x, _	term variable
q, r	qubit symbols
U_{2^n}, V_{2^n}	unitary symbols
l	label
i, j, k, n, n_a, n_b	indices

Typ, τ	::=			Types
	|	**qbit**	M	qbit, opaque qubit type
	|	**qref** $\langle q \rangle$		qref q, qubit reference type
	|	**fun** $(\tau_1; \tau_2)$		$\tau_1 \to \tau_2$
	|	**cmd** (τ)		τ cmd
	|	**prod** $(\overline{l_i \hookrightarrow \tau_i}^{i \in 1..n})$		$\times_{i=1}^{n} \tau_i$
	|	**sum** $(\overline{l_i \hookrightarrow \tau_i}^{i \in 1..n})$	M	$+_{l \in L} \tau_l$
	|	**bool**		bool
	|	**unit**		unit
Exp, e	::=			Expressions
	|	x		x
	|	**let** $(e_1; x.e_2)$	bind x in e_2	let x be e_1 in e_2
	|	$\lambda \{\tau\}(x.e)$	bind x in e	$\lambda(x : \tau)e$
	|	**ap** $(e_1; e_2)$		$e_1(e_2)$
	|	**cmd** (m)		cmd m, encapsulation
	|	**qloc** $\langle q \rangle$	M	$\& q$, qubit location
	|	**tpl** $(\overline{l_i \hookrightarrow e_i}^{i \in 1..n})$		$\langle e_i \rangle_{i=1..n}$, tuple
	|	**proj** $\langle l_i \rangle(e)$		$e \cdot i$, projection
	|	**in** $\langle l_i \rangle\{\overline{\tau_i}^{i \in 1..n}\}(e)$	M	$l \cdot e$
	|	**case** $(e; \overline{l_i \hookrightarrow x_i.e_i}^{i \in 1..n})$	M	case $e\,\{l \cdot x_l \hookrightarrow e_l\}_{l \in L}$
	|	**true**		true
	|	**false**		false
	|	**if** $(e; e_1; e_2)$		if e then e_1 else e_2
	|	**not** e		$\neg\, e$
	|	**triv**		$\langle\rangle$
	|	$[e_1/x]e_2$	M	substitution
	|	(e)	M	parentheses
Cmd, m	::=			Commands
	|	**ret** (e)		ret e, return
	|	**bnd** $(e; x.m)$	bind x in m	bnd $x \leftarrow e; m$, sequencing
	|	**newqref** $(x.m)$	bind x in m	new x in m, new qubit reference
	|	**gateap** $\langle U_{2^n} \rangle(e)$		U (e), gate application
	|	**diagap** $\langle U_{2^n}, V_{2^n} \rangle(e_1; e_2)$		D(U , V)(e_1, e_2), block diagonal
	|	**meas** (e)		meas(e), measurement
	|	**dcl** $(q.m)$	M	dcl q in m, new (opaque) qubit
	|	**gateapm** $\langle U_{2^n}, q \rangle$	M	gate application (opaque)
	|	**measm** $\langle q \rangle$	M	measure qbit (opaque)
	|	$[e/x]m$	M	substitution
Sugar, s	::=			Derived forms
	|	$\{x \leftarrow m_1; m_2\}$		
	|	$\{m_1; m_2\}$		
	|	**do** e		
	|	**proc** $(x : \tau)m$		
	|	**call** $e_1(e_2)$		
	|	$\tau_1 \Rightarrow \tau_2$		
Γ	::=			Typing context

