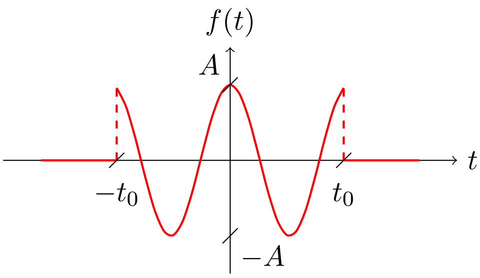
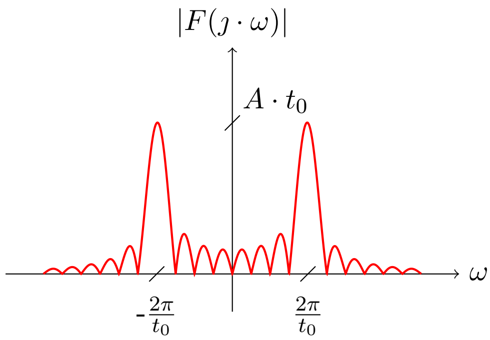
Signal Theory in practise





$$f(t) = A \cdot \Pi\left(\frac{t}{2 \cdot t_0}\right) \cdot cos\left(\frac{2\pi}{t_0} \cdot t\right)$$

$$\begin{aligned} F(\jmath\omega) = A \cdot t_0 \cdot [\ & Sa\left(\omega \cdot t_0 + 2\pi\right) \\ & -Sa\left(\omega \cdot t_0 - 2\pi\right)] \end{aligned}$$

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Chapter 1

Fundamental concepts and measures

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1.1.2 Energy of a signal

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Chapter 2

Analysis of periodic signals using orthogonal series

2.1 Trigonometric Fourier series

2.2 Complex exponential Fourier series

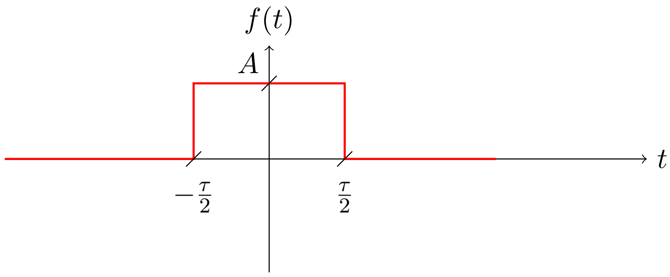
2.3 Computing the power of a signal – the Parseval's theorem

Chapter 3

Analysis of non-periodic signals. Fourier Transformation and Transform

3.1 Calculation of Fourier Transform by definition

Task 1. Compute the Fourier transform of a rectangular impulse shown below. Compute and draw magnitude and phase spectra.



First of all, describe the $f(t)$ signal using elementary signals:

$$f(t) = A \cdot \Pi(\frac{t}{\tau}) \tag{3.1}$$

The Fourier transform is defined as:

$$F(\jmath\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-\jmath \cdot \omega \cdot t} \cdot dt \tag{3.2}$$

For the given $f(t)$ signal we get:

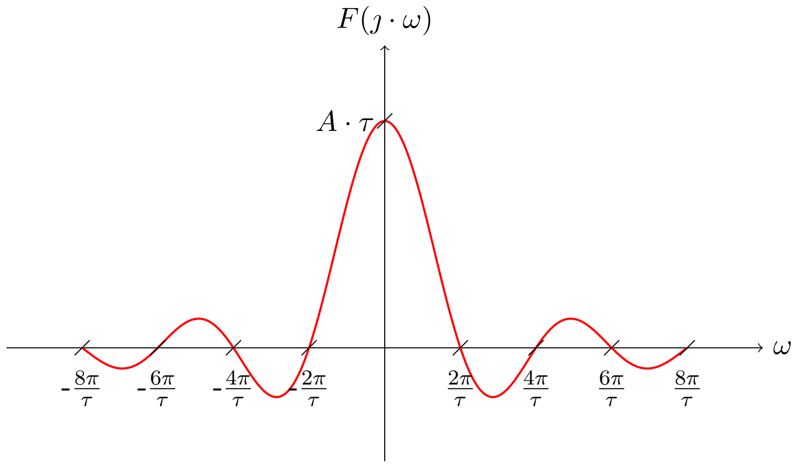
$$\begin{aligned} F(\jmath\omega) &= \int_{-\infty}^{\infty} f(t) \cdot e^{-\jmath \cdot \omega \cdot t} \cdot dt = \\ &= \int_{-\infty}^{\infty} A \cdot \Pi(\frac{t}{\tau}) \cdot e^{-\jmath \cdot \omega \cdot t} \cdot dt = \\ &= \int_{-\infty}^{-\frac{\tau}{2}} 0 \cdot e^{-\jmath \cdot \omega \cdot t} \cdot dt + \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A \cdot e^{-\jmath \cdot \omega \cdot t} \cdot dt + \int_{\frac{\tau}{2}}^{\infty} 0 \cdot e^{-\jmath \cdot \omega \cdot t} \cdot dt = \end{aligned}$$

