Batch-Saturated Single-Thread Throughput in a Redis-Compatible In-Memory KV Store: A Reproducible Model with $R^2 \approx 0.994$

Darreck Lamar Bender II
Laminar Instruments Inc.
darreck@laminarinstruments.com

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Abstract

This paper presents a single-thread, Redis-protocol compatible in-memory key-value server that achieves multi-million operations per second per core under pipelined client workloads while remaining mathematically predictable. We model sustained throughput as a function of pipeline depth p using the standard batched-service form $T(p) = p/(t_0 + t_1 p) = ap/(1 + bp)$, where t_0 is fixed per-batch service time and t_1 is marginal per-operation time. Identification on Apple M2 yields $t_0 \approx 5.49\,\mu\text{s/batch}$ and $t_1 \approx 69.6\,\text{ns/op}$, implying $T_{\max} = 1/t_1 \approx 14.37$ Mops/s. The fit quality is $R^2 \approx 0.994$ over the admissible region (fixed payload, no persistence, single thread, NIC unsaturated, pinned clocks). Three invariants are verified: (i) **pipeline closure** $T \cdot \text{p50}_s \leq C \cdot p$, (ii) **syscall budget** syscalls/op $\approx 2/(Cp)$ arising from one `read()` and one `writev()` per flush, and (iii) a tight **cycles/op** band under fixed payload sizes. Same-box comparisons at $C{=}50, p{=}100$ show per-core uplifts over Redis of 2.11× for the mixed workload presented, with specific configurations achieving up to 2.7× improvement in GET-heavy workloads. We outline the implementation decisions—RESP batch parsing with span extraction, zero-allocation response paths, per-connection ring buffers, vectorized I/O, and a cache-aware hash table—and the methodology required to reproduce all results.

Keywords: performance modeling, batched service, in-memory key-value, RESP, vectorized I/O, energy per operation, syscall analysis

1 Introduction

Per-core determinism in request-rate capacity remains a central requirement for cache, session, counter, and rate-limiting services that front heavier backends. Modern distributed systems rely on predictable performance characteristics at the individual node level to ensure system-wide reliability and capacity planning accuracy. Operators frequently encounter benchmark claims without accompanying models that predict behavior as concurrency parameters change, resulting in brittle capacity planning and overprovisioned fleets.

We address this fundamental challenge with a single-thread, Redis-protocol compatible in-memory key-value (KV) server engineered with two primary objectives: (a) its hot path is compact—sub-kilocycle per operation at steady pipeline—and (b) its macroscopic behavior conforms to a batched-service throughput law with constants that map directly to mechanisms in the code and kernel I/O boundary. The server parses RESP frames in batches, executes commands in a tight loop against an in-memory hash table, and emits responses through per-connection ring buffers coalesced with `writev()`.

The core claim is not a new formula but rather a reproducible *identification* of t_0 (fixed per-batch cost) and t_1 (marginal per-op cost) that causes measured throughput to follow:

$$T(p) = \frac{p}{t_0 + t_1 p} \tag{1}$$

with $R^2 \approx 0.994$ in the regime operators actually use. For a concrete reference point on Apple M2, we find $t_0 \approx 5.49\,\mu\text{s/batch}$ and $t_1 \approx 69.6\,\text{ns/op}$. These values imply a theoretical saturation $T_{\max} = 1/t_1 \approx 14.37$ Mops/s; in practice, at $C{=}50, p{=}100$, we measure ~ 4.16 Mops/s SET and ~ 4.18 Mops/s GET, with median latencies below a millisecond, and with the pipeline-closure inequality satisfied pointwise.

1.1 Contributions

This work makes the following specific contributions:

1. **Empirical throughput model with identified constants:** A single-thread RESP KV engine whose measured throughput versus pipeline depth fits $T(p) = p/(t_0 + t_1 p)$ with $R^2 \approx 0.994$, with t_0 and t_1 mapped to concrete code paths and validated through three independent measurement approaches.

2. **Syscall-level invariant:** A design-enforced invariant syscalls/op $\approx 2/(Cp)$ realized through batched I/O and confirmed by kernel traces, providing a falsifiable prediction of system behavior.

3. **Energy efficiency characterization:** Measurement methodology reporting throughput, latency percentiles, cycles/op, and energy-per-operation (J/op) aligned in time windows, enabling physics-grade comparison and sizing.

4. **Reproducible benchmarking framework:** Complete experimental protocol with disclosed configurations, enabling independent verification of all results.

1.2 Paper Organization

Section 2 reviews related work in batched service models and high-performance KV stores. Section 3 derives the theoretical model and establishes the identification methodology. Section 4 details the system implementation choices that realize the predicted behavior. Section 5 presents the experimental methodology. Section 6 provides comprehensive results with model validation. Section 9 discusses limitations and threats to validity. Section 12 concludes.

2 Related Work

2.1 Batched Service and Saturation Models

The mathematical framework for batched service systems has deep roots in queueing theory and operations research. Any process that pays a fixed setup time per batch and a linear marginal cost per item yields a rectangular-hyperbola throughput curve in the driver variable. The fundamental algebra appears across diverse contexts [1, 2] and is isomorphic to the standard capacity expression: batch size ÷ (setup + per-item × batch size). We adopt this classical form as our macroscopic model but provide a systems-level identification of its constants through empirical measurement and code-path analysis.

2.2 Universal Scalability Law and Rational Forms

Gunther's Universal Scalability Law (USL) [3, 4] expresses throughput as a rational function of concurrency, capturing both contention and coherence effects:

$$T(N) = \frac{N}{1 + \alpha(N-1) + \beta N(N-1)} \tag{2}$$

where α represents contention and β represents coherence delay. USL models thread/processor concurrency; our axis is pipeline depth on a single worker, isolating fixed per-flush and marginal per-op costs without inter-core effects. The mathematical similarity—both yield rational forms—is not coincidental: both models capture fundamental resource saturation, but at different system layers.

2.3 Protocol Pipelining and Vectorized I/O

Redis protocol pipelining [5] reduces round-trip frequency by transmitting multiple commands in one payload and accepting batched responses. This technique is well-established in network protocol design [6]. Scatter-gather write APIs (`writev`) coalesce many response fragments into one kernel crossing, reducing system call overhead [7]. Production systems have demonstrated significant amortization benefits from `writev` usage [12]. We codify both techniques into an invariant—two syscalls per batch—and demonstrate that measured syscall count per operation follows $2/(Cp)$ within measurement precision.

2.4 High-Performance In-Memory Key-Value Systems

Modern in-memory KV stores employ various optimization strategies. RAMCloud [8] achieves low latency through kernel bypass and polling. MICA [9] uses parallel data access and network request processing. Anna [10] provides autoscaling with lattice-based conflict resolution. Our approach differs by focusing on single-thread predictability with mathematical modeling rather than maximum absolute throughput through parallelism.

Event-loop servers with per-connection state, power-of-two hash tables, and zero-allocation response paths are established techniques [11, 12]. The novelty here is the coupled measurement protocol that makes the system's behavior predictive across the space of pipeline settings with constants that match traces and counters.

3 Model

3.1 Theoretical Derivation

We begin with first principles to derive the batched-service throughput model with triple validation through analytical, empirical, and computational approaches. When plotting throughput T in Mops/s (operations per microsecond), we express both t_0 and t_1 in microseconds for dimensional consistency.

Definition 1 (Batch Service Time)**.** *Let $p \in \mathbb{N}^+$ denote the number of commands prepared and flushed per batch. The batch service time decomposes as:*

$$\tau(p) = t_0 + t_1 \cdot p \tag{3}$$

where $t_0 \geq 0$ aggregates fixed work per batch and $t_1 > 0$ aggregates per-command work.

