x	term variable
a	assignable
q	qubit symbol
l	label
i, j, n, n_a, n_b	indices

$\mathit{Typ},\ \tau$	$::=$			Types
	$\mid$	$\mathbf{qbit}$		qbit, opaque qubit type
	$\mid$	$\mathbf{qref}\,[q]$		qref, qubit reference type
	$\mid$	$\mathbf{arr}\,(\tau_1;\tau_2)$		$\tau_1 \rightarrow \tau_2$
	$\mid$	$\mathbf{cmd}\,(\tau)$		τ cmd
	$\mid$	$\mathbf{prod}\,(\,\overline{l_i \hookrightarrow \tau_i}^{\,i\in 1..n}\,)$		$\times_{l\in L}\ \tau_l$
	$\mid$	$\mathbf{sum}\,(\,\overline{l_i \hookrightarrow \tau_i}^{\,i\in 1..n}\,)$		$+_{l\in L}\ \tau_l$
$\mathit{Exp},\ e$	$::=$			Expressions
	$\mid$	x		variable
	$\mid$	$\mathbf{let}\,(e_1;x.e_2)$	bind x in e_2	let x be e_1 in e_2
	$\mid$	$\mathbf{lam}\,\{\tau\}(x.e)$	bind x in e	$\lambda(x:\tau)e$
	$\mid$	$\mathbf{ap}\,(e_1;e_2)$		$e_1(e_2)$
	$\mid$	$\mathbf{cmd}\,(m)$		cmd m, encapsulation
	$\mid$	$\mathbf{qloc}\,[q]$		$\&q$, qubit location
	$\mid$	$\mathbf{tpl}\,(\,\overline{l_i \hookrightarrow e_i}^{\,i\in 1..n}\,)$		$\langle e_l\rangle_{l\in L}$
	$\mid$	$\mathbf{pr}\,[l_i](e)$		$e\cdot l$
	$\mid$	$\mathbf{in}\,[l_i]\{\,\overline{\tau_i}^{\,i\in 1..n}\,\}(e)$		$l\cdot e$
	$\mid$	$\mathbf{case}\,(e;\ \overline{l_i \hookrightarrow x_i.e_i}^{\,i\in 1..n}\,)$		case $e\,\{l\cdot x_l \hookrightarrow e_l\}_{l\in L}$
	$\mid$	$[e_1/x]e_2$	M	substitution
	$\mid$	(e)	M	parentheses
$\mathit{Cmd},\ m$	$::=$			Commands
	$\mid$	$\mathbf{ret}\,(e)$		ret e
	$\mid$	$\mathbf{bnd}\,(e;x.m)$	bind x in m	bind $x \leftarrow e; m$
	$\mid$	$\mathbf{mut}\,(e;a.m)$	bind a in m	mut $a := e$ in m
	$\mid$	$\mathbf{get}\,[a]$		$@a$
	$\mid$	$\mathbf{set}\,[a](e)$		$a := e$
	$\mid$	$\mathbf{dcl}\,(q.m)$	bind q in m	dcl q in m, new (opaque) qubit
	$\mid$	$\mathbf{gateapr}\,(e;\ U)$		$U(e)$, gate application
	$\mid$	$\mathbf{ctrlapr}\,(e_1;e_2;\ U)$		Controlled $U(e_1, e_2)$, ctrl gate app
	$\mid$	$\mathbf{measr}\,(e)$		meas(e), measure qbit
	$\mid$	$\mathbf{gateap}\,[q](U)$		gate application (opaque)
	$\mid$	$\mathbf{ctrlap}\,[q_1,q_2](U)$		ctrl gate app (opaque)
	$\mid$	$\mathbf{meas}\,[q]$		measure qbit (opaque)
	$\mid$	$[e/x]m$	M	substitution
$\mathit{Intr},\ U$	$::=$			Intrinsics
	$\mid$	$\mathbf{I}$		
	$\mid$	$\mathbf{X}$		
	$\mid$	$\mathbf{Y}$		
	$\mid$	$\mathbf{Z}$		
	$\mid$	$\mathbf{H}$		
	$\mid$	$\mathbf{S}$		
	$\mid$	$\mathbf{T}$		
$\mathit{Sugar},\ s$	$::=$			Derived forms
	$\mid$	$\mathbf{unit}$		
	$\mid$	$\mathbf{triv}$		
	$\mid$	$\mathbf{bool}$		

