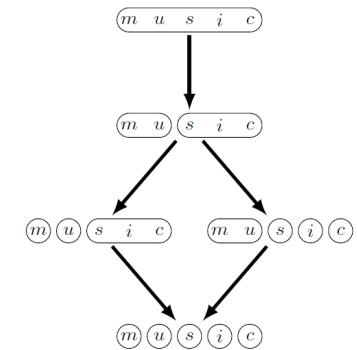
**Non-constrained**  
partitions



Admissible partitions  
according to a **total order**



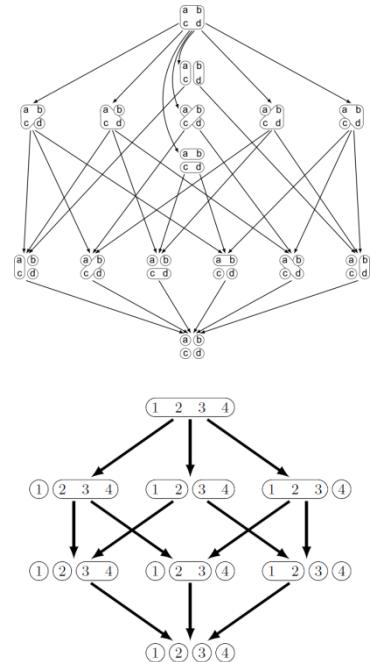
Admissible partitions  
according to a **hierarchy**



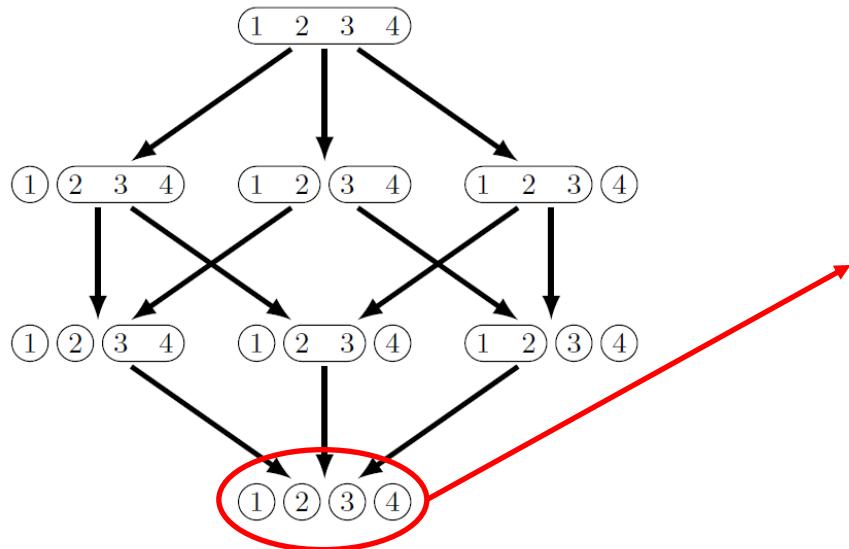
←      →  
Less constrained      More constrained  
More complex      Less complex

# My Approach

- P0** To characterize the aggregation process  
→ The algebra of possible partitions
- P1** To preserve the system's semantics  
→ A constrained partitioning method  
To aggregate according to several dimension  
→ Some constraints expressing the system's topology
- P2** To evaluate and compare the representations  
→ Some measures of complexity and information  
To offer several granularity levels  
→ The optimization of a compromise
- P3** To compute the best representations  
→ A generic algorithm of constrained optimization

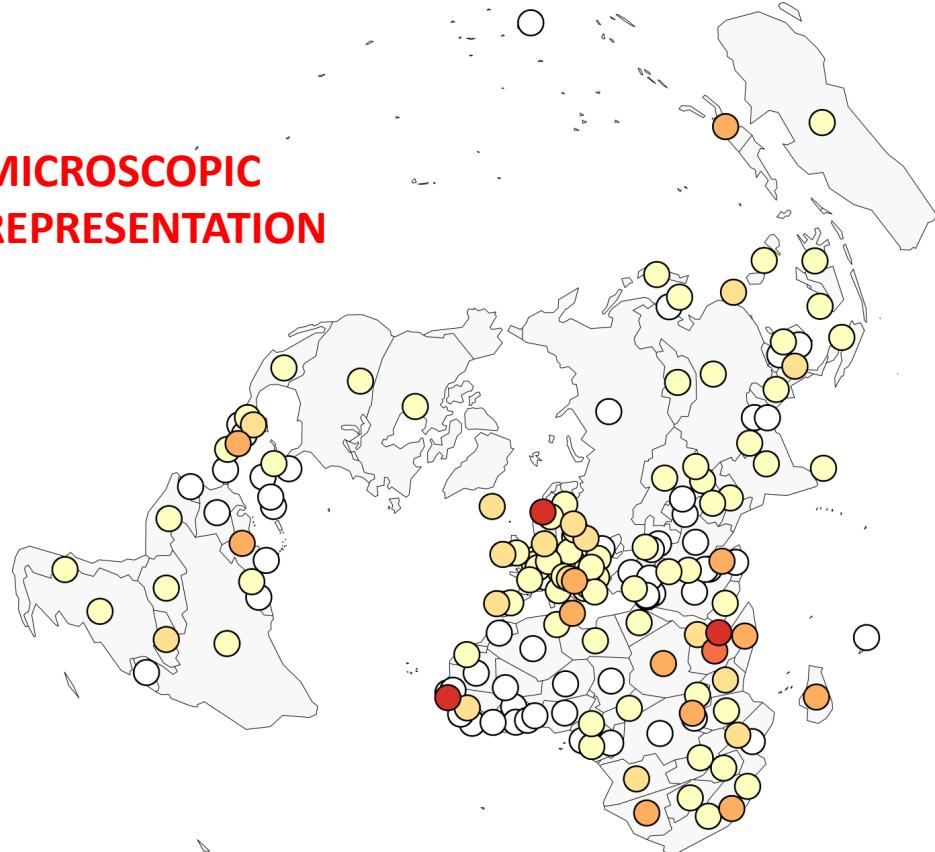


# Objectives and Difficulties



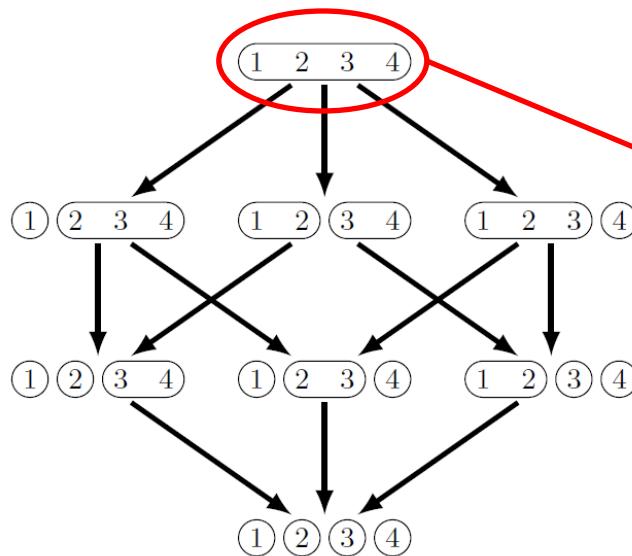
Which **admissible partition**  
is the **best partition** for a  
particular dataset?

MICROSCOPIC  
REPRESENTATION

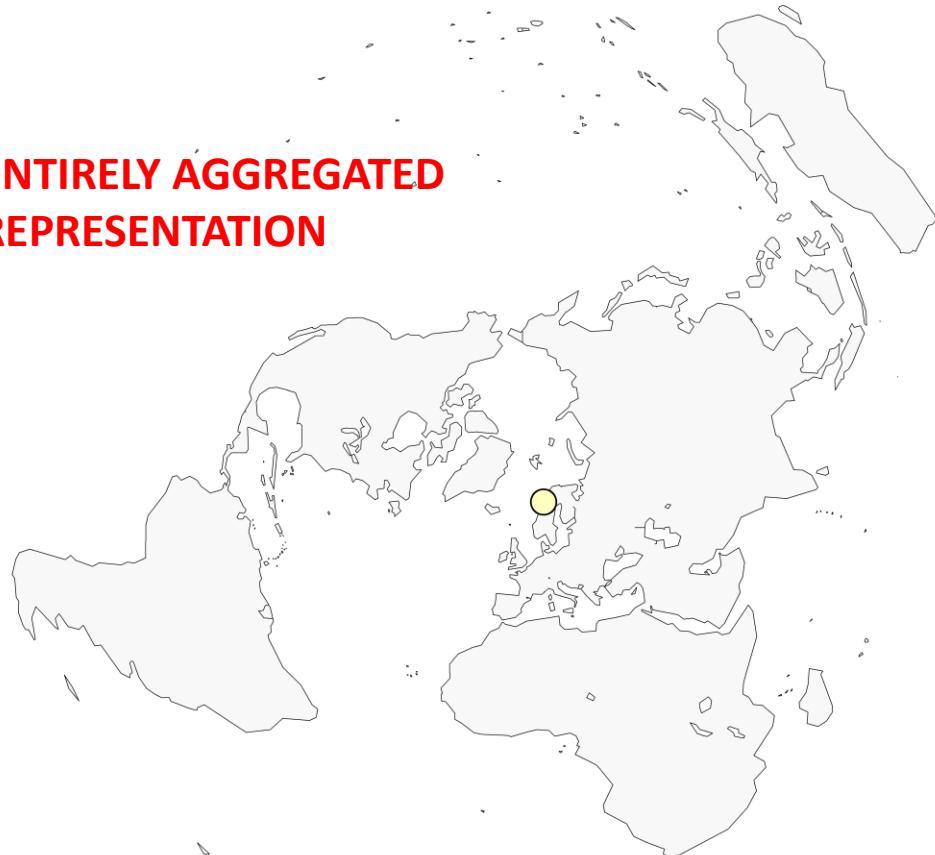


→ TOO COMPLEX TO SCALE-UP

# Objectives and Difficulties



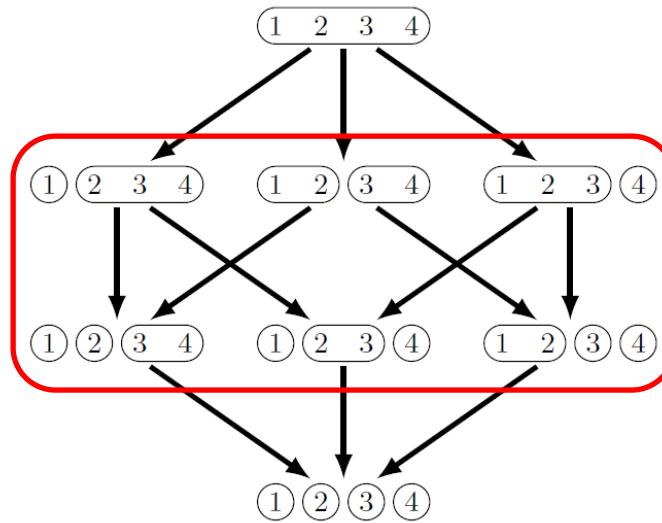
ENTIRELY AGGREGATED  
REPRESENTATION



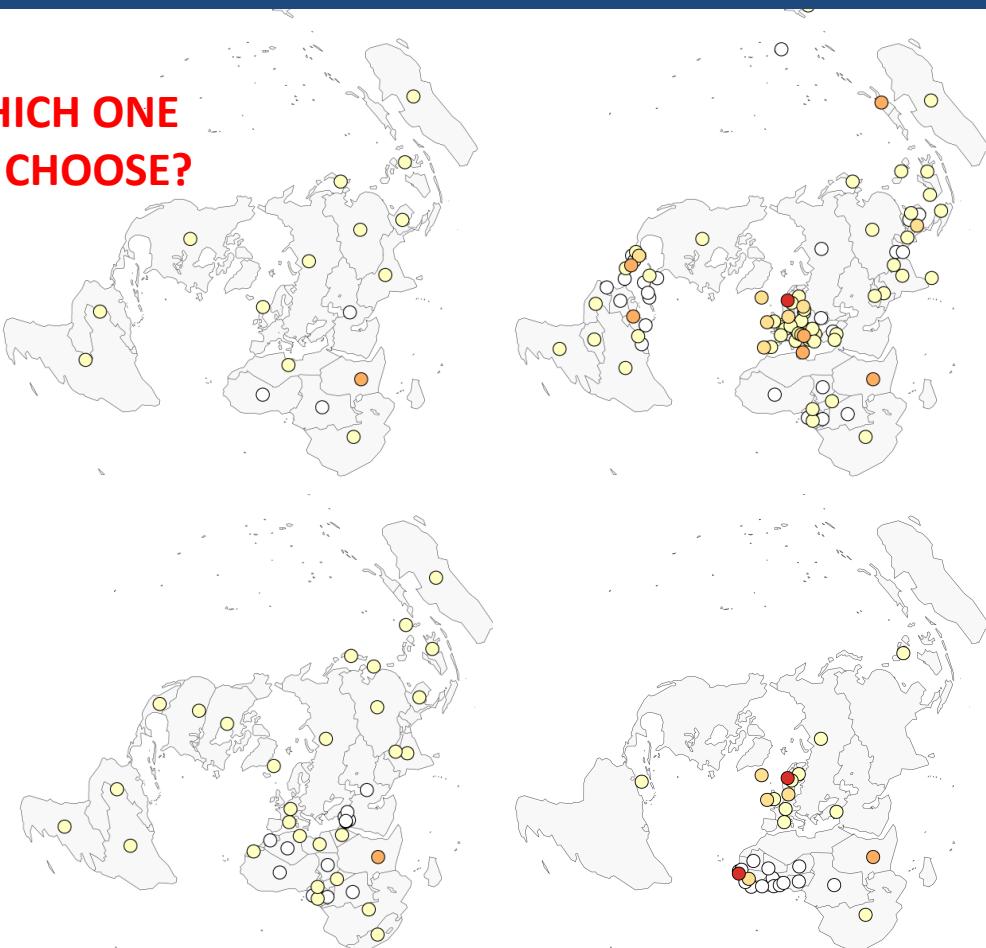
Which **admissible partition**  
is the **best partition** for a  
particular dataset?

→ GIVES TOO FEW INFORMATION

# Objectives and Difficulties

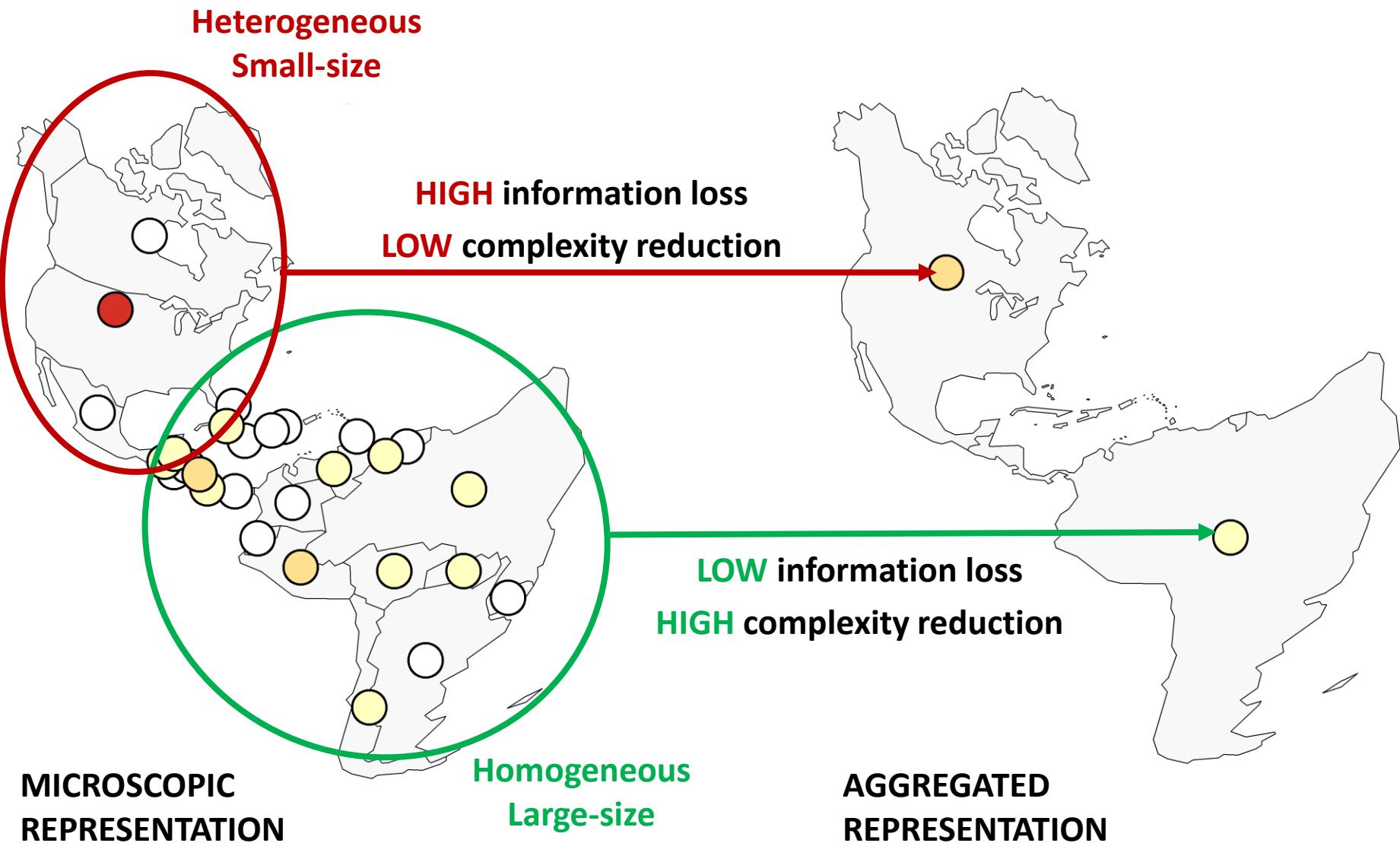


WHICH ONE  
TO CHOOSE?

