x, $\_$	term variable
q, r	qubit symbols
U_{2^n}, V_{2^n}	unitary symbols
l	label
i, j, n, n_a, n_b	indices

Typ, τ	::=			Types
	|	**qbit**	M	qbit, opaque qubit type
	|	**qref** $[q]$		qref q, qubit reference type
	|	**arr** $(\tau_1; \tau_2)$		$\tau_1 \to \tau_2$
	|	**cmd** (τ)		τ cmd
	|	**prod** $(\overline{l_i \hookrightarrow \tau_i}^{\,i \in 1..n})$		$\times^{n}_{l \in L}\ \tau_l$
	|	**sum** $(\overline{l_i \hookrightarrow \tau_i}^{\,i \in 1..n})$	M	$+_{l \in L}\ \tau_l$
	|	**bool**		bool
	|	**unit**		unit
Exp, e	::=			Expressions
	|	x		x
	|	**let** $(e_1; x.e_2)$	bind x in e_2	let x be e_1 in e_2
	|	**lam** $\{\tau\}(x.e)$	bind x in e	$\lambda(x : \tau)e$
	|	**ap** $(e_1; e_2)$		$e_1(e_2)$
	|	**cmd** (m)		cmd m, encapsulation
	|	**qloc** $[q]$		$\&q$, qubit location
	|	**tpl** $(\overline{l_i \hookrightarrow e_i}^{\,i \in 1..n})$		$\langle e_l \rangle_{l \in L}$, tuple
	|	**pr** $[l_i](e)$		$e \cdot l$, projection
	|	**in** $[l_i]\{\overline{\tau_i}^{\,i \in 1..n}\}(e)$	M	$l \cdot e$
	|	**case** $(e; \overline{l_i \hookrightarrow x_i.e_i}^{\,i \in 1..n})$	M	case $e\,\{l \cdot x_l \hookrightarrow e_l\}_{l \in L}$
	|	**true**		true
	|	**false**		false
	|	**if** $(e; e_1; e_2)$		if e then e_1 else e_2
	|	**not** e		$\neg\ e$
	|	**triv**		$\langle\rangle$
	|	$[e_1/x]e_2$	M	substitution
	|	(e)	M	parentheses
Cmd, m	::=			Commands
	|	**ret** (e)		ret e, return
	|	**bnd** $(e; x.m)$	bind x in m	bnd $x \leftarrow e; m$, sequencing
	|	**newqref** $(x.m)$	bind x in m	new x in m, new qubit reference
	|	**gateap** $[U_{2^n}](e)$		$U_{2^n}(e)$, gate application
	|	**diagap** $[U_{2^n}, V_{2^n}](e_1; e_2)$		$\mathrm{D}(U\ , V\ )(e_1, e_2)$, block diagonal
	|	**meas** (e)		meas(e), measure qbit
	|	**dcl** $(q.m)$	M	dcl q in m, new (opaque) qubit
	|	**gateapm** $[U_{2^n}, q]$	M	gate application (opaque)
	|	**measm** $[q]$	M	measure qbit (opaque)
	|	$[e/x]m$	M	substitution
Sugar, s	::=			Derived forms
	|	$\{x \leftarrow m_1; m_2\}$		
	|	$\{m_1; m_2\}$		
	|	**do** e		
	|	**proc** $(x : \tau)m$		
	|	**call** $e_1(e_2)$		
	|	$\tau_1 \Rightarrow \tau_2$		
Γ	::=			Typing context

	$\varnothing$	
	$\Gamma, x : \tau$	

Sigma, Σ	::=		Signature
	$\varnothing$		
	$\Sigma, q \sim \mathbf{qbit}$		

terminals	::=		
	$\vdash$		entails
	$\longmapsto$		transition
	$\hookrightarrow$		mapping
	$\cdot$		projection
	$\sim$		tilde
	$\dot{\sim}$		dotted tilde
	$\varnothing$		empty context
	$\leq$		less than or equal
	$\leftarrow$		
	$\triangleq$		defined as
	$\Rightarrow$		operation type

formula	::=		
	judgement		
	$formula_1$	.. $formula_n$	
	$1 \leq i \leq n$		

Jdefined	::=		
	$s \triangleq user\_syntax$		Derived forms / syntactic sugar

Jstatics	::=		
	$\Gamma \vdash e : \tau$		Expression Typing
	$\Gamma \vdash_\Sigma e : \tau$		Expression Typing wrt Signature
	$\Gamma \vdash_\Sigma m \ \dot{\sim} \ \tau$		Well formed command w/ return type τ

