Analyse multi-échelles des systèmes complexes

Théorie de l'information et optimisation combinatoire

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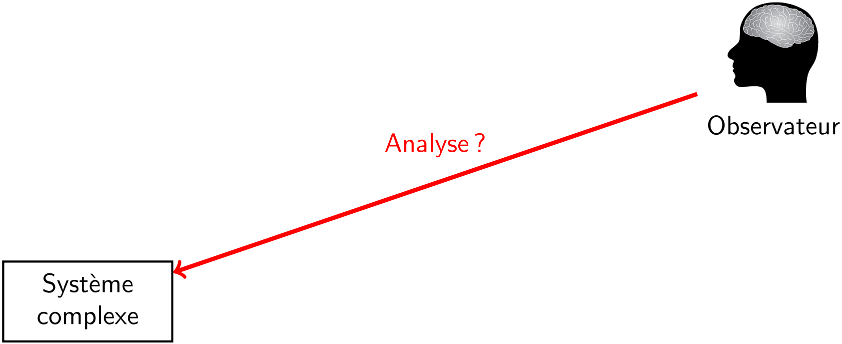
Master et Doctorat (2009-2013)

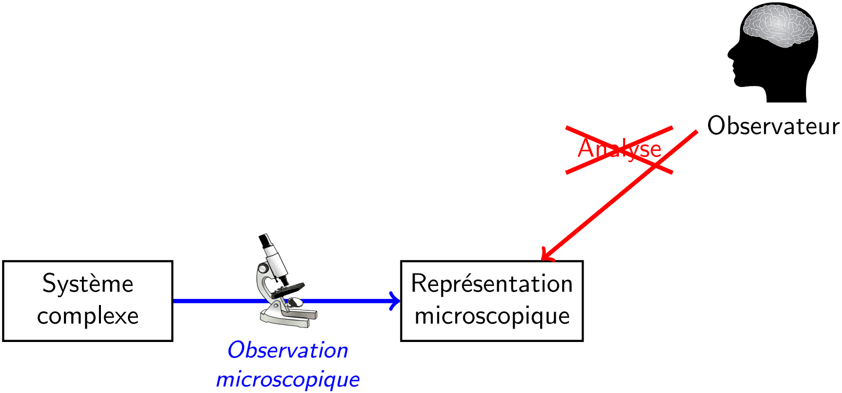
Max Planck Institute for Mathematics in the Sciences

Post-doctorat (2013-today)

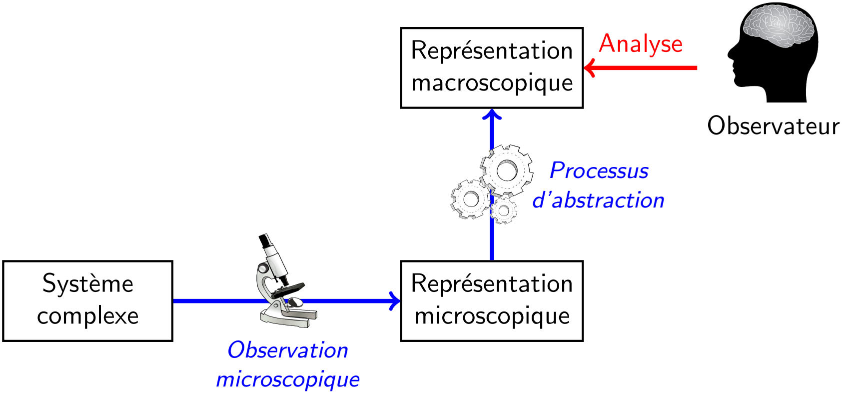
Problème



Problème

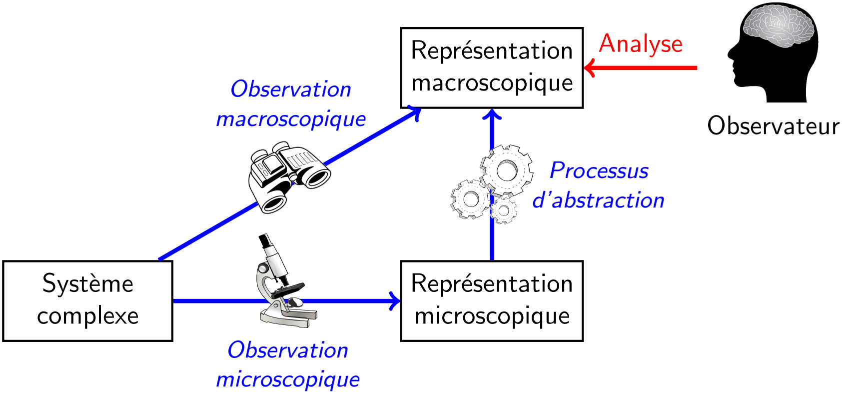

Observateur
~~Analyse~~
Système complexe
Observation microscopique
Représentation microscopique

Problème



Premier problème : engendrer des abstractions pertinentes pour l'analyse à partir de données microscopiques.

Problème

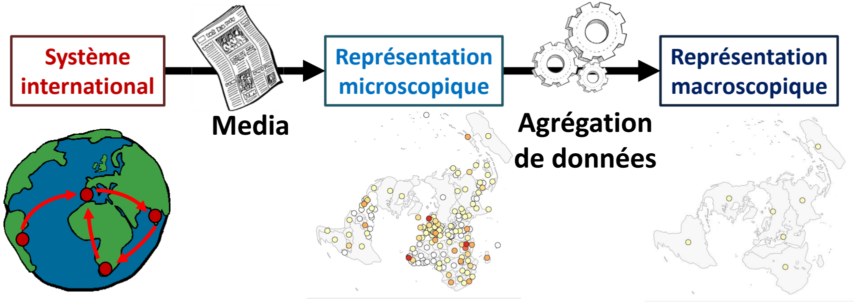


Premier problème : engendrer des abstractions pertinentes pour l'analyse à partir de données microscopiques.

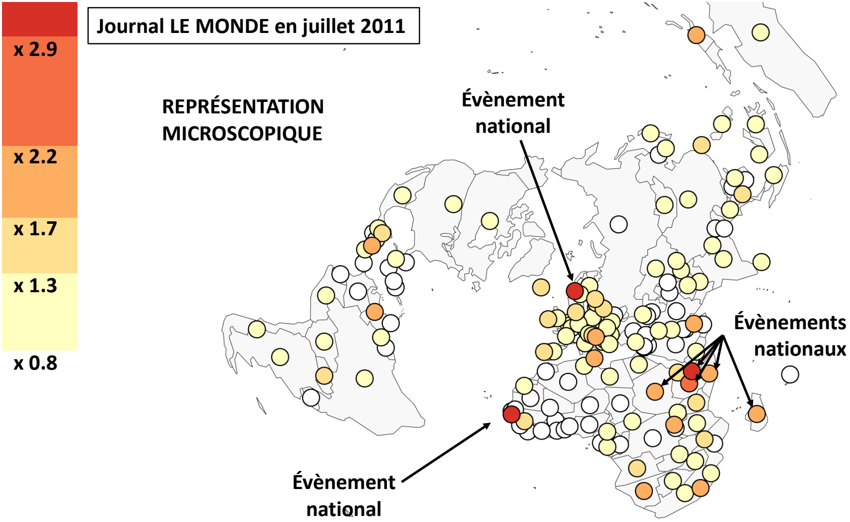
Second problème : engendrer des abstractions sans passer par l'observation microscopique du système.

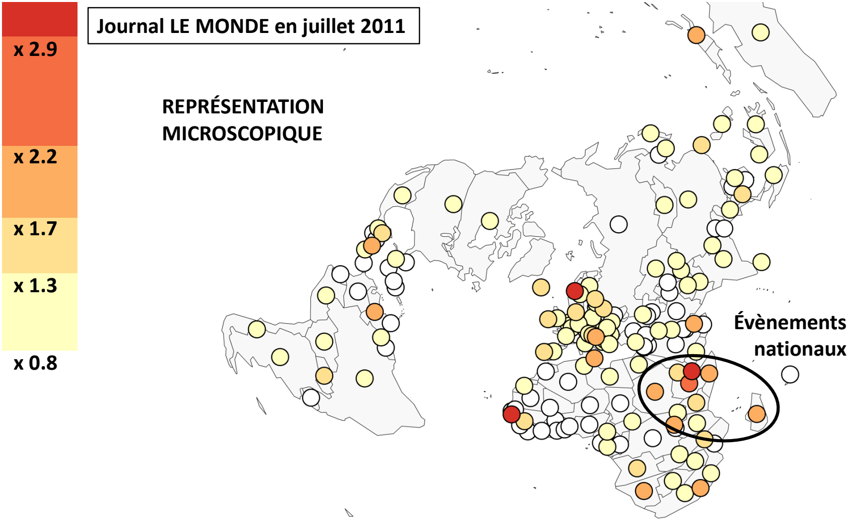
Hypothèse : les médias constituent un instrument d'observation adéquat du niveau national

[Grasland *et al.*, 2011]



Détection d'évènements médiatiques

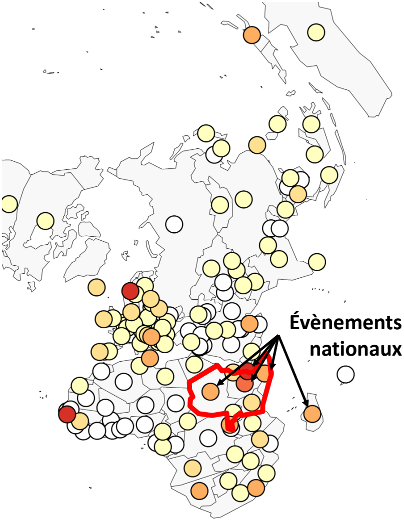



x 2.9
x 2.2
x 1.7
x 1.3
x 0.8
Journal LE MONDE en juillet 2011
REPRÉSENTATION
MICROSCOPIQUE
Évènements
nationaux

π_1 π_2 π_3 **Espace**

Temps t

	USA	Libye	Syrie	France	Israël	...	Total
2 mai	25	12	11	10	4	...	142
9 mai	14	6	12	12	5	...	108
16 mai	20	11	12	6	9	...	142
23 mai	15	9	6	13	5	...	120
30 mai	10	16	17	9	4	...	137
6 juin	14	16	16	9	4	...	114
13 juin	15	14	17	9	6	...	119
20 juin	17	13	12	12	7	...	123
27 juin	7	6	7	20	2	...	103
4 juill.	12	13	8	10	6	...	129
11 juill.	21	10	10	14	3	...	107
18 juill.	7	3	8	4	5	...	61
25 juill.	16	7	6	13	4	...	128
1 août	21	1	9	7	4	...	88
...	...	...	...	...	...	...	...
Total	423	308	260	248	153	...	3520



Agrégation de données géographiques

Espace π_1 π_2 π_3

Temps t

	USA	Agrégat	Israël	...	Total
2 mai	25	13+11+10	4	...	**142**
9 mai	14	6+12+12	5	...	**108**
16 mai	20	11+12+6	9	...	**142**
23 mai	15	9+6+13	5	...	**120**
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27 juin	7	6+7+20	2	...	**103**
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...	...	...	...	...	**...**
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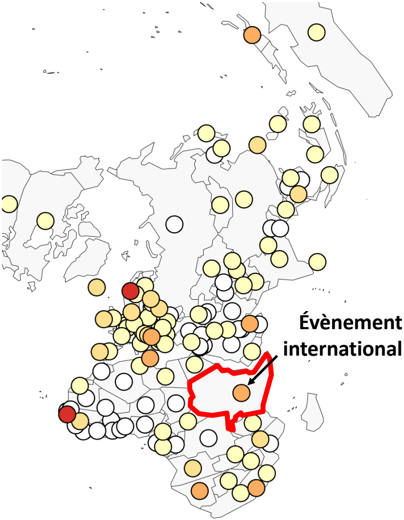
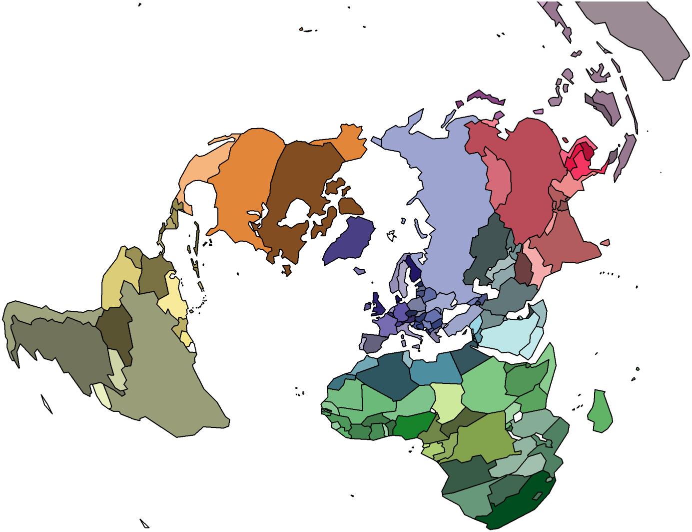