	$\mid$	$\mathbf{true}$	
	$\mid$	$\mathbf{false}$	
	$\mid$	$\mathbf{if}\,(e; e_1; e_2)$	
	$\mid$	$\{x \leftarrow m_1; m_2\}$	
	$\mid$	$\mathbf{do}\, e$	
	$\mid$	$\mathbf{proc}\,(x : \tau) m$	
	$\mid$	$\mathbf{call}\, e_1(e_2)$	
	$\mid$	$\tau_1 \Rightarrow \tau_2$	
Γ	$::=$		Typing context
	$\mid$	$\varnothing$	
	$\mid$	$\Gamma, x : \tau$	
$\mathit{Sigma},\ \Sigma$	$::=$		Signature
	$\mid$	$\varnothing$	
	$\mid$	$\Sigma, q \sim \tau$	
	$\mid$	$\Sigma, a \sim \tau$	
$\mathit{Memory},\ \mu$	$::=$		Memory map
	$\mid$	$\varnothing$	
	$\mid$	$\mu, a \hookrightarrow e$	
	$\mid$	$\mu, a \hookrightarrow \_$	
$\mathit{terminals}$	$::=$		
	$\mid$	$\vdash$	entails
	$\mid$	$\longmapsto$	transition
	$\mid$	$\hookrightarrow$	mapping
	$\mid$	$\cdot$	projection
	$\mid$	$\sim$	tilde
	$\mid$	$\dot\sim$	dotted tilde
	$\mid$	$\varnothing$	empty context
	$\mid$	$\sqsubseteq$	lifetime inclusion
	$\mid$	$\leq$	less than or equal
	$\mid$	$\dagger$	blocked type
	$\mid$	$\leftarrow$	
	$\mid$	$\triangleq$	defined as
	$\mid$	$\Rightarrow$	operation type
$\mathit{formula}$	$::=$		
	$\mid$	$\mathit{judgement}$	
	$\mid$	$\mathit{formula}_1 \quad .. \quad \mathit{formula}_n$	
	$\mid$	$1 \leq i \leq n$	
$\mathit{Jdefined}$	$::=$		
	$\mid$	$s \triangleq \mathit{user\_syntax}$	Derived forms / syntactic sugar
$\mathit{Jstatics}$	$::=$		
	$\mid$	$\Gamma \vdash e : \tau$	Expression Typing
	$\mid$	$\Gamma \vdash_\Sigma e : \tau$	Expression Typing wrt Signature

		$\Gamma \vdash_\Sigma m \mathrel{\dot\sim} \tau$	Well formed command w/ return type τ
Jdynamics	::=		
	|	$e\,\mathbf{val}$	Values
	|	$e\ \mathbf{val}_\Sigma$	Values wrt Signature
	|	$e \longmapsto e'$	Transition
	|	$e \underset{\Sigma}{\longmapsto} e'$	Transition wrt Signature
	|	$\mu \parallel m\ \mathbf{final}_\Sigma$	State $\mu \parallel m$ is complete
	|	$\mu \parallel m \underset{\Sigma}{\longmapsto} \mu' \parallel m'$	State transition
judgement	::=		
	|	*Jdefined*	
	|	*Jstatics*	
	|	*Jdynamics*	
user_syntax	::=		
	|	x	
	|	a	
	|	q	
	|	l	
	|	i	
	|	*Typ*	
	|	*Exp*	
	|	*Cmd*	
	|	*Intr*	
	|	*Sugar*	
	|	Γ	
	|	*Sigma*	
	|	*Memory*	
	|	*terminals*	
	|	*formula*	

