

**UNIVERSITY OF ASIA PACIFIC**  
**Department of Civil Engineering**

**Fall 2023**

**CE 419**

**Structural Engineering VII**

**Project Title: Calculation of Stiffness and Natural Frequency by Etabs and Staad Pro**

**Group Members:**

Name	Registration Number
Usayed Islam	20105062
Lamia Ahmed Sadia	20105084

**Submitted to:**

**Dr. Iftekhar Anam**

**Professor, Department of Civil Engineering,  
University of Asia Pacific**

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## ***Manual Calculation (Project-1,2)***

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### **Horizontal Stiffness (Project-01)**

Shape Function  $\psi = \sin(\frac{\pi x}{L})$

Deflection at A  $\neq 0$ , C = 0

So,

$$\psi(x) = \sin\left(\frac{\pi x}{L}\right); \psi(0) = 0; \psi(L) = 0$$

$$\psi'(x) = \left(\frac{\pi}{L}\right) \cos\left(\frac{\pi x}{L}\right); \psi(0) = 1; \psi(L) = 0$$

$$\psi''(x) = -\left(\frac{\pi}{L}\right)^2 \sin\left(\frac{\pi x}{L}\right)$$

Here,

$$R = \frac{84+62}{2} = 73$$

E=8475 ksi

=1229499 ksf

$$I_{AB} = \frac{12 * 50^3}{12} ft^4 = 125000 ft^4$$

$$I_{AB} = \frac{\pi * 60^3}{64} ft^4 = 636174 ft^4$$

N=2, (Gauss)

$$k^* = \int_0^L EI(\psi''(x))^2 dx$$

$$= \int_0^{80} EI_{AB}(\psi''(x))^2 dx + \int_{80}^{95} EI_{BC}(\psi''(x))^2 dx$$

n	$\pm \xi_i$	A <sub>i</sub>
1	0.0	2.0
2	$\pm 0.57735$	1.0
3	$\pm 0.77460$	0.55556
	0.0	0.88889
4	$\pm 0.86114$	0.34785
	$\pm 0.33998$	0.65215
5	$\pm 0.90618$	0.23693
	$\pm 0.53847$	0.47863
	0.0	0.56889

$$So, \xi_1 = 0.57735; \xi_2 = -0.57735$$

$$A_1 = 1; A_2 = 1$$

We know,  $\int_0^l f(x)dx = \frac{b-a}{2} \sum A_i f(x)$

And

$$x_i = \frac{b+a}{2} + \frac{b-a}{2} \xi$$

For AB [0~80] portion,

$$x_1 = \frac{80+0}{2} + \frac{80-0}{2} * 0.57735 = 63.094$$

$$x_2 = \frac{0.3 + 0}{2} - \frac{0.3 - 0}{2} * 0.57735 = 16.906$$

For AB [80~95] portion,

$$x_1 = \frac{95 + 80}{2} + \frac{95 - 80}{2} * 0.57735 = 91.83$$

$$x_2 = \frac{95 + 80}{2} - \frac{95 - 80}{2} * 0.57735 = 83.16$$

Now,

$$\begin{aligned} k^* &= \int_0^L EI(\psi''(x))^2 dx \\ &= \int_0^{80} EI_{AB}(\psi''(x))^2 dx + \int_{80}^{95} EI_{BC}(\psi''(x))^2 dx \\ &= 1220400 * 125000 * \frac{80}{2} \int_0^{80} \left\{ -1 * \left( \frac{\pi}{80^2} \right)^2 \sin\left(\frac{\pi * 63.094}{80}\right) \right\}^2 dx + \\ &\quad \int_0^{80} \left\{ -1 * \left( \frac{\pi}{80^2} \right)^2 \sin\left(\frac{\pi * 16.90}{80}\right) \right\}^2 dx + 1220400 * 636174 * \frac{95-80}{2} * \\ &\quad \int_{80}^{95} \left\{ -1 * \left( \frac{\pi}{15^2} \right)^2 \sin\left(\frac{\pi * 91.83}{15}\right) \right\}^2 dx + \int_{80}^{95} \left\{ -1 * \left( \frac{\pi}{15^2} \right)^2 \sin\left(\frac{\pi * 83.16}{15}\right) \right\}^2 dx \\ &= 1.257 * 10^{10} \text{ kip/ft} \end{aligned}$$

## Natural Frequency (Project-02)

We know, *Natural Frequency*,  $\omega = \sqrt{\frac{k^*}{m^*}}$

$$m^* = \int_0^L \mu(\psi(x))^2 dx$$

$$\begin{aligned} &= \left( \int_0^L \mu(\psi(x))^2 dx \right) \\ &= \left( \int_0^{80} \mu_{AB}(\psi(x))^2 dx + \int_{80}^{90} \mu_{BC}(\psi(x))^2 dx \right) \end{aligned}$$

$$P=173 \frac{k}{ft}$$

$$\mu = \frac{173}{32.2} = 5.37 \frac{k * s^2}{ft^2}$$

For AB[0~80] portion,

$$\begin{aligned} &\int_0^{80} \mu_{AB}(\psi(x))^2 dx \\ &= \int_0^{80} 5.37 * \sin\left(\frac{\pi x}{80}\right)^2 dx \\ &= 214.8 K \frac{s^2}{ft} \end{aligned}$$

For BC[80~95] portion,

$$\int_{80}^{95} \mu_{BC}(\psi(x))^2 dx$$

$$= \int_{80}^{95} 5.37 * \sin\left(\frac{\pi x}{15}\right)^2 dx$$

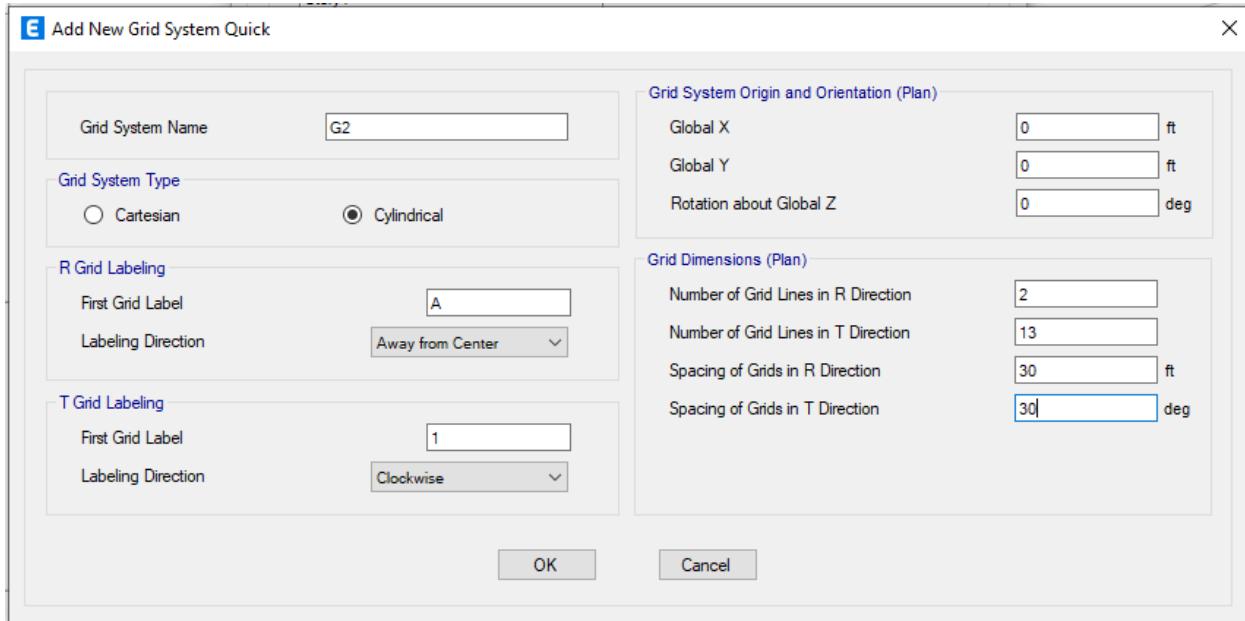
$$= 40.275 K \frac{s^2}{ft}$$

$$m^* = 214.8 + 40.275 = 255.075 K \frac{s^2}{ft}$$

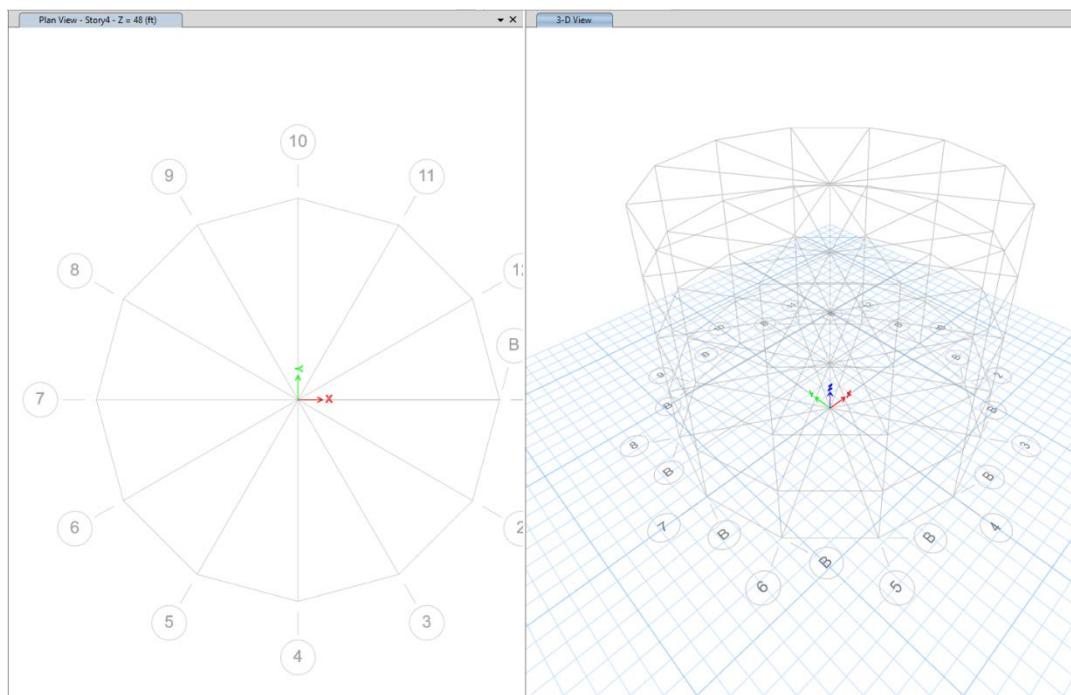
$$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{1.256 * 10^{10}}{255.075}} = 7017.15 \text{ rad/sec}$$

## Steps From Etabs(Project-01)

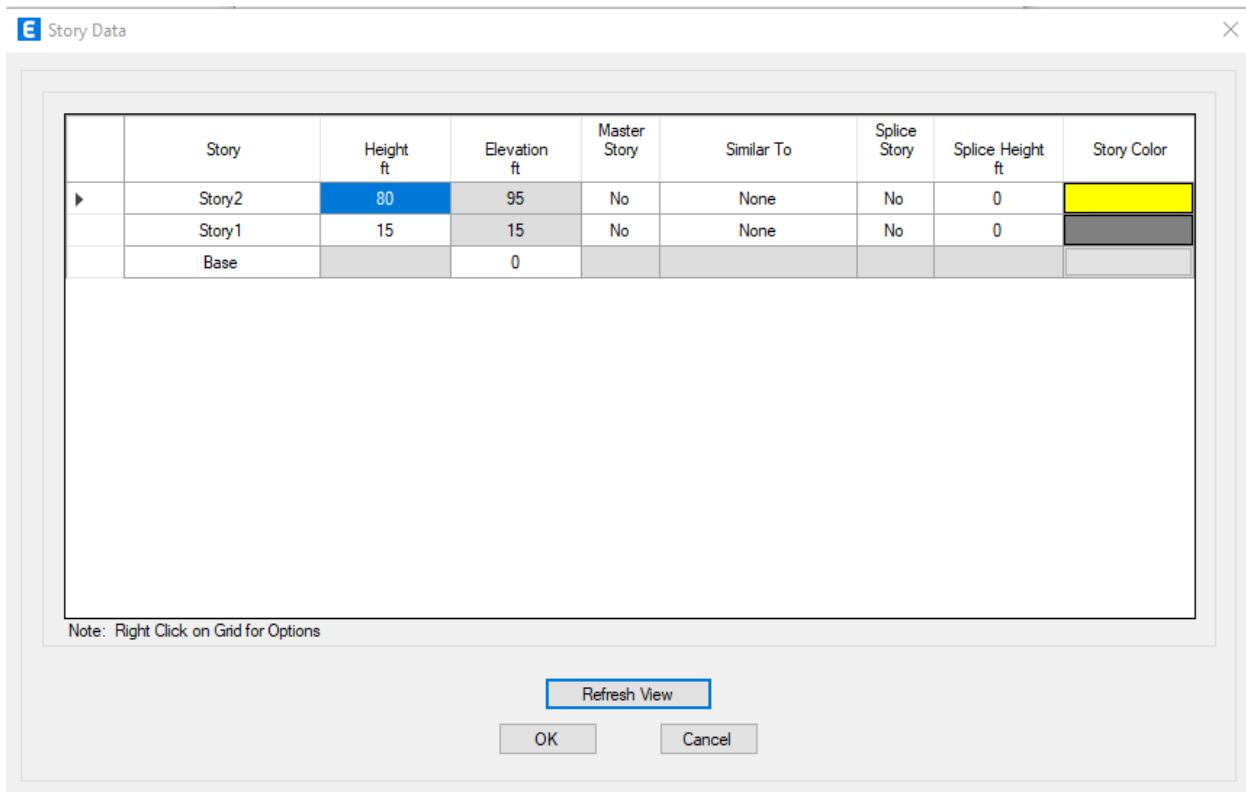
### Step 01:



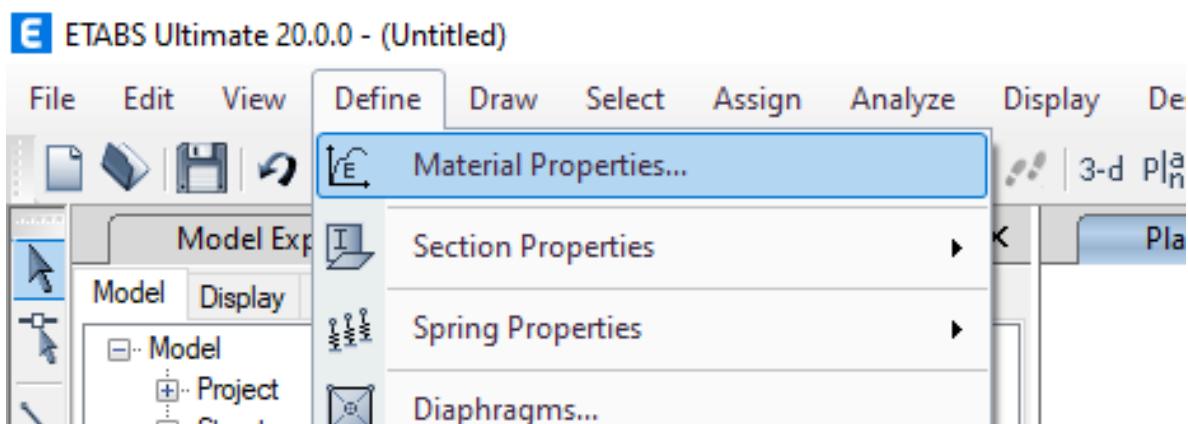
### Step 02:



### Step 03:



### Step 04:



## Step 05:

**E Material Property Data** X

**General Data**

Material Name	Eo-8475
Material Type	Concrete
Directional Symmetry Type	Isotropic
Material Display Color	<span style="background-color: magenta; width: 20px; height: 15px; display: inline-block;"></span> Change...
Material Notes	<a href="#">Modify/Show Notes...</a>

**Material Weight and Mass**

Specify Weight Density       Specify Mass Density

Weight per Unit Volume	150	lb/ft <sup>3</sup>
Mass per Unit Volume	4.662	lb·s <sup>2</sup> /ft <sup>4</sup>

**Mechanical Property Data**

Modulus of Elasticity, E	8475000	lb/in <sup>2</sup>
Poisson's Ratio, U	0.2	
Coefficient of Thermal Expansion, A	0.0000055	1/F
Shear Modulus, G	3531250	lb/in <sup>2</sup>

**Design Property Data**

[Modify/Show Material Property Design Data...](#)

**Advanced Material Property Data**

[Nonlinear Material Data...](#)      [Material Damping Properties...](#)

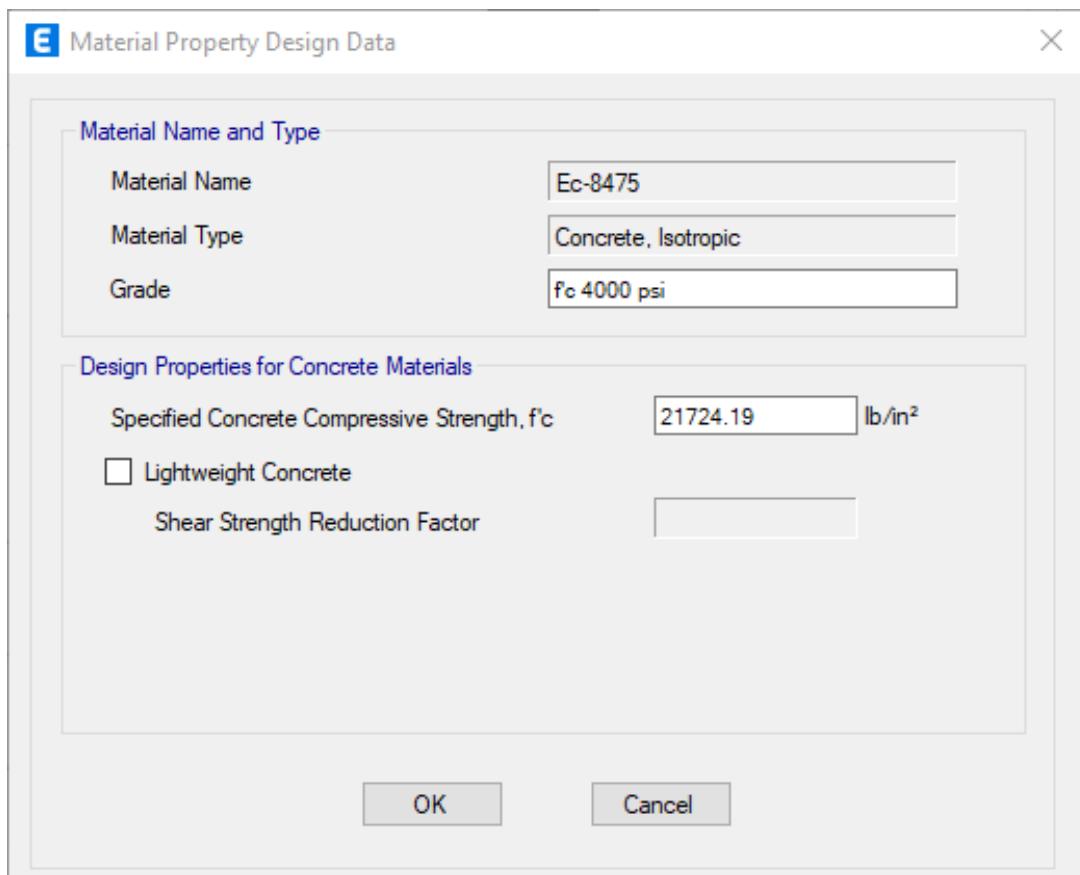
[Time Dependent Properties...](#)

**Modulus of Rupture for Cracked Deflections**

Program Default (Based on Concrete Slab Design Code)       User Specified

[OK](#) [Cancel](#)

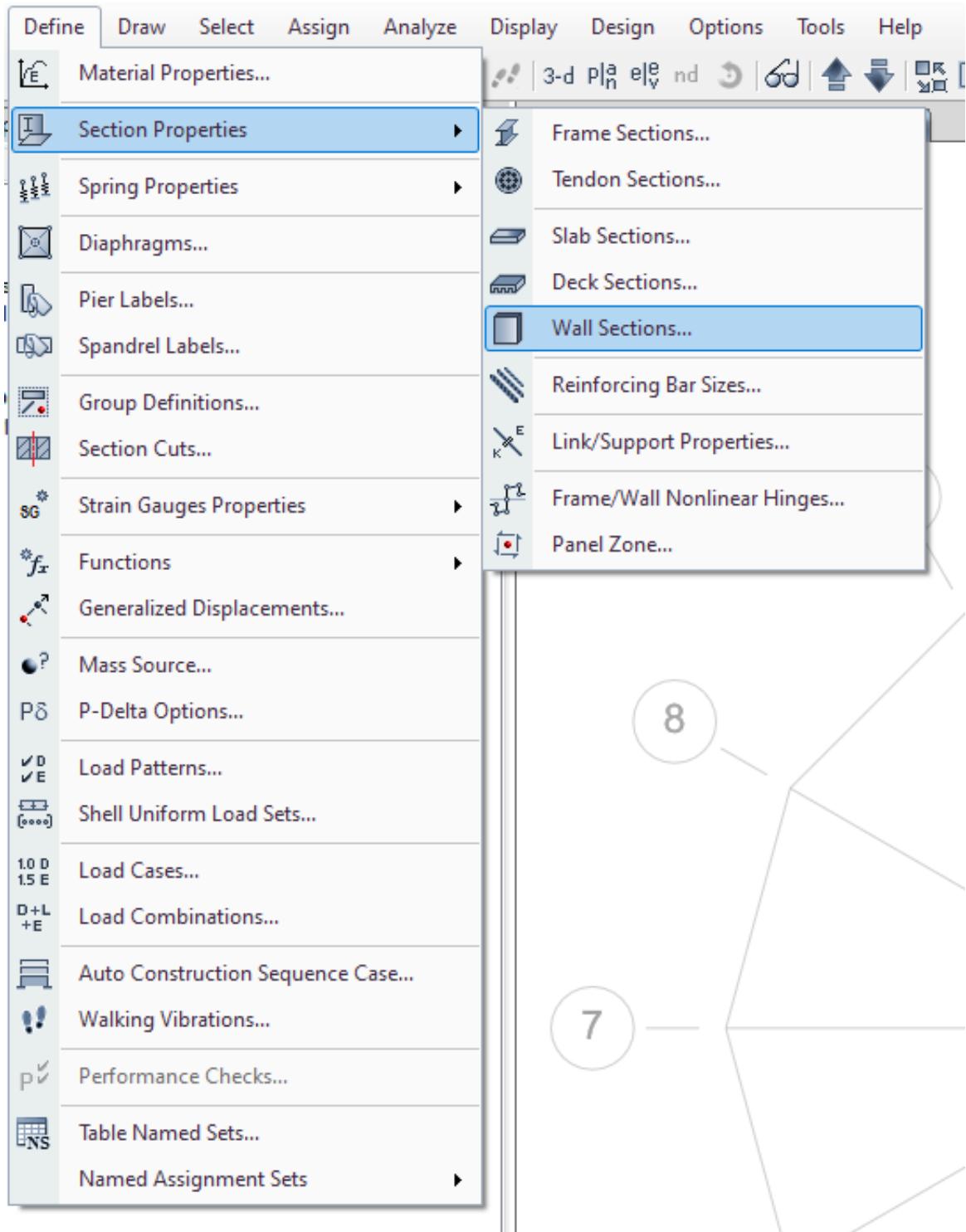
## Step 06:



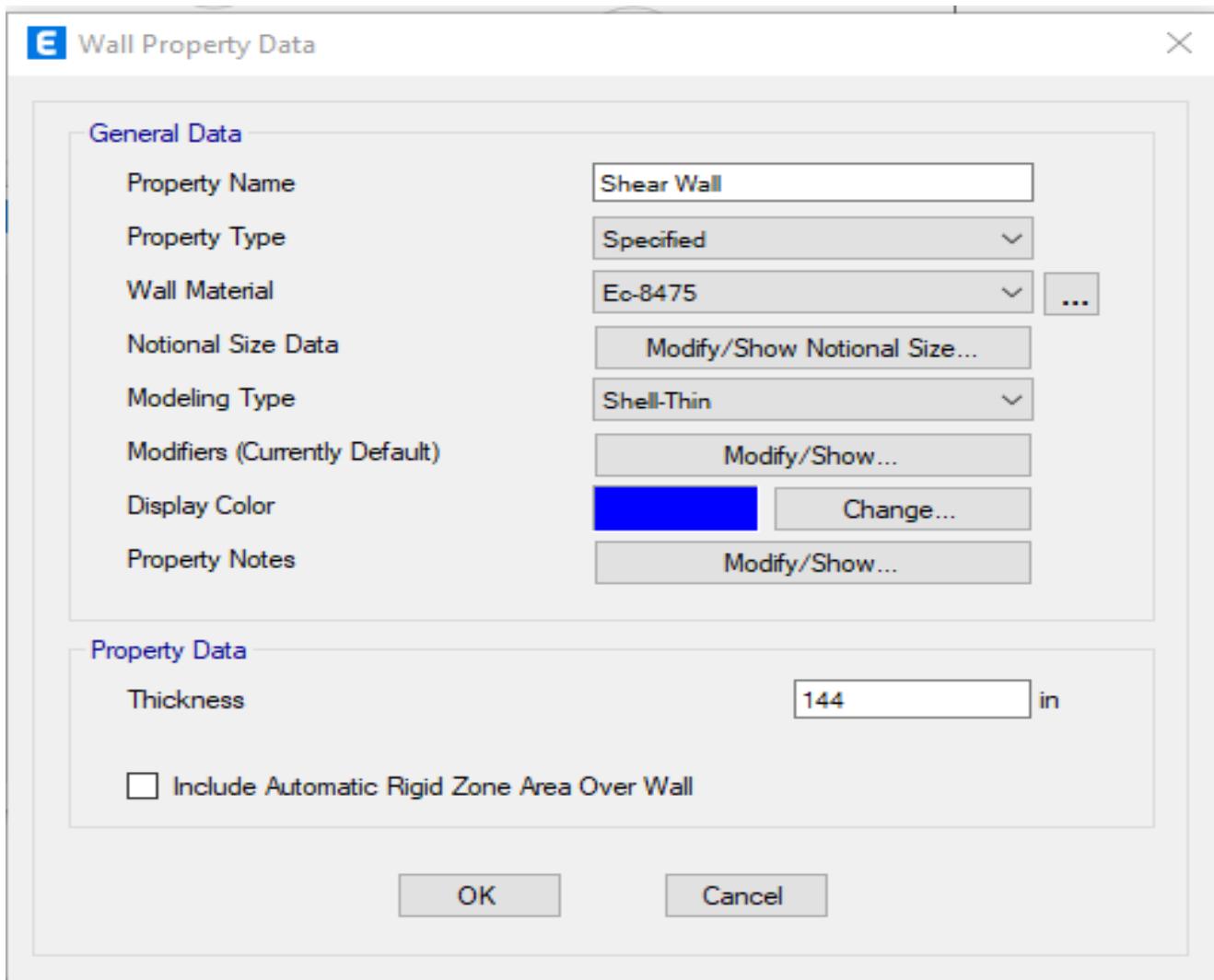
$$8475 * 1000 = 57500 \sqrt{f'c}$$

$$\Rightarrow f'c = 21724.19 \frac{lb}{lb^2}$$

## Step 07:

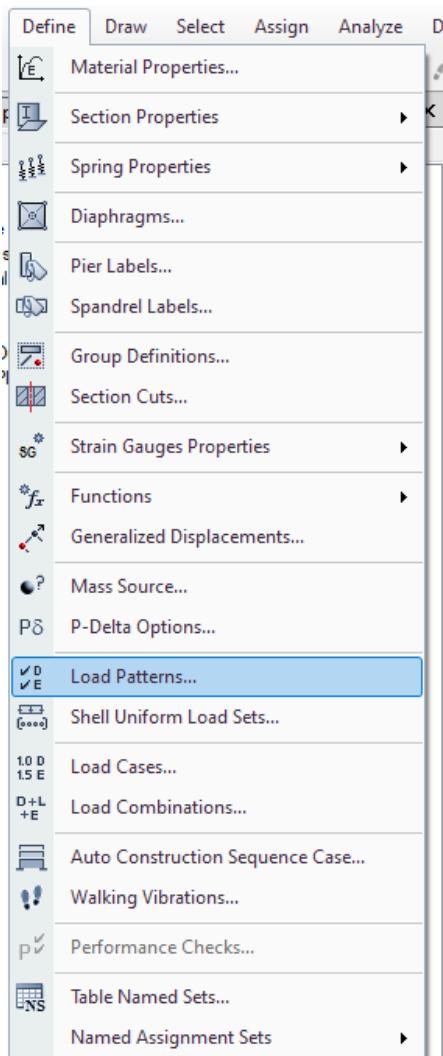


## Step 08:

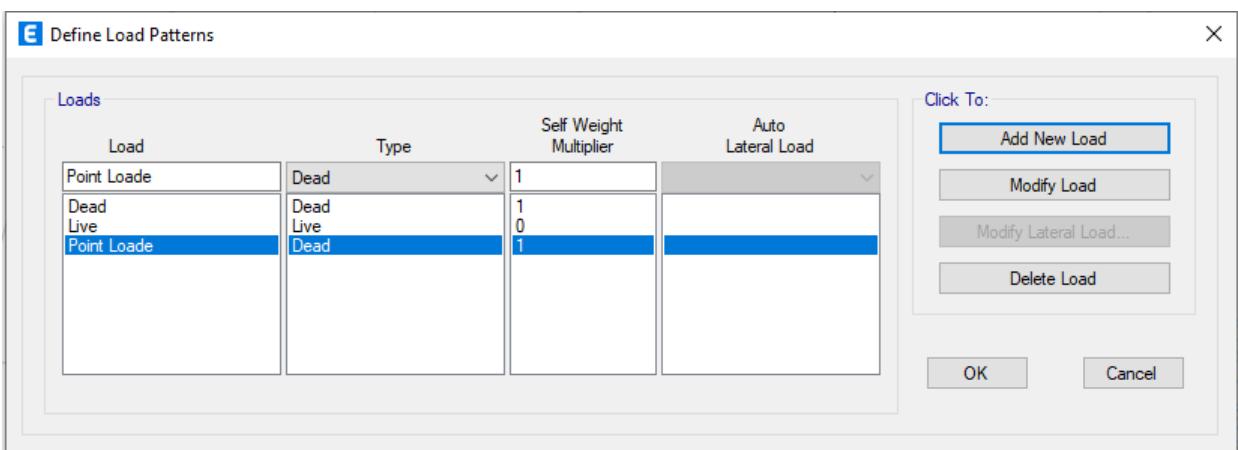


$$12 \times 12 = 144$$

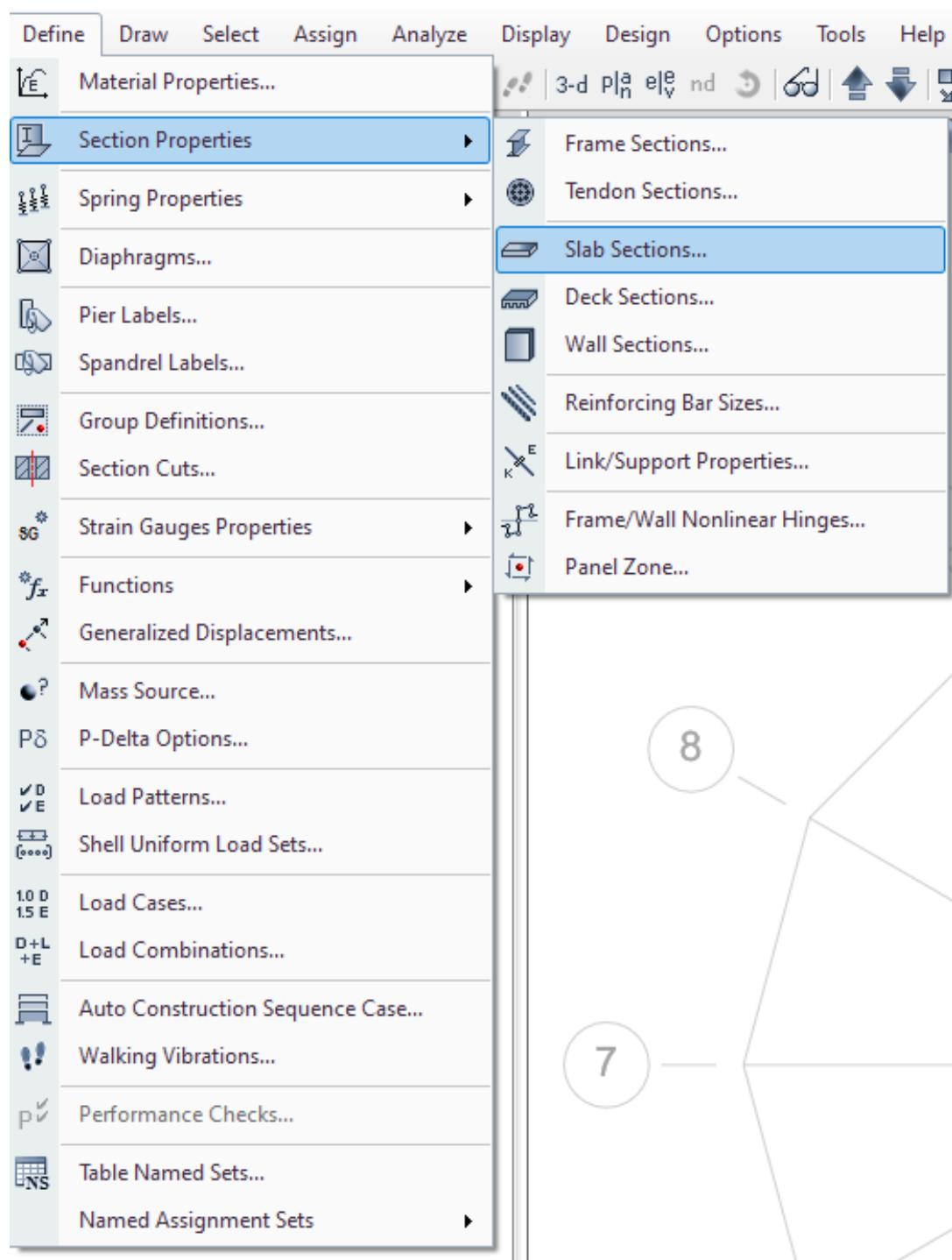
## Step 09:



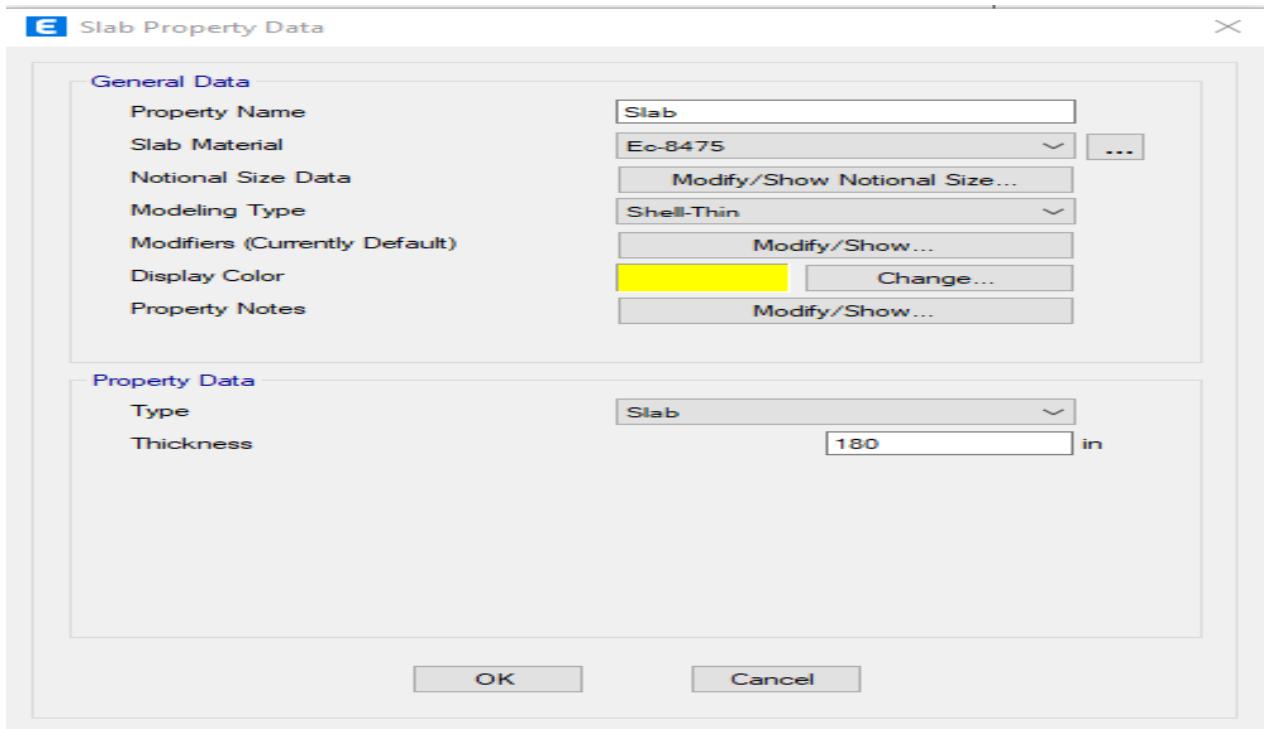
## Step 10:



## Step 11:

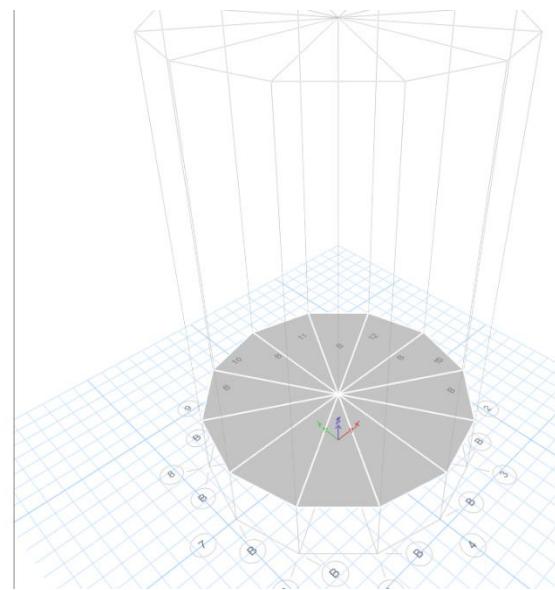
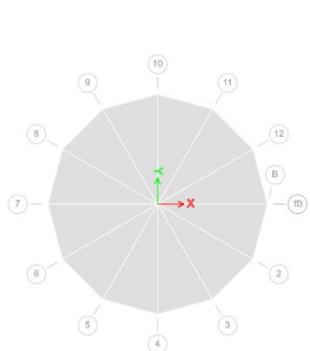


## Step 12:

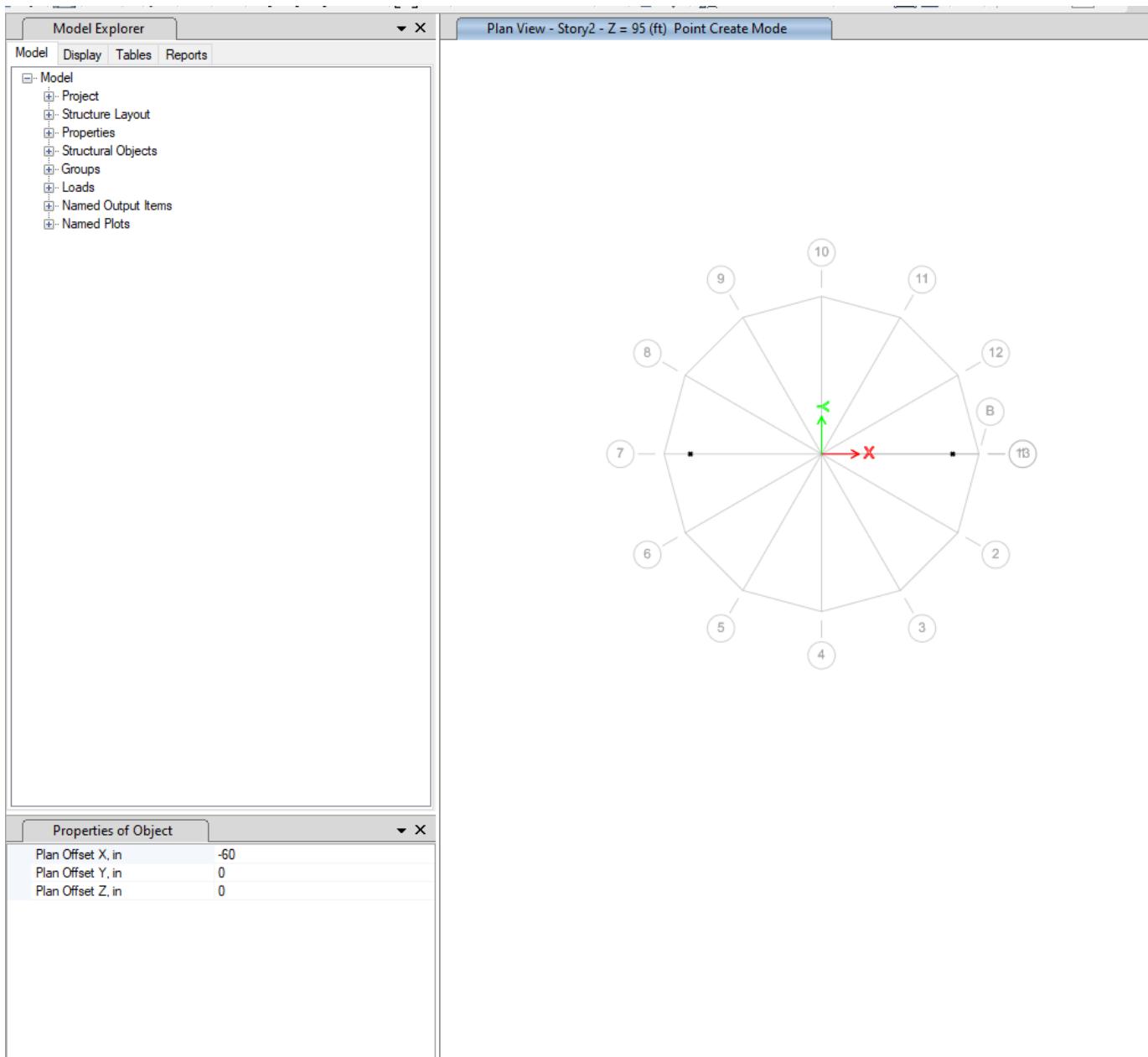


$$15 * 12 = 180$$

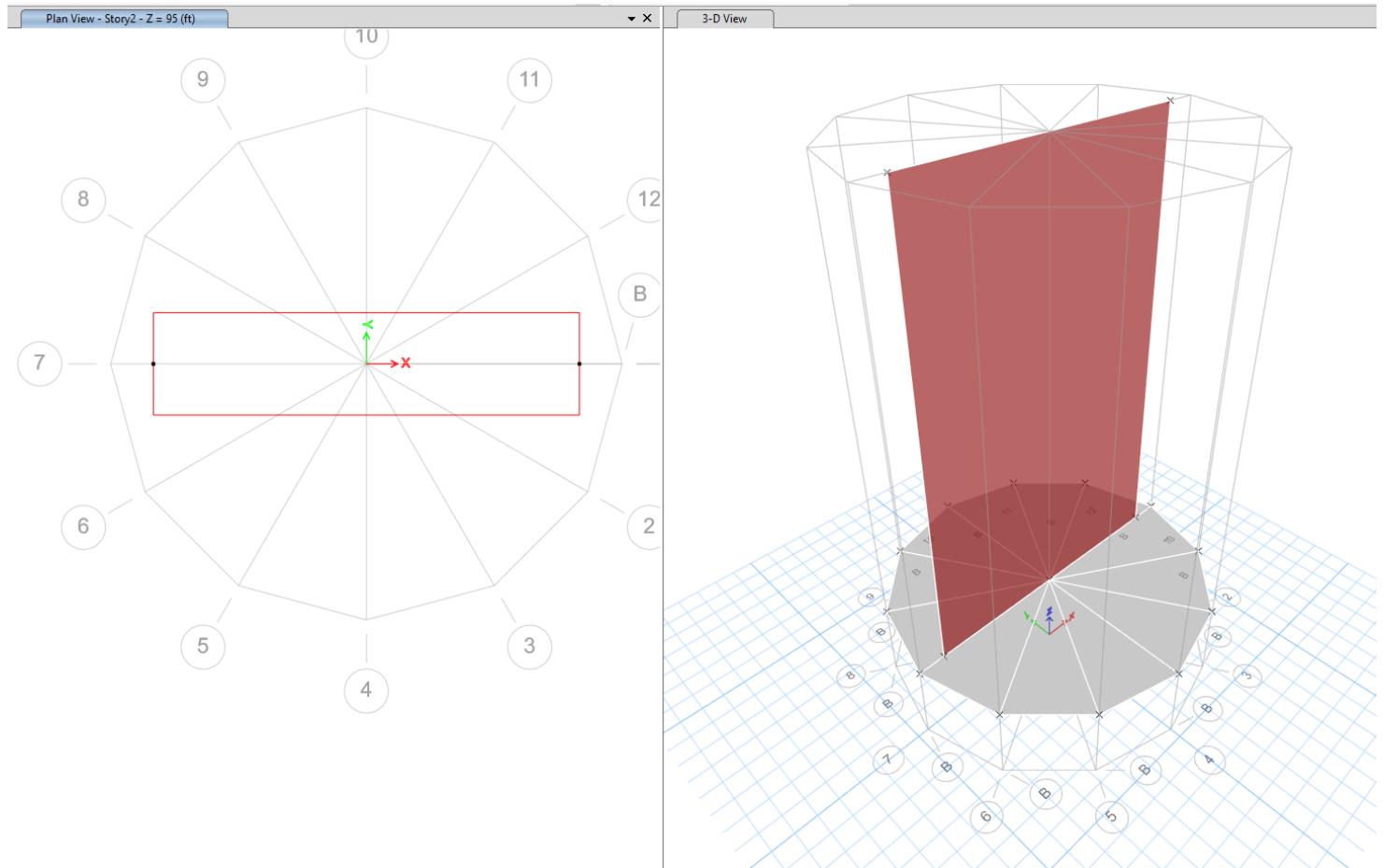
## Step 13:



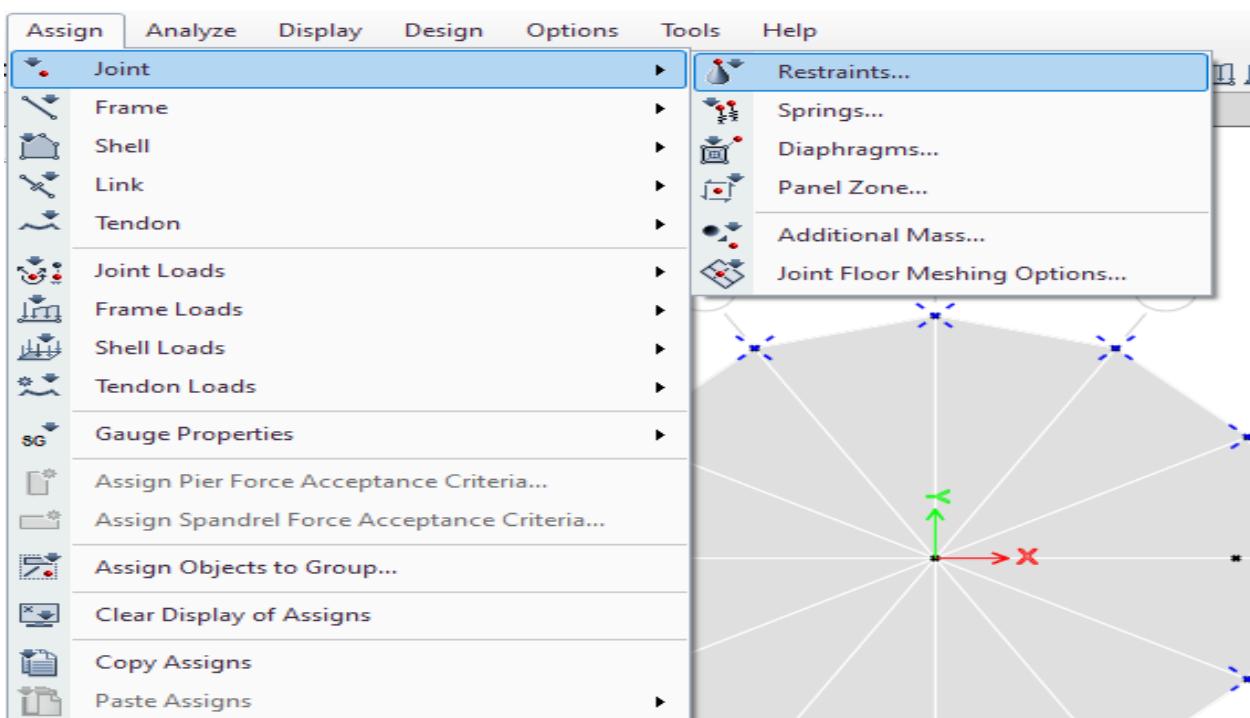
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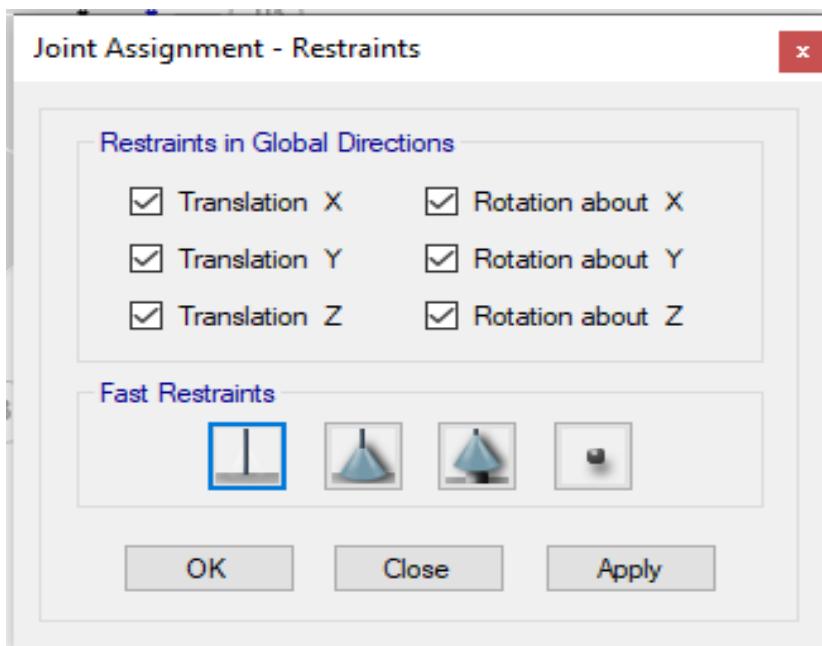
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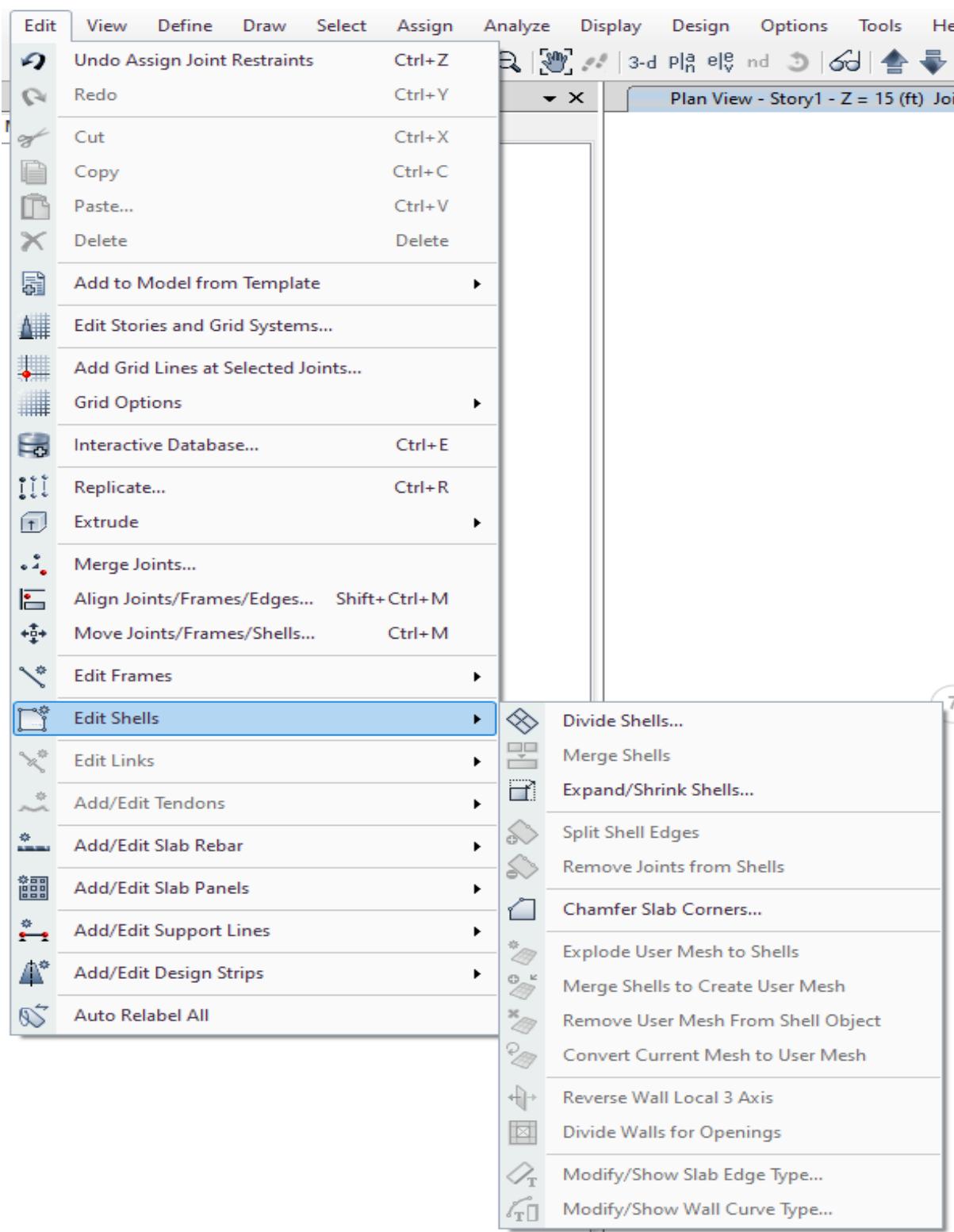
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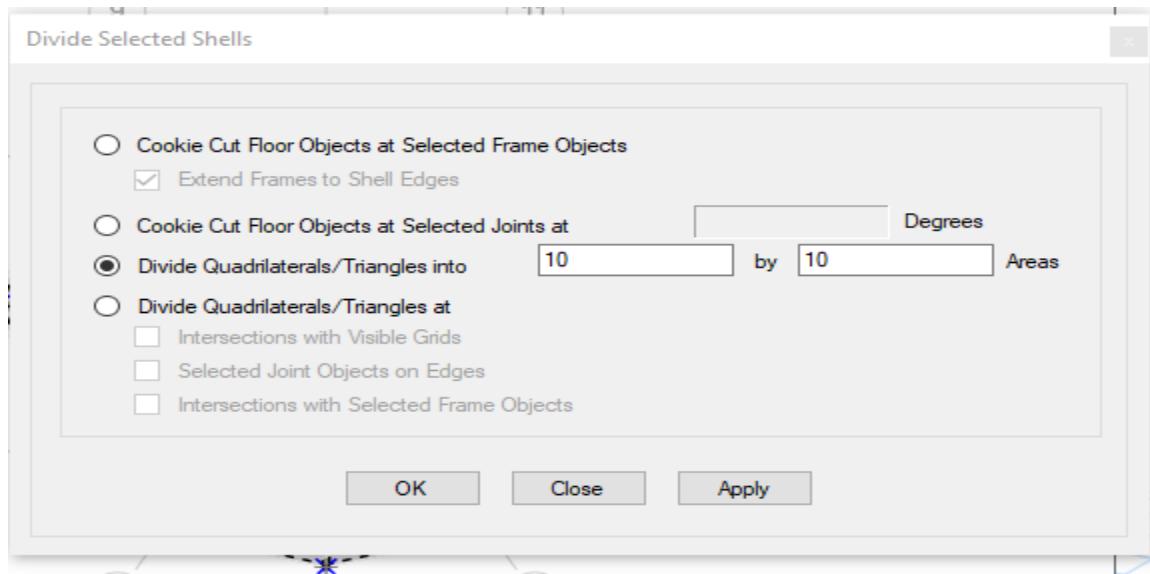
## Step 17



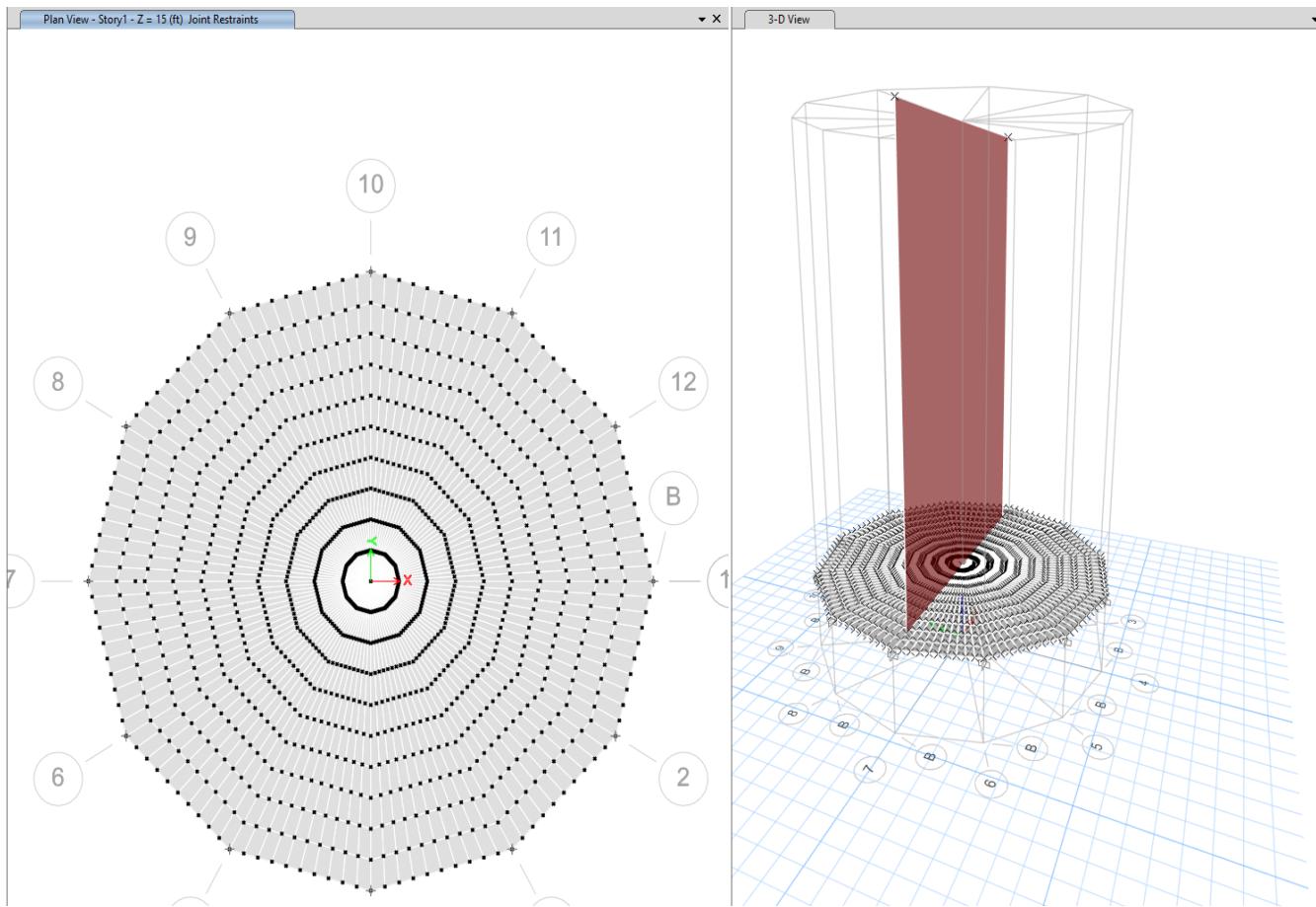
## Step 18:



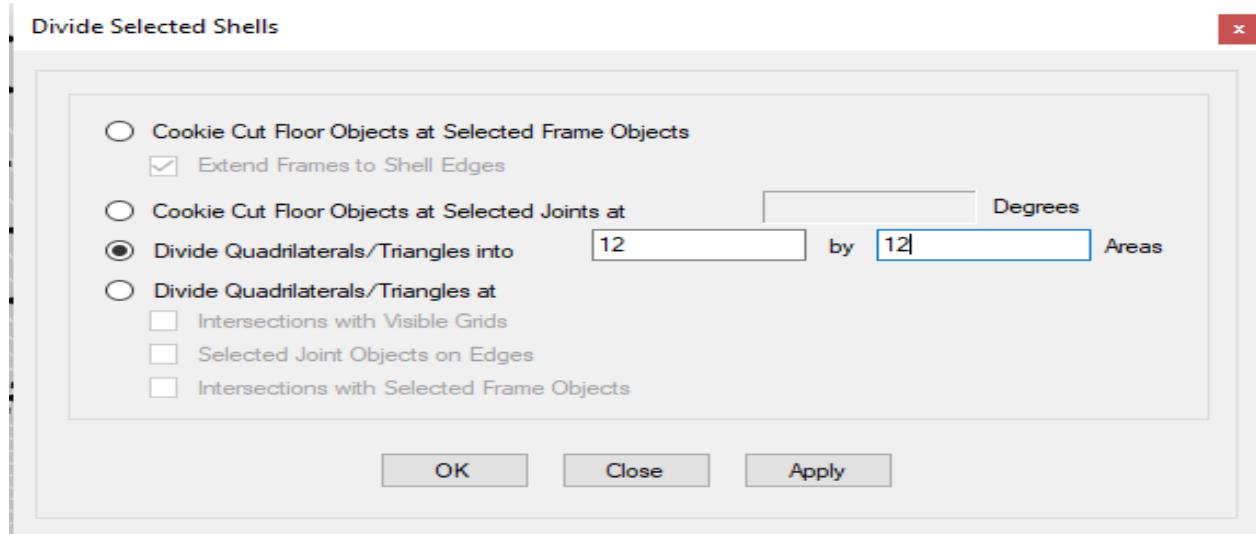
## Step 19:



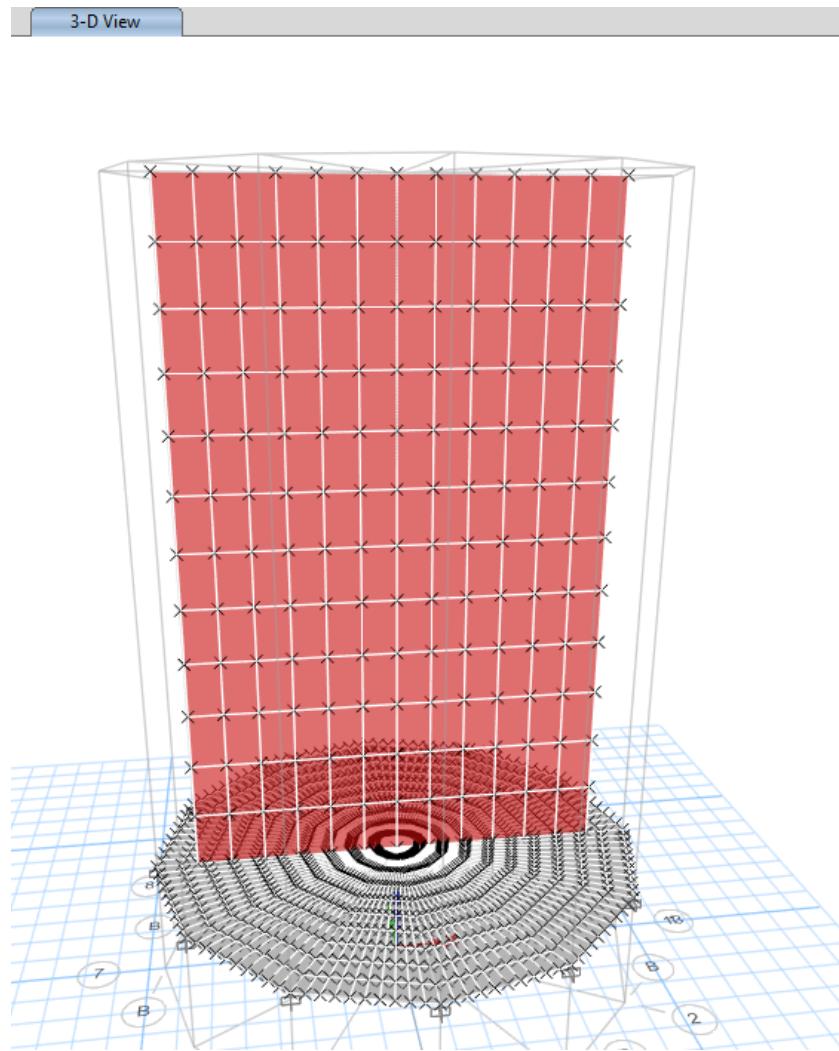
## Step 20:



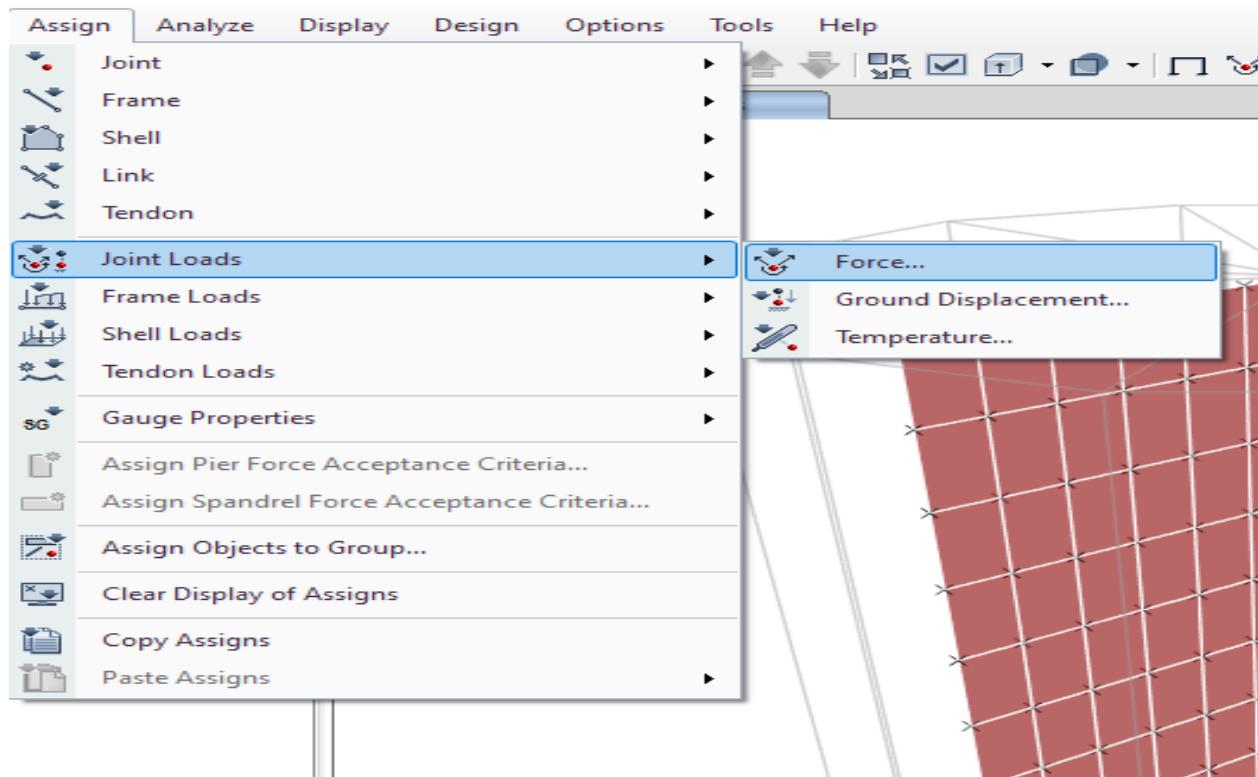
## Step 21:



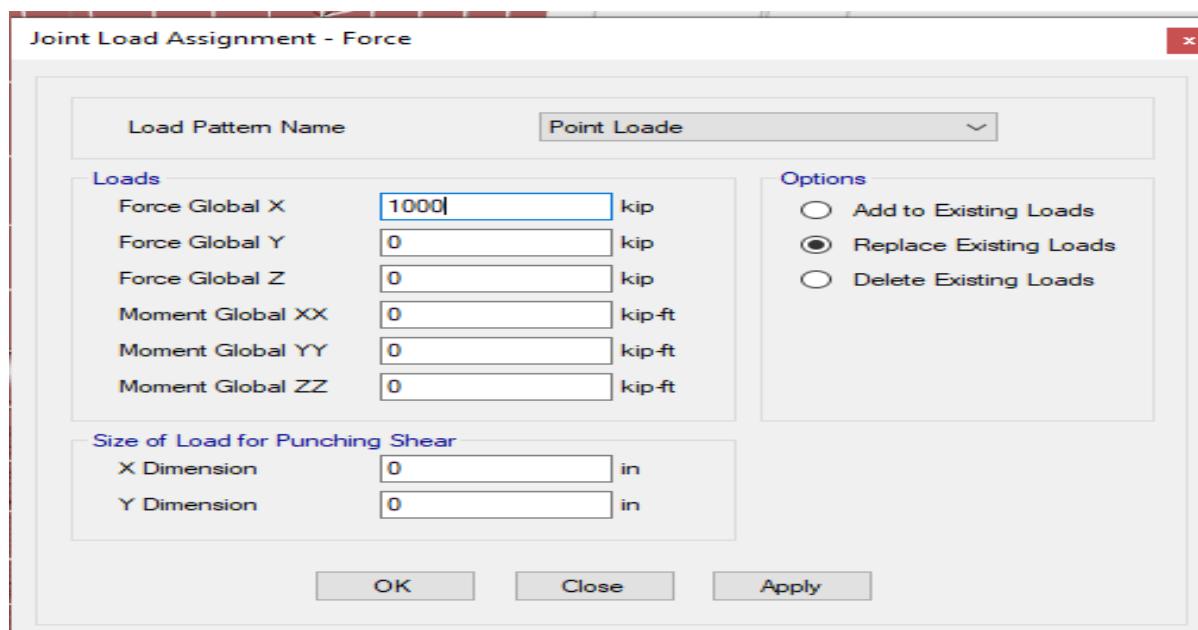
## Step 22:



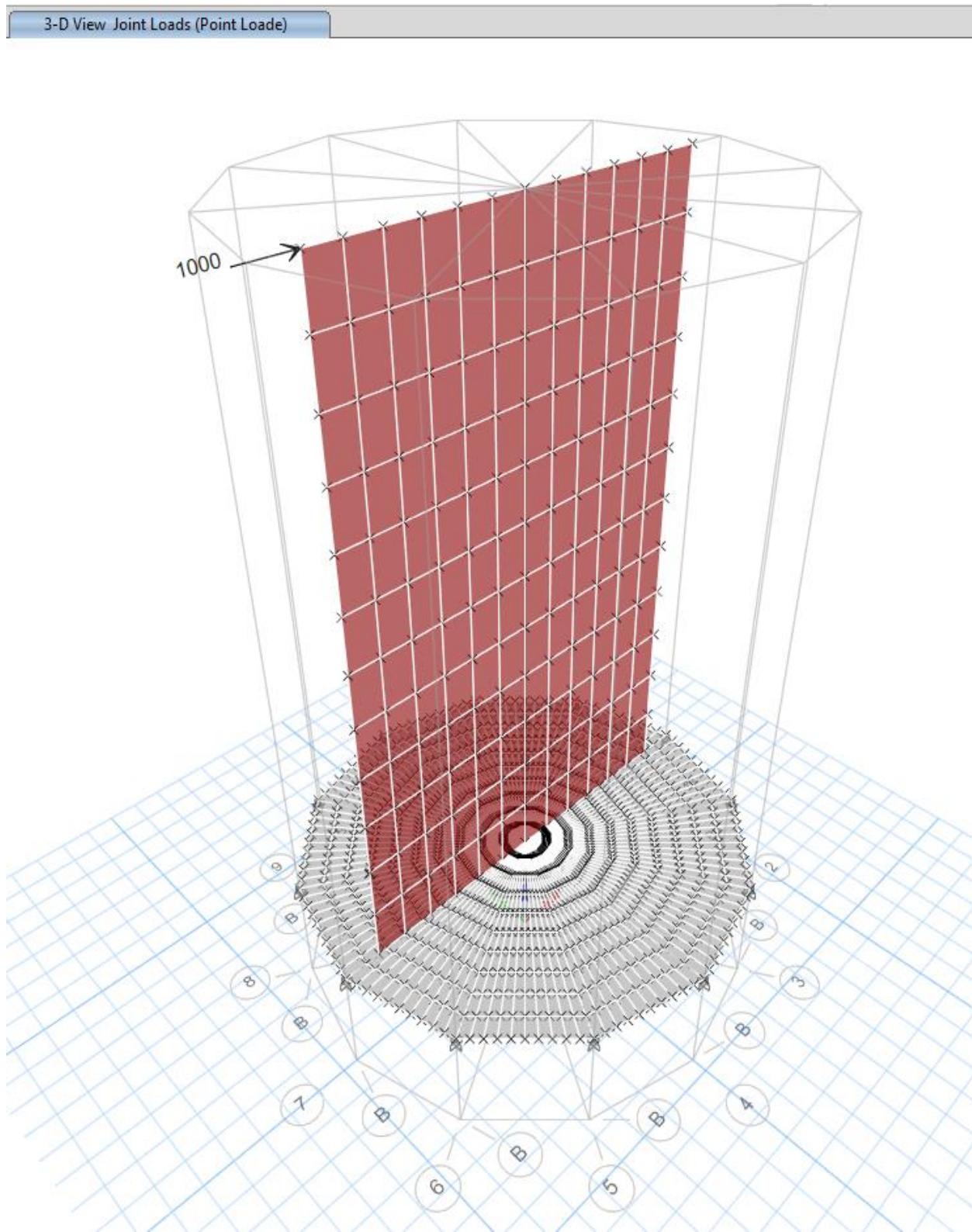
## Step 23:



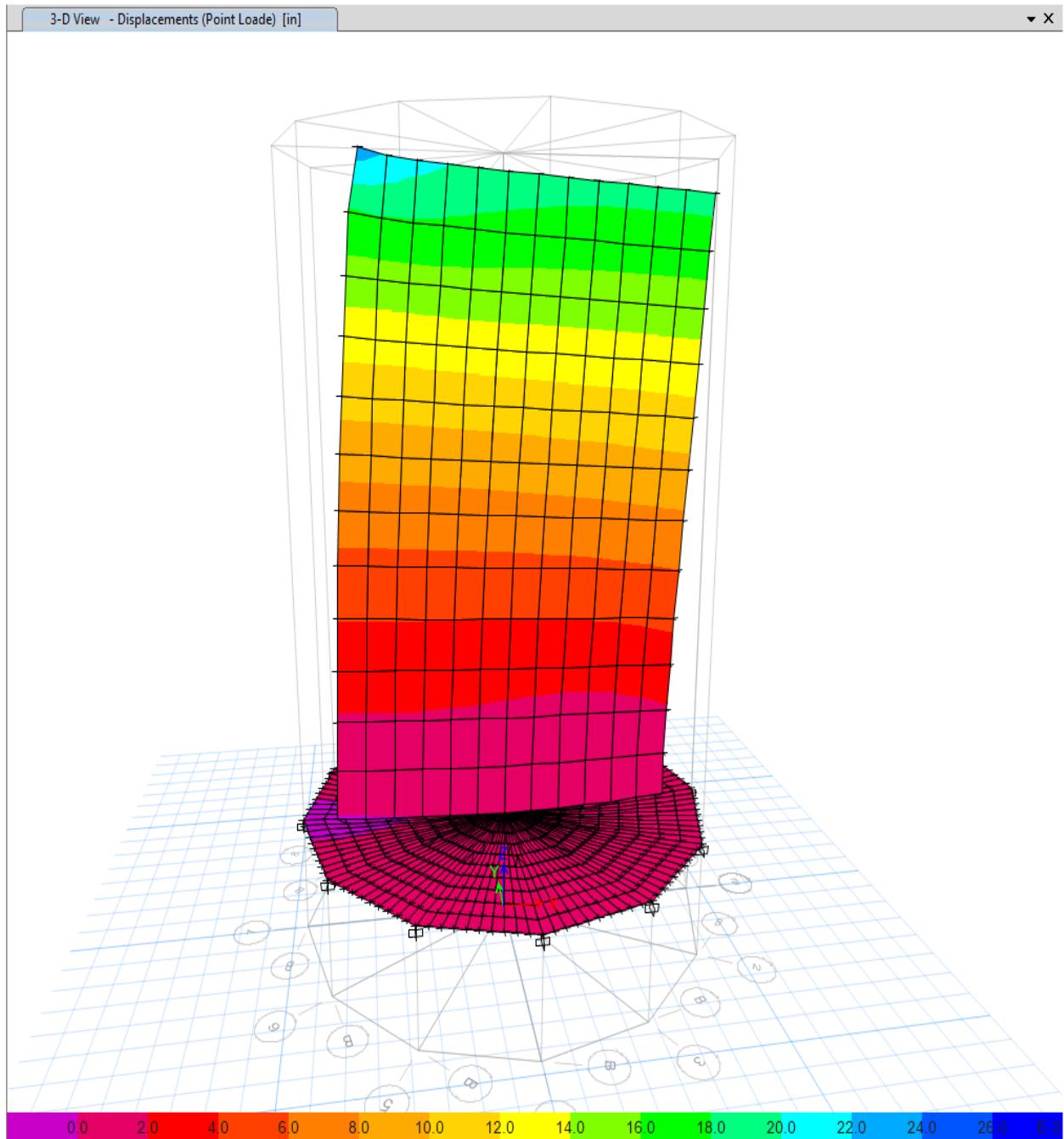
## Step 24:



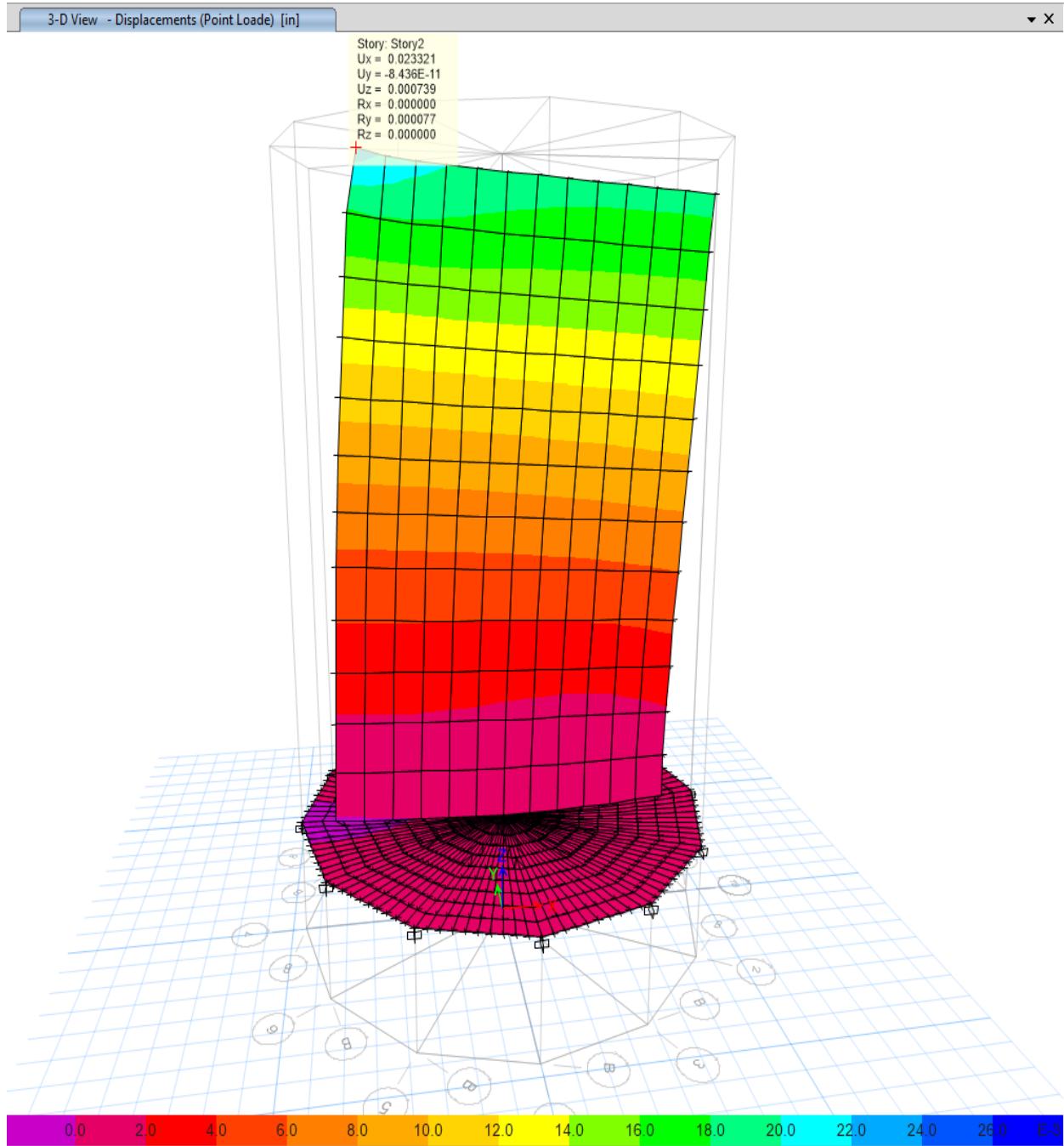
## Step 25:



## Step 26:



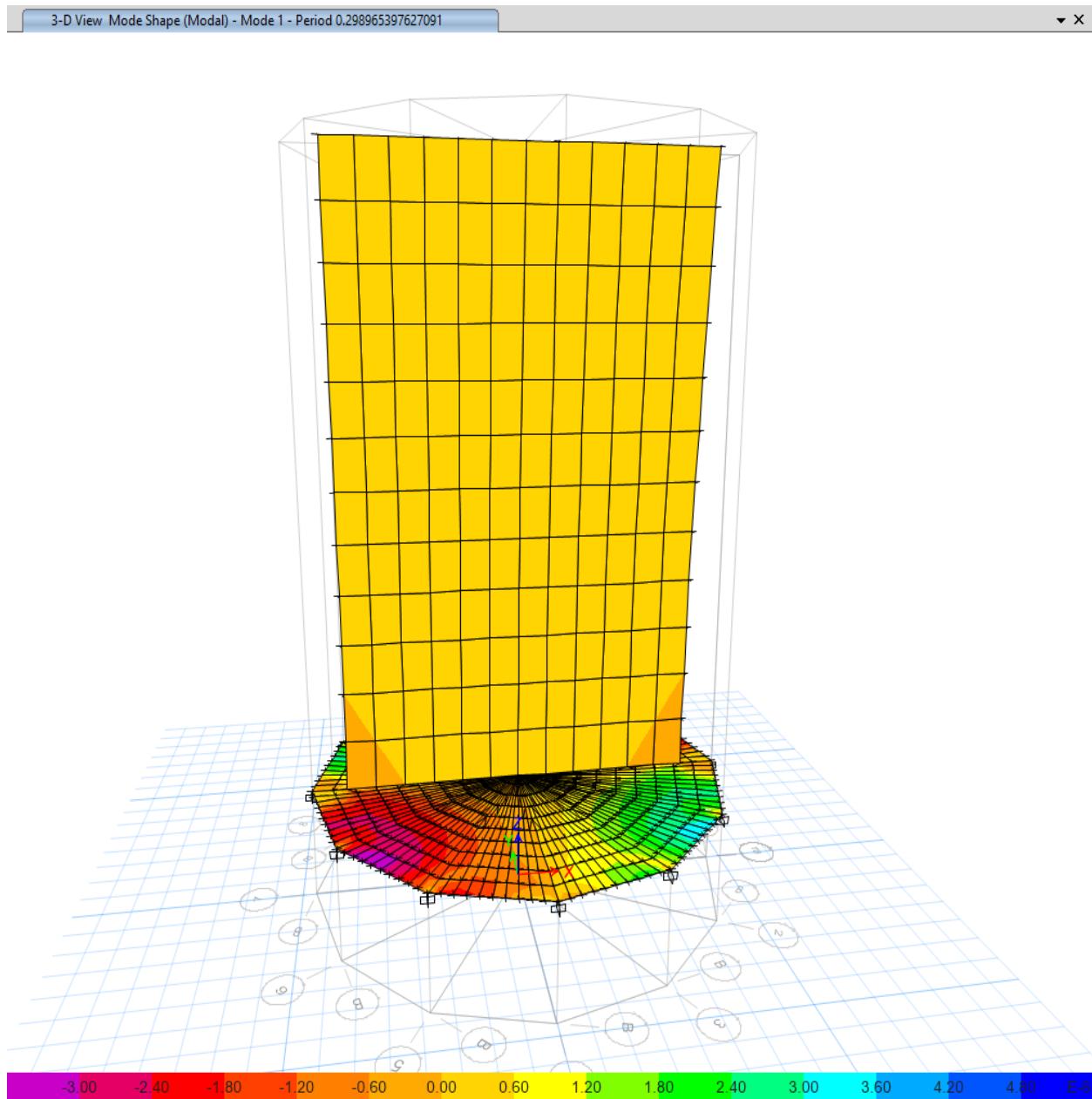
## Step 27:



$$\text{Stiffness} = \frac{1000}{0.023321} = 42.87 * 10^3 \text{ Kip/ft}$$

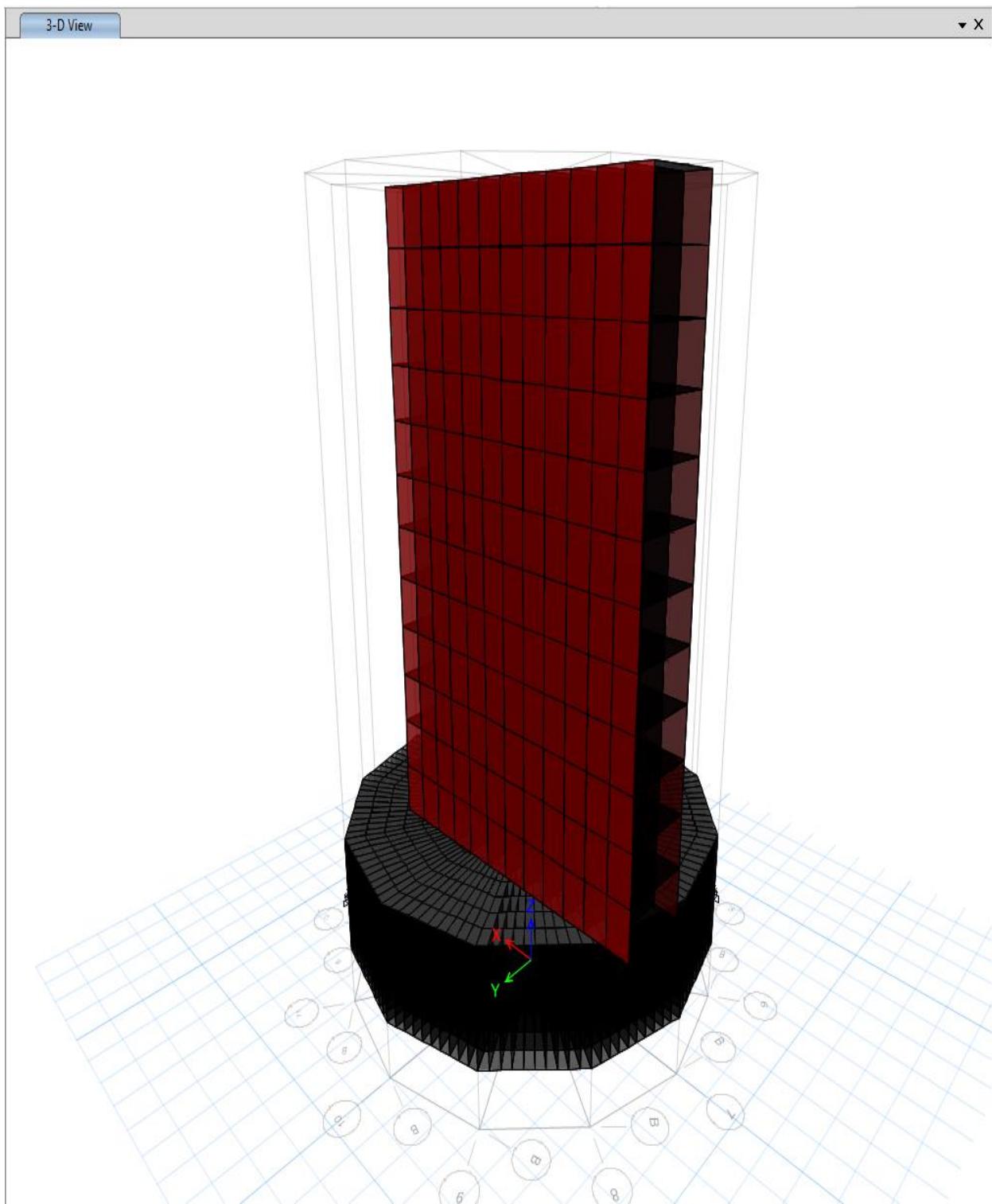
## Steps From Etabs (Project-02)

### Step 01:



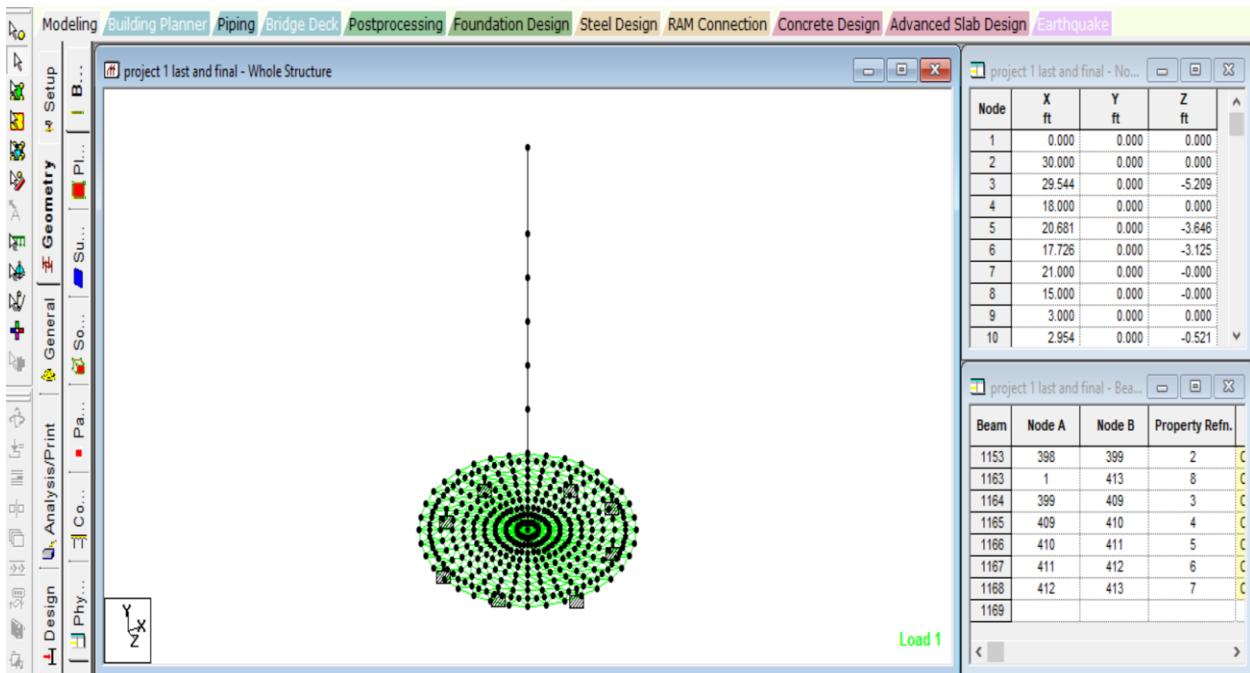
$$\text{Frequency} = \frac{2\pi}{0.299} = 21.01 \text{ rad/sec}$$

