Teoria Sygnałów w zadaniach

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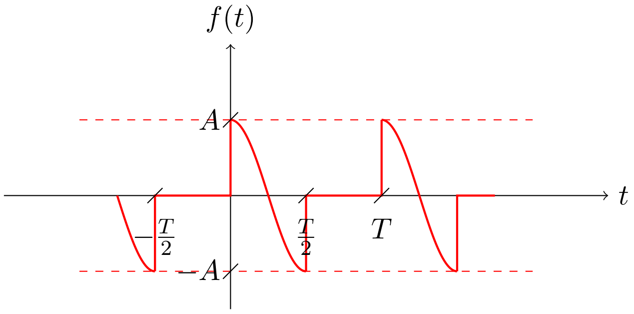
Zadanie 1. Wyznacz wszystkie współczynniki zespolonego szeregu fouriera dla okresowego sygnału $f(t)$ będącego przekształceniem sygnału cosinusoidalnego przedstawionego na rysunku.



W pierwszej kolejności należy opisać sygnał za pomocą wzoru:

$$f(x) = \begin{cases} A \cdot cos\left(\frac{2\pi}{T} \cdot t\right) & t \in \left(0 + k \cdot T; \frac{T}{2} + k \cdot T\right) \\ 0 & t \in \left(\frac{T}{2} + k \cdot T; T + k \cdot T\right) \end{cases} \wedge k \in C \tag{1}$$

Współczynnik F_0 wyznaczamy ze wzoru

$$F_0 = \frac{1}{T} \int_T f(t) \cdot dt \tag{2}$$

Podstawiamy do wzoru wzór naszej funkcji w pierwszym okresie $k = 0$

$$\begin{aligned}
F_0 &= \frac{1}{T} \int_T f(t) \cdot dt \\
&= \frac{1}{T} \left(\int_0^{\frac{T}{2}} A \cdot cos\left(\frac{2\pi}{T} \cdot t\right) \cdot dt + \int_{\frac{T}{2}}^{T} 0 \cdot dt \right) \\
&= \frac{1}{T} \left(A \cdot \int_0^{\frac{T}{2}} cos\left(\frac{2\pi}{T} \cdot t\right) \cdot dt + 0 \right) \\
&= \frac{A}{T} \cdot \int_0^{\frac{T}{2}} cos\left(\frac{2\pi}{T} \cdot t\right) \cdot dt \\
&= \left\{ \begin{array}{ll} z & = \frac{2\pi}{T} \cdot t \\ dz & = \frac{2\pi}{T} \cdot dt \\ dt & = \frac{1}{\frac{2\pi}{T}} \cdot dz \\ dt & = \frac{T}{2\pi} \cdot dz \end{array} \right\} \\
&= \frac{A}{T} \cdot \int_0^{\frac{T}{2}} cos\left(z\right) \cdot \frac{T}{2\pi} \cdot dz \\
&= \frac{A}{T} \cdot \frac{T}{2\pi} \cdot \int_0^{\frac{T}{2}} cos\left(z\right) \cdot dz \\
&= \frac{A}{T} \cdot \frac{T}{2\pi} \cdot sin\left(z\right)\Big|_0^{\frac{T}{2}} \\
&= \frac{A}{2\pi} \cdot sin\left(\frac{2\pi}{T} \cdot t\right)\Bigg|_0^{\frac{T}{2}} \\
&= \frac{A}{2\pi} \cdot \left(sin\left(\frac{2\pi}{T} \cdot \frac{T}{2}\right) - sin\left(\frac{2\pi}{T} \cdot 0\right) \right)
\end{aligned}$$

$$
\begin{aligned}
&= \frac{A}{2\pi} \cdot (sin\,(pi) - sin\,(0)) \\
&= \frac{A}{2\pi} \cdot (0 - 0) \\
&= \frac{A}{2\pi} \cdot 0 \\
&= 0
\end{aligned}
$$

Wartość współczynnika F_0 wynosi 0

Współczynnik F_k wyznaczamy ze wzoru

$$F_k = \frac{1}{T} \int_T f(t) \cdot e^{-\jmath \cdot k \cdot \frac{2\pi}{T} \cdot t} \cdot dt \tag{3}$$

