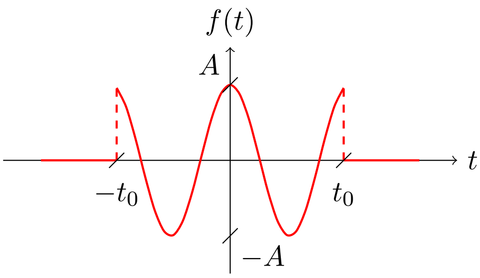
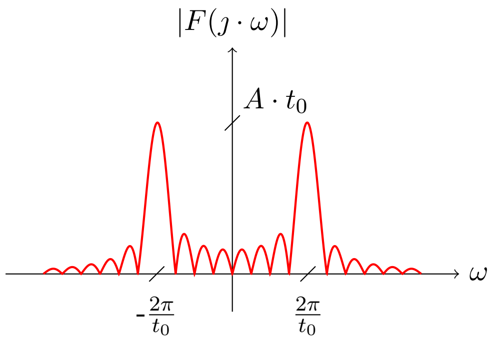
Signal Theory in practise





$$f(t) = A \cdot \Pi\left(\frac{t}{2 \cdot t_0}\right) \cdot cos\left(\frac{2\pi}{t_0} \cdot t\right)$$

$$F(\jmath\omega) = A \cdot t_0 \cdot [\ Sa\left(\omega \cdot t_0 + 2\pi\right) - Sa\left(\omega \cdot t_0 - 2\pi\right)]$$

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Chapter 1

Fundamental concepts and measures

1.1 Basic signal metrics

1.1.1 Mean value of a signal

1.1.2 Energy of a signal

1.1.3 Power and effective value of a signal

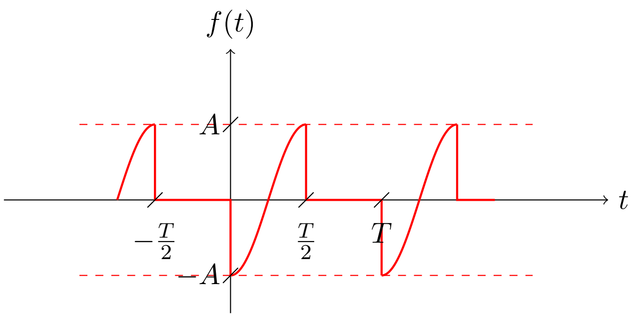
Chapter 2

Analysis of periodic signals using orthogonal series

2.1 Trigonometric Fourier series

2.2 Complex exponential Fourier series

Task 1. Wyznacz wszystkie współczynniki zespolonego szeregu fouriera dla okresowego sygnału $f(t)$ będącego przekształceniem sygnału sinusoidalnego przedstawionego na rysunku.



W pierwszej kolejności należy opisać sygnał za pomocą wzoru:

$$f(x) = \begin{cases} A \cdot cos\left(\frac{2\pi}{T} \cdot t\right) & t \in \left(0 + k \cdot T; \frac{T}{2} + k \cdot T\right) \\ 0 & t \in \left(\frac{T}{2} + k \cdot T; T + k \cdot T\right) \end{cases} \wedge k \in C \tag{2.1}$$