$$
\begin{aligned}
&= \int_{-\infty}^{-\frac{\tau}{2}} 0 \cdot dt + \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A \cdot e^{-\jmath \cdot \omega \cdot t} \cdot dt + \int_{\frac{\tau}{2}}^{\infty} 0 \cdot dt = \\
&= 0 + \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A \cdot e^{-\jmath \cdot \omega \cdot t} \cdot dt + 0 = \\
&= A \cdot \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \cdot e^{-\jmath \cdot \omega \cdot t} \cdot dt = \\
&= \left\{ \begin{matrix} z & = -\jmath \cdot \omega \cdot t \\ dz & = -\jmath \cdot \omega \cdot dt \\ dt & = \frac{1}{-\jmath \cdot \omega} \cdot dz \end{matrix} \right\} = \\
&= A \cdot \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \cdot e^{z} \cdot \frac{1}{-\jmath \cdot \omega} \cdot dz = \\
&= A \cdot \frac{1}{-\jmath \cdot \omega} \cdot \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \cdot e^{z} \cdot dz = \\
&= A \cdot \frac{1}{-\jmath \cdot \omega} \cdot e^{z}|_{-\frac{\tau}{2}}^{\frac{\tau}{2}} = \\
&= A \cdot \frac{1}{-\jmath \cdot \omega} \cdot e^{-\jmath \cdot \omega \cdot t}\Big|_{-\frac{\tau}{2}}^{\frac{\tau}{2}} = \\
&= \frac{A}{-\jmath \cdot \omega} \cdot \left(e^{-\jmath \cdot \omega \cdot \frac{\tau}{2}} - e^{-\jmath \cdot \omega \cdot (-\frac{\tau}{2})} \right) = \\
&= \frac{A}{\jmath \cdot \omega} \cdot \left(e^{\jmath \cdot \omega \cdot \frac{\tau}{2}} - e^{-\jmath \cdot \omega \cdot \frac{\tau}{2}} \right) = \\
&= \left\{ sin(x) = \frac{e^{\jmath \cdot x} - e^{-\jmath \cdot x}}{2 \cdot \jmath} \right\} = \\
&= \frac{2 \cdot A}{\omega} \cdot sin\left(\omega \cdot \frac{\tau}{2} \right) = \\
&= \left\{ \frac{sin(x)}{x} = Sa(x) \right\} = \\
&= A \cdot \tau \cdot Sa\left(\omega \cdot \frac{\tau}{2} \right)
\end{aligned}
$$

The Fourier transform of the $f(t) = A \cdot \Pi(\frac{t}{\tau})$ is equal to $F(\jmath\omega) = A \cdot \tau \cdot Sa\left(\omega \cdot \frac{\tau}{2}\right)$.

