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Exercise 8

Problem 2.

Given the measurement model

$$y = \sqrt{x_1^2 + x_2^2} + n \qquad n \sim N(0, 0.1^2)$$

where $y \in \mathbb{R}$ is the measurement, $x = (x_1, x_2)^T \in \mathbb{R}^2$ is the unknown parameter, and $n \in \mathbb{R}$ is observational noise. The prior is given by

$$\pi(x_1, x_2) = C \exp\left(-\frac{1}{2}\left(|x_1 - 1| + (x_2 - 1)^2\right)\right)$$

where C is the normalization constant.

To find posterior distribution $\pi(x|y)$, we use Bayes theorem.

$$\pi(x|y) \propto \pi(y|x)\,\pi(x)$$

The liklihood $\pi(y|x)$ given the noise $n \sim N(0, 0.1^2)$ is

$$\pi(y|x) = \frac{1}{\sqrt{2\pi \cdot 0.1^2}} \exp\left(-\frac{\left(y - \sqrt{x_1^2 + x_2^2}\right)^2}{2 \cdot 0.1^2}\right)$$

Therefore, the posterior distribution is proportional to:

$$\pi(x|y) \propto \exp\left(-\frac{\left(y - \sqrt{x_1^2 + x_2^2}\right)^2}{2 \cdot 0.1^2}\right)$$

$$\exp\left(-\frac{1}{2}\left(|x_1 - 1| + (x_2 - 1)^2\right)\right)$$

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