What If Linear Equations Could Be Solved Instantly?

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Abstract

This paper presents a collection of constant-time (O(1)) methods for solving linear equations and matrix inversion. The work begins with the 2X method for solving 2x2 linear systems through direct computation, followed by a formal proof. A symbolic and practical O(1) method for computing 2x2 matrix inverses is then presented. Finally, a diagrammatic and algebraic method is introduced for solving 3x3 linear systems (Gitter or 4X method). All content is drawn exclusively from the original material.

Introduction

For decades, Gaussian and Gauss-Jordan elimination have stood as the standard tools for solving linear systems. These methods, though reliable, come with a price: **time.** Specifically, they involve computational complexity on the order of **O(n^3)**, which becomes a bottleneck in real-time and high-precision applications. This paper introduces and formalizes three alternative methods:

- **The 2X Method** (for 2×2 systems)
- **A direct constant-time Matrix Inversion**
- **The Gitter / 4X Method** (for 3×3 systems)

These methods provide **performance in O(1)**, achieved through **visual cross-structure logic**, not row manipulation or pivoting. By replacing multi-step algorithms with single-step visual-algebraic identities, they aim to **outperform traditional methods both in speed and conceptual clarity.**

1. The 2X Method for 2x2 Linear Systems

The 2X method offers a direct approach for solving a 2x2 linear system in the form:

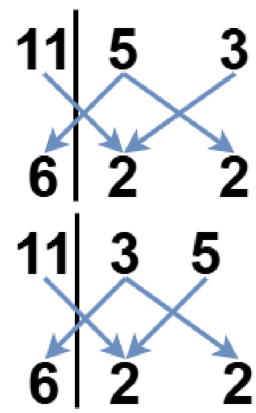
Let us begin with a standard 2×2 system:

$$5x + 3y = 11$$
$$2x + 2y = 6$$

Rewritten in augmented form:

$$\begin{array}{r|cc} 11 & 5 & 3 \\ 6 & 2 & 2 \end{array}$$

X-Shape Construction



To solve for **y**, draw an **X** across the matrix:

y = (5×6 - 2×11) / (5×2 - 3×2) = 2

To solve for **x**, switch the 5 and 3 in the first row:

$$\begin{array}{r|cc} 11 & 3 & 5 \\ 6 & 2 & 2 \end{array}$$

x = (3×6 - 2×11) / (3×2 - 5×2) = 1

2. Proof of the 2X Method

A detailed proof is constructed to show the validity of the 2X method. The system is algebraically manipulated:

2.1. Generalized Form

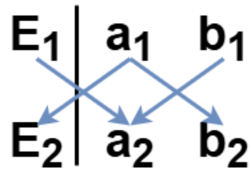
Given:

$$a1\ x + b1\ y = E1$$
$$a2\ x + b2\ y = E2$$

Write as:

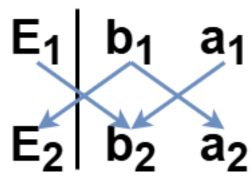
E1 |	**a1**	**b1**
E2 |	**a2**	**b2**

For the first Solution:



x = (b1×E2 - b2×E1) / (b1×a2 – a1×b2)

and for the second:



y = (a1×E2 - a2×E1) / (a1×b2 – b1×a2)

Rewrite in vector form:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{a_1 \cdot b_2 - b_1 \cdot a_2} \cdot \begin{pmatrix} b_2 \cdot E_1 - b_1 \cdot E_2 \\ a_1 \cdot E_2 - a_2 \cdot E\ 1 \end{pmatrix} = \frac{1}{det\,(M)} \cdot adj\,(M) \cdot \begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = M^{-1} \cdot \begin{pmatrix} E_1 \\ E_2 \end{pmatrix} \quad \textbf{[I]}$$

hence:

$M \cdot \vec{x} = \vec{E}$ is the original equation.

The proof confirms that the cross-pattern method yields valid and equivalent values for x and y in every algebraically consistent case where the denominator is non-zero.

3. O(1) Method for Matrix Inverse

To explain the method let’s begin with a simple example:

given:

$$M = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

Interpret the matrix as representing a linear transformation, applied to an abstract vector, and reformulate it in the following form:

$$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}, \text{ with } \quad M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

solve for x and y in terms of **A** and **B** using the previous method by drawing the X's :

$$x = \frac{B - 3 \cdot A}{4 - 6} = \frac{3 \cdot A}{2} - \frac{B}{2} \quad \text{and } y = \frac{2 \cdot B - 4 \cdot A}{6 - 4} = -2 \cdot A + B$$

This means:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{3 \cdot A}{2} - \frac{B}{2} \\ -2 \cdot A + B \end{pmatrix} = \begin{bmatrix} \frac{3}{2} & \frac{-1}{2} \\ -2 & 1 \end{bmatrix} \cdot \begin{pmatrix} A \\ b \end{pmatrix} \Leftrightarrow \begin{bmatrix} \frac{3}{2} & \frac{-1}{2} \\ -2 & 1 \end{bmatrix} = M^{-1}$$

This inversion technique avoids row operations or Gaussian elimination, and instead applies a fixed, deterministic pattern. It requires calculating only a few multiplications and subtractions and is applicable for higher dimensions.

