Option Pricing Analysis for Hoa Phat Group (HPG)

Financial Derivatives Project

Financial Derivatives Pricing

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1 Data Collection and Initial Analysis

1.1 Data Download

Data for American Airlines (AAL) stock was downloaded from Yahoo Finance with maximum available price history from December 2007 to June 2025. The data begins on December 14, 2007 with an opening price of 2,587.907 VND and a closing price of 2,616.661 VND.

1.2 Price Visualization

Figure 1 shows the historical price movement of HPG stock over the entire period. Notable features include significant price fluctuations over the 18-year period, with current price levels around 2,587.907 VND as of June 2025.

Figure 1: HPG Stock Price History (2007-2025)

1.3 Log Returns Analysis

Daily log returns were calculated using the formula:

$$r_t = \ln\left(\frac{P_t}{p_{t-1}}\right) \tag{1}$$

Figure 2 presents the time series of log returns, highlighting the volatility clustering phenomenon typical in financial markets.

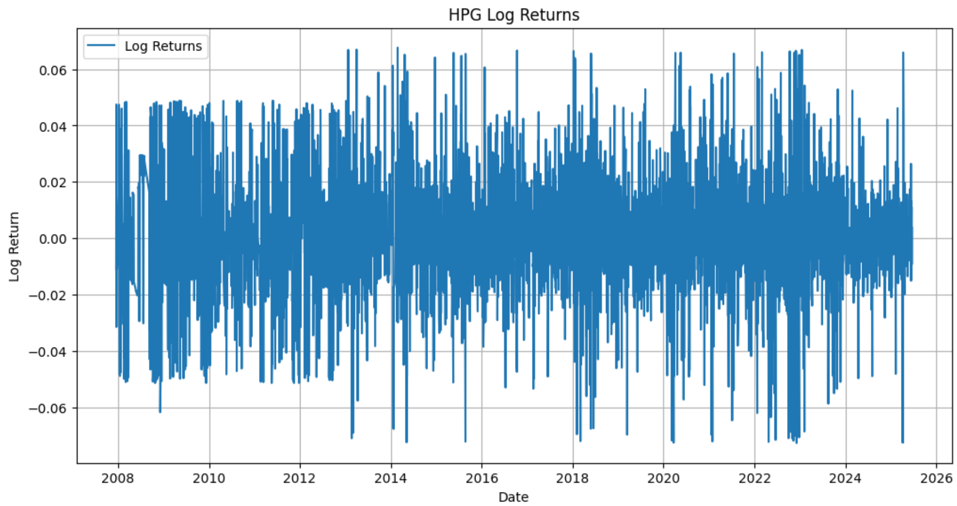


Figure 2: HPG Daily Log Returns (2007-2025)

2 Normality Testing for Log Returns

2.1 Visual Analysis

2.1.1 QQ Plot

Figure 3 shows the QQ plot for HPG log returns compared against theoretical normal distribution quantiles.

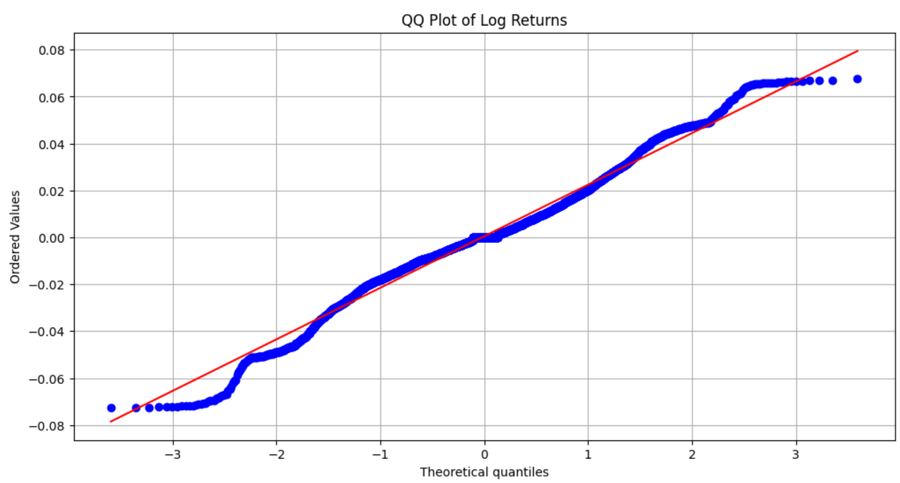


Figure 3: QQ Plot of HPG Log Returns

QQ Plot Interpretation:
- If points follow the straight diagonal line, data is normally distributed.
- Points deviating from the line (especially at tails) indicate non-normality.
- Financial returns typically show deviations at both tails, suggesting heavier tails.
- Based on the QQ plot above, we can visually assess if HPG returns show fat tails.

