Review

Machinery health prognostics: A systematic review from data acquisition to RUL prediction

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ABSTRACT

Machinery prognostics is one of the major tasks in condition based maintenance (CBM), which aims to predict the remaining useful life (RUL) of machinery based on condition information. A machinery prognostic program generally consists of four technical processes, i.e., data acquisition, health indicator (HI) construction, health stage (HS) division, and RUL prediction. Over recent years, a significant amount of research work has been undertaken in each of the four processes. And much literature has made an excellent overview on the last process, i.e., RUL prediction. However, there has not been a systematic review that covers the four technical processes comprehensively. To fill this gap, this paper provides a review on machinery prognostics following its whole program, i.e., from data acquisition to RUL prediction. First, in data acquisition, several prognostic datasets widely used in academic literature are introduced systematically. Then, commonly used HI construction approaches and metrics are discussed. After that, the HS division process is summarized by introducing its major tasks and existing approaches. Afterwards, the advancements of RUL prediction are reviewed including the popular approaches and metrics. Finally, the paper provides discussions on current situation, upcoming challenges as well as possible future trends for researchers in this field.

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Contents

1. Introduction

Condition based maintenance (CBM) is a maintenance strategy which monitors the health condition of machinery in real time and makes an optimal maintenance decision based on condition monitoring information [1,2]. This strategy is effective in reducing unnecessary maintenance operations and improving the reliability of machinery, thus becoming more and more popular in recent years. Health prognostics is one of the major tasks in CBM, which aims to predict the remaining useful life (RUL) of machinery based on the historical and on-going degradation trends observed from condition monitoring information [3–5]. As shown in Fig. 1, a machinery health prognostic program is generally composed of four technical processes [6], i.e., data acquisition, health indicator (HI) construction, health stage (HS) division and RUL prediction. At first, measured data, such as vibration signals, are acquired from sensors to monitor the health condition of machinery. Then, from the measured data, HIs are constructed using signal processing techniques, artificial intelligent (AI) techniques, etc., to represent the health condition of machinery. After that, according to the varying degradation trends of HIs, the whole lifetime of machinery is divided into two or more different HSs. Finally, in the HS which presents obvious degradation trend, the RUL is predicted with the analysis of the degradation trends and a pre-specified failure threshold (FT).

Machinery health prognostics has attracted more and more attention from academic researchers and industrial operators in recent years. Fig. 2 shows the variation of publication numbers over time on the topic of machinery prognostics in the past 20 years, which is counted based on the search result from the Web of Science. It is seen that the publication number pre-

Section 1: Data acquisition

Measured data, such as vibration signals, are acquired from sensors to monitor the health condition of machinery.

Section 2: HI construction

From the measured data, HIs are constructed using signal processing techniques, AI techniques, etc., to represent the health condition of machinery.

Section 3: HS division

According to the varying degradation trends of HIs, the lifetime of machinery is divided into two or more different HSs.

