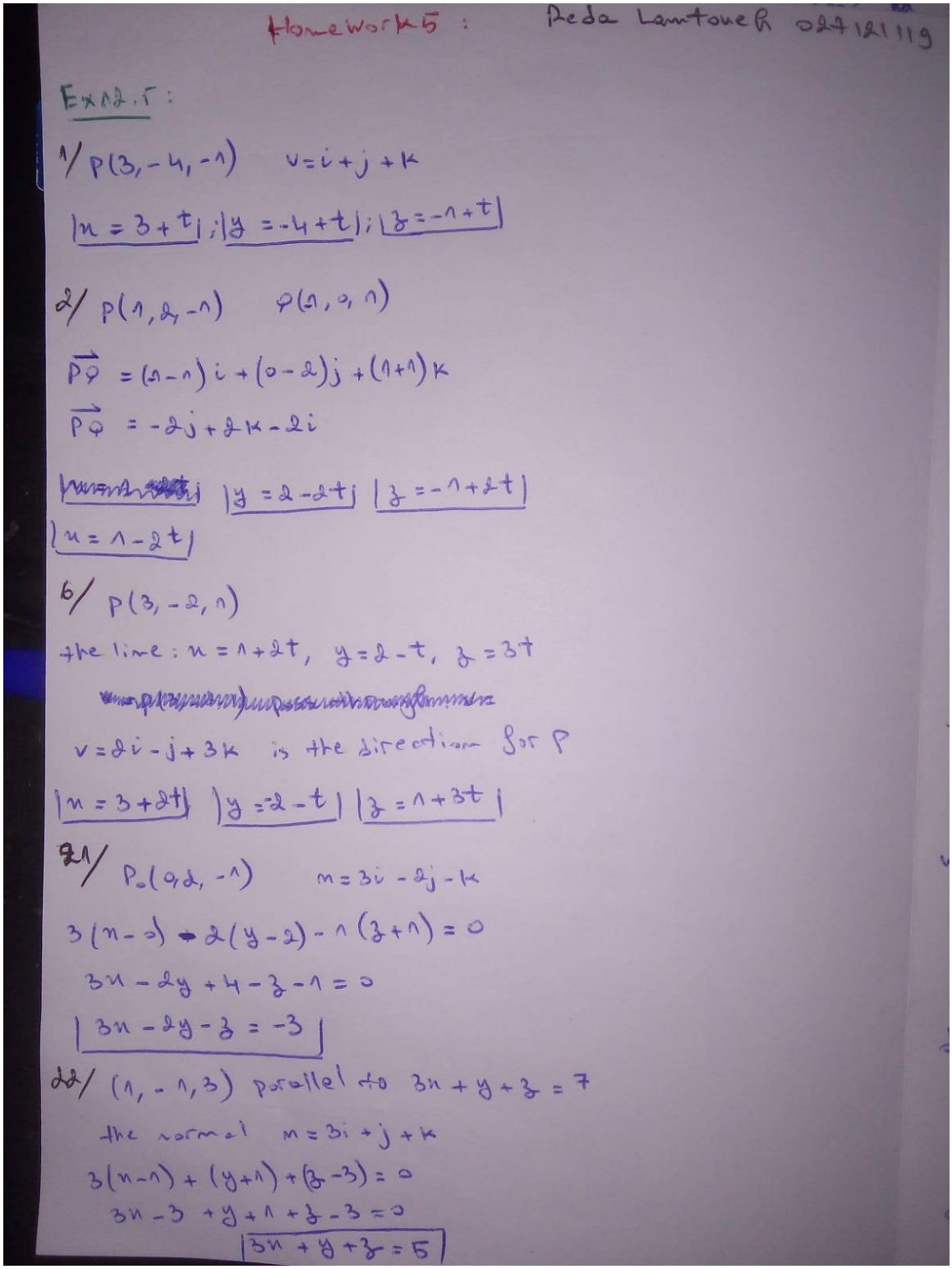
homework5 RedaLamtoueh 027121119



Homework 5 : Reda Lamtoueh 027121119

Exo2.5 :