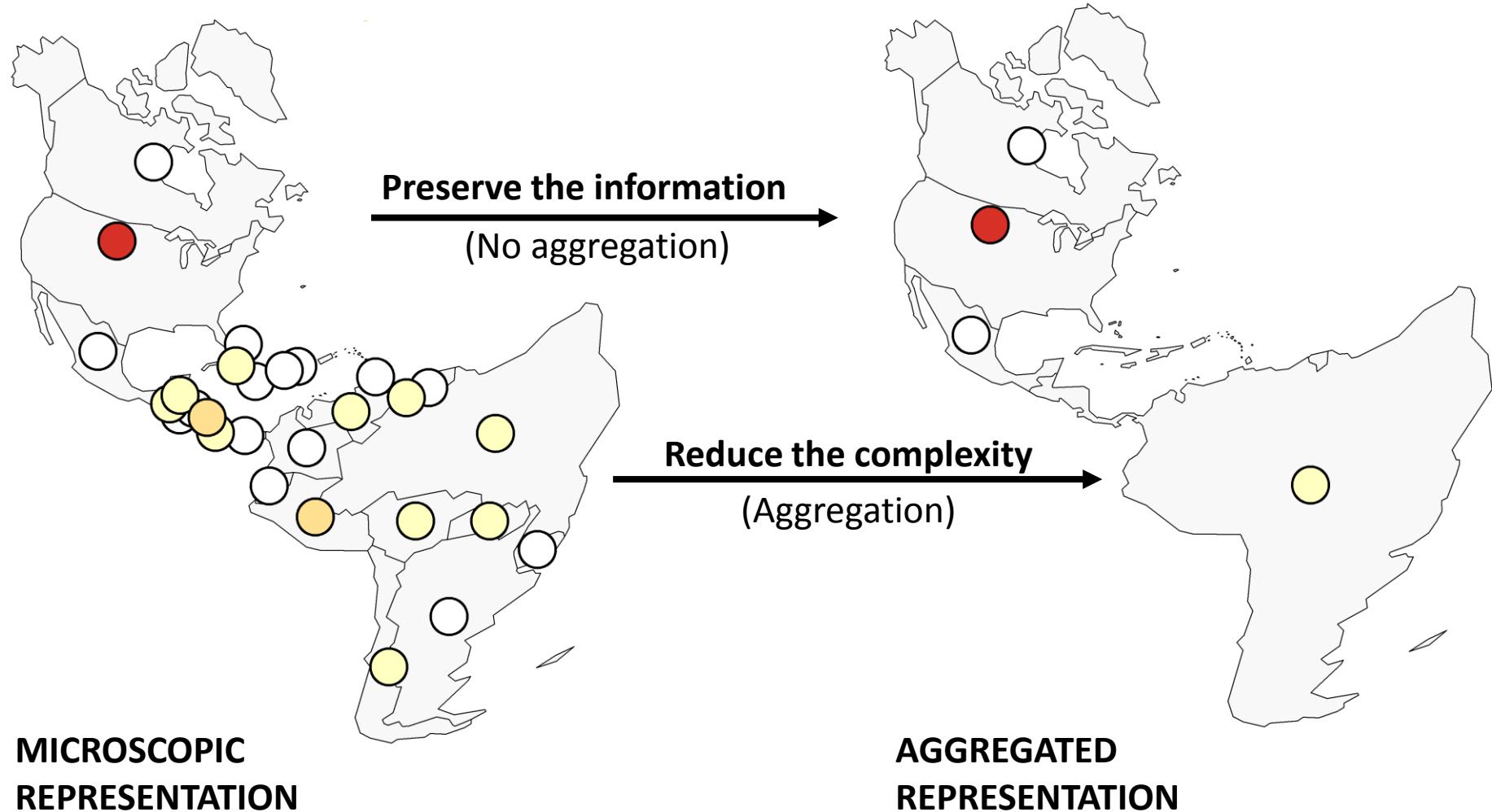


Which **admissible partition**  
is the **best partition** for a  
particular dataset?

# Complexity and Information



# Complexity and Information



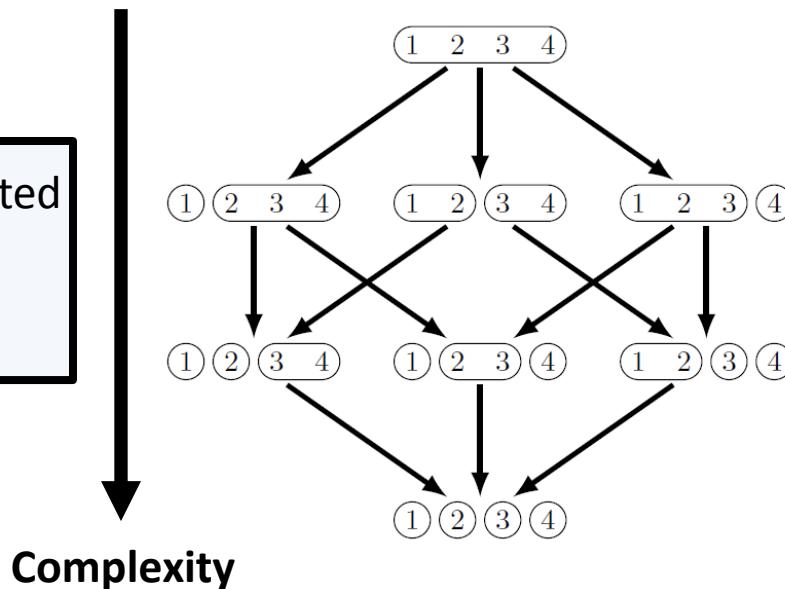
# Quality Measures

[Lamarche-Perrin *et al.*, ECCS 2012]

**Complexity** depends on the **tasks** we want to fulfill and the **description tools** that are available to do so

[Bonabeau and Dossal, 1997]

Number of represented aggregates:  
 $T(\mathcal{X}) = |\mathcal{X}|$



**Information loss** is measured by the **KL-divergence** between two probabilistic distributions

[Kullback et Leibler, 1951]

Information

Loss

Kullback-Leibler Divergence  

$$D(\mathcal{X}) = \sum_{X \in \mathcal{X}} \sum_{x \in X} \frac{v(x)}{v(\Omega)} \log_2 \left( \frac{v(x)}{v(X)} \right)$$

# Measures Decomposability

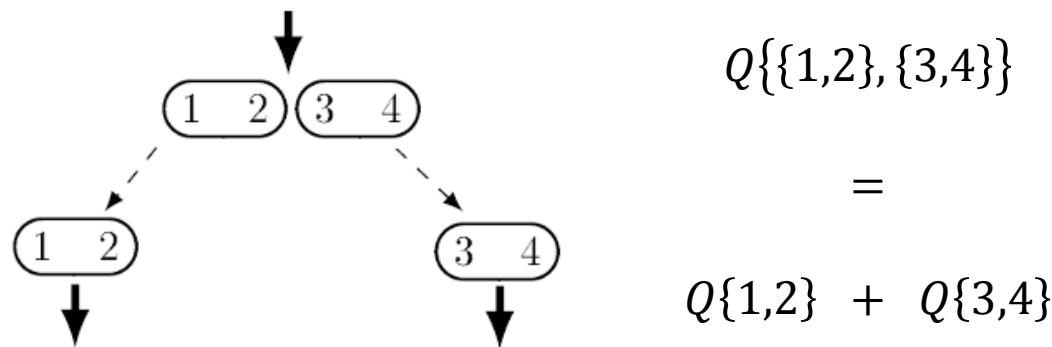
Number of represented aggregates:  
 $T(\mathcal{X}) = |\mathcal{X}|$

Kullback-Leibler Divergence

$$D(\mathcal{X}) = \sum_{x \in \mathcal{X}} \sum_{\Omega \in X} \frac{v(x)}{v(\Omega)} \log_2 \left( \frac{v(x) |X|}{v(\Omega)} \right)$$

**Additive Decomposability:** The quality of a partition is the sum of the qualities of its parts

[Jackson *et al.*, 2005] [Csiszár, 2008]

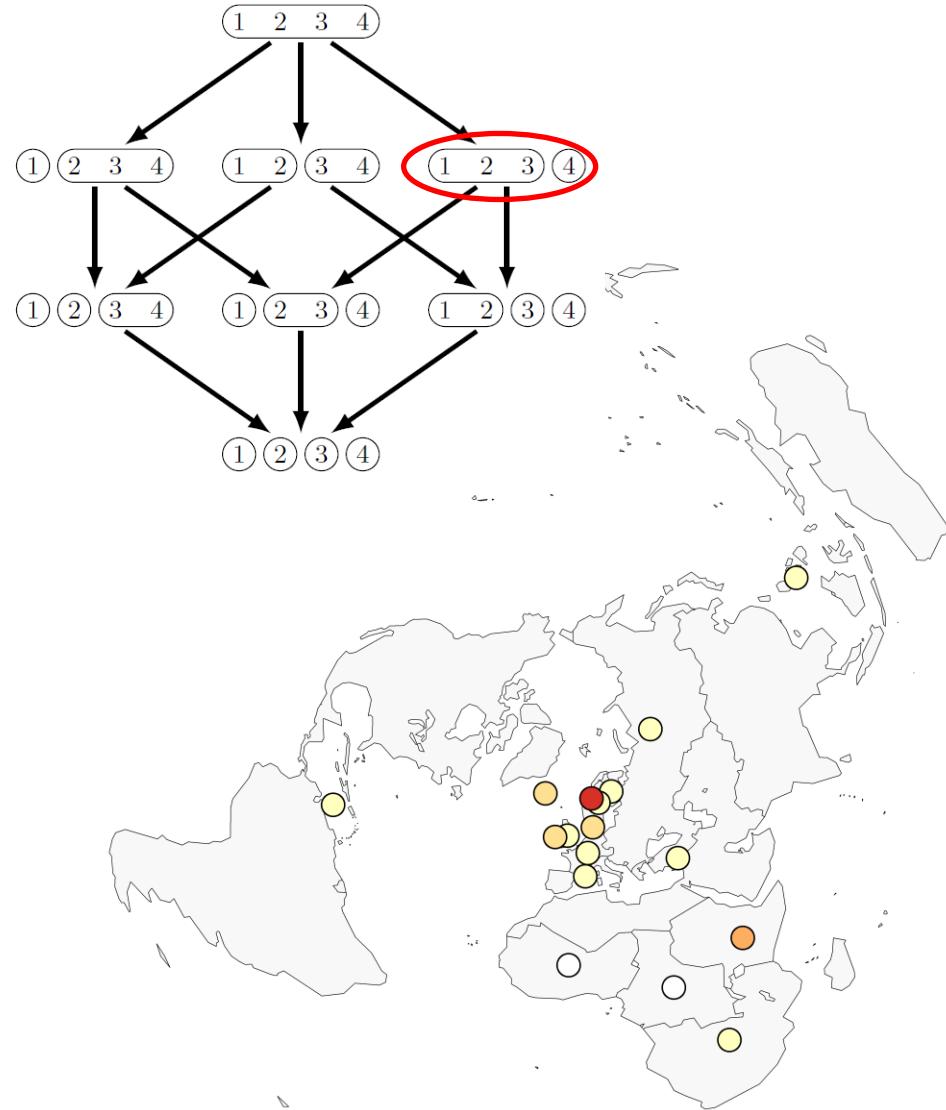
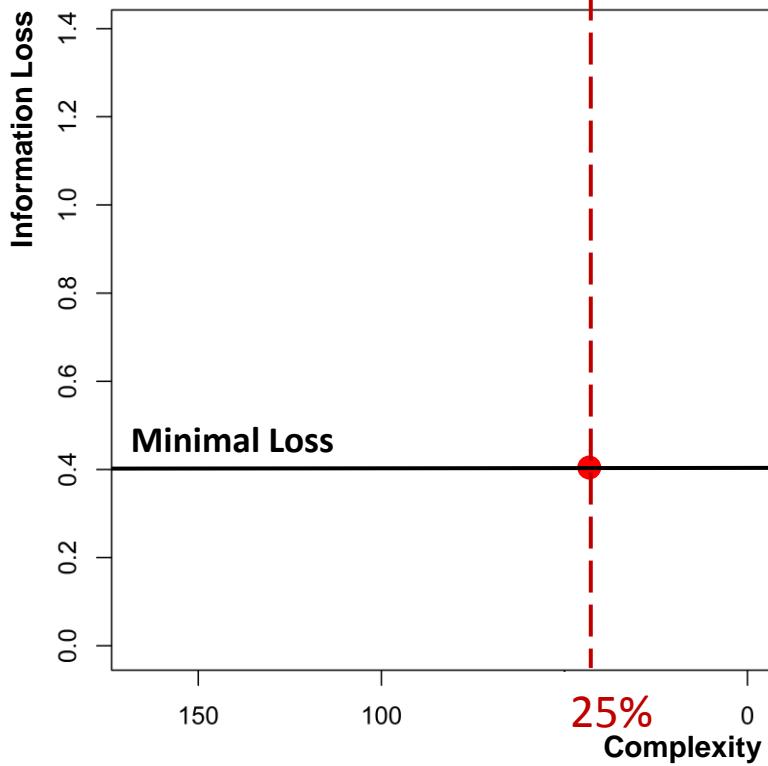


# Optimizing the Partition Qualities

Two criteria that should be optimized

Compromise of Quality:

$$CQL_\alpha = \alpha \frac{\Delta T}{\Delta T_{\max}} - (1 - \alpha) \frac{D}{D_{\max}}$$

