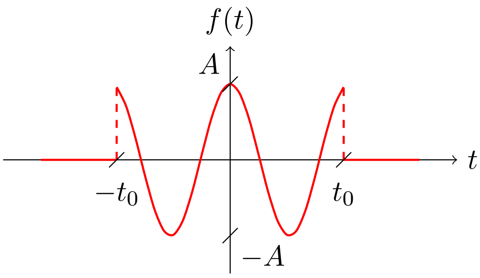
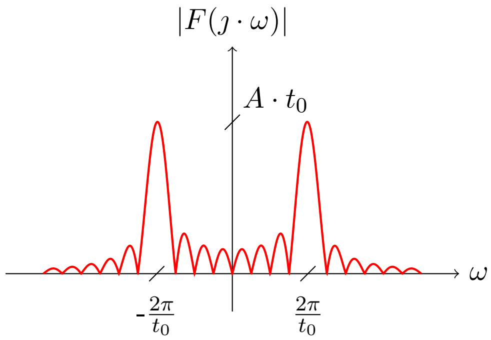
Teoria Sygnałów w zadaniach





$$f(t) = A \cdot \Pi\left(\frac{t}{2 \cdot t_0}\right) \cdot cos\left(\frac{2\pi}{t_0} \cdot t\right)$$

$$F(\jmath\omega) = A \cdot t_0 \cdot [\ Sa\,(\omega \cdot t_0 + 2\pi) \\ -Sa\,(\omega \cdot t_0 - 2\pi)]$$

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Rozdział 1

Podstawowe własności sygnałów

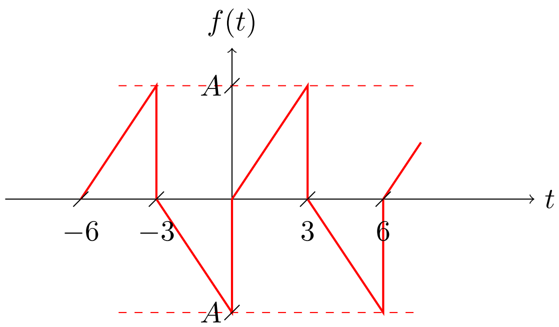
1.1 Podstawowe własności sygnałów

1.1.1 Wartość średnia

1.1.2 Energia sygnału

1.1.3 Moc sygnału

Zadanie 1. Calculate the average power and the effective value (RMS) for the periodic signal given below



First of all, the definition of $f(t)$ signal has to be derived. This is periodic piecewise function, piecewise linear function to be precise. The simplest form of linear function is:

$$f(t) = m \cdot t + b \tag{1.1}$$

In the first interval of the first period (e.g. $t \in (0;3)$), linear function crosses two points: $(0,0)$ and $(3,A)$. So, in order to derive m and b, the following system of the equations has to be solved.

$$\begin{cases} 0 = m \cdot 0 + b \\ A = m \cdot 3 + b \end{cases}$$

$$\begin{cases} 0 = b \\ A = m \cdot a \cdot T + b \end{cases}$$

$$\begin{cases} 0 = b \\ A = m \cdot 3 + 0 \end{cases}$$

$$\begin{cases} 0 = b \\ \frac{A}{3} = m \end{cases}$$

As a result we get:

$$f(t) = \frac{A}{3} \cdot t$$

In the second interval of the first period (e.g. $t \in (3; 6)$), linear function crosses other two points:: $(3, 0)$ oraz $(6, -A)$. So, in order to derive m i b, the following system of the equations has to be solved.