Section 4: RUL prediction

In the HS which presents obvious degradation trends, the RUL is predicted with the analysis of degradation trends and a pre-specified FT.
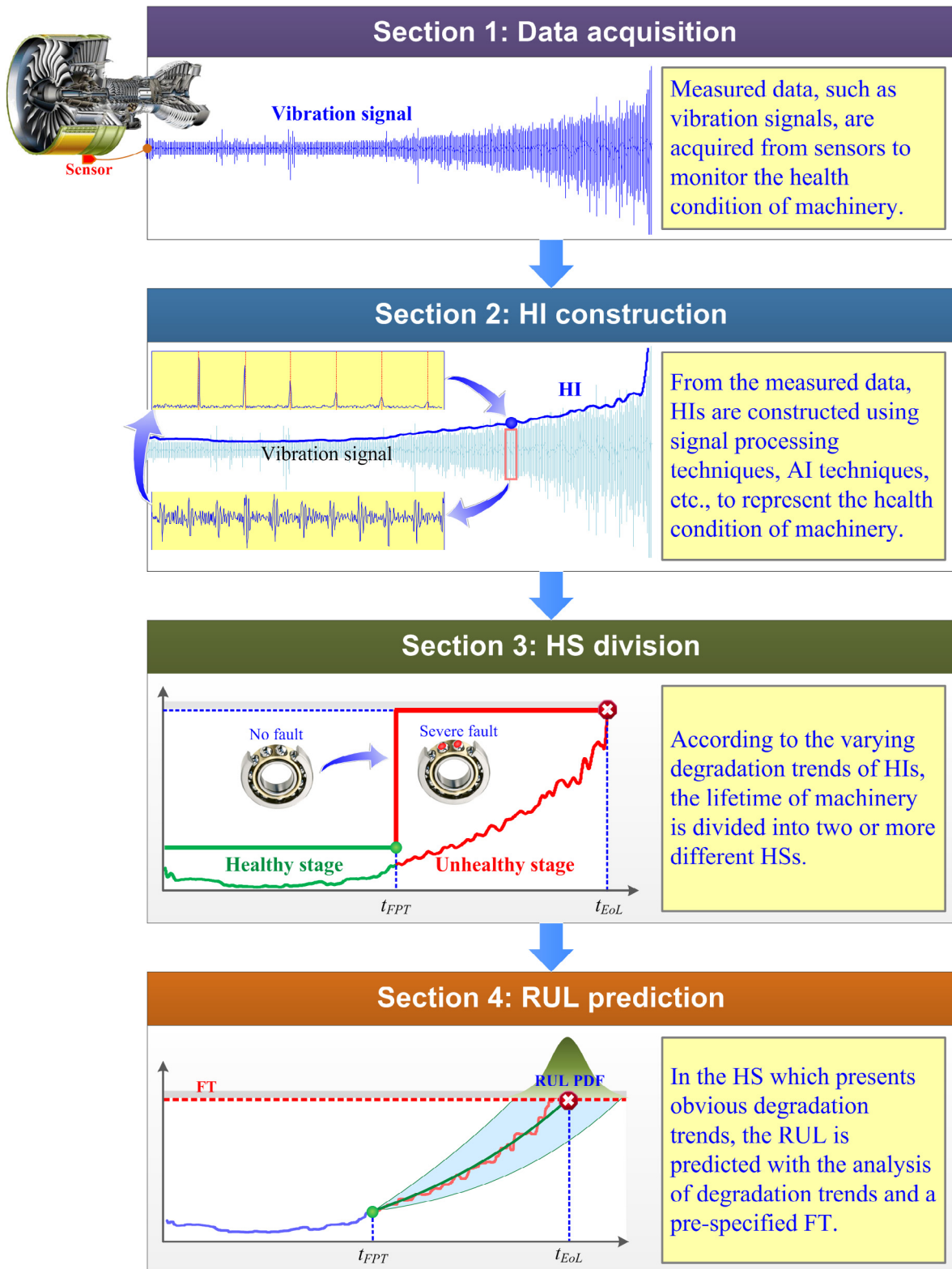
Fig. 1. Four technical processes in a machinery health prognostic program.
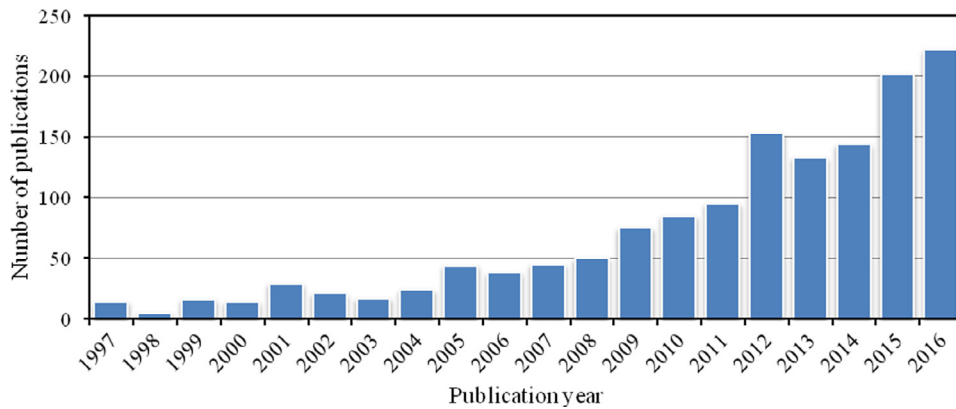
Fig. 2. Histogram of publication numbers on machinery prognostics in the past 20 years.

sents a rapid increasing trend since 2006. The total number of the publications from 1997 to 2011 is 572, while the number of publications in the last five years is 854, which is much larger than the total number in the first 15 years. There are also some excellent review papers among these publications. For example, Heng et al. [7] focused on the prognostics of rotating machinery and gave a brief review on the RUL prediction approaches about their merits and weaknesses up to 2008. Sikorska et al. [8] presented a classification of the RUL prediction approaches and discussed their advantages and disadvantages from the perspective of industrial applications up to 2009. Lee et al. [9] provided a review on the system design of prognostics and health management (PHM) up to 2010, and gave a tutorial for the selection of RUL prediction approaches by comparing their advantages and disadvantages. Si et al. [10] mainly focused on the statistical data-driven RUL prediction approaches and gave a comprehensive review about their basic theories and methodologies up to 2011. Kan et al. [11] reviewed the RUL prediction approaches that can be applied under the non-linear and non-stationary conditions up to 2015.

The aforementioned papers have given interesting reviews related to machinery prognostics. However, they have the following limitations. (1) Most of them [7–10] were published five years ago. It is observed from Fig. 2 that lots of papers have been published in recent five years. Therefore, a new review is required to cover the advancements of this field in recent years. (2) Although Refs. [10,11] were published in recent years, Ref. [10] just concentrated on the statistical data-driven approaches and Ref. [11] just focused on the non-linear and non-stationary issues. (3) These papers just reviewed the last technical process of machinery prognostics, i.e., RUL prediction. The other three processes, i.e., data acquisition, HI construction and HS division, however, were generally ignored by the existing reviews. In conclusion, it lacks a systematic review covering the whole program of the machinery prognostics about its advancements in recent years. This paper fills these gaps and gives a systematic overview about the four technical processes of machinery prognostics in order. Compared with the existing review papers, the major contributions of this paper are as follows.