	$\mid$	$\varnothing$	
	$\mid$	$\Gamma, x : \tau$	
Sigma, Σ	::=		Signature
	$\mid$	$\varnothing$	
	$\mid$	Σ, q	
terminals	::=		
	$\mid$	$\vdash$	entails
	$\mid$	$\longmapsto$	transition
	$\mid$	$\hookrightarrow$	mapping
	$\mid$	$\cdot$	projection
	$\mid$	$\sim$	tilde
	$\mid$	$\dot{\sim}$	dotted tilde
	$\mid$	$\varnothing$	empty context
	$\mid$	$\leq$	less than or equal
	$\mid$	$\leftarrow$	
	$\mid$	$\triangleq$	defined as
	$\mid$	$\Rightarrow$	operation type
	$\mid$	$\langle$	
	$\mid$	$\rangle$	
	$\mid$	λ	
formula	::=		
	$\mid$	*judgement*	
	$\mid$	$formula_1 \;\; .. \;\; formula_n$	
	$\mid$	$1 \leq i \leq n$	
Jdefined	::=		
	$\mid$	$s \triangleq user\_syntax$	Derived forms / syntactic sugar
Jstatics	::=		
	$\mid$	$\Gamma \vdash e : \tau$	Expression Typing
	$\mid$	$\Gamma \vdash_{\Sigma} e : \tau$	Expression Typing wrt Signature
	$\mid$	$\Gamma \vdash_{\Sigma} m \;\dot{\sim}\; \tau$	Well formed command w/ return type τ
Jdynamics	::=		
	$\mid$	$e\,\mathbf{val}$	Values
	$\mid$	$e\ \texttt{val}_{\Sigma}$	Values wrt Signature
	$\mid$	$e \longmapsto e'$	Transition
	$\mid$	$e \underset{\Sigma}{\longmapsto} e'$	Transition wrt Signature
	$\mid$	$m\ \texttt{final}_{\Sigma}$	State m is complete
	$\mid$	$m \underset{\Sigma}{\longmapsto} m'$	State transition
judgement	::=		
	$\mid$	*Jdefined*	
	$\mid$	*Jstatics*	
	$\mid$	*Jdynamics*	

$$
\begin{array}{lll}
\mathit{user\_syntax} & ::= & \\
& | & x \\
& | & q \\
& | & U_{2^n} \\
& | & l \\
& | & i \\
& | & \mathit{Typ} \\
& | & \mathit{Exp} \\
& | & \mathit{Cmd} \\
& | & \mathit{Sugar} \\
& | & \Gamma \\
& | & \mathit{Sigma} \\
& | & \mathit{terminals} \\
& | & \mathit{formula}
\end{array}
$$

$\boxed{s \triangleq \mathit{user\_syntax}}$ Derived forms / syntactic sugar

$$\frac{}{\{x \leftarrow m_1; m_2\} \triangleq \mathbf{bnd}\,(\mathbf{cmd}\,(m_1); x.m_2)} \quad \textsc{SeqComp}$$

$$\frac{}{\{m_1; m_2\} \triangleq \mathbf{bnd}\,(\mathbf{cmd}\,(m_1); \_.m_2)} \quad \textsc{SeqCompU}$$

$$\frac{}{\mathbf{do}\, e \triangleq \mathbf{bnd}\,(e; x.\mathbf{ret}\,(x))} \quad \textsc{Do}$$

$$\frac{}{\mathbf{proc}\,(x:\tau)m \triangleq \lambda\,\{\tau\}(x.\mathbf{cmd}\,(m))} \quad \textsc{Procedure}$$

$$\frac{}{\mathbf{call}\, e_1(e_2) \triangleq \mathbf{do}\,(\mathbf{ap}\,(e_1; e_2))} \quad \textsc{Call}$$

$$\frac{}{\tau_1 \Rightarrow \tau_2 \triangleq \mathbf{fun}\,(\tau_1; \mathbf{cmd}\,(\tau_2))} \quad \textsc{OperationType}$$