1/ P(3, -4, -1) $v = i + j + k$

$x = 3 + t$; $y = -4 + t$; $z = -1 + t$

2/ P(1, 2, -1) Q(1, 0, 1)

$\overrightarrow{PQ} = (1-1)i + (0-2)j + (1+1)k$

$\overrightarrow{PQ} = -2j + 2k - 2i$

$y = 2 - 2t$ $z = -1 + 2t$

$x = 1 - 2t$

6/ P(3, -2, 1)

the line : $x = 1 + 2t$, $y = 2 - t$, $z = 3t$

$v = 2i - j + 3k$ is the direction for P

$x = 3 + 2t$ $y = -2 - t$ $z = 1 + 3t$

21/ $P_0(0, 2, -1)$ $n = 3i - 2j - k$

$3(x - 0) - 2(y - 2) - 1(z + 1) = 0$

$3x - 2y + 4 - z - 1 = 0$

$3x - 2y - z = -3$

22/ (1, -1, 3) parallel to $3x + y + z = 7$

the normal $n = 3i + j + k$

$3(x - 1) + (y + 1) + (z - 3) = 0$

$3x - 3 + y + 1 + z - 3 = 0$

$3x + y + z = 5$

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33/ $P(0,0,12)$, $x = 4t$, $y = -2t$, $z = 2t$

normal $\mathbf{v} = 4i - 2j + 2k$

the point $P(0,0,0)$ lies on the plane

$\overrightarrow{PQ} = 12k$

$$d = \left| 12k \times \frac{4i - 2j + 2k}{\sqrt{4^2 + 2^2 + 2^2}} \right|$$

$$\Rightarrow 12k \times (4i - 2j + 2k) = \begin{vmatrix} i & j & k \\ 0 & 0 & 12 \\ 4 & -2 & 2 \end{vmatrix} = -12 \begin{vmatrix} i & j \\ 4 & -2 \end{vmatrix}$$

$$= -12(-2i - 4j)$$

$$= 24i + 48j$$

$$d = \left| \frac{24i + 48j}{\sqrt{24}} \right| = \frac{\sqrt{24^2 + 48^2}}{\sqrt{24}} = \frac{24\sqrt{5}}{\sqrt{24}}$$

39/ $P(2,-3,4)$, the plane $x + 2y + 2z = 13$

the normal $n = i + 2j + 2k$

we can check easily that the point $P(13,0,0)$ lies on the plane

$$\overrightarrow{PQ} = (13-2)i + (0+3)j + (0-4)k$$

$$= 11i + 3j - 4k$$

$$d = \left| \frac{\overrightarrow{PQ} \cdot n}{|n|} \right| = \frac{11 + 6 - 8}{\sqrt{1+4+4}}$$

$$d = \frac{9}{3} \Rightarrow \boxed{d = 3}$$

18/ $5x + y - z = 10$, $x - 2y + 3z = -1$

$n_1 = 5i + j - k$, $n_2 = i - 2j + 3k$

$$\theta = \cos^{-1}\left(\frac{n_1 \cdot n_2}{|n_1||n_2|}\right) = \cos^{-1}\left(\frac{5 - 2 - 3}{\sqrt{27} \times \sqrt{14}}\right) = \cos^{-1}(0) = \frac{\pi}{2}$$

Ex 12.6:

1/ $x^2 + y^2 + 4z^2 = 10$

$\frac{x^2}{10} + \frac{y^2}{10} + \frac{2}{5}z^2 = 1$ Ellipsoid. (c)

6/ $x = -y^2 - z^2$

$y^2 + z^2 = -x$ paraboloid. (e)

12/ $9x^2 + 4y^2 + 2z^2 = 36$

$\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{18} = 1$ Ellipsoid (e)