Współczynnik F_0 wyznaczamy ze wzoru

$$F_0 = \frac{1}{T} \int_T f(t) \cdot dt \tag{2.2}$$

Podstawiamy do wzoru wzór naszej funkcji w pierwszym okresie $k = 0$

$$F_0 = \frac{1}{T} \int_T f(t) \cdot dt =$$

$$
\begin{aligned}
&= \frac{1}{T}\left(\int_0^{\frac{T}{2}} A \cdot cos\left(\frac{2\pi}{T}\cdot t\right)\cdot dt + \int_{\frac{T}{2}}^{T} 0 \cdot dt\right) = \\
&= \frac{1}{T}\left(A \cdot \int_0^{\frac{T}{2}} cos\left(\frac{2\pi}{T}\cdot t\right)\cdot dt + 0\right) = \\
&= \frac{A}{T} \cdot \int_0^{\frac{T}{2}} cos\left(\frac{2\pi}{T}\cdot t\right)\cdot dt = \\
&= \begin{Bmatrix} z & = \frac{2\pi}{T}\cdot t \\ dz & = \frac{2\pi}{T}\cdot dt \\ dt & = \frac{1}{\frac{2\pi}{T}}\cdot dz \\ dt & = \frac{T}{2\pi}\cdot dz \end{Bmatrix} = \\
&= \frac{A}{T} \cdot \int_0^{\frac{T}{2}} cos\left(z\right) \cdot \frac{T}{2\pi}\cdot dz = \\
&= \frac{A}{T} \cdot \frac{T}{2\pi} \cdot \int_0^{\frac{T}{2}} cos\left(z\right)\cdot dz = \\
&= \frac{A}{T} \cdot \frac{T}{2\pi} \cdot sin\left(z\right)\Big|_0^{\frac{T}{2}} = \\
&= \frac{A}{2\pi} \cdot sin\left(\frac{2\pi}{T}\cdot t\right)\Bigg|_0^{\frac{T}{2}} = \\
&= \frac{A}{2\pi} \cdot \left(sin\left(\frac{2\pi}{T}\cdot\frac{T}{2}\right) - sin\left(\frac{2\pi}{T}\cdot 0\right)\right) = \\
&= \frac{A}{2\pi} \cdot \left(sin\left(pi\right) - sin\left(0\right)\right) = \\
&= \frac{A}{2\pi} \cdot (0-0) = \\
&= \frac{A}{2\pi} \cdot 0 = \\
&= 0
\end{aligned}
$$

Wartość współczynnika F_0 wynosi 0

Współczynnik F_k wyznaczamy ze wzoru

$$F_k = \frac{1}{T}\int_T f(t) \cdot e^{-\jmath\cdot k \cdot \frac{2\pi}{T}\cdot t}\cdot dt \tag{2.3}$$

