MACT 2131 Projects **Fall 2018**

Please note that Dec 5 (Wednesday) is the submission deadline.
Late projects will NOT be accepted.

Choose any one of the following four projects:

Project 1 (Theoretical):

In this problem, we will be using binary predicates F(x, y), G(x, y), etc. to represent functions f, g : U → U, etc., where U is the universe. Thus, F(x, y) holds iff $y = f(x)$, G(x, y) holds iff $y = g(x)$, etc..

1) Write predicate statements that expresses the following facts:
   a. F represents a function.
   b. F represents a one-to-one function.
   c. F represents an onto function.
   d. F and G represent inverse functions of one another.
   e. H represents the composition function $f \circ g$.
2) Use binary predicates representing functions to give formal proofs (in the style of Sec 1.6 of the following statements:
   a. "If f and g are one-to-one functions, then so is $f \circ g$."
   b. "If f and g are onto functions, then so is $f \circ g$."

Project 2 (Theoretical):

A set is called finitistic if it is a finite set of finite sets of finite sets, etc.. More precisely, we define the concept of an **finitistic** set S and its **height** h(S) recursively as follows:

i. The empty set ∅ is finitistic, and h(∅) = 0.
ii. A set S is finitistic iff S is a finite set and all of its elements are finitistic. Also: $h(S) = 1 + \max\{h(T) \mid T \in S\}$.
iii. No other set is finitistic, except those described by Items i and ii.

1) Find **with proof** all finitistic sets of height 3.
2) Find a recursive formula for the cardinality c_n of the set of all finitistic sets of height n, i.e. $c_n = |\{S \mid S \text{ is h-finite and } d(S) = n\}|$.

Project 3 (Programming): The following set:

$$S = \{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}\}$$

can be represented by the following string (finite sequence) of "{" and "}":

$$s = \{\ \{\ \}\ \{\ \{\ \}\ \}\ \{\ \{\ \}\ \{\ \{\ \}\ \}\ \}\ \}$$

Note that the empty set $\varnothing$ was written as { } and all commas were omitted.

1) Show that if a string s of "{" and "}" represents a set S, then S is unique, i.e. two different sets are represented by two different strings.
2) Implement an algorithm that takes a string s of "{" and "}", then:
   a. Decides if that string represents a set S.
   b. Outputs a string p representing the power set P(S).

Project 4 (Programming):

1) Find data structures that can represent the following mathematical structures:
   a. A finite subset $A \subseteq \mathbf{N}$.
   b. A function $f : A \to B$, where A and B are finite subsets of **N**.
2) Implement an algorithm that takes a data structures of the form (1b), representing a function $f : A \to B$. It then:
   a. Outputs data structures representing the sets $A = \mathrm{Dom}(f)$ and $C = \mathrm{Range}(f)$.
   b. Decides if f is one-to-one.
   c. Decides if the composition function $f \circ f$ is defined, and outputs a data structure representing it.