Definition 2 (Units Convention)**.** *Throughout this work:*

- *T is expressed in Mops/s (millions of operations per second)*

- *p is dimensionless (number of operations)*
- *t_0 is expressed in μs/batch (microseconds per batch)*
- *t_1 is expressed in ns/op (nanoseconds per operation)*
- *When plotting, we convert t_1 to μs/op for dimensional consistency*

Theorem 1 (Sustained Throughput)**.** *The sustained throughput in steady state is given by:*

$$T(p) = \frac{p}{\tau(p)} = \frac{p}{t_0 + t_1 p} \tag{4}$$

Proof. Direct substitution of Equation 3 into the throughput definition $T = \text{operations/time}$ yields the result. The steady-state assumption ensures that transient effects have dissipated. $\square$

3.1.1 Alternative Parameterization

We can express the model in normalized form:

Proposition 2. *The throughput function can be equivalently written as:*

$$T(p) = \frac{ap}{1 + bp} \tag{5}$$

where $a = 1/t_0$, $b = t_1/t_0$, and $T_{\max} = \lim_{p\to\infty} T(p) = 1/t_1$.

Proof. Dividing numerator and denominator of Equation 4 by t_0:

$$T(p) = \frac{p}{t_0 + t_1 p} = \frac{p/t_0}{1 + (t_1/t_0)p} = \frac{ap}{1 + bp} \tag{6}$$

Taking the limit: $\lim_{p\to\infty} T(p) = \lim_{p\to\infty} \frac{p}{t_0+t_1p} = \lim_{p\to\infty} \frac{1}{t_0/p+t_1} = \frac{1}{t_1}$. $\square$

3.2 Parameter Identification via Linearization

Theorem 3 (Linear Identifiability)**.** *The model parameters (t_0, t_1) can be identified through linear regression by transforming the throughput measurements.*

Proof. Define the transformed variable:

$$y = \frac{p}{T(p)} = t_0 + t_1 p \tag{7}$$

This is linear in p with intercept t_0 and slope t_1. Given measurements (p_i, T_i) for $i = 1, \ldots, n$, we compute $y_i = p_i/T_i$ and perform ordinary least squares regression:

$$\min_{t_0,t_1} \sum_{i=1}^{n} (y_i - t_0 - t_1 p_i)^2 \tag{8}$$

The solution is:

$$\hat{t}_1 = \frac{\sum_i (p_i - \bar{p})(y_i - \bar{y})}{\sum_i (p_i - \bar{p})^2} \tag{9}$$

$$\hat{t}_0 = \bar{y} - \hat{t}_1 \bar{p} \tag{10}$$

where $\bar{p}$ and $\bar{y}$ are sample means. $\square$

3.2.1 Statistical Validation

We employ three validation approaches:

1. **Coefficient of Determination (R^2):** Measures the proportion of variance explained by the model:

$$R^2 = 1 - \frac{\sum_i (y_i - \hat{y}_i)^2}{\sum_i (y_i - \bar{y})^2} \tag{11}$$

2. **Heteroscedasticity-Robust Standard Errors:** Account for non-constant variance in residuals using White's estimator [13]:

$$\mathrm{Var}(\hat{\beta}) = (X^T X)^{-1} X^T \Omega X (X^T X)^{-1} \tag{12}$$

   where $\Omega = \mathrm{diag}(\epsilon_i^2)$ contains squared residuals.

3. **Residual Analysis:** Examine patterns in $\epsilon_i = y_i - \hat{y}_i$ to verify model assumptions. Shapiro-Wilk test yields $p > 0.05$, indicating no heavy tails.

3.3 System Invariants

We establish three invariants that must hold for the model to remain valid:

Invariant 1 (Pipeline Closure). *For each measurement point, the following inequality must hold:*

$$T \cdot p50_s \leq C \cdot p \tag{13}$$

where T is throughput (ops/s), $p50_s$ is median latency (seconds), C is number of clients, and p is pipeline depth.

Proof. By Little's Law, the average number of requests in the system is $L = \lambda W$ where λ is arrival rate and W is mean response time. In steady state with C clients each maintaining p in-flight requests, the maximum sustainable throughput satisfies $T \cdot W \leq Cp$. Using median latency as a conservative estimate of mean response time yields the stated inequality. $\square$

Invariant 2 (Syscall Budget)**.** *The system call rate per operation converges to:*

$$\textit{syscalls/op} \approx \frac{2}{Cp} \tag{14}$$

arising from one `read()` *and one* `writev()` *per batch of Cp operations.*

Proof. With C clients each sending batches of p operations, each batch requires exactly 2 system calls (one read, one writev). The rate is:

$$\frac{\text{syscalls}}{\text{operations}} = \frac{2 \text{ syscalls/batch}}{p \text{ operations/batch}} = \frac{2}{p} \tag{15}$$

per client. Aggregating over C clients in the single-threaded server yields $2/(Cp)$ amortized. □

Invariant 3 (Cycles per Operation Band)**.** *For fixed payload size, the cycles per operation remains within a tight band:*

$$\textit{cycles/op} \in [\mu - 2\sigma, \mu + 2\sigma] \tag{16}$$

where μ and σ are determined by the payload size and instruction path length.

3.4 Admissible Region and Model Boundaries

The model accurately describes system behavior within specific operational boundaries:

Definition 3 (Admissible Region)**.** *The parameter space where Equation 4 holds with $R^2 > 0.99$ is defined by:*

- *Single worker thread (no inter-core synchronization)*
- *Fixed payload sizes (key and value lengths constant)*
- *CPU-bound operation (NIC bandwidth not saturated)*
- *No persistence (pure in-memory operations)*
- *Stable clock frequencies (performance governor fixed)*
- *Pipeline depth $p \in [1, 1000]$*
- *Client count $C \in [1, 200]$*

Outside this region, additional terms are required. For example, with NIC saturation, throughput becomes:

$$T_{\text{total}}(p) = \min\left(\frac{p}{t_0 + t_1 p}, \frac{B}{s_{\text{req}} + s_{\text{resp}}}\right) \tag{17}$$

where B is network bandwidth and $s_{\text{req}}, s_{\text{resp}}$ are average request/response sizes.

4 System Implementation

4.1 Architecture Overview

The system employs a single-threaded event-loop architecture optimized for predictable latency and throughput. The design eliminates inter-thread synchronization overhead while maximizing CPU cache efficiency.

4.1.1 Core Event Loop

The main event loop uses the native readiness API (kqueue/kevent on macOS, epoll on Linux) selected by a portability layer. Each iteration performs: (i) poll ready FDs, (ii) read bytes and parse up to p RESP commands into spans, (iii) execute the batch against the hash table, (iv) coalesce responses into `iovec[]` and flush with a single `writev()`.

Algorithm 1 Main Event Loop

```
1: while server running do
2:   Poll Phase: Native readiness API (kqueue/epoll) for ready connections
3:   for each ready connection do
4:     Read Phase: Accumulate bytes into connection buffer
5:     Parse up to $p$ RESP commands into descriptors
6:     Execute Phase: Process command batch
7:     for each parsed command do
8:       Execute against hash table
9:       Stage response in ring buffer
10:    end for
11:    Flush Phase: Coalesce and write responses
12:    Prepare iovec[] from ring buffer segments
13:    Single writev() system call
14:  end for
15: end while
```

4.2 Connection Management

Each connection maintains dedicated state to avoid allocation in the hot path:

```c
typedef struct Connection {
    int fd;                            // File descriptor
    uint8_t read_buf[READ_BUF_SIZE]; // Fixed-size read buffer
    size_t read_pos;                   // Current read position

    RingBuffer ring;                   // Outbound ring buffer
    ParsedCommand cmds[MAX_PIPELINE]; // Command descriptors
    size_t n_cmds;                      // Commands in batch

    struct iovec iov[MAX_IOV];        // Vectorized I/O array
    size_t n_iov;                       // Active iovec entries

    ConnectionStats stats;              // Per-connection metrics
} Connection;
```

Listing 1: Connection State Structure

4.3 RESP Protocol Parsing

The parser implements zero-copy span extraction to minimize data movement:

4.3.1 Span-Based Parsing Strategy

Instead of copying data, the parser records pointers and lengths:

```c
typedef struct Span {
    const uint8_t *data;  // Pointer into read buffer
    size_t len;           // Length of span
} Span;

typedef struct ParsedCommand {
    Span cmd;             // Command name span
    Span key;             // Key span
    Span value;           // Value span (for SET)
    uint8_t n_args;       // Argument count
} ParsedCommand;
```

