



Programming Project

Implementation Report

Section [44094]
Group [4]

Students	
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I. Task Distribution

The table below shows task distribution among team members.

<i>Student</i>	<i>Tasks</i>
<i>Latifah Alessa</i>	Simple backtracking, Forward checking with MRV
<i>Lamees Alturki</i>	Generate grid, Forward checking

II. Overview

Tenner Grid puzzles pose an intriguing challenge in combinatorial problem solving, where the objective is to fill a grid with numbers while adhering to certain constraints.

Problem Formulation:

Variables: cells represented by it's coordinate on a matrix e.g $\{C_{0,0}, C_{0,1}, \dots\}$.

Domain: the set from 0-9

Constraints:

1. Numbers appear only once in a row.
2. Numbers may be repeated in columns.
3. Numbers in the columns must add up to the given sums.
4. Numbers in contiguous cells must be different.

This report presents the design and implementation of a program capable of generating and solving 3 by 10 sized Tenner Grid puzzles by modeling them as Constraint Satisfaction Problems.

III. Description

i. Backtracking

The recursive Step of this method is iterating over possible values from 0 to 9 and tries to assign them to the current cell if they satisfy the puzzle's rules. It then recursively calls itself to move to the next cell, repeat the process and if none of the values can be placed in the current cell without violating the rules, it backtracks by resetting the cell value to -1 and tries the next value in the previous recursive call. And so, on until it returns true if the algorithm has reached the end of the grid, indicating that the puzzle has been solved.

```
1  Function simpleBacktracking(grid, currentRow, currentColumn):
2      Start timer
3
4      If currentRow == LastRow AND currentColumn == LastColumn:
5          Return true
6
7      If currentColumn == LastColumn:
8          currentColumn = FirstColumn
9          currentRow = currentRow + 1
10
11     If grid[currentRow][currentColumn] is not empty:
12         Return simpleBacktracking(grid, currentRow, currentColumn + 1)
13
14     For value from 1 to 9:
15         If value satisfies rules for currentRow, currentColumn:
16             Set grid[currentRow][currentColumn] = value
17             If simpleBacktracking(grid, currentRow, currentColumn + 1):
18                 Stop timer
19                 Return true
20
21         Unset grid[currentRow][currentColumn]
22
23     Stop timer
24     Return false
```

Figure 1: shows the pseudocode of the backtracking algorithm.

ii. Forward checking

Firstly, the algorithm iterates through the grid to find the first empty cell, once an empty cell is found the algorithm iterates over the domain of possible values of said cell and checks if a number from the domain can be placed in the cell while satisfying the grid's rules and domain constraints. Otherwise, if no empty cell is found, it means the grid is already solved.

Before recursively calling the ForwardChecking function, the algorithm calls forwardCheckdomain to update the domains of neighboring cells based on the value just assigned to the current cell. Then if placing the value in the current cell is valid, the algorithm recursively calls itself to move to the next empty cell. If this recursive call returns true, then the puzzle has been solved. But if no valid value can be placed in the current cell, or if the recursive call returns false, indicating that no solution was found with the current assignment, the algorithm backtracks by resetting the current cell's value to -1. It also restores the domains of neighboring cells affected by the previous assignment.

```
1  Function ForwardChecking(grid, domains):
2      Start timer
3
4      FindEmptyCell(grid) // Find the first empty cell in the grid
5
6      If grid is full:
7          Stop timer
8          Return true // Grid is solved
9
10     For num from 0 to 9:
11         If num is valid according to domain and rules:
12             Assign num to the empty cell
13             Update domain and constraints
14             If ForwardChecking(grid, domains):
15                 Stop timer
16                 Return true // If the rest of the grid can be filled
17             Backtrack // Undo the assignment and restore domain
18     Return false // No valid number found for the empty cell
```

Figure 2: shows the pseudocode of the Forward checking algorithm.