Podstawiamy do wzoru wzór naszej funkcji w pierwszym okresie $k = 0$

$$
\begin{aligned}
F_k &= \frac{1}{T} \int_T f(t) \cdot e^{-\jmath \cdot k \cdot \frac{2\pi}{T} \cdot t} \cdot dt \\
&= \frac{1}{T} \left(\int_0^{\frac{T}{2}} A \cdot cos\left(\frac{2\pi}{T} \cdot t\right) \cdot e^{-\jmath \cdot k \cdot \frac{2\pi}{T} \cdot t} \cdot dt + \int_{\frac{T}{2}}^{T} 0 \cdot e^{-\jmath \cdot k \cdot \frac{2\pi}{T} \cdot t} \cdot dt \right) \\
&= \frac{1}{T} \left(A \cdot \int_0^{\frac{T}{2}} cos\left(\frac{2\pi}{T} \cdot t\right) \cdot e^{-\jmath \cdot k \cdot \frac{2\pi}{T} \cdot t} \cdot dt + \int_{\frac{T}{2}}^{T} 0 \cdot dt \right) \\
&= \left\{ cos\,(x) = \tfrac{e^{\jmath \cdot x} + e^{-\jmath \cdot x}}{2} \right\} \\
&= \frac{1}{T} \left(A \cdot \int_0^{\frac{T}{2}} \frac{e^{\jmath \cdot \frac{2\pi}{T} \cdot t} + e^{-\jmath \cdot \frac{2\pi}{T} \cdot t}}{2} \cdot e^{-\jmath \cdot k \cdot \frac{2\pi}{T} \cdot t} \cdot dt + 0 \right) \\
&= \frac{A}{2 \cdot T} \cdot \int_0^{\frac{T}{2}} \left(e^{\jmath \cdot \frac{2\pi}{T} \cdot t} + e^{-\jmath \cdot \frac{2\pi}{T} \cdot t} \right) \cdot e^{-\jmath \cdot k \cdot \frac{2\pi}{T} \cdot t} \cdot dt \\
&= \frac{A}{2 \cdot T} \cdot \int_0^{\frac{T}{2}} \left(e^{\jmath \cdot \frac{2\pi}{T} \cdot t - \jmath \cdot k \cdot \frac{2\pi}{T} \cdot t} + e^{-\jmath \cdot \frac{2\pi}{T} \cdot t - \jmath \cdot k \cdot \frac{2\pi}{T} \cdot t} \right) \cdot dt \\
&= \frac{A}{2 \cdot T} \cdot \int_0^{\frac{T}{2}} \left(e^{\jmath \cdot \frac{2\pi}{T} \cdot (1-k) \cdot t} + e^{-\jmath \cdot \frac{2\pi}{T} \cdot (1+k) \cdot t} \right) \cdot dt \\
&= \frac{A}{2 \cdot T} \cdot \left(\int_0^{\frac{T}{2}} e^{\jmath \cdot \frac{2\pi}{T} \cdot (1-k) \cdot t} \cdot dt + \int_0^{\frac{T}{2}} e^{-\jmath \cdot \frac{2\pi}{T} \cdot (1+k) \cdot t} \cdot dt \right) \\
&= \left\{ \begin{matrix} z_1 = \jmath \cdot \frac{2\pi}{T} \cdot (1-k) \cdot t & z_2 = -\jmath \cdot \frac{2\pi}{T} \cdot (1+k) \cdot t \\ dz_1 = \jmath \cdot \frac{2\pi}{T} \cdot (1-k) \cdot dt & dz_2 = -\jmath \cdot \frac{2\pi}{T} \cdot (1+k) \cdot dt \\ dt = \frac{1}{\jmath \cdot \frac{2\pi}{T} \cdot (1-k)} \cdot dz_1 & dt = \frac{1}{-\jmath \cdot \frac{2\pi}{T} \cdot (1+k)} \cdot dz_2 \end{matrix} \right\} \\
&= \frac{A}{2 \cdot T} \cdot \left(\int_0^{\frac{T}{2}} e^{z_1} \cdot \frac{1}{\jmath \cdot \frac{2\pi}{T} \cdot (1-k)} \cdot dz_1 + \int_0^{\frac{T}{2}} e^{z_2} \cdot \frac{1}{-\jmath \cdot \frac{2\pi}{T} \cdot (1+k)} \cdot dz_2 \right) \\
&= \frac{A}{2 \cdot T} \cdot \left(\frac{1}{\jmath \cdot \frac{2\pi}{T} \cdot (1-k)} \cdot \int_0^{\frac{T}{2}} e^{z_1} \cdot dz_1 + \frac{1}{-\jmath \cdot \frac{2\pi}{T} \cdot (1+k)} \cdot \int_0^{\frac{T}{2}} e^{z_2} \cdot dz_2 \right) \\