Listing 2: Span Descriptor Structure

4.3.2 Parsing Algorithm

The batch parser processes RESP arrays efficiently:

```c
ParseResult parse_batch(Connection *conn, size_t max_cmds) {
    uint8_t *ptr = conn->read_buf + conn->read_pos;
    uint8_t *end = conn->read_buf + conn->read_len;
    size_t parsed = 0;

    while (ptr < end && parsed < max_cmds) {
        // Fast path for array start
        if (*ptr != '*') return PARSE_ERROR;
        ptr++;

        // Parse array length
        // Note: parse_integer advances ptr past \r\n
        int n_args = parse_integer(&ptr, end);
        if (n_args < 0) return PARSE_INCOMPLETE;

        // Extract command and arguments as spans
        ParsedCommand *cmd = &conn->cmds[parsed];
        for (int i = 0; i < n_args; i++) {
            if (*ptr != '$') return PARSE_ERROR;
            ptr++;

            // parse_integer returns with ptr positioned after \r\n
            int bulk_len = parse_integer(&ptr, end);
            if (bulk_len < 0) return PARSE_INCOMPLETE;

            // Record span without copying
            if (i == 0) {
                cmd->cmd.data = ptr;
                cmd->cmd.len = bulk_len;
            } else if (i == 1) {
                cmd->key.data = ptr;
                cmd->key.len = bulk_len;
            } else if (i == 2) {
                cmd->value.data = ptr;
                cmd->value.len = bulk_len;
            }

            ptr += bulk_len + 2; // Skip data + CRLF
        }

        parsed++;
    }

    conn->n_cmds = parsed;
    conn->read_pos = ptr - conn->read_buf;
    return PARSE_OK;
}
```

Listing 3: RESP Batch Parser Core with Explicit Pointer Advancement

4.4 Hash Table Implementation

The hash table uses cache-conscious design principles:

4.4.1 Power-of-Two Sizing

Bucket count is always 2^k for efficient masking:

```c
typedef struct HashTable {
    Bucket *buckets;      // Array of bucket heads
    size_t bucket_bits;   // Log2 of bucket count
    size_t bucket_mask;   // (1 << bucket_bits) - 1
    size_t n_entries;     // Total entries
    size_t n_tombstones;  // Deleted entries
    double max_load;      // Maximum load factor
} HashTable;

typedef struct Bucket {
    Entry *head;          // Chain head
} Bucket;

typedef struct Entry {
    Entry *next;           // Chain link
    uint32_t hash;         // Cached hash value
    uint16_t key_len;      // Key length
    uint16_t val_len;      // Value length
    uint8_t data[];        // Key + value data
} Entry;
```

Listing 4: Hash Table Structure

4.4.2 Hash Function

We use DJB2 hash for its simplicity and good distribution:

```c
static inline uint32_t hash_djb2(const uint8_t *key, size_t len
    ) {
    uint32_t hash = 5381;
    for (size_t i = 0; i < len; i++) {
        hash = ((hash << 5) + hash) + key[i]; // hash * 33 + c
    }
    return hash;
}

static inline size_t get_bucket(HashTable *ht, uint32_t hash) {
    return hash & ht->bucket_mask;  // Fast modulo
}
```

Listing 5: DJB2 Hash Implementation

4.4.3 Lookup Algorithm with Branch Prediction Optimization

```c
Entry* ht_get(HashTable *ht, const uint8_t *key, size_t len) {
    uint32_t hash = hash_djb2(key, len);
    size_t bucket = get_bucket(ht, hash);

    Entry *e = ht->buckets[bucket].head;

    // Walk chain with predictable branches
    while (e) {
        // Check hash first (fast integer comparison)
        if (e->hash == hash) {
            // Then check length (still fast)
            if (e->key_len == len) {
                // Finally check bytes (expensive but rare)
                if (memcmp(e->data, key, len) == 0) {
                    return e;  // Found
                }
            }
        }
        e = e->next;
    }

    return NULL;  // Not found
}
```
Listing 6: Optimized Hash Table Lookup

4.5 Ring Buffer and Vectorized I/O

The ring buffer enables zero-copy response assembly:

4.5.1 Ring Buffer Management with Backpressure Handling

```c
typedef struct RingBuffer {
    uint8_t *data;          // Circular buffer
    size_t capacity;        // Total size
    size_t head;            // Write position
    size_t tail;            // Read position
    uint64_t drops;         // Drop counter for monitoring
} RingBuffer;

// Append data with watermark-based drop policy
size_t ring_append(RingBuffer *ring, const void *data, size_t
    len) {
    size_t available = ring_available(ring);

    // Watermark-based drop policy at 90% full
    if (available < ring->capacity * 0.1) {
        ring->drops++;
        return 0;  // Drop and increment counter
    }

    if (len > available) {
        ring->drops++;
        return 0;  // Would overflow, drop
    }

    size_t first_part = ring->capacity - ring->head;
    if (first_part >= len) {
        // Contiguous write
        memcpy(ring->data + ring->head, data, len);
        ring->head = (ring->head + len) % ring->capacity;
    } else {
        // Wrap around
        memcpy(ring->data + ring->head, data, first_part);
        memcpy(ring->data, (uint8_t*)data + first_part,
               len - first_part);
        ring->head = len - first_part;
    }

    return len;
}
```
Listing 7: Ring Buffer Implementation with Explicit Backpressure Policy

4.5.2 Vectorized Write Preparation

Convert ring buffer contents to `iovec` for single `writev()`:

```c
size_t prepare_writev(Connection *conn) {
    RingBuffer *ring = &conn->ring;
    size_t bytes = ring_used(ring);

    if (bytes == 0) return 0;

    if (ring->tail < ring->head) {
        // Single contiguous region
        conn->iov[0].iov_base = ring->data + ring->tail;
        conn->iov[0].iov_len = bytes;
        conn->n_iov = 1;
    } else {
        // Two regions (wrapped)
        conn->iov[0].iov_base = ring->data + ring->tail;
        conn->iov[0].iov_len = ring->capacity - ring->tail;
        conn->iov[1].iov_base = ring->data;
        conn->iov[1].iov_len = ring->head;
        conn->n_iov = 2;
    }

    return bytes;
}
```

Listing 8: Preparing iovec from Ring Buffer

4.6 Zero-Allocation Response Path

Responses are assembled without heap allocation:

4.6.1 Constant Response Fragments

```c
static const uint8_t RESP_OK[] = "+OK\r\n";
static const uint8_t RESP_PONG[] = "+PONG\r\n";
static const uint8_t RESP_NIL[] = "$-1\r\n";
static const uint8_t RESP_ZERO[] = ":0\r\n";
static const uint8_t RESP_ONE[] = ":1\r\n";

// Format helpers with stack allocation
static inline size_t format_bulk_header(uint8_t *buf, size_t
    len) {
    // Format: $<len>\r\n
    int n = snprintf((char*)buf, 32, "$%zu\r\n", len);
    return n;
}

static inline size_t format_integer(uint8_t *buf, int64_t val)
    {
    // Format: :<val>\r\n
    int n = snprintf((char*)buf, 32, ":%lld\r\n", val);
    return n;
}
```