P1 : Sémantique des agrégats géographiques

Expert



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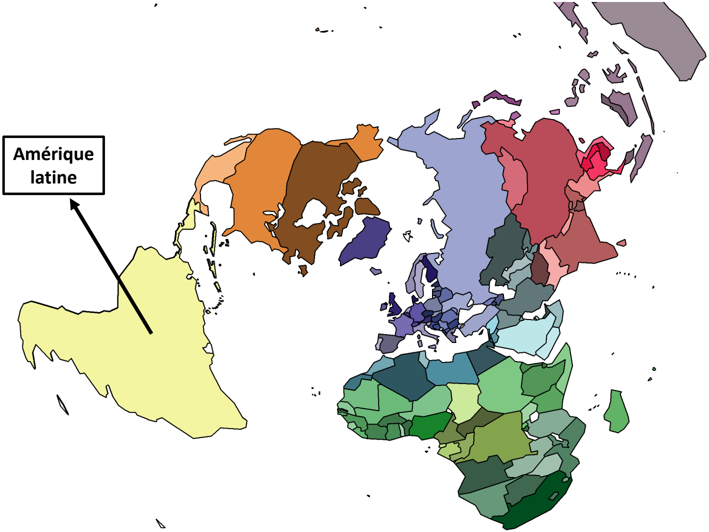
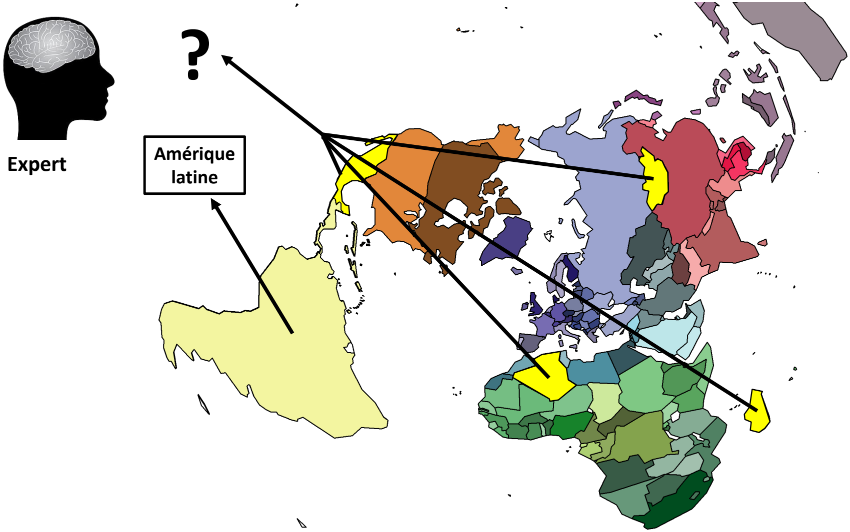
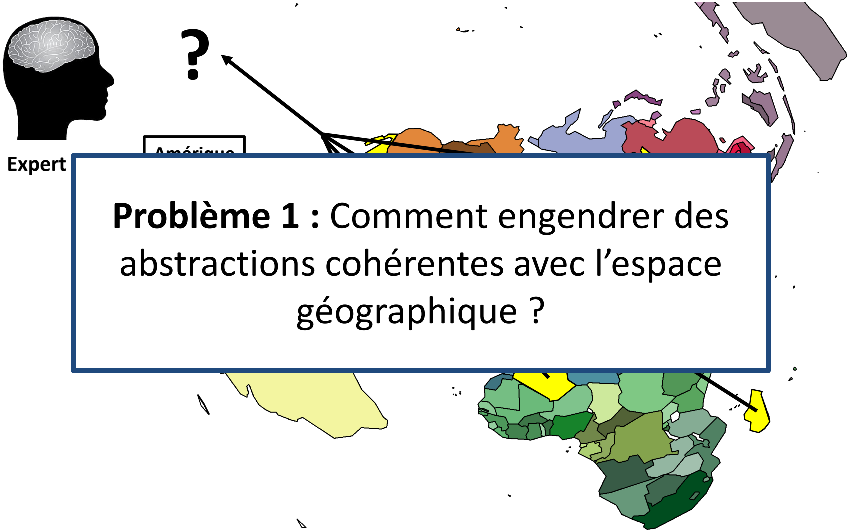
Expert



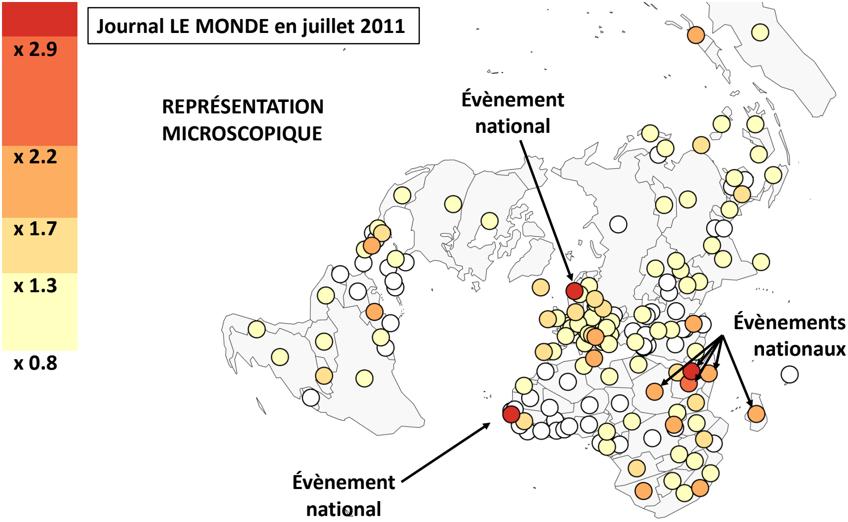
P1 : Sémantique des agrégats géographiques



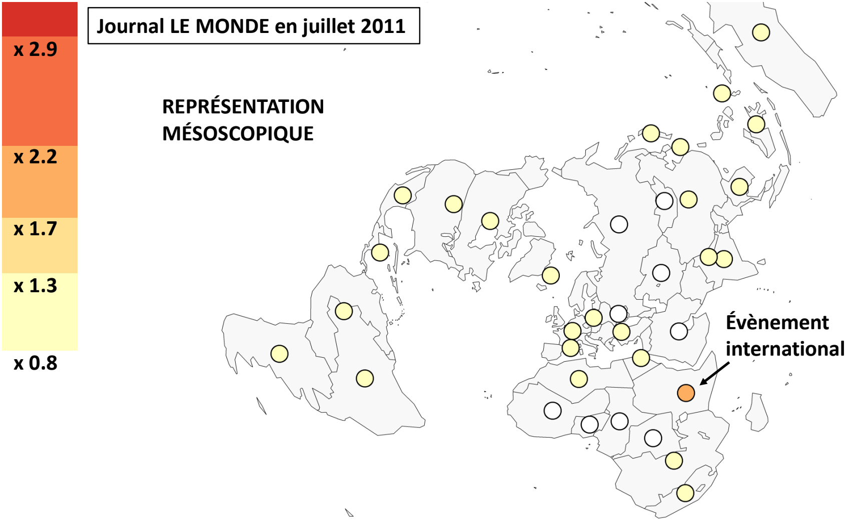


Problème 1 : Comment engendrer des abstractions cohérentes avec l'espace géographique ?

P2 : Granularité de l'information



P2 : Granularité de l'information



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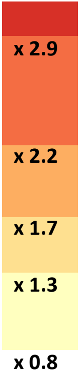

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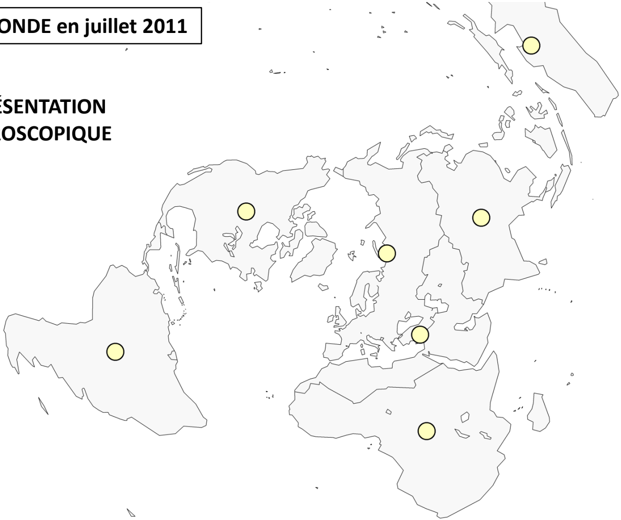
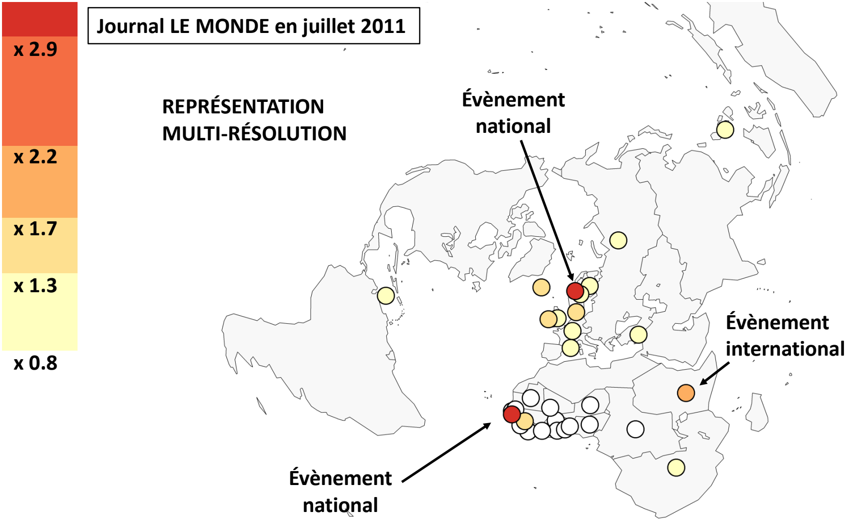
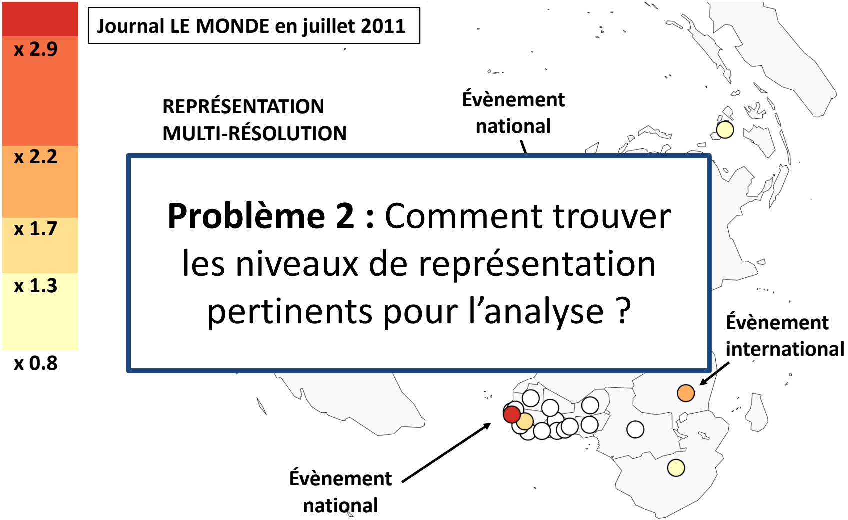
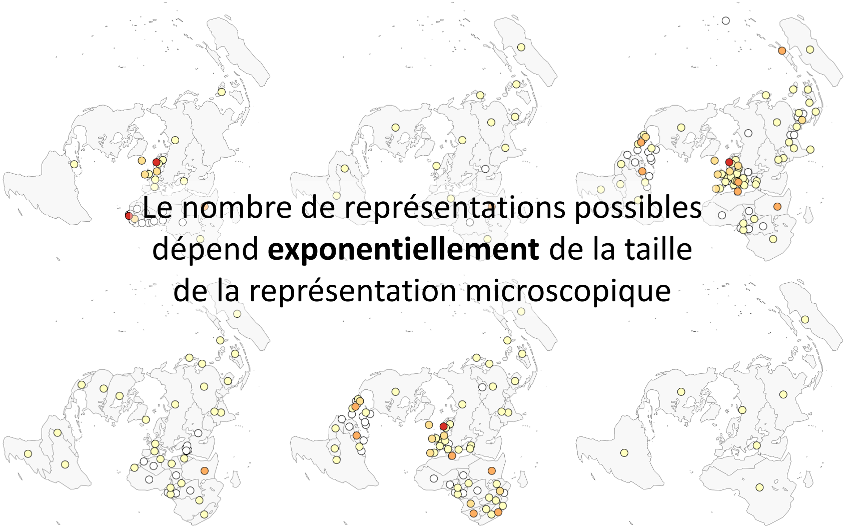
REPRÉSENTATION MACROSCOPIQUE



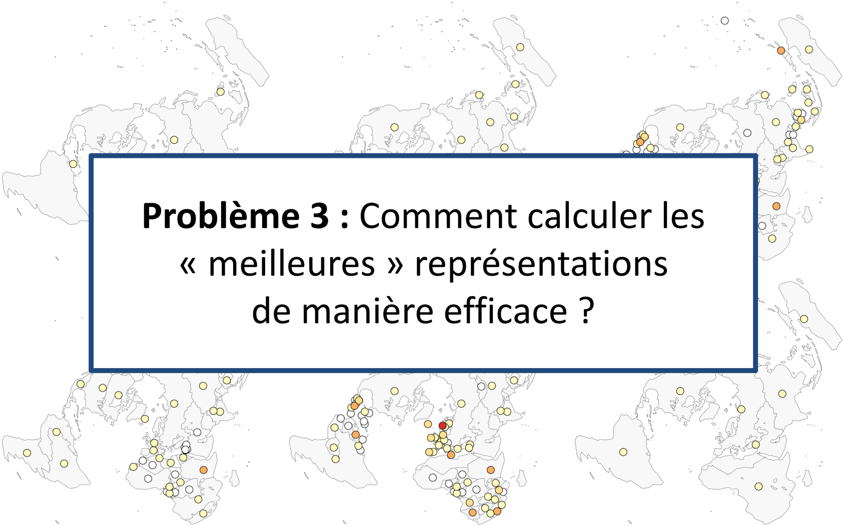

x 2.9
x 2.2
x 1.7
x 1.3
x 0.8
Journal LE MONDE en juillet 2011
REPRÉSENTATION
MULTI-RÉSOLUTION
Évènement
national
Évènement
international
Évènement
national



Problème 2 : Comment trouver les niveaux de représentation pertinents pour l'analyse ?



Le nombre de représentations possibles dépend **exponentiellement** de la taille de la représentation microscopique



Problème 3 : Comment calculer les « meilleures » représentations de manière efficace ?

L'approche multi-échelles

P0 Caractériser le processus d'agrégation

P1 Quantifier les niveaux de représentations

P2 Préserver la structure du système

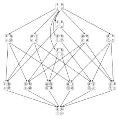
P3 Calculer les représentations optimales

L'approche multi-échelles

P0

Caractériser le processus d'agrégation

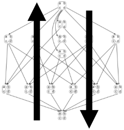
→ Algèbre des partitions



P1

Quantifier les niveaux de représentations

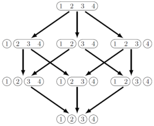
→ Mesure de complexité et d'information



P2

Préserver la structure du système

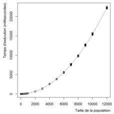
→ Contraintes combinatoires



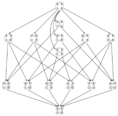
P3

Calculer les représentations optimales

→ Algorithmes d'optimisation combinatoire



P0 Caractériser le processus d'agrégation
→ Algèbre des partitions



P1 Quantifier les niveaux de représentations
→ Mesure de complexité et d'information

P2 Préserver la structure du système
→ Contraintes combinatoires

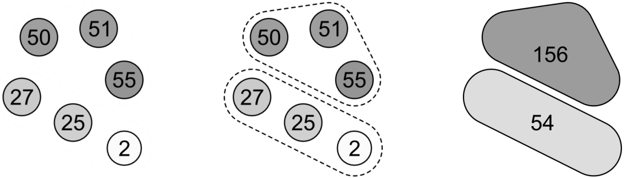
P3 Calculer les représentations optimales
→ Algorithmes d'optimisation combinatoire

