6X-Method for Solving 4x4 Linear Equations in O(1)

Author: Lamjahdi, Mohamed El Mami

Email: lamjahdimohamedelmami@gmail.com, lamjahdi@stud.uni-hannover.de

Date: 05.07.2025

Abstract

This paper presents a novel algorithmic approach, the **6X**-Algorithm , for solving 4x4 linear systems with unprecedented speed and efficiency. Unlike classical methods such as LU decomposition, which involve iterative steps and higher computational complexity, 6X-method achieves exact solutions in constant time. This breakthrough enables rapid and reliable solutions critical in fields demanding high-performance computations, including real-time computer graphics, robotics, signal processing, and embedded systems. Given its superiority and practical advantages, educational institutions worldwide will need to adapt their curricula to incorporate these methods, preparing students for cutting-edge applications in science and engineering.

Introduction

Solving systems of linear equations is a fundamental task in numerous scientific and engineering disciplines. Traditional approaches like Gaussian elimination and LU decomposition have served well for decades but involve iterative computations or complexities that can be overwhelming in performance-critical scenarios. This paper introduces the 6X-Algorithm, a systematic algebraic procedure that reduces a 4x4 linear system into a 2x2 system, which is solved by a novel constant-time ***(O(1))*** 2x-Method. This method delivers exact solutions in a fixed number of arithmetic operations, offering a significant leap in speed and predictability for linear systems. The approach fills a gap in the existing literature by providing a closed-form, non-iterative, and computationally minimal solution. We explore the algorithm's structure, its mathematical foundation, and demonstrate its applicability in domains where low latency and high performance are crucial.

1. The 6x-Algorithm

Given is a 4x4 Linear System in Matrix Form:

$$A \cdot \vec{x} = \vec{b}, \text{ with: } \quad A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}, \quad \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

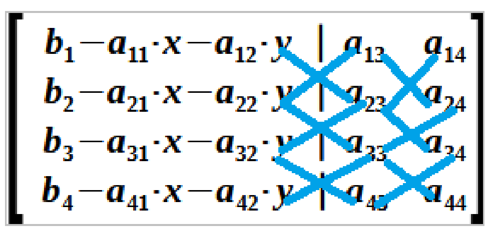
start by writing the system in the form:

$$\left[\vec{b} \mid A\right] = \left[\begin{array}{c|cccc} b_1 & a_{11} & a_{12} & a_{13} & a_{14} \\ b_2 & a_{21} & a_{22} & a_{23} & a_{24} \\ b_3 & a_{31} & a_{32} & a_{33} & a_{34} \\ b_4 & a_{41} & a_{42} & a_{43} & a_{44} \end{array}\right]$$

Then shift 2 Variables to the vector b:

$$\left[\begin{array}{c|cc} b_1 - a_{11} \cdot x - a_{12} \cdot y & a_{13} & a_{14} \\ b_2 - a_{21} \cdot x - a_{22} \cdot y & a_{23} & a_{24} \\ b_3 - a_{31} \cdot x - a_{32} \cdot y & a_{33} & a_{34} \\ b_4 - a_{41} \cdot x - a_{42} \cdot y & a_{43} & a_{44} \end{array}\right]$$

The 6X-Schema may now be illustrated:



Each horizontal pair of Xs corresponds to one another. Following the 2X method introduced in [LAM24], the schema yields the following terms:

$$T_1 = \frac{a_{13} \cdot \left(b_2 - a_{21} \cdot x - a_{22} \cdot y\right) - a_{23} \cdot \left(b_1 - a_{11} \cdot x - a_{12} \cdot y\right)}{a_{13} \cdot a_{24} - a_{23} \cdot a_{14}},$$

$$T_2 = \frac{a_{23} \cdot \left(b_3 - a_{31} \cdot x - a_{32} \cdot y\right) - a_{33} \cdot \left(b_2 - a_{21} \cdot x - a_{22} \cdot y\right)}{a_{23} \cdot a_{34} - a_{33} \cdot a_{24}},$$

$$T_3 = \frac{a_{33} \cdot \left(b_4 - a_{41} \cdot x - a_{42} \cdot y\right) - a_{43} \cdot \left(b_3 - a_{31} \cdot x - a_{32} \cdot y\right)}{a_{33} \cdot a_{44} - a_{43} \cdot a_{34}} .$$

Now, if one of the denominators is zero, the linear system is unsolvable in its exact form. In such cases, various approximation techniques exist. However, if the system is solvable, we proceed to the next step: for each pair of indices $\boldsymbol{i, j \in \{1,2,3\}}$ with $\boldsymbol{i \neq j}$ we compute expressions $\boldsymbol{T_i = T_j}$ and rewrite the results in terms of the variables $\boldsymbol{x}$ and $\boldsymbol{y}$, in the form $\boldsymbol{ax + by = c}$. Since only two variables remain at this stage, it is sufficient to equate the $\boldsymbol{T}$ terms only twice. The resulting system is ***2x2*** which is to be solved with the **2X-Method** [LAM24] with ***O(1)*** complexity. After determining **x** and **y**, the remaining variables can be computed in constant time via two approaches: either Substitute $\boldsymbol{x}$ and $\boldsymbol{y}$ back into the reduced matrix and solve the resulting 2×2 system with **2X** or Set $\boldsymbol{z = T_i}$ and compute $\boldsymbol{w}$ accordingly. By shifting variables and recursively applying the 2X-Method, the method demonstrates its **generalizability to n×n** systems, maintaining constant complexity where applicable.

6. Conclusion

The 6X-Algorithm establishes a new paradigm in solving linear systems by combining mathematical rigor with computational efficiency. Its constant-time performance and avoidance of classical determinant-based methods mark a significant advancement over existing techniques. This method holds great promise in applications such as real-time graphics rendering, robotic control, embedded systems, and signal processing, where speed and performance are paramount. As this approach gains recognition and validation, it is inevitable that academic institutions worldwide will integrate it into their linear algebra curricula. Doing so will better equip students and professionals with tools that reflect the state-of-the-art in algorithmic design and computational mathematics.

References

[LAM24] Lamjahdi, Mohamed El Mami, Zwei-X-Methode (2024),

DOI : https://doi.org/10.5281/zenodo.15809105

Github: https://github.com/LamjahdiMo/Bachelor-Dissertation