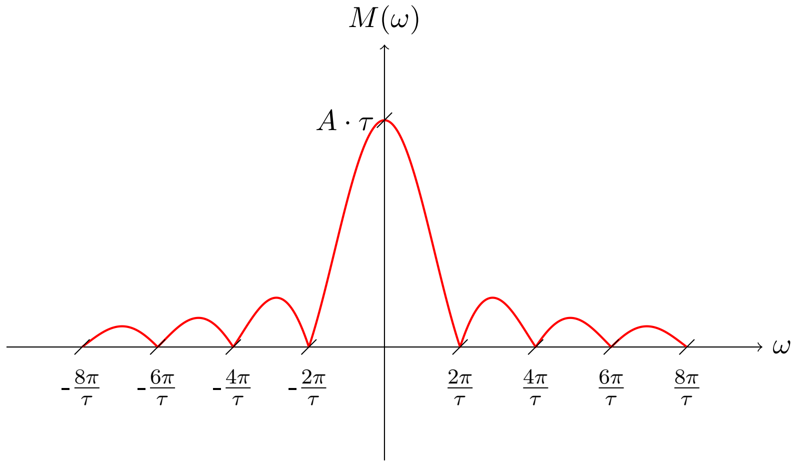
Draw complex spectrum of the $f(t) = A \cdot \Pi(\frac{t}{\tau})$:

$$
F(\jmath\omega) = A \cdot \tau \cdot Sa\left(\omega \cdot \frac{\tau}{2} \right) \tag{3.3}
$$



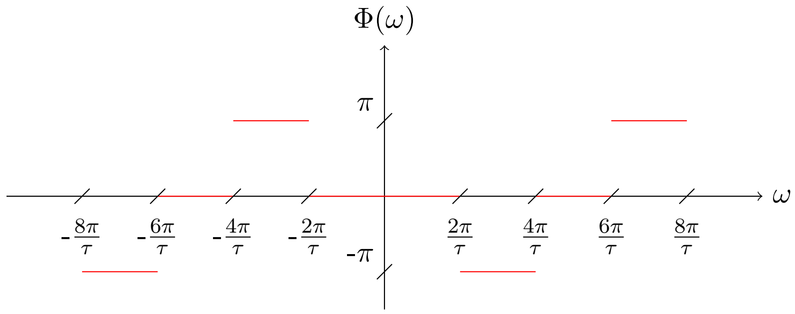
The magnitude spectrum is defined as:

$$M(\omega) = |F(j \cdot \omega)| \tag{3.4}$$

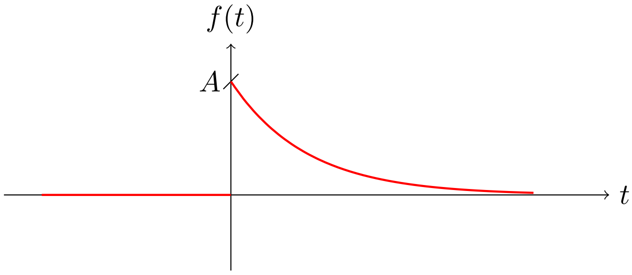


The phase spectrum is defined as:

$$\Phi(\omega) = arctan2(\frac{Im\{F(\jmath \cdot \omega)\}}{Re\{F(\jmath \cdot \omega)\}}) \tag{3.5}$$



Task 2. Compute the Fourier transform of a impulse shown below. Compute and draw magnitude and phase spectra.



The signal $f(t)$, as a piecewise function, is given by:

$$f(t) = \begin{cases} 0 & dla \quad t \in (-\infty; 0) \\ A \cdot e^{-a \cdot t} & dla \quad t \in (0; \infty) \end{cases} \tag{3.6}$$

The Fourier transform is defined as:

$$F(\jmath\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-\jmath \cdot \omega \cdot t} \cdot dt \tag{3.7}$$

For the given $f(t)$ signal we get:

$$\begin{aligned}
F(\jmath\omega) &= \int_{-\infty}^{\infty} f(t) \cdot e^{-\jmath \cdot \omega \cdot t} \cdot dt = \\
&= \int_{-\infty}^{0} 0 \cdot e^{-\jmath \cdot \omega \cdot t} \cdot dt + \int_{0}^{\infty} A \cdot e^{-a \cdot t} \cdot e^{-\jmath \cdot \omega \cdot t} \cdot dt = \\
&= \int_{-\infty}^{0} 0 \cdot dt + \int_{0}^{\infty} A \cdot e^{-a \cdot t} \cdot e^{-\jmath \cdot \omega \cdot t} \cdot dt = \\
&= 0 + \int_{0}^{\infty} A \cdot e^{-a \cdot t} \cdot e^{-\jmath \cdot \omega \cdot t} \cdot dt = \\
&= \int_{0}^{\infty} A \cdot e^{-a \cdot t} \cdot e^{-\jmath \cdot \omega \cdot t} \cdot dt = \\
&= A \cdot \int_{0}^{\infty} e^{-(a + \jmath \cdot \omega) \cdot t} \cdot dt = \\
&= \lim_{\tau \to \infty} A \cdot \int_{0}^{\tau} e^{-(a + \jmath \cdot \omega) \cdot t} \cdot dt = \\
&= \left\{ \begin{aligned} z &= -(a + \jmath \cdot \omega) \cdot t \\ dz &= -(a + \jmath \cdot \omega) \cdot dt \\ dt &= \tfrac{1}{-(a + \jmath \cdot \omega)} \cdot dz \end{aligned} \right\} = \\
&= \lim_{\tau \to \infty} A \cdot \int_{0}^{\tau} e^{z} \cdot \frac{1}{-(a + \jmath \cdot \omega)} \cdot dz = \\
&= A \cdot \frac{1}{-(a + \jmath \cdot \omega)} \cdot \lim_{\tau \to \infty} \int_{0}^{\tau} e^{z} \cdot dz = \\
&= A \cdot \frac{1}{-(a + \jmath \cdot \omega)} \cdot \lim_{\tau \to \infty} e^{z}\Big|_{0}^{\tau} = \\
&= \frac{A}{-(a + \jmath \cdot \omega)} \cdot \lim_{\tau \to \infty} e^{-(a + \jmath \cdot \omega) \cdot t}\Big|_{0}^{\tau} =
\end{aligned}$$

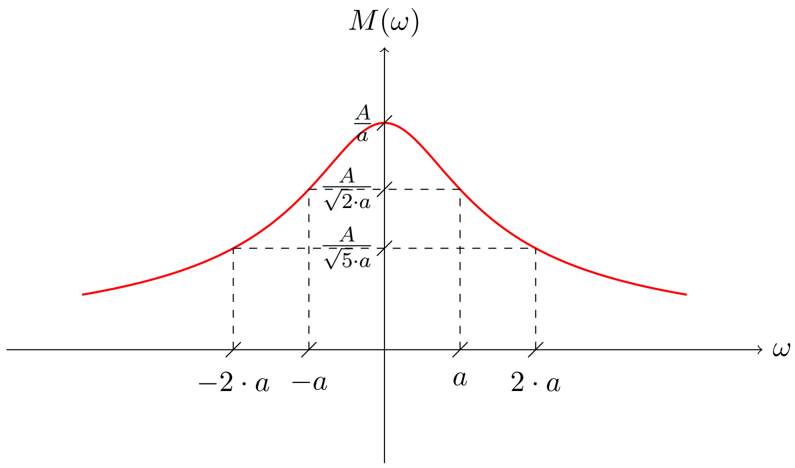
$$
\begin{aligned}
&= \frac{A}{-(a+\jmath\cdot\omega)}\cdot\lim_{\tau\to\infty}\left(e^{-(a+\jmath\cdot\omega)\cdot\tau}-e^{-(a+\jmath\cdot\omega)\cdot 0}\right)=\\
&= \frac{A}{-(a+\jmath\cdot\omega)}\cdot\lim_{\tau\to\infty}\left(e^{-(a+\jmath\cdot\omega)\cdot\tau}-e^{0}\right)=\\
&= \frac{A}{-(a+\jmath\cdot\omega)}\cdot\lim_{\tau\to\infty}\left(e^{-(a+\jmath\cdot\omega)\cdot\tau}-1\right)=\\
&= \frac{A}{-(a+\jmath\cdot\omega)}\cdot\left(\lim_{\tau\to\infty}e^{-(a+\jmath\cdot\omega)\cdot\tau}-1\right)=\\
&= \frac{A}{-(a+\jmath\cdot\omega)}\cdot\left(\lim_{\tau\to\infty}e^{-a\cdot\tau+\jmath\cdot\omega\cdot\tau}-1\right)=\\
&= \frac{A}{-(a+\jmath\cdot\omega)}\cdot\left(\lim_{\tau\to\infty}e^{-a\cdot\tau}\cdot e^{\jmath\cdot\omega\cdot\tau}-1\right)=\\
&= \frac{A}{-(a+\jmath\cdot\omega)}\cdot\left(\lim_{\tau\to\infty}e^{-a\cdot\tau}\cdot\lim_{\tau\to\infty}e^{\jmath\cdot\omega\cdot\tau}-1\right)=\\
&= \frac{A}{-(a+\jmath\cdot\omega)}\cdot\left(0\cdot\lim_{\tau\to\infty}e^{\jmath\cdot\omega\cdot\tau}-1\right)=\\
&= \frac{A}{-(a+\jmath\cdot\omega)}\cdot(0-1)=\\
&= \frac{A}{a+\jmath\cdot\omega}
\end{aligned}
$$

