Practice IMC

Seed: 871306

Problem 1 *Find all positive integers* $n, k_1, \ldots, k_n$ *such that* $k_1 + \cdots + k_n = 5n - 4$ *and*

$$\frac{1}{k_1} + \cdots + \frac{1}{k_n} = 1.$$

Problem 2 *Let* B *be a set of more than* $2^{n+1}/n$ *distinct points with coordinates of the form* $(\pm 1, \pm 1, \ldots, \pm 1)$ *in* n*-dimensional space with* $n \geq 3$*. Show that there are three distinct points in* B *which are the vertices of an equilateral triangle.*

Problem 3 *Prove that any monic polynomial (a polynomial with leading coefficient 1) of degree* n *with real coefficients is the average of two monic polynomials of degree* n *with* n *real roots.*

Problem 4 *Let* n *be a positive integer. Determine, in terms of* n*, the largest integer* m *with the following property: There exist real numbers* $x_1, \ldots, x_{2n}$ *with* $-1 < x_1 < x_2 < \cdots < x_{2n} < 1$ *such that the sum of the lengths of the* n *intervals*

$$[x_1^{2k-1}, x_2^{2k-1}], [x_3^{2k-1}, x_4^{2k-1}], \ldots, [x_{2n-1}^{2k-1}, x_{2n}^{2k-1}]$$

is equal to 1 for all integers k *with* $1 \leq k \leq m$*.*

Problem 5 *Can an arc of a parabola inside a circle of radius 1 have a length greater than 4?*