Processus d'agrégation

Population $\Omega = \{x_1, \dots, x_N\}$

Dénombrement $v : \Omega \to \mathbb{N}$

Partitionnement → **Agrégation** →



Représentation microscopique

$$\Pr(X = x) = \frac{v(x)}{\sum_{x \in \Omega} v(x)}$$

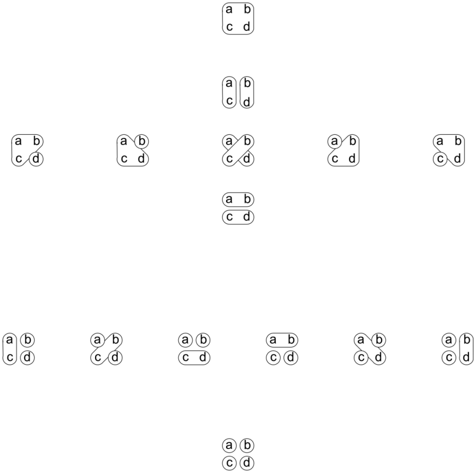
Représentation partitionnée

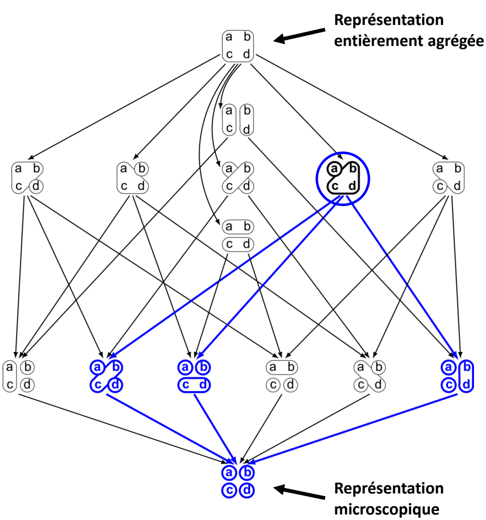
$$\Omega = A_1 \cup \dots \cup A_k$$
$$A_i \cap A_j = \emptyset$$

Représentation agrégée

$$\Pr(\hat{X} = A) = \sum_{x \in A} \Pr(X = x)$$

Poset des partitions admissibles





Structure algébrique

Ordre partiel sur l'ensemble des partitions possibles

→ **relations de raffinement**

L'approche multi-échelles

P0 Caractériser le processus d'agrégation

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→ Mesure de complexité et d'information

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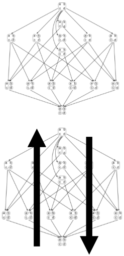
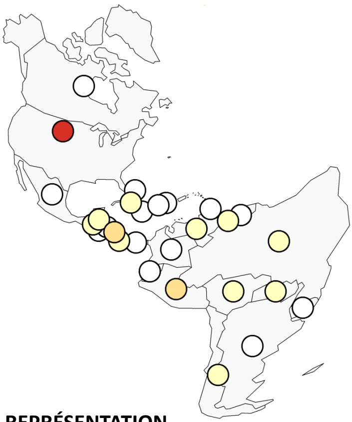
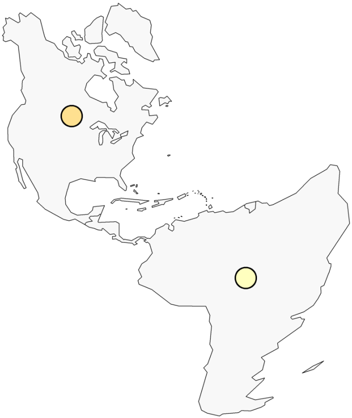


Complexité et information

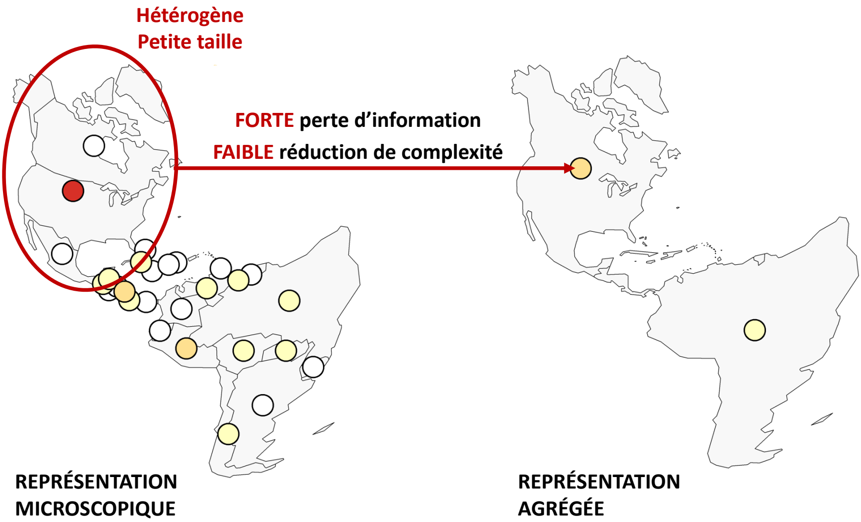


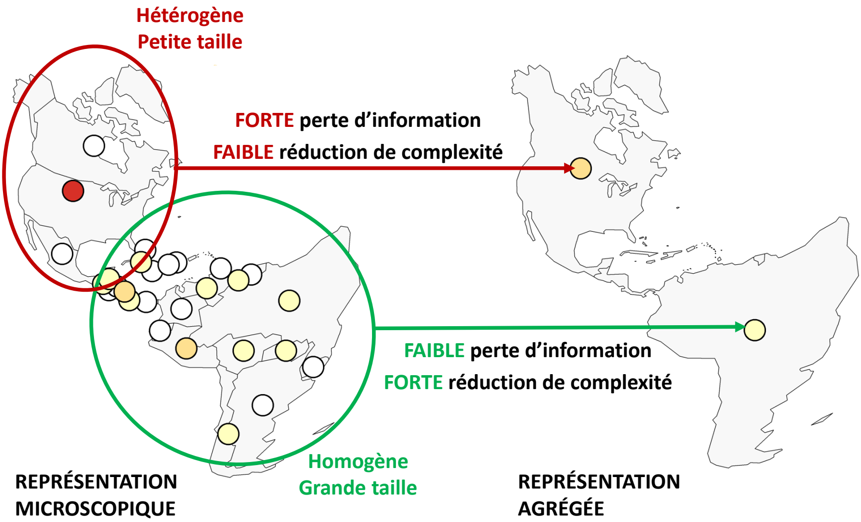
REPRÉSENTATION MICROSCOPIQUE



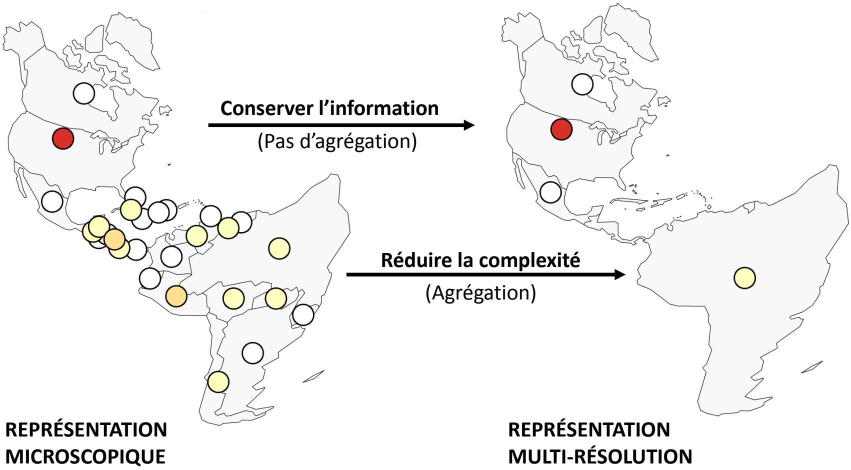
REPRÉSENTATION AGRÉGÉE

Complexité et information




Hétérogène
Petite taille
FORTE perte d'information
FAIBLE réduction de complexité
FAIBLE perte d'information
FORTE réduction de complexité
Homogène
Grande taille
REPRÉSENTATION MICROSCOPIQUE
REPRÉSENTATION AGRÉGÉE

Complexité et information



Quantifier la complexité et l'information

- La **complexité** dépend de la tâche à accomplir et des outils de description disponibles [Bonabeau et Dessalles, 1997]
  - Nombre de paramètres $\mathrm{C}(A_1, \ldots, A_k) = k$
  - Entropie de Shannon $\mathrm{H}(\hat{X}) = \sum_{A_i} -\Pr\left(\hat{X} = A_i\right) \log_2 \Pr\left(\hat{X} = A_i\right)$

- La **perte d'information** est mesurée par la divergence entre deux distributions de probabilité [Kullback et Leibler, 1951]
  - Divergence de Kullback-Leibler $\mathrm{D}(X||X') = \sum_{x\in\Omega} \Pr(X = x) \log_2 \frac{\Pr(X=x)}{\Pr(X'=x)}$
  - Information de désagrégation $\mathrm{H}(\mathrm{X}|\hat{X}) = \sum_{A_i} \Pr\left(\hat{X} = A_i\right) \mathrm{H}\left(\mathrm{X}|\hat{X} = A_i\right)$

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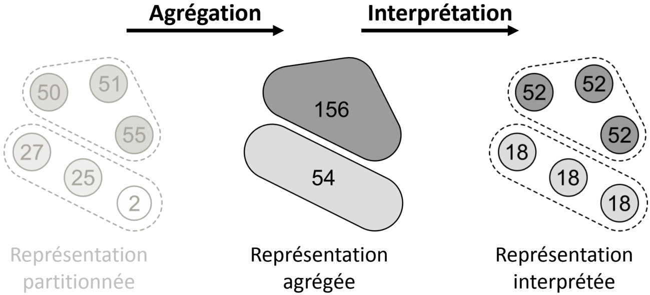
Le processus d'agrégation

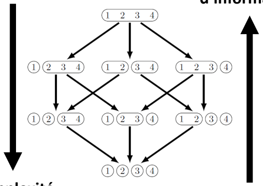
Population $\Omega = \{x_1, \dots, x_N\}$

Dénombrement $v : \Omega \to \mathbb{N}$



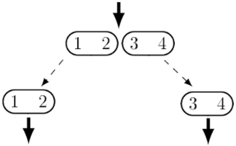
$\Pr(\hat{X} = A) = \sum_{x \in A} \Pr(X = x)$ $\Pr(X' = x) = \Pr(X = x \mid \hat{X} = A_x)$

Monotonicité



$$\mathcal{X}_1 < \mathcal{X}_2 \quad \Rightarrow \quad \begin{array}{l} \mathrm{D}(\mathcal{X}_1) < \mathrm{D}(\mathcal{X}_1) \\ \mathrm{C}(\mathcal{X}_1) < \mathrm{C}(\mathcal{X}_1) \end{array}$$

Décomposabilité



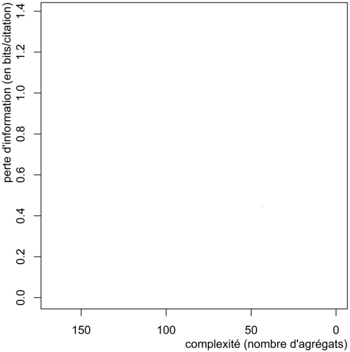
$$\mathrm{D}(\mathcal{X}) = \textstyle\sum_{A \in \mathcal{X}} \mathrm{D}(A)$$
$$\mathrm{C}(\mathcal{X}) = \textstyle\sum_{A \in \mathcal{X}} \mathrm{C}(A)$$

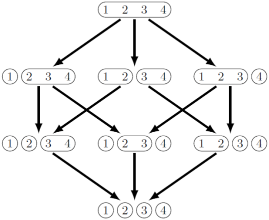
Optimisation des mesures

Deux objectifs à optimiser...

Compromis de qualité :

$$Q_\beta(\mathcal{X}) = \frac{C(\mathcal{X})}{C(\{\Omega\})} + \beta \frac{D(\mathcal{X})}{D(\{\Omega\})}$$





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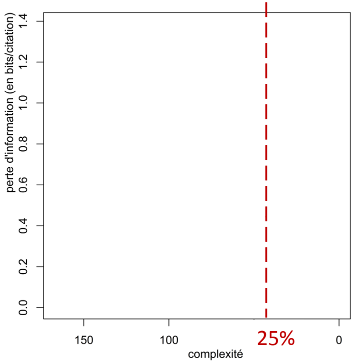

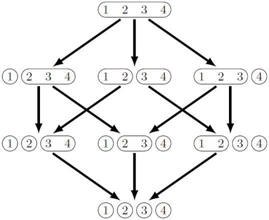

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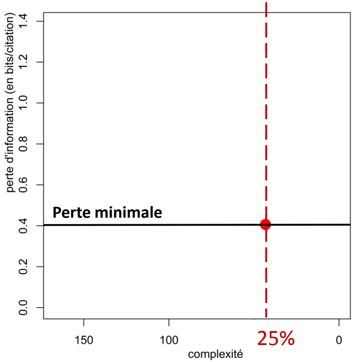

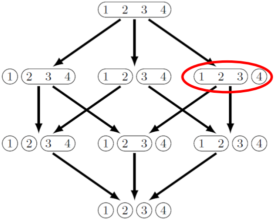

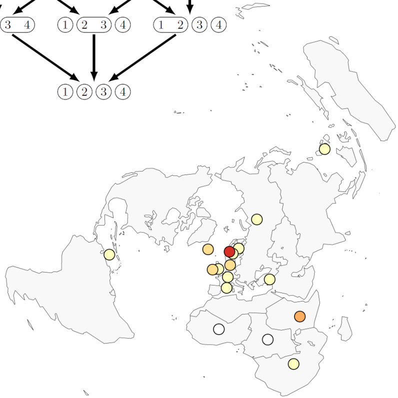

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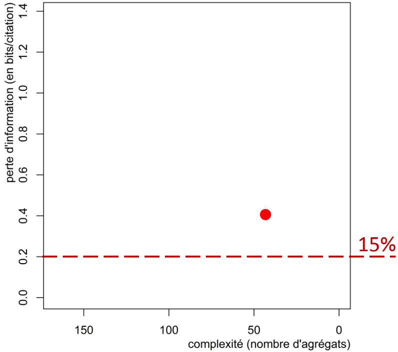
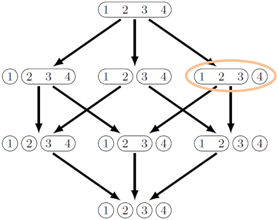
$$Q_\beta(\mathcal{X}) = \frac{C(\mathcal{X})}{C(\{\Omega\})} + \beta \frac{D(\mathcal{X})}{D(\{\Omega\})}$$



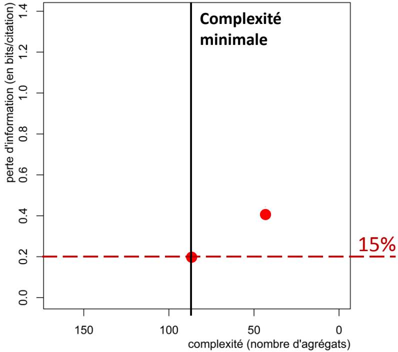


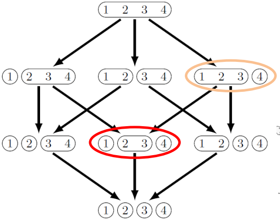
Optimisation des mesures

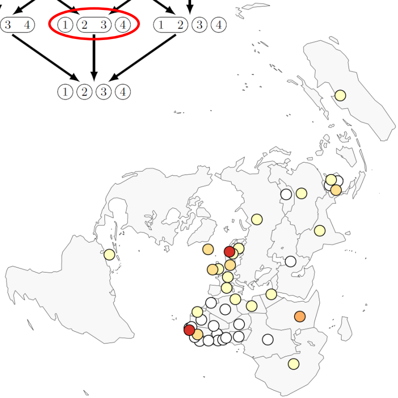
Deux objectifs à optimiser...