$$\begin{cases} 0 = m \cdot 3 + b \\ -A = m \cdot 6 + b \end{cases}$$

$$\begin{cases} -3 \cdot m = b \\ -A = 6 \cdot m - 3 \cdot m \end{cases}$$

$$\begin{cases} -3 \cdot m = b \\ -A = 3 \cdot m \end{cases}$$

$$\begin{cases} -3 \cdot m = b \\ -\frac{A}{3} = m \end{cases}$$

$$\begin{cases} -3 \cdot (-\frac{A}{3}) = b \\ -\frac{A}{3} = m \end{cases}$$

$$\begin{cases} A = b \\ -\frac{A}{3} = m \end{cases}$$

As a result, second interval of the first period is described by:

$$f(t) = -\frac{A}{3} \cdot t + A$$

As a result the piecewise linear function in the first period is given by:

$$f(t) = \begin{cases} \frac{A}{3} \cdot t & for \quad t \in (0; 3) \\ -\frac{A}{3} \cdot t + A & for \quad t \in (3; 6) \end{cases}$$

For the whole periodic signal $f(t)$ we get:

$$f(t) = \begin{cases} \frac{A}{3} \cdot (t - k \cdot 6) & for \quad t \in (0 + k \cdot 6; 3 + k \cdot 6) \\ -\frac{A}{3} \cdot (t - k \cdot 6) + A & for \quad t \in (3 + k \cdot 6; 6 + k \cdot 6) \end{cases} \wedge k \in I$$

The average power for periodic signals is defined by:

$$P = \frac{1}{T} \cdot \int_T |f(t)|^2 \cdot dt \tag{1.2}$$

In our case we get:

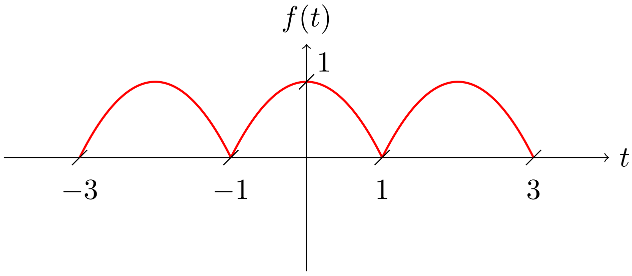
$$
\begin{aligned}
P &= \frac{1}{T} \cdot \int_T |f(t)|^2 \cdot dt = \\
&= \frac{1}{6} \cdot \left(\int_0^3 \left| \frac{A}{3} \cdot t \right|^2 \cdot dt + \int_3^6 \left| -\frac{A}{3} \cdot t + A \right|^2 \cdot dt \right) = \\
&= \frac{1}{6} \cdot \int_0^3 \left(\frac{A}{3} \cdot t \right)^2 \cdot dt + \frac{1}{6} \cdot \int_3^6 \left(-\frac{A}{3} \cdot t + A \right)^2 \cdot dt = \\
&= \frac{1}{6} \cdot \int_0^3 \frac{A^2}{9} \cdot t^2 \cdot dt + \frac{1}{6} \cdot \int_3^6 \left(\left(-\frac{A}{3} \cdot t \right)^2 - 2 \cdot \frac{A}{3} \cdot t \cdot A + A^2 \right) \cdot dt = \\
&= \frac{A^2}{54} \cdot \int_0^3 t^2 \cdot dt + \frac{1}{6} \cdot \int_3^6 \frac{A^2}{9} \cdot t^2 \cdot dt - \frac{1}{6} \cdot \int_3^6 \frac{2 \cdot A^2}{3} \cdot t \cdot dt + \frac{1}{6} \cdot \int_3^6 A^2 \cdot dt = \\
&= \frac{A^2}{54} \cdot \left. \frac{t^3}{3} \right|_0^3 + \frac{A^2}{54} \cdot \int_3^6 t^2 \cdot dt - \frac{2 \cdot A^2}{18} \cdot \int_3^6 t^2 \cdot dt + \frac{A^2}{6} \cdot \int_3^6 dt = \\
&= \frac{A^2}{162} \cdot \left(3^3 - 0^3 \right) + \frac{A^2}{54} \cdot \left. \frac{t^3}{3} \right|_3^6 - \frac{2 \cdot A^2}{18} \cdot \left. \frac{t^2}{2} \right|_3^6 + \frac{A^2}{6} \cdot t|_3^6 = \\
&= \frac{A^2}{162} \cdot 27 + \frac{A^2}{162} \cdot \left(6^3 - 3^3 \right) - \frac{2 \cdot A^2}{36} \cdot \left(6^2 - 3^2 \right) + \frac{A^2}{6} \cdot (6 - 3) = \\
&= \frac{A^2}{6} + \frac{A^2}{162} \cdot 189 - \frac{2 \cdot A^2}{36} \cdot 27 + \frac{A^2}{6} \cdot 3 = \\
&= \frac{A^2}{6} + \frac{7 \cdot A^2}{6} - \frac{9 \cdot A^2}{6} + \frac{3 \cdot A^2}{6} = \\
&= \frac{2 \cdot A^2}{6} = \\
&= \frac{A^2}{3}
\end{aligned}
$$