Podstawiamy do wzoru wzór naszej funkcji w pierwszym okresie $k = 0$

$$
\begin{aligned}
F_k &= \frac{1}{T}\int_T f(t) \cdot e^{-\jmath\cdot k \cdot \frac{2\pi}{T}\cdot t}\cdot dt = \\
&= \frac{1}{T}\left(\int_0^{\frac{T}{2}} A \cdot cos\left(\frac{2\pi}{T}\cdot t\right)\cdot e^{-\jmath\cdot k \cdot \frac{2\pi}{T}\cdot t}\cdot dt + \int_{\frac{T}{2}}^{T} 0 \cdot e^{-\jmath\cdot k \cdot \frac{2\pi}{T}\cdot t}\cdot dt\right) = \\
&= \frac{1}{T}\left(A \cdot \int_0^{\frac{T}{2}} cos\left(\frac{2\pi}{T}\cdot t\right)\cdot e^{-\jmath\cdot k \cdot \frac{2\pi}{T}\cdot t}\cdot dt + \int_{\frac{T}{2}}^{T} 0 \cdot dt\right) = \\
&= \left\{cos\left(x\right) = \frac{e^{\jmath\cdot x} + e^{-\jmath\cdot x}}{2}\right\} = \\
&= \frac{1}{T}\left(A \cdot \int_0^{\frac{T}{2}} \frac{e^{\jmath\cdot\frac{2\pi}{T}\cdot t} + e^{-\jmath\cdot\frac{2\pi}{T}\cdot t}}{2} \cdot e^{-\jmath\cdot k \cdot \frac{2\pi}{T}\cdot t}\cdot dt + 0\right) =
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{A}{2\cdot T}\cdot\int_0^{\frac{T}{2}}\left(e^{\jmath\cdot\frac{2\pi}{T}\cdot t}+e^{-\jmath\cdot\frac{2\pi}{T}\cdot t}\right)\cdot e^{-\jmath\cdot k\cdot\frac{2\pi}{T}\cdot t}\cdot dt =\\
&= \frac{A}{2\cdot T}\cdot\int_0^{\frac{T}{2}}\left(e^{\jmath\cdot\frac{2\pi}{T}\cdot t-\jmath\cdot k\cdot\frac{2\pi}{T}\cdot t}+e^{-\jmath\cdot\frac{2\pi}{T}\cdot t-\jmath\cdot k\cdot\frac{2\pi}{T}\cdot t}\right)\cdot dt =\\
&= \frac{A}{2\cdot T}\cdot\int_0^{\frac{T}{2}}\left(e^{\jmath\cdot\frac{2\pi}{T}\cdot(1-k)\cdot t}+e^{-\jmath\cdot\frac{2\pi}{T}\cdot(1+k)\cdot t}\right)\cdot dt =\\
&= \frac{A}{2\cdot T}\cdot\left(\int_0^{\frac{T}{2}} e^{\jmath\cdot\frac{2\pi}{T}\cdot(1-k)\cdot t}\cdot dt+\int_0^{\frac{T}{2}} e^{-\jmath\cdot\frac{2\pi}{T}\cdot(1+k)\cdot t}\cdot dt\right) =\\
&= \left\{\begin{matrix} z_1=\jmath\cdot\frac{2\pi}{T}\cdot(1-k)\cdot t & z_2=-\jmath\cdot\frac{2\pi}{T}\cdot(1+k)\cdot t\\ dz_1=\jmath\cdot\frac{2\pi}{T}\cdot(1-k)\cdot dt & dz_2=-\jmath\cdot\frac{2\pi}{T}\cdot(1+k)\cdot dt\\ dt=\frac{1}{\jmath\cdot\frac{2\pi}{T}\cdot(1-k)}\cdot dz_1 & dt=\frac{1}{-\jmath\cdot\frac{2\pi}{T}\cdot(1+k)}\cdot dz_2\end{matrix}\right\} =\\
&= \frac{A}{2\cdot T}\cdot\left(\int_0^{\frac{T}{2}} e^{z_1}\cdot\frac{1}{\jmath\cdot\frac{2\pi}{T}\cdot(1-k)}\cdot dz_1+\int_0^{\frac{T}{2}} e^{z_2}\cdot\frac{1}{-\jmath\cdot\frac{2\pi}{T}\cdot(1+k)}\cdot dz_2\right) =\\
&= \frac{A}{2\cdot T}\cdot\left(\frac{1}{\jmath\cdot\frac{2\pi}{T}\cdot(1-k)}\cdot\int_0^{\frac{T}{2}} e^{z_1}\cdot dz_1+\frac{1}{-\jmath\cdot\frac{2\pi}{T}\cdot(1+k)}\cdot\int_0^{\frac{T}{2}} e^{z_2}\cdot dz_2\right) =\\