Compromis de qualité :

$$Q_\beta(\mathcal{X}) = \frac{C(\mathcal{X})}{C(\{\Omega\})} + \beta \frac{D(\mathcal{X})}{D(\{\Omega\})}$$







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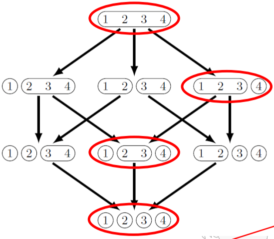
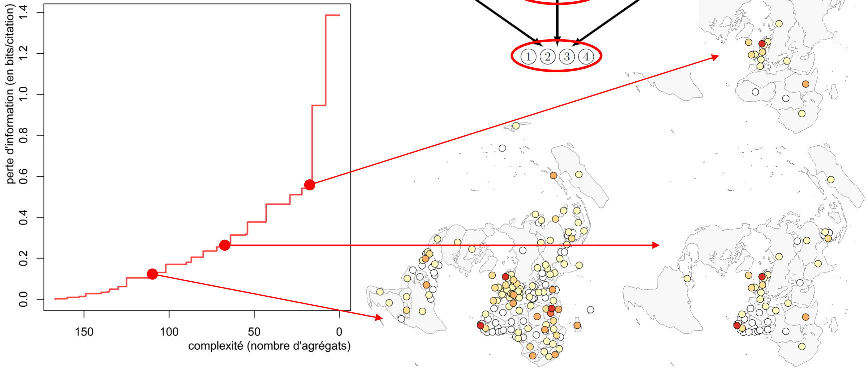
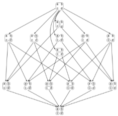


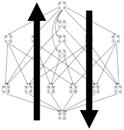


L'approche multi-échelles

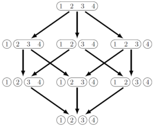
P0 Caractériser le processus d'agrégation
→ Algèbre des partitions



P1 Quantifier les niveaux de représentations
→ Mesure de complexité et d'information



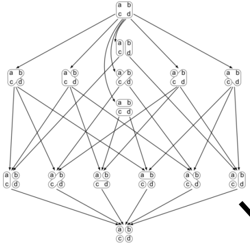
P2 Préserver la structure du système
→ Contraintes combinatoires



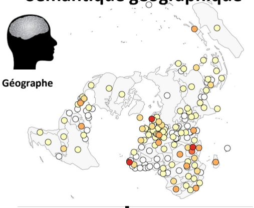
P3 Calculer les représentations optimales
→ Algorithmes d'optimisation combinatoire

Problème et objectif

Ensemble des partitions possibles

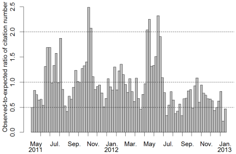


Sémantique géographique

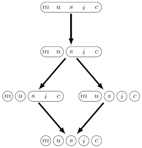


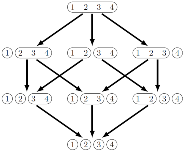
Géographe

Sémantique temporelle

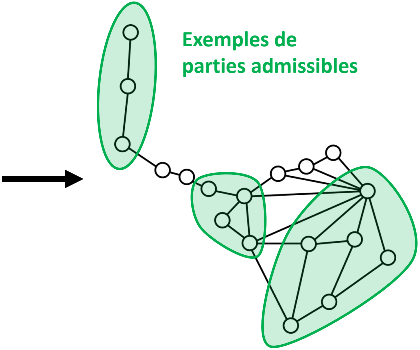


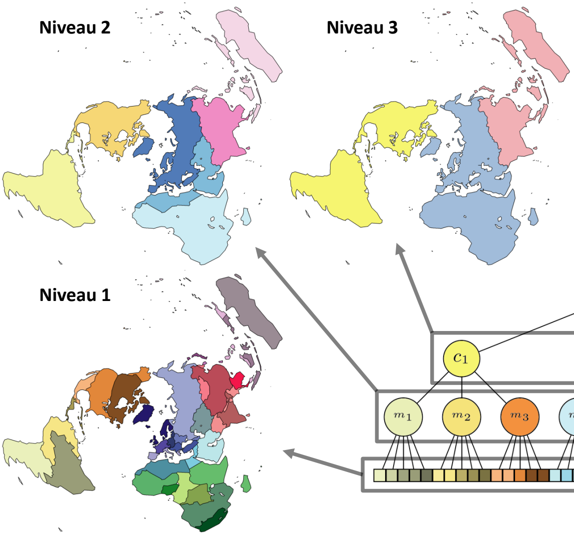
Contraindre l'ensemble des partitions admissibles



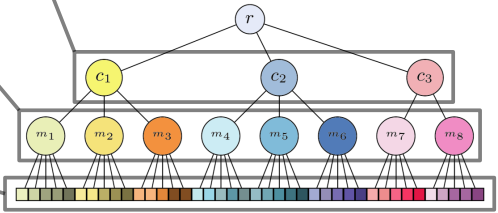


Parties admissibles : ensembles de pays connexes vis-à-vis du graphe de voisinage

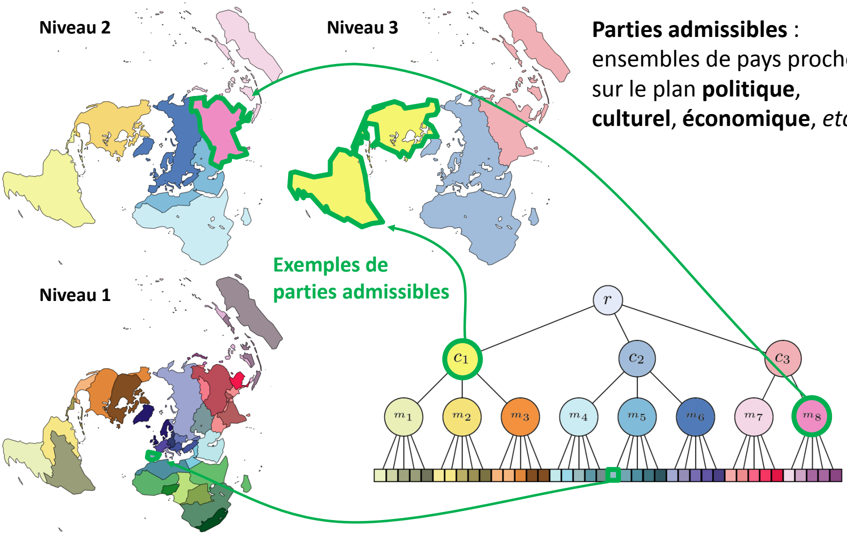




Parties admissibles : ensembles de pays proches sur le plan **politique**, **culturel**, **économique**, *etc.*

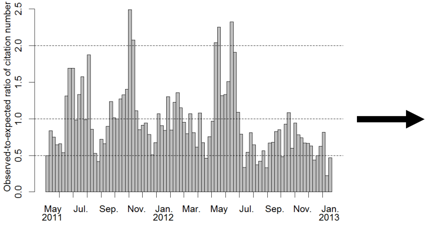


Conserver une hiérarchie territoriale lors de l'agrégation

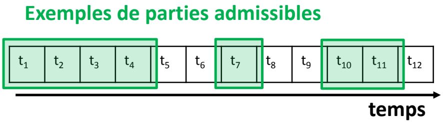


Parties admissibles : ensembles de pays proches sur le plan **politique**, **culturel**, **économique**, *etc.*

Conserver un ordre total lors de l'agrégation

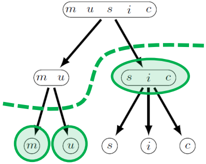


Parties admissibles :
intervalles de temps

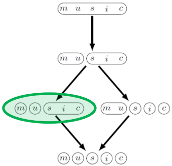


Contraintes combinatoires

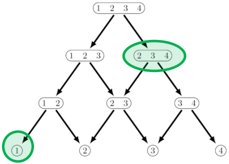
Parties admissibles
(nœuds de la hiérarchie)



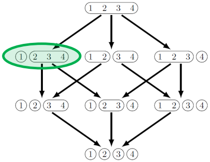
Partitions admissibles
(coupe dans la hiérarchie)



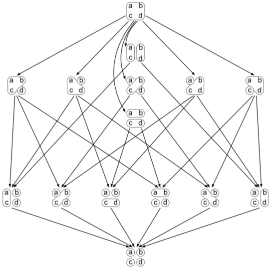
Parties admissibles
(intervalles de temps)



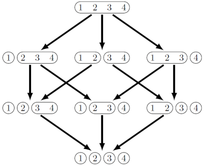
Partitions admissibles
(séquences d'intervalles)



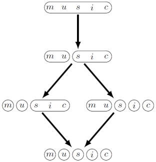
Partitions
non-contraintes



Partitions admissibles
selon un **ordre total**



Partitions admissibles
selon une **hiérarchie**

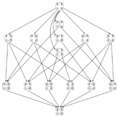


Moins contraint — **Plus contraint**

Plus complexe — **Moins complexe**

L'approche multi-échelles

P0 Caractériser le processus d'agrégation
→ Algèbre des partitions



P1 Quantifier les niveaux de représentations
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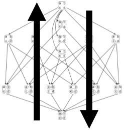

P2 Préserver la structure du système
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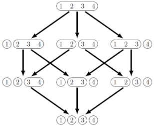
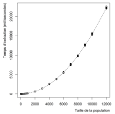


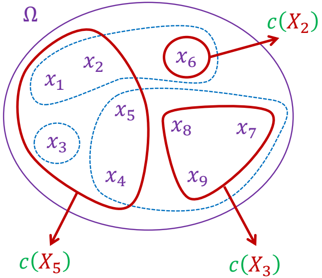
P3 Calculer les représentations optimales
→ Algorithmes d'optimisation combinatoire



The Set Partitioning Problem

Given:

- a set of individuals $\Omega = \{x_1, \dots, x_n\}$
- a set of admissible parts $\mathcal{P} = \{X_1, \dots, X_m\} \subset 2^\Omega$
- a cost function $c : \mathcal{P} \to \mathbb{R}$
- the corresponding set of admissible partitions $\mathfrak{P} = \{\mathcal{X} \subset \mathcal{P}$ such that $\mathcal{X}$ is a partition of $\Omega\}$



Problem: Find an admissible partition that minimizes the cost function:

$$\mathcal{X}^* = \arg\min_{\mathcal{X} \in \mathfrak{P}} \left(\sum_{X \in \mathcal{X}} c(X) \right)$$

→ NP-complete!

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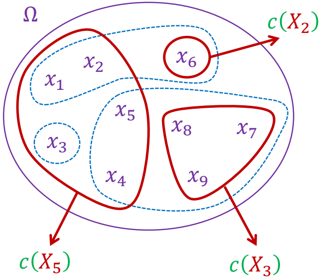
Additional assumptions



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→ NP-complete!

Applications and Special Versions of the SPP

Multilevel Geographical Analysis

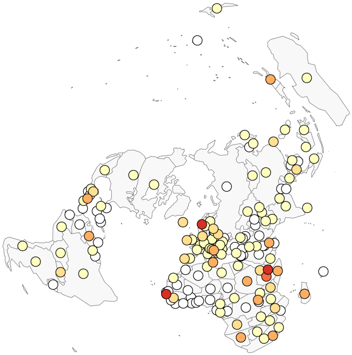
- Ω = territorial units
- $\mathcal{P}$ = admissible aggregates
- c = compression rate
- $\mathfrak{P}$ = aggregated representations

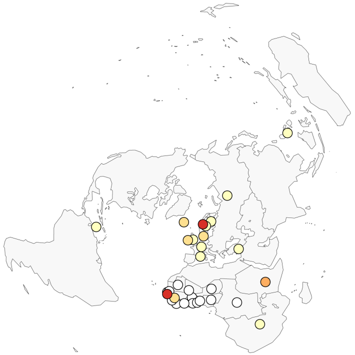
Hierarchical SPP

- Assumption: $\mathcal{P}$ forms a hierarchy
- Result: $\mathcal{O}(n)$ depth-first search
  [Pons *et al.*, 2011] [Lamarche-Perrin *et al.*, 2014]

Graph SPP

- Assumption: $\mathcal{P}$ are connected parts of a graph
- Result: NP-complete [Becker *et al.*, 1998]





Applications and Special Versions of the SPP

Multilevel Geographical Analysis

Time Series Analysis

- $\Omega =$ ordered data points
- $\mathcal{P} =$ time intervals
- $c =$ compression rate
- $\mathfrak{P} =$ aggregated time series

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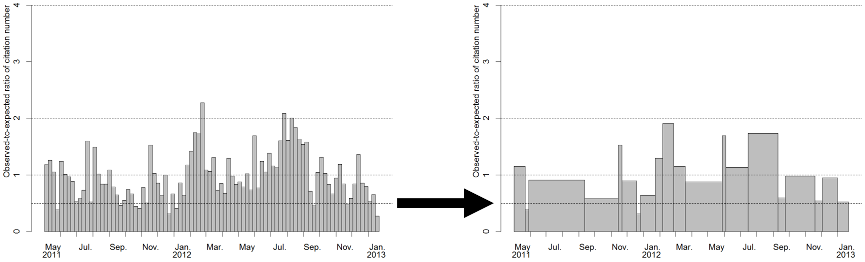
  [Anily *et al.*, 1991] [Jackson *et al.*, 2005]



Applications and Special Versions of the SPP

Multilevel Geographical Analysis

Time Series Analysis

Coalition Structure Generation

- $\Omega =$ agents
- $\mathcal{P} =$ feasible teams
- $c =$ interaction costs
- $\mathfrak{P} =$ coalition structures

Hierarchical SPP

- Assumption: $\mathcal{P}$ forms a hierarchy
- Result: $\mathcal{O}(n)$ depth-first search
  [Pons *et al.*, 2011] [Lamarche-Perrin *et al.*, 2014]

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Complete SPP

- Assumption: $\mathcal{P}$ contains all parts
- Result: $\mathcal{O}(3^n)$ dynamic programming
  [Yeh, 1986] [Lehmann *et al.*, 2006]

Applications and Special Versions of the SPP

Multilevel Geographical Analysis

Time Series Analysis

Coalition Structure Generation

Community Detection

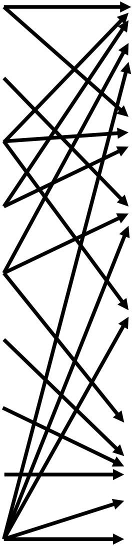
Distributed System Monitoring

Load Balancing Problem

Database Optimization

Image Processing

Combinatorial Auctions



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- Assumption: $\mathcal{P}$ contains all parts
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[Yeh, 1986] [Lehmann *et al.*, 2006]

Ordered x Hierarchical SPP [Dosimont *et al.*, 2014]

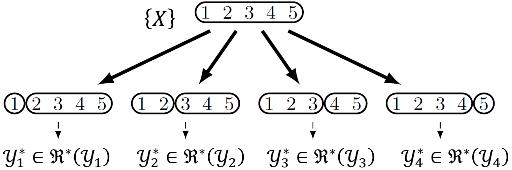
Array SPP [Muthukrishnan *et al.*, 2005]

SPP with Size Bounds [Rothkopf *et al.*, 1998]

Cyclic SPP [Rothkopf *et al.*, 1998]

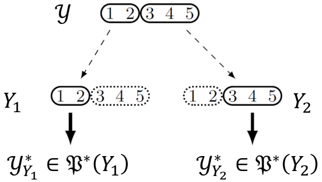
DIVIDE...