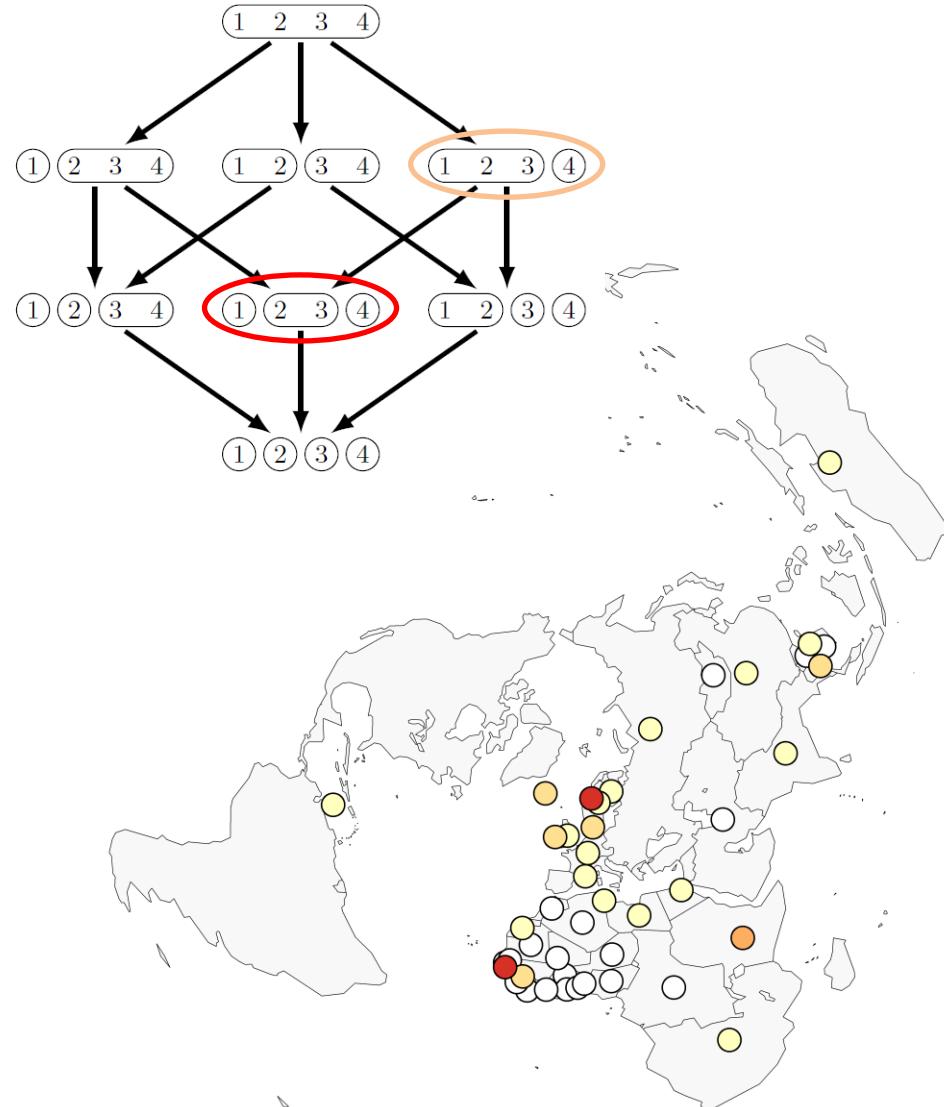
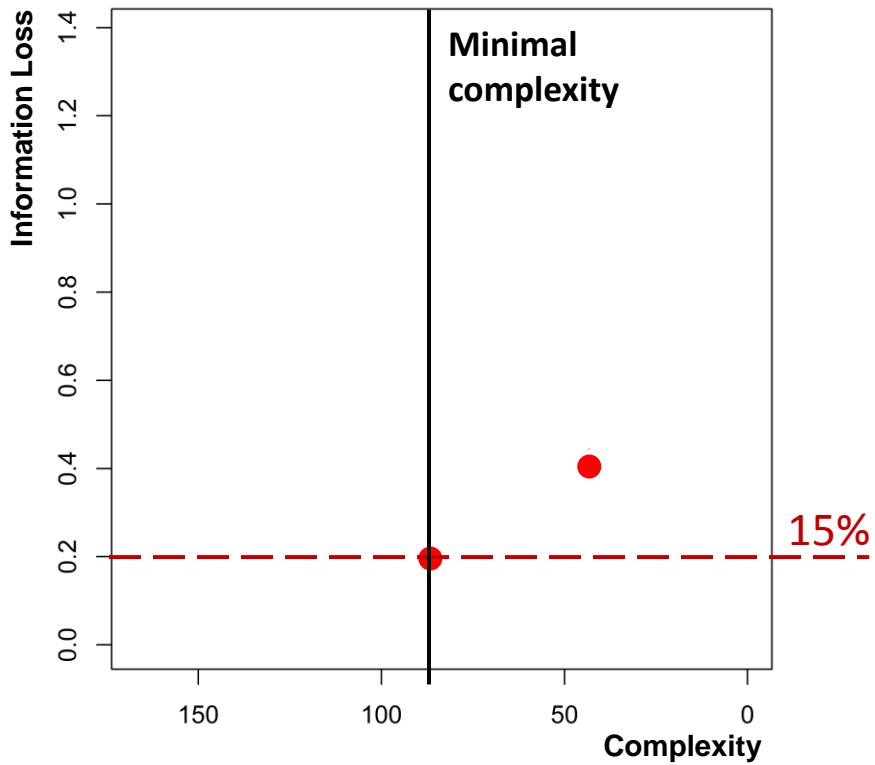


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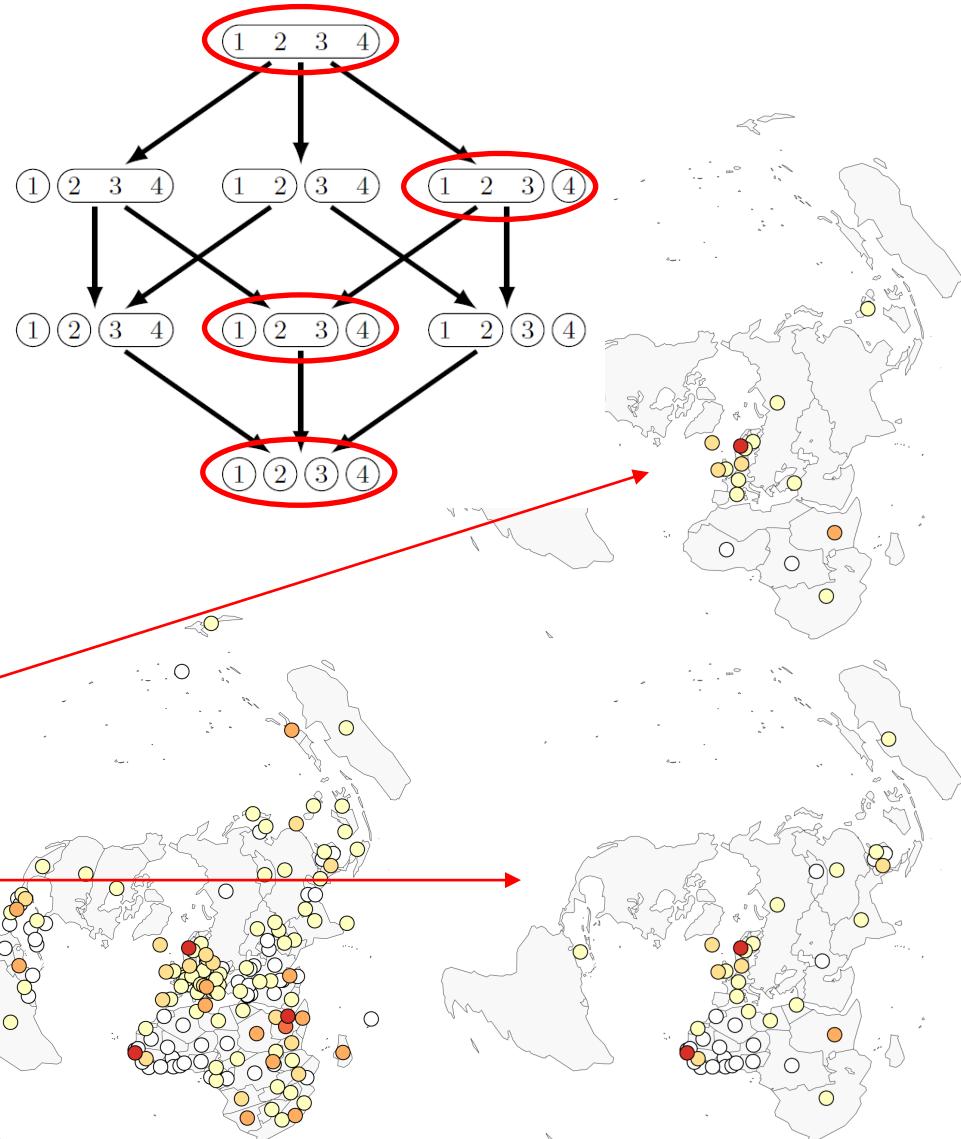
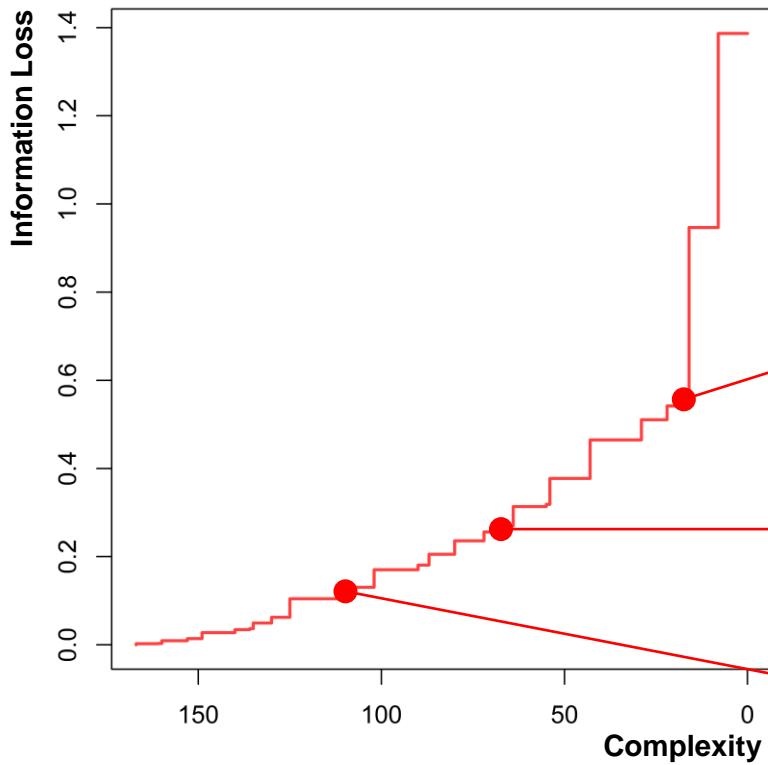


# Optimizing the Partition Qualities

Two criteria that should be optimized

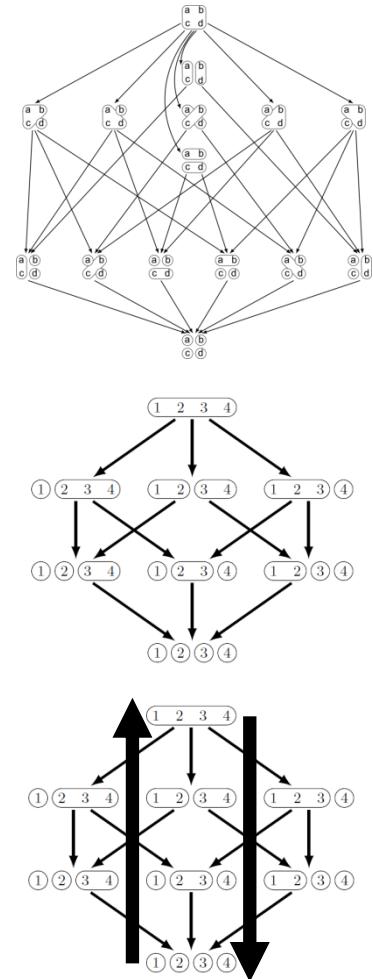
Compromise of Quality:

$$CQL_\alpha = \alpha \frac{\Delta T}{\Delta T_{\max}} - (1 - \alpha) \frac{D}{D_{\max}}$$



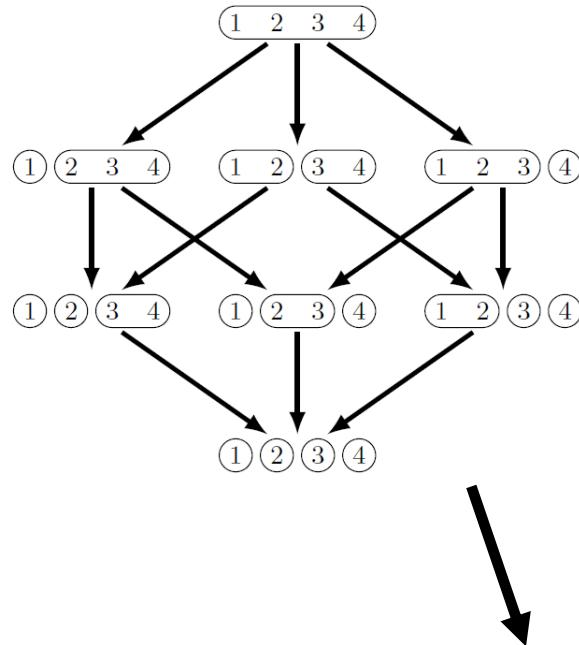
# My Approach

- P0** To characterize the aggregation process  
→ The algebra of possible partitions
- P1** To preserve the system's semantics  
→ A constrained partitioning method  
To aggregate according to several dimension  
→ Some constraints expressing the system's topology
- P2** To evaluate and compare the representations  
→ Some measures of complexity and information  
To offer several granularity levels  
→ The optimization of a compromise
- P3** To compute the best representations  
→ A generic algorithm of constrained optimization



# Objectives

**Set of admissible partitions**



**Quality Measures**

$$\Delta T(\mathcal{X}) = |\Omega| - |\mathcal{X}|$$

$$D(\mathcal{X}) = \sum_{X \in \mathcal{X}} \sum_{x \in X} \frac{v(x)}{v(\Omega)} \log_2 \left( \frac{v(x)|X|}{v(X)} \right)$$

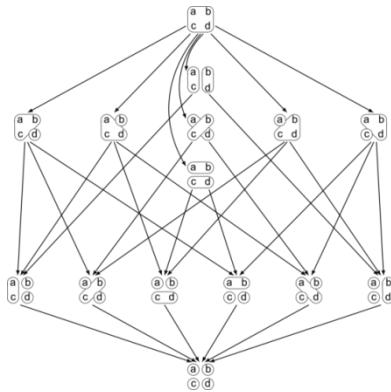
$$CQL_\alpha = \alpha \frac{\Delta T}{\Delta T_{\max}} - (1 - \alpha) \frac{D}{D_{\max}}$$



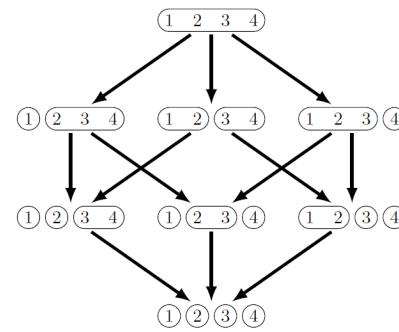
**The Admissible Optimal Partitions Problem:** Which admissible partitions optimize a given compromise of quality?  
 → A well-known constrained optimization problem

# Exponential Algorithmic Complexity

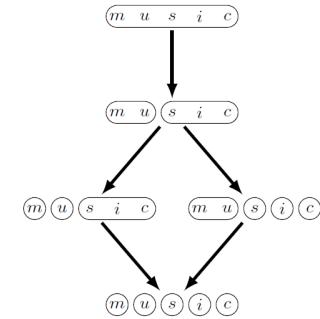
**Non-constrained Set**



**Ordered Set**



**Hierarchical Set**



**Number of admissible partitions**  
( $n$  = size of the system)

$$\Theta(e^n \log n)$$

$$\Theta(2^n)$$

$$\Theta(c^n)$$

[Berend and Tassa, 2010]

[Lamarche-Perrin *et al.*, IAT 2013]

# Toward an Efficient Algorithm

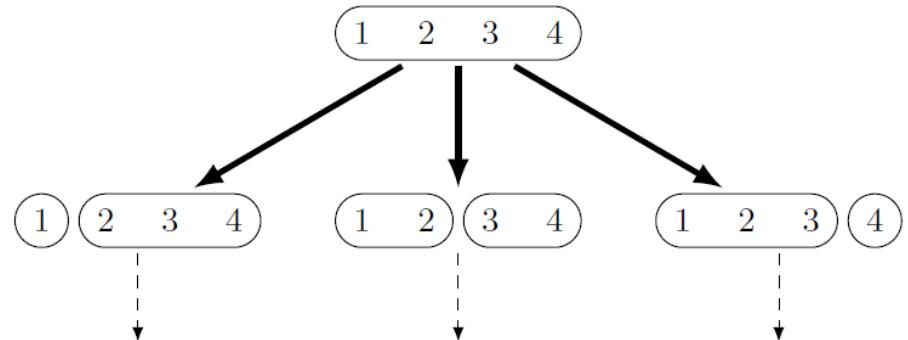
- **Classical clustering techniques** uses some heuristics to find local optima [Halkidi *et al.*, 2001]
- **In our case:**
  - The **admissibility constraints** allow to reduce the complexity of the optimization problem
  - The **algebraic properties** of the quality measures allow to efficiently compare the partitions

→ We look for the global optima

# Idea of the Algorithm

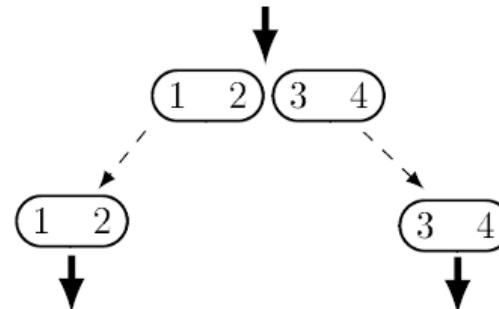
DIVIDE...