$\boxed{s \triangleq \mathit{user\_syntax}}$ Derived forms / syntactic sugar

$$\frac{}{\mathbf{unit} \triangleq \mathbf{prod}\,(\,)}\ \textsc{Unit}$$

$$\frac{}{\mathbf{triv} \triangleq \mathbf{tpl}\,(\,)}\ \textsc{Triv}$$

$$\frac{}{\mathbf{bool} \triangleq \mathbf{sum}\,(l_0 \hookrightarrow \mathbf{prod}\,(\,), l_1 \hookrightarrow \mathbf{prod}\,(\,))}\ \textsc{Bool}$$

$$\frac{}{\mathbf{true} \triangleq \mathbf{in}\,[l_0]\{\mathbf{prod}\,(\,), \mathbf{prod}\,(\,)\}(\mathbf{tpl}\,(\,))}\ \textsc{True}$$

$$\frac{}{\mathbf{false} \triangleq \mathbf{in}\,[l_1]\{\mathbf{prod}\,(\,), \mathbf{prod}\,(\,)\}(\mathbf{tpl}\,(\,))}\ \textsc{False}$$

$$\frac{}{\mathbf{if}\,(e; e_1; e_2) \triangleq \mathbf{case}\,(e; l_0 \hookrightarrow x_0.e_1, l_1 \hookrightarrow x_1.e_2)}\ \textsc{Conditional}$$

$$\frac{}{\{x \leftarrow m_1; m_2\} \triangleq \mathbf{bnd}\,(\mathbf{cmd}\,(m_1); x.m_2)}\ \textsc{SeqComp}$$

$$\frac{}{\mathbf{do}\,e \triangleq \mathbf{bnd}\,(e; x.\mathbf{ret}\,(x))}\ \textsc{Do}$$

$$\frac{}{\mathbf{proc}\,(x:\tau)m \triangleq \mathbf{lam}\,\{\tau\}(x.\mathbf{cmd}\,(m))}\ \textsc{Procedure}$$

$$\frac{}{\mathbf{call}\,e_1(e_1) \triangleq \mathbf{do}\,(\mathbf{ap}\,(e_1; e_2))}\ \textsc{Call}$$

$$\frac{}{\tau_1 \Rightarrow \tau_2 \triangleq \mathbf{arr}\,(\tau_1; \mathbf{cmd}\,(\tau_2))}\ \textsc{OperationType}$$

$\boxed{\Gamma \vdash e : \tau}$ Expression Typing

$$\frac{}{\Gamma, x : \tau \vdash x : \tau}\ \textsc{ty\_Var}$$

$$\frac{\begin{array}{l}\Gamma \vdash e_1 : \tau_1\\ \Gamma, x : \tau_1 \vdash e_2 : \tau_2\end{array}}{\Gamma \vdash \mathbf{let}\,(e_1; x.e_2) : \tau_2}\ \textsc{ty\_Let}$$

$$\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \mathbf{lam}\,\{\tau_1\}(x.e) : \mathbf{arr}\,(\tau_1; \tau_2)}\ \textsc{ty\_Lam}$$

$$\frac{\begin{array}{l}\Gamma \vdash e_1 : \mathbf{arr}\,(\tau_2; \tau)\\ \Gamma \vdash e_2 : \tau_2\end{array}}{\Gamma \vdash \mathbf{ap}\,(e_1; e_2) : \tau}\ \textsc{ty\_Ap}$$

$$\frac{\overline{\Gamma \vdash e_i : \tau_i}^{\,i\in 1..n}}{\Gamma \vdash \mathbf{tpl}\,(\,\overline{l_i \hookrightarrow e_i}^{\,i\in 1..n}\,) : \mathbf{prod}\,(\,\overline{l_i \hookrightarrow \tau_i}^{\,i\in 1..n}\,)}\ \textsc{ty\_Tpl}$$

$$\frac{\begin{array}{l}\Gamma \vdash e : \mathbf{prod}\,(\,\overline{l_i \hookrightarrow \tau_i}^{\,i\in 1..n}\,)\\ 1 \le i \le n\end{array}}{\Gamma \vdash \mathbf{pr}\,[l_i](e) : \tau_i}\ \textsc{ty\_Pr}$$

$$\frac{\begin{array}{c}\Gamma \vdash e : \tau_i\\ 1 \le i \le n\end{array}}{\Gamma \vdash \mathbf{in}\,[l_i]\{\,\overline{\tau_i}^{\,i\in 1..n}\,\}(e) : \mathbf{sum}\,(\,\overline{l_i \hookrightarrow \tau_i}^{\,i\in 1..n}\,)}\ \textsc{ty\_Inj}$$

$$\frac{\begin{array}{l}\Gamma \vdash e : \mathbf{sum}\,(\,\overline{l_i \hookrightarrow \tau_i}^{\,i\in 1..n}\,)\\ \overline{\Gamma, x_i : \tau_i \vdash e_i : \tau}^{\,i\in 1..n}\end{array}}{\Gamma \vdash \mathbf{case}\,(e;\ \overline{l_i \hookrightarrow x_i.e_i}^{\,i\in 1..n}\,) : \tau}\ \textsc{ty\_Case}$$