4. The Gitter (4X) Method for 3x3 Systems

A symbolic method is presented to solve 3x3 systems directly. The system is structured as:

$$\begin{aligned} x + y + z &= 6 \\ 2x + y - z &= 1 \\ -x + 2y + 2z &= 9 \end{aligned}$$

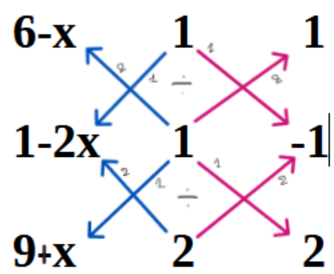
Start by expressing it in the form:

6 | 1 1 1

1 | 2 1 -1

9 | -1 2 2

The adjudicate for each solution can be computed, and the determinant determined as shown in [I]. However, this method remains suboptimal. A more efficient approach involves isolating one variable on the left-hand side and plotting 4X as illustrated in the figure below.



This approach allows for a significantly more efficient extraction of the solutions. The first two marked X's above yield the first function of the variable x, while the two marked X's below yield the second function of x. Solving for x is accomplished by equating these two functions. To extract y and z easily, the matrix can be reduced to a 2×2 form using the first solution, and the X-Drawing method can then be applied. This method enables solution extraction in constant time, eliminating the need for lengthy iterations or complex computations.

5. Validation

The method was verrified in laboratory of Institut Institute of Manufacturing Engineering and Machine Tools as a part of bachelor thesis in the leibniz University of Hanover. The work deals with the idealization of control techniques to improve the rolling process, particularly to cope with transfer stagnation and external disturbances. A central approach was the use of the Smith estimator, which displaces delays caused by interface latencies by imitating the process, thus allowing for more precise control. In combination with a PID controller, the imitation proved to be particularly suitable. Simulations with Simulink and TwinCAT confirm the results. Two innovative estimators were developed as part of the work: the M.A-Estimator (Mean Parameter Asynchronous Estimator) and the M.E-Estimator (Mean Value Real-Time Estimator). Both estimators work in parallel with the control system and improve the responsiveness of the process in the event of disturbances. The M.A-Estimator optimizes the rolling force by calculating asynchronous input signals, while the M.E-Estimator uses real-time measurements to dynamically modify the control parameters. The verification of these approaches is carried out through tests and simulations, which ensure their suitability for minimizing dead times and increasing stability in the process. Special attention is paid to the verification of the M.E-Estimator, which has proven to be particularly suitable for improving systems in estimation by means of **Bernoulli estimation*** and the Two-X method. These methods allow for a fast and precise determination for unsolvable matrices, especially in 2x2 systems. The estimator thus represents an ideal extension of the control techniques and proves to be the best method for controlling processes with inactivity phases. Overall, the work confirms that the O(1)-methods presented in this paper allow for an overwhelming improvement in performance. These innovative approaches contribute to minimizing transfer stagnation by providing precise predictions during dead time which significantly increases the quality and productivity of the rolling process.

***Bernoulli-Approximation:**

The main reason for the unsolvability of linear systems is division by zero. This also occurs in the Gauss algorithm, where the pivot is divided by zero. One considers an unsolvable system with abstract variables and constants greater than 3×3, and applies the Gauss-Jordan algorithm until a division by a zero pivot is reached. Then, one steps back and performs a limit calculation of the division, which can only be done using a Bernoulli limit approach.

This method was implemented in a Python environment for the prediction of monotonically increasing and decreasing data, as well as partially oriented data with randomized middle values, and achieved a precision of over 80%. Since Gauss-Jordan elimination is not optimal, this method must be generalized to 2×2 and 3×3 cases. The generalization results in a limit value of 1 in the case of unsolvability. Subsequently, the Bernoulli approximation with 1 as the limit value was integrated into the 2X method's estimator — but only in the case of matrix unsolvability (i.e., in the case distinction when the second X or the denominator equals zero).

6. Conclusion

This work presents three constant-time methods that fundamentally simplify the process of solving linear equations and computing inverses. Their simplicity, speed, and elegance make them highly suitable for real-time and embedded applications.

References

[LAM24] Lamjahdi, Mohamed El Mami, Zwei-X-Methode (2024),

Github: https://github.com/LamjahdiMo/Bachelor-Dissertation

DOI : https://doi.org/10.5281/zenodo.15786689