- This paper reasonably divides a whole program of machinery prognostics into four technical processes, i.e., data acquisition, HI construction, HS division and RUL prediction, and reviews them systematically in order.
- This paper summarizes typical public datasets for prognostics, including the data sources, properties and applications, aiming to provide guidance for researchers to select suitable datasets according to their research requirements.
- This paper reviews the HI construction and the HS division processes, which are significant for prognostics but always ignored by the existing review papers.
- This paper gives a comprehensive review through analyzing large amount of references to provide a systematic perspective for researchers as well as a basic tutorial for beginners.

The remaining of this paper is organized as follows. Section 2 discusses the data acquisition process and presents a brief summary about the datasets widely used in prognostics in existing publications. Section 3 addresses the HI construction process from two aspects, i.e., HI construction approaches and metrics for evaluating prognostic HIs. In Section 4, the HS division process is discussed in detail. Section 5 concentrates on the RUL prediction process and reviews the commonly used prediction approaches and metrics. Conclusions are drawn in Section 6 with discussions on future challenges as well as opportunities for machinery prognostics.

2. Data acquisition

Data acquisition is a process of capturing and storing different kinds of monitoring data from various sensors installed on the monitored equipment. It is the first process of machinery prognostics, which provides basic condition monitoring information for following processes. A data acquisition system is composed of sensors, data transmission devices and data storage devices. Various sensors are employed to capture different types of monitoring data which are able to reflect the degradation

process of machinery. The commonly used sensors include accelerometers, acoustic emission sensors, infrared thermometers, current sensors, etc. The captured data are transmitted into a PC or portable devices through a data transmission device and stored into a memory location for further analysis. With the rapid development of sensor and communication technologies, more and more advanced data acquisition devices have been designed and applied into modern industries. However, it is still difficult to acquire run-to-failure data of machinery in high quality for academic research due to the following reasons.

- Machinery generally expresses a long-term degradation process from health to failure, which may take several months or even many years. It is time consuming and expensive to capture the whole run-to-failure data during such a long-term degradation process.
- Practically, machinery is not allowed to run to failure since an unexpected failure may lead to breakdown of the entire machine or even catastrophic accidents. In such cases, run-to-failure data are hard to be captured in industrial fields.
- Machinery, such as wind turbine gearboxes, automotive gearboxes and aircraft engines, always works under tough environment. Lots of interferences from the outside environment are mixed into the monitoring data, thus decreasing the quality of the data.
- Many monitoring data are captured during the out-of-service period, such as the downtime or the restart time. These measurements generally present distinct behaviors compared with the measurements captured under in-service period, thus further decreasing the quality of the monitoring data.
- Few military or commercial institutions which can collect run-to-failure data would like to publish their data because of military secret or commercial competition. Therefore, these limited data sources are accessible only to a few academics privileged to cooperate with these institutions.

Due to the above obstacles, most prognostic datasets in existing publications are always acquired from accelerated degradation test beds instead of real industrial equipment. To facilitate the development of prognostic approaches, the Prognostic Center of Excellence of National Aeronautics and Space Administration (NASA) has established a prognostic data repository [12] which collects lots of datasets specially generated for demonstration of prognostic approaches. To provide some references and guidelines in data acquisition, this paper selects four typical mechanical prognostic datasets from the repository of NASA. These datasets are introduced systematically, including their data sources, properties and applications. Major technical details of data acquisition about these datasets are also provided including the sensor type, the sensor location, the sampling strategy and the storage strategy. It is expected to provide some references for researchers on the selection of the sensor type and location, and how to make their own sampling and storage strategies. It also aims to provide a guideline for researchers in the data selection to evaluate their prognostic methods.

2.1. Turbofan engine degradation simulation dataset

2.1.1. Introduction of the dataset

This dataset [13] is composed of multiple run-to-failure data of turbofan engines simulated using a thermo-dynamical simulation model whose diagram is shown in Fig. 3. This model has 14 inputs and 58 outputs. The 14 inputs include fuel flow and 13 health parameters which allow the user to simulate the effects of faults and degradations in major rotating components of turbofan engines. Totally 21 parameters of the 58 outputs are utilized to measure the system response under different health states and operational conditions.

This dataset includes five subsets which are generated under different operational conditions and health states. One subset was employed as the challenge data in the IEEE PHM 2008 conference [14]. The other four subsets were packed into another version [15]. Each subset is composed of 26 columns, recording the unit number, time, three operational parameters
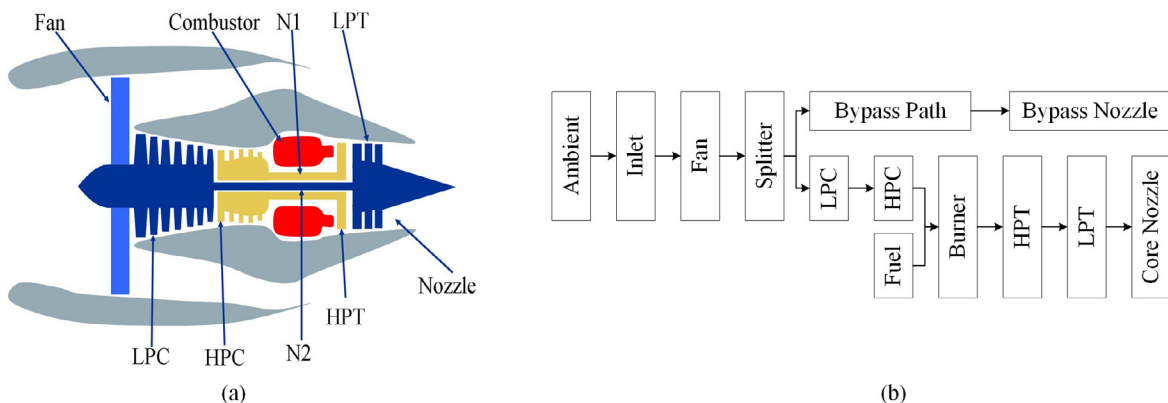

Fig. 3. (a) Simplified diagram of the simulated engine and (b) various models and their connections in the simulation.

and 21 output parameters, respectively. Each subset is further divided into a training set and a testing set. The ground truth RULs of the testing set in the challenge data of IEEE PHM 2008 have not been revealed until now for competition.

