ICS635 Homework 2: by Lambert Leong

Problem 1

1. null hypothesis - Oliver's blood is at the crime scene. Probability Oliver was at the crime and the other person is AB blood type

$$P(O|D) = \frac{P(D|O)P(O)}{P(D)}$$

Probability Oliver was not at the crime and that of 2 random people, 1 person is type O and the other is type AB

$$P(\tilde{O}|D) = \frac{P(D|\tilde{O})P(\tilde{O})}{P(D)}$$

Likelihood Ratio

$$\frac{P(O|D)}{P(\tilde{O}|D)} = \frac{P(D|O)P(O)}{P(D|\tilde{O})P(\tilde{O})}$$
$$\frac{P(O|D)}{P(\tilde{O}|D)} = \frac{.01}{2 * (.6) * (.01)}$$
$$\frac{P(O|D)}{P(\tilde{O}|D)} = .83$$

The likelihood ratio is not sigificanti enought to accept the null hypothesis, which states that Oliver's blood is at the crime scene

Problem 2

1.

$$\begin{aligned}\widehat{\theta}_{MLE} &= \frac{N_1}{N} \\ &= \frac{1}{1} \\ &= 1\end{aligned}$$

2.

$$\begin{aligned}\widehat{\theta}_{MAP} &= \frac{N_1 + a - 1}{(N_0 + N_1) + a + b - 2} \\ &= \frac{1 + 1 - 1}{0 + 1 + 1 + 1 - 2} \\ &= \frac{1}{1}\end{aligned}$$

3.

$$
\begin{aligned}
\bar{\theta} &= \frac{N_1 + a}{N_0 + N_1 + a + b} \\
&= \frac{1+1}{0+1+1+1} \\
&= \frac{2}{3}
\end{aligned}
$$

4.

$$
\begin{aligned}
p(\theta|x) &\propto p(x|\theta)p(\theta) \\
&= \theta^x (1-\theta)^{1-x} \frac{\Gamma(N_1 + \alpha + N_0 + \beta))}{\Gamma(N_1+\alpha)\Gamma(N_0+\beta)} \theta^{N_1+\alpha-1}(1-\theta)^{N_0+\beta-1} \\
&= \frac{\Gamma(N_1 + \alpha + N_0 + \beta))}{\Gamma(N_1+\alpha)\Gamma(N_0+\beta)} \int_0^1 \theta^{x+N_1+\alpha-1}(1-\theta)^{N_0+\beta-x} d\theta \\
&= \frac{\Gamma(N_1 + \alpha + N_0 + \beta))}{\Gamma(N_1+\alpha)\Gamma(N_0+\beta)} \frac{\Gamma(x+N_1+\alpha)\Gamma(N_0+\beta-x+1)}{\Gamma(N_1+\alpha+N_0+\beta+1)} \\
&= \frac{\Gamma(1+1+0+1))}{\Gamma(1+1)\Gamma(0+1)} \frac{\Gamma(1+1+1)\Gamma(0+1-0+1)}{\Gamma(1+1+0+1+1)} \\
&= \frac{\Gamma(3))}{\Gamma(2)\Gamma(1)} \frac{\Gamma(3)\Gamma(2)}{\Gamma(4)}
\end{aligned}
$$

Or

$$p(\theta|x) \propto Beta(\theta|N_1 + a, N_0 + b)$$

5.

$$
\begin{aligned}
p(x_2 = 1|x = 1, \theta) &= Beta(\theta|x_2 = 1, x = 1) \\
&= \theta^{x_2 - 1}(1-\theta)^{x-1}
\end{aligned}
$$

6.

$$
\begin{aligned}
\widehat{\theta}_{MLE} &= \frac{N_1}{N} \\
&= \frac{5}{5} \\
&= 1
\end{aligned}
$$

$$
\begin{aligned}
\widehat{\theta}_{MAP} &= \frac{N_1 + a - 1}{(N_0 + N_1) + a + b - 2} \\
&= \frac{5 + 1 - 1}{0 + 5 + 1 + 1 - 2} \\
&= \frac{5}{5} \\
&= 1
\end{aligned}
$$

$$
\begin{aligned}
\bar{\theta} &= \frac{N_1 + a}{N_0 + N_1 + a + b} \\
&= \frac{5 + 1}{0 + 5 + 1 + 1} \\
&= \frac{6}{7}
\end{aligned}
$$

$$
\begin{aligned}
p(\theta|x) &\propto p(x|\theta)p(\theta) \\
&= \theta^x (1-\theta)^{1-x} \frac{\Gamma(N_1 + \alpha + N_0 + \beta))}{\Gamma(N_1 + \alpha)\Gamma(N_0 + \beta)} \theta^{N_1 + \alpha - 1} (1-\theta)^{N_0 + \beta - 1} \\
&= \frac{\Gamma(N_1 + \alpha + N_0 + \beta))}{\Gamma(N_1 + \alpha)\Gamma(N_0 + \beta)} \int_0^1 \theta^{x + N_1 + \alpha - 1} (1-\theta)^{N_0 + \beta - x} d\theta \\
&= \frac{\Gamma(N_1 + \alpha + N_0 + \beta))}{\Gamma(N_1 + \alpha)\Gamma(N_0 + \beta)} \frac{\Gamma(x + N_1 + \alpha)\Gamma(N_0 + \beta - x + 1)}{\Gamma(N_1 + \alpha + N_0 + \beta + 1)} \\
&= \frac{\Gamma(5 + 1 + 0 + 1))}{\Gamma(5 + 1)\Gamma(0 + 1)} \frac{\Gamma(1 + 5 + 1)\Gamma(0 + 1 - 0 + 1)}{\Gamma(5 + 1 + 0 + 1 + 1)} \\
&= \frac{\Gamma(7))}{\Gamma(6)\Gamma(1)} \frac{\Gamma(7)\Gamma(2)}{\Gamma(8)}
\end{aligned}
$$

7.

Problem 3

1.

$$\theta_{spam} = \frac{spam}{(spam + normal)} = \frac{3}{(3+4)} = \frac{3}{7}$$
$$\theta_{secret|spam} = \frac{secret\ in\ spam}{(total\ spam)} = \frac{2}{(2+1)} = \frac{2}{3}$$
$$\theta_{secret|non-spam} = \frac{secret\ in\ non-spam}{(total\ non-spam)} = \frac{1}{(2+2)} = \frac{1}{4}$$
$$\theta_{sport|non-spam} = \frac{sport\ in\ non-spam}{(total\ non-spam)} = \frac{2}{(2+2)} = \frac{1}{2}$$
$$\theta_{dollar|spam} = \frac{dollar\ in\ spam}{(total\ spam)} = \frac{1}{(1+2)} = \frac{1}{3}$$

Problem 4

1. The tradeoff between bias and variance is something to consider when training complex models. Increasing the data set may introduce more variance for which complex models may have trouble fitting during training. Thusly, the training error may increase. This is not necessarily a bad thing because the model may still have the ability to generalize to unseen data during training.

2.

Problem 5

1.