&= \frac{A}{2\cdot T}\cdot\left(\frac{T}{\jmath\cdot 2\pi\cdot(1-k)}\cdot\int_0^{\frac{T}{2}} e^{z_1}\cdot dz_1-\frac{T}{\jmath\cdot 2\pi\cdot(1+k)}\cdot\int_0^{\frac{T}{2}} e^{z_2}\cdot dz_2\right) =\\
&= \frac{A}{2\cdot T}\cdot\frac{T}{\jmath\cdot 2\pi}\cdot\left(\frac{1}{(1-k)}\cdot\int_0^{\frac{T}{2}} e^{z_1}\cdot dz_1-\frac{1}{(1+k)}\cdot\int_0^{\frac{T}{2}} e^{z_2}\cdot dz_2\right) =\\
&= \frac{A}{\jmath\cdot 4\pi}\cdot\left(\frac{1}{(1-k)}\cdot e^{z_1}\Big|_0^{\frac{T}{2}}-\frac{1}{(1+k)}\cdot e^{z_2}\Big|_0^{\frac{T}{2}}\right) =\\
&= \frac{A}{\jmath\cdot 4\pi}\cdot\left(\frac{1}{(1-k)}\cdot e^{\jmath\cdot\frac{2\pi}{T}\cdot(1-k)\cdot t}\Big|_0^{\frac{T}{2}}-\frac{1}{(1+k)}\cdot e^{-\jmath\cdot\frac{2\pi}{T}\cdot(1+k)\cdot t}\Big|_0^{\frac{T}{2}}\right) =\\
&= \frac{A}{\jmath\cdot 4\pi}\cdot\left(\frac{1}{(1-k)}\cdot\left(e^{\jmath\cdot\frac{2\pi}{T}\cdot(1-k)\cdot\frac{T}{2}}-e^{\jmath\cdot\frac{2\pi}{T}\cdot(1-k)\cdot 0}\right)-\frac{1}{(1+k)}\cdot\left(e^{-\jmath\cdot\frac{2\pi}{T}\cdot(1+k)\cdot\frac{T}{2}}-e^{-\jmath\cdot\frac{2\pi}{T}\cdot(1+k)\cdot 0}\right)\right) =\\
&= \frac{A}{\jmath\cdot 4\pi}\cdot\left(\frac{1}{(1-k)}\cdot\left(e^{\jmath\cdot\pi\cdot(1-k)}-e^{0}\right)-\frac{1}{(1+k)}\cdot\left(e^{-\jmath\cdot\pi\cdot(1+k)}-1\right)\right) =\\
&= \frac{A}{\jmath\cdot 4\pi}\cdot\left(\frac{1}{(1-k)}\cdot\left(e^{\jmath\cdot\pi}\cdot e^{-\jmath\cdot k\cdot\pi}-1\right)-\frac{1}{(1+k)}\cdot\left(e^{-\jmath\cdot\pi}\cdot e^{-\jmath\cdot k\cdot\pi}-1\right)\right) =\\
&= \frac{A}{\jmath\cdot 4\pi}\cdot\left(\frac{1}{(1-k)}\cdot\left(-1\cdot e^{-\jmath\cdot k\cdot\pi}-1\right)-\frac{1}{(1+k)}\cdot\left(-1\cdot e^{-\jmath\cdot k\cdot\pi}-1\right)\right) =\\
&= \frac{A}{\jmath\cdot 4\pi}\cdot\left(\frac{1}{(1-k)}\cdot\left(-\cdot e^{-\jmath\cdot k\cdot\pi}-1\right)-\frac{1}{(1+k)}\cdot\left(-\cdot e^{-\jmath\cdot k\cdot\pi}-1\right)\right) =\\
&= \frac{A}{\jmath\cdot 4\pi}\cdot\left(\frac{\left(-e^{-\jmath\cdot k\cdot\pi}-1\right)\cdot(1+k)}{(1-k)\cdot(1+k)}-\frac{\left(-e^{-\jmath\cdot k\cdot\pi}-1\right)\cdot(1-k)}{(1-k)\cdot(1+k)}\right) =\\
&= \frac{A}{\jmath\cdot 4\pi}\cdot\left(\frac{-\cdot e^{-\jmath\cdot k\cdot\pi}-1-k\cdot e^{-\jmath\cdot k\cdot\pi}-k}{(1-k)\cdot(1+k)}-\frac{-\cdot e^{-\jmath\cdot k\cdot\pi}-1+k\cdot e^{-\jmath\cdot k\cdot\pi}+k}{(1-k)\cdot(1+k)}\right) =\\
&= \frac{A}{\jmath\cdot 4\pi}\cdot\left(\frac{-\cdot e^{-\jmath\cdot k\cdot\pi}-1-k\cdot e^{-\jmath\cdot k\cdot\pi}-k+e^{-\jmath\cdot k\cdot\pi}+1-k\cdot e^{-\jmath\cdot k\cdot\pi}-k}{1-k^2}\right) =\\
&= \frac{A}{\jmath\cdot 4\pi}\cdot\left(\frac{-2\cdot k\cdot e^{-\jmath\cdot k\cdot\pi}-2\cdot k}{1-k^2}\right) =\\
&= -\frac{A\cdot k}{\jmath\cdot 2\pi}\cdot\left(\frac{e^{-\jmath\cdot k\cdot\pi}+1}{1-k^2}\right)
\end{aligned}
$$