2.1.2. Properties of the dataset

The properties of this dataset regarding the demonstration of prognostic approaches are concluded as follows.

- This is a simulated dataset acquired from commercial software instead of real turbofan engines. Although the degradation processes are simulated as realistic as possible with the consideration of many variability sources, they are still different from real cases.
- Each subset includes not less than one hundred training and testing units. Therefore, this dataset is an ideal candidate for prognostic algorithms which require large numbers of training units.
- This dataset contains 21 observations which are composed of different types of features, such as the temperature, pressure, speed and bleed. Therefore, it is a typical prognostic issue of multi-sensor information fusion.
- This dataset includes six different operational conditions. The time-varying operational conditions cause fluctuation of the degradation trends. Thus, it is suitable for the study of the relationship between time-varying operational conditions and degradation trends.

2.1.3. Applications of the dataset

Because of the above mentioned properties, this dataset has been employed in research of various aspects related to machinery prognostics [16]. Some researchers [17–22] took advantage of the massive available run-to-failure data and developed advanced prognostic approaches by learning the relationship between feature vectors and RULs using AI techniques. Some researchers [23–27] made full use of the abundant training units and developed prognostic approaches based on unit-to-unit similarity. Some publications [28–30] utilized the diversity of the features and conducted prognostics based on multi-feature fusion. Some researchers [31] proposed prognostic approaches considering the time-varying operational conditions of this dataset. Some papers [32–35] also used this dataset to conduct research work related to HS division. Others [36–40] used this dataset to develop new prognostic approaches.

2.2. FEMTO bearing dataset

2.2.1. Introduction of the dataset

This dataset was shared in the IEEE international conference of PHM 2012 for prognostic challenge [41], and was provided by Franche-Comté Electronics Mechanics Thermal Science and Optics–Sciences and Technologies institute [42]. This dataset is composed of 17 run-to-failure data of rolling element bearings acquired from a PRONOSTIA platform. To conduct accelerated degradation tests in a few hours, a high-level radial force larger than the bearings' maximum dynamic load was applied on the testing bearings. During the tests, the rotating speed of the bearing was kept stable. Two accelerometers and a thermocouple were used to capture the vibration signals and the temperatures of the bearings. The bearing useful life is considered to end when the amplitude of the vibration signal exceeds 20 g.

2.2.2. Properties of the dataset

- This dataset is more realistic than the former one. Different from the simulation degradation dataset of turbofan engines, this dataset was acquired from a real experimental platform. The bearings were naturally degraded without being seeded a fault in advance.
- There are only two training units for each operational condition. In addition, the fault patterns and the lifetimes of different units are various even under the same condition. This phenomenon can be seen from Fig. 4(a) which presents the degradation processes of seven bearings under the same operational condition. It is hard to learn useful information about the degradation patterns of testing units from the training ones, which increases the difficulty of RUL prediction.
- The frequency resolution of the vibration signals is too low to be analyzed in detail. The time length of each sample is 0.1 s, which means that the frequency resolution is 10 Hz. Traditional fault diagnostic methods based on frequency analysis do not work with such rough frequency spectra.
- An incipient fault in one component easily propagates to other components through frequent contacts, resulting in the simultaneous occurrence of different kinds of fault patterns, as shown in Fig. 4(b). This behavior introduces this dataset into a multi-fault prognostic issue.

2.2.3. Applications of the dataset

To reveal the degradation processes of the bearings, most researchers [43–59] attempted to construct HIs from vibration signals of this dataset using signal processing techniques or AI techniques. Some researchers [60–67] also applied this dataset to the study of HS division. Other researchers concentrated on the RUL prediction of the bearings, and developed various RUL prediction approaches [68–79].
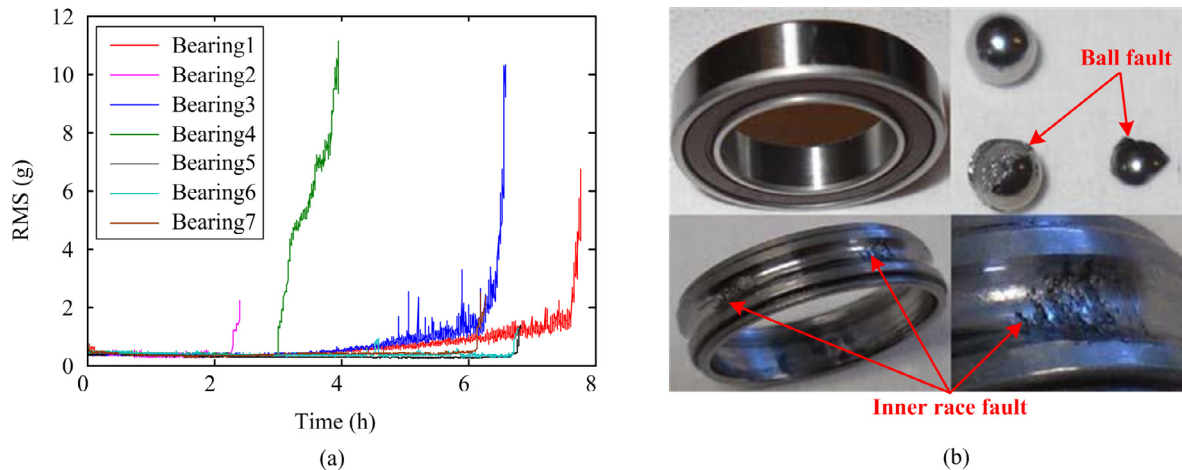
Fig. 4. (a) Root mean square (RMS) values of seven bearings under the same operational condition and (b) photographs of bearings before and after tests.