$\boxed{\Gamma \vdash e : \tau}$ Expression Typing

$$\frac{}{\Gamma, x : \tau \vdash x : \tau} \quad \textsc{ty\_Var}$$

$$\frac{\begin{array}{l}\Gamma \vdash e_1 : \tau_1 \\ \Gamma, x : \tau_1 \vdash e_2 : \tau_2\end{array}}{\Gamma \vdash \mathbf{let}\,(e_1; x.e_2) : \tau_2} \quad \textsc{ty\_Let}$$

$$\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda\,\{\tau_1\}(x.e) : \mathbf{fun}\,(\tau_1; \tau_2)} \quad \textsc{ty\_Lam}$$

$$\frac{\begin{array}{l}\Gamma \vdash e_1 : \mathbf{fun}\,(\tau_2; \tau) \\ \Gamma \vdash e_2 : \tau_2\end{array}}{\Gamma \vdash \mathbf{ap}\,(e_1; e_2) : \tau} \quad \textsc{ty\_Ap}$$

$$\frac{\overline{\Gamma \vdash e_i : \tau_i}^{\,i\in 1..n}}{\Gamma \vdash \mathbf{tpl}\,(\,\overline{l_i \hookrightarrow e_i}^{\,i\in 1..n}\,) : \mathbf{prod}\,(\,\overline{l_i \hookrightarrow \tau_i}^{\,i\in 1..n}\,)} \quad \textsc{ty\_Tpl}$$

$$\frac{\begin{array}{l}\Gamma \vdash e : \mathbf{prod}\,(\,\overline{l_i \hookrightarrow \tau_i}^{\,i\in 1..n}\,) \\ 1 \leq i \leq n\end{array}}{\Gamma \vdash \mathbf{proj}\,\langle l_i\rangle(e) : \tau_i} \quad \textsc{ty\_Pr}$$

$$\frac{\Gamma \vdash e : \tau_i \qquad 1 \leq i \leq n}{\Gamma \vdash \mathbf{in}\,\langle l_i \rangle \{\, \overline{\tau_i}^{\,i \in 1..n} \,\}(e) : \mathbf{sum}\,(\, \overline{l_i \hookrightarrow \tau_i}^{\,i \in 1..n} \,)} \quad \text{TY\_INJ}$$

$$\frac{\Gamma \vdash e : \mathbf{sum}\,(\, \overline{l_i \hookrightarrow \tau_i}^{\,i \in 1..n} \,) \qquad \overline{\Gamma, x_i : \tau_i \vdash e_i : \tau}^{\,i \in 1..n}}{\Gamma \vdash \mathbf{case}\,(e;\, \overline{l_i \hookrightarrow x_i.e_i}^{\,i \in 1..n} \,) : \tau} \quad \text{TY\_CASE}$$

$\boxed{\Gamma \vdash_\Sigma e : \tau}$ Expression Typing wrt Signature

$$\frac{\Gamma \vdash_\Sigma m \ \dot{\sim}\ \tau}{\Gamma \vdash_\Sigma \mathbf{cmd}\,(m) : \mathbf{cmd}\,(\tau)} \quad \text{TYS\_CMD}$$

$$\frac{}{\Gamma \vdash_{\Sigma, q} \mathbf{qloc}\,\langle q \rangle : \mathbf{qref}\,\langle q \rangle} \quad \text{TYS\_QLOC}$$

$\boxed{\Gamma \vdash_\Sigma m \ \dot{\sim}\ \tau}$ Well formed command w/ return type τ

$$\frac{\Gamma \vdash_\Sigma e : \tau}{\Gamma \vdash_\Sigma \mathbf{ret}\,(e) \ \dot{\sim}\ \tau} \quad \text{CMD\_RET}$$

$$\frac{\Gamma \vdash_\Sigma e : \mathbf{cmd}\,(\tau) \qquad \Gamma, x : \tau \vdash_\Sigma m \ \dot{\sim}\ \tau'}{\Gamma \vdash_\Sigma \mathbf{bnd}\,(e; x.m) \ \dot{\sim}\ \tau'} \quad \text{CMD\_BND}$$

$$\frac{\Gamma, x : \mathbf{qref}\,\langle q \rangle \vdash_{\Sigma, q} m \ \dot{\sim}\ \tau}{\Gamma \vdash_\Sigma \mathbf{newqref}\,(x.m) \ \dot{\sim}\ \tau} \quad \text{CMD\_NEWQREF}$$