&= \frac{A}{2 \cdot T} \cdot \left(\frac{T}{\jmath \cdot 2\pi \cdot (1-k)} \cdot \int_0^{\frac{T}{2}} e^{z_1} \cdot dz_1 - \frac{T}{\jmath \cdot 2\pi \cdot (1+k)} \cdot \int_0^{\frac{T}{2}} e^{z_2} \cdot dz_2 \right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{A}{2\cdot T}\cdot\frac{T}{\jmath\cdot 2\pi}\cdot\left(\frac{1}{(1-k)}\cdot\int_0^{\frac{T}{2}} e^{z_1}\cdot dz_1-\frac{1}{(1+k)}\cdot\int_0^{\frac{T}{2}} e^{z_2}\cdot dz_2\right)\\
&= \frac{A}{\jmath\cdot 4\pi}\cdot\left(\frac{1}{(1-k)}\cdot e^{z_1}\Big|_0^{\frac{T}{2}}-\frac{1}{(1+k)}\cdot e^{z_2}\Big|_0^{\frac{T}{2}}\right)\\
&= \frac{A}{\jmath\cdot 4\pi}\cdot\left(\frac{1}{(1-k)}\cdot e^{\jmath\cdot\frac{2\pi}{T}\cdot(1-k)\cdot t}\Big|_0^{\frac{T}{2}}-\frac{1}{(1+k)}\cdot e^{-\jmath\cdot\frac{2\pi}{T}\cdot(1+k)\cdot t}\Big|_0^{\frac{T}{2}}\right)\\
&= \frac{A}{\jmath\cdot 4\pi}\cdot\left(\frac{1}{(1-k)}\cdot\left(e^{\jmath\cdot\frac{2\pi}{T}\cdot(1-k)\cdot\frac{T}{2}}-e^{\jmath\cdot\frac{2\pi}{T}\cdot(1-k)\cdot 0}\right)-\frac{1}{(1+k)}\cdot\left(e^{-\jmath\cdot\frac{2\pi}{T}\cdot(1+k)\cdot\frac{T}{2}}-e^{-\jmath\cdot\frac{2\pi}{T}\cdot(1+k)\cdot 0}\right)\right)\\
&= \frac{A}{\jmath\cdot 4\pi}\cdot\left(\frac{1}{(1-k)}\cdot\left(e^{\jmath\cdot\pi\cdot(1-k)}-e^{0}\right)-\frac{1}{(1+k)}\cdot\left(e^{-\jmath\cdot\pi\cdot(1+k)}-1\right)\right)\\
&= \frac{A}{\jmath\cdot 4\pi}\cdot\left(\frac{1}{(1-k)}\cdot\left(e^{\jmath\cdot\pi}\cdot e^{-\jmath\cdot k\cdot\pi}-1\right)-\frac{1}{(1+k)}\cdot\left(e^{-\jmath\cdot\pi}\cdot e^{-\jmath\cdot k\cdot\pi}-1\right)\right)\\
&= \frac{A}{\jmath\cdot 4\pi}\cdot\left(\frac{1}{(1-k)}\cdot\left(-1\cdot e^{-\jmath\cdot k\cdot\pi}-1\right)-\frac{1}{(1+k)}\cdot\left(-1\cdot e^{-\jmath\cdot k\cdot\pi}-1\right)\right)\\
&= \frac{A}{\jmath\cdot 4\pi}\cdot\left(\frac{1}{(1-k)}\cdot\left(-\cdot e^{-\jmath\cdot k\cdot\pi}-1\right)-\frac{1}{(1+k)}\cdot\left(-\cdot e^{-\jmath\cdot k\cdot\pi}-1\right)\right)\\
&= \frac{A}{\jmath\cdot 4\pi}\cdot\left(\frac{\left(-e^{-\jmath\cdot k\cdot\pi}-1\right)\cdot(1+k)}{(1-k)\cdot(1+k)}-\frac{\left(-e^{-\jmath\cdot k\cdot\pi}-1\right)\cdot(1-k)}{(1-k)\cdot(1+k)}\right)\\
&= \frac{A}{\jmath\cdot 4\pi}\cdot\left(\frac{-\cdot e^{-\jmath\cdot k\cdot\pi}-1-k\cdot e^{-\jmath\cdot k\cdot\pi}-k}{(1-k)\cdot(1+k)}-\frac{-\cdot e^{-\jmath\cdot k\cdot\pi}-1+k\cdot e^{-\jmath\cdot k\cdot\pi}+k}{(1-k)\cdot(1+k)}\right)\\
&= \frac{A}{\jmath\cdot 4\pi}\cdot\left(\frac{-\cdot e^{-\jmath\cdot k\cdot\pi}-1-k\cdot e^{-\jmath\cdot k\cdot\pi}-k+e^{-\jmath\cdot k\cdot\pi}+1-k\cdot e^{-\jmath\cdot k\cdot\pi}-k}{1-k^2}\right)\\
&= \frac{A}{\jmath\cdot 4\pi}\cdot\left(\frac{-2\cdot k\cdot e^{-\jmath\cdot k\cdot\pi}-2\cdot k}{1-k^2}\right)\\
&= -\frac{A\cdot k}{\jmath\cdot 2\pi}\cdot\left(\frac{e^{-\jmath\cdot k\cdot\pi}+1}{1-k^2}\right)
\end{aligned}
$$

