Practice IMC

Seed: 618247

Problem 1 *A club consisting of 11 men and 12 women needs to choose a committee from among its members so that the number of women on the committee is one more than the number of men on the committee. The committee could have as few as 1 member or as many as 23 members. Let N be the number of such committees that can be formed. Find the sum of the prime numbers that divide N.*

Problem 2 *Let a, b, c be real numbers greater than or equal to 1. Prove that*

$$\min\left(\frac{10a^2-5a+1}{b^2-5b+10}, \frac{10b^2-5b+1}{c^2-5c+10}, \frac{10c^2-5c+1}{a^2-5a+10}\right) \le abc$$

Problem 3 *Given that a, b, c, d, e are real numbers such that*

$a+b+c+d+e = 8,$

$a^2+b^2+c^2+d^2+e^2 = 16.$

Determine the maximum value of e.

Problem 4 *Let*

$$\begin{array}{cccc} a_{1,1} & a_{1,2} & a_{1,3} & \cdots \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdots \\ a_{3,1} & a_{3,2} & a_{3,3} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{array}$$

be a doubly infinite array of positive integers, and suppose each positive integer appears exactly eight times in the array. Prove that $a_{m,n} > mn$ for some pair of positive integers (m, n).

Problem 5 *Let $\mathbb{Z}$ be the set of integers. Find all functions $f : \mathbb{Z} \to \mathbb{Z}$ such that*

$$xf(2f(y)-x) + y^2 f(2x - f(y)) = \frac{f(x)^2}{x} + f(yf(y))$$

for all $x, y \in \mathbb{Z}$ with $x \neq 0$.