Branching the state space according to the covering relation

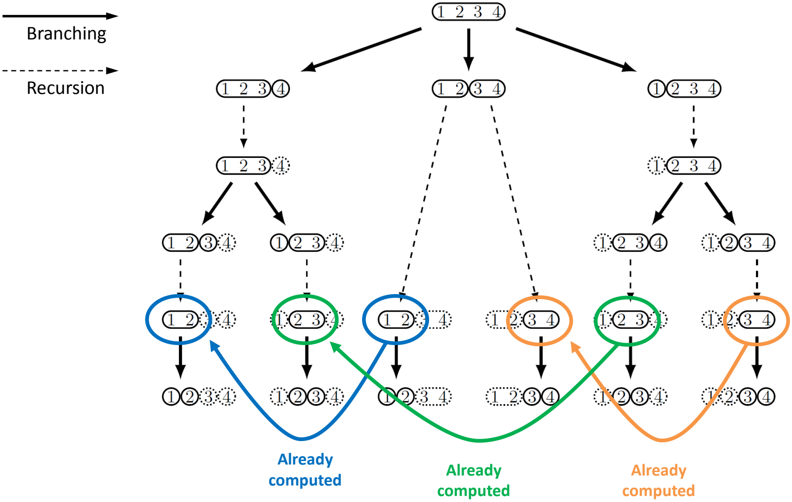


...AND CONQUER

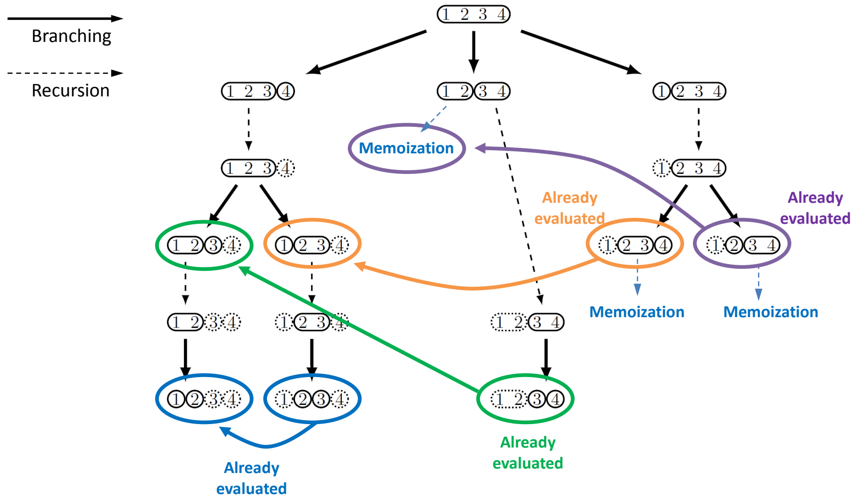
Recursion according to the principle of optimality



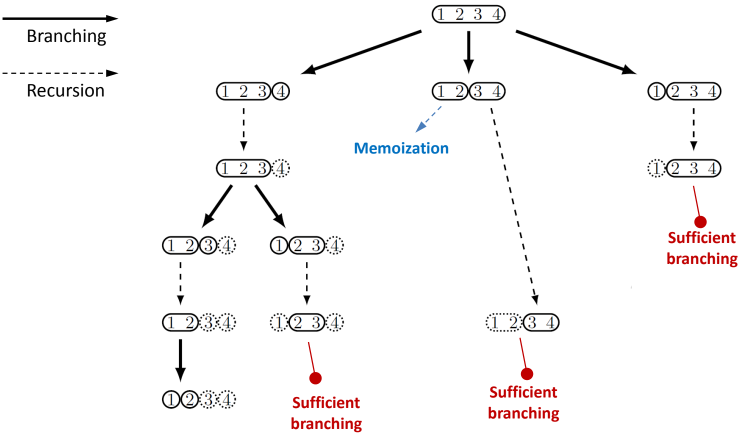
Memoization



Non-redudant Branching



Non-redudant Branching



The Generic Algorithm

A Generic Algorithm to Solve the SPP

Global Inputs:

- c a cost function;
- $\mathcal{P}$ a set of admissible parts defining admissible partitions;
- $\mathfrak{L}$ a set of locally-optimal admissible partitions of parts on which the algorithm has already been applied.

Local Inputs:

- X an admissible part;
- $\overline{\mathcal{X}}$ the complementary partition of X inherited from the "higher" call ($\overline{\mathcal{X}}$ is a partition of $\Omega \backslash X$);
- $\mathfrak{D}$ the set of admissible partitions which refinements have already been evaluated during "higher" calls.

Output:

- $\mathcal{X}^*$ a locally-optimal admissible partition of X.

- If the algorithm has already been applied to part X, return the locally-optimal partition recorded in $\mathfrak{L}$.
- Initialization: $\mathcal{X}^* \leftarrow \{\{X\}\}$ and $\mathfrak{D}' \leftarrow \mathfrak{D}$.
- For each $\mathcal{Y} \in \mathfrak{C}(\{X\})$ such that $\overline{\mathcal{X}} \cup \mathcal{Y}$ does not refine any partition in $\mathfrak{D}$, do the following:
  - For each part $Y \in \mathcal{Y}$, call the algorithm with local inputs $X \leftarrow Y$, $\overline{\mathcal{X}} \leftarrow \overline{\mathcal{X}} \cup \mathcal{Y} \backslash \{Y\}$, and $\mathfrak{D} \leftarrow \mathfrak{D}'$ to compute a locally-optimal partition $\mathcal{Y}_Y^* \in \mathfrak{P}^*(Y)$.
  - $\mathcal{Y}^* \leftarrow \bigcup_{Y \in \mathcal{Y}} \mathcal{Y}_Y^*$.
  - If $c(\mathcal{Y}^*) > c(\mathcal{X}^*)$, then $\mathcal{X}^* \leftarrow \mathcal{Y}^*$.
  - $\mathfrak{D}' \leftarrow \mathfrak{D}' \cup \{\mathcal{Y}\}$.
- Return $\mathcal{X}^*$ and record this result in $\mathfrak{L}$.

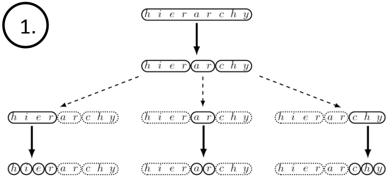
Generic: solve any instance of the SPP
→ but inefficient for special versions

Designing dedicated implementations:

1. Analysing the generic execution
2. Building appropriate data structures
3. Deriving a specialized algorithm

Application to the Hierarchical SPP

1.



2.

Data Structure

- Set of parts: rooted tree
- Optimal partition: cut of the tree
- Algorithm: depth-first search

3.

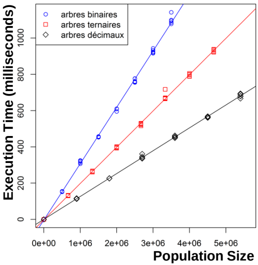
Algorithm 1 for the HSPP

Require: A tree with a label $cost$ on each node representing the cost of the corresponding admissible part.

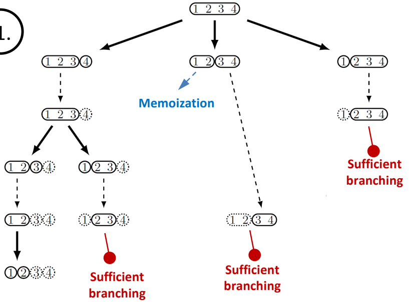
Ensure: Each node of the tree has a Boolean label $optimalCut$ representing an optimal partition (see above).

procedure SOLVEHSPP($node$)
  if $node$ has no child **then**
    $node.optimalCost \leftarrow node.cost$
    $node.optimalCut \leftarrow true$
  else
    $MCost \leftarrow node.cost$
    $\mu Cost \leftarrow 0$
    for each $child$ of $node$ **do**
      SOLVEHSPP($child$)
      $\mu Cost \leftarrow \mu Cost + child.optimalCost$
    $node.optimalCost \leftarrow \max(\mu Cost, MCost)$
    $node.optimalCut \leftarrow (\mu Cost < MCost)$

Linear Complexity



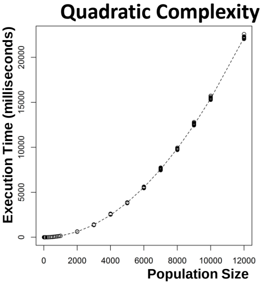
1.



2.

Data Structure

- Set of parts: triangular matrix
- Optimal partition: array of cuts
- Algorithm: dynamic programming

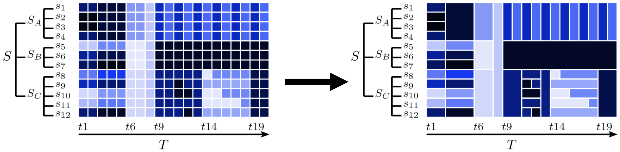


3.

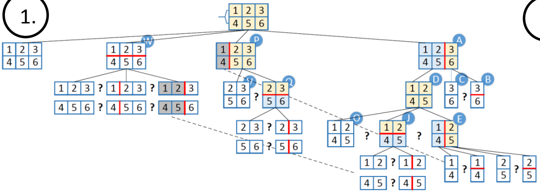
Algorithm 2 for the OSPP

Require: A matrix $cost$ recording the costs of intervals.
Ensure: The vector $optimalCut$ represents an optimal partition (see text above).

for $j \in [\![1, n]\!]$ **do**
  $optimalCost[j] \leftarrow cost[1, j]$
  $optimalCut[j] \leftarrow 1$
  for $cut \in [\![2, j]\!]$ **do**
    $\mu Cost \leftarrow optimalCost[cut - 1] + cost[cut, j]$
    if $\mu Cost > optimalCost[j]$ **then**
      $optimalCost[j] \leftarrow \mu Cost$
      $optimalCut[j] \leftarrow cut$



1.



3.

Algorithm 1 computes a hierarchy-and-order-consistent partition that maximizes the parametrized information criterion

procedure *node*.COMPUTEOPTIMALPARTITION(p)

  for each *child* **do** ▷ Recursion
    child.COMPUTEOPTIMALPARTITION(p)

  for $i = |T| - 1, \ldots, 0$ **do** ▷ Iteration
    for $j = i, \ldots, |T| - 1$ **do**

      $cut[i, j] \leftarrow j$ ▷ No cut
      $pIC[i, j] \leftarrow p.gain[i, j] - (1 - p).loss[i, j]$

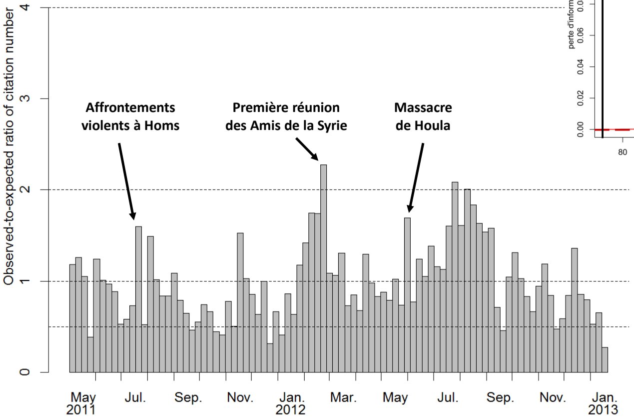
      if has children **then** ▷ Spatial cut?
        $pIC_s \leftarrow 0$
        for each *child* **do**
          $pIC_s \leftarrow pIC_s + child.pIC[i, j]$
        if $pIC_s > pIC[i, j]$ **then**
          $cut[i, j] \leftarrow -1$
          $pIC[i, j] \leftarrow pIC_s$

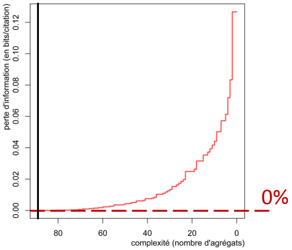
      for $cut_t = i, \ldots, j - 1$ **do** ▷ Temporal cut?
        $pIC_t \leftarrow pIC[i, cut] + pIC[cut + 1, j]$
        if $pIC_t > pIC[i, j]$ **then**
          $cut[i, j] \leftarrow cut_t$
          $pIC[i, j] \leftarrow pIC_t$

2. **Data Structure**

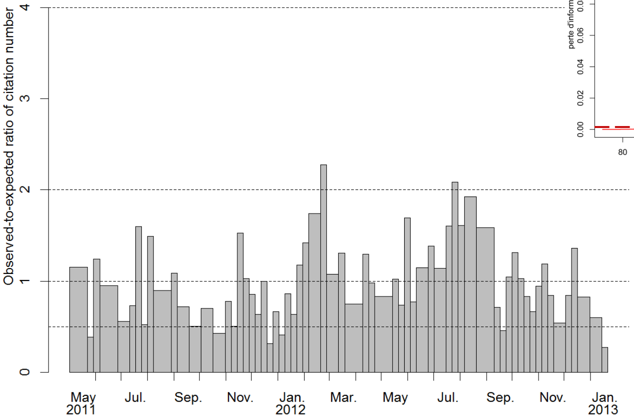
- Set of parts: rooted tree of triangular matrices
- Optimal partition: cut of the tree and arrays of cuts
- Algorithm: depth-first search and dynamic programming

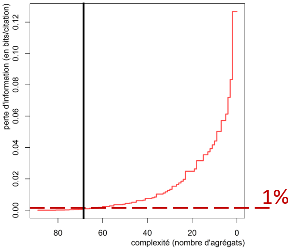
La Syrie vue par Le Monde en 2011 et 2012



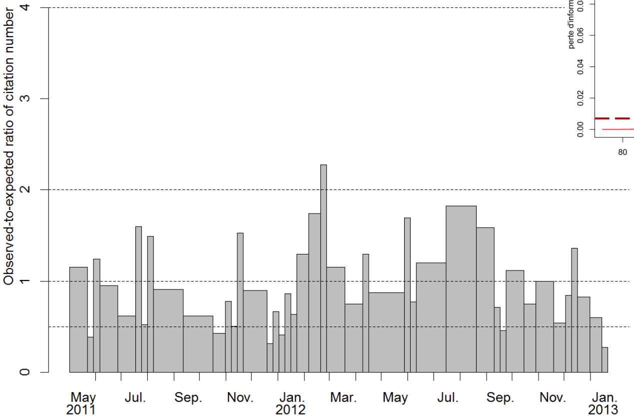


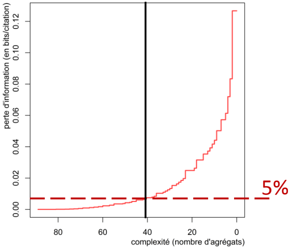
La Syrie vue par Le Monde en 2011 et 2012