Wartość współczynnika F_k wynosi $-\frac{A\cdot k}{\jmath\cdot 2\pi}\cdot\left(\frac{e^{-\jmath\cdot k\cdot\pi}+1}{1-k^2}\right)$.
Dla $k=1$ i $k=-1$ trzeba wyzanczyć wartość współczynnika raz jeszcze wprost ze wzoru

$$
\begin{aligned}
F_1 &= \frac{1}{T}\int_T f(t)\cdot e^{-\jmath\cdot 1\cdot\frac{2\pi}{T}\cdot t}\cdot dt = \\
&= \frac{1}{T}\left(\int_0^{\frac{T}{2}} A\cdot cos\left(\frac{2\pi}{T}\cdot t\right)\cdot e^{-\jmath\cdot 1\cdot\frac{2\pi}{T}\cdot t}\cdot dt + \int_{\frac{T}{2}}^{T} 0\cdot e^{-\jmath\cdot 1\cdot\frac{2\pi}{T}\cdot t}\cdot dt\right) = \\
&= \frac{1}{T}\left(A\cdot\int_0^{\frac{T}{2}} cos\left(\frac{2\pi}{T}\cdot t\right)\cdot e^{-\jmath\cdot\frac{2\pi}{T}\cdot t}\cdot dt + \int_{\frac{T}{2}}^{T} 0\cdot dt\right) = \\
&= \left\{cos\left(x\right) = \tfrac{e^{\jmath\cdot x}+e^{-\jmath\cdot x}}{2}\right\} = \\
&= \frac{1}{T}\left(A\cdot\int_0^{\frac{T}{2}} \frac{e^{\jmath\cdot\frac{2\pi}{T}\cdot t}+e^{-\jmath\cdot\frac{2\pi}{T}\cdot t}}{2}\cdot e^{-\jmath\cdot\frac{2\pi}{T}\cdot t}\cdot dt + 0\right) = \\
&= \frac{A}{2\cdot T}\cdot\int_0^{\frac{T}{2}}\left(e^{\jmath\cdot\frac{2\pi}{T}\cdot t}+e^{-\jmath\cdot\frac{2\pi}{T}\cdot t}\right)\cdot e^{-\jmath\cdot\frac{2\pi}{T}\cdot t}\cdot dt = \\
&= \frac{A}{2\cdot T}\cdot\int_0^{\frac{T}{2}}\left(e^{\jmath\cdot\frac{2\pi}{T}\cdot t-\jmath\cdot\frac{2\pi}{T}\cdot t}+e^{-\jmath\cdot\frac{2\pi}{T}\cdot t-\jmath\cdot\frac{2\pi}{T}\cdot t}\right)\cdot dt = \\
&= \frac{A}{2\cdot T}\cdot\int_0^{\frac{T}{2}}\left(e^{\jmath\cdot\frac{2\pi}{T}\cdot(1-1)\cdot t}+e^{-\jmath\cdot\frac{2\pi}{T}\cdot(1+1)\cdot t}\right)\cdot dt = \\
&= \frac{A}{2\cdot T}\cdot\left(\int_0^{\frac{T}{2}} e^{\jmath\cdot\frac{2\pi}{T}\cdot 0\cdot t}\cdot dt + \int_0^{\frac{T}{2}} e^{-\jmath\cdot\frac{2\pi}{T}\cdot 2\cdot t}\cdot dt\right) = \\
&= \frac{A}{2\cdot T}\cdot\left(\int_0^{\frac{T}{2}} e^{0}\cdot dt + \int_0^{\frac{T}{2}} e^{-\jmath\cdot\frac{4\pi}{T}\cdot t}\cdot dt\right) = \\
&= \frac{A}{2\cdot T}\cdot\left(\int_0^{\frac{T}{2}} 1\cdot dt + \int_0^{\frac{T}{2}} e^{-\jmath\cdot\frac{4\pi}{T}\cdot t}\cdot dt\right) = \\
&= \frac{A}{2\cdot T}\cdot\left(\int_0^{\frac{T}{2}} dt + \int_0^{\frac{T}{2}} e^{-\jmath\cdot\frac{4\pi}{T}\cdot t}\cdot dt\right) = \\
&= \left\{\begin{aligned} z &= -\jmath\cdot\tfrac{4\pi}{T}\cdot t \\ dz &= -\jmath\cdot\tfrac{4\pi}{T}\cdot dt \\ dt &= \tfrac{1}{-\jmath\cdot\frac{4\pi}{T}}\cdot dz \end{aligned}\right\} = \\
&= \frac{A}{2\cdot T}\cdot\left(\int_0^{\frac{T}{2}} dt + \int_0^{\frac{T}{2}} e^{z}\cdot\frac{1}{-\jmath\cdot\frac{4\pi}{T}}\cdot dz\right) = \\
&= \frac{A}{2\cdot T}\cdot\left(\int_0^{\frac{T}{2}} dt + \frac{1}{-\jmath\cdot\frac{4\pi}{T}}\cdot\int_0^{\frac{T}{2}} e^{z}\cdot dz\right) = \\
&= \frac{A}{2\cdot T}\cdot\left(t\Big|_0^{\frac{T}{2}} - \frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot e^{z}\Big|_0^{\frac{T}{2}}\right) = \\
&= \frac{A}{2\cdot T}\cdot\left(\left(\frac{T}{2}-0\right) - \frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot e^{-\jmath\cdot\frac{4\pi}{T}\cdot t}\Big|_0^{\frac{T}{2}}\right) = \\
&= \frac{A}{2\cdot T}\cdot\left(\frac{T}{2} - \frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot\left(e^{-\jmath\cdot\frac{4\pi}{T}\cdot\frac{T}{2}} - e^{-\jmath\cdot\frac{4\pi}{T}\cdot 0}\right)\right) = \\
&= \frac{A}{2\cdot T}\cdot\left(\frac{T}{2} - \frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot\left(e^{-\jmath\cdot 2\pi} - e^{0}\right)\right) =
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{A}{2 \cdot T} \cdot \left(\frac{T}{2} - \frac{1}{\jmath \cdot \frac{4\pi}{T}} \cdot (1 - 1) \right) = \\
&= \frac{A}{2 \cdot T} \cdot \left(\frac{T}{2} - \frac{1}{\jmath \cdot \frac{4\pi}{T}} \cdot 0 \right) = \\
&= \frac{A}{2 \cdot T} \cdot \left(\frac{T}{2} - 0 \right) = \\
&= \frac{A}{2 \cdot T} \cdot \frac{T}{2} = \\
&= \frac{A}{4}
\end{aligned}
$$