Wartość współczynnika F_k wynosi $-\frac{A\cdot k}{\jmath\cdot 2\pi}\cdot\left(\frac{e^{-\jmath\cdot k\cdot\pi}+1}{1-k^2}\right)$.
Dla $k=1$ i $k=-1$ trzeba wyzanczyć wartość współczynnika raz jeszcze wprost ze wzoru

$$
\begin{aligned}
F_1 &= \frac{1}{T}\int_T f(t)\cdot e^{-\jmath\cdot 1\cdot\frac{2\pi}{T}\cdot t}\cdot dt\\
&= \frac{1}{T}\left(\int_0^{\frac{T}{2}} A\cdot cos\left(\frac{2\pi}{T}\cdot t\right)\cdot e^{-\jmath\cdot 1\cdot\frac{2\pi}{T}\cdot t}\cdot dt+\int_{\frac{T}{2}}^{T} 0\cdot e^{-\jmath\cdot 1\cdot\frac{2\pi}{T}\cdot t}\cdot dt\right)\\
&= \frac{1}{T}\left(A\cdot\int_0^{\frac{T}{2}} cos\left(\frac{2\pi}{T}\cdot t\right)\cdot e^{-\jmath\cdot\frac{2\pi}{T}\cdot t}\cdot dt+\int_{\frac{T}{2}}^{T} 0\cdot dt\right)\\
&= \left\{cos\left(x\right)=\tfrac{e^{\jmath\cdot x}+e^{-\jmath\cdot x}}{2}\right\}\\
&= \frac{1}{T}\left(A\cdot\int_0^{\frac{T}{2}}\frac{e^{\jmath\cdot\frac{2\pi}{T}\cdot t}+e^{-\jmath\cdot\frac{2\pi}{T}\cdot t}}{2}\cdot e^{-\jmath\cdot\frac{2\pi}{T}\cdot t}\cdot dt+0\right)\\
&= \frac{A}{2\cdot T}\cdot\int_0^{\frac{T}{2}}\left(e^{\jmath\cdot\frac{2\pi}{T}\cdot t}+e^{-\jmath\cdot\frac{2\pi}{T}\cdot t}\right)\cdot e^{-\jmath\cdot\frac{2\pi}{T}\cdot t}\cdot dt\\
&= \frac{A}{2\cdot T}\cdot\int_0^{\frac{T}{2}}\left(e^{\jmath\cdot\frac{2\pi}{T}\cdot t-\jmath\cdot\frac{2\pi}{T}\cdot t}+e^{-\jmath\cdot\frac{2\pi}{T}\cdot t-\jmath\cdot\frac{2\pi}{T}\cdot t}\right)\cdot dt
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{A}{2\cdot T}\cdot\int_0^{\frac{T}{2}}\left(e^{\jmath\cdot\frac{2\pi}{T}\cdot(1-1)\cdot t}+e^{-\jmath\cdot\frac{2\pi}{T}\cdot(1+1)\cdot t}\right)\cdot dt\\
&= \frac{A}{2\cdot T}\cdot\left(\int_0^{\frac{T}{2}} e^{\jmath\cdot\frac{2\pi}{T}\cdot 0\cdot t}\cdot dt+\int_0^{\frac{T}{2}} e^{-\jmath\cdot\frac{2\pi}{T}\cdot 2\cdot t}\cdot dt\right)\\