Decomposition according  
to the refinement relation



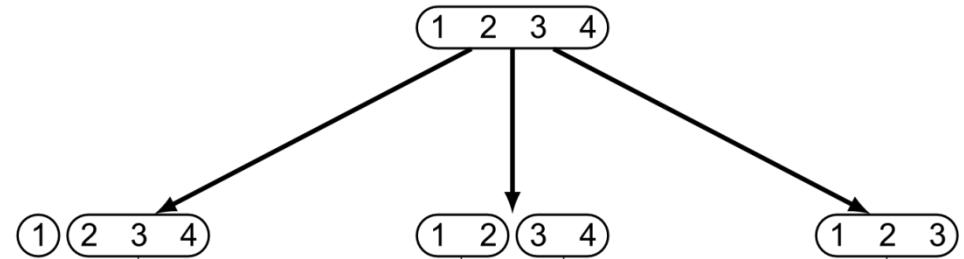
...AND CONQUER

Recursion according to the  
decomposability of measures

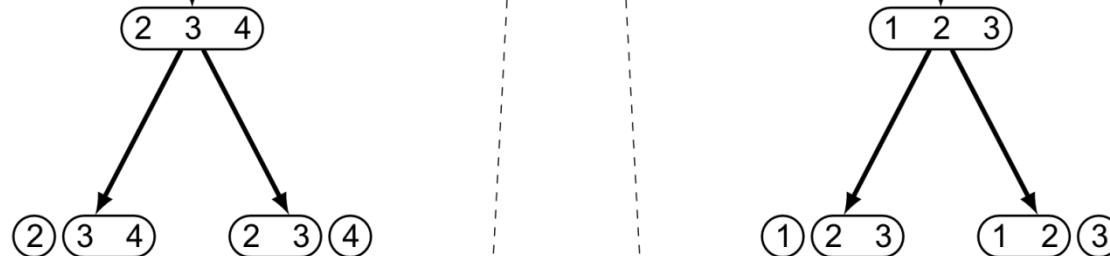


# Execution of the Algorithm

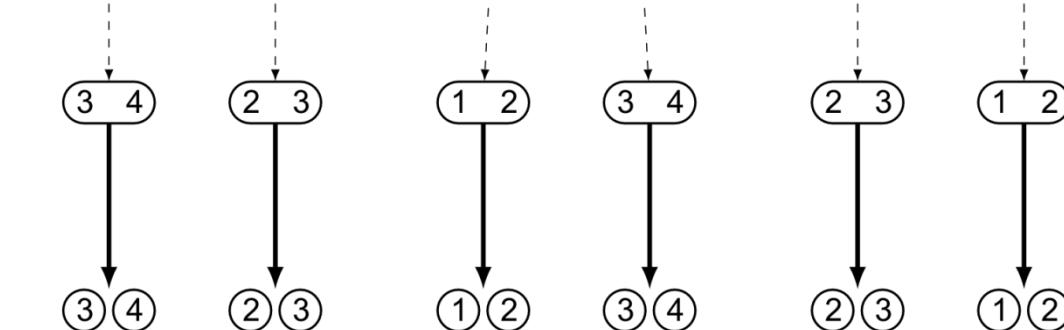
Decomposition



Recursion

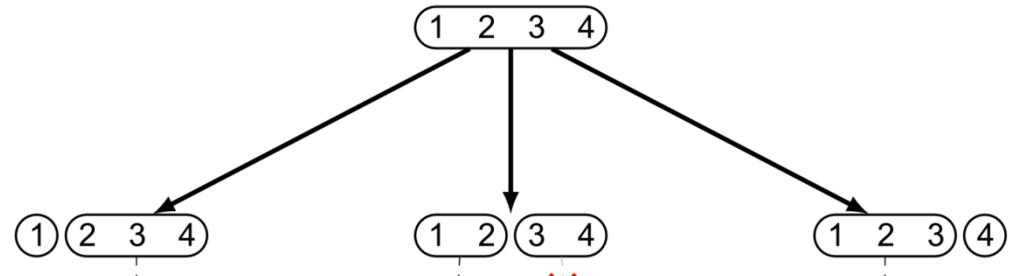


Decomposition



# Execution of the Algorithm

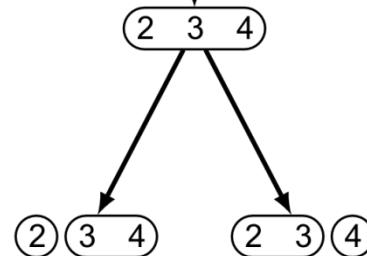
**Decomposition**



**Recursion**

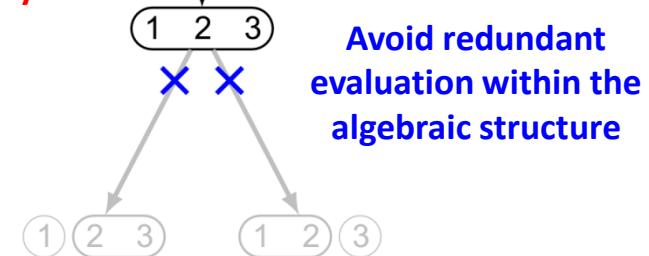
**Record the intermediary results**

**Decomposition**

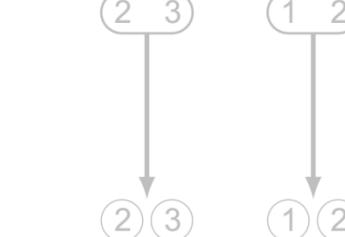


**Recursion**

**Decomposition**



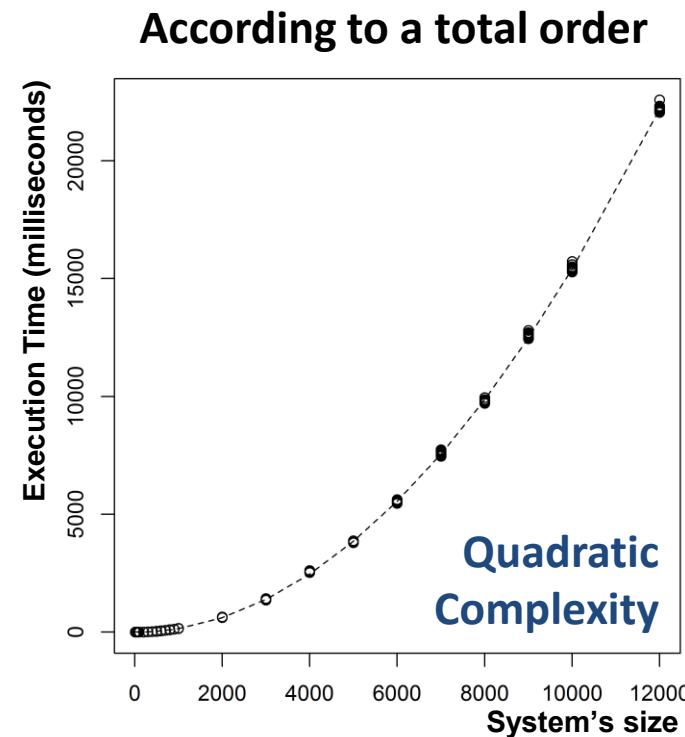
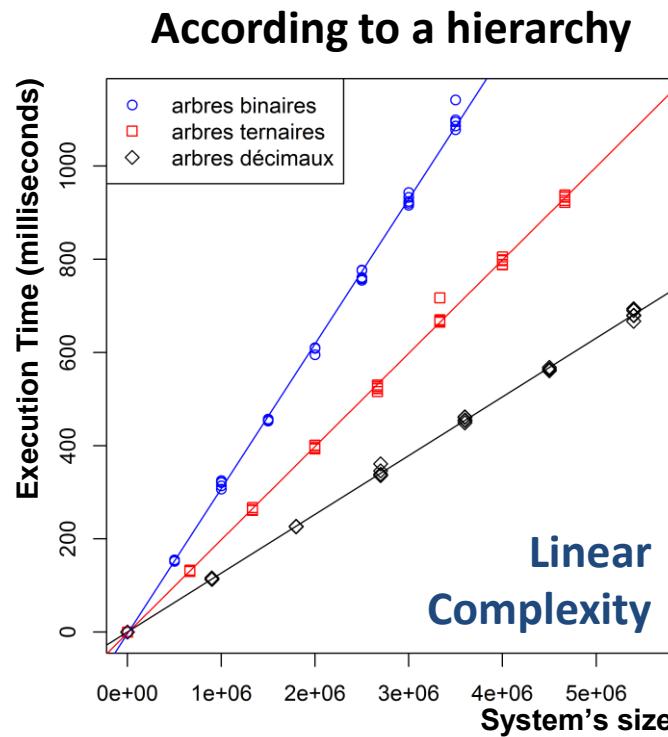
**Avoid redundant evaluation within the algebraic structure**



# Complexity of the Algorithm

The algorithm spatial and temporal complexity directly depend on the algebraic properties of the set of admissible partitions

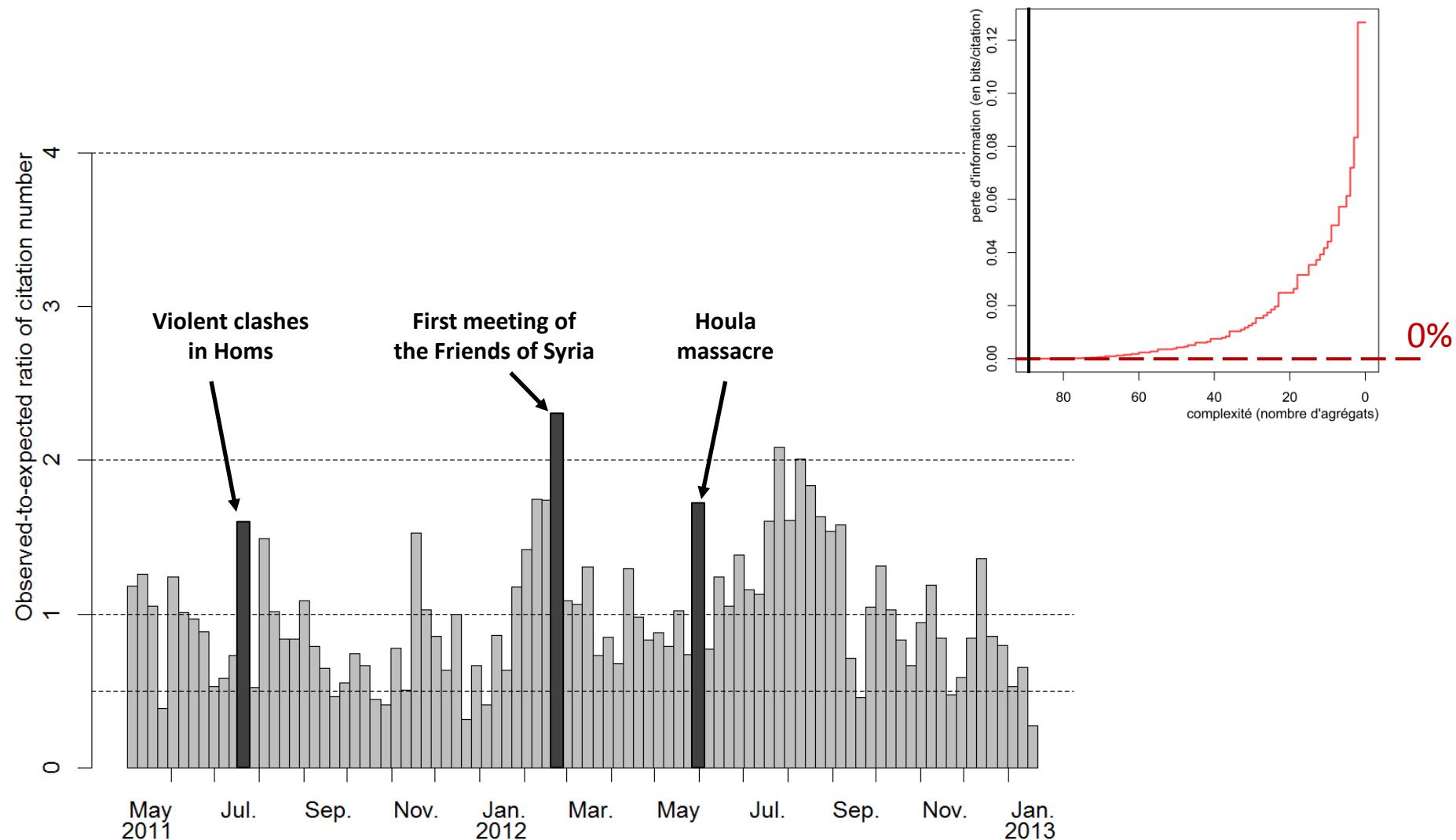
**the more constrained, the less complex**



# **EXPERIMENTS AND RESULTS**

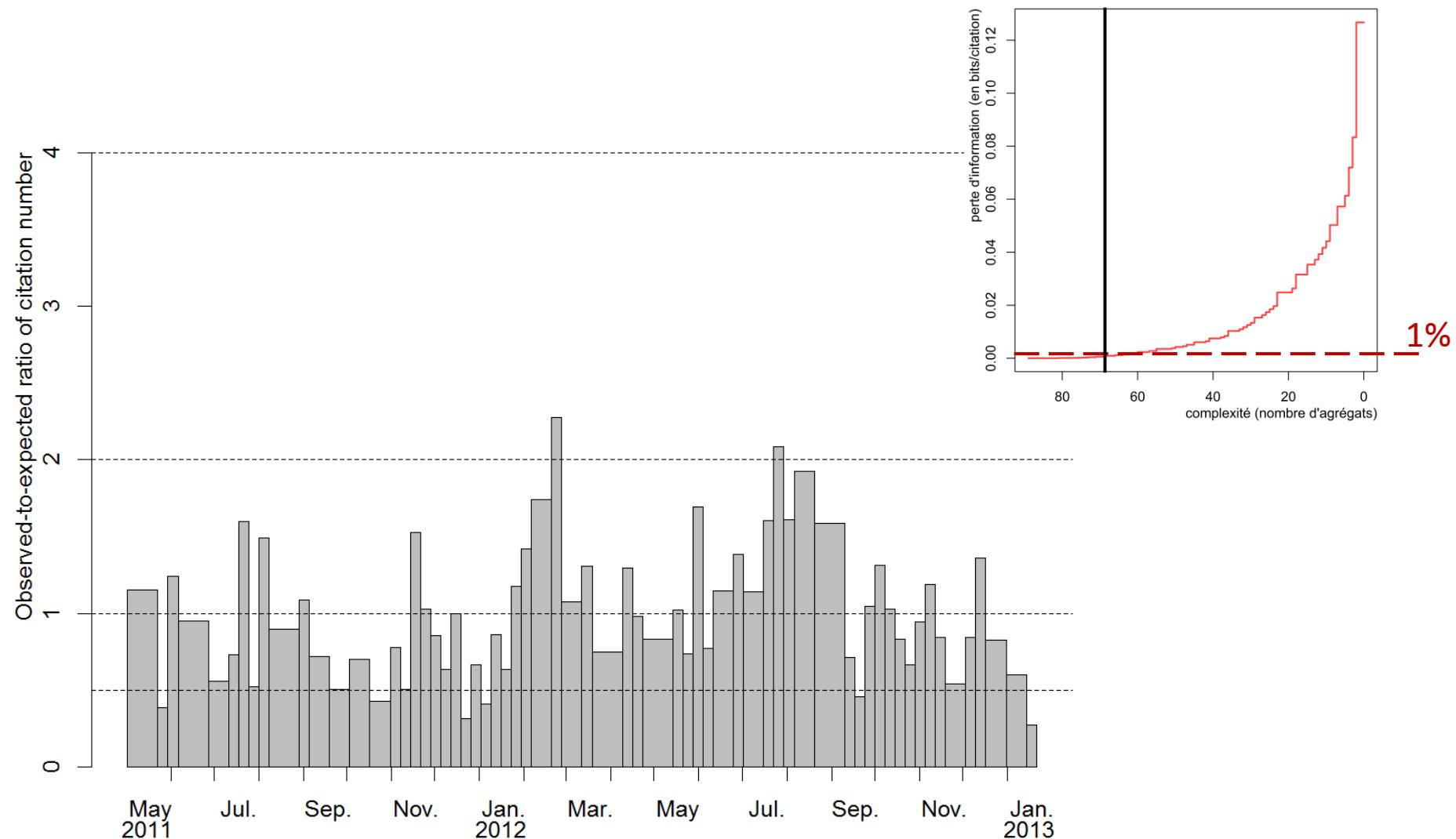
# The Syrian civil war according to LE MONDE