La Syrie vue par Le Monde en 2011 et 2012





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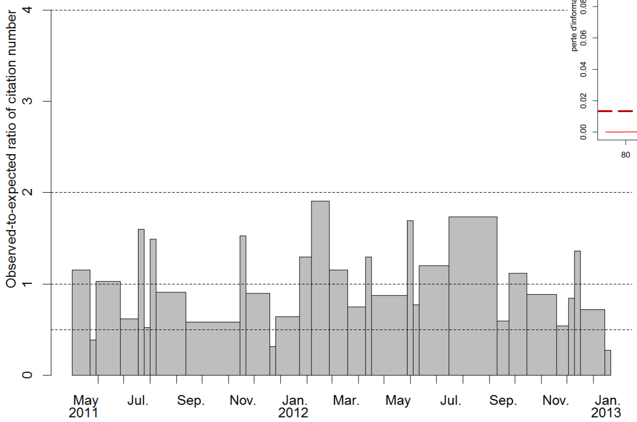
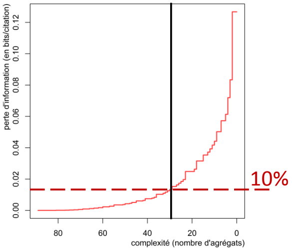
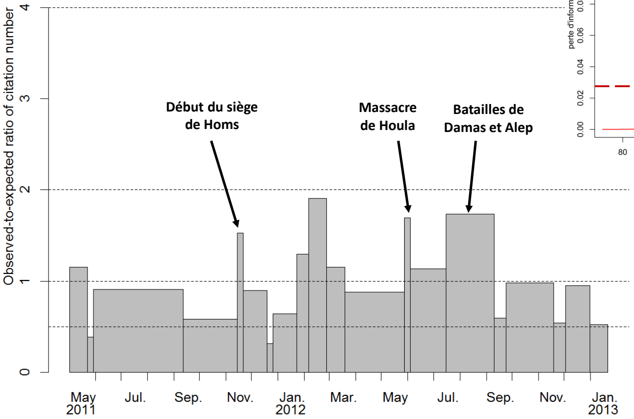
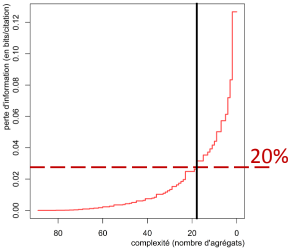




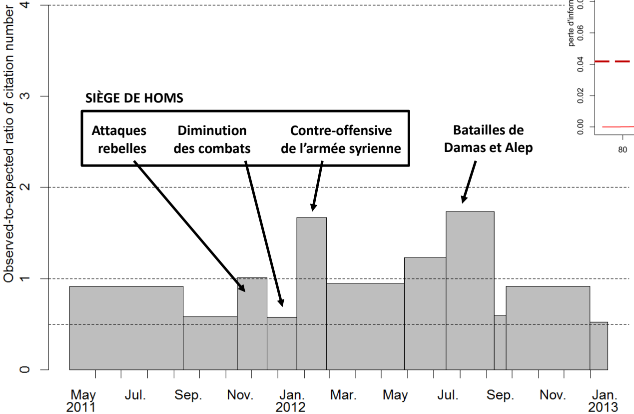
La Syrie vue par Le Monde en 2011 et 2012

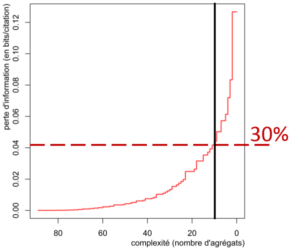




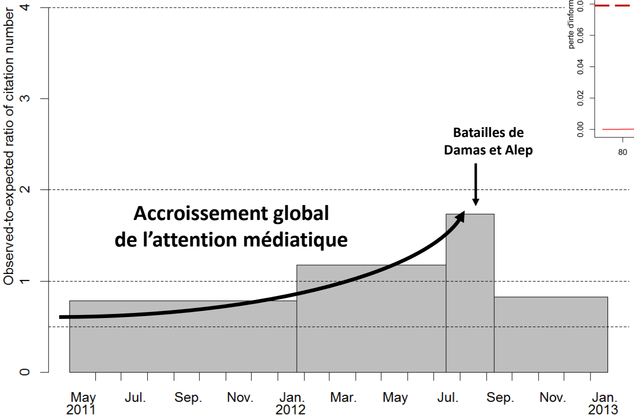
La Syrie vue par Le Monde en 2011 et 2012

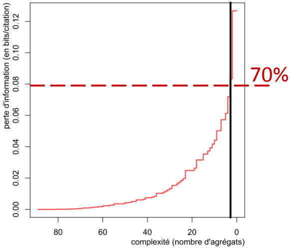
Source : Wikipedia
Timeline of the Syrian civil war
Siege of Homs



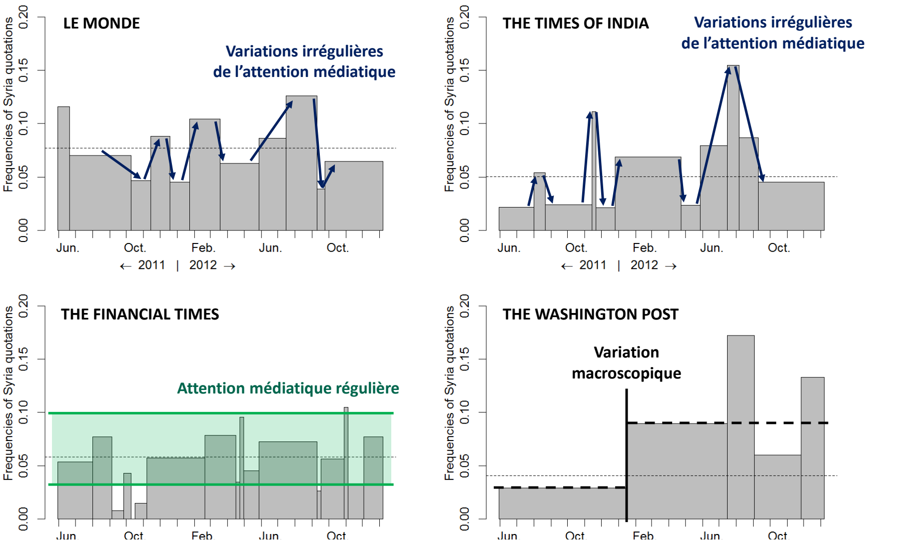


La Syrie vue par Le Monde en 2011 et 2012



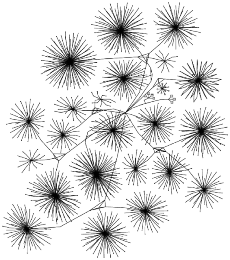


La Syrie vue par 4 journaux différents

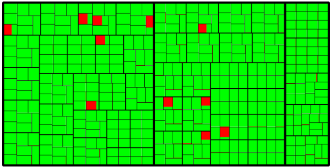


Agrégation de traces d'exécution

Structure hiérarchique de la grille de calcul
[Schnorr *et al.*, 2013]

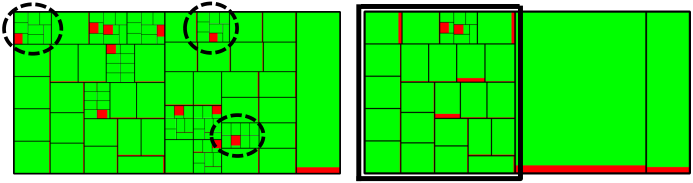


Représentation treemap microscopique

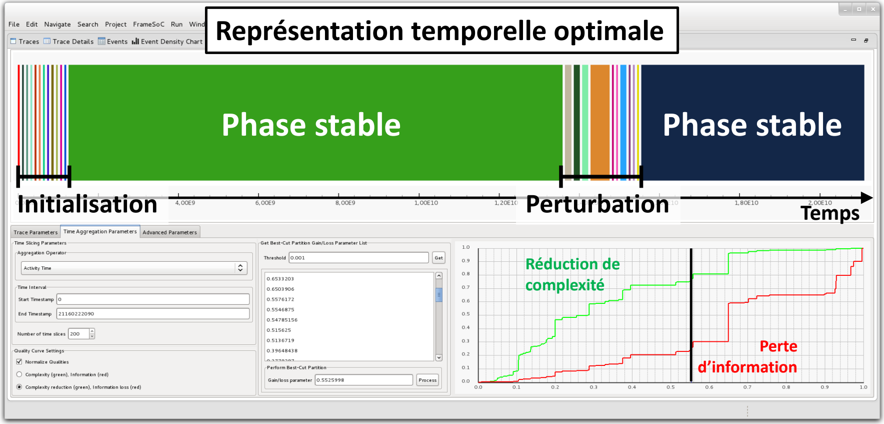


Représentations treemap agrégées

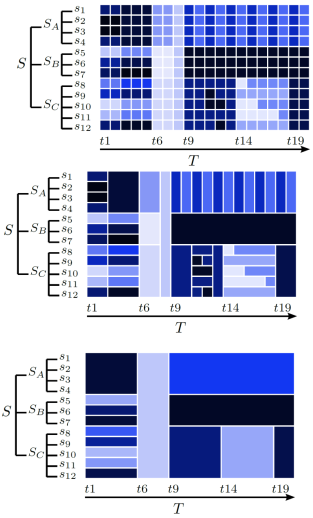
Détection d'anomalies multi-échelle

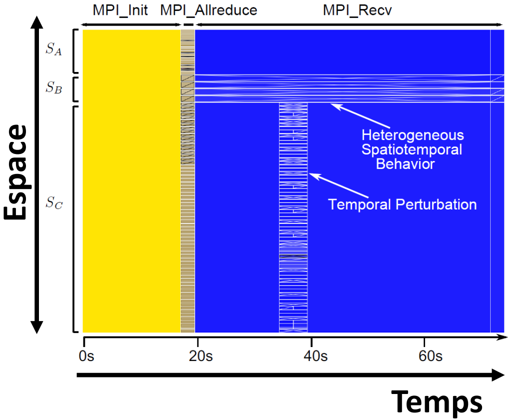


Agrégation de traces d'exécution

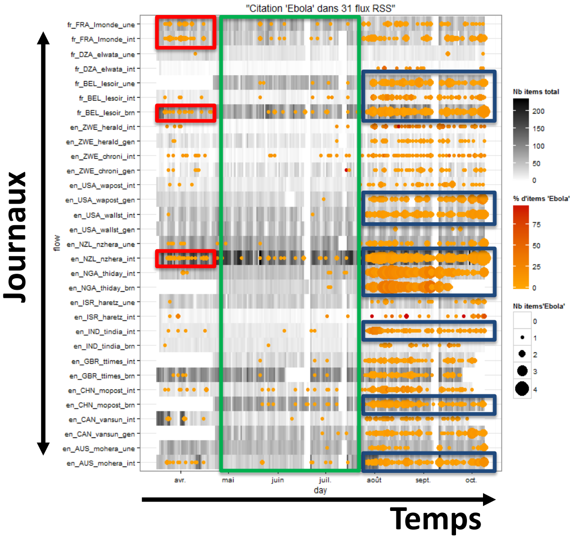


Agrégation spatio-temporelle



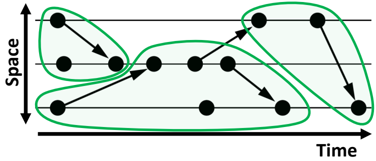


Agrégation médiatique

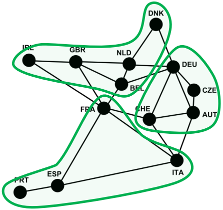


Application Perspectives

Partitioning of Interaction Diagrams [Mattern, 1989]



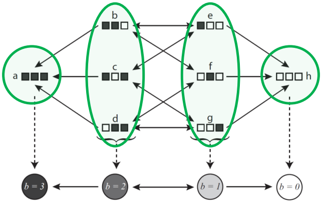
Partitioning of Graphs



Partitioning of Interaction Matrices

	ESP	FRA	GBR	BEL	CHE
ESP	X	12	11	10	4
FRA	14	X	12	12	5
GBR	20	11	X	6	9
BEL	15	9	6	X	5
CHE	10	16	17	9	X

Partitioning the State Space of Dynamical Systems [Banisch *et al.*, 2013]



General Setting

General Setting

$$X^0 \xrightarrow{\quad T^t \quad} X^t \xrightarrow{\quad T^\tau \quad} X^{t+\tau}$$

- Initial State $X^0 \in \Sigma$ within a finite state space Σ
- Current State $X^t \in \Sigma$ with current time $t \in \mathbb{N}$
- Future State $X^{t+\tau} \in \Sigma$ with prediction horizon $\tau \in \mathbb{N}$
- Transition Kernel $T(X^{t+1}|X^t)$ (Markovian and time-homogeneous)

General Setting

$$
\begin{tikzcd}
 & \phi(X^t) \arrow[r, dashed, "\text{prediction}"] & \psi(X^{t+\tau}) \\
X^0 \arrow[r, "T^t"] & X^t \arrow[u, "\phi"] \arrow[r, "T^\tau"] & X^{t+\tau} \arrow[u, "\psi"]
\end{tikzcd}
$$

- Initial State $X^0 \in \Sigma$ within a finite state space Σ
- Current State $X^t \in \Sigma$ with current time $t \in \mathbb{N}$
- Future State $X^{t+\tau} \in \Sigma$ with prediction horizon $\tau \in \mathbb{N}$
- Transition Kernel $T(X^{t+1}|X^t)$ (Markovian and time-homogeneous)
- Pre-measurement $\phi : \Sigma \to \mathcal{S}_\phi$ defined by $\Pr(\phi(X)|X)$
- Post-measurement $\psi : \Sigma \to \mathcal{S}_\psi$ defined by $\Pr(\psi(X)|X)$

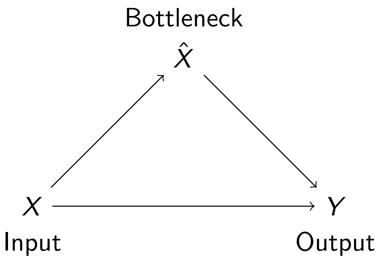
$$
\begin{tikzcd}
 & \phi(X^t) \arrow[r, dashed, "\text{prediction}"] & \psi(X^{t+\tau}) \\
X^0 \arrow[r, "T^t"] & X^t \arrow[u, "\phi"] \arrow[r, "T^\tau"] & X^{t+\tau} \arrow[u, "\psi"]
\end{tikzcd}
$$

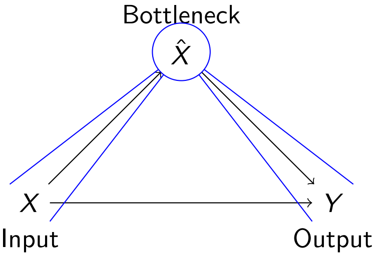
The Optimal Prediction Problem

Given a transition kernel T, an initial distribution X^0, a time t, an horizon τ, and a post-measurement ψ, **find** a pre-measurement ϕ such that $\phi(X^t)$ is "efficient" to predict $\psi(X^{t+\tau})$.

