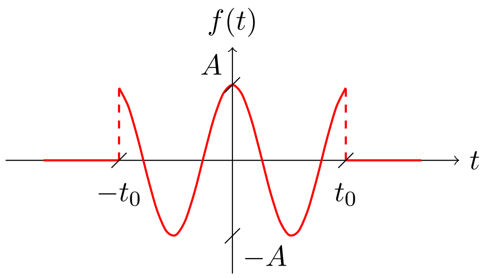
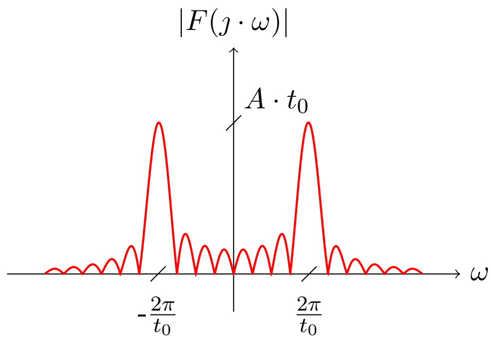
Signal Theory in practise





$$f(t) = A \cdot \Pi\left(\frac{t}{2 \cdot t_0}\right) \cdot cos\left(\frac{2\pi}{t_0} \cdot t\right)$$

$$F(\jmath\omega) = A \cdot t_0 \cdot [\ Sa\left(\omega \cdot t_0 + 2\pi\right) - Sa\left(\omega \cdot t_0 - 2\pi\right)]$$

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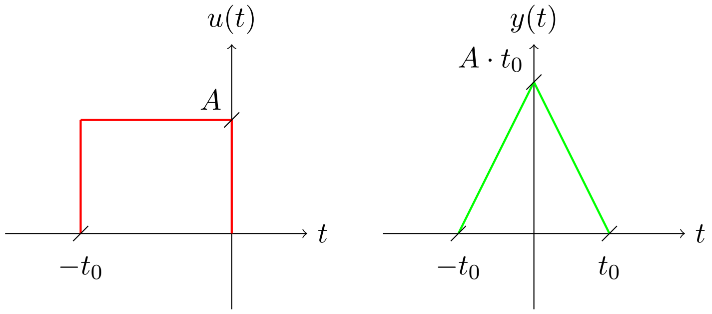
Task 1.

Calculate the impulse response $h(t)$ of an LTI system for input $u(t)$ and output $y(t)$ signals given below. Exploit transforms: $\Pi(t) \xrightarrow{\mathcal{F}} Sa\left(\frac{\omega}{2}\right)$ and $\Lambda(t) \xrightarrow{\mathcal{F}} Sa^2\left(\frac{\omega}{2}\right)$.



We know that for the LTI systems: $Y(\jmath\omega) = U(\jmath\omega) \cdot H(\jmath\omega)$ and $h(t) \xrightarrow{\mathcal{F}} H(\jmath\omega)$. So, $h(t) = \mathcal{F}^{-1}\{H(\jmath\omega)\}$ and $H(\jmath\omega) = \frac{Y(\jmath\omega)}{U(\jmath\omega)}$.

In order to derive frequency response $H(\jmath\omega)$, the Fourier transforms of the $u(t)$ and $y(t)$ signals have to be computed:

$$u(t) = A \cdot \Pi\left(\frac{t + \frac{t_0}{2}}{t_0}\right) \qquad y(t) = A \cdot t_0 \cdot \Lambda\left(\frac{t}{t_0}\right)$$

$$u(t) \xrightarrow{\mathcal{F}} U(\jmath\omega) \qquad y(t) \xrightarrow{\mathcal{F}} Y(\jmath\omega)$$

$$\Pi(t) \xrightarrow{\mathcal{F}} Sa\left(\frac{\omega}{2}\right) \qquad \Lambda(t) \xrightarrow{\mathcal{F}} Sa^2\left(\frac{\omega}{2}\right)$$

$$\Pi\left(\frac{1}{t_0} \cdot t\right) \xrightarrow{\mathcal{F}} t_0 \cdot Sa\left(\frac{\omega \cdot t_0}{2}\right) \qquad \Lambda\left(\frac{1}{t_0} \cdot t\right) \xrightarrow{\mathcal{F}} t_0 \cdot Sa^2\left(\frac{\omega \cdot t_0}{2}\right)$$

$$\Pi\left(\frac{t+\frac{t_0}{2}}{t_0}\right) \xrightarrow{\mathcal{F}} t_0 \cdot Sa\left(\frac{\omega \cdot t_0}{2}\right) \cdot e^{\jmath \cdot \omega \cdot \frac{t_0}{2}} \qquad A \cdot t_0 \cdot \Lambda\left(\frac{t}{t_0}\right) \xrightarrow{\mathcal{F}} A \cdot t_0^2 \cdot Sa^2\left(\frac{\omega \cdot t_0}{2}\right)$$

$$A \cdot \Pi\left(\frac{t+\frac{t_0}{2}}{t_0}\right) \xrightarrow{\mathcal{F}} A \cdot t_0 \cdot Sa\left(\frac{\omega \cdot t_0}{2}\right) \cdot e^{\jmath \cdot \omega \cdot \frac{t_0}{2}}$$

Now, frequency response $H(\jmath\omega)$ can be derived:

$$\begin{aligned} H(\jmath\omega) &= \frac{Y(\jmath\omega)}{U(\jmath\omega)} = \\ &= \frac{A \cdot t_0^2 \cdot Sa^2\left(\frac{\omega \cdot t_0}{2}\right)}{A \cdot t_0 \cdot Sa\left(\frac{\omega \cdot t_0}{2}\right) \cdot e^{\jmath \cdot \omega \cdot \frac{t_0}{2}}} = \\ &= t_0 \cdot Sa\left(\frac{\omega \cdot t_0}{2}\right) \cdot e^{-\jmath \cdot \omega \cdot \frac{t_0}{2}} \end{aligned}$$

Finally, the impulse response $h(t)$ can be calculated:

$$\begin{aligned} h(t) &\xrightarrow{\mathcal{F}} H(\jmath\omega) \\ ? &\xrightarrow{\mathcal{F}} t_0 \cdot Sa\left(\frac{\omega \cdot t_0}{2}\right) \cdot e^{-\jmath \cdot \omega \cdot \frac{t_0}{2}} \\ \Pi(t) &\xrightarrow{\mathcal{F}} Sa\left(\frac{\omega}{2}\right) \\ \Pi\left(\frac{1}{t_0} \cdot t\right) &\xrightarrow{\mathcal{F}} t_0 \cdot Sa\left(\frac{\omega \cdot t_0}{2}\right) \\ \Pi\left(\frac{t-\frac{t_0}{2}}{t_0}\right) &\xrightarrow{\mathcal{F}} t_0 \cdot Sa\left(\frac{\omega \cdot t_0}{2}\right) \cdot e^{-\jmath \cdot \omega \cdot \frac{t_0}{2}} \end{aligned}$$

The impulse response of the system is equal to $h(t) = \Pi\left(\frac{t-\frac{t_0}{2}}{t_0}\right)$.

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