2.1.2 Histogram with Normal Distribution

Figure 4 displays the histogram of log returns with a normal distribution overlay.

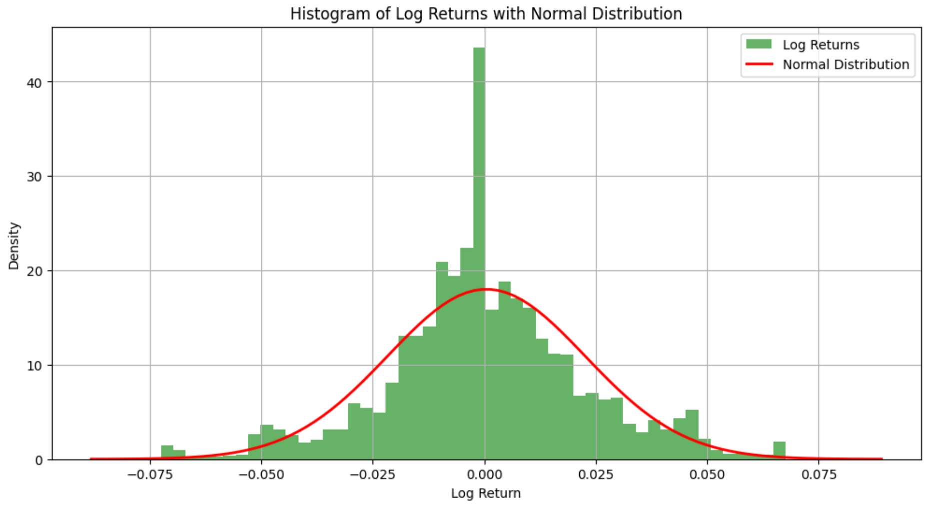


Figure 4: Histogram of HPG Log Returns with Normal Distribution Curve

Histogram Interpretation:

- Compare the actual distribution (green bars) with the theoretical normal curve (red line).
- Financial returns typically show higher peak (excess kurtosis) and fatter tails.
- If the green bars are higher in the center and at extremes than the red line, this indicates non normality.
- The central spike indicates a higher concentration of small daily returns, while the divergence at the ends indicates more outliers than a Gaussian model would expect.

2.2 Statistical Tests

2.2.1 Jarque-Bera Test

The Jarque-Bera test examines the goodness-of-fit of data to a normal distribution by analyzing skewness and kurtosis.

Statistic	**Value**
Test Statistic	128.2534
p-value	0.0000

Table 1: Jarque-Bera Test Results

2.2.2 Kolmogorov-Smirnov Test

The Kolmogorov-Smirnov test compares the empirical distribution function with the cumulative distribution function of a reference distribution.

Statistic	**Value**
Test Statistic	0.060128
p-value	0.0000

Table 2: Kolmogorov-Smirnov Test Results

2.2.3 Anderson-Darling Test

The Anderson-Darling test gives more weight to the tails of the distribution than the Kolmogorov-Smirnov test. Particularly relevant for financial data where extreme events are important.

Statistic	**Value**
Test Statistic	29.7003

Table 3: Anderson-Darling Test Results

2.3 Normality Test Conclusion

Based on the visual evidence and statistical tests, HPG log returns exhibit significant deviations from normality. The QQ plot shows heavy tails, indicating more extreme values than would be expected in a normal distribution. The Jarque-Bera, Kolmogorov-Smirnov, and Anderson-Darling tests all reject the null hypothesis of normality with p-values near zero. This non-normality is typical of financial returns and has important implications for option pricing models.