[Giraud, Grasland, Lamarche-Perrin *et al.*, ECTQG 2013]



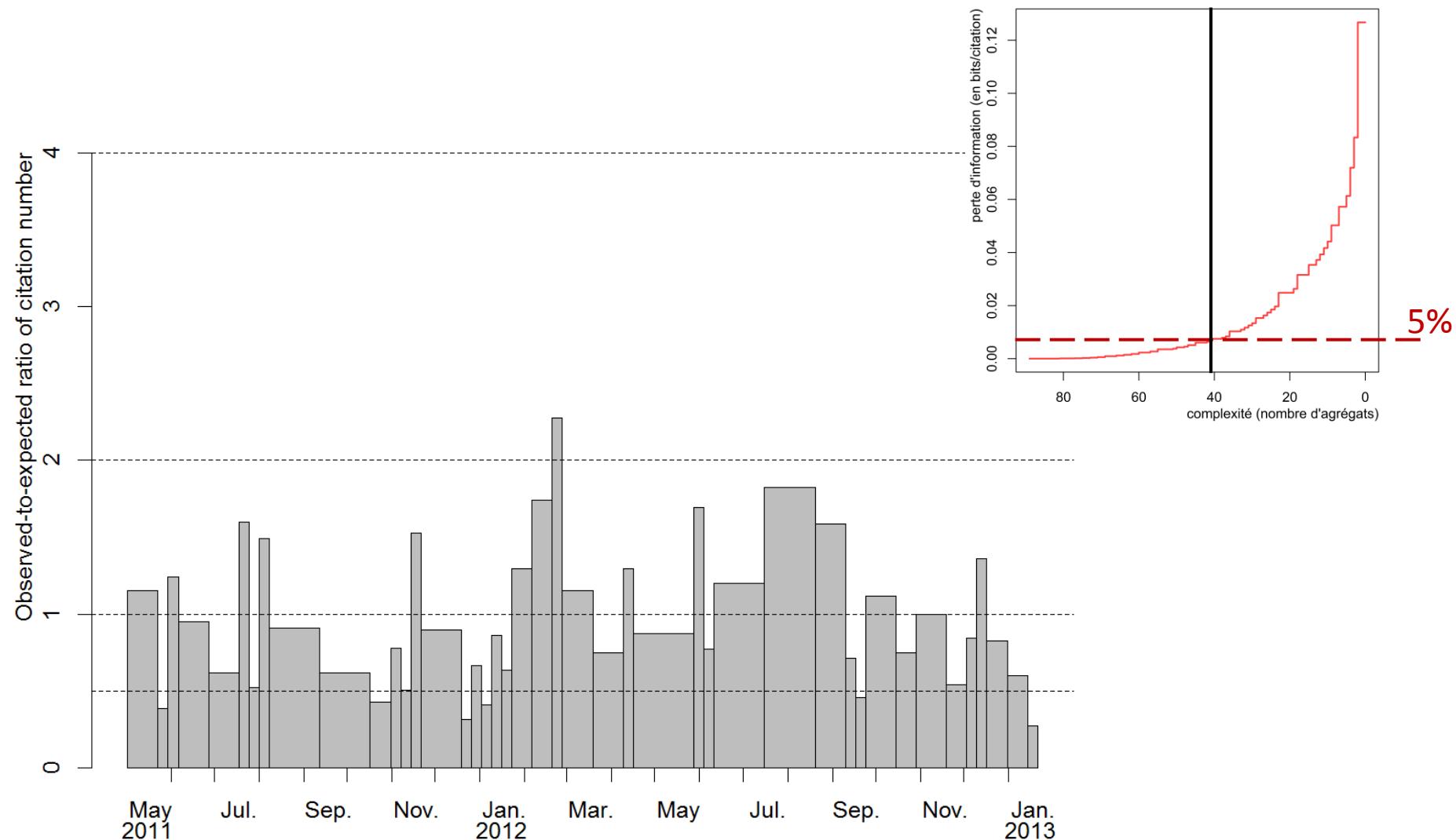
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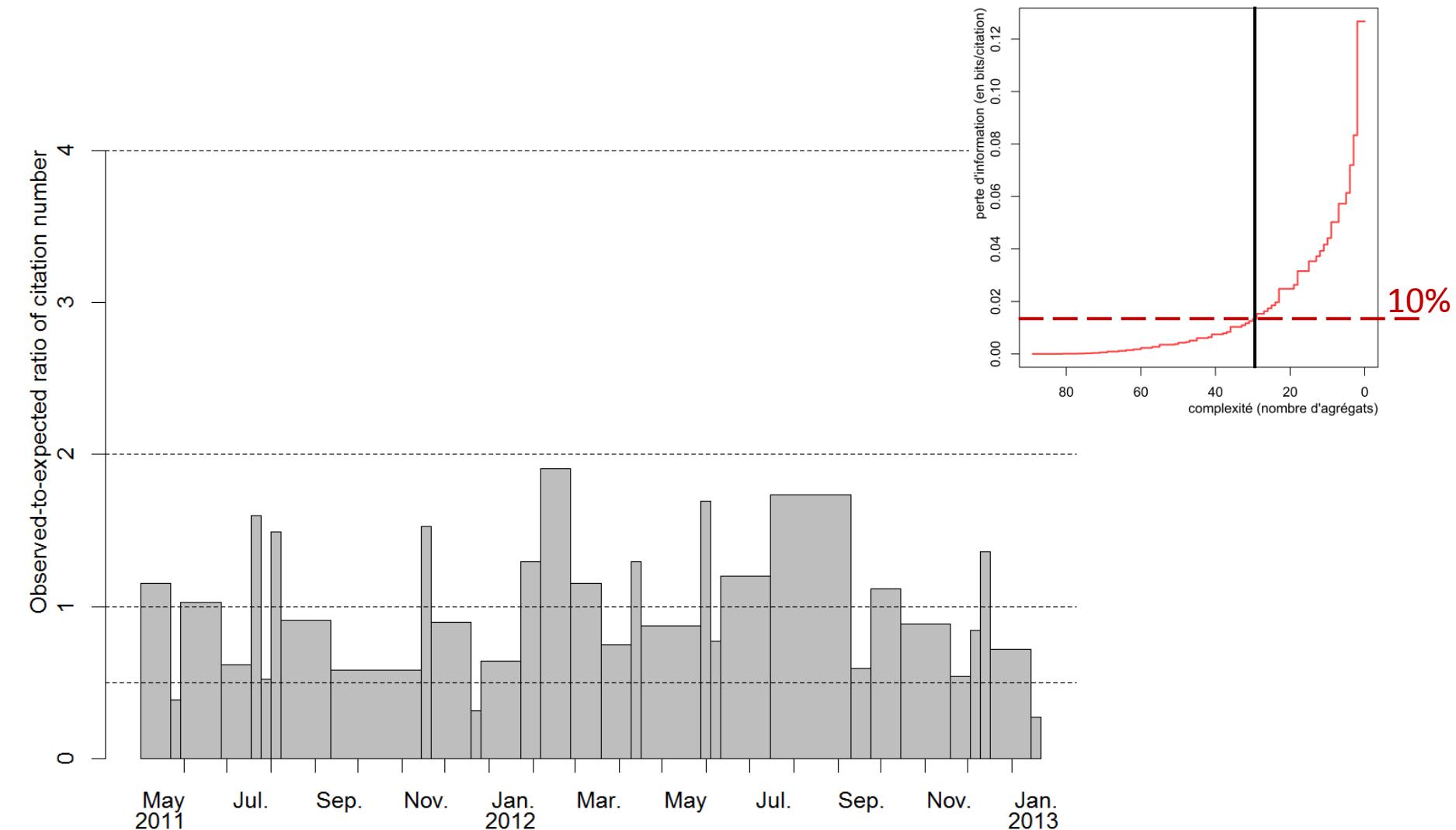
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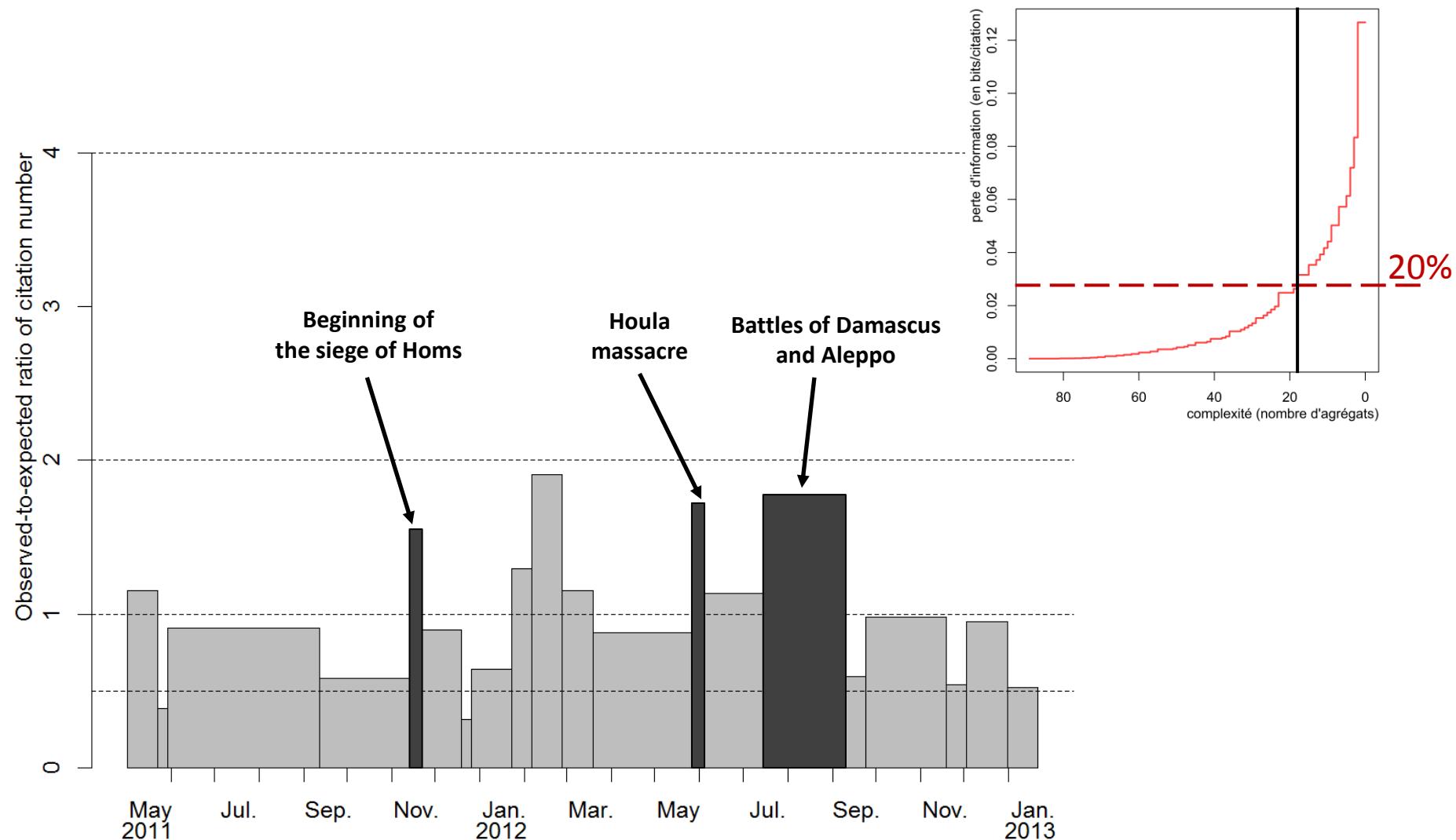
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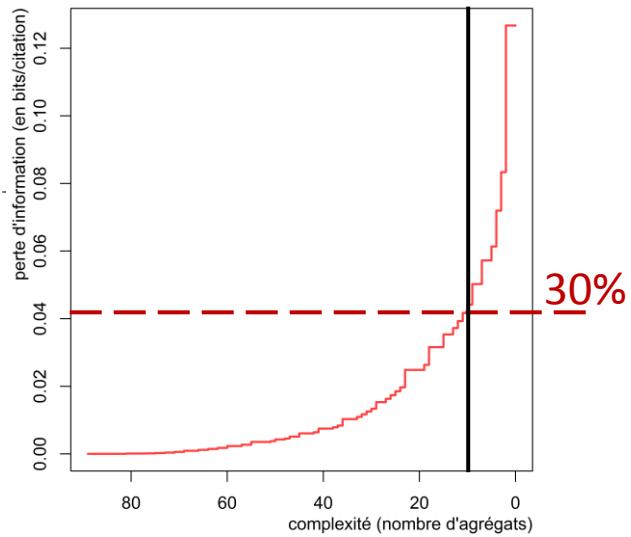
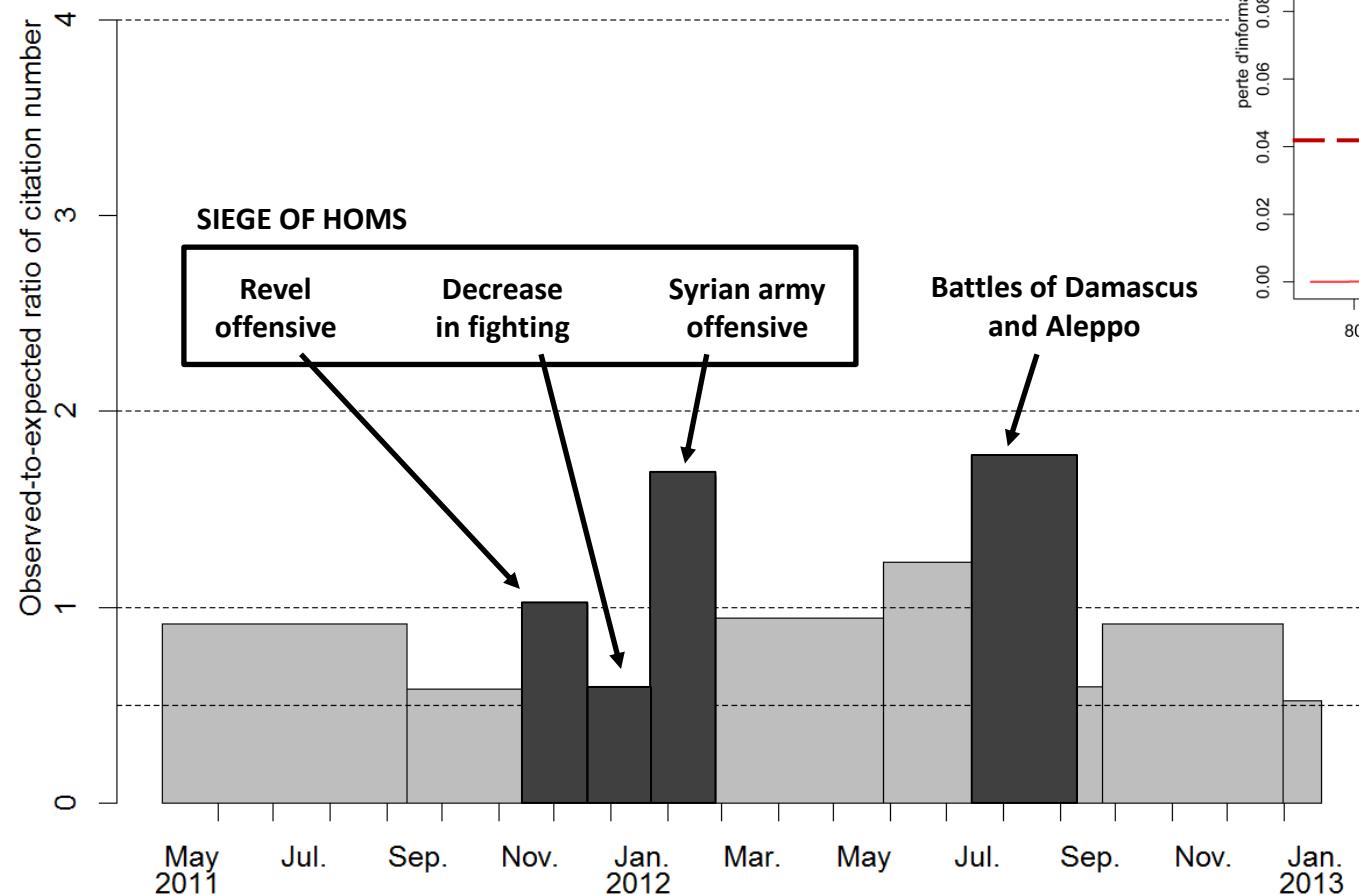