iii. Forward checking with MRV

This algorithm is almost identical to Forward Checking which we described above, but the main difference lies in the introduction of the concept of MRV, which refers to the unassigned cell with the fewest remaining values in its domain. The algorithm aims to prioritize such cells because they are likely to lead to quicker pruning of search branches.

```
1  Function ForwardCheckingMRV(grid, domains):
2      Start timer
3
4      FindUnassignedVariableWithMRV(grid, domains) // Find the unassigned variable with
5                                                    // the fewest remaining values
6
7      If grid is full:
8          Stop timer
9          Return true // Grid is solved
10
11     For num from 0 to 9:
12         If num is valid according to domain and rules:
13             Assign num to the empty cell
14             Update domain and constraints
15             If ForwardCheckingMRV(grid, domains):
16                 Stop timer
17                 Return true // If the rest of the grid can be filled
18             Backtrack // Undo the assignment and restore domain
19     Return false // No valid number found for the empty cell
```

Figure 3: shows the pseudocode of the Forward checking with MRV algorithm.

IV. Sample Run

Welcome to the Tenner Grid generator and solver

```
-----Initial State-----  
| 7 0 4 -1 2 9 -1 5 1 8 |  
| 2 -1 5 0 6 4 -1 -1 -1 -1 |  
| 0 8 4 -1 -1 1 9 5 -1 6 |  
| 9 9 13 6 15 14 22 18 6 23 |
```

~~~~~Backtracking~~~~~

```
-----Final State-----  
| 7 0 4 3 2 9 6 5 1 8 |  
| 2 1 5 0 6 4 7 8 3 9 |  
| 0 8 4 3 7 1 9 5 2 6 |  
| 9 9 13 6 15 14 22 18 6 23 |
```

consistency: 587

assignments: 103

Time Used: 0.004 milliseconds

~~~~~ForwardChecking~~~~~

```
-----Final State-----  
| 7 0 4 3 2 9 6 5 1 8 |  
| 2 1 5 0 6 4 7 8 3 9 |  
| 0 8 4 3 7 1 9 5 2 6 |  
| 9 9 13 6 15 14 22 18 6 23 |
```

consistency: 465

assignments: 18

Time Used: 0.0086 milliseconds

~~~~~ForwardCheckingMRV~~~~~

```
-----Final State-----  
| 7 0 4 3 2 9 6 5 1 8 |  
| 2 1 5 0 6 4 7 8 3 9 |  
| 0 8 4 3 7 1 9 5 2 6 |  
| 9 9 13 6 15 14 22 18 6 23 |
```

consistency: 632

assignments: 24

Time Used: 0.0128 milliseconds

## V. Analyzing The Algorithms

### iv. Comparison

-----Initial State-----  
| 3 0 1 2 9 8 4 7 5 6 |  
| 6 4 3 -1 5 1 2 9 8 0 |  
| 1 -1 8 2 4 -1 5 0 -1 9 |  
| 10 11 12 11 18 15 11 16 16 15 |

~~~~~Backtracking~~~~~

-----Final State-----
| 3 0 1 2 9 8 4 7 5 6 |
| 6 4 3 7 5 1 2 9 8 0 |
| 1 7 8 2 4 6 5 0 3 9 |
| 10 11 12 11 18 15 11 16 16 15 |

consistency: 174

assignments: 27

Time Used: 3.0E-4 milliseconds

~~~~~ForwardChecking~~~~~

-----Final State-----  
| 3 0 1 2 9 8 4 7 5 6 |  
| 6 4 3 7 5 1 2 9 8 0 |  
| 1 7 8 2 4 6 5 0 3 9 |  
| 10 11 12 11 18 15 11 16 16 15 |

consistency: 174

assignments: 4

Time Used: 9.0E-4 milliseconds

~~~~~ForwardCheckingMRV~~~~~

-----Final State-----
| 3 0 1 2 9 8 4 7 5 6 |
| 6 4 3 7 5 1 2 9 8 0 |
| 1 7 8 2 4 6 5 0 3 9 |
| 10 11 12 11 18 15 11 16 16 15 |

consistency: 174

assignments: 4

Time Used: 0.001 milliseconds

-----Initial State-----
| -1 5 -1 -1 3 -1 -1 6 -1 8 |
| -1 -1 7 0 6 1 -1 3 -1 5 |
| 0 -1 -1 1 4 2 -1 6 8 3 |
| 2 14 25 2 13 5 22 15 21 16 |

~~~~~Backtracking~~~~~

-----Final State-----  
| 0 5 9 1 3 2 7 6 4 8 |  
| 2 4 7 0 6 1 8 3 9 5 |  
| 0 5 9 1 4 2 7 6 8 3 |  
| 2 14 25 2 13 5 22 15 21 16 |

consistency: 2777

assignments: 465

Time Used: 8.0E-4 milliseconds