2.3. IMS bearing dataset

2.3.1. Introduction of the dataset

This dataset was provided by the center for Intelligent Maintenance Systems (IMS), University of Cincinnati [80], and shared on the website of the Prognostic Data Repository of NASA [81]. It is composed of three data subsets of bearing degradation tests. During the tests, four Rexnord ZA-2115 double row bearings were installed on a shaft. Accelerometers were installed on the bearing housings. An oil circulation system was designed to lubricate the bearings. A magnetic plug was installed in the oil feedback pipe to collect debris. The test was stopped adaptively by an electrical switch when the accumulated debris exceeded a certain level. After the tests, the bearings were checked and their fault patterns were recorded in detail.

2.3.2. Properties of the dataset

Although the IMS bearing dataset and the FEMTO bearing dataset were both acquired from accelerated degradation tests of bearings, they present distinct properties. Here, we emphasize the major similarities and differences between these two datasets.

- The bearings in this dataset experienced longer and more complicated degradation processes than those in the former dataset, which increases the reality of this dataset as well as the difficulty of RUL prediction. As shown in Fig. 5(a), the lifetimes of the bearings in this dataset are above 7 days, which are longer than those of the former. In addition, the bearings experienced "increase-decrease-increase" degradation trends. This behavior is due to the "self-healing" nature of the damage [82]. First, the amplitude of vibration increases because of the impact caused by the initial surface defect, such as spalling or cracks. Then the initial defect is smoothed by continuous rolling contact leading to the decrease of the impact amplitude. When the damage spreads over a broader area, the vibration amplitude increases again. The fluctuant degradation trends bring great challenges for RUL prediction.
- Similar to the former dataset, the available units in this dataset are limited. In addition, the lifetimes of the run-to-failure units have distinct discrepancies (varying from 7 days to 35 days), as shown in Fig. 5(a). It means that little information is available for the training of prognostic models, thus increasing the difficulty of RUL prediction.
- The frequency resolution of the vibration signals is high enough for fault diagnosis using frequency analysis techniques. According to the data instruction [81], each file is made of 20,480 data samples with a sampling frequency 20 kHz. However, another survey [83] has pointed out that the sampling frequency may be 20.48 kHz. It means that the frequency resolution is about 1 Hz, ensuring the availability of fault diagnosis based on frequency analysis. Therefore, operators can extract frequency-domain features to monitor the degradation processes of specific components, i.e., the rollers, outer race and inner race.
- At the end of the tests, the fault pattern of each bearing was observed and recorded in detail, as shown in Fig. 5(b). It provides valuable information for researchers to study the relationship between different fault patterns and their degradation trends.

2.3.3. Applications of the dataset

After the publication of the dataset, it has been widely used to demonstrate the approaches for condition monitoring and prognostics of rolling element bearings. Because of the high frequency resolution of the vibration signals, some researchers
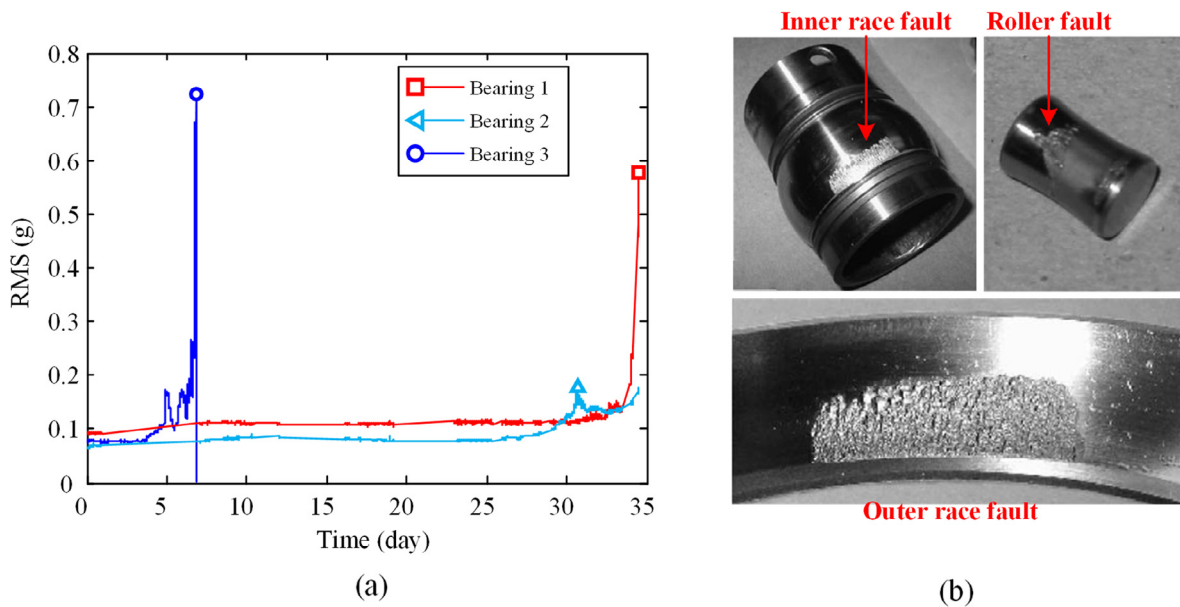
Fig. 5. (a) RMS values of three run-to-failure units and (b) different faults of rolling element bearings.