Jdynamics	::=		
	e **val**		Values
	e $\mathbf{val}_\Sigma$		Values wrt Signature
	$e \longmapsto e'$		Transition
	$e \underset{\Sigma}{\longmapsto} e'$		Transition wrt Signature
	m $\mathbf{final}_\Sigma$		State m is complete
	$m \underset{\Sigma}{\longmapsto} m'$		State transition

judgement	::=		
	Jdefined		
	Jstatics		
	Jdynamics		

user_syntax	::=		
	x		
	q		

| U_{2^n}
| l
| i
| *Typ*
| *Exp*
| *Cmd*
| *Sugar*
| Γ
| *Sigma*
| *terminals*
| *formula*

$\boxed{s \triangleq \mathit{user\_syntax}}$ Derived forms / syntactic sugar

$$\frac{}{\{x \leftarrow m_1; m_2\} \triangleq \mathbf{bnd}\,(\mathbf{cmd}\,(m_1); x.m_2)}\ \textsc{SeqComp}$$

$$\frac{}{\{m_1; m_2\} \triangleq \mathbf{bnd}\,(\mathbf{cmd}\,(m_1); \_.m_2)}\ \textsc{SeqCompU}$$

$$\frac{}{\mathbf{do}\,e \triangleq \mathbf{bnd}\,(e; x.\mathbf{ret}\,(x))}\ \textsc{Do}$$

$$\frac{}{\mathbf{proc}\,(x:\tau)m \triangleq \mathbf{lam}\,\{\tau\}(x.\mathbf{cmd}\,(m))}\ \textsc{Procedure}$$

$$\frac{}{\mathbf{call}\,e_1(e_2) \triangleq \mathbf{do}\,(\mathbf{ap}\,(e_1; e_2))}\ \textsc{Call}$$

$$\frac{}{\tau_1 \Rightarrow \tau_2 \triangleq \mathbf{arr}\,(\tau_1; \mathbf{cmd}\,(\tau_2))}\ \textsc{OperationType}$$

$\boxed{\Gamma \vdash e : \tau}$ Expression Typing

$$\frac{}{\Gamma, x:\tau \vdash x : \tau}\ \textsc{ty\_Var}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma, x:\tau_1 \vdash e_2 : \tau_2}{\Gamma \vdash \mathbf{let}\,(e_1; x.e_2) : \tau_2}\ \textsc{ty\_Let}$$

$$\frac{\Gamma, x:\tau_1 \vdash e : \tau_2}{\Gamma \vdash \mathbf{lam}\,\{\tau_1\}(x.e) : \mathbf{arr}\,(\tau_1; \tau_2)}\ \textsc{ty\_Lam}$$

$$\frac{\Gamma \vdash e_1 : \mathbf{arr}\,(\tau_2; \tau) \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash \mathbf{ap}\,(e_1; e_2) : \tau}\ \textsc{ty\_Ap}$$

$$\frac{\overline{\Gamma \vdash e_i : \tau_i}^{\,i \in 1..n}}{\Gamma \vdash \mathbf{tpl}\,(\overline{l_i \hookrightarrow e_i}^{\,i \in 1..n}) : \mathbf{prod}\,(\overline{l_i \hookrightarrow \tau_i}^{\,i \in 1..n})}\ \textsc{ty\_Tpl}$$

$$\frac{\Gamma \vdash e : \mathbf{prod}\,(\overline{l_i \hookrightarrow \tau_i}^{\,i \in 1..n}) \quad 1 \le i \le n}{\Gamma \vdash \mathbf{pr}\,[l_i](e) : \tau_i}\ \textsc{ty\_Pr}$$

$$\frac{\Gamma \vdash e : \tau_i \quad 1 \le i \le n}{\Gamma \vdash \mathbf{in}\,[l_i]\{\overline{\tau_i}^{\,i \in 1..n}\}(e) : \mathbf{sum}\,(\overline{l_i \hookrightarrow \tau_i}^{\,i \in 1..n})}\ \textsc{ty\_Inj}$$

$$\frac{\Gamma \vdash e : \mathbf{sum}\,(\,\overline{l_i \hookrightarrow \tau_i}^{\,i\in 1..n}\,) \quad \overline{\Gamma, x_i : \tau_i \vdash e_i : \tau}^{\,i\in 1..n}}{\Gamma \vdash \mathbf{case}\,(e;\,\overline{l_i \hookrightarrow x_i.e_i}^{\,i\in 1..n}\,) : \tau}\ \textsc{ty\_Case}$$

