Sheet 8

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Problem 3(a):

we have given a transition matrix P.

$$P = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$$

To find invariant probability distribution π for the markov chain with P, we need to solve the equation.

$$\pi P = \pi$$

with that, we can setting up the equation,

$$\pi_1 = 1/3\,\pi_1 + 1/3\,\pi_2 + 1/3\,\pi_3$$
$$\pi_2 = 1/3\,\pi_1 + 1/3\,\pi_2 + 1/3\,\pi_3$$
$$\pi_3 = 1/3\,\pi_1 + 1/3\,\pi_2 + 1/3\,\pi_3$$

Simplifying left hand side of each equation.

$$2/3\,\pi_1 = 1/3\,\pi_2 + 1/3\,\pi_3 \quad \text{— (I)}$$
$$2/3\,\pi_2 = 1/3\,\pi_1 + 1/3\,\pi_3 \quad \text{— (II)}$$
$$2/3\,\pi_3 = 1/3\,\pi_1 + 1/3\,\pi_2 \quad \text{— (III)}$$

[subtructing π_i on each left hand side], Example: $\pi_1 - 1/3\,\pi_1 = 1/3\,\pi_2 + 1/3\,\pi_3$

From equation (I),

$$2/3\,\pi_1 = 1/3\,\pi_2 + 1/3\,\pi_3$$
$$\Rightarrow 2\pi_1 = \pi_2 + \pi_3 \quad \text{(multply by 3)}$$

(IV) $\therefore \pi_1 = 1/2\,\pi_2 + 1/2\,\pi_3$ (Divided by 2)

Substituting π_1 value to equation (ii)

$$\frac{2}{3}\pi_2 = \frac{1}{3}\left(\frac{1}{2}\pi_2 + \frac{1}{2}\pi_3\right) + \frac{1}{3}\pi_3$$

$$\frac{2}{3}\pi_2 = \frac{1}{6}\pi_2 + \frac{1}{6}\pi_3 + \frac{1}{3}\pi_3$$

$$\frac{2}{3}\pi_2 = \frac{1}{6}\pi_2 + \frac{1}{2}\pi_3 \quad \text{[combine } \pi_3 \text{ terms]}$$

$$4\pi_2 = \pi_2 + 3\pi_3 \quad \text{(multiply by 6)}$$

$$3\pi_2 = 3\pi_3$$

$$\therefore \pi_2 = \pi_3$$

Since $\pi_2 = \pi_3$, we can replace π_3 by π_2 in equation (iv)

$$\pi_1 = \frac{1}{2}\pi_2 + \frac{1}{2}\pi_3$$
$$= \frac{1}{2}\pi_2 + \frac{1}{2}\pi_2$$
$$= \pi_2$$

Thus, $\pi_1 = \pi_2 = \pi_3$

Sine π is probability distribution, we have the normalized Condition, $\pi_1 + \pi_2 + \pi_3 = 1$

As we can see $\pi_1 = \pi_2 = \pi_3$, Let $\pi_1 = \pi_2 = \pi_3 = \pi$

so, $\pi + \pi + \pi = 1$

$3\pi = 1$, $\Rightarrow \pi = 1/3$

Hence, The invariant probability distribution is, $\pi = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$.