[80] applied this dataset to the research of health condition monitoring of rolling element bearings based on signal processing techniques. Some publications [84–88] also used this dataset to conduct the study of HI construction from vibration signals. Due to the complexity of the degradation processes, this dataset was treated as an optimal candidate for the research of HS division [89,90]. Some researchers [91–95] developed RUL prediction approaches based on this dataset as well.

2.4. Milling dataset

2.4.1. Introduction of the dataset

This dataset includes 16 run-to-failure data, as shown in Fig. 6, acquired from tool wear experiments of a milling machine [96,97]. The experiments were conducted under a cutting speed of 200 m/min with different cut depths and feeds. Two types of material were used in the work pieces. These operation parameters were chosen according to industrial applicability and recommended manufacturer's settings. Two acoustic emission sensors and two vibration sensors were mounted on the spin-
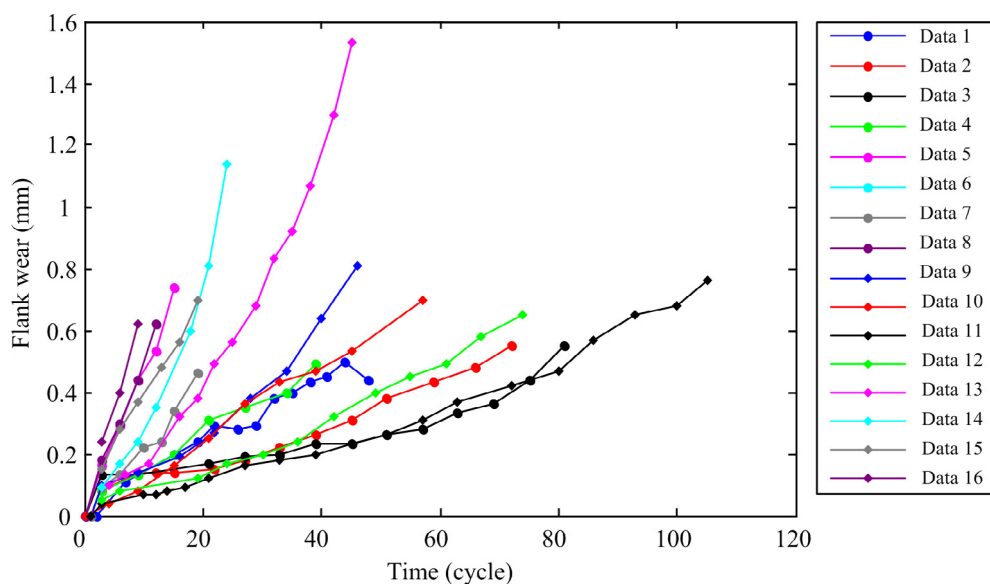
Fig. 6. Flank wear of 16 run-to-failure tools.

dle and the table of the milling machine, respectively. The RMS values of the acoustic emission signals and vibration signals as well as the current signals were recorded in a frequency of 250 Hz. The flank wear was observed with the help of a microscope at irregular intervals. The experiments were stopped when the flank wear exceeded the FT.

2.4.2. Properties of the dataset

- The tool wear processes were conducted under operational conditions close to industrial applications using a real milling machine instead of simulation models or experimental platforms. Therefore, the degradation processes of this dataset are the most realistic among these datasets.
- Different kinds of signals including acoustic emission signals, vibration signals and electric current were captured from different locations of the test rig. As a result, this dataset is applicable for research related to multi-sensor fusion for machinery prognostics.
- Totally eight different operational conditions were generated with random combination of three operational parameters, i.e., work piece materials, cut depths and feeds, ensuring the suitability of this dataset for the study of tool wear processes under different operational conditions.
- Different from the above three datasets, the health states, i.e., the degrees of flank wear, were observed directly in this dataset. Therefore, it provided important information for researchers to study the relationship between health states and measurements, such as RMS values of vibration signals.

2.4.3. Applications of the dataset

Because of the availability of both the flank wear and the monitoring signals, this dataset was employed to research the relationship between the health states and the measurements in [98]. Some researchers [99,100] took advantage of the multi-sensor signals and developed new HIs for the RUL prediction of the machine tools. Some publications [97,101] also applied this dataset to the research of health condition monitoring or HS division. This dataset was also applied to the research of RUL prediction approaches in [102].