$$\frac{\Gamma \vdash_\Sigma e : \mathbf{prod}\,(\, \overline{l_i \hookrightarrow \mathbf{qref}\,\langle q_i \rangle}^{\,i \in 1..n} \,)}{\Gamma \vdash_\Sigma \mathbf{gateap}\,\langle U_{2^n} \rangle(e) \ \dot{\sim}\ \mathbf{unit}} \quad \text{CMD\_GATEAPREF}$$

$$\frac{\Gamma \vdash_\Sigma e_1 : \mathbf{qref}\,\langle q \rangle \qquad \Gamma \vdash_\Sigma e_2 : \mathbf{prod}\,(\, \overline{l_i \hookrightarrow \mathbf{qref}\,\langle r_i \rangle}^{\,i \in 1..n} \,)}{\Gamma \vdash_\Sigma \mathbf{diagap}\,\langle U_{2^n}, V_{2^n} \rangle(e_1; e_2) \ \dot{\sim}\ \mathbf{unit}} \quad \text{CMD\_DIAGAPREF}$$

$$\frac{\Gamma \vdash_\Sigma e : \mathbf{qref}\,\langle q \rangle}{\Gamma \vdash_\Sigma \mathbf{meas}\,(e) \ \dot{\sim}\ \mathbf{bool}} \quad \text{CMD\_MEASREF}$$

$$\frac{\Gamma \vdash_{\Sigma, q} m \ \dot{\sim}\ \tau}{\Gamma \vdash_\Sigma \mathbf{dcl}\,(q.m) \ \dot{\sim}\ \tau} \quad \text{CMD\_DCL}$$

$$\frac{}{\Gamma \vdash_{\Sigma, q} \mathbf{gateapm}\,\langle U_{2^n}, q \rangle \ \dot{\sim}\ \mathbf{unit}} \quad \text{CMD\_GATEAP}$$

$$\frac{}{\Gamma \vdash_{\Sigma, q} \mathbf{measm}\,\langle q \rangle \ \dot{\sim}\ \mathbf{bool}} \quad \text{CMD\_MEAS}$$

$\boxed{e\ \mathbf{val}}$ Values

$$\frac{}{\lambda\,\{\tau\}(x.e)\ \mathbf{val}} \quad \text{V\_LAM}$$

$$\frac{\overline{e_i\ \mathbf{val}}^{\,i \in 1..n}}{\mathbf{tpl}\,(\, \overline{l_i \hookrightarrow e_i}^{\,i \in 1..n} \,)\ \mathbf{val}} \quad \text{V\_TPL}$$

$$\frac{e\ \mathbf{val}}{\mathbf{in}\,\langle l_i \rangle \{\, \overline{\tau_i}^{\,i \in 1..n} \,\}(e)\ \mathbf{val}} \quad \text{V\_INJ}$$

$\boxed{e\ \mathtt{val}_\Sigma}$ Values wrt Signature

$$\frac{}{\mathbf{cmd}\,(m)\ \mathtt{val}_\Sigma}\quad \text{VS\_CMD}$$

$$\frac{}{\mathbf{qloc}\,\langle q\rangle\ \mathtt{val}_{\Sigma,q}}\quad \text{VS\_QLOC}$$

$\boxed{e \longmapsto e'}$ Transition

$$\frac{e_1 \longmapsto e_1'}{\mathbf{let}\,(e_1; x.e_2) \longmapsto \mathbf{let}\,(e_1'; x.e_2)}\quad \text{TR\_LET}$$

$$\frac{e_1\ \mathbf{val}}{\mathbf{let}\,(e_1; x.e_2) \longmapsto [e_1/x]e_2}\quad \text{TR\_LETINSTR}$$

$$\frac{e_1 \longmapsto e_1'}{\mathbf{ap}\,(e_1; e_2) \longmapsto \mathbf{ap}\,(e_1'; e_2)}\quad \text{TR\_APL}$$