The Fourier transform of the $f(t)=A\cdot\Pi(\frac{t}{\tau})$ is equal to $F(\jmath\omega)=\frac{A}{a+\jmath\cdot\omega}$.
Let's explicitly determine the real and imaginary part:

$$
\begin{aligned}
F(\jmath\omega) &= \frac{A}{(a+\jmath\cdot\omega)}=\\
&= \frac{A}{(a+\jmath\cdot\omega)}\cdot\frac{(a-\jmath\cdot\omega)}{(a-\jmath\cdot\omega)}=\\
&= \frac{A\cdot(a-\jmath\cdot\omega)}{(a^2+\omega^2)}=\\
&= \frac{A\cdot a}{(a^2+\omega^2)}-\jmath\cdot\frac{A\cdot\omega}{(a^2+\omega^2)}
\end{aligned}
$$

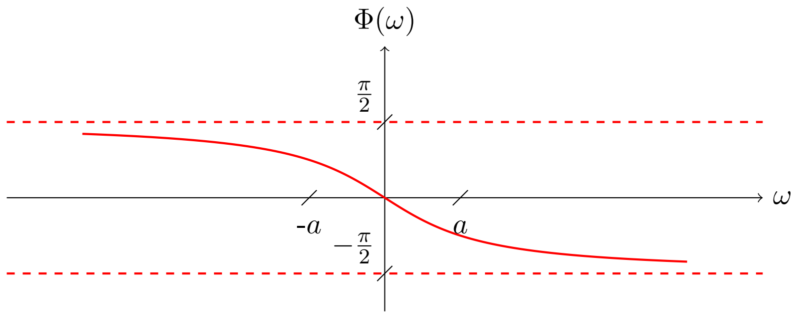
The magnitude spectrum is defined as:

$$
\begin{aligned}
M(\omega) = |F(j\omega)| &=\\
&= \sqrt{\left(\frac{A\cdot a}{(a^2+\omega^2)}\right)^2+\left(\frac{-A\cdot\omega}{(a^2+\omega^2)}\right)^2}=\\
&= \sqrt{\frac{A^2\cdot(a^2+\omega^2)}{(a^2+\omega^2)^2}}=\\
&= \sqrt{\frac{A^2}{(a^2+\omega^2)}}=\\
&= \frac{A}{\sqrt{a^2+\omega^2}}
\end{aligned}
$$

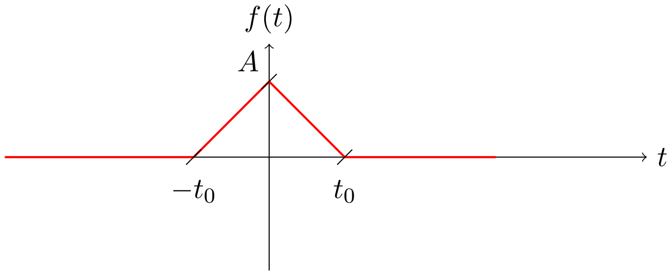


The phase spectrum is defined as:

$$\begin{aligned}\Phi(\omega) &= arctan2(\frac{Im\{F(\jmath\omega)\}}{Re\{F(\jmath\omega)\}}) = \\ &= arctan2\left(\frac{\left(\frac{-A\cdot\omega}{(a^2+\omega^2)}\right)}{\left(\frac{A\cdot a}{(a^2+\omega^2)}\right)}\right) = \\ &= arctan2\left(\frac{-A\cdot\omega}{(a^2+\omega^2)}\cdot\frac{(a^2+\omega^2)}{A\cdot a}\right) = \\ &= arctan2\left(-\frac{\omega}{a}\right)\end{aligned}$$



Task 3. Compute the Fourier transform of a triangle impulse shown below.