~~~~~ForwardChecking~~~~~

-----Final State-----
| 0 5 9 1 3 2 7 6 4 8 |
| 2 4 7 0 6 1 8 3 9 5 |
| 0 5 9 1 4 2 7 6 8 3 |
| 2 14 25 2 13 5 22 15 21 16 |

consistency: 2338

assignments: 83

Time Used: 0.0012 milliseconds

~~~~~ForwardCheckingMRV~~~~~

-----Final State-----  
| 0 5 9 1 3 2 7 6 4 8 |  
| 2 4 7 0 6 1 8 3 9 5 |  
| 0 5 9 1 4 2 7 6 8 3 |  
| 2 14 25 2 13 5 22 15 21 16 |

consistency: 2463

assignments: 85

Time Used: 0.0013 milliseconds

-----Initial State-----  
 | 8 7 5 9 1 6 3 0 2 4 |  
 | -1 3 -1 -1 -1 -1 -1 -1 -1 |  
 | -1 -1 -1 -1 0 -1 4 -1 6 -1 |  
 | 16 17 15 21 3 18 15 6 15 9 |

~~~~~Backtracking~~~~~

-----Final State-----
 | 8 7 5 9 1 6 3 0 2 4 |
 | 6 3 1 4 2 9 8 5 7 0 |
 | 2 7 9 8 0 3 4 1 6 5 |
 | 16 17 15 21 3 18 15 6 15 9 |

consistency: 69419
 assignments: 11863
 Time Used: 9.0E-4 milliseconds

~~~~~ForwardChecking~~~~~

-----Final State-----  
 | 8 7 5 9 1 6 3 0 2 4 |  
 | 6 3 1 4 2 9 8 5 7 0 |  
 | 2 7 9 8 0 3 4 1 6 5 |  
 | 16 17 15 21 3 18 15 6 15 9 |

consistency: 42820  
 assignments: 2156  
 Time Used: 0.0016 milliseconds

~~~~~ForwardCheckingMRV~~~~~

-----Final State-----
 | 8 7 5 9 1 6 3 0 2 4 |
 | 6 3 1 4 2 9 8 5 7 0 |
 | 2 7 9 8 0 3 4 1 6 5 |
 | 16 17 15 21 3 18 15 6 15 9 |

consistency: 42430
 assignments: 2160
 Time Used: 0.0015 milliseconds

-----Initial State-----
 | -1 9 -1 -1 -1 -1 -1 -1 -1 |
 | -1 -1 -1 1 -1 -1 5 6 3 0 |
 | 9 -1 -1 -1 -1 -1 2 8 -1 5 |
 | 19 17 19 6 14 18 7 21 8 6 |

~~~~~Backtracking~~~~~

-----Final State-----  
 | 3 9 8 2 5 6 0 7 4 1 |  
 | 7 2 4 1 9 8 5 6 3 0 |  
 | 9 6 7 3 0 4 2 8 1 5 |  
 | 19 17 19 6 14 18 7 21 8 6 |

consistency: 589769  
 assignments: 97941  
 Time Used: 0.0051 milliseconds

~~~~~ForwardChecking~~~~~

-----Final State-----
 | 3 9 8 2 5 6 0 7 4 1 |
 | 7 2 4 1 9 8 5 6 3 0 |
 | 9 6 7 3 0 4 2 8 1 5 |
 | 19 17 19 6 14 18 7 21 8 6 |

consistency: 383379
 assignments: 17808
 Time Used: 0.0025 milliseconds

~~~~~ForwardCheckingMRV~~~~~

-----Final State-----  
 | 3 9 8 2 5 6 0 7 4 1 |  
 | 7 2 4 1 9 8 5 6 3 0 |  
 | 9 6 7 3 0 4 2 8 1 5 |  
 | 19 17 19 6 14 18 7 21 8 6 |

consistency: 361626  
 assignments: 15690  
 Time Used: 0.0038 milliseconds