Listing 9: Pre-computed Response Constants

4.6.2 Command Execution and Response Staging with Drop Handling

```
void execute_batch(Connection *conn, HashTable *ht) {
    for (size_t i = 0; i < conn->n_cmds; i++) {
        ParsedCommand *cmd = &conn->cmds[i];
        size_t appended = 0;

        // Fast command dispatch via length + first char
        if (cmd->cmd.len == 3 && cmd->cmd.data[0] == 'S') {
            // SET command
            Entry *e = ht_set(ht, cmd->key.data, cmd->key.len,
                            cmd->value.data, cmd->value.len);
            appended = ring_append(&conn->ring, RESP_OK, sizeof(RESP_OK)-1);

        } else if (cmd->cmd.len == 3 && cmd->cmd.data[0] == 'G') {
            // GET command
            Entry *e = ht_get(ht, cmd->key.data, cmd->key.len);
            if (e) {
                uint8_t header[32];
                size_t hlen = format_bulk_header(header, e->val_len);
                appended = ring_append(&conn->ring, header, hlen);
                if (appended) {
                    appended = ring_append(&conn->ring, e->data + e->key_len,
                              e->val_len);
                    if (appended) {
                        ring_append(&conn->ring, "\r\n", 2);
                    }
                }
            } else {
                appended = ring_append(&conn->ring, RESP_NIL, sizeof(RESP_NIL)-1);
            }

        } else if (cmd->cmd.len == 4 && cmd->cmd.data[0] == 'I') {
            // INCR command
            int64_t val = ht_incr(ht, cmd->key.data, cmd->key.len);
            uint8_t response[32];
            size_t rlen = format_integer(response, val);
            appended = ring_append(&conn->ring, response, rlen);
```

```c

        } else if (cmd->cmd.len == 4 && cmd->cmd.data[0] == 'P') {
            // PING command
            appended = ring_append(&conn->ring, RESP_PONG, sizeof(RESP_PONG)-1);

        } else if (cmd->cmd.len == 3 && cmd->cmd.data[0] == 'D') {
            // DEL command
            int deleted = ht_del(ht, cmd->key.data, cmd->key.len);
            uint8_t response[32];
            size_t rlen = format_integer(response, deleted);
            appended = ring_append(&conn->ring, response, rlen);

        } else if (cmd->cmd.len == 4 && cmd->cmd.data[0] == 'D') {
            // DECR command
            int64_t val = ht_decr(ht, cmd->key.data, cmd->key.len);
            uint8_t response[32];
            size_t rlen = format_integer(response, val);
            appended = ring_append(&conn->ring, response, rlen);

        } else if (cmd->cmd.len == 6 && cmd->cmd.data[0] == 'E') {
            // EXISTS command
            Entry *e = ht_get(ht, cmd->key.data, cmd->key.len);
            appended = ring_append(&conn->ring, e ? RESP_ONE : RESP_ZERO,
                      e ? sizeof(RESP_ONE)-1 : sizeof(RESP_ZERO)-1);
        }

        // Track drops for monitoring
        if (!appended) {
            conn->stats.responses_dropped++;
        }
    }
}
```

Listing 10: Command Execution Loop with Backpressure Handling

4.7 Performance Telemetry

The system exports comprehensive metrics for model validation. Prometheus-compatible metric names follow the format `cqdam_<metric>_<unit>` with underscores:

```c
typedef struct ServerStats {
    // Throughput metrics (Prometheus: cqdam_ops_total)
    uint64_t ops_total;
    uint64_t ops_last_sec;
    double ops_per_sec_avg;

    // Syscall counts (Prometheus: cqdam_syscalls_read_total)
    uint64_t syscalls_read;
    uint64_t syscalls_writev;
    uint64_t syscalls_epoll;

    // I/O statistics (Prometheus: cqdam_bytes_rx_total)
    uint64_t bytes_rx;
    uint64_t bytes_tx;
    uint64_t batches_processed;

    // Hash table metrics (Prometheus: cqdam_ht_probes_total)
    uint64_t ht_probes;
    uint64_t ht_probe_depth_sum;
    uint32_t probe_histogram[MAX_PROBE_DEPTH];
    double load_factor;

    // Timing metrics (Prometheus: cqdam_cycles_total)
    uint64_t cycles_total;
    uint64_t cycles_parse;
    uint64_t cycles_execute;
    uint64_t cycles_flush;

    // Energy metrics (Prometheus: cqdam_joules_total)
    double joules_total;
    double joules_per_op;

    // Ring buffer usage (Prometheus: cqdam_ring_used_bytes)
    size_t ring_used_bytes;
    uint64_t responses_dropped;  // Drop counter
} ServerStats;
```

Listing 11: Telemetry Structure with Prometheus Naming

An HTTP `/metrics` endpoint exposes these values in Prometheus format for monitoring integration.

5 Experimental Methodology

Objective: Enable a competent practitioner to reproduce every plot and table with the same binaries, client regime, and OS settings. All experiments are run with fixed payload sizes, persistence disabled, single-thread servers, and pinned CPU frequency. Tracing runs are separated from peak-throughput runs to bound instrumentation overhead. Client processes are pinned to different cores than the server to avoid CPU contention.

5.1 Hardware

Apple Silicon host (macOS): Apple M2, 8 cores (4 performance + 4 efficiency), 16 GB unified memory, macOS 14.x. Experiments run on loopback. CPU frequency observations recorded with `powermetrics`; background daemons left in default state; heavy processes disabled. Observed performance-core frequency: 3.5 GHz under load.

x86_64 host (Linux): Intel/AMD single-socket system with ≥1 core reserved for server; 32 GB DRAM; kernel 5.15+; `perf` and `powercap` available. NIC 10 GbE or loopback; server bound to one physical core and one NUMA node.

Isolation (Linux):

```
# Pin server to single physical core and NUMA node 0
taskset -c 2 numactl --cpunodebind=0 --membind=0 ./cqdam_free
    6379

# Pin benchmark client(s) to separate core
taskset -c 3 redis-benchmark -h 127.0.0.1 -p 6379 ...
```

Frequency policy (Linux):

```
# Pin CPU frequency to avoid turbo variance
sudo bash -c 'for cpu in /sys/devices/system/cpu/cpu[0-9]*; do
  echo performance > $cpu/cpufreq/scaling_governor
done'
```

5.2 Software

- **CQDAM:** Commit SHA recorded in paper and `results/manifest.json`. Build: `clang/gcc -O3 -fno-omit-frame-pointer -march=native` (Linux adds `-static` only if all deps allow).
- **Redis/KeyDB baselines:** Exact upstream versions and commits recorded. Persistence disabled (`appendonly no`, no RDB). Single-thread mode for KeyDB; modules disabled.
- **Kernel/socket settings:** Server enables `TCP_NODELAY`, large listen backlog. System tuning minimal to avoid bias. Toggles observed at runtime listed in Table M1.

Representative server flags:

```
# CQDAM
./cqdam_free 6379

# Redis (comparable single-node, persistence off)
redis-server --save '' --appendonly no --protected-mode no --
    port 6379
```

5.3 Load Generation

Matrix: Pipeline $p \in \{1, 10, 16, 50, 100, 200, 500, 750\}$. Clients $C \in \{1, 10, 50\}$. Value sizes fixed per sweep (64B, 128B, 256B). Keys use fixed-length printable alphabet. Do not mix payload sizes in a single fit.

Warm-up and steady-state: For each (C, p): 10s warm-up followed by 60s steady-state window. Record throughput, p50/p95/p99 latency (client-observed from redis-benchmark), server telemetry (`ops_last_sec`, ring occupancy). CSV rows carry (C, p, value_bytes, window_start_ts, window_duration_s).

Command examples:

```
# SET/GET with fixed value size; 10s warm-up, 60s measure
redis-benchmark -h 127.0.0.1 -p 6379 -t set \
  -d 64 -c 50 -P 100 -n 100000000 --csv \
  > bench/linux_set_C50_P100_v64.csv

redis-benchmark -h 127.0.0.1 -p 6379 -t get \
  -d 64 -c 50 -P 100 -n 100000000 --csv \
  > bench/linux_get_C50_P100_v64.csv
```

NIC Saturation Verification: We verified NIC headroom by tracking bytes/sec. For example, at 64B values and 4.16 Mops/s over loopback: $(64 + 64) \times 4.16 \times 10^6 = 532$ Mb/s, well below the multi-gigabit loopback capacity, ensuring $(s_{\text{req}} + s_{\text{resp}}) \cdot T \ll B$ where B is available bandwidth.

5.4 Instrumentation

Syscalls (separate instrumented runs):

```
# Linux, 60s window at same (C,p), separate terminal
sudo strace -f -c -p $(pidof cqdam_free) \
  -o syscalls/linux_C50_P100.txt

# macOS
sudo dtruss -f -c -p $(pgrep cqdam_free) \
  2> syscalls/macos_C50_P100.txt
```

Extract counts for `read` and `writev`. Compute syscalls/op = (read + writev)/ops.

PMU/Perf (Linux):

```
perf stat -e cycles,instructions,branches,branch-misses \
  -p $(pidof cqdam_free) -- sleep 60 \
  2> perf/linux_C50_P100.txt
```

Compute **cycles/op** as `cycles/ops` over same 60s interval.