First of all, describe the $f(t)$ signal using elementary signals:

$$f(t) = A \cdot \Lambda(\frac{t}{t_0}) \tag{3.8}$$

The Fourier transform is defined as:

$$F(\jmath\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-\jmath \cdot \omega \cdot t} \cdot dt \tag{3.9}$$

In order to integrate the $f(t)$ signal, we need to describe it as a piecewise signal.

The simplest form of a linear function is:

$$f(t) = m \cdot t + b \tag{3.10}$$

In the first interval (e.g. $t \in (-t_0; 0)$), linear function crosses two points: $(-t_0, 0)$ and $(0, A)$. So, in order to derive m and b, the following system of the equations has to be solved.

$$\begin{cases} 0 = m \cdot (-t_0) + b \\ A = m \cdot 0 + b \end{cases}$$

$$\begin{cases} -b = m \cdot (-t_0) \\ A = b \end{cases}$$

$$\begin{cases} \frac{b}{t_0} = m \\ A = b \end{cases}$$

$$\begin{cases} A = b \\ \frac{A}{t_0} = m \end{cases}$$

As a result we get:

$$f(t) = \frac{A}{t_0} \cdot t + A$$

In the second interval (e.g. $t \in (0; t_0)$), linear function crosses two points: $(0; A)$ and $(t_0, 0)$. So, in order to derive m and b, the following system of the equations has to be solved.

$$\begin{cases} 0 = m \cdot t_0 + b \\ A = m \cdot 0 + b \end{cases}$$

$$\begin{cases} -b = m \cdot t_0 \\ A = b \end{cases}$$

$$\begin{cases} -\frac{b}{t_0} = m \\ A = b \end{cases}$$

$$\begin{cases} A = b \\ -\frac{A}{t_0} = m \end{cases}$$

As a result we get:

$$f(t) = -\frac{A}{t_0} \cdot t + A$$

As a result the piecewise linear function is given by:

$$f(t) = A \cdot \Lambda(\frac{t}{t_0}) = \begin{cases} 0 & for \quad t \in (-\infty; -t_0) \\ \frac{A}{t_0} \cdot t + A & for \quad t \in (-t_0; 0) \\ -\frac{A}{t_0} \cdot t + A & for \quad t \in (0; t_0) \\ 0 & for \quad t \in (t_0; \infty) \end{cases} \tag{3.11}$$

For the given $f(t)$ signal we get:

$$\begin{aligned}
F(\jmath\omega) &= \int_{-\infty}^{\infty} f(t) \cdot e^{-\jmath\cdot\omega\cdot t} \cdot dt = \\
&= \int_{-\infty}^{-t_0} 0 \cdot e^{-\jmath\cdot\omega\cdot t} \cdot dt + \int_{-t_0}^{0} \left(\frac{A}{t_0} \cdot t + A\right) \cdot e^{-\jmath\cdot\omega\cdot t} \cdot dt = \\
&+ \int_{0}^{t_0} \left(-\frac{A}{t_0} \cdot t + A\right) \cdot e^{-\jmath\cdot\omega\cdot t} \cdot dt + \int_{t_0}^{\infty} 0 \cdot e^{-\jmath\cdot\omega\cdot t} \cdot dt = \\
&= \int_{-\infty}^{-t_0} 0 \cdot dt + \int_{-t_0}^{0} \frac{A}{t_0} \cdot t \cdot e^{-\jmath\cdot\omega\cdot t} \cdot dt + \int_{-t_0}^{0} A \cdot e^{-\jmath\cdot\omega\cdot t} \cdot dt = \\
&+ \int_{0}^{t_0} -\frac{A}{t_0} \cdot t \cdot e^{-\jmath\cdot\omega\cdot t} \cdot dt + \int_{0}^{t_0} A \cdot e^{-\jmath\cdot\omega\cdot t} \cdot dt + \int_{t_0}^{\infty} 0 \cdot dt = \\
&= 0 + \frac{A}{t_0} \cdot \int_{-t_0}^{0} t \cdot e^{-\jmath\cdot\omega\cdot t} \cdot dt + A \cdot \int_{-t_0}^{0} e^{-\jmath\cdot\omega\cdot t} \cdot dt = \\
&- \frac{A}{t_0} \cdot \int_{0}^{t_0} t \cdot e^{-\jmath\cdot\omega\cdot t} \cdot dt + A \cdot \int_{0}^{t_0} e^{-\jmath\cdot\omega\cdot t} \cdot dt + 0 = \\
&= \begin{Bmatrix} u & = t & dv & = e^{-\jmath\cdot\omega\cdot t} \cdot dt \\ du & = dt & v & = \frac{1}{-\jmath\cdot\omega} \cdot e^{-\jmath\cdot\omega\cdot t} \end{Bmatrix} = \\
&= \frac{A}{t_0} \cdot \left(t \cdot \frac{1}{-\jmath \cdot \omega} \cdot e^{-\jmath\cdot\omega\cdot t} \Big|_{-t_0}^{0} - \int_{-t_0}^{0} \frac{1}{-\jmath \cdot \omega} \cdot e^{-\jmath\cdot\omega\cdot t} \cdot dt \right) = \\
&+ A \cdot \left(\frac{1}{-\jmath \cdot \omega} \cdot e^{-\jmath\cdot\omega\cdot t} \Big|_{-t_0}^{0} \right) =
\end{aligned}$$