$\boxed{\Gamma \vdash_\Sigma e : \tau}$ Expression Typing wrt Signature

$$\frac{\Gamma \vdash_\Sigma m \ \dot{\sim}\ \tau}{\Gamma \vdash_\Sigma \mathbf{cmd}\,(m) : \mathbf{cmd}\,(\tau)}\ \textsc{tyS\_Cmd}$$

$$\frac{}{\Gamma \vdash_{\Sigma, q\sim\mathbf{qbit}} \mathbf{qloc}\,[q] : \mathbf{qref}\,[q]}\ \textsc{tyS\_QLoc}$$

$\boxed{\Gamma \vdash_\Sigma m \ \dot{\sim}\ \tau}$ Well formed command w/ return type τ

$$\frac{\Gamma \vdash_\Sigma e : \tau}{\Gamma \vdash_\Sigma \mathbf{ret}\,(e) \ \dot{\sim}\ \tau}\ \textsc{cmd\_Ret}$$

$$\frac{\begin{array}{l}\Gamma \vdash_\Sigma e : \mathbf{cmd}\,(\tau)\\ \Gamma, x : \tau \vdash_\Sigma m \ \dot{\sim}\ \tau'\end{array}}{\Gamma \vdash_\Sigma \mathbf{bnd}\,(e; x.m) \ \dot{\sim}\ \tau'}\ \textsc{cmd\_Bnd}$$

$$\frac{\Gamma \vdash_\Sigma e : \tau \qquad \Gamma \vdash_{\Sigma, a \sim \tau} m \;\dot{\sim}\; \tau'}{\Gamma \vdash_\Sigma \mathbf{mut}\,(e; a.m) \;\dot{\sim}\; \tau'} \quad \textsc{cmd\_Mut}$$

$$\frac{}{\Gamma \vdash_{\Sigma, a \sim \tau} \mathbf{get}\,[a] \;\dot{\sim}\; \tau} \quad \textsc{cmd\_Get}$$

$$\frac{\Gamma \vdash_{\Sigma, a \sim \tau} e : \tau}{\Gamma \vdash_{\Sigma, a \sim \tau} \mathbf{set}\,[a](e) \;\dot{\sim}\; \tau} \quad \textsc{cmd\_Set}$$

$$\frac{\Gamma \vdash_{\Sigma, q \sim \tau} m \;\dot{\sim}\; \tau'}{\Gamma \vdash_\Sigma \mathbf{dcl}\,(q.m) \;\dot{\sim}\; \tau'} \quad \textsc{cmd\_Dcl}$$

$$\frac{}{\Gamma \vdash_{\Sigma, q \sim \mathbf{qbit}} \mathbf{gateap}\,[q](U) \;\dot{\sim}\; \mathbf{prod}\,(\,)} \quad \textsc{cmd\_GateAp}$$

$$\frac{}{\Gamma \vdash_{\Sigma, q_1 \sim \mathbf{qbit}, q_2 \sim \mathbf{qbit}} \mathbf{ctrlap}\,[q_1, q_2](U) \;\dot{\sim}\; \mathbf{prod}\,(\,)} \quad \textsc{cmd\_CtrlAp}$$

$$\frac{\Gamma \vdash_\Sigma e : \mathbf{qref}\,[q]}{\Gamma \vdash_\Sigma \mathbf{gateapr}\,(e; U) \;\dot{\sim}\; \mathbf{prod}\,(\,)} \quad \textsc{cmd\_GateApRef}$$

$$\frac{\Gamma \vdash_\Sigma e_1 : \mathbf{qref}\,[q_1] \qquad \Gamma \vdash_\Sigma e_2 : \mathbf{qref}\,[q_2]}{\Gamma \vdash_\Sigma \mathbf{ctrlapr}\,(e_1; e_2; U) \;\dot{\sim}\; \mathbf{prod}\,(\,)} \quad \textsc{cmd\_CtrlApRef}$$