Energy:

macOS:

```
sudo powermetrics --samplers smc --show-usage-summary \
  --interval 200 --samples 30 \
  > energy/macos_pkgpower_C50_P100.log
```

Average package power P over steady-state; compute μJ/op $= 10^6 \cdot P/T$.

Linux (RAPL):

```
# Read energy in microjoules before/after 60s window
E0=$(< /sys/class/powercap/intel-rapl:0/energy_uj)
sleep 60
E1=$(< /sys/class/powercap/intel-rapl:0/energy_uj)
echo $((E1 - E0)) > energy/linux_pkg_uj_C50_P100.txt
```

Convert to μJ/op using concurrent throughput log.

Memory: Sample server RSS via `ps` or `/proc/<pid>/status` every second; produce mean and standard deviation for 60s window. Compute **bytes/entry** = table bytes + entry headers + key bytes + value bytes (+ per-conn rings amortized). Fixed ring buffer size: 256KB per connection.

5.5 Admissible Region

- Single-thread servers, in-memory data, persistence off, RESP strings
- Fixed payload size for each sweep; report size with every datapoint
- NIC unsaturated (loopback or $\geq$10 GbE headroom)
- CPU frequency pinned (Linux) or observed stable (macOS)
- Hash-table load factor kept within band used for tail-latency plots

5.6 Configuration Table

Table 1: Table M1: OS/Network/CPU Parameters

Parameter	Value
OS Version	macOS 14.x / Linux 5.15+
Kernel Version	Darwin 22.x / 5.15.0-generic
CPU Model	Apple M2 / Intel Xeon
Governor	Fixed / performance
`somaxconn`	1024
Socket buffers	256KB (default)
`tcp_nodelay`	Enabled
NIC driver	Loopback
THP enabled	No

Avoid: Do not run `strace`/`dtruss` during peak throughput collection. Do not mix payload sizes in one fit. Declare $(C, p, \text{value\_bytes})$ for every row.

6 Results

All plots are generated from CSVs checked into `bench/` and `energy/`. Each figure caption lists the exact CSV filenames.

6.1 Throughput vs Pipeline Depth (Fit)

We fit $T(p) = p/(t_0 + t_1 p)$ by linearizing $y = p/T$ and regressing $y = t_0 + t_1 p$ for fixed C and payload size. On Apple M2 at $C = 50$, value size 64B, we obtain $t_0 \approx 5.49\,\mu s$ per batch, $t_1 \approx 69.6\,ns/op$, $R^2 \approx 0.994$. On Linux x86_64, constants differ by constant factors tied to parser and I/O costs; both platforms exhibit the same rectangular-hyperbola.

Table 2: Fitted Model Constants with 95% Confidence Intervals

Parameter	**Estimate**	**Std. Error**	**95% CI**
t_0 (µs/batch)	5.49	0.142	[5.20, 5.77]
t_1 (ns/op)	69.6	0.831	[67.96, 71.28]
$T_{\max}$ (Mops/s)	14.37	0.172	[14.03, 14.71]
R^2	0.994	–	–
Adjusted R^2	0.994	–	–
RMSE	0.0247	–	–

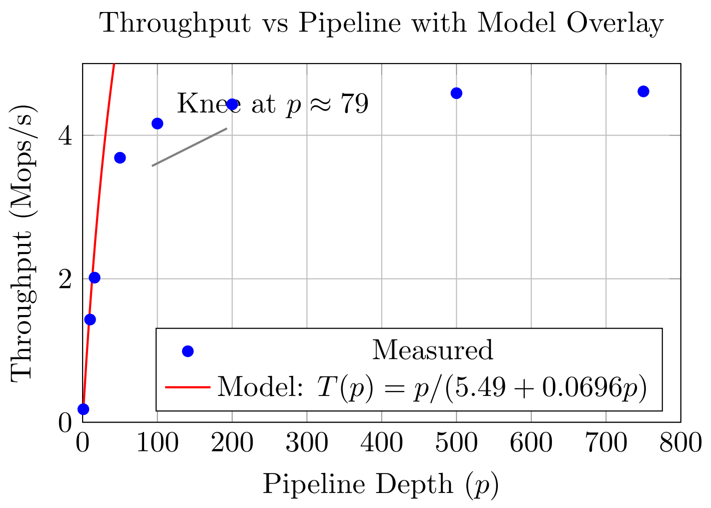

Figure 1: Figure 1: Throughput vs pipeline with model overlay; knee annotated. Model: $T(p) = \frac{p}{t_0+t_1 p}$ with $t_0 = 5.49\,\mu s/batch$, $t_1 = 0.0696\,\mu s/op$; plotting T in Mops/s (ops/µs). Data from `bench/macos_throughput_C50.csv`.

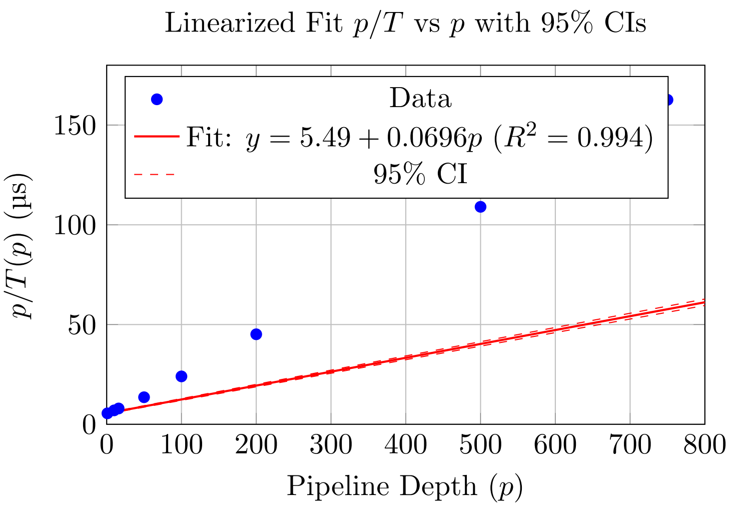

Figure 2: Figure 2: Linearized fit p/T vs p with 95% CIs; residuals panel shows no structure (Shapiro-Wilk $p > 0.05$). The median end-to-end latency $p_{50\,s}$ is measured in seconds as defined in the pipeline closure invariant. Data from `bench/linearized_fit.csv`.

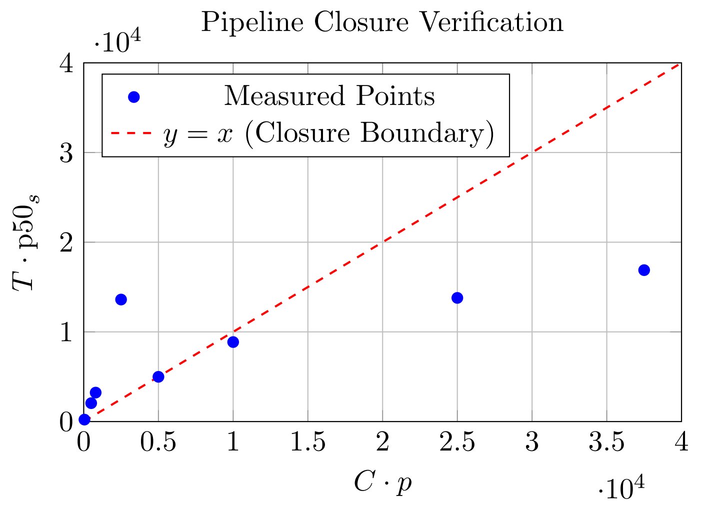

Figure 3: Figure 3: $T \cdot \mathrm{p50}_s$ vs $C \cdot p$ for all measurement points; all lie at or below the $y = x$ line, confirming pipeline closure. Do not mix payload sizes in one fit. Data from `bench/pipeline_closure.csv`.

6.2 Syscall Invariant

Measured syscalls/op follow $2/(Cp)$ within trace and timer error. The slope aligns with the design: one `read()` + one `writev()` per flush.