$$
\begin{aligned}
&-\frac{A}{t_0}\cdot\left(t\cdot\frac{1}{-\jmath\cdot\omega}\cdot e^{-\jmath\cdot\omega\cdot t}\Big|_0^{t_0}-\int_0^{t_0}\frac{1}{-\jmath\cdot\omega}\cdot e^{-\jmath\cdot\omega\cdot t}\cdot dt\right)=\\
&+A\cdot\left(\frac{1}{-\jmath\cdot\omega}\cdot e^{-\jmath\cdot\omega\cdot t}\Big|_0^{t_0}\right)=\\
&=\frac{A}{t_0}\cdot\left(0\cdot e^{-\jmath\cdot\omega\cdot 0}-(-t_0)\cdot\frac{1}{-\jmath\cdot\omega}\cdot e^{-\jmath\cdot\omega\cdot(-t_0)}+\frac{1}{\jmath\cdot\omega}\left(\frac{1}{-\jmath\cdot\omega}\cdot e^{-\jmath\cdot\omega\cdot t}\Big|_{-t_0}^{0}\right)\right)=\\
&+\frac{A}{-\jmath\cdot\omega}\cdot\left(e^{-\jmath\cdot\omega\cdot 0}-e^{-\jmath\cdot\omega\cdot(-t_0)}\right)=\\
&-\frac{A}{t_0}\cdot\left(t_0\cdot\frac{1}{-\jmath\cdot\omega}\cdot e^{-\jmath\cdot\omega\cdot t_0}-0\cdot e^{-\jmath\cdot\omega\cdot 0}+\frac{1}{\jmath\cdot\omega}\left(\frac{1}{-\jmath\cdot\omega}\cdot e^{-\jmath\cdot\omega\cdot t}\Big|_0^{t_0}\right)\right)=\\
&+\frac{A}{-\jmath\cdot\omega}\cdot\left(e^{-\jmath\cdot\omega\cdot t_0}-e^{-\jmath\cdot\omega\cdot 0}\right)=\\
&=\frac{A}{t_0}\cdot\left(0-t_0\cdot\frac{1}{\jmath\cdot\omega}\cdot e^{\jmath\cdot\omega\cdot t_0}-\frac{1}{\jmath^2\cdot\omega^2}\left(e^{-\jmath\cdot\omega\cdot 0}-e^{-\jmath\cdot\omega\cdot(-t_0)}\right)\right)=\\
&-\frac{A}{\jmath\cdot\omega}\cdot\left(1-e^{\jmath\cdot\omega\cdot t_0}\right)=\\
&-\frac{A}{t_0}\cdot\left(t_0\cdot\frac{1}{-\jmath\cdot\omega}\cdot e^{-\jmath\cdot\omega\cdot t_0}-0-\frac{1}{\jmath^2\cdot\omega^2}\left(e^{-\jmath\cdot\omega\cdot t_0}-e^{-\jmath\cdot\omega\cdot 0}\right)\right)=\\
&-\frac{A}{\jmath\cdot\omega}\cdot\left(e^{-\jmath\cdot\omega\cdot t_0}-1\right)=\\
&=-\frac{A}{\jmath\cdot\omega}\cdot e^{\jmath\cdot\omega\cdot t_0}-\frac{A}{t_0\cdot\jmath^2\cdot\omega^2}+\frac{A}{t_0\cdot\jmath^2\cdot\omega^2}\cdot e^{\jmath\cdot\omega\cdot t_0}-\frac{A}{\jmath\cdot\omega}+\frac{A}{\jmath\cdot\omega}\cdot e^{\jmath\cdot\omega\cdot t_0}=\\
&+\frac{A}{\jmath\cdot\omega}\cdot e^{-\jmath\cdot\omega\cdot t_0}+\frac{A}{t_0\cdot\jmath^2\cdot\omega^2}\cdot e^{-\jmath\cdot\omega\cdot t_0}-\frac{A}{t_0\cdot\jmath^2\cdot\omega^2}-\frac{A}{\jmath\cdot\omega}\cdot e^{-\jmath\cdot\omega\cdot t_0}+\frac{A}{\jmath\cdot\omega}=\\
&=-\frac{2\cdot A}{t_0\cdot\jmath^2\cdot\omega^2}+\frac{A}{t_0\cdot\jmath^2\cdot\omega^2}\cdot\left(e^{\jmath\cdot\omega\cdot t_0}+e^{-\jmath\cdot\omega\cdot t_0}\right)=\\
&=\frac{2\cdot A}{t_0\cdot\omega^2}-\frac{A}{t_0\cdot\omega^2}\cdot\left(e^{\jmath\cdot\omega\cdot t_0}+e^{-\jmath\cdot\omega\cdot t_0}\right)=\\
&=\left\{cos(x)=\tfrac{e^{\jmath\cdot x}+e^{-\jmath\cdot x}}{2}\right\}=\\
&=\frac{2\cdot A}{t_0\cdot\omega^2}-\frac{2\cdot A}{t_0\cdot\omega^2}\cdot cos(\omega\cdot t_0)=\\
&=\frac{2\cdot A}{t_0\cdot\omega^2}\cdot(1-cos(\omega\cdot t_0))=\\
&=\begin{Bmatrix}sin^2(x)=\frac{1}{2}-\frac{1}{2}\cdot cos(2\cdot x)\\ cos(2\cdot x)=1-2\cdot sin^2(x)\end{Bmatrix}=\\
&=\frac{2\cdot A}{t_0\cdot\omega^2}\cdot(1-1+2\cdot sin^2(\frac{\omega\cdot t_0}{2}))=\\
&=\frac{4\cdot A}{t_0\cdot\omega^2}\cdot sin^2(\frac{\omega\cdot t_0}{2})=\\
&=\frac{A\cdot t_0}{\frac{t_0^2\cdot\omega^2}{4}}\cdot sin^2(\frac{\omega\cdot t_0}{2})=\\
&=\left\{\tfrac{sin(x)}{x}=Sa(x)\right\}=\\
&=A\cdot t_0\cdot Sa^2(\frac{\omega\cdot t_0}{2})
\end{aligned}
$$

The Fourier transform of the $f(t)=A\cdot\Lambda(\frac{t}{t_0})$ is equal to $F(\jmath\omega)=A\cdot t_0\cdot Sa^2(\frac{\omega\cdot t_0}{2})$.

3.2 Exploiting properties of the Fourier transform

3.3 Calculating energy of the signal from its Fourier transform. The Parseval's theorem

Chapter 4

Processing of signals by linear and time invariant (LTI) systems

4.1 Linear convolution

4.2 Filters

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