The Information Bottleneck Method (Tishby et al., 1999))

$$\underset{\text{Input}}{X} \longrightarrow \underset{\text{Output}}{Y}$$


Bottleneck
$\hat{X}$
X
Input
Y
Output



- **Minimize** Bottleneck Complexity $min_{\Pr(\hat{X}|X)} \ I(X; \hat{X})$
- **Maximize** Bottleneck Capacity $max_{\Pr(\hat{X}|X)} \ I(\hat{X}; Y)$

- **Minimize** the IB-variational

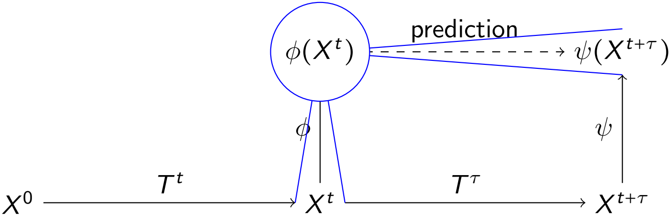
$$min_{\Pr(\hat{X}|X)} \ I(X; \hat{X}) - \beta \ I(\hat{X}; Y) \quad \text{with } \beta \in \mathbb{R}^+$$

The Optimal Prediction Problem

$$\begin{tikzcd}
 & \phi(X^t) \arrow[rr, dashed, "\text{prediction}"] & & \psi(X^{t+\tau}) \\
X^0 \arrow[r, "T^t"] & X^t \arrow[u, "\phi"] \arrow[rr, "T^\tau"] & & X^{t+\tau} \arrow[u, "\psi"']
\end{tikzcd}$$

- **Minimize** Measurement Complexity $min_\phi \ I(X^t; \phi(X^t))$
- **Maximize** Predictive Capacity $max_\phi \ I(\phi(X^t); \psi(X^{t+\tau}))$

The Optimal Prediction Problem



- **Minimize** Measurement Complexity $\quad min_\phi \; I(X^t; \phi(X^t))$
- **Maximize** Predictive Capacity $\quad max_\phi \; I(\phi(X^t); \psi(X^{t+\tau}))$

The Optimal Prediction Problem

Given a time t, an horizon τ, a post-measurement ψ, and a trade-off parameter $\beta \in \mathbb{R}^+$, **find** a pre-measurement ϕ with minimal complexity and maximal anticipatory capacity :

$$\min_\phi \;\; I(X^t; \phi(X^t)) \; - \; \beta \;\; I(\phi(X^t); \psi(X^{t+\tau}))$$

Constraining the Set of Feasible Measurements

$$
\begin{tikzcd}
 & \{\phi_1(X^t), & \phi_2(X^t), & \dots, & \phi_k(X^t)\} \arrow[r, dashed, "\text{prediction}"] & \psi(X^{t+\tau}) \\
X^0 \arrow[rrrr, "T^t"] & & & & X^t \arrow[ulll, "\phi_1"] \arrow[ull, "\phi_2"] \arrow[u, "\phi_k"] \arrow[r, "T^\tau"] & X^{t+\tau} \arrow[u, "\psi"']
\end{tikzcd}
$$

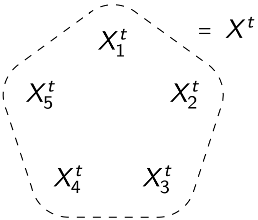
The Constrained Optimal Prediction Problem

Given a time t, an horizon τ, a post-measurement ψ, a trade-off parameter $\beta \in \mathbb{R}^+$, and a set of feasible pre-measurements $\Phi = \{\phi_1, \dots, \phi_k\}$, **find** a feasible pre-measurement ϕ with minimal complexity and maximal anticipatory capacity :

$$\min_{\phi \subset \Phi} \; I(X^t; \phi(X^t)) \; - \; \beta \; I(\phi(X^t); \psi(X^{t+\tau}))$$

Generic Measurement in Agent-based Systems

- Set of Agents $\Omega = \{1, \dots, N\}$
- State of ith Agent $X_i^t \in S$
- System State $X^t = (X_1^t, \dots, X_N^t) \in \Sigma = S^N$
- Transitions Kernel $T(X_i^{t+1} | X^t)$



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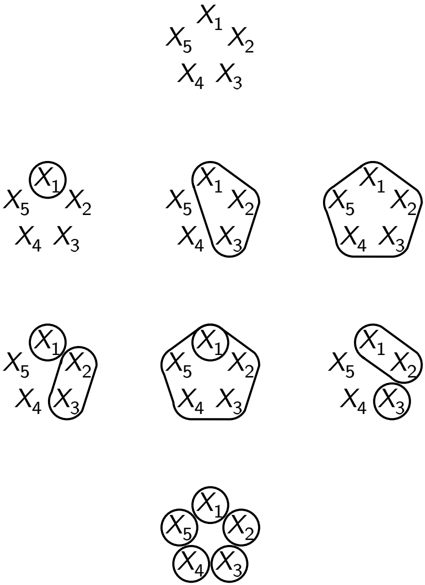
Definition (Generic Measurement μ)

A family of measurements $(\mu_A : \Sigma \to \mathcal{S}_\mu)_{A \subset \Omega}$ *such that :*

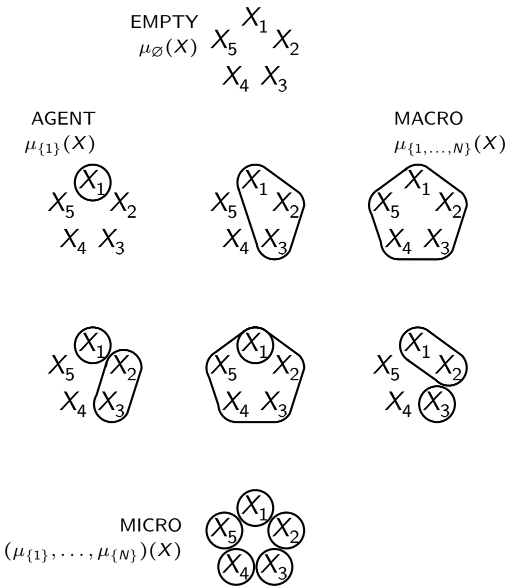
$$\forall A \subset \Omega, \quad \Pr(\mu_A(X) | X) = \Pr(\mu_A(X) | (X_i)_{i \in A})$$

- Feasible Measurement $(\mu_{A_1}, \ldots, \mu_{A_k}) : \Sigma \to (\mathcal{S}_\mu)^k$
  with $\{A_1, \ldots, A_k\} \subset 2^\Omega$

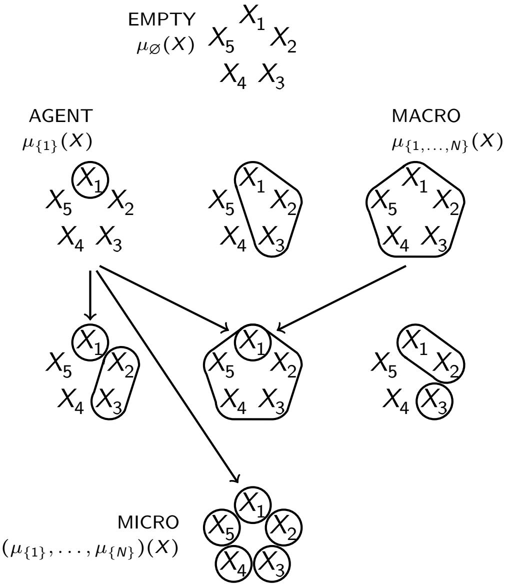
Example of Feasible Measurements



Example of Feasible Measurements



Example of Feasible Measurements



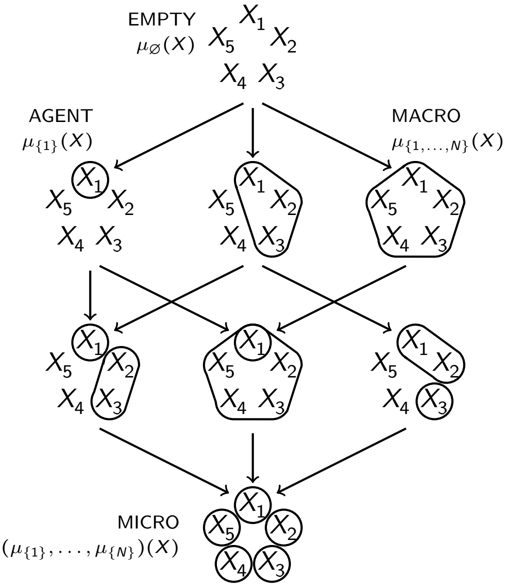
Definition 1 (Refinement)

$\phi_1 < \phi_2$
iff $X \rightarrow \phi_1(X) \rightarrow \phi_2(X)$ is aMarkov chain
iff $I(X; \phi_2(X) | \phi_1(X)) = 0$

Theorem 2 (Monotonicity)

$\phi_1 \subset \phi_2 \;\Rightarrow$
$I(X^t; \phi_1(X^t)) \geq I(X^t; \phi_2(X^t))$ *and*
$I(\phi_1(X^t); \psi(X^{t+\tau})) \geq I(\phi_2(X^t); \psi(X^{t+\tau}))$

Example of Feasible Measurements



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$I(\phi_1(X^t); \psi(X^{t+\tau})) \geq I(\phi_2(X^t); \psi(X^{t+\tau}))$

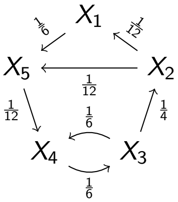
Definition 3 (Additivity)

μ additive iff $\forall A \cap B = \varnothing$,
$H(\mu_{A\cup B}(X) \mid \mu_A(X), \mu_B(X)) = 0$
and $H(\mu_A(X) \mid \mu_{A\cup B}(X), \mu_B(X)) = 0$

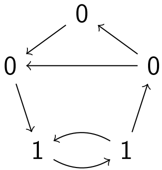
Application to the Voter Model (Banisch & Lima, 2012)

- Set of Agents $\Omega = \{1, \ldots, N\}$
- State of ith Agent $X_i^t \in \{0, 1\}$
- System State $X^t = (X_1^t, \ldots, X_N^t) \in \{0, 1\}^N$
- Transitions Kernel $T(X_i^{t+1} | X^t)$ given by an interaction graph :

$$\text{arc } (i, j) \text{ selected} \quad \Rightarrow \quad X_j^{t+1} = X_i^t \quad \text{and} \quad \forall k \neq j, \quad X_k^{t+1} = X_k^t$$



$t = 0$



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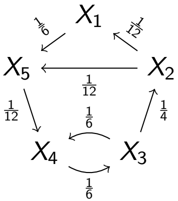
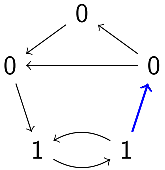
- Transitions Kernel $T(X_i^{t+1} | X^t)$ given by an interaction graph :

arc (i, j) selected $\Rightarrow$ $X_j^{t+1} = X_i^t$ and $\forall k \neq j,$ $X_k^{t+1} = X_k^t$



$t = 0$ $\rightarrow$ arc $(3, 2)$



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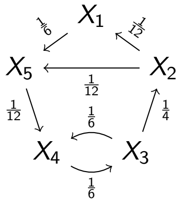
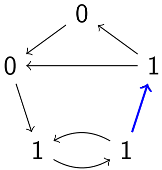


$t = 1$



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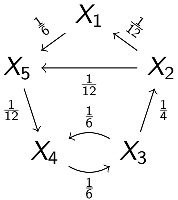
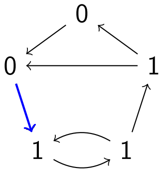


$t = 1$ $\rightarrow$ arc $(5, 4)$



Application to the Voter Model (Banisch & Lima, 2012)

- Set of Agents $\Omega = \{1, \ldots, N\}$
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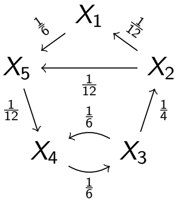
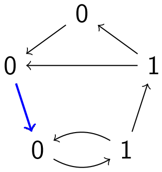


$t = 2$



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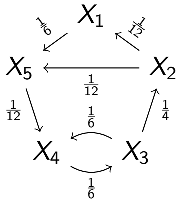
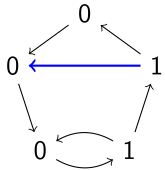


$t = 2 \quad \rightarrow \quad \text{arc } (2, 5)$



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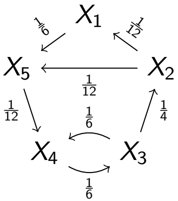
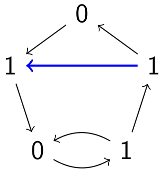


$t = 3$



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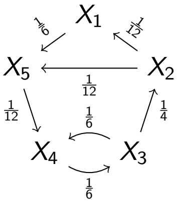
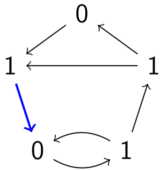


$t = 3 \quad \rightarrow \quad \text{arc } (5, 4)$



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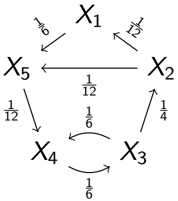
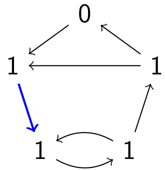


$t = 5$



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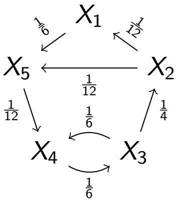
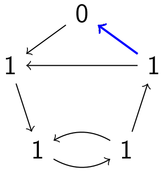


$t = 5 \quad \rightarrow \quad \text{arc } (2, 1)$



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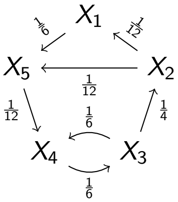
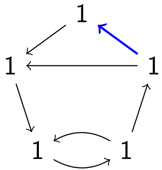


$t = 6$



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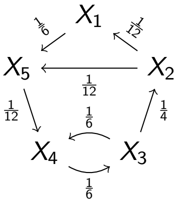

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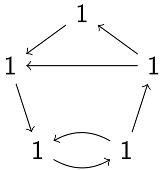
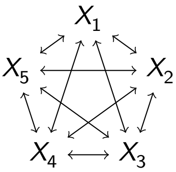


The Complete Graph



- All arcs are equally likely $\quad \forall (i,j) \in \Omega^2, \quad \Pr(\text{arc } (i,j)) = \frac{1}{N\,(N-1)}$
- Uniform Initial State $\quad \forall x \in \{0,1\}^N, \quad p(X^0 = x) = 2^{-N}$
- Two Equiprobable Attractors $\quad X^\infty \in \{(0,\dots,0),(1,\dots,1)\}$

Aggregated-state Measurement in the Voter Model

- Aggregated-state (Generic) Measurement $\forall A \subset \Omega, \quad \eta_A(x) = \sum_{i \in A} x_i$

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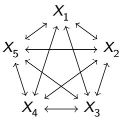
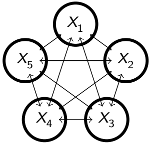
EMPTY