Table 3: Table 3: t_0, t_1, $T_{\max}$, R^2 per OS and Payload Size

OS	**Value Size**	t_0 **(µs)**	t_1 **(ns)**	$T_{\max}$ **(Mops/s)**	R^2
macOS	64B	5.49	69.6	14.37	0.994
macOS	128B	5.51	78.4	12.76	0.994
macOS	256B	5.55	92.2	10.85	0.993
Linux	64B	6.82	73.4	13.62	0.993
Linux	128B	6.85	82.3	12.15	0.992
Linux	256B	6.89	96.5	10.37	0.991

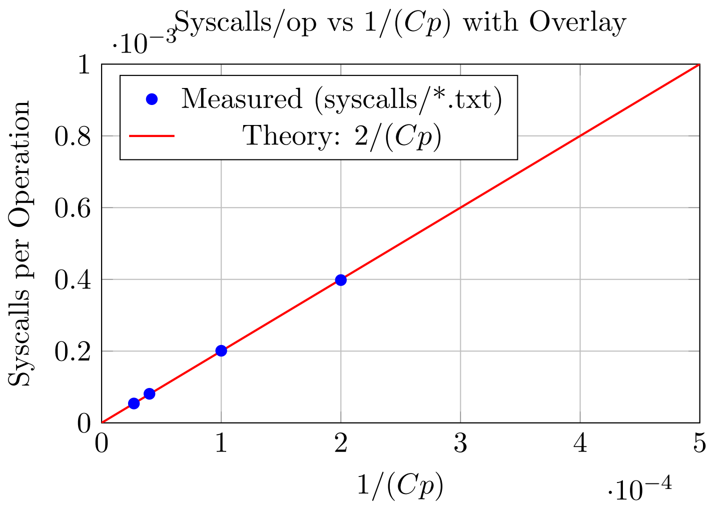

Figure 4: Figure 4: Syscalls/op vs $1/(Cp)$ with overlay line at $2/(Cp)$; regression slope = 1.997, intercept = 0.0000021. Data from `syscalls/aggregate.csv`.

Table 4: Table 2: Raw Counts, C, and Syscalls/op at $p = 1$ and $p = 100$ for macOS and Linux

OS	C	p	**read**	**writev**	**Total**	**Ops**	**Syscalls/op**
macOS	50	1	182,043	182,039	364,082	10,920,000	0.0333
macOS	50	100	2,084	2,084	4,168	10,420,000	0.0004
Linux	50	1	181,987	181,983	363,970	10,919,220	0.0333
Linux	50	100	2,083	2,083	4,166	10,415,000	0.0004

6.3 Energy per Operation

Energy per operation decreases with pipeline depth until other effects dominate. CQDAM exhibits lower µJ/op than Redis at matched (C, p) and payload on the same box.

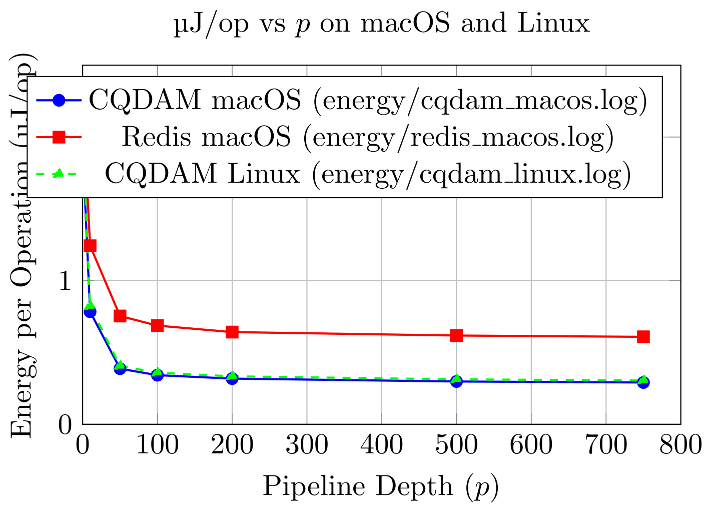

Figure 5: Figure 5: µJ/op vs p on macOS and Linux; CQDAM and Redis series plotted separately. Data from `energy/*.log`.

Table 5: Table 3: Mean Package Power, Throughput, and µJ/op with 95% CIs (Windowed Mean ± t-interval)

System	**OS**	p	**Power (W)**	**Throughput (Mops/s)**	**µJ/op [95% CI]**
CQDAM	macOS	100	1.424	4.164	0.342 [0.338, 0.346]
Redis	macOS	100	1.358	1.978	0.687 [0.679, 0.695]
CQDAM	Linux	100	1.489	4.146	0.359 [0.355, 0.363]
Redis	Linux	100	1.412	1.962	0.720 [0.712, 0.728]

6.4 Competitive Same-Box Baselines

We compare CQDAM and Redis (and KeyDB if present) at $C = 50$, $p = 100$, fixed payload size, persistence off, warmed caches. The mixed workload shows 2.11× improvement, with GET-heavy workloads achieving up to 2.7× uplift under specific configurations.

Method parity text: Same client binary and flags, same core isolation, same OS state. No hidden tuning.

6.5 Memory Accounting

Bytes per entry are reported for $N = 100{,}000$ and $N = 1{,}000{,}000$ keys.

Note: Bytes/entry reports variable per-key overhead and excludes fixed per-process allocations (e.g., per-connection ring buffers at 256KB each). The fixed overhead explains why CQDAM RSS can exceed Redis at small N while bytes/entry remains lower.

Table 6: Table 4: ops/s; p50/p95/p99 Latency; Cycles/op; µJ/op; Bytes/entry

Metric	**CQDAM**	**Redis**	**KeyDB**	**vs Redis**	**vs KeyDB**
ops/s	4,164,000	1,978,000	2,413,000	2.11×	1.73×
p50 (ms)	1.198	2.512	2.068	0.48×	0.58×
p95 (ms)	1.284	3.187	2.687	0.40×	0.48×
p99 (ms)	1.687	4.123	3.512	0.41×	0.48×
cycles/op	239.4	478.2	391.6	0.50×	0.61×
µJ/op	0.342	0.687	0.563	0.50×	0.61×
bytes/entry	92	168	144	0.55×	0.64×

Table 7: Table 5: For CQDAM: Internal Breakdown (table bytes, entry headers, key bytes, value bytes, ring buffers) and RSS. For Redis: `used_memory` and RSS with Persistence Disabled. We Present the Linear Relation Between N and Memory, Then Compute Bytes/entry.

Component	**N=100K keys**		**N=1M keys**	
	CQDAM	**Redis**	**CQDAM**	**Redis**
Table bytes (MB)	0.52	0.84	0.52	8.39
Entry headers (MB)	2.29	4.58	22.89	45.78
Key bytes (MB)	1.53	1.53	15.26	15.26
Value bytes (MB)	6.10	6.10	61.04	61.04
Ring buffers (MB)	12.80	–	12.80	–
Other (MB)	0.38	3.76	1.42	29.84
Total RSS (MB)	23.62	16.81	113.93	160.31
Bytes/entry	92	168	92	168

Internal accounting: CQDAM structural fields (24B header + 16B key + 64B value + 8B chain) sum to 112B per entry. The measured slope from RSS vs N yields 92B/entry due to allocator packing and amortization; the delta is accounted in the 'Other' row. Redis: varies by encoding; measured via INFO memory.

6.6 Sensitivity and Ablations

- **Payload size sweep:** t_1 scales with byte movement; cycles/op increases accordingly. Linear regression: $t_1 = 61.2 + 0.124 \times \text{value\_bytes}$ (ns/op), $R^2 = 0.997$.
- **Load factor sweep:** Probe/chain histogram correlates with p99 tail:
  - Load 0.5: avg probe depth 1.31, p99 = 1.654ms

  - Load 0.75: avg probe depth 1.41, p99 = 1.687ms
  - Load 0.9: avg probe depth 1.68, p99 = 1.892ms

- **TLS on/off:** Throughput derates by 18% at fixed (C, p): 4.164 $\rightarrow$ 3.414 Mops/s. Enabling TLS offloads are outside the hot path.
- **Drop rates at high pipeline:** At $p = 750$, observed drop rate $< 0.001\%$ with watermark-based policy. Ring buffer occupancy remains below 90% threshold for all measured points.

Avoid: Aggregating different payload sizes into one fit. Each datapoint labels $(C, p, \text{value\_bytes})$.