The average power equals to $\frac{A^2}{3}$.
The effective value (RMS) is defined by:

$$RMS = \sqrt{P} \tag{1.3}$$

Therefore, effective value (RMS) equals to $\frac{A}{\sqrt{3}}$.

Zadanie 2. Compute the average power for the following periodic signal $f(t)$



Signal in the range $t \in (-1; 1)$ is described as:

$$f(t) = 1 - t^2 \tag{1.4}$$

The average power for periodic signals is defined by:

$$P = \frac{1}{T} \cdot \int_T |f(t)|^2 \cdot dt \tag{1.5}$$

In this case period T is equal to 2.

$$\begin{aligned}
P &= \frac{1}{T} \cdot \int_T |f(t)|^2 \cdot dt = \\
&= \frac{1}{2} \cdot \int_{-1}^{1} \left|1 - t^2\right|^2 \cdot dt = \\
&= \frac{1}{2} \cdot \int_{-1}^{1} \left(1 - t^2\right)^2 \cdot dt = \\
&= \frac{1}{2} \cdot \int_{-1}^{1} \left(1 - 2 \cdot t^2 + t^4\right) \cdot dt = \\
&= \frac{1}{2} \cdot \left[\int_{-1}^{1} 1 \cdot dt + \int_{-1}^{1} (-2) \cdot t^2 \cdot dt + \int_{-1}^{1} t^4 \cdot dt\right] = \\
&= \frac{1}{2} \cdot \left[t\Big|_{-1}^{1} - 2 \cdot \left.\frac{t^3}{3}\right|_{-1}^{1} + \left.\frac{t^5}{5}\right|_{-1}^{1} \right] = \\
&= \frac{1}{2} \cdot \left[(1 - (-1)) - \frac{2}{3} \cdot (1 - (-1)) + \frac{1}{5} \cdot (1 - (-1))\right] = \\
&= \frac{1}{2} \cdot \left[2 - \frac{4}{3} + \frac{2}{5}\right] = \\
&= \frac{1}{2} \cdot \left[\frac{30}{15} - \frac{20}{15} + \frac{6}{15}\right] = \\
&= \frac{1}{2} \cdot \frac{16}{15} = \\
&= \frac{8}{15}
\end{aligned}$$

The average power equals to $\frac{8}{15}$.

Rozdział 2

Analiza sygnałów okresowych za pomocą szeregów ortogonalnych

2.1 Trygonometryczny szerego Fouriera

2.2 Zespolony szerego Fouriera

2.3 Obliczenia mocy sygnałów - twierdzenie Parsevala

Rozdział 3

Analiza sygnałów nieokresowych. Transformata Fouriera

3.1 Wyznaczanie transformaty Fouriera z definicji

3.2 Wykorzystanie twierdzeń do obliczeń transformaty Fouriera

3.3 Obliczenia energii sygnału za pomocą transformaty Fouriera. Twierdzenie Parsevala

Rozdział 4

Przetwarzanie sygnałów za pomocą układów LTI

4.1 Obliczanie splotu ze wzoru

4.2 Filtry

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