Wartość współczynnika F_1 wynosi $\frac{A}{4}$.

$$
\begin{aligned}
F_{-1} &= \frac{1}{T} \int_T f(t) \cdot e^{-\jmath \cdot (-1) \cdot \frac{2\pi}{T} \cdot t} \cdot dt = \\
&= \frac{1}{T} \left(\int_0^{\frac{T}{2}} A \cdot cos \left(\frac{2\pi}{T} \cdot t \right) \cdot e^{-\jmath \cdot (-1) \cdot \frac{2\pi}{T} \cdot t} \cdot dt + \int_{\frac{T}{2}}^{T} 0 \cdot e^{-\jmath \cdot (-1) \cdot \frac{2\pi}{T} \cdot t} \cdot dt \right) = \\
&= \frac{1}{T} \left(A \cdot \int_0^{\frac{T}{2}} cos \left(\frac{2\pi}{T} \cdot t \right) \cdot e^{\jmath \cdot \frac{2\pi}{T} \cdot t} \cdot dt + \int_{\frac{T}{2}}^{T} 0 \cdot dt \right) = \\
&= \left\{ cos\,(x) = \tfrac{e^{\jmath \cdot x} + e^{-\jmath \cdot x}}{2} \right\} = \\
&= \frac{1}{T} \left(A \cdot \int_0^{\frac{T}{2}} \frac{e^{\jmath \cdot \frac{2\pi}{T} \cdot t} + e^{-\jmath \cdot \frac{2\pi}{T} \cdot t}}{2} \cdot e^{\jmath \cdot \frac{2\pi}{T} \cdot t} \cdot dt + 0 \right) = \\
&= \frac{A}{2 \cdot T} \cdot \int_0^{\frac{T}{2}} \left(e^{\jmath \cdot \frac{2\pi}{T} \cdot t} + e^{-\jmath \cdot \frac{2\pi}{T} \cdot t} \right) \cdot e^{\jmath \cdot \frac{2\pi}{T} \cdot t} \cdot dt = \\
&= \frac{A}{2 \cdot T} \cdot \int_0^{\frac{T}{2}} \left(e^{\jmath \cdot \frac{2\pi}{T} \cdot t + \jmath \cdot \frac{2\pi}{T} \cdot t} + e^{-\jmath \cdot \frac{2\pi}{T} \cdot t + \jmath \cdot \frac{2\pi}{T} \cdot t} \right) \cdot dt = \\
&= \frac{A}{2 \cdot T} \cdot \int_0^{\frac{T}{2}} \left(e^{\jmath \cdot \frac{2\pi}{T} \cdot (1+1) \cdot t} + e^{-\jmath \cdot \frac{2\pi}{T} \cdot (-1+1) \cdot t} \right) \cdot dt = \\
&= \frac{A}{2 \cdot T} \cdot \left(\int_0^{\frac{T}{2}} e^{\jmath \cdot \frac{2\pi}{T} \cdot 2 \cdot t} \cdot dt + \int_0^{\frac{T}{2}} e^{-\jmath \cdot \frac{2\pi}{T} \cdot 0 \cdot t} \cdot dt \right) = \\
&= \frac{A}{2 \cdot T} \cdot \left(\int_0^{\frac{T}{2}} e^{\jmath \cdot \frac{4\pi}{T} \cdot t} \cdot dt + \int_0^{\frac{T}{2}} e^{0} \cdot dt \right) = \\
&= \frac{A}{2 \cdot T} \cdot \left(\int_0^{\frac{T}{2}} e^{\jmath \cdot \frac{4\pi}{T} \cdot t} \cdot dt + \int_0^{\frac{T}{2}} 1 \cdot dt \right) = \\
&= \frac{A}{2 \cdot T} \cdot \left(\int_0^{\frac{T}{2}} e^{\jmath \cdot \frac{4\pi}{T} \cdot t} \cdot dt + \int_0^{\frac{T}{2}} dt \right) = \\
&= \left\{ \begin{array}{c} z = \jmath \cdot \frac{4\pi}{T} \cdot t \\ dz = \jmath \cdot \frac{4\pi}{T} \cdot dt \\ dt = \frac{1}{\jmath \cdot \frac{4\pi}{T}} \cdot dz \end{array} \right\} = \\
&= \frac{A}{2 \cdot T} \cdot \left(\int_0^{\frac{T}{2}} e^{z} \cdot \frac{1}{\jmath \cdot \frac{4\pi}{T}} \cdot dz + \int_0^{\frac{T}{2}} dt \right) = \\
&= \frac{A}{2 \cdot T} \cdot \left(\frac{1}{\jmath \cdot \frac{4\pi}{T}} \cdot \int_0^{\frac{T}{2}} e^{z} \cdot dz + \int_0^{\frac{T}{2}} dt \right) =
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{A}{2\cdot T}\cdot\left(\frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot e^{z}\Big|_0^{\frac{T}{2}} + t\Big|_0^{\frac{T}{2}}\right) = \\
&= \frac{A}{2\cdot T}\cdot\left(\frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot e^{\jmath\cdot\frac{4\pi}{T}\cdot t}\Big|_0^{\frac{T}{2}} + \left(\frac{T}{2}-0\right)\right) = \\
&= \frac{A}{2\cdot T}\cdot\left(\frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot\left(e^{\jmath\cdot\frac{4\pi}{T}\cdot\frac{T}{2}} - e^{\jmath\cdot\frac{4\pi}{T}\cdot 0}\right) + \frac{T}{2}\right) = \\
&= \frac{A}{2\cdot T}\cdot\left(\frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot\left(e^{\jmath\cdot 2\pi} - e^{0}\right) + \frac{T}{2}\right) = \\
&= \frac{A}{2\cdot T}\cdot\left(\frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot(1-1) + \frac{T}{2}\right) = \\
&= \frac{A}{2\cdot T}\cdot\left(\frac{1}{\jmath\cdot\frac{4\pi}{T}}\cdot 0 + \frac{T}{2}\right) = \\
&= \frac{A}{2\cdot T}\cdot\left(0 + \frac{T}{2}\right) = \\
&= \frac{A}{2\cdot T}\cdot\frac{T}{2} = \\
&= \frac{A}{4}
\end{aligned}
$$