&= \frac{A}{2\cdot T}\cdot\left(\int_0^{\frac{T}{2}} e^{0}\cdot dt+\int_0^{\frac{T}{2}} e^{-\jmath\cdot\frac{4\pi}{T}\cdot t}\cdot dt\right)\\
&= \frac{A}{2\cdot T}\cdot\left(\int_0^{\frac{T}{2}} 1\cdot dt+\int_0^{\frac{T}{2}} e^{-\jmath\cdot\frac{4\pi}{T}\cdot t}\cdot dt\right)\\
&= \frac{A}{2\cdot T}\cdot\left(\int_0^{\frac{T}{2}} dt+\int_0^{\frac{T}{2}} e^{-\jmath\cdot\frac{4\pi}{T}\cdot t}\cdot dt\right)\\
&= \left\{\begin{matrix} z=-\jmath\cdot\frac{4\pi}{T}\cdot t\\ dz=-\jmath\cdot\frac{4\pi}{T}\cdot dt\\ dt=\frac{1}{-\jmath\cdot\frac{4\pi}{T}}\cdot dz\end{matrix}\right\}\\
&= \frac{A}{2\cdot T}\cdot\left(\int_0^{\frac{T}{2}} dt+\int_0^{\frac{T}{2}} e^{z}\cdot\frac{1}{-\jmath\cdot\frac{4\pi}{T}}\cdot dz\right)\\
&= \frac{A}{2\cdot T}\cdot\left(\int_0^{\frac{T}{2}} dt+\frac{1}{-\jmath\cdot\frac{4\pi}{T}}\cdot\int_0^{\frac{T}{2}} e^{z}\cdot dz\right)\\
&= \frac{A}{2\cdot T}\cdot\left(t\Big|_0^{\frac{T}{2}}-\frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot e^{z}\Big|_0^{\frac{T}{2}}\right)\\
&= \frac{A}{2\cdot T}\cdot\left(\left(\frac{T}{2}-0\right)-\frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot e^{-\jmath\cdot\frac{4\pi}{T}\cdot t}\Big|_0^{\frac{T}{2}}\right)\\
&= \frac{A}{2\cdot T}\cdot\left(\frac{T}{2}-\frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot\left(e^{-\jmath\cdot\frac{4\pi}{T}\cdot\frac{T}{2}}-e^{-\jmath\cdot\frac{4\pi}{T}\cdot 0}\right)\right)\\
&= \frac{A}{2\cdot T}\cdot\left(\frac{T}{2}-\frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot\left(e^{-\jmath\cdot 2\pi}-e^{0}\right)\right)\\
&= \frac{A}{2\cdot T}\cdot\left(\frac{T}{2}-\frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot(1-1)\right)\\
&= \frac{A}{2\cdot T}\cdot\left(\frac{T}{2}-\frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot 0\right)\\
&= \frac{A}{2\cdot T}\cdot\left(\frac{T}{2}-0\right)\\
&= \frac{A}{2\cdot T}\cdot\frac{T}{2}\\
&= \frac{A}{4}
\end{aligned}
$$