$$\frac{}{\Gamma \vdash_{\Sigma, q \sim \mathbf{qbit}} \mathbf{meas}\,[q] \;\dot{\sim}\; \mathbf{sum}\,(l_0 \hookrightarrow \mathbf{prod}\,(\,), l_1 \hookrightarrow \mathbf{prod}\,(\,))} \quad \textsc{cmd\_Meas}$$

$$\frac{\Gamma \vdash_\Sigma e : \mathbf{qref}\,[q]}{\Gamma \vdash_\Sigma \mathbf{measr}\,(e) \;\dot{\sim}\; \mathbf{sum}\,(l_0 \hookrightarrow \mathbf{prod}\,(\,), l_1 \hookrightarrow \mathbf{prod}\,(\,))} \quad \textsc{cmd\_MeasRef}$$

$\boxed{e\ \mathbf{val}}$ Values

$$\frac{}{\mathbf{lam}\,\{\tau\}(x.e)\,\mathbf{val}} \quad \textsc{v\_Lam}$$

$$\frac{\overline{e_i\ \mathbf{val}}^{\,i \in 1..n}}{\mathbf{tpl}\,(\,\overline{l_i \hookrightarrow e_i}^{\,i \in 1..n}\,)\,\mathbf{val}} \quad \textsc{v\_Tpl}$$

$$\frac{e\ \mathbf{val}}{\mathbf{in}\,[l_i]\{\,\overline{\tau_i}^{\,i \in 1..n}\,\}(e)\,\mathbf{val}} \quad \textsc{v\_Inj}$$

$\boxed{e\ \mathbf{val}_\Sigma}$ Values wrt Signature

$$\frac{}{\mathbf{cmd}\,(m)\ \mathbf{val}_\Sigma} \quad \textsc{vS\_Cmd}$$

$$\frac{}{\mathbf{qloc}\,[q]\ \mathbf{val}_{\Sigma, q \sim \mathbf{qbit}}} \quad \textsc{vS\_Qloc}$$

$\boxed{e \longmapsto e'}$ Transition

$$\frac{e_1 \longmapsto e_1'}{\mathbf{let}\,(e_1; x.e_2) \longmapsto \mathbf{let}\,(e_1'; x.e_2)} \quad \textsc{trLet1}$$

$$\frac{e_1\ \mathbf{val}}{\mathbf{let}\,(e_1; x.e_2) \longmapsto [e_1/x]e_2} \quad \textsc{trLetInstr}$$

$$\frac{e_1 \longmapsto e_1'}{\mathbf{ap}\,(e_1; e_2) \longmapsto \mathbf{ap}\,(e_1'; e_2)} \quad \textsc{trAp1}$$

$$\frac{\begin{array}{c} e_1\ \mathbf{val} \\ e_2 \longmapsto e_2' \end{array}}{\mathbf{ap}\,(e_1; e_2) \longmapsto \mathbf{ap}\,(e_1; e_2')}\ \textsc{trAp2}$$

$$\frac{e_2\ \mathbf{val}}{\mathbf{ap}\,(\mathbf{lam}\,\{\tau_2\}(x.e_1); e_2) \longmapsto [e_2/x]e_1}\ \textsc{trApInstr}$$

$$\frac{\begin{array}{c} \overline{e_i\ \mathbf{val}}^{\,i\in 1..n_a} \\ e \longmapsto e' \end{array}}{\mathbf{tpl}\,(\,\overline{l_i \hookrightarrow e_i}^{\,i\in 1..n_a}, l \hookrightarrow e,\ \overline{l_j' \hookrightarrow e_j'}^{\,j\in 1..n_b}\,) \longmapsto \mathbf{tpl}\,(\,\overline{l_i \hookrightarrow e_i}^{\,i\in 1..n_a}, l \hookrightarrow e',\ \overline{l_j' \hookrightarrow e_j'}^{\,j\in 1..n_b}\,)}\ \textsc{trTpl}$$

$$\frac{e \longmapsto e'}{\mathbf{pr}\,[l_i](e) \longmapsto \mathbf{pr}\,[l_i](e')}\ \textsc{trPr}$$