2.5. Epilog

This section first gives a simple introduction about the data acquisition process, and then summarizes four commonly used prognostic datasets in the field of machinery prognostics. The data sources, properties and applications are introduced respectively, aiming to provide a guideline for researchers to select suitable datasets according to their requirements. It should be mentioned that the datasets selected in this paper are just four popular ones which have been published in the prognostic data repository of NASA. There are still some other published datasets [103,104] and unpublished ones [105–107]. In addition, some international conferences such as the PHM conferences sponsored by the PHM society [108] and the IEEE PHM conferences sponsored by the IEEE reliability society [109] often organize prognostic challenge, which provide valuable datasets and competition opportunities for researchers from both academic institutions and industrial fields. Thanks to the open share of these datasets, researchers can demonstrate their algorithms or methods using the same benchmark. This is valuable for the development of basic theories and methodologies of machinery prognostics. However, most of the existing data are generated through simulation or in the laboratory environment. These data are different from the natural degradation data from industrial fields. The industrial issues may be more complicated and obscure than those from simulations or experiments. Prognostic approaches which work well for simulations and experiments may be invalid for real industrial issues. Therefore, more realistic prognostic datasets are still needed in the future research.

3. HI construction

The damage degree of machinery, such as the crack lengths and the wear areas, is generally unable to be observed directly due to the following reasons. (1) Frequent shutdown is not allowed during the operating process of industrial equipment. (2) Although it is allowed to stop the equipment sometimes, incipient damages are always at the micro scale and hard to be measured without the help of professional instruments. (3) Some inside faults in complicated components, such as the roller fault in rolling element bearings, are hard to be observed without destruction. To estimate the health states of machinery in real time, different kinds of condition monitoring signals, such as vibration signals and acoustic emission signals, are generally captured from running equipment, as mentioned in Section 2. These monitoring signals contain lots of health condition information as well as measurement noise. To further reveal the degradation processes of machinery, some HIs such as RMS, kurtosis and skewness are extracted from the monitoring signals.

HI construction plays a significant role in machinery prognostics. A suitable HI is expected to simplify the prognostic modeling and produce accurate prediction results. There are two major issues related to the HI construction. (1) How to construct HIs from monitoring signals? (2) How to evaluate the suitability of the constructed HIs for RUL prediction? These two issues will be discussed in the following two subsections, respectively.

3.1. HI construction approaches

HIs can be categorized into two different classes according to their construction strategies: physics HIs (PHIs) and virtual HIs (VHIs) [40]. PHIs are related to the physics of failures and generally extracted from monitoring signals using statistical methods or signal processing methods, such as the RMS of vibration signals. In contrast, VHIs are generally constructed by fusing multiple PHIs or multi-sensor signals. They lose physics meanings and just present a virtual description about the degradation trends of machinery. Publications related to these two kinds of HIs are displayed in the following two subsections respectively.

3.1.1. PHIs

Several traditional PHIs of rotating machinery were summarized in [110,111]. RMS is the most widely used PHI in the RUL prediction of machinery. Li et al. [62,69] used kurtosis to select the FPT and employed RMS to predict the RUL of rolling element bearings. Huang et al. [70,112] also predicted the RUL of bearings using RMS. Malhi et al. [113] extracted RMS and peak values from the wavelet coefficients to predict the RUL of bearings. The following Refs. [114,115] also applied RMS to the RUL prediction of machinery. The kurtosis value extracted from band-pass filtered vibration signals was applied to the RUL prediction of bearings [116] as well. Moreover, some PHIs were extracted from the frequency domain of the vibration signals. Gasperin et al. [106] extracted the power density of the gear-mesh frequency from the envelope spectrum to predict the RUL of gears. Gebraeel et al. [117,118] calculated the average amplitude of the defective frequency and its harmonics as a PHI of thrust bearings. Hu et al. [119] calculated the average of the frequency amplitudes within a spectrum band as the HI of oil sand pumps.

Some researchers constructed new PHIs based on statistical characteristics of signals in time domain. Medjaher et al. [48] calculated the correlation coefficient between two series of vibration signals captured at different time periods as a PHI. Boškoski et al. [54] constructed a PHI for the RUL prediction of bearings, which described the statistical complexity of the envelope signals. Lin et al. [120] calculated the percentage of a residual error signal exceeding a baseline to estimate the development of cracks in gear teeth. Hanachi et al. [121] developed two PHIs with explicit physical meaning to monitor the performance deterioration of gas turbine engines. Li et al. [122] proposed a PHI for health condition monitoring of bearings by calculating the mathematical morphology pattern spectrum of signals. Li et al. [123] calculated the energy ratio between the residual signals acquired from autoregressive (AR) filtering and original signals as a PHI of bearings. Some new PHIs were also developed from the entropy of time-domain signals [46,51,68,84,90,124]. Some researchers extracted new PHIs from the frequency domain of signals. Loutas et al. [44] calculated the spectral flatness as a PHI of bearings. Soualhi et al. [53] analyzed vibration signals using Hilbert–Huang transform and extracted defective frequencies as PHIs. Marka et al. [125] and Hu et al. [126] calculated the average-log-ratio of the rotational-harmonic amplitudes to monitor the gear damage process. Nie et al. [58] proposed a PHI for rolling element bearings through sparse representation of spectra. To enhance the superiority of PHIs for prognostics, some researchers constructed PHIs using the accumulative strategy. Javed et al. [45] designed two PHIs based on trigonometric functions and used the accumulative strategy to improve the monotonicity of the PHIs. Porotsky et al. [55] attempted to extract PHIs through accumulation of measurements. Wang et al. [127] proposed a moving-average wear degradation index through logarithm transformation of the accumulative frequency spectra.

