1. Exercise sheet

Due by: Friday, 19 April 2024, 23:59 (CEST)
Please refer to **Assignment Submission Guideline** on Moodle

Problem 1. In this exercise, you are asked to implement the forward Euler scheme to simulate nonlinear dynamics. Consider the *Van der Pol* oscillator, given by

$$\begin{aligned}\dot{x}(t) &= y(t)\\ \dot{y}(t) &= -x(t) + \big(1 - (x(t))^2\big)y(t)\end{aligned}$$

from initial value $x(0) = y(0) = 0.1$. Implement the forward Euler scheme for this system up to time $t = 30$ with uniform time discretisation.

- Plot $(x(t), y(t))$ for $0 \leq t \leq 30$ with $\Delta t = 0.01$.
- Change the time step to $\Delta t = 0.2$ and $\Delta t = 0.3$, then plot the results.
- **(Optional, not to be marked)** Play further with the time step until it finds numerical instability. Also simulate the oscillator with different initial values.

Problem 2. In this exercise, the nonlinear term is removed to obtain an analytic solution. Consider the harmonic oscillator:

$$\begin{aligned}\dot{x}(t) &= y(t)\\ \dot{y}(t) &= -x(t).\end{aligned}$$

- Simulate the harmonic oscillator from initial value $x(0) = y(0) = 1.0$ using the forward Euler scheme with $\Delta t = 0.01$. Obtain the values of $y(5)$ and $y(10)$.
- Given the initial condition $x(0) = x_0$ and $y(0) = y_0$, the solution $y(t)$ is given by
$$y(t) = -x_0 \sin(t) + y_0 \cos(t).$$
Solve x_0 and y_0 from the values of $y(5)$ and $y(10)$ you obtained in the previous step. Is it reasonably close to the true value?
- Increase the time step Δt to 0.1 and repeat the procedure. How is the result different from the previous one?
- **(Optional, not to be marked)** Play with the time step. Decrease Δt towards zero and plot the relationship of Δt vs. the error $|x_0 - 1|$.