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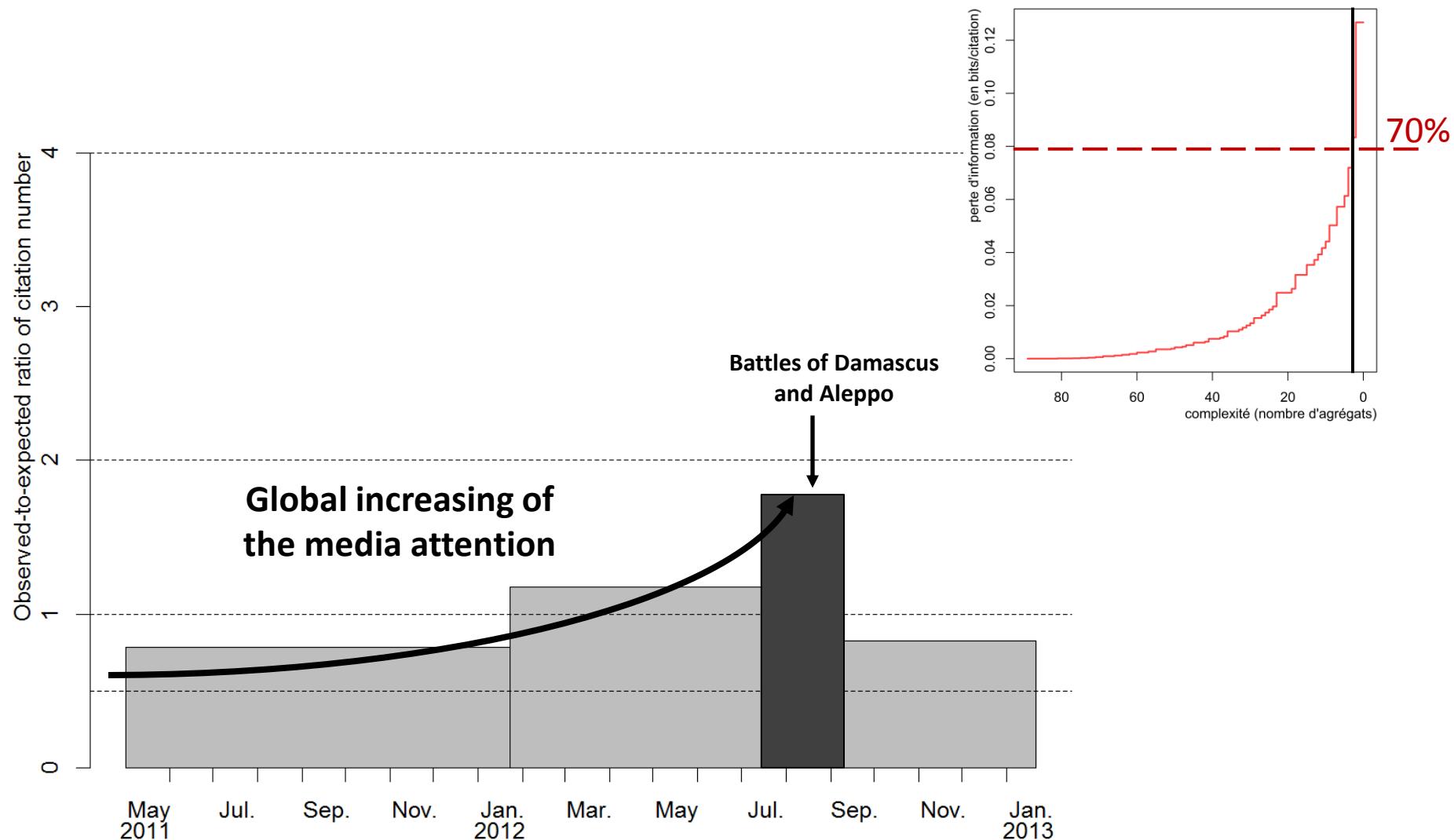
Source: Wikipedia

*Timeline of the Syrian civil war*  
Siege of Homs



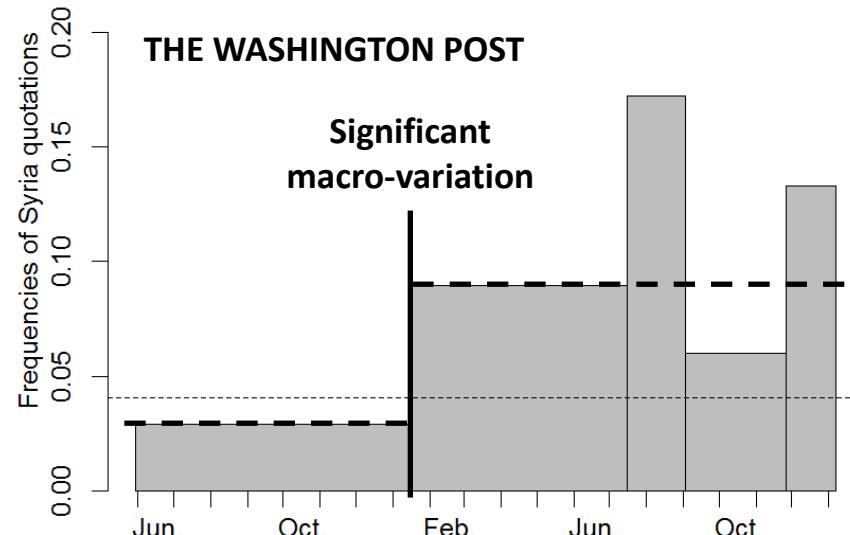
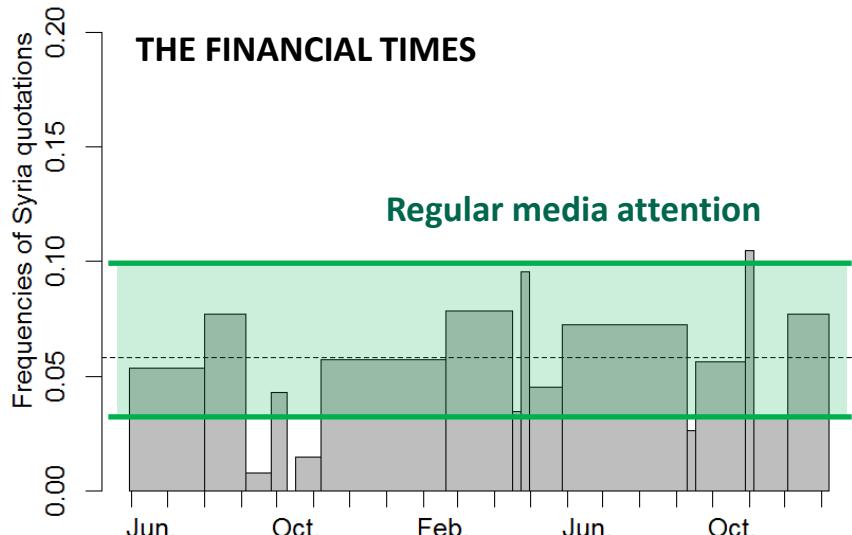
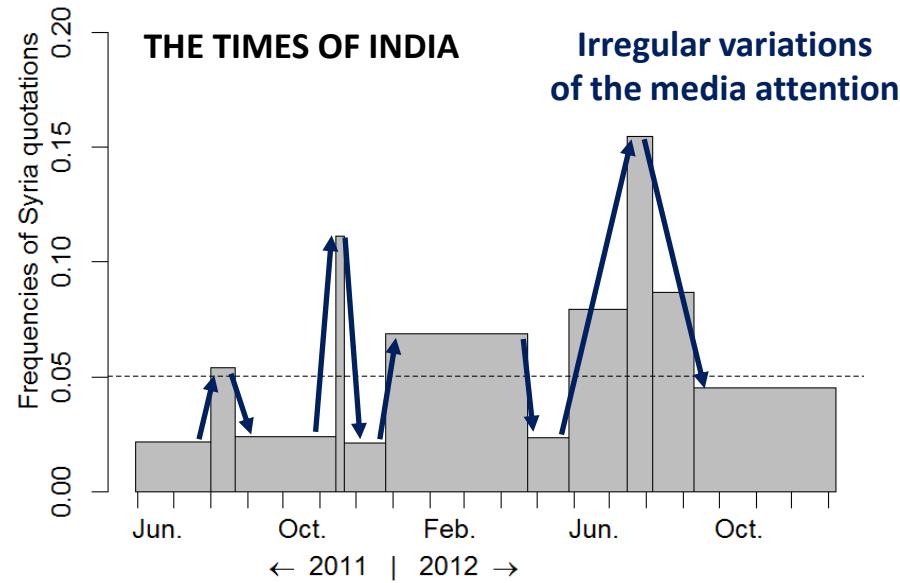
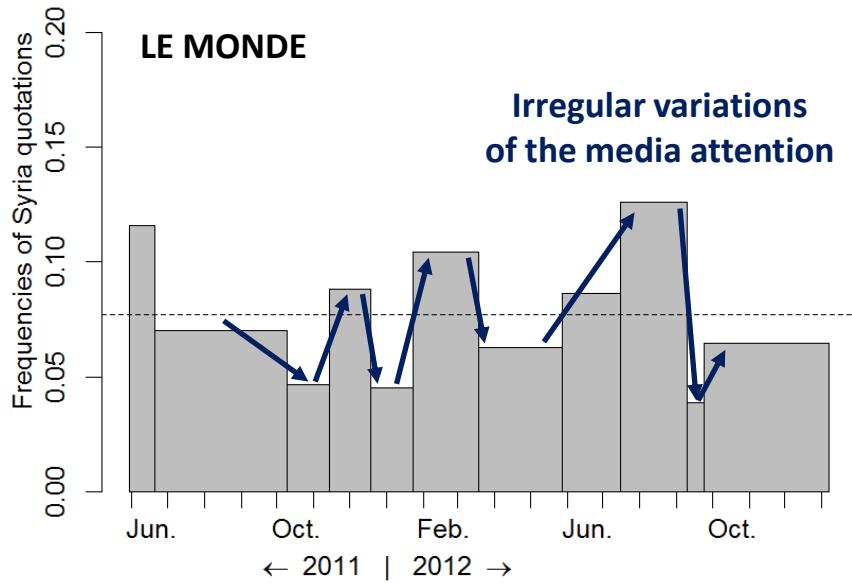
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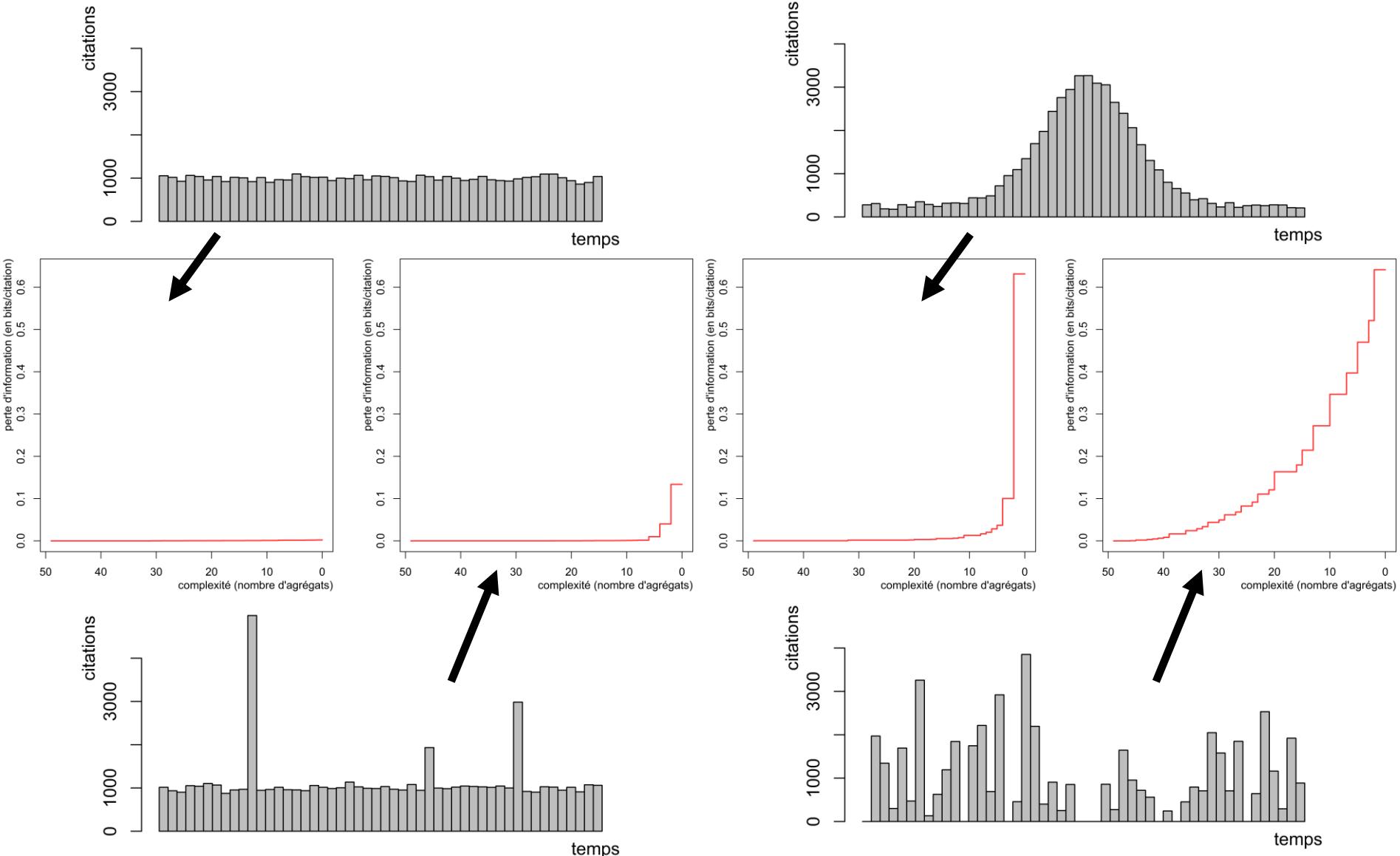


# The Syrian civil war according to 4 newspapers

[Giraud, Grasland, Lamarche-Perrin *et al.*, ECTQG 2013]



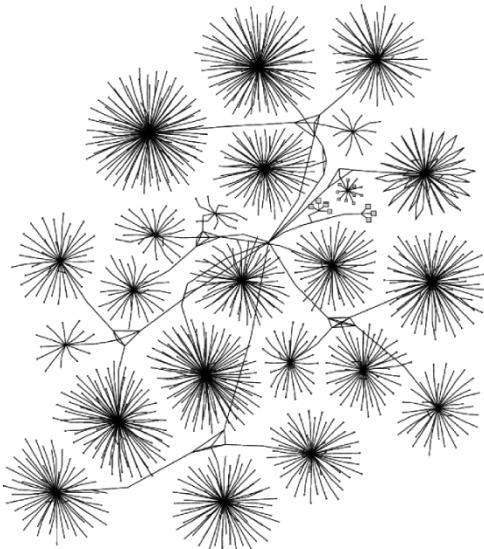
# Characterizing Different Datasets



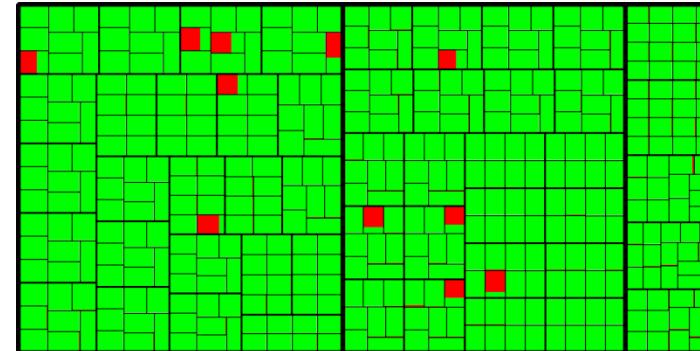
# Aggregation of Execution Traces

[Lamarche-Perrin, Schnorr *et al.*, TSI 2013]