3.1.2. VHIs

As one of the most popular dimension reduction techniques, principal component analysis (PCA) is often applied to the VHI construction process. Widodo et al. [93] used PCA to reduce the dimension of feature sets and further calculated the deviations between unknown states and the healthy state as a VHI. Benkedjouh et al. [128] used PCA combined with isometric feature mapping to construct a VHI for cutting tools. Wang et al. [52] used PCA to fuse multiple features and calculated the T^2 statistic as a VHI of bearings. Other publications related to PCA based VHIs are Refs. [43,59,87,129]. The self-organizing map (SOM) technique was introduced into the VHI construction by Qiu et al. [85]. After that, this technique has been widely used in the VHI construction [49,50,56,57,105,130–132]. Some publications also used the Mahalanobis distance as a VHI. Wang et al. [71] fused multiple time-domain features by calculating their Mahalanobis distance from the healthy stage. Jin et al. [67] calculated the energies of wavelet coefficients and fused them by calculating their Mahalanobis distance. Kumar et al. [133] also constructed a VHI based on Mahalanobis distance for condition monitoring of bearings. Other publications related to the VHI are as follows. Yu [86,100,134] constructed several VHIs and applied them to health condition monitoring of bearings and machine tools. Some researchers [21,23,24,40] employed a linear data transformation method to construct a VHI by fusing multiple features. Giantomassi et al. [28] constructed a VHI by fusing multiple original features via a multi-layer perception network. Liu et al. [29] constructed a VHI with the consideration of two properties, i.e., monotonicity and consistency of FTs. Liao et al. [135] used the genetic programming algorithm to generate multiple VHIs with the integration of multiple features and selected the optimal one from them. Ocak et al. [136] fused multiple features using the hidden Markov model (HMM) and calculated the probabilities of the HMM in the healthy stage as a VHI of bearings. Shen et al. [107] constructed a VHI using the fuzzy support vector data description technique and improved the trendability of the VHI by introducing the operating time information into it. Li et al. [137] used the symbolic dynamic filtering technique to extract features and fused them using the cumulative sum chart to monitor the health conditions of bearings. Pan et al. [138] built an assessment model utilizing fuzzy *c*-means and extracted a VHI by calculating the subjection of the tested data

to the healthy stage. Bechhoefer et al. [139] constructed a VHI for the health condition monitoring of gears by fusing six features based on Rayleigh probability density functions (PDFs). Liu et al. [140] proposed a VHI for bearings through phase space reconstitution combined with approximate diagonalization of Eigen-matrices. Guo et al. [141] constructed a VHI for bearing prognostics by fusing multiple features using the recurrent neural network.

3.2. Metrics for prognostic HIs

Subsection 3.1 answers the first question, i.e., how to construct HIs from monitoring signals. In this subsection, the second question, i.e., how to evaluate the suitability of the constructed HIs for RUL prediction, will be discussed. The performance of prognostic HIs has great influence on the complexity of prognostic modeling and the prediction accuracy. Thus, selecting a suitable HI is prerequisite for accurate prognostics. Many researchers have proposed various metrics for the evaluation of prognostic HIs. A summary of these metrics is given in Table 1. Metrics of HIs can be classified into five categories according to the different independent variables of their functions: (1) metrics depending on a single HI, (2) metrics depending on a HI and time, (3) metrics depending on a HI and HS sequence, (4) metrics depending on multiple HIs, and (5) hybrid metrics. These metrics evaluate the HIs from different aspects. And different available information is required in the calculation processes of the five different categories. Readers are suggested to choose metrics from the following list according to both their requirement and available information in practice.

3.2.1. Metrics depending on a single HI

- *Monotonicity*

In real applications, the degradation processes of machinery are irreversible, i.e., a fault component is unable to recover by itself without manual repair. To coincide with the irreversible degradation processes, a suitable HI should have a monotonic increasing or decreasing trend. This property is named as monotonicity [29,142]. It is the inherent property of a HI itself without considering its relationships with other factors and often expressed using a formula depending on the sequence of the HI.

Two monotonicity metrics were proposed based on the derivatives of the HI sequence. One is described as follows [45,47,135]:

$$Mon_1(X) = \frac{1}{K-1}|No.\ of\ d/dx > 0 - No.\ of\ d/dx < 0|, \tag{1}$$

where $X = \{x_k\}_{k=1:K}$ is the HI sequence with x_k representing the HI value at time t_k; K is the total number of the HI values included in the sequence; $d/dx = x_{k+1} - x_k$ denotes the difference of the HI sequence; *No. of* $d/dx > 0$ and *No. of* $d/dx < 0$ represent the number of the positive differences and the negative differences