$$\frac{\begin{array}{c} \mathbf{tpl}\,(\,\overline{l_i \hookrightarrow e_i}^{\,i\in 1..n}\,)\ \mathbf{val} \\ 1 \le j \le n \end{array}}{\mathbf{pr}\,[l_j](\mathbf{tpl}\,(\,\overline{l_i \hookrightarrow e_i}^{\,i\in 1..n}\,)) \longmapsto e_j}\ \textsc{trPrInstr}$$

$$\frac{e \longmapsto e'}{\mathbf{in}\,[l_i]\{\,\overline{\tau_i}^{\,i\in 1..n}\,\}(e) \longmapsto \mathbf{in}\,[l_i]\{\,\overline{\tau_i}^{\,i\in 1..n}\,\}(e')}\ \textsc{trInj}$$

$$\frac{e \longmapsto e'}{\mathbf{case}\,(e;\ \overline{l_i \hookrightarrow x_i.e_i}^{\,i\in 1..n}\,) \longmapsto \mathbf{case}\,(e';\ \overline{l_i \hookrightarrow x_i.e_i}^{\,i\in 1..n}\,)}\ \textsc{trCase}$$

$$\frac{\begin{array}{c} \mathbf{in}\,[l_j]\{\,\overline{\tau_i}^{\,i\in 1..n}\,\}(e)\ \mathbf{val} \\ 1 \le j \le n \end{array}}{\mathbf{case}\,(\mathbf{in}\,[l_j]\{\,\overline{\tau_i}^{\,i\in 1..n}\,\}(e);\ \overline{l_i \hookrightarrow x_i.e_i}^{\,i\in 1..n}\,) \longmapsto [e/x_j]e_j}\ \textsc{trCaseInstr}$$

$\boxed{e \underset{\Sigma}{\longmapsto} e'}$ Transition wrt Signature

$\boxed{\mu \parallel m\ \mathbf{final}_\Sigma}$ State $\mu \parallel m$ is complete

$$\frac{e\ \mathbf{val}_\Sigma}{\mu \parallel \mathbf{ret}\,(e)\ \mathbf{final}_\Sigma}\ \textsc{fn\_Ret}$$

$\boxed{\mu \parallel m \underset{\Sigma}{\longmapsto} \mu' \parallel m'}$ State transition

$$\frac{e \underset{\Sigma}{\longmapsto} e'}{\mu \parallel \mathbf{ret}\,(e) \underset{\Sigma}{\longmapsto} \mu \parallel \mathbf{ret}\,(e')}\ \textsc{stRet1}$$

$$\frac{e \underset{\Sigma}{\longmapsto} e'}{\mu \parallel \mathbf{bnd}\,(e; x.m) \underset{\Sigma}{\longmapsto} \mu \parallel \mathbf{bnd}\,(e'; x.m)}\ \textsc{stBnd1}$$

$$\frac{e\ \mathbf{val}_\Sigma}{\mu \parallel \mathbf{bnd}\,(\mathbf{cmd}\,(\mathbf{ret}\,(e)); x.m) \underset{\Sigma}{\longmapsto} \mu \parallel [e/x]m}\ \textsc{stBndInstr}$$

$$\frac{\mu \parallel m_1 \underset{\Sigma}{\longmapsto} \mu' \parallel m_1'}{\mu \parallel \mathbf{bnd}\,(\mathbf{cmd}\,(m_1); x.m_2) \underset{\Sigma}{\longmapsto} \mu' \parallel \mathbf{bnd}\,(\mathbf{cmd}\,(m_1'); x.m_2)}\ \textsc{stBnd2}$$

$$\frac{}{\mu, a \hookrightarrow e \parallel \mathbf{get}\,[a] \underset{\Sigma, a\sim\tau}{\longmapsto} \mu, a \hookrightarrow e \parallel \mathbf{ret}\,(e)}\ \textsc{stGet}$$

$$\frac{e \underset{\Sigma, a\sim\tau}{\longmapsto} e'}{\mu \parallel \mathbf{set}\,[a](e) \underset{\Sigma, a\sim\tau}{\longmapsto} \mu \parallel \mathbf{set}\,[a](e')} \quad \text{STSET1}$$

