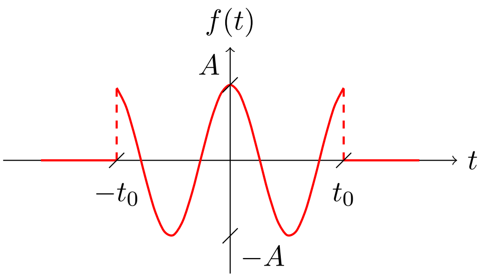
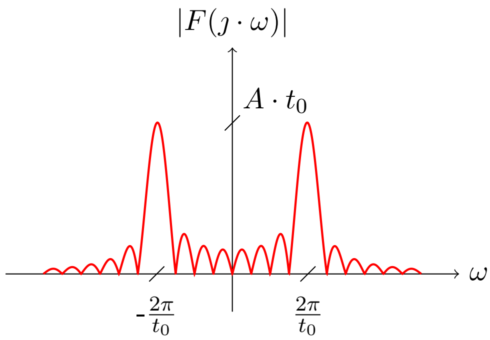
Signal Theory in practise





$$f(t) = A \cdot \Pi\left(\frac{t}{2 \cdot t_0}\right) \cdot cos\left(\frac{2\pi}{t_0} \cdot t\right)$$

$$F(\jmath\omega) = A \cdot t_0 \cdot [\ Sa\,(\omega \cdot t_0 + 2\pi) - Sa\,(\omega \cdot t_0 - 2\pi)]$$

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Chapter 1

Fundamental concepts and measures

1.1 Basic signal metrics

1.1.1 Mean value of a signal

1.1.2 Energy of a signal

1.1.3 Power and effective value of a signal

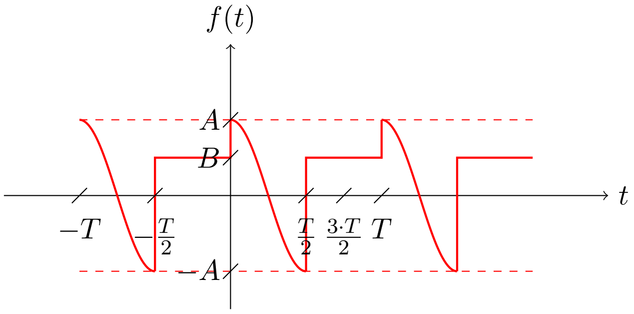
Chapter 2

Analysis of periodic signals using orthogonal series

2.1 Trigonometric Fourier series

2.2 Complex exponential Fourier series

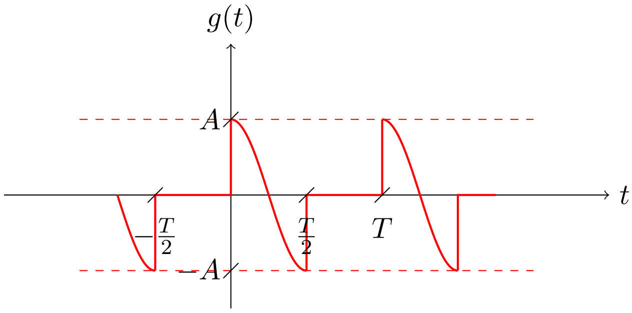
Task 1. Calculate coefficients of the periodic signal $f(t)$ shown below for the expansion into a complex exponential Fourier series. Use knowledge about linearity of complex exponential Fourier series and about the effect of signal shift in time on the complex exponential Fourier series.

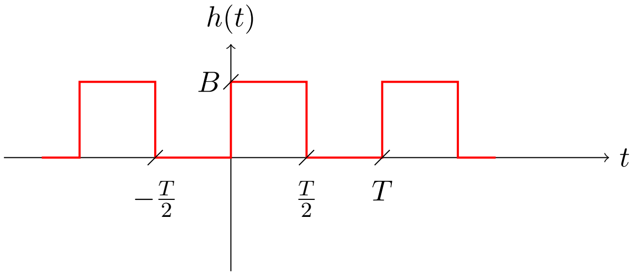


Periodic signal $f(t)$, as a piecewise function, is given by:

$$f(x) = \begin{cases} A \cdot cos\left(\frac{2\pi}{T} \cdot t\right) & t \in \left(0 + k \cdot T; \frac{T}{2} + k \cdot T\right) \\ B & t \in \left(\frac{T}{2} + k \cdot T; T + k \cdot T\right) \end{cases} \wedge k \in Z \tag{2.1}$$