### 3D View:



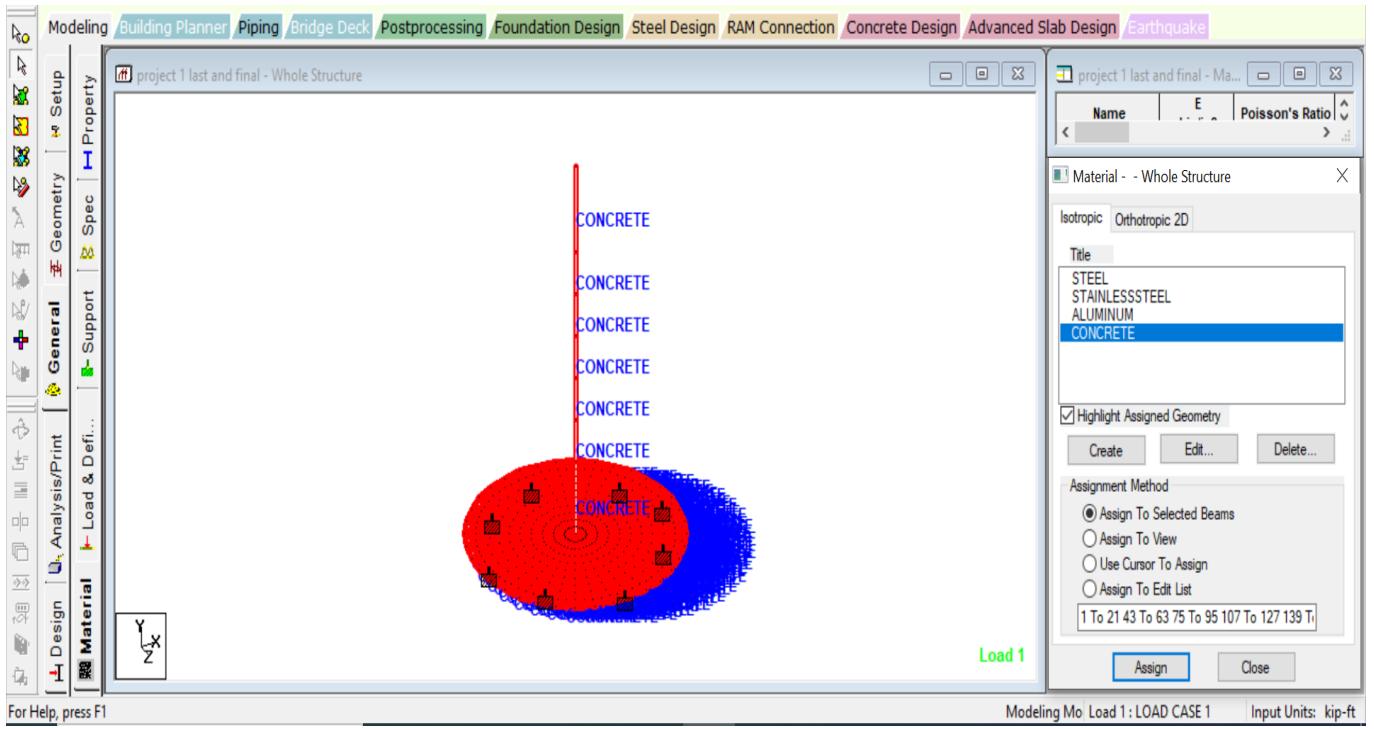
## Steps From Staad Pro (Project-01)

### Step 01:



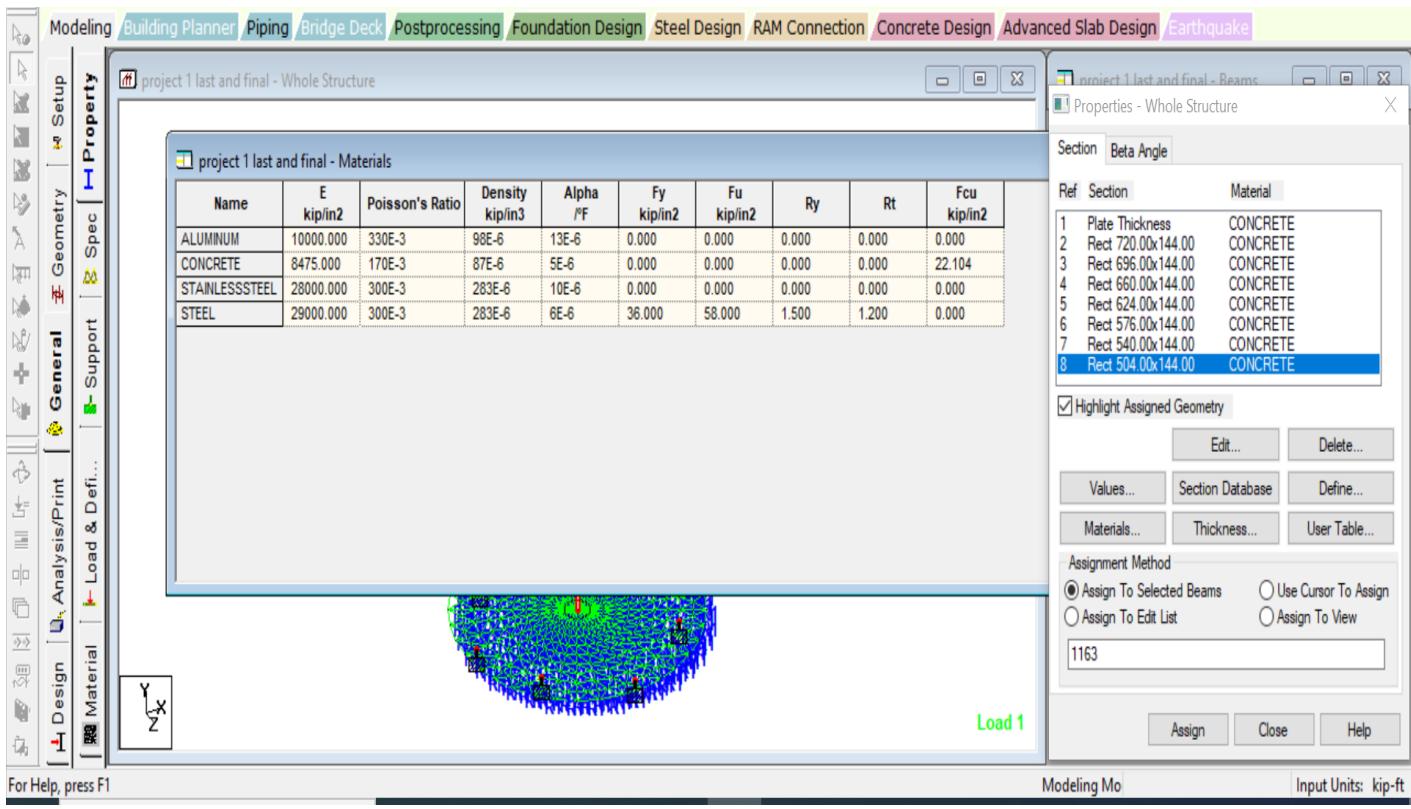
Geometric Property Including 413 nodes

## Step 02:



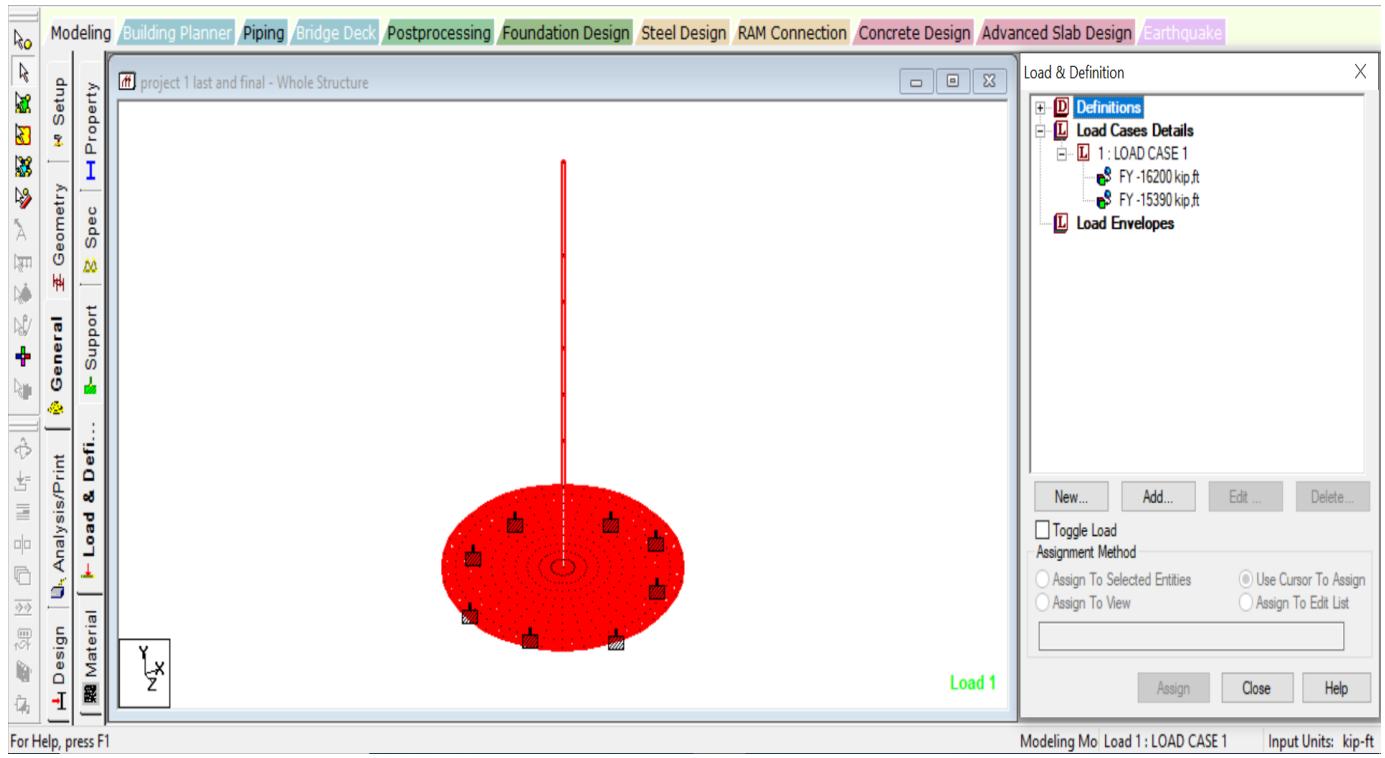
Using Concrete as material other materials were not included.

## Step 03:



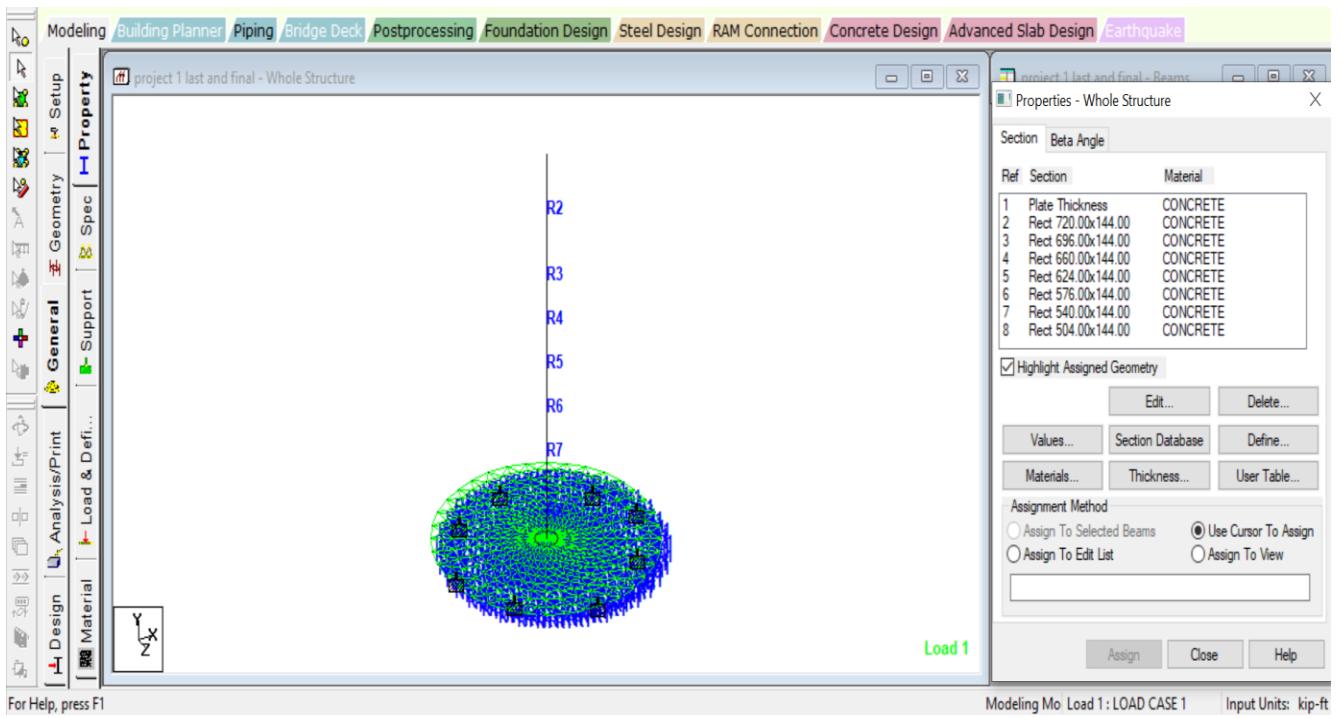
Modulus of Elasticity of Concrete Assignee= 8475 kip/in<sup>2</sup>