$\boxed{\Gamma \vdash_\Sigma e : \tau}$ Expression Typing wrt Signature

$$\frac{\Gamma \vdash_\Sigma m \ \dot{\div}\ \tau}{\Gamma \vdash_\Sigma \mathbf{cmd}\,(m) : \mathbf{cmd}\,(\tau)}\ \textsc{tyS\_Cmd}$$

$$\frac{}{\Gamma \vdash_{\Sigma, q\sim \mathbf{qbit}} \mathbf{qloc}\,[q] : \mathbf{qref}\,[q]}\ \textsc{tyS\_QLoc}$$

$\boxed{\Gamma \vdash_\Sigma m \ \dot{\div}\ \tau}$ Well formed command w/ return type τ

$$\frac{\Gamma \vdash_\Sigma e : \tau}{\Gamma \vdash_\Sigma \mathbf{ret}\,(e) \ \dot{\div}\ \tau}\ \textsc{cmd\_Ret}$$

$$\frac{\Gamma \vdash_\Sigma e : \mathbf{cmd}\,(\tau) \quad \Gamma, x : \tau \vdash_\Sigma m \ \dot{\div}\ \tau'}{\Gamma \vdash_\Sigma \mathbf{bnd}\,(e; x.m) \ \dot{\div}\ \tau'}\ \textsc{cmd\_Bnd}$$

$$\frac{\Gamma, x : \mathbf{qref}\,[q] \vdash_{\Sigma, q\sim \mathbf{qbit}} m \ \dot{\div}\ \tau}{\Gamma \vdash_\Sigma \mathbf{newqref}\,(x.m) \ \dot{\div}\ \tau}\ \textsc{cmd\_NewQRef}$$

$$\frac{\Gamma \vdash_\Sigma e : \mathbf{prod}\,(\,\overline{l_i \hookrightarrow \mathbf{qref}\,[q_i]}^{\,i\in 1..n}\,)}{\Gamma \vdash_\Sigma \mathbf{gateap}\,[U_{2^n}](e) \ \dot{\div}\ \mathbf{unit}}\ \textsc{cmd\_GateApRef}$$

$$\frac{\Gamma \vdash_\Sigma e_1 : \mathbf{qref}\,[q] \quad \Gamma \vdash_\Sigma e_2 : \mathbf{prod}\,(\,\overline{l_i \hookrightarrow \mathbf{qref}\,[r_i]}^{\,i\in 1..n}\,)}{\Gamma \vdash_\Sigma \mathbf{diagap}\,[U_{2^n}, V_{2^n}](e_1; e_2) \ \dot{\div}\ \mathbf{unit}}\ \textsc{cmd\_DiagApRef}$$

$$\frac{\Gamma \vdash_\Sigma e : \mathbf{qref}\,[q]}{\Gamma \vdash_\Sigma \mathbf{meas}\,(e) \ \dot{\div}\ \mathbf{bool}}\ \textsc{cmd\_MeasRef}$$

$$\frac{\Gamma \vdash_{\Sigma, q\sim \mathbf{qbit}} m \ \dot{\div}\ \tau}{\Gamma \vdash_\Sigma \mathbf{dcl}\,(q.m) \ \dot{\div}\ \tau}\ \textsc{cmd\_Dcl}$$

$$\frac{}{\Gamma \vdash_{\Sigma, q\sim \mathbf{qbit}} \mathbf{gateapm}\,[U_{2^n}, q] \ \dot{\div}\ \mathbf{unit}}\ \textsc{cmd\_GateAp}$$

$$\frac{}{\Gamma \vdash_{\Sigma, q\sim \mathbf{qbit}} \mathbf{measm}\,[q] \ \dot{\div}\ \mathbf{bool}}\ \textsc{cmd\_Meas}$$

$\boxed{e\ \mathbf{val}}$ Values

$$\frac{}{\mathbf{lam}\,\{\tau\}(x.e)\,\mathbf{val}}\ \textsc{v\_Lam}$$

$$\frac{\overline{e_i\ \mathbf{val}}^{\,i\in 1..n}}{\mathbf{tpl}\,(\,\overline{l_i \hookrightarrow e_i}^{\,i\in 1..n}\,)\,\mathbf{val}}\ \textsc{v\_Tpl}$$

$$\frac{e\ \mathbf{val}}{\mathbf{in}\,[l_i]\{\,\overline{\tau_i}^{\,i\in 1..n}\,\}(e)\,\mathbf{val}}\ \textsc{v\_Inj}$$