If we look carefully, signal $f(t)$ may be decomposed into two signals $g(t)$ and $h(t)$ for which we have already calculated Fourier series coefficients. The signals are given below:





To be precise, the $f(t)$ signal will be the sum of $g(t)$ and $h(t)$ shifted in time by $\frac{T}{2}$:

$$f(t) = g(t) + h\left(t - \frac{T}{2}\right) \tag{2.2}$$

Based on linearity of complex exponential Fourier series and about the effect of signal shift in time on the complex exponential Fourier series, we can write:

$$F_k = G_k + H_k \cdot e^{-\jmath \cdot k \cdot \frac{2\pi}{T} \cdot \frac{T}{2}} F_k = G_k + H_k \cdot e^{-\jmath \cdot k \cdot \pi} F_k = G_k + H_k \cdot (-1)^k \tag{2.3}$$

From previous tasks we know, that coefficients for the expansion into a complex exponential Fourier series of $g(t)$ and $h(t)$ signals are equal to:

$$G_0 = 0$$
$$G_1 = \frac{A}{4}$$
$$G_{-1} = \frac{A}{4}$$
$$G_k = \jmath \cdot \frac{A \cdot k}{2\pi} \cdot \left(\frac{(-1)^k + 1}{1 - k^2}\right)$$

$$H_0 = \frac{B}{2}$$
$$H_k = \jmath \cdot \frac{B}{k \cdot 2\pi} \cdot \left((-1)^k - 1\right)$$

Right now we know everything to calculate F_k coefficients:

$$F_k = G_k + H_k \cdot (-1)^k$$

$$\begin{aligned} F_0 &= G_0 + H_0 \cdot (-1)^0 = \\ &= 0 + \frac{B}{2} \cdot 1 = \\ &= \frac{B}{2} \end{aligned} \tag{2.4}$$

$$\begin{aligned} F_1 &= G_1 + H_1 \cdot (-1)^1 = \\ &= \frac{A}{4} + \jmath \cdot \frac{B}{1 \cdot 2\pi} \cdot \Big((-1)^1 - 1\Big) \cdot (-1) = \\ &= \frac{A}{4} + \jmath \cdot \frac{B}{\pi} \end{aligned} \tag{2.5}$$

$$\begin{aligned} F_{-1} &= G_{-1} + H_{-1} \cdot (-1)^{-1} = \\ &= \frac{A}{4} + \jmath \cdot \frac{B}{(-1) \cdot 2\pi} \cdot \Big((-1)^{-1} - 1\Big) \cdot (-1) = \\ &= \frac{A}{4} - \jmath \cdot \frac{B}{\pi} \end{aligned} \tag{2.6}$$

$$\begin{aligned} F_k &= G_k + H_k \cdot (-1)^k = \\ &= \jmath \cdot \frac{A \cdot k}{2\pi} \cdot \left(\frac{(-1)^k + 1}{1 - k^2}\right) + \jmath \cdot \frac{B}{k \cdot 2\pi} \cdot \Big((-1)^k - 1\Big) \cdot (-1)^k = \\ &= \jmath \cdot \left[\frac{A \cdot k}{2\pi} \cdot \left(\frac{(-1)^k + 1}{1 - k^2}\right) + \frac{B}{k \cdot 2\pi} \cdot \Big(1 - (-1)^k\Big)\right] \end{aligned} \tag{2.7}$$

2.3 Computing the power of a signal – the Parseval's theorem

Chapter 3

Analysis of non-periodic signals. Fourier Transformation and Transform

3.1 Calculation of Fourier Transform by definition

3.2 Exploiting properties of the Fourier transform

3.3 Calculating energy of the signal from its Fourier transform. The Parseval's theorem

Chapter 4

Processing of signals by linear and time invariant (LTI) systems

4.1 Linear convolution

4.2 Filters

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