$\eta_\varnothing(x) = 0$



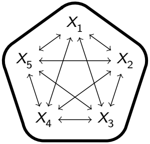
MICRO

$(\eta_{\{1\}}, \ldots, \eta_{\{N\}})(x)$



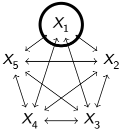
MACRO

$\eta_\Omega(x) = x_1 + \ldots + x_N$



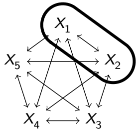
AGENT

$\eta_{\{1\}}(x) = x_1$



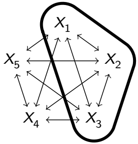
SIZE2

$\eta_{\{1,2\}}(x) = x_1 + x_2$

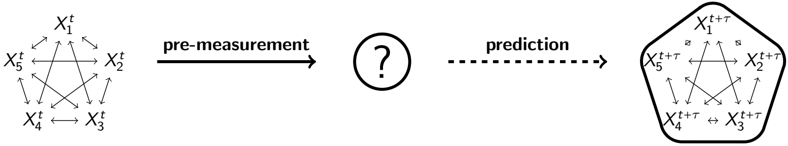


SIZE3

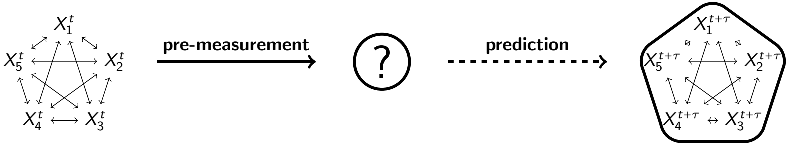
$\eta_{\{1,2,3\}}(x) = x_1 + x_2 + x_3$



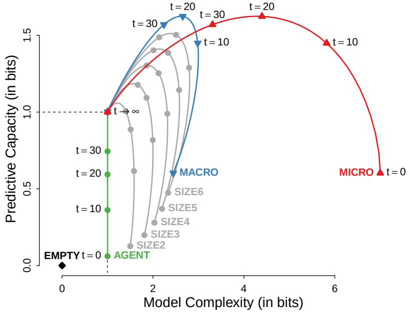
Predicting the Macroscopic Measurement



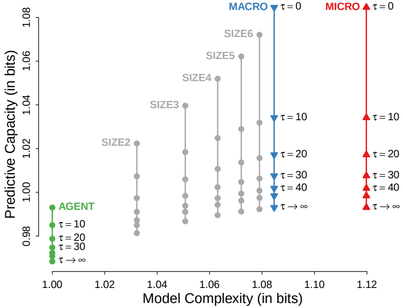
Predicting the Macroscopic Measurement



Fixed size $N = 7$
Fixed horizon $\tau = 3$



Fixed size $N = 7$
Fixed time $t = 100$



Information Bottleneck Diagram

- Tridimensional Parameter Space $(t, \tau, \beta) \in \mathbb{N} \times \mathbb{N} \times \mathbb{R}^+$

Information Bottleneck Diagram

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- Given two pre-measurements ϕ_1 and ϕ_2 :

The border is...	when...	ϕ_1 opt. region	ϕ_2 opt. region
(1) strictly positive	$H_1 < H_2$ and $I_1 < I_2$	$[0, \beta^{t,\tau}_{\phi_1,\phi_2}]$	$[\beta^{t,\tau}_{\phi_1,\phi_2}, +\infty[$
	$H_1 > H_2$ and $I_1 > I_2$	$[\beta^{t,\tau}_{\phi_1,\phi_2}, +\infty[$	$[0, \beta^{t,\tau}_{\phi_1,\phi_2}]$

with $H_i = I(X^t; \phi_i(X^t))$, $I_i = I(\phi_i(X^t); \psi(X^{t+\tau}))$ and $\beta^{t,\tau}_{\phi_1,\phi_2} = \frac{H_2 - H_1}{I_2 - I_1}$.

Information Bottleneck Diagram

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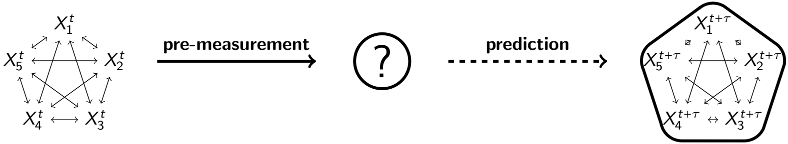
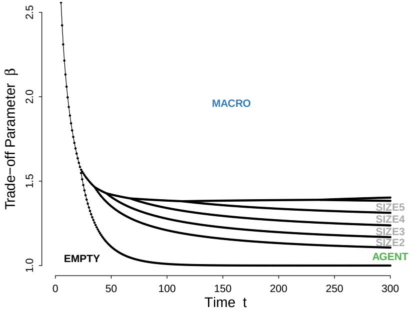
	$H_1 > H_2$ and $I_1 > I_2$	$[\beta^{t,\tau}_{\phi_1,\phi_2}, +\infty[$	$[0, \beta^{t,\tau}_{\phi_1,\phi_2}]$
(2) null	$H_1 = H_2$ and $I_1 < I_2$	$\{0\}$	$[0, +\infty[$
	$H_1 = H_2$ and $I_1 > I_2$	$[0, +\infty[$	$\{0\}$
(3) infinite	$H_1 < H_2$ and $I_1 = I_2$	$[0, +\infty[$	$\{+\infty\}$
	$H_1 > H_2$ and $I_1 = I_2$	$\{+\infty\}$	$[0, +\infty[$
(4) defined nowhere	$H_1 < H_2$ and $I_1 > I_2$	$[0, +\infty[$	$\emptyset$
	$H_1 > H_2$ and $I_1 < I_2$	$\emptyset$	$[0, +\infty[$
(5) defined everywhere	$H_1 = H_2$ and $I_1 = I_2$	$[0, +\infty[$	$[0, +\infty[$

with $H_i = I(X^t; \phi_i(X^t))$, $I_i = I(\phi_i(X^t); \psi(X^{t+\tau}))$ and $\beta^{t,\tau}_{\phi_1,\phi_2} = \frac{H_2 - H_1}{I_2 - I_1}$.

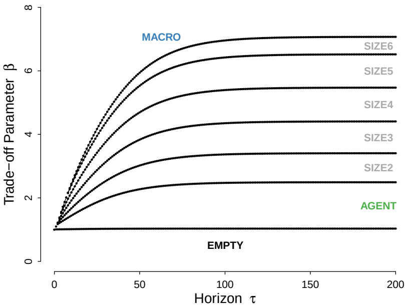
Predicting the Macroscopic Measurement



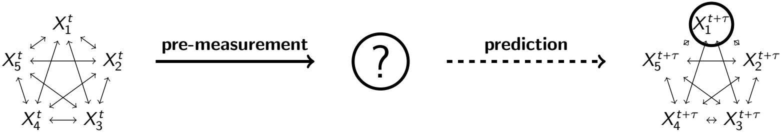
Fixed size $N = 7$
Fixed horizon $\tau = 3$



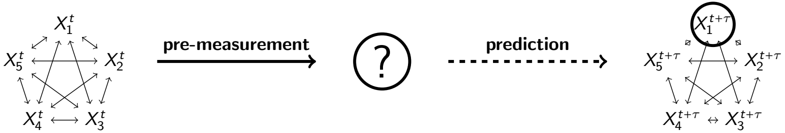
Fixed size $N = 7$
Fixed time $t = 100$



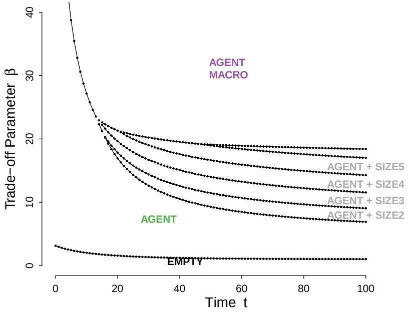
Predicting the Agent Measurement



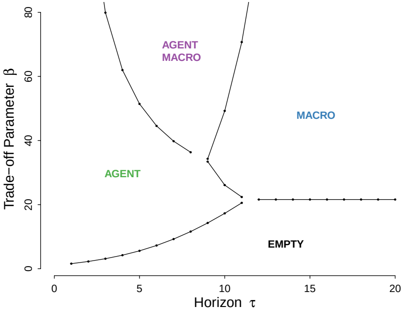
Predicting the Agent Measurement



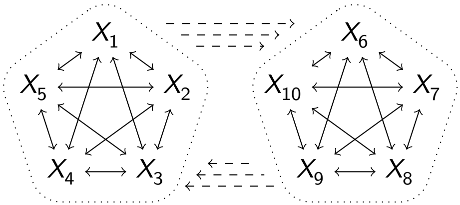
Fixed size $N = 7$
Fixed horizon $\tau = 3$



Fixed size $N = 7$
Fixed time $t = 0$



The Two-community Voter Model



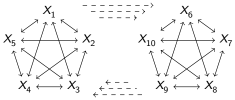
First Community Ω_1 Second Community Ω_2

Coupling Parameter $\rho = \dfrac{\Pr(\text{inter edge})}{\Pr(\text{intra edge})} < 1$

The Two-community Voter Model

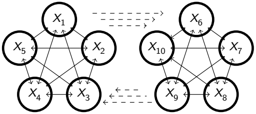
EMPTY

$\eta_{\varnothing}$



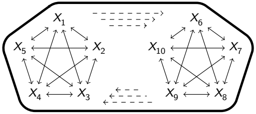
MICRO

$\{\eta_{\{1\}}, \ldots, \eta_{\{N\}}\}$



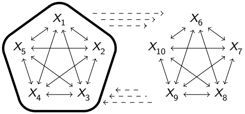
MACRO

η_{Ω}



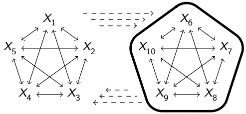
MESO1

η_{Ω_1}



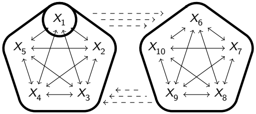
MESO2

η_{Ω_2}

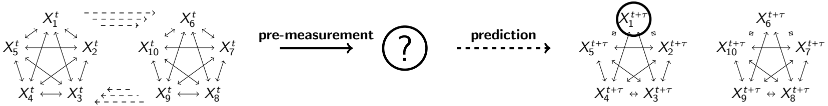


AGENT+MESO1+MESO2

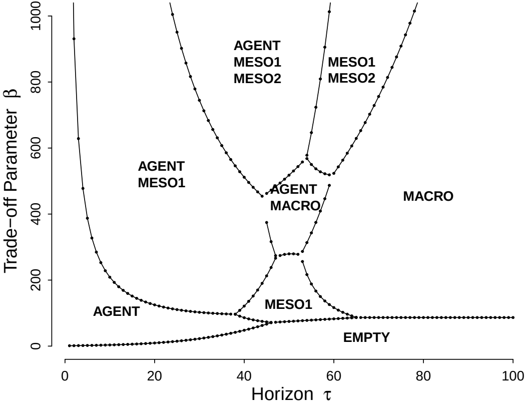
$(\eta_{\{1\}}, \eta_{\Omega_1}, \eta_{\Omega_2})$



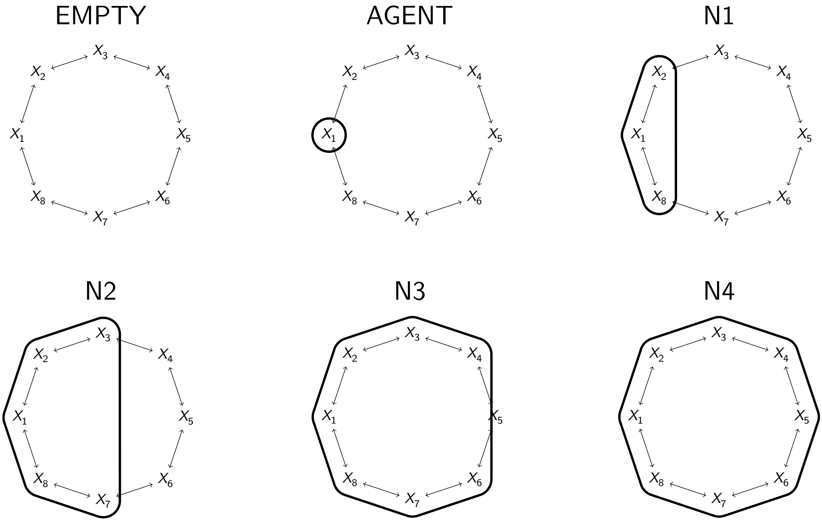
Predicting the Agent Measurement



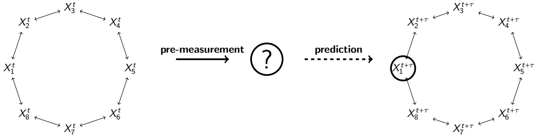
Fixed size and coupling $N_1 = 10,\ N_2 = 10,\ \rho = \frac{1}{5}$
Fixed time $t = 0$



The Ring Voter Model

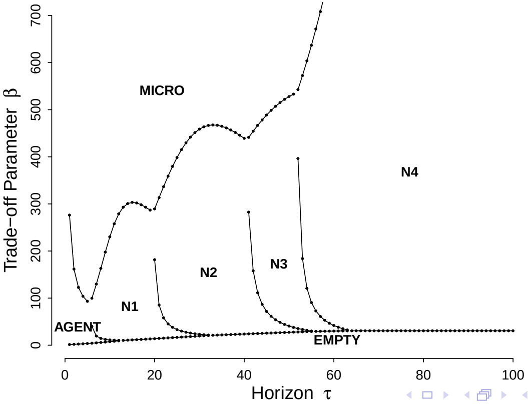


Predicting the Agent Measurement in the Ring



Fixed size $N = 9$

Fixed time $t = 0$



Application Perspectives

- Application to efficient prediction of **population dynamics in ecology :** different species compete for survival in the environment
    - Multilevel measurements regarding the geographical space (related to geographical data aggregation)
    - Multilevel measurements regarding the structure of inter-species mutual dependencies (webs of food, reproduction dynamics)
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- Application to efficient prediction of **trade flows in economy** : countries exchange products on a global scale
    - Multilevel measurements regarding the network of international trade (related to graph theory and community modelling)
    - Multilevel measurements regarding the structure of products (production chains, economic fields)
    - Complexity should also be used to model data collection costs

Merci pour votre attention

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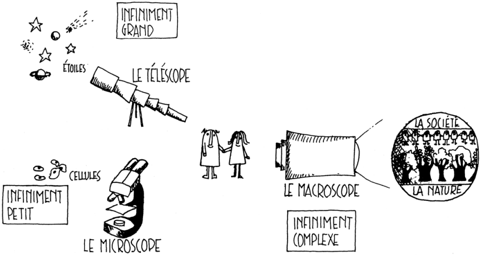

« Aujourd'hui nous sommes confrontés à un autre infini : l'infiniment complexe. Mais cette fois, plus d'instrument. »

Joël de Rosnay, *Le macroscope*, 1975