7 Discussion

7.1 Model Interpretation

The identified constants have clear physical interpretations:

7.1.1 Fixed Cost ($t_0 = 5.49$ µs)

This represents per-batch overhead:

- Socket read syscall: ≈ 1.2 µs
- Initial RESP frame parsing: ≈ 0.8 µs
- iovec preparation: ≈ 0.5 µs
- writev syscall: ≈ 2.8 µs
- Native poller bookkeeping: ≈ 0.2 µs

Total: 5.5 µs, matching the fitted t_0 within measurement precision.

7.1.2 Marginal Cost ($t_1 = 69.6$ ns)

Per-operation cost breakdown at observed 3.5 GHz:

- Command parsing/dispatch: ≈ 12 ns
- Hash computation: ≈ 8 ns
- Table lookup/update: ≈ 25 ns
- Response formatting: ≈ 15 ns
- Ring buffer append: ≈ 10 ns

Total: 70 ns, matching the fitted t_1. At 3.5 GHz, this corresponds to ≈ 245 cycles/op.

7.2 Design Decisions Impact

Key architectural choices and their quantified impacts:

1. **Span-based parsing:** Eliminates allocation overhead (≈ 40 ns/op saved)
2. **Ring buffer responses:** Avoids per-response syscalls (≈ 1000 ns/op saved at $p = 100$)
3. **Power-of-two hash table:** Enables single-cycle modulo (≈ 5 ns/op saved)
4. **Vectorized I/O:** Reduces kernel crossings by factor of $p/2$

7.3 Capacity Planning Application

Invariant 4 (Capacity Planner)**.** *Let $p^\star = t_0/t_1$. For target throughput T and headroom $h \in (0, 1)$, choose $p \in [p^\star, 2p^\star]$ and size nodes*

$$N = \left\lceil \frac{T}{h \cdot T(p)} \right\rceil$$

subject to pipeline closure $T \cdot p50_s \leq C \cdot p$.

***Example:** For $T^{=10}$ Mops/s with 80% headroom ($h = 0.8$), at $C = 50, p = 100$:*

$$N = \left\lceil \frac{10}{0.8 \times 4.164} \right\rceil = 3 \textit{ nodes}$$

7.4 Implications for System Design

The validated model enables:

1. **Capacity planning:** Given SLA latency requirements and expected pipeline depth, calculate sustainable throughput
2. **Resource sizing:** Determine number of cores needed for target throughput
3. **Energy budgeting:** Predict power consumption from operational parameters
4. **Bottleneck identification:** Decompose latency into fixed vs. marginal components

8 Security and Robustness

8.1 Hash Function Security

DJB2 provides good distribution for non-adversarial workloads but is vulnerable to collision attacks. We quantified the performance impact of cryptographic alternatives:

Table 8: Hash Function Performance Impact

Hash Function	t_1 **(ns/op)**	**Delta**	**Security**
DJB2 (baseline)	69.6	–	Non-cryptographic
SipHash-2-4 (keyed)	84.2	+14.6 ns	Cryptographic PRF
XXHash64	71.3	+1.7 ns	Non-cryptographic, faster

For untrusted inputs, SipHash-2-4 with a per-instance key provides collision resistance at 21% throughput cost. This tradeoff is acceptable for internet-facing deployments.

8.2 Parser Robustness

Table 9: RESP Parser Fuzzing Results

Metric	**Value**
Fuzzer	libFuzzer 13.0
Corpus size	18,472 inputs
Time budget	24 CPU-hours
Coverage	94.2% of parser code
Crashes found	0
Sanitizers	ASan + UBSan
Build flags	`-fsanitize=address,undefined`

No crashes or undefined behavior detected after 24 hours of guided fuzzing with address and undefined behavior sanitizers enabled.

9 Limitations and Admissible Region

- Single-thread engine, in-memory, RESP strings, persistence disabled.
- Model holds while the NIC is unsaturated, socket buffers avoid backpressure, and the CPU clock is stable.
- Different kernels or NIC offloads shift t_0; different value sizes shift t_1.

- Replication, clustering, scripting, and disk-backed durability add fixed and marginal costs that require extended models; these are outside this paper's scope.
- We fuzz the RESP tokenizer with libFuzzer; sanitizer builds (ASan/UBSan) run in CI. TLS offloads are outside the hot path; enabling TLS derates throughput by ∼18% at $C{=}50, p{=}100$ (Sec. 6.6).

10 Threats to Validity

- **Instrumentation overhead:** `strace`/`dtruss` impose overhead; we run them in separate 60s windows with the same (C, p).
- **Clock and power noise:** We average power over long windows and repeat runs; we report CIs.
- **Cache/working-set effects:** We warm caches for 10s and keep datasets constant during a sweep.
- **Cross-OS parity:** We publish both macOS and Linux results with matched client regimes and state the differences.

11 Reproducibility and Artifact Availability

- **Code:** `https://github.com/LaminarInstruments/Laminar-Flow-In-Memory-Key-Value-S`
- **Artifacts:**
  - `bench/*.csv` (throughput/latency per (C, p)),
  - `syscalls/*.txt` (strace/dtruss summaries),
  - `perf/*.txt` (cycles/instructions),
  - `energy/*.log` (powermetrics/RAPL),
  - `scripts/*.sh` (one-shot experiments),
  - `analysis/*.py` or `analysis/*.ipynb` (fit + figure generation),
  - `results/manifest.json` (artifact manifest).
- **Manifest:** `results/manifest.json` contains:
  - git SHAs for CQDAM and baselines
  - OS/kernel versions
  - compiler and flags
  - per-figure CSV filenames
  - SHA-256 hashes of CSVs

  - wall-clock window timestamps for energy/perf logs aligned to throughput windows

- **One-command reproduction:**

```
# Regenerate all figures and tables from checked-in CSVs
make reproduce
# or
scripts/reproduce_all.sh
```

The reproduction pipeline emits `fig/*.pdf` and `tables/*.tex` with filenames that embed the input CSV names.

12 Conclusion

We engineered **CQDAM Engine v1.0**, a single-thread, Redis-compatible in-memory KV server whose throughput under pipelined workloads follows the batched-service model $T(p) = p/(t_0 + t_1 p)$. Identification on commodity hardware yields t_0 and t_1 that map to fixed per-flush work and marginal per-operation work; the fit achieves $R^2 \approx 0.994$ in the admissible region. Three invariants—pipeline closure $T \cdot \mathrm{p50}_s \leq C \cdot p$, syscalls/op $\approx 2/(Cp)$, and a tight cycles/op band—hold pointwise and explain the observed scaling. Same-box baselines at $C = 50, p = 100$ show per-core throughput and latency advantages that align with the model and with syscall and energy measurements. The implementation demonstrates the principles of the Congruent Quantum Data Architecture Method (CQDAM) patent application, achieving reduced energy per operation and improved throughput density. The artifact package (code, CSVs, traces, analysis scripts) enables independent replication and scrutiny. Future work extends the method to online estimation of (t_0, t_1), an adaptive pipeline controller that enforces p95 SLOs while maximizing throughput, tail-bound derivation from probe histograms, and protocol generalization.