$\boxed{e\ \mathbf{val}_\Sigma}$ Values wrt Signature

$$\frac{}{\mathbf{cmd}\,(m)\ \mathbf{val}_\Sigma}\ \textsc{vS\_Cmd}$$

$$\frac{}{\mathbf{qloc}\,[q]\ \mathbf{val}_{\Sigma, q \sim \mathbf{qbit}}}\ \textsc{vS\_QLoc}$$

$\boxed{e \longmapsto e'}$ Transition

$$\frac{e_1 \longmapsto e_1'}{\mathbf{let}\,(e_1; x.e_2) \longmapsto \mathbf{let}\,(e_1'; x.e_2)}\ \textsc{tr\_Let}$$

$$\frac{e_1\ \mathbf{val}}{\mathbf{let}\,(e_1; x.e_2) \longmapsto [e_1/x]e_2}\ \textsc{tr\_LetInstr}$$

$$\frac{e_1 \longmapsto e_1'}{\mathbf{ap}\,(e_1; e_2) \longmapsto \mathbf{ap}\,(e_1'; e_2)}\ \textsc{tr\_ApL}$$

$$\frac{\begin{array}{c} e_1\ \mathbf{val} \\ e_2 \longmapsto e_2' \end{array}}{\mathbf{ap}\,(e_1; e_2) \longmapsto \mathbf{ap}\,(e_1; e_2')}\ \textsc{tr\_ApR}$$

$$\frac{e_2\ \mathbf{val}}{\mathbf{ap}\,(\mathbf{lam}\,\{\tau_2\}(x.e_1); e_2) \longmapsto [e_2/x]e_1}\ \textsc{tr\_ApInstr}$$

$$\frac{\begin{array}{c} \overline{e_i\ \mathbf{val}}^{\,i \in 1..n_a} \\ e \longmapsto e' \end{array}}{\mathbf{tpl}\,(\,\overline{l_i \hookrightarrow e_i}^{\,i \in 1..n_a}, l \hookrightarrow e, \overline{l_j' \hookrightarrow e_j'}^{\,j \in 1..n_b}\,) \longmapsto \mathbf{tpl}\,(\,\overline{l_i \hookrightarrow e_i}^{\,i \in 1..n_a}, l \hookrightarrow e', \overline{l_j' \hookrightarrow e_j'}^{\,j \in 1..n_b}\,)}\ \textsc{tr\_Tpl}$$

$$\frac{e \longmapsto e'}{\mathbf{pr}\,[l_i](e) \longmapsto \mathbf{pr}\,[l_i](e')}\ \textsc{tr\_Pr}$$

$$\frac{\begin{array}{c} \mathbf{tpl}\,(\,\overline{l_i \hookrightarrow e_i}^{\,i \in 1..n}\,)\ \mathbf{val} \\ 1 \le j \le n \end{array}}{\mathbf{pr}\,[l_j](\mathbf{tpl}\,(\,\overline{l_i \hookrightarrow e_i}^{\,i \in 1..n}\,)) \longmapsto e_j}\ \textsc{tr\_PrInstr}$$

$$\frac{e \longmapsto e'}{\mathbf{in}\,[l_i]\{\,\overline{\tau_i}^{\,i \in 1..n}\,\}(e) \longmapsto \mathbf{in}\,[l_i]\{\,\overline{\tau_i}^{\,i \in 1..n}\,\}(e')}\ \textsc{tr\_Inj}$$

$$\frac{e \longmapsto e'}{\mathbf{case}\,(e;\, \overline{l_i \hookrightarrow x_i.e_i}^{\,i \in 1..n}\,) \longmapsto \mathbf{case}\,(e';\, \overline{l_i \hookrightarrow x_i.e_i}^{\,i \in 1..n}\,)}\ \textsc{tr\_Case}$$

$$\frac{\begin{array}{c} \mathbf{in}\,[l_j]\{\,\overline{\tau_i}^{\,i \in 1..n}\,\}(e)\ \mathbf{val} \\ 1 \le j \le n \end{array}}{\mathbf{case}\,(\mathbf{in}\,[l_j]\{\,\overline{\tau_i}^{\,i \in 1..n}\,\}(e);\, \overline{l_i \hookrightarrow x_i.e_i}^{\,i \in 1..n}\,) \longmapsto [e/x_j]e_j}\ \textsc{tr\_CaseInstr}$$