$$\frac{\begin{array}{c} e_1\ \mathbf{val} \\ e_2 \longmapsto e_2' \end{array}}{\mathbf{ap}\,(e_1; e_2) \longmapsto \mathbf{ap}\,(e_1; e_2')}\quad \text{TR\_APR}$$

$$\frac{e_2\ \mathbf{val}}{\mathbf{ap}\,(\lambda\,\{\tau_2\}(x.e_1); e_2) \longmapsto [e_2/x]e_1}\quad \text{TR\_APINSTR}$$

$$\frac{\begin{array}{c} \overline{e_i\ \mathbf{val}}^{\,i\in 1..n_a} \\ e \longmapsto e' \end{array}}{\mathbf{tpl}\,(\,\overline{l_i \hookrightarrow e_i}^{\,i\in 1..n_a}, l_k \hookrightarrow e,\ \overline{l_j' \hookrightarrow e_j'}^{\,j\in 1..n_b}\,) \longmapsto \mathbf{tpl}\,(\,\overline{l_i \hookrightarrow e_i}^{\,i\in 1..n_a}, l_k \hookrightarrow e',\ \overline{l_j' \hookrightarrow e_j'}^{\,j\in 1..n_b}\,)}\quad \text{TR\_TPL}$$

$$\frac{e \longmapsto e'}{\mathbf{proj}\,\langle l_i\rangle(e) \longmapsto \mathbf{proj}\,\langle l_i\rangle(e')}\quad \text{TR\_PR}$$

$$\frac{\begin{array}{c} \mathbf{tpl}\,(\,\overline{l_i \hookrightarrow e_i}^{\,i\in 1..n}\,)\ \mathbf{val} \\ 1 \le j \le n \end{array}}{\mathbf{proj}\,\langle l_j\rangle(\mathbf{tpl}\,(\,\overline{l_i \hookrightarrow e_i}^{\,i\in 1..n}\,)) \longmapsto e_j}\quad \text{TR\_PRINSTR}$$

$$\frac{e \longmapsto e'}{\mathbf{in}\,\langle l_i\rangle\{\,\overline{\tau_i}^{\,i\in 1..n}\,\}(e) \longmapsto \mathbf{in}\,\langle l_i\rangle\{\,\overline{\tau_i}^{\,i\in 1..n}\,\}(e')}\quad \text{TR\_INJ}$$

$$\frac{e \longmapsto e'}{\mathbf{case}\,(e;\ \overline{l_i \hookrightarrow x_i.e_i}^{\,i\in 1..n}\,) \longmapsto \mathbf{case}\,(e';\ \overline{l_i \hookrightarrow x_i.e_i}^{\,i\in 1..n}\,)}\quad \text{TR\_CASE}$$

$$\frac{\begin{array}{c} \mathbf{in}\,\langle l_j\rangle\{\,\overline{\tau_i}^{\,i\in 1..n}\,\}(e)\ \mathbf{val} \\ 1 \le j \le n \end{array}}{\mathbf{case}\,(\mathbf{in}\,\langle l_j\rangle\{\,\overline{\tau_i}^{\,i\in 1..n}\,\}(e);\ \overline{l_i \hookrightarrow x_i.e_i}^{\,i\in 1..n}\,) \longmapsto [e/x_j]e_j}\quad \text{TR\_CASEINSTR}$$

$\boxed{e \underset{\Sigma}{\longmapsto} e'}$ Transition wrt Signature

$\boxed{m\ \mathtt{final}_\Sigma}$ State m is complete

$$\frac{e\ \mathtt{val}_\Sigma}{\mathbf{ret}\,(e)\ \mathtt{final}_\Sigma}\quad \text{FN\_RET}$$