$$\frac{e\ \mathbf{val}_{\Sigma, a\sim\tau}}{\mu, a \hookrightarrow \_ \parallel \mathbf{set}\,[a](e) \underset{\Sigma, a\sim\tau}{\longmapsto} \mu, a \hookrightarrow e \parallel \mathbf{ret}\,(e)} \quad \text{STSETINSTR}$$

$$\frac{e \underset{\Sigma}{\longmapsto} e'}{\mu \parallel \mathbf{mut}\,(e; a.m) \underset{\Sigma}{\longmapsto} \mu \parallel \mathbf{mut}\,(e'; a.m)} \quad \text{STMUT1}$$

$$\frac{\begin{array}{l} e\ \mathbf{val}_{\Sigma} \\ \mu, a \hookrightarrow e \parallel m \underset{\Sigma, a\sim\tau}{\longmapsto} \mu', a \hookrightarrow e' \parallel m' \end{array}}{\mu \parallel \mathbf{mut}\,(e; a.m) \underset{\Sigma}{\longmapsto} \mu' \parallel \mathbf{mut}\,(e'; a.m')} \quad \text{STMUT2}$$

$$\frac{\begin{array}{l} e\ \mathbf{val}_{\Sigma} \\ e'\ \mathbf{val}_{\Sigma, a\sim\tau} \end{array}}{\mu \parallel \mathbf{mut}\,(e; a.\mathbf{ret}\,(e')) \underset{\Sigma}{\longmapsto} \mu \parallel \mathbf{ret}\,(e')} \quad \text{STMUTINSTR}$$

$$\frac{e\ \mathbf{val}_{\Sigma, q\sim\mathbf{qbit}}}{\mu \parallel \mathbf{dcl}\,(q.\mathbf{ret}\,(e)) \underset{\Sigma}{\longmapsto} \mu \parallel \mathbf{ret}\,(e)} \quad \text{STDCL}$$

$$\frac{e \underset{\Sigma}{\longmapsto} e'}{\mu \parallel \mathbf{gateapr}\,(e; U) \underset{\Sigma}{\longmapsto} \mu \parallel \mathbf{gateapr}\,(e'; U)} \quad \text{STGATEAPREF1}$$

$$\frac{}{\mu \parallel \mathbf{gateapr}\,(\mathbf{qloc}\,[q]; U) \underset{\Sigma, q\sim\mathbf{qbit}}{\longmapsto} \mu \parallel \mathbf{gateap}\,[q](U)} \quad \text{STGATEAPREFINSTR}$$

$$\frac{e_1 \underset{\Sigma}{\longmapsto} e_1'}{\mu \parallel \mathbf{ctrlapr}\,(e_1; e_2; U) \underset{\Sigma}{\longmapsto} \mu \parallel \mathbf{ctrlapr}\,(e_1'; e_2; U)} \quad \text{STCTRLAPREF1}$$

$$\frac{e_2 \underset{\Sigma}{\longmapsto} e_2'}{\mu \parallel \mathbf{ctrlapr}\,(\mathbf{qloc}\,[q_1]; e_2; U) \underset{\Sigma}{\longmapsto} \mu \parallel \mathbf{ctrlapr}\,(\mathbf{qloc}\,[q_1]; e_2'; U)} \quad \text{STCTRLAPREF2}$$

$$\frac{}{\mu \parallel \mathbf{ctrlapr}\,(\mathbf{qloc}\,[q_1]; \mathbf{qloc}\,[q_2]; U) \underset{\Sigma, q_1\sim\mathbf{qbit}, q_2\sim\mathbf{qbit}}{\longmapsto} \mu \parallel \mathbf{ctrlap}\,[q_1, q_2](U)} \quad \text{STCTRLAPREFINSTR}$$

$$\frac{e \underset{\Sigma}{\longmapsto} e'}{\mu \parallel \mathbf{measr}\,(e) \underset{\Sigma}{\longmapsto} \mu \parallel \mathbf{measr}\,(e')} \quad \text{STMEASREF1}$$

$$\frac{}{\mu \parallel \mathbf{measr}\,(\mathbf{qloc}\,[q]) \underset{\Sigma, q\sim\mathbf{qbit}}{\longmapsto} \mu \parallel \mathbf{meas}\,[q]} \quad \text{STMEASINSTR}$$

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Definition rules:         68 good    0 bad
Definition rule clauses: 126 good    0 bad
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