## Step 04:



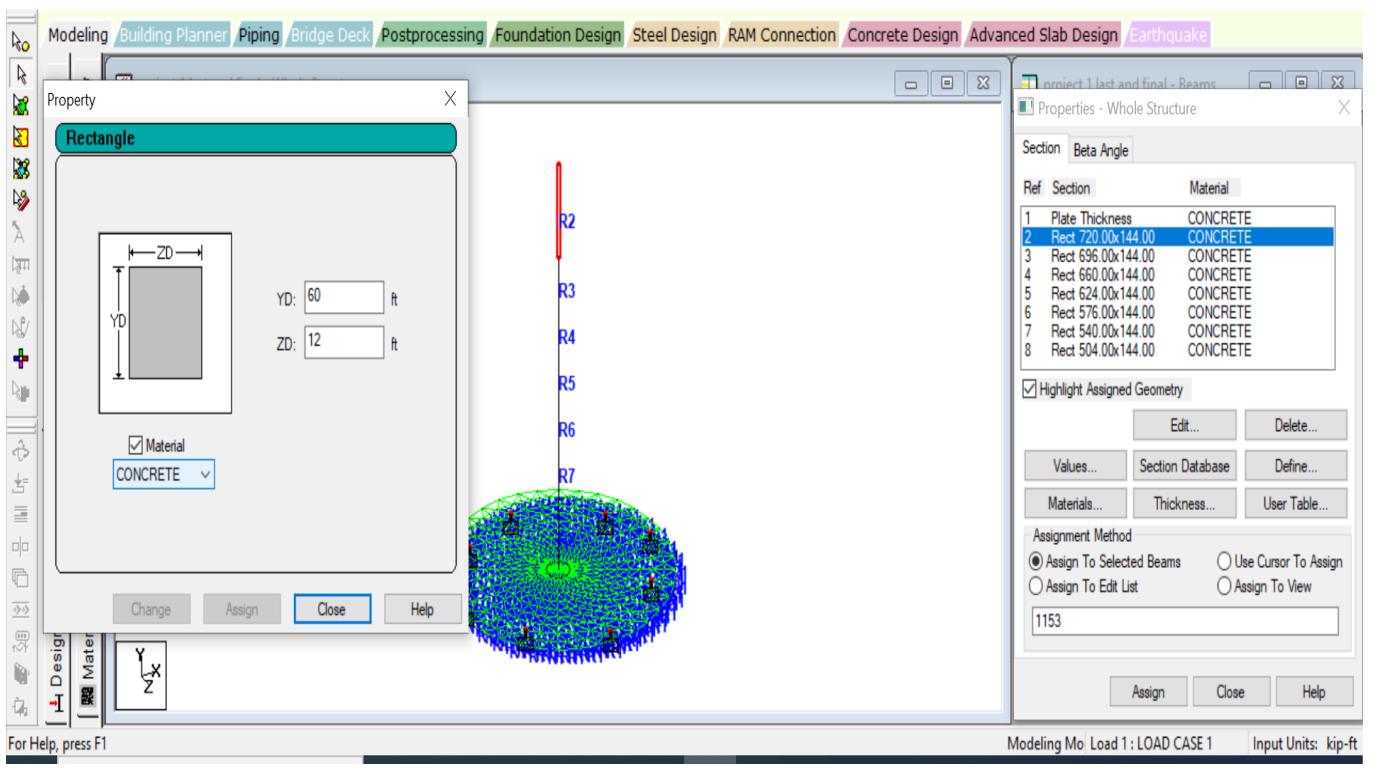
*Two Load was assigned 16200 kip/ft (slab weight) and 15490 kip/ft (Pile cap weight)*

## Step 05:



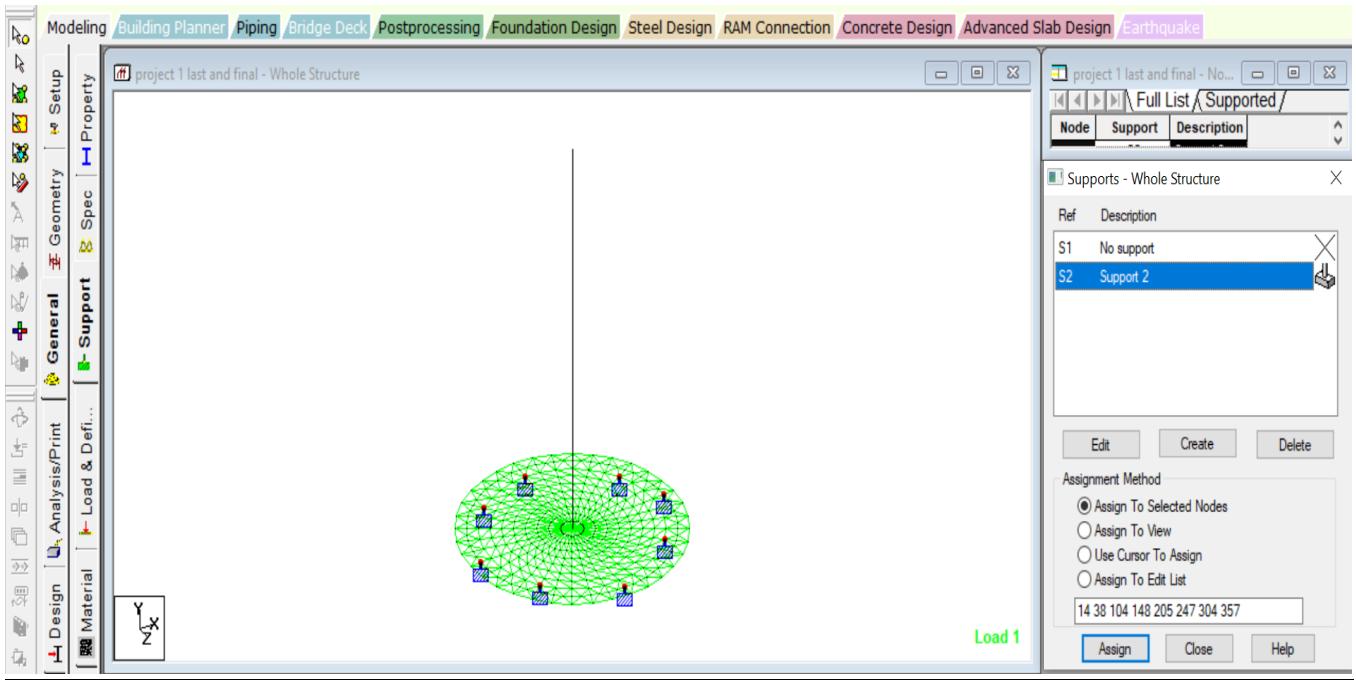
Different Plate Thickness as pile was a variable section.

## Step 06:



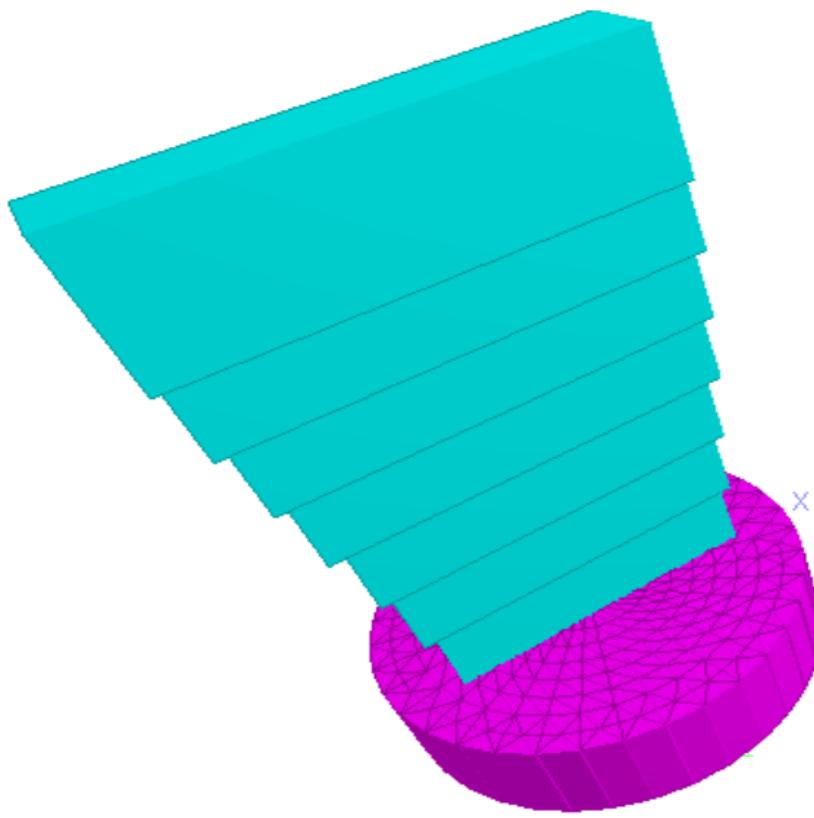
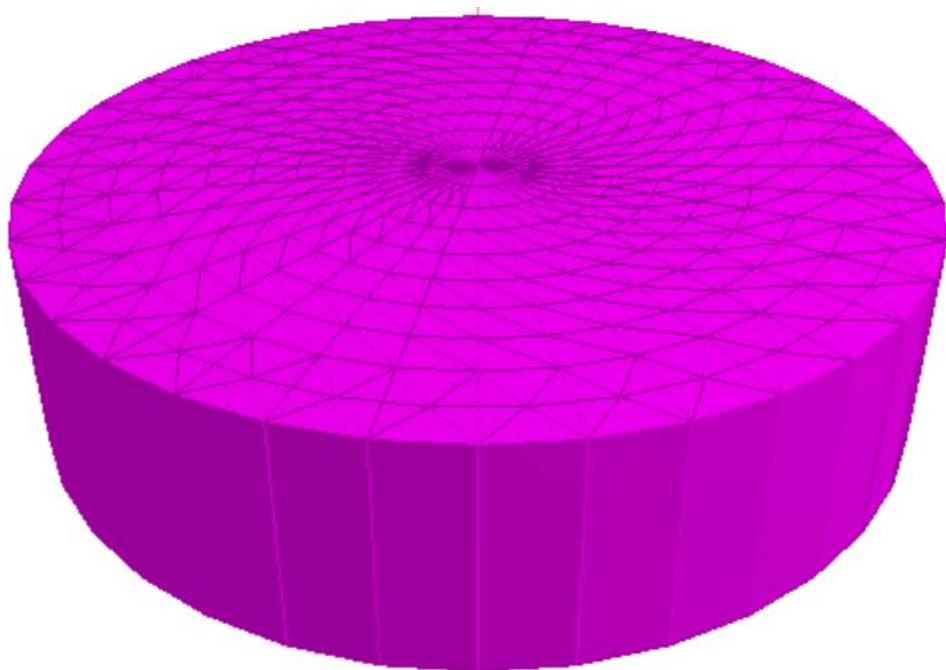
Pile cross section (60\*12)

## Step 07:

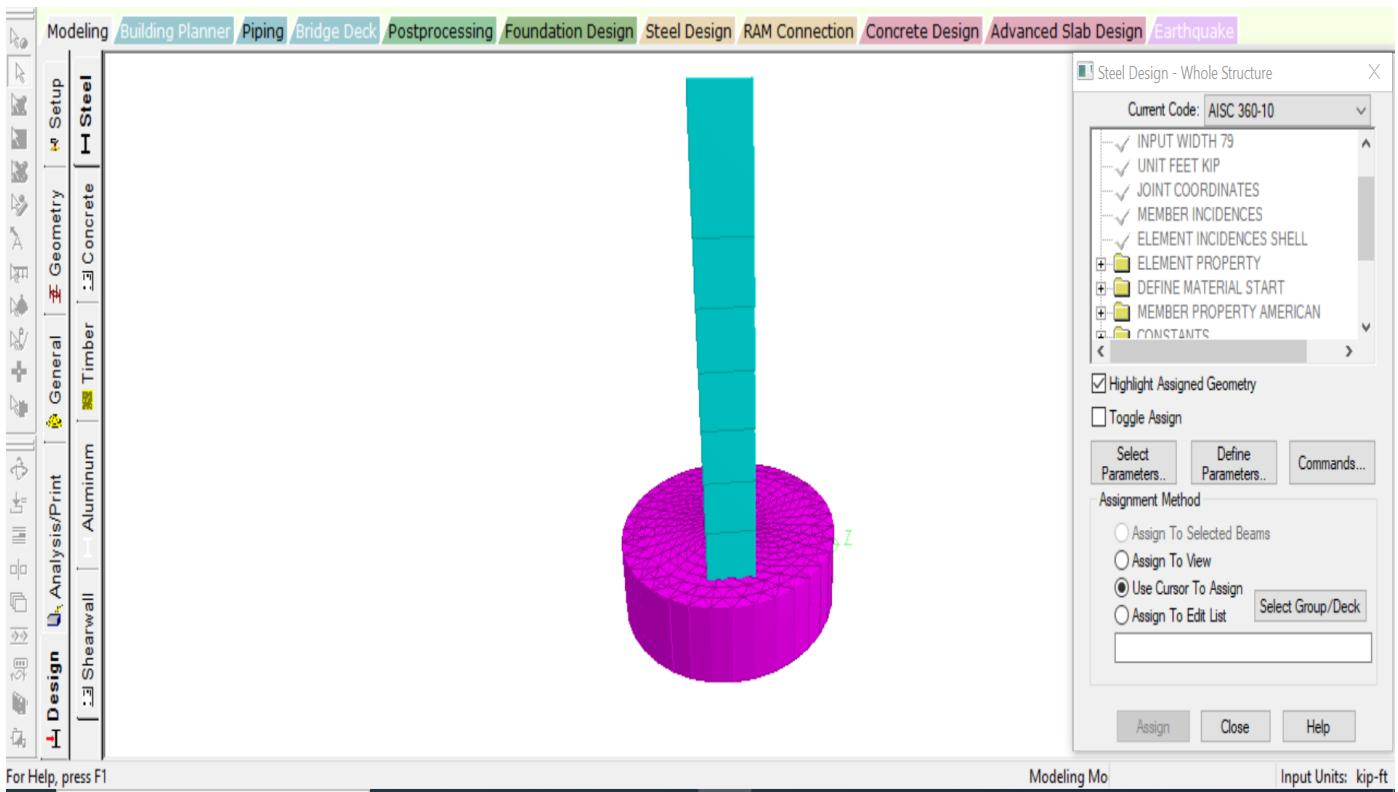


8 Fixed Support was included in pile cap section.

3D View:



## 3D View:



## Analysis Data:

```

****TOTAL APPLIED LOAD ( KIP FEET ) SUMMARY (LOADING 1 )
    SUMMATION FORCE-X =          0.00
    SUMMATION FORCE-Y =      -31590.00
    SUMMATION FORCE-Z =          0.00

SUMMATION OF MOMENTS AROUND THE ORIGIN-
MX= 0.0000000E+00 MY= 0.0000000E+00 MZ= 0.0000000E+00

****TOTAL REACTION LOAD( KIP FEET ) SUMMARY (LOADING 1 )
    SUMMATION FORCE-X =          0.00
    SUMMATION FORCE-Y =      31590.00
    SUMMATION FORCE-Z =          0.00

SUMMATION OF MOMENTS AROUND THE ORIGIN-
MX= -1.9402554E-10 MY= -6.5872507E-13 MZ= 1.2417635E-09

MAXIMUM DISPLACEMENTS ( INCH /RADIAN) (LOADING 1)
    MAXIMUMS AT NODE
    X = -1.08819E-03   398
    Y = -9.76744E-02   398
    Z = 2.91341E-03   398
    RX= 8.32207E-05   108
    RY= -2.30287E-21   319
    RZ= 8.25163E-05    17

```

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Maximum Horizontal Displacement was,  
 $2.9134 \times 10^{-3}$  inch.

And Maximum Force was, 31590 kip.

So,

$$\text{Horizontal Stiffness} = \frac{f}{u} = \frac{31590 * 12}{2.9134 * 10^{-3}} = 1.3 * 10^8 \text{ k/ft}$$

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### *Result Comparison (project-01)*

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The horizontal stiffness from manual calculation is  $1.257 * 10^{10}$  k/ft, but using Q4 elements in Etabs it is  $42.87 * 10^3$  k/ft and using Q4 Element in Staad pro it is  $1.3 * 10^8$  k/ft.

The Variation is huge because in Etabs we do not include Axial forces (self-weight of span). Only a horizontal force of 1000 k has been added because of the limitation of Etabs.

On the other hand, Staad pro cannot mesh properly a vertical member (pile). Inaccuracy of shape function can differ the result also.

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## ***Result Comparison (Project-02)***

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The natural frequency for manual calculation is 7017.15 rad/s and Natural frequency for Etabs is 21.01 rad/s. Unfortunately, we could not calculate natural frequency for Staad pro. Without any Earthquake data it is not possible to calculate natural frequency in Staad pro.

In Etabs Horizontal stiffness was different than manual calculation so, inaccuracy occurs.

Moreover, mass calculation in Etabs was not accurate as self-weight of Pile cap was not assigned in Etabs.