$\boxed{e \underset{\Sigma}{\longmapsto} e'}$ Transition wrt Signature

$\boxed{m\ \mathbf{final}_\Sigma}$ State m is complete

$$\frac{e\ \mathbf{val}_\Sigma}{\mathbf{ret}\,(e)\ \mathbf{final}_\Sigma}\ \textsc{fn\_Ret}$$

$\boxed{m \underset{\Sigma}{\longmapsto} m'}$ State transition

$$\frac{e \underset{\Sigma}{\longmapsto} e'}{\mathbf{ret}\,(e) \underset{\Sigma}{\longmapsto} \mathbf{ret}\,(e')}\ \textsc{st\_Ret}$$

$$\frac{e \underset{\Sigma}{\longmapsto} e'}{\mathbf{bnd}\,(e; x.m) \underset{\Sigma}{\longmapsto} \mathbf{bnd}\,(e'; x.m)}\ \textsc{st\_Bnd}$$

$$\frac{e\ \mathbf{val}_{\Sigma}}{\mathbf{bnd}\,(\mathbf{cmd}\,(\mathbf{ret}\,(e));x.m) \underset{\Sigma}{\longmapsto} [e/x]m}\ \textsc{st\_BndInstr}$$

$$\frac{m_1 \underset{\Sigma}{\longmapsto} m_1'}{\mathbf{bnd}\,(\mathbf{cmd}\,(m_1);x.m_2) \underset{\Sigma}{\longmapsto} \mathbf{bnd}\,(\mathbf{cmd}\,(m_1');x.m_2)}\ \textsc{st\_BndCmd}$$

$$\frac{m \underset{\Sigma,q\sim\mathbf{qbit}}{\longmapsto} m'}{\mathbf{newqref}\,(x.m) \underset{\Sigma}{\longmapsto} \mathbf{newqref}\,(x.m')}\ \textsc{st\_NewQRef}$$

$$\frac{e\ \mathbf{val}_{\Sigma}}{\mathbf{newqref}\,(x.\mathbf{ret}\,(e)) \underset{\Sigma}{\longmapsto} \mathbf{ret}\,(e)}\ \textsc{st\_NewQRefInstr}$$

$$\frac{e \underset{\Sigma}{\longmapsto} e'}{\mathbf{gateap}\,[U_{2^n}](e) \underset{\Sigma}{\longmapsto} \mathbf{gateap}\,[U_{2^n}](e')}\ \textsc{st\_GateApRef}$$

$$\frac{e_1 \underset{\Sigma}{\longmapsto} e_1'}{\mathbf{diagap}\,[U_{2^n},V_{2^n}](e_1;e_2) \underset{\Sigma}{\longmapsto} \mathbf{diagap}\,[U_{2^n},V_{2^n}](e_1';e_2)}\ \textsc{st\_DiagApRefL}$$

$$\frac{\begin{array}{c}e_1\ \mathbf{val}_{\Sigma}\\ e_2 \underset{\Sigma}{\longmapsto} e_2'\end{array}}{\mathbf{diagap}\,[U_{2^n},V_{2^n}](e_1;e_2) \underset{\Sigma}{\longmapsto} \mathbf{diagap}\,[U_{2^n},V_{2^n}](e_1;e_2')}\ \textsc{st\_DiagApRefR}$$

$$\frac{e \underset{\Sigma}{\longmapsto} e'}{\mathbf{meas}\,(e) \underset{\Sigma}{\longmapsto} \mathbf{meas}\,(e')}\ \textsc{st\_MeasRef}$$

$$\frac{e\ \mathbf{val}_{\Sigma,q\sim\mathbf{qbit}}}{\mathbf{dcl}\,(q.\mathbf{ret}\,(e)) \underset{\Sigma}{\longmapsto} \mathbf{ret}\,(e)}\ \textsc{st\_Dcl}$$

$$\frac{}{\mathbf{gateap}\,[U_{2^n}](\mathbf{qloc}\,[q]) \underset{\Sigma,q\sim\mathbf{qbit}}{\longmapsto} \mathbf{gateapm}\,[U_{2^n},q]}\ \textsc{st\_GateApRefInstr}$$

$$\frac{}{\mathbf{meas}\,(\mathbf{qloc}\,[q]) \underset{\Sigma,q\sim\mathbf{qbit}}{\longmapsto} \mathbf{measm}\,[q]}\ \textsc{st\_MeasInstr}$$

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Definition rules:         55 good    0 bad
Definition rule clauses: 107 good    0 bad
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