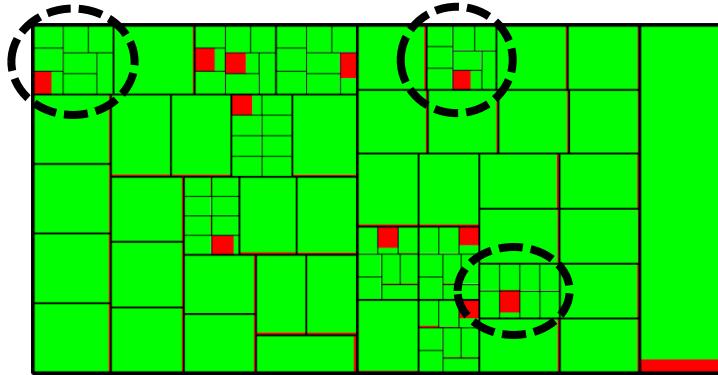
Hierarchical structure  
of the grid computing  
[Schnorr *et al.*, 2013]



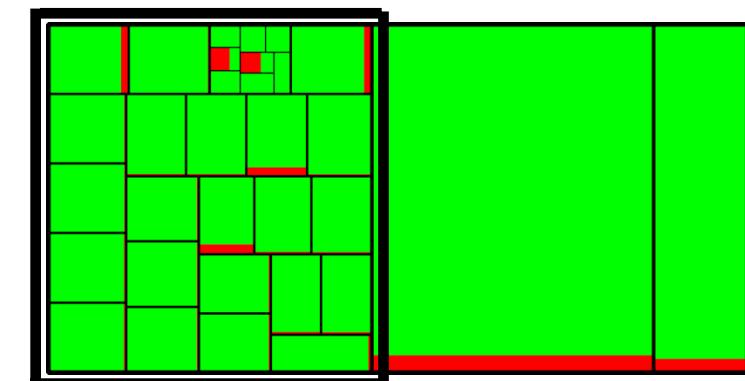
Microscopic treemap  
representation



Detection of  
multi-scale  
anomalies

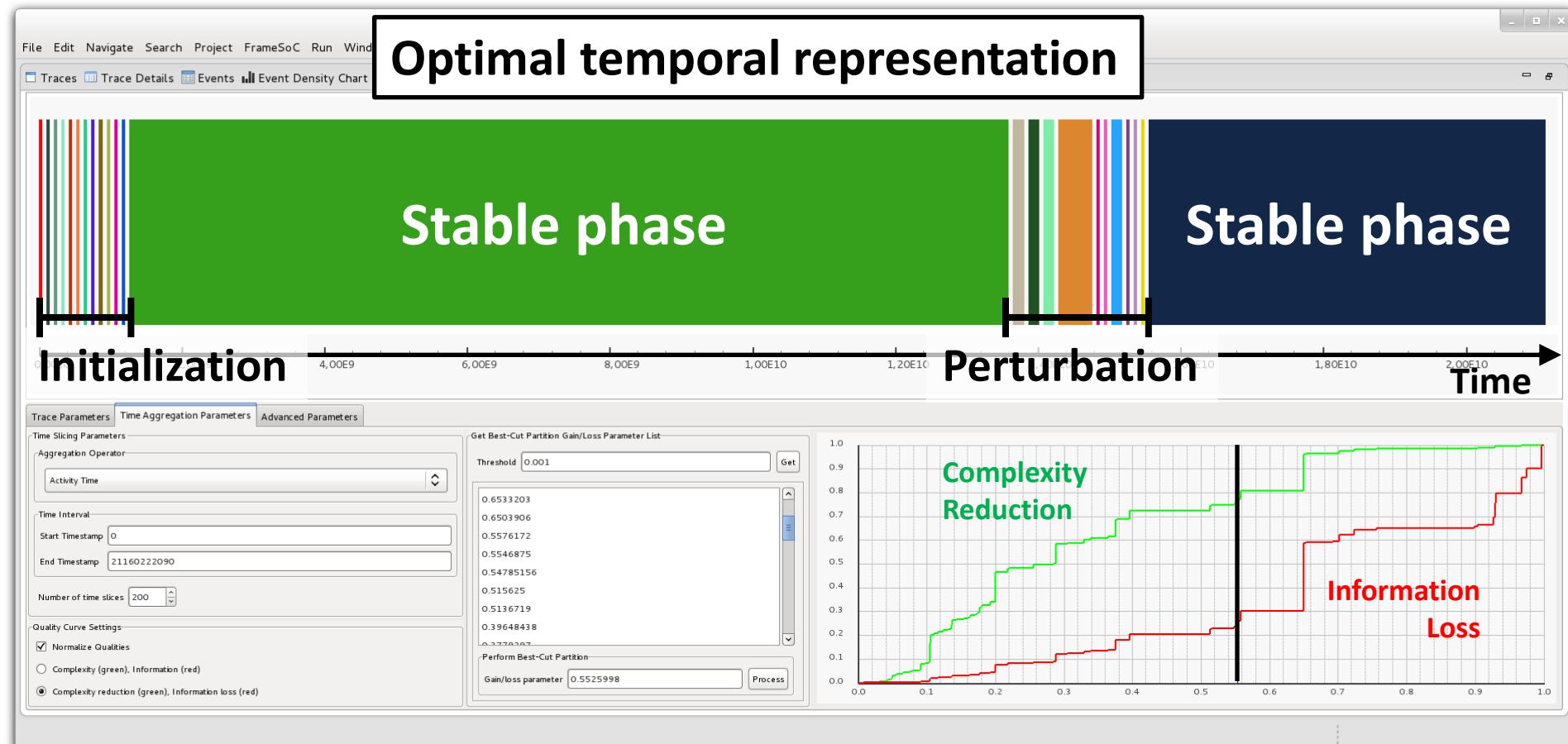


Aggregated treemap representations



# Aggregation of Execution Traces

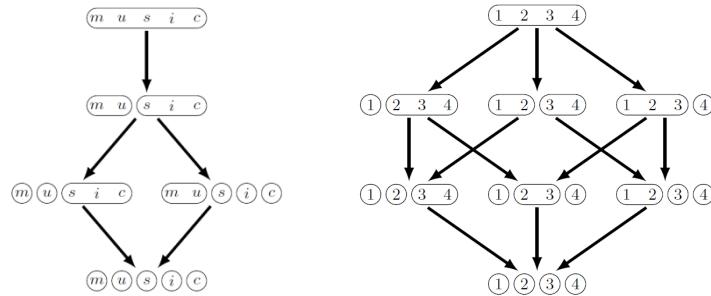
[Pagano, Dosimont *et al.*, 2013]



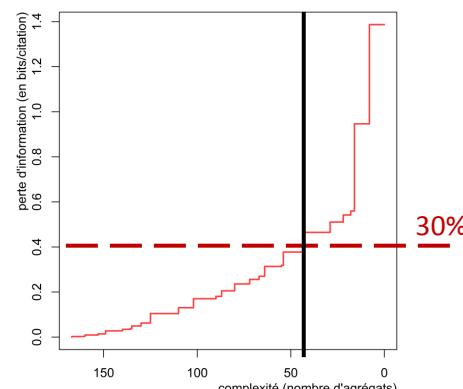
# **OUTCOMES AND PERSPECTIVES**

# Summary of the Contributions

**P1** Algebraic structures that express the system's semantic properties



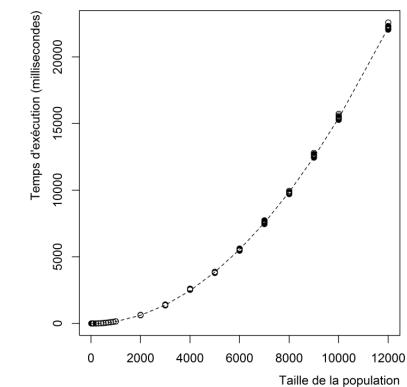
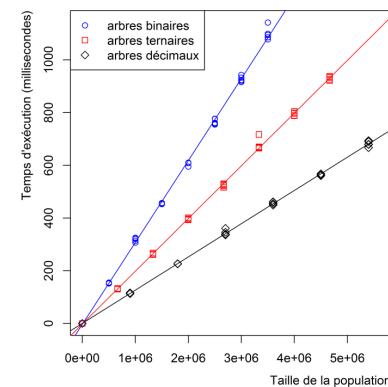
**P2** Graphs of quality to choose the representations granularity



**P2** A compromise of quality to generate multi-resolution representations

$$CQL_\alpha = \alpha \frac{\Delta T}{\Delta T_{\max}} - (1 - \alpha) \frac{D}{D_{\max}}$$

**P3** A generic aggregation algorithm with polynomial complexity

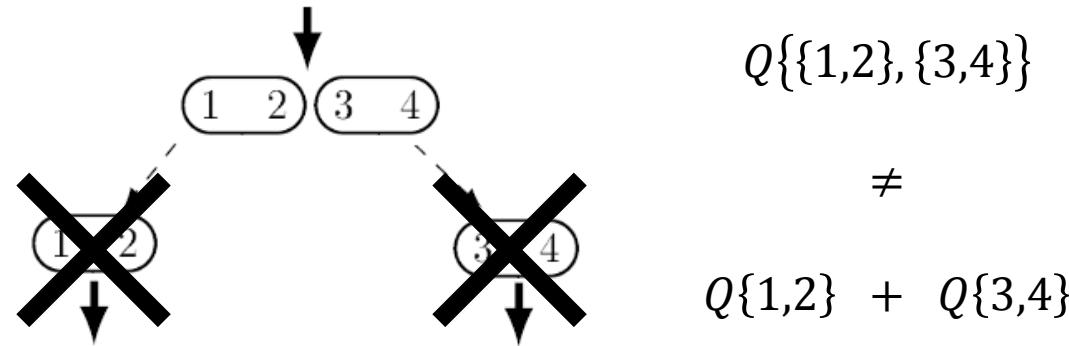


# Summary of the Hypotheses

## **Microscopic** observation tools

- P0** Aggregation according to **partitions**
- P1** Hierarchical or ordered systems
- P2** Decomposable quality measures

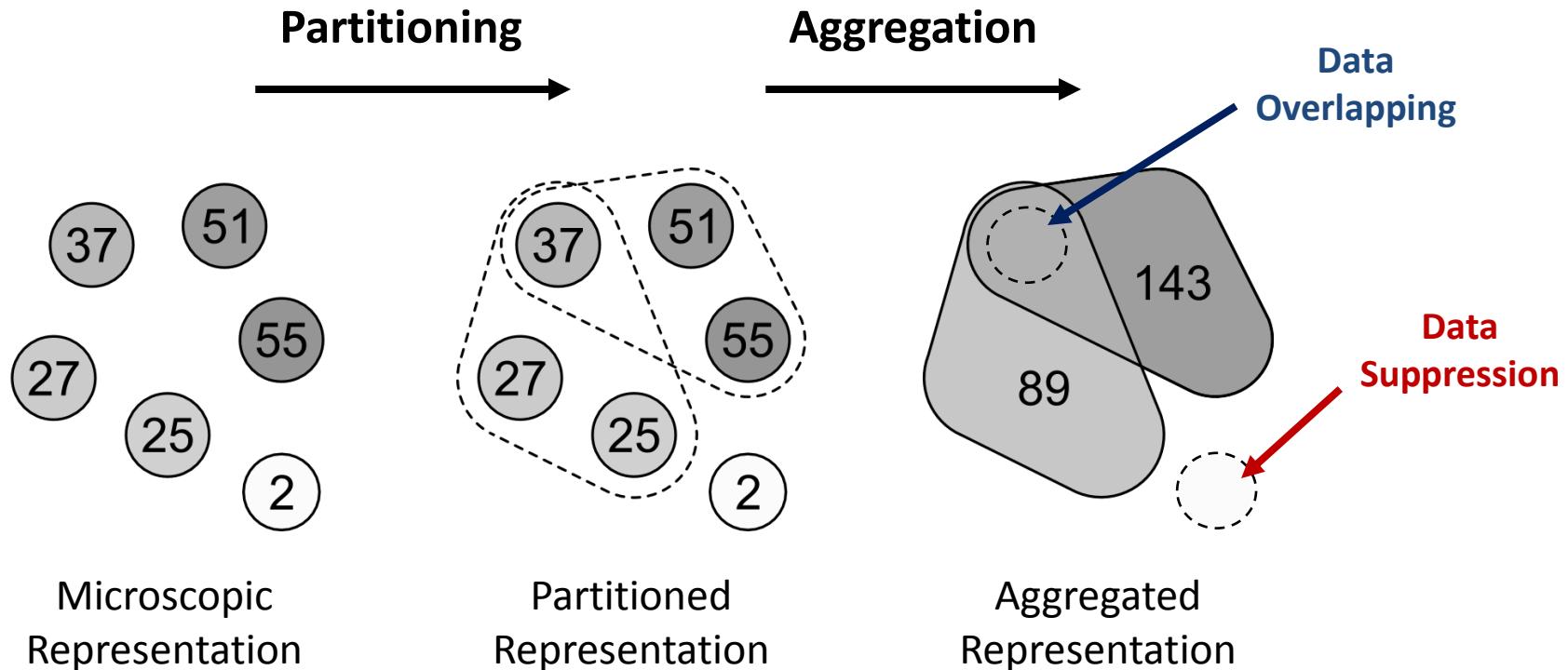
# Non-decomposable Measures



- The recursive approach is no longer possible
- The quality of a part cannot be defined

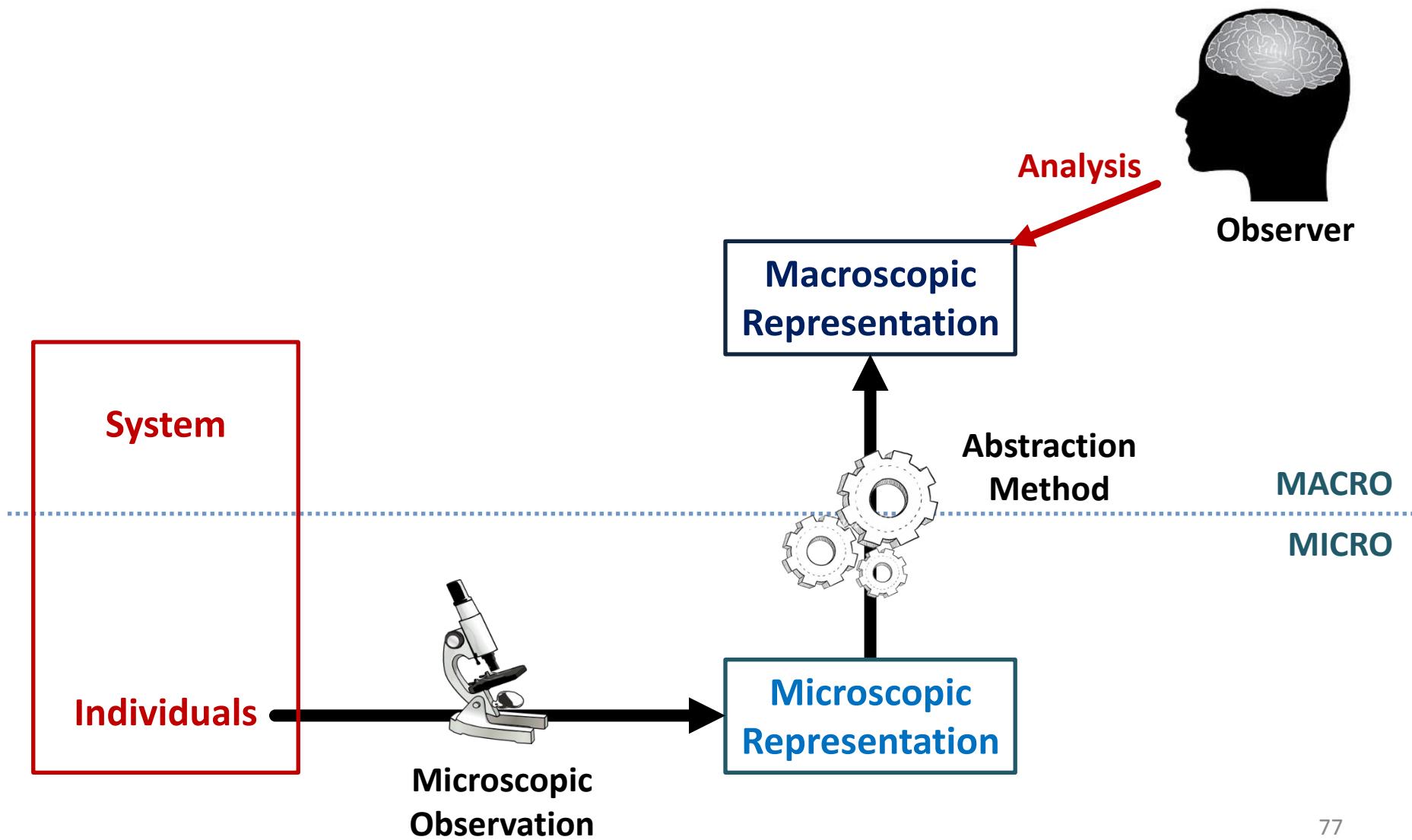
→ Do non-decomposable quality measures have a meaning to evaluate the aggregation process?