3 Volatility Estimation

3.1 Historical Volatility

$$\boldsymbol{\sigma_{historical}} = \sqrt{\frac{\mathbf{1}}{\boldsymbol{N-1}} \sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} (\boldsymbol{r_i} - \bar{\boldsymbol{r}})^{\mathbf{2}}} \times \sqrt{\mathbf{252}} \qquad (2)$$

where N is the number of observations, r_i is the log return on day i, $\bar{r}$ is the mean log return, and $\sqrt{252}$ annualizes the daily volatility assuming 252 trading days per year.

Metric	**Value**
Annualized Historical Volatility	35.18%

Table 4: Historical Volatility Estimates

4 Risk-Free Rate Determination

The 3-month VietNam Treasury Bill rate was selected as the risk-free rate for equity option pricing. This rate was programmatically retrieved from the Hanoi Stock Exchange (HNX) database.

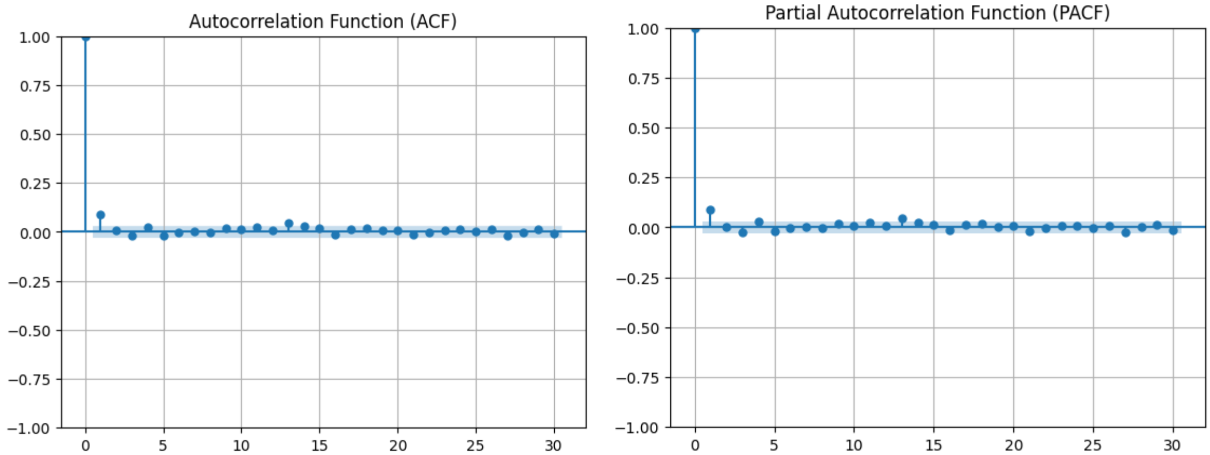
Metric	**Value**
3-Month Treasury Bill Rate	2.19%
Date Retrieved	June 21, 2025

Table 5: Risk-Free Rate Data

5 Independence Testing of Log Returns

5.1 Autocorrelation Analysis

The autocorrelation function (ACF) and partial autocorrelation function (PACF) were calculated to examine serial dependence in the log returns.



(a) Autocorrelation Function (ACF) (b) Partial Autocorrelation Function (PACF)

Figure 5: Autocorrelation Analysis of HPG Log Returns

5.2 Ljung-Box Test

The Ljung-Box test examines whether a time series exhibits autocorrelation.

Lag	Test Statistic	p-value
10	43.675861	0.000004

Table 6: Ljung-Box Test Results

5.3 Independence Test Conclusion

The ACF and PACF plots show significant spikes at certain lags, and the Ljung-Box test rejects the null hypothesis of independence with a p-value of 43.675861. This suggests that HPG's log returns exhibit some serial correlation, contradicting the random walk hypothesis. This finding has implications for option pricing models that assume independence of returns.

