

Table: 2 Billion rows

1 page: 4 KB (4096 Bytes)

1 pointer: 8 Bytes

fan out: 66

a) 1st level: Level 0

fan out 66 \rightarrow 65 keys

\rightarrow 66 pointers

$$\Rightarrow 65 \times 128 + 66 \times 8 = 8848 \text{ Bytes per node}$$

$$\Rightarrow \text{pages/node} = \frac{8848}{4096} = \lceil 2.16 \text{ pages} \rceil = 3 \text{ pages}$$

2nd level: Level 1

$$66 \text{ nodes} \times 3 \text{ pages/nodes} = 198 \text{ pages}$$

3rd level: Level 2

$$66^2 \times 3 \text{ pages} = 13068 \text{ pages}$$

10th level: Level 9

$$66^{10} \times 3 \text{ pages}$$

b) Levels needed for B+ tree

$$L = \log_{66} 2 \text{ Billion} = 5.11$$

$$\Rightarrow \text{round up} = 6 \text{ levels}$$

Space calculation:

Level 1: 1 node

Level 2: 66 nodes

$$\text{Level 3: } 66 * 66 = 4356$$

$$\text{Level 4: } 66 * 66 * 66 = 287,496$$

$$\text{Level 5: } 66 * 66 * 66 * 66 = 19,000,000$$

$$\text{Level 6: } 66^5 =$$

Space required:

$$\text{Level 1: } 1 * \text{page size} = 4 \text{ KB}$$

$$\text{Level 2: } 66 \text{ nodes} * \text{page size} = 264 \text{ KB}$$

$$\text{Level 3: } 4356 * \text{page size} = 17424 \text{ KB}$$

$$\text{Level 4: } 287,496 * \text{page size} = 1,149,984 \text{ KB}$$

$$\text{Level 5: } 19,000,000 * \text{page size} = 72.479 \text{ GB}$$

$$\text{Level 6: } 66^5 * \text{page size} = 4777.27 \text{ GB}$$

c) 6 Levels: Worst case, No Ram optimize

$$(3 \times 6) + 1 \rightarrow \text{equality search}$$

$$= 19 \text{ I/Os}$$

$$\boxed{3 \times 6 + \text{I/O cost (Result)}}$$

* Actual case (given 48 GB Ram)

\rightarrow 4 levels on Ram

\rightarrow 2 levels on disk

$$\Rightarrow \boxed{\begin{matrix} (3 \times 2) & + & \text{I/O cost} \\ \downarrow & & \downarrow \\ \text{pages} & & \text{disk} \end{matrix}} \text{ (Result)}$$