$\boxed{m \underset{\Sigma}{\longmapsto} m'}$ State transition

$$\frac{e \underset{\Sigma}{\longmapsto} e'}{\mathbf{ret}\,(e) \underset{\Sigma}{\longmapsto} \mathbf{ret}\,(e')} \quad \textsc{st\_Ret}$$

$$\frac{e \underset{\Sigma}{\longmapsto} e'}{\mathbf{bnd}\,(e; x.m) \underset{\Sigma}{\longmapsto} \mathbf{bnd}\,(e'; x.m)} \quad \textsc{st\_Bnd}$$

$$\frac{e\ \mathtt{val}_{\Sigma}}{\mathbf{bnd}\,(\mathbf{cmd}\,(\mathbf{ret}\,(e)); x.m) \underset{\Sigma}{\longmapsto} [e/x]m} \quad \textsc{st\_BndInstr}$$

$$\frac{m_1 \underset{\Sigma}{\longmapsto} m_1'}{\mathbf{bnd}\,(\mathbf{cmd}\,(m_1); x.m_2) \underset{\Sigma}{\longmapsto} \mathbf{bnd}\,(\mathbf{cmd}\,(m_1'); x.m_2)} \quad \textsc{st\_BndCmd}$$

$$\frac{m \underset{\Sigma,q}{\longmapsto} m'}{\mathbf{newqref}\,(x.m) \underset{\Sigma}{\longmapsto} \mathbf{newqref}\,(x.m')} \quad \textsc{st\_NewQRef}$$

$$\frac{e\ \mathtt{val}_{\Sigma}}{\mathbf{newqref}\,(x.\mathbf{ret}\,(e)) \underset{\Sigma}{\longmapsto} \mathbf{ret}\,(e)} \quad \textsc{st\_NewQRefInstr}$$

$$\frac{e \underset{\Sigma}{\longmapsto} e'}{\mathbf{gateap}\,\langle U_{2^n}\rangle(e) \underset{\Sigma}{\longmapsto} \mathbf{gateap}\,\langle U_{2^n}\rangle(e')} \quad \textsc{st\_GateApRef}$$

$$\frac{e_1 \underset{\Sigma}{\longmapsto} e_1'}{\mathbf{diagap}\,\langle U_{2^n}, V_{2^n}\rangle(e_1; e_2) \underset{\Sigma}{\longmapsto} \mathbf{diagap}\,\langle U_{2^n}, V_{2^n}\rangle(e_1'; e_2)} \quad \textsc{st\_DiagApRefL}$$

$$\frac{\begin{array}{c} e_1\ \mathtt{val}_{\Sigma} \\ e_2 \underset{\Sigma}{\longmapsto} e_2' \end{array}}{\mathbf{diagap}\,\langle U_{2^n}, V_{2^n}\rangle(e_1; e_2) \underset{\Sigma}{\longmapsto} \mathbf{diagap}\,\langle U_{2^n}, V_{2^n}\rangle(e_1; e_2')} \quad \textsc{st\_DiagApRefR}$$

$$\frac{e \underset{\Sigma}{\longmapsto} e'}{\mathbf{meas}\,(e) \underset{\Sigma}{\longmapsto} \mathbf{meas}\,(e')} \quad \textsc{st\_MeasRef}$$

$$\frac{e\ \mathtt{val}_{\Sigma,q}}{\mathbf{dcl}\,(q.\mathbf{ret}\,(e)) \underset{\Sigma}{\longmapsto} \mathbf{ret}\,(e)} \quad \textsc{st\_Dcl}$$

$$\frac{}{\mathbf{gateap}\,\langle U_{2^n}\rangle(\mathbf{qloc}\,\langle q\rangle) \underset{\Sigma,q}{\longmapsto} \mathbf{gateapm}\,\langle U_{2^n}, q\rangle} \quad \textsc{st\_GateApRefInstr}$$

$$\frac{}{\mathbf{meas}\,(\mathbf{qloc}\,\langle q\rangle) \underset{\Sigma,q}{\longmapsto} \mathbf{measm}\,\langle q\rangle} \quad \textsc{st\_MeasInstr}$$

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Definition rules:         55 good    0 bad
Definition rule clauses: 107 good    0 bad
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