Wartość współczynnika F_{-1} wynosi $\frac{A}{4}$.
Tak wiec ostatecznie współczynniki zespolonego szeregu fouriera

$$
\begin{aligned}
F_0 &= 0 \\
F_1 &= \frac{A}{4} \\
F_{-1} &= \frac{A}{4} \\
F_k &= -\frac{A\cdot k}{\jmath\cdot 2\pi}\cdot\left(\frac{e^{-\jmath\cdot k\cdot\pi}+1}{1-k^2}\right)
\end{aligned}
$$

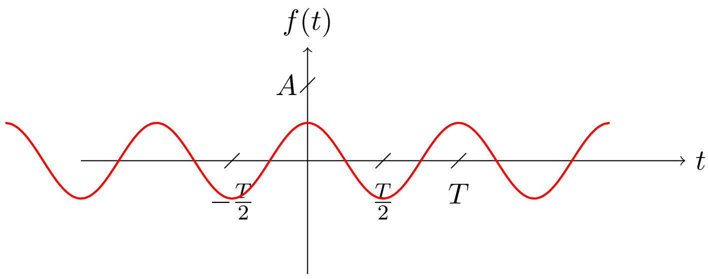
Możemy wyznaczyć kilka wartości współczynników F_k

k	-5	-4	-3	-2	-1	0	1	2	3	4	5
F_k	0	$\jmath\cdot\frac{4\cdot A}{15\cdot\pi}$	0	$\jmath\cdot\frac{2\cdot A}{3\cdot\pi}$	$\frac{A}{4}$	0	$\frac{A}{4}$	$-\jmath\cdot\frac{2\cdot A}{3\cdot\pi}$	0	$-\jmath\cdot\frac{4\cdot A}{15\cdot\pi}$	0
$\lvert F_k\rvert$	0	$\frac{4\cdot A}{15\cdot\pi}$	0	$\frac{2\cdot A}{3\cdot\pi}$	$\frac{A}{4}$	0	$\frac{A}{4}$	$\frac{2\cdot A}{3\cdot\pi}$	0	$\frac{4\cdot A}{15\cdot\pi}$	0
$Arg\{F_k\}$	0	π	0	π	0	0	0	$-\pi$	0	$-\pi$	0