```

-----Initial State-----
| 5 -1 9 -1 6 4 -1 0 -1 -1 |
|-1 -1 1 -1 9 8 2 -1 -1 4 |
| 5 7 4 -1 -1 0 -1 -1 9 2 |
|16 14 14 13 16 12 8 15 20 7 |

```

~~~~~Backtracking~~~~~

```

-----Final State-----
| 5 7 9 2 6 4 3 0 8 1 |
| 6 0 1 5 9 8 2 7 3 4 |
| 5 7 4 6 1 0 3 8 9 2 |
|16 14 14 13 16 12 8 15 20 7 |

```

consistency: 6138

assignments: 1075

Time Used: 7.0E-4 milliseconds

~~~~~ForwardChecking~~~~~

```

-----Final State-----
| 5 7 9 2 6 4 3 0 8 1 |
| 6 0 1 5 9 8 2 7 3 4 |
| 5 7 4 6 1 0 3 8 9 2 |
|16 14 14 13 16 12 8 15 20 7 |

```

consistency: 4653

assignments: 196

Time Used: 0.002 milliseconds

~~~~~ForwardCheckingMRV~~~~~

```

-----Final State-----
| 5 7 9 2 6 4 3 0 8 1 |
| 6 0 1 5 9 8 2 7 3 4 |
| 5 7 4 6 1 0 3 8 9 2 |
|16 14 14 13 16 12 8 15 20 7 |

```

consistency: 4232

assignments: 168

Time Used: 0.0019 milliseconds

| | No. consistency checks | | | | | Median |
|-------------------------|------------------------|---------|---------|---------|---------|--------|
| | Game #1 | Game #2 | Game #3 | Game #4 | Game #5 | |
| Backtracking | 174 | 2777 | 6138 | 589769 | 69419 | 6138 |
| Forward checking | 174 | 2338 | 4653 | 383379 | 42820 | 4653 |
| FC + MRV | 174 | 2463 | 4232 | 361626 | 42430 | 4232 |

v. Analysis

It is observed that Simple Backtracking still solves the Tenner Grid problem despite being the least efficient among the three tested algorithms. Since, it requires the highest median number of consistency checks, indicating that it explores a larger portion of the search space before finding a solution. This suggests that Simple Backtracking may struggle with larger or more complex instances of the Tenner Grid problem due to its exhaustive search approach. Forward Checking improves upon Simple Backtracking by pruning the search space more aggressively. As a result, fewer consistency checks are required to find a solution to the Tenner Grid problem. However, it was still outperformed by Forward Checking with MRV, suggesting that there is still room for improvement in reducing the search space further.

Forward Checking with MRV stands out as the most efficient algorithm for solving the Tenner Grid problem in terms of the median number of consistency checks required. By prioritizing variables with the fewest remaining values in their domains, this algorithm effectively reduces the search space, leading to quicker convergence to a solution. This makes it particularly well suited for larger or more complex instances of the Tenner Grid problem, where efficiency is crucial.

To conclude, while all three algorithms can solve the Tenner Grid problem, Forward Checking with MRV emerges as the most efficient option, followed by Forward Checking and then Simple Backtracking. Depending on the size and complexity of the problem instance, choosing the appropriate algorithm can significantly impact the efficiency and scalability of the solution.

VI. Student Peer Evaluation

The table below illustrate Student-Peer-Evaluation across two different aspects.
 (1: fully satisfied, 0.5: partially satisfied, 0: not satisfied)

| Name of evaluator | | Evaluation for each student | |
|-------------------|---------|---|--------|
| | | <i>Level of commitment</i> | |
| | | Latifah | Lamees |
| | Latifah | | 1 |
| | Lamees | 1 | |
| | | <i>Professional behavior towards team</i> | |
| | | Latifah | Lamees |
| | Latifah | | 1 |
| | Lamees | 1 | |