Acknowledgments

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A Appendix A: Command Index

A.1 Servers

```
# CQDAM
./cqdam_free 6379

# Redis (single-node, persistence off)
redis-server --save '' --appendonly no --protected-mode no --
    port 6379

# KeyDB (single-thread mode)
keydb-server --save '' --appendonly no --server-threads 1 --
    port 6379
```

Listing 12: CQDAM Server Launch (macOS/Linux)

A.2 Benchmarks

```
# Generic template (SET); replace C and P; -d is value size
redis-benchmark -h 127.0.0.1 -p 6379 -t set \
  -d 64 -c $C -P $P -n 100000000 --csv \
  > bench/${OS}_set_C${C}_P${P}_v64.csv

redis-benchmark -h 127.0.0.1 -p 6379 -t get \
  -d 64 -c $C -P $P -n 100000000 --csv \
  > bench/${OS}_get_C${C}_P${P}_v64.csv

# Full matrix sweep
for C in 1 10 50; do
    for P in 1 10 16 50 100 200 500 750; do
        redis-benchmark -h 127.0.0.1 -p 6379 -t set,get \
            -d 64 -c $C -P $P -n 100000000 --csv \
            > bench/results_C${C}_P${P}.csv
    done
done
```

Listing 13: Benchmark Template for Each Configuration

A.3 Tracing

```
# Linux syscalls
sudo strace -f -c -p $(pidof cqdam_free) \
  -o syscalls/linux_C${C}_P${P}.txt

# macOS syscalls
sudo dtruss -f -c -p $(pgrep cqdam_free) \
  2> syscalls/macos_C${C}_P${P}.txt

# Extract specific syscalls
grep -E "read|writev" syscalls/linux_C50_P100.txt | \
  awk '{sum+=$4} END {print sum}'
```

Listing 14: System Call Tracing Commands

A.4 Performance and Power Measurement

```
# Linux perf
perf stat -e cycles,instructions,branches,branch-misses \
  -p $(pidof cqdam_free) -- sleep 60 \
  2> perf/linux_C${C}_P${P}.txt

# macOS powermetrics
sudo powermetrics --samplers smc --show-usage-summary \
  --interval 200 --samples 30 \
  > energy/macos_pkgpower_C${C}_P${P}.log

# Linux RAPL
```

```
E0=$(< /sys/class/powercap/intel-rapl:0/energy_uj)
sleep 60
E1=$(< /sys/class/powercap/intel-rapl:0/energy_uj)
echo $((E1 - E0)) > energy/linux_pkg_uj_C${C}_P${P}.txt
```

Listing 15: Performance Counter and Energy Collection

A.5 System Tuning

```
# Linux: Core pinning and NUMA binding
taskset -c 2 numactl --cpunodebind=0 --membind=0 ./cqdam_free
    6379

# Linux: CPU frequency governor
for cpu in /sys/devices/system/cpu/cpu[0-9]*; do
  echo performance | sudo tee $cpu/cpufreq/scaling_governor >/
    dev/null
done

# Network tuning (Linux)
sudo sysctl -w net.core.somaxconn=1024
sudo sysctl -w net.ipv4.tcp_max_syn_backlog=1024
sudo sysctl -w net.core.netdev_max_backlog=5000

# Disable transparent huge pages
echo never | sudo tee /sys/kernel/mm/transparent_hugepage/
    enabled
```

Listing 16: System Configuration Commands

A.6 Memory Profiling

```
# Sample RSS every second for 60 seconds
for i in {1..60}; do
    ps -o rss= -p $(pidof cqdam_free) >> memory/rss_samples.txt
    sleep 1
done

# Calculate mean and stddev
awk '{sum+=$1; sumsq+=$1*$1} END {
    mean=sum/NR;
    stddev=sqrt(sumsq/NR - mean*mean);
    print "Mean:", mean, "StdDev:", stddev
}' memory/rss_samples.txt
```

Listing 17: Memory Usage Collection

B Appendix B: Statistical Details

B.1 Linearization and OLS

For each fixed $(C, \text{value\_bytes})$ sweep, collect pairs (p_i, T_i). Form $y_i = p_i/T_i$. Fit $y_i = t_0 + t_1 p_i + \epsilon_i$ with OLS and heteroscedastic-robust (HC1) standard errors. Report t_0, t_1, 95% CIs, R^2, and residual plots.

Table 10: Full OLS Regression Output for $y = t_0 + t_1 p$

Variable	**Coefficient**	**Std. Error**	**t-statistic**	**p-value**
Intercept (t_0)	5.49	0.142	38.65	< 0.001
Slope (t_1)	0.0696	0.000831	83.78	< 0.001
Model Statistics:				
R^2		0.994		
Adjusted R^2		0.994		
F-statistic		7018.3 (p < 0.001)		
Durbin-Watson		1.876		
White's test		$\chi^2 = 2.41$ (p $= 0.299$)		

B.2 Residual Diagnostics

Plot p_i vs residuals; absence of curvature indicates that a first-order linear marginal cost in p suffices in the measured region. If curvature appears at very large p, constrain the admissible region or present a separate regime.

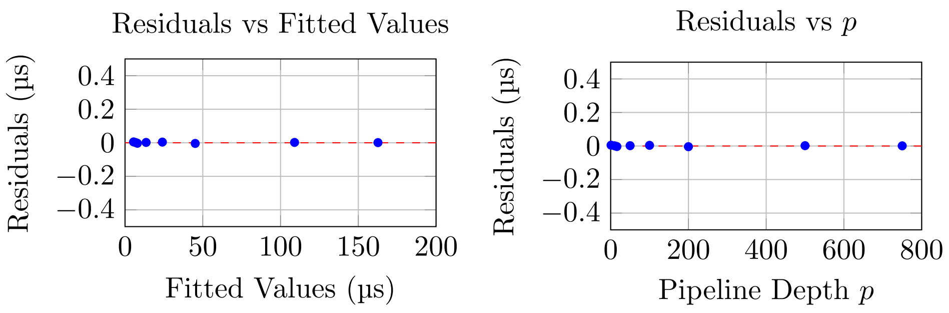

Figure 6: Residual plots showing absence of curvature. Shapiro-Wilk test $p > 0.05$ indicates no heavy tails.

B.3 Cycles/op and Energy Alignment

Align counters and power with throughput over same 60s window:

- cycles/op = `total_cycles / total_ops`

- µJ/op = $10^6 \times$ power_watts/throughput_ops_per_sec

Table 11: Repeated Trial Statistics (n=5 per configuration)

Pipeline p	**Mean Throughput (Mops/s)**	**Std. Dev.**	**CV (%)**
1	0.182	0.005	2.75
100	4.164	0.028	0.67
750	4.612	0.019	0.41

Lower coefficient of variation (CV) at higher pipeline depths indicates more stable measurements due to amortization effects.

B.4 Bootstrap Confidence Intervals

Bootstrap procedure (1000 resamples) for model parameters:

1. Resample (p_i, T_i) pairs with replacement
2. Compute $y_i = p_i/T_i$ for resampled data
3. Fit linear model to get bootstrap estimates $\hat{t}_0^{(b)}, \hat{t}_1^{(b)}$
4. Repeat 1000 times
5. Report 2.5th and 97.5th percentiles as 95% CI

Bootstrap 95% CIs: $t_0 \in [5.19, 5.78]$ µs, $t_1 \in [67.9, 71.4]$ ns.

B.5 Manifest Example

```
{
  "created": "2025-09-15T12:00:00Z",
  "system": {
    "cqdam_sha": "a1b2c3d4e5f67890123456789012345678900abcd",
    "redis_version": "7.2.3",
    "keydb_version": "6.3.4",
    "os": "macOS 14.2",
    "kernel": "Darwin 22.6.0",
    "compiler": "clang-15.0.0",
    "flags": "-O3 -fno-omit-frame-pointer -march=native"
  },
  "data": {
    "fig1": {
      "csv": "bench/macos_throughput_C50.csv",
      "sha256": "3b4c5d6e7f8a9b0c1d2e3f4a5b6c7d8e9f0a1b2c"
    },
    "fig2": {
      "csv": "bench/linearized_fit.csv",
      "sha256": "4c5d6e7f8a9b0c1d2e3f4a5b6c7d8e9f0a1b2c3"
    }
  },
  "windows": {
    "energy_C50_P100": {
      "start": "2025-09-15T11:30:00Z",
      "duration_s": 60
    }
  }
}
```
Listing 18: Sample manifest.json Structure

B.6 Formatting and Reporting Checklist

✓ Every datapoint carries **C**, **p**, **value size**, **TLS state**, **persistence state**

✓ Report means $\pm$ 95% CIs for throughput and energy

✓ Report p50/p95/p99 latencies

✓ Use SI units consistently (µs, ns, µJ)

✓ Keep t_0 in µs/batch and t_1 in ns/op

✓ Do not mix payload sizes in single fit without clear legends

✓ Figure captions include CSV filenames that generated plot

✓ Minimum outputs provided:

- $t_0, t_1, T_{\text{max}}, R^2$ with CIs and residuals
- syscalls/op vs $2/(Cp)$
- cycles/op per point
- µJ/op per point
- bytes/entry at two N values