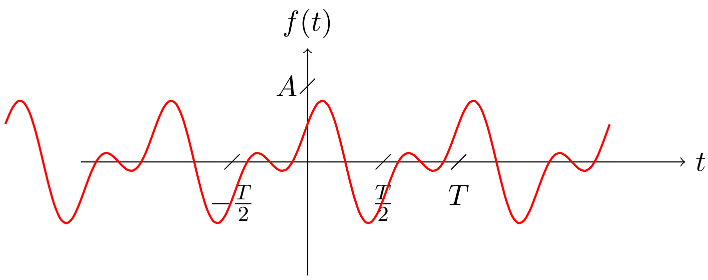
Podstawiając to wzoru aproksymacyjnego funkcje $f(t)$ możemy wyrazić jako

$$f(t) = \sum_{k=-\infty}^{\infty} F_k\cdot e^{\jmath\cdot k\cdot\frac{2\pi}{T}\cdot t} \tag{2.4}$$

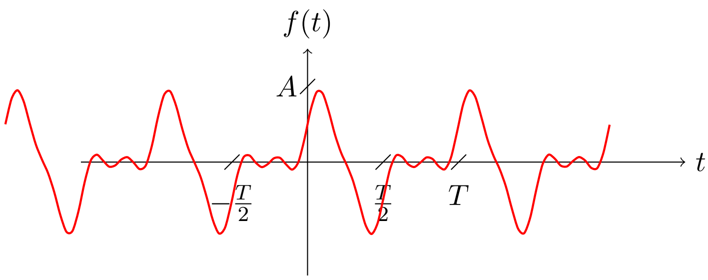
W przypadku sumowania od $k_{min} = -1$ do $k_{max} = 1$ otrzymujemy



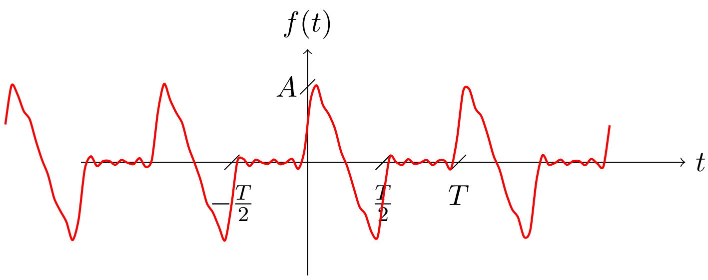
W przypadku sumowania od $k_{min} = -2$ do $k_{max} = 2$ otrzymujemy



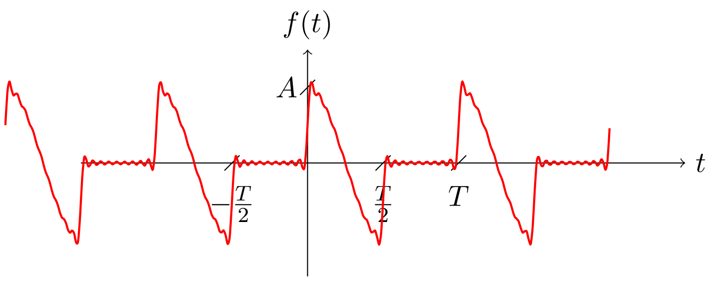
W przypadku sumowania od $k_{min} = -4$ do $k_{max} = 4$ otrzymujemy



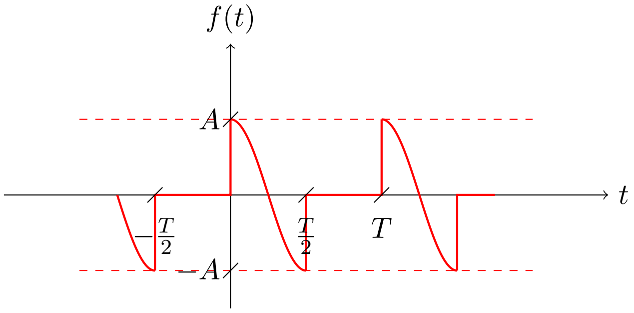
W przypadku sumowania od $k_{min} = -10$ do $k_{max} = 10$ otrzymujemy



W przypadku sumowania od $k_{min} = -20$ do $k_{max} = 20$ otrzymujemy



W granicy sumowania od $k_{min} = -\infty$ do $k_{max} = \infty$ otrzymujemy oryginalny sygnał.



2.3 Computing the power of a signal – the Parseval's theorem

Chapter 3

Analysis of non-periodic signals. Fourier Transformation and Transform

3.1 Calculation of Fourier Transform by definition

3.2 Exploiting properties of the Fourier transform

3.3 Calculating energy of the signal from its Fourier transform. The Parseval's theorem

Chapter 4

Processing of signals by linear and time invariant (LTI) systems

4.1 Linear convolution

4.2 Filters

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