# Non-disjoint and Non-covering Parts

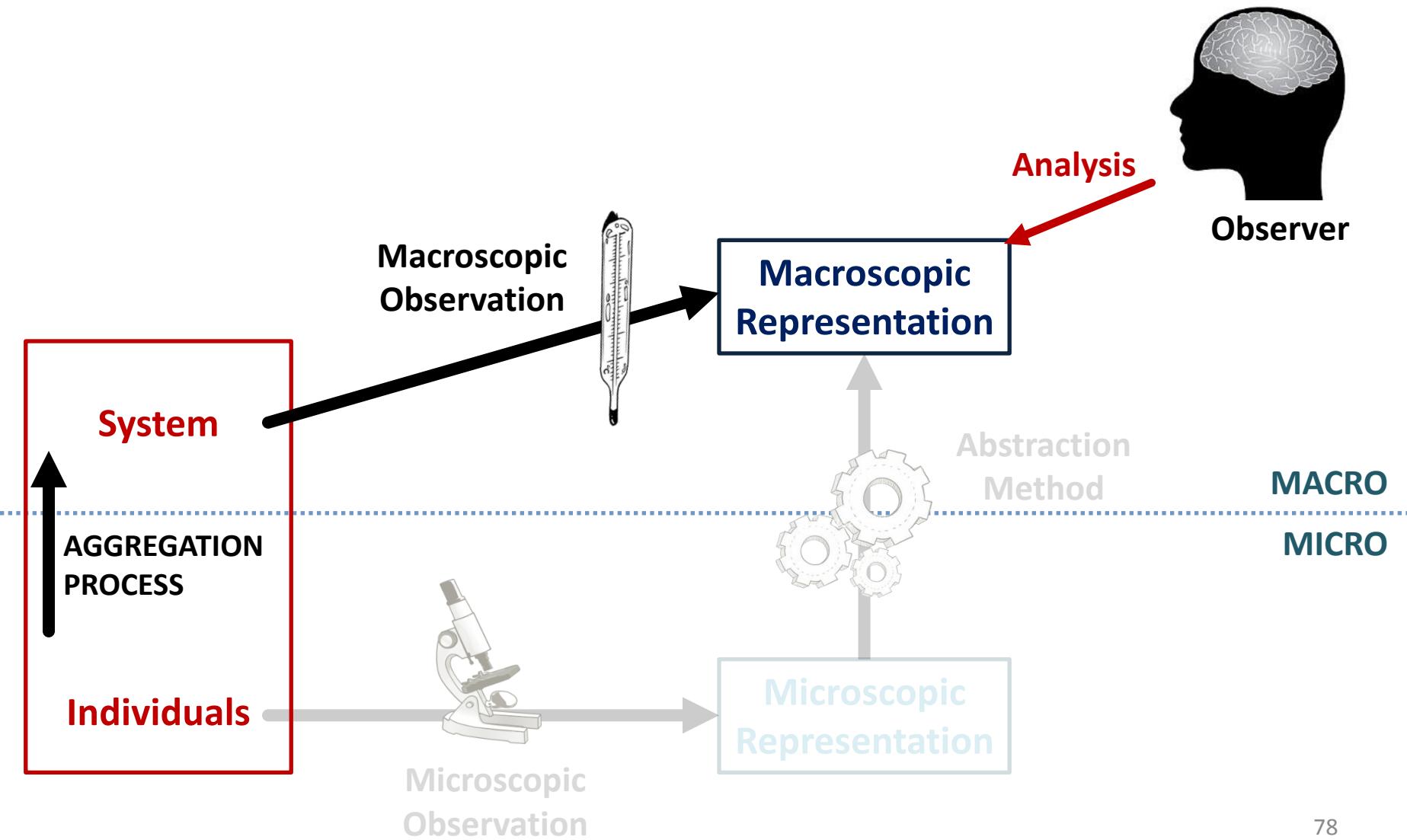


→ Effect on the algorithmic complexity?

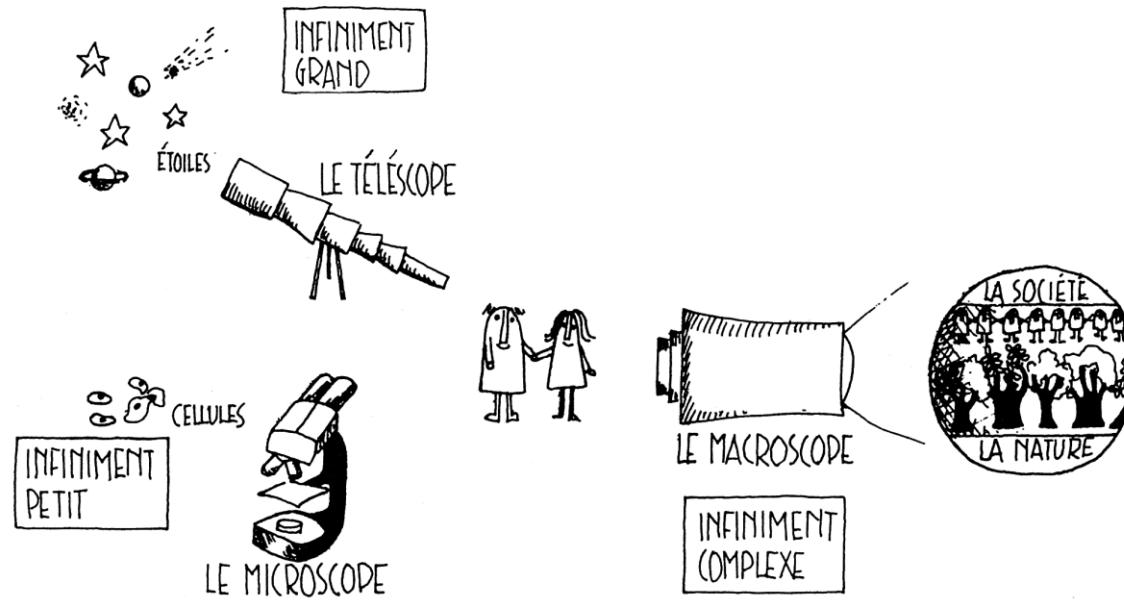
# Macroscopic Observation



# Macroscopic Observation



# THANKS FOR YOUR ATTENTION



« Aujourd’hui nous sommes confrontés à un autre infini : l’infiniment complexe. Mais cette fois, plus d’instrument. »

Joël de Rosnay, *Le macroscope*, 1975

# LIST OF PUBLICATIONS

## Peer-reviewed Journals (being accepted)

Lamarche-Perrin, Demazeau et Vincent. **Building the Best Macroscopic Representations of Complex Multi-Agent Systems.** *Transaction on Computational Collective Intelligence (TCCI)*, 2014.

Lamarche-Perrin, Schnorr, Vincent et Demazeau. **Agrégation de traces pour la visualisation de grands systèmes distribués.** *Technique et Science Informatiques (TSI)*, 2014.

## International Peer-reviewed Conferences with Proceedings

Lamarche-Perrin, Demazeau et Vincent. **The Best-partitions Problem: How to Build Meaningful Aggregations.** *Intelligent Agent Technology (IAT)*, Atlanta, 2013.

Giraud, Grasland, Lamarche-Perrin, Demazeau et Vincent. **Identification of International Media Events by Spatial and Temporal Aggregation of RSS Flows of Newspapers.** *European Colloquium in Theoretical and Quantitative Geography (ECTQG)*, Dourdan, 2013.

Lamarche-Perrin, Demazeau and Vincent. **How to Build the Best Macroscopic Description of your Multi-agent System? Practical Applications of Agents and Multi-Agent Systems (PAAMS)**, Salamanca, 2013.

## French Peer-reviewed Conferences with Proceedings

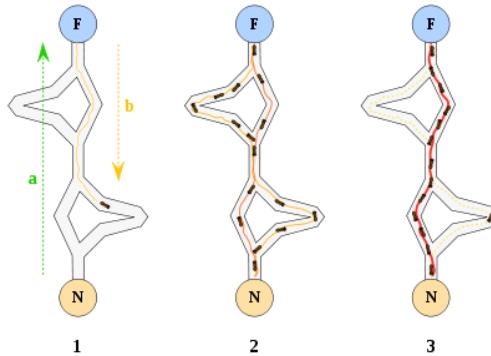
Lamarche-Perrin, Demazeau et Vincent. **Organisation, agrégation et visualisation d'informations médiatiques.** *Colloque annuel du Collège des Sciences du Territoire*, Paris, 2011.

Lamarche-Perrin, Demazeau et Vincent. **Observation macroscopique et émergence dans les SMA de très grande taille.** *Journées Francophones des Systèmes Multi-Agents (JFSMA)*, Valenciennes, 2011.

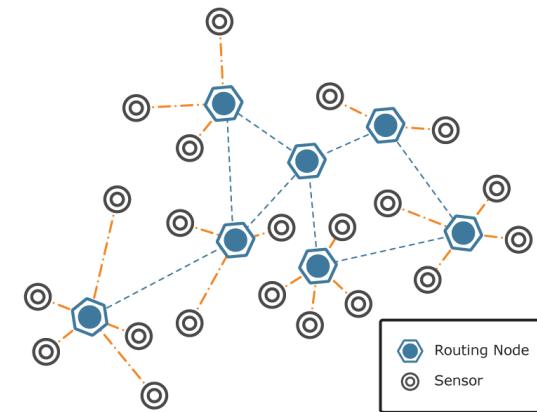
# Application Perspectives

## Multi-agent Systems

[Lamarche-Perrin *et al.*, JFSMA 2011]

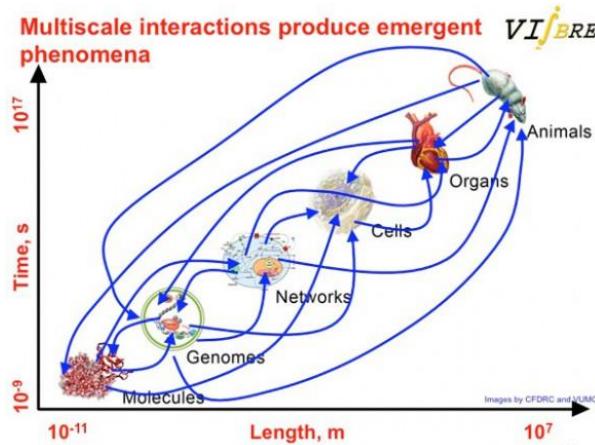


## Sensor Networks



## Multi-scale simulation

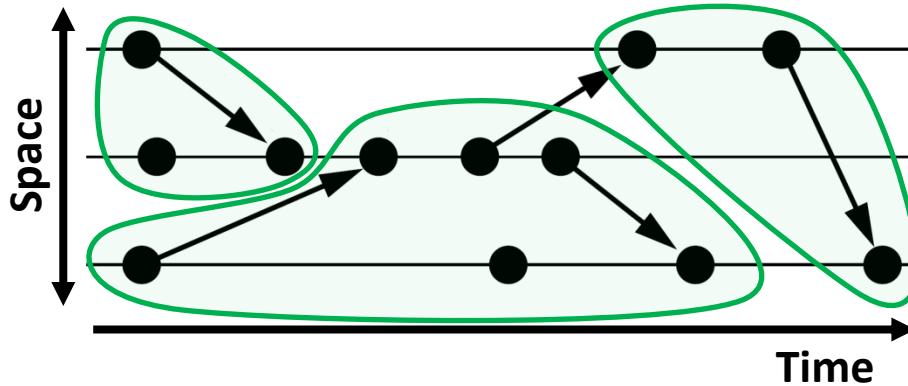
[Gil-Quijano *et al.*, 2012]



# Other Topological Structures

## Aggregation of Events

[Mattern, 1989]

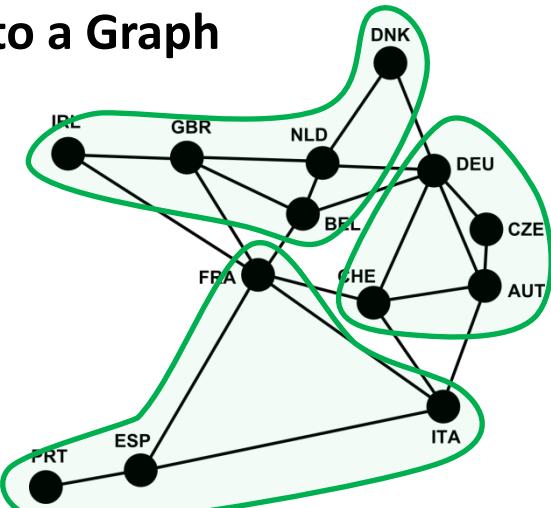


## Aggregation of an Interaction Matrix

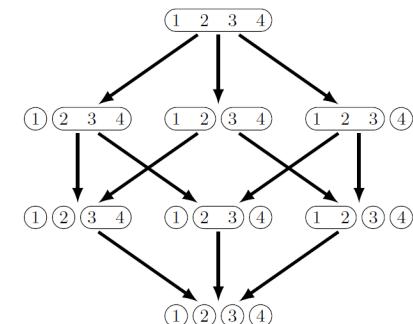
	ESP	FRA	GBR	BEL	CHE
ESP	X	12	11	10	4
FRA	14	X	12	12	5
GBR	20	11	X	6	9
BEL	15	9	6	X	5
CHE	10	16	17	9	X

[Lamarche-Perrin *et al.*, CIST 2011]

## Aggregation according to a Graph



## Corresponding Algebras



# Algorithmic Complexity

Find the complexity classes that are associated to other dimensions, other semantics, other topologies, *etc.*

Set	Admissible Parts	Admissible Partitions	Temporal Complexity	Spatial Complexity
Non-constrained	$\Theta(2^n)$	$\Theta(e^{n \log n})$	$\Theta(3^n)$	$\Theta(2^n)$
Ordered	$\Theta(n^2)$	$\Theta(2^n)$	$\Theta(n^2)$	$\Theta(n^2)$
Hierarchical	$O(n)$	$O(c^n)$	$O(n)$	$O(n)$
Other topologies	?	?	?	?

# Multidimensional Aggregation

$\{a, b, c, d, e\}$				
$\{a, b\}$	$\{c, d, e\}$			
$\{a\}$	$\{b\}$	$\{c\}$	$\{d\}$	$\{e\}$

$(a, 1)$	$(b, 1)$	$(c, 1)$	$(d, 1)$	$(e, 1)$
$(a, 2)$	$(b, 2)$	$(c, 2)$	$(d, 2)$	$(e, 2)$
$(a, 3)$	$(b, 3)$	$(c, 3)$	$(d, 3)$	$(e, 3)$
$(a, 4)$	$(b, 4)$	$(c, 4)$	$(d, 4)$	$(e, 4)$

$\{\underline{1}\}$
$\{\underline{2}\}$
$\{\underline{3}\}$
$\{\underline{4}\}$

$\{a, b, c, d, e\}$		
$\{a, b\}$		
$\{a\}$	$\{b\}$	$\{c, d, e\}$

$(a, 1)$	$(b, 1)$	$(\{c, d, e\}, \{1\})$
$(\{a\}, \{2, 3, 4\})$	$(\{b\}, \{2, 3, 4\})$	$(\{c, d, e\}, \{2, 3, 4\})$

$\{\underline{1}\}$
$\{2, 3, 4\}$

Microscopic Representation

Aggregated Representation

# Aggregation of Causal Structures