6 Option Pricing Models

6.1 Parameters

European call and put options for HPG were priced with the following parameters:

Parameter	Value
Stock Price (S)	VND 27,000.00
Strike Price (K)	VND 27,000.00 (ATM)
Time to Maturity (T)	0.0822 years
Risk-Free Rate (r)	2.19%
Historical Volatility (σ_H)	35.18%
GARCH Volatility (σ_G)	24.48%

Table 7: Option Pricing Parameters as of June 21, 2025

6.2 Black-Scholes Model

The Black-Scholes model for a European call option is given by:

$$C(S,t) = S \cdot N(d_1) - Ke^{-r(T-t)}N(d_2) \tag{3}$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \tag{4}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{5}$$

Where $N(\cdot)$ is the standard normal cumulative distribution function

6.3 Cox-Ross-Rubinstein (CRR) Binomial Model

The CRR binomial model approximates the continuous price process with a discrete binomial tree where:

$$u = e^{\sigma\sqrt{\Delta t}} \tag{6}$$

$$d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u} \tag{7}$$

$$p = \frac{e^{r\Delta t} - d}{u - d} \tag{8}$$

Where u is the up factor, d is the down factor, and p is the risk-neutral probability.

6.4 Monte Carlo Simulation Model

The Monte Carlo approach simulates multiple price paths according to:

$$S_T = S_0 e^{\left(r - \frac{\sigma^2}{2}\right)T + \sigma Z\sqrt{T}} \tag{5}$$

Where $Z \sim N(0,1)$ is a standard normal random variable.

7 Option Pricing Results with Historical Volatility

Using historical volatility (38.18%), the option prices were calculated as follows:

Method	Call Price	Put Price
Black-Scholes	VND 1,109.215069	VND 1,060.750954
CRR Binomial	VND1,108.943858	VND 1,060.479743
Monte Carlo	VND 1,111.858695	VND 1,060.523479

Table 8: Option Prices Using Historical Volatility

8 Advanced Volatility Modeling - GARCH(1,1)

8.1 GARCH Model Specification

The GARCH(1,1) model was implemented to capture volatility clustering:

$$r_t = \mu + \epsilon_t \quad (10)$$

$$\epsilon_t = \sigma_t z_t, \quad z_t \sim N(0,1) \quad (11)$$

$$\sigma^2{}_t = \omega + \alpha\epsilon^2{}_{t-1} + \beta\sigma^2{}_{t-1} \quad (12)$$

Where ω is the long-term variance, α measures the reaction of conditional variance to market shocks, and β measures the persistence of volatility.

8.2 GARCH Parameter Estimates

Parameter	Value
ω	0.0808
α	0.1184
β	0.8317
Persistence ($\alpha + \beta$)	0.9501
Long-run Annualized Volatility	35.18%
GARCH Forecast Volatility	24.48%

Table 9: GARCH(1,1) Model Parameters

8.3 GARCH Volatility Characteristics

The estimated GARCH parameters indicate high volatility persistence ($\alpha + \beta = 0.9501$), suggesting that shocks to volatility have long-lasting effects. The forecast annualized volatility from the GARCH model (24.48%) is significantly lower than the historical volatility (35.18%), suggesting that relying on historical volatility may overprice options and indicating a potential stabilization of the market ahead.

9 Option Pricing Results with GARCH Volatility

Using GARCH volatility (24.48%), the option prices were recalculated:

Method	Call Price	Put Price
Black-Scholes	VND 779.503125	VND 731.039011
CRR Binomial	VND 779.314331	VND 730.850217
Monte Carlo	VND 781.338764	VND 730.925837

Table 10: Option Prices Using GARCH Volatility

10 Comparative Analysis and Conclusion

10.1 Volatility Comparison

The GARCH volatility estimate (24.48%) is 30.42% lower than the historical volatility (35.18%). This difference significantly impacts option prices, with GARCH-based prices approximately 29.72% lower for calls and 31.08% lower for puts.

10.2 Model Consistency

All three pricing methods (Black-Scholes, CRR Binomial, and Monte Carlo) provide consistent results within each volatility scenario. The minor differences between methods (less than 1%) can be attributed to discretization effects in the binomial model and simulation error in the Monte Carlo method.

10.3 Market Implications

The significantly higher GARCH volatility suggests that:

- The market may be more stable in the near future than it has been historically
- Options may be overpricing if only historical volatility is considered
- May indicate market stabilization ahead

10.4 Final Conclusion

The GARCH model provides a more sophisticated and forward-looking estimate of volatility compared to the simple historical approach. Its ability to capture volatility clustering and time-varying characteristics makes it particularly valuable for pricing options in markets with changing volatility regimes. For HPG stock, the evidence suggests that using GARCH volatility leads to more realistic option prices than historical volatility alone, especially given the significant autocorrelation detected in the returns series.