Wartość współczynnika F_1 wynosi $\frac{A}{4}$.

$$
\begin{aligned}
F_{-1} &= \frac{1}{T}\int_T f(t)\cdot e^{-\jmath\cdot(-1)\cdot\frac{2\pi}{T}\cdot t}\cdot dt\\
&= \frac{1}{T}\left(\int_0^{\frac{T}{2}} A\cdot cos\left(\frac{2\pi}{T}\cdot t\right)\cdot e^{-\jmath\cdot(-1)\cdot\frac{2\pi}{T}\cdot t}\cdot dt+\int_{\frac{T}{2}}^{T} 0\cdot e^{-\jmath\cdot(-1)\cdot\frac{2\pi}{T}\cdot t}\cdot dt\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{T}\left(A\cdot\int_0^{\frac{T}{2}} cos\left(\frac{2\pi}{T}\cdot t\right)\cdot e^{\jmath\cdot\frac{2\pi}{T}\cdot t}\cdot dt+\int_{\frac{T}{2}}^{T} 0\cdot dt\right)\\
&= \left\{cos\left(x\right)=\tfrac{e^{\jmath\cdot x}+e^{-\jmath\cdot x}}{2}\right\}\\
&= \frac{1}{T}\left(A\cdot\int_0^{\frac{T}{2}} \frac{e^{\jmath\cdot\frac{2\pi}{T}\cdot t}+e^{-\jmath\cdot\frac{2\pi}{T}\cdot t}}{2}\cdot e^{\jmath\cdot\frac{2\pi}{T}\cdot t}\cdot dt+0\right)\\
&= \frac{A}{2\cdot T}\cdot\int_0^{\frac{T}{2}}\left(e^{\jmath\cdot\frac{2\pi}{T}\cdot t}+e^{-\jmath\cdot\frac{2\pi}{T}\cdot t}\right)\cdot e^{\jmath\cdot\frac{2\pi}{T}\cdot t}\cdot dt\\
&= \frac{A}{2\cdot T}\cdot\int_0^{\frac{T}{2}}\left(e^{\jmath\cdot\frac{2\pi}{T}\cdot t+\jmath\cdot\frac{2\pi}{T}\cdot t}+e^{-\jmath\cdot\frac{2\pi}{T}\cdot t+\jmath\cdot\frac{2\pi}{T}\cdot t}\right)\cdot dt\\
&= \frac{A}{2\cdot T}\cdot\int_0^{\frac{T}{2}}\left(e^{\jmath\cdot\frac{2\pi}{T}\cdot(1+1)\cdot t}+e^{-\jmath\cdot\frac{2\pi}{T}\cdot(-1+1)\cdot t}\right)\cdot dt\\
&= \frac{A}{2\cdot T}\cdot\left(\int_0^{\frac{T}{2}} e^{\jmath\cdot\frac{2\pi}{T}\cdot 2\cdot t}\cdot dt+\int_0^{\frac{T}{2}} e^{-\jmath\cdot\frac{2\pi}{T}\cdot 0\cdot t}\cdot dt\right)\\
&= \frac{A}{2\cdot T}\cdot\left(\int_0^{\frac{T}{2}} e^{\jmath\cdot\frac{4\pi}{T}\cdot t}\cdot dt+\int_0^{\frac{T}{2}} e^{0}\cdot dt\right)\\
&= \frac{A}{2\cdot T}\cdot\left(\int_0^{\frac{T}{2}} e^{\jmath\cdot\frac{4\pi}{T}\cdot t}\cdot dt+\int_0^{\frac{T}{2}} 1\cdot dt\right)\\
&= \frac{A}{2\cdot T}\cdot\left(\int_0^{\frac{T}{2}} e^{\jmath\cdot\frac{4\pi}{T}\cdot t}\cdot dt+\int_0^{\frac{T}{2}} dt\right)\\
&= \left\{\begin{array}{c} z=\jmath\cdot\frac{4\pi}{T}\cdot t\\ dz=\jmath\cdot\frac{4\pi}{T}\cdot dt\\ dt=\frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot dz\end{array}\right\}\\
&= \frac{A}{2\cdot T}\cdot\left(\int_0^{\frac{T}{2}} e^{z}\cdot\frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot dz+\int_0^{\frac{T}{2}} dt\right)\\
&= \frac{A}{2\cdot T}\cdot\left(\frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot\int_0^{\frac{T}{2}} e^{z}\cdot dz+\int_0^{\frac{T}{2}} dt\right)\\
&= \frac{A}{2\cdot T}\cdot\left(\frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot e^{z}\Big|_0^{\frac{T}{2}}+t\Big|_0^{\frac{T}{2}}\right)\\
&= \frac{A}{2\cdot T}\cdot\left(\frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot e^{\jmath\cdot\frac{4\pi}{T}\cdot t}\Big|_0^{\frac{T}{2}}+\left(\frac{T}{2}-0\right)\right)\\
&= \frac{A}{2\cdot T}\cdot\left(\frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot\left(e^{\jmath\cdot\frac{4\pi}{T}\cdot\frac{T}{2}}-e^{\jmath\cdot\frac{4\pi}{T}\cdot 0}\right)+\frac{T}{2}\right)\\
&= \frac{A}{2\cdot T}\cdot\left(\frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot\left(e^{\jmath\cdot 2\pi}-e^{0}\right)+\frac{T}{2}\right)\\
&= \frac{A}{2\cdot T}\cdot\left(\frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot(1-1)+\frac{T}{2}\right)\\
&= \frac{A}{2\cdot T}\cdot\left(\frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot 0+\frac{T}{2}\right)\\
&= \frac{A}{2\cdot T}\cdot\left(0+\frac{T}{2}\right)\\
&= \frac{A}{2\cdot T}\cdot\frac{T}{2}
\end{aligned}
$$

$$= \frac{A}{4}$$

Wartość współczynnika F_{-1} wynosi $\frac{A}{4}$.
Tak wiec ostatecznie współczynniki zespolonego szeregu fouriera

$$\begin{aligned} F_0 &= 0 \\ F_1 &= \frac{A}{4} \\ F_{-1} &= \frac{A}{4} \\ F_k &= -\frac{A \cdot k}{\jmath \cdot 2\pi} \cdot \left(\frac{e^{-\jmath \cdot k \cdot \pi} + 1}{1 - k^2} \right) \end{aligned} \tag{4}$$

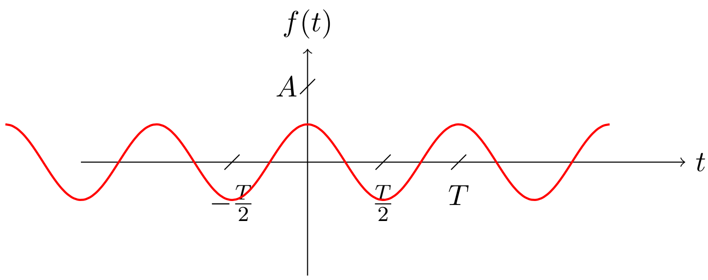
Możemy wyznaczyć kilka wartości współczynników F_k

k	-5	-4	-3	-2	-1	0	1	2	3	4	5
F_k	0	$\jmath \cdot \frac{4 \cdot A}{15 \cdot \pi}$	0	$\jmath \cdot \frac{2 \cdot A}{3 \cdot \pi}$	$\frac{A}{4}$	0	$\frac{A}{4}$	$-\jmath \cdot \frac{2 \cdot A}{3 \cdot \pi}$	0	$-\jmath \cdot \frac{4 \cdot A}{15 \cdot \pi}$	0
$\lvert F_k \rvert$	0	$\frac{4 \cdot A}{15 \cdot \pi}$	0	$\frac{2 \cdot A}{3 \cdot \pi}$	$\frac{A}{4}$	0	$\frac{A}{4}$	$\frac{2 \cdot A}{3 \cdot \pi}$	0	$\frac{4 \cdot A}{15 \cdot \pi}$	0
$Arg\{F_k\}$	0	π	0	π	0	0	0	$-\pi$	0	$-\pi$	0

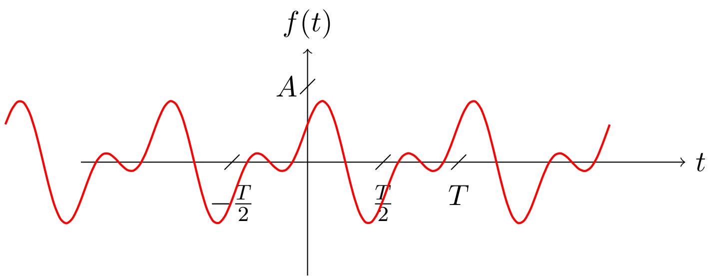
Podstawiając to wzoru aproksymacyjnego funkcje $f(t)$ możemy wyrazić jako

$$f(t) = \sum_{k=-\infty}^{\infty} F_k \cdot e^{\jmath \cdot k \cdot \frac{2\pi}{T} \cdot t} \tag{5}$$

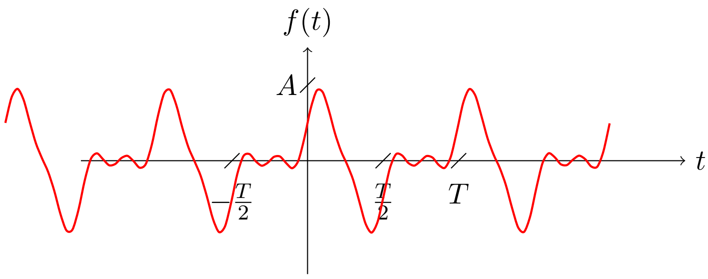
W przypadku sumowania od $k_{min} = -1$ do $k_{max} = 1$ otrzymujemy



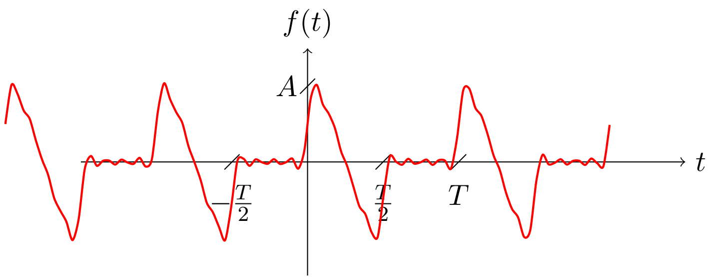
W przypadku sumowania od $k_{min} = -2$ do $k_{max} = 2$ otrzymujemy



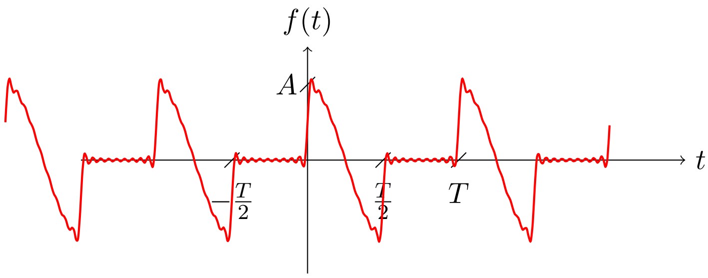
W przypadku sumowania od $k_{min} = -4$ do $k_{max} = 4$ otrzymujemy



W przypadku sumowania od $k_{min} = -10$ do $k_{max} = 10$ otrzymujemy



W przypadku sumowania od $k_{min} = -20$ do $k_{max} = 20$ otrzymujemy



W granicy sumowania od $k_{min} = -\infty$ do $k_{max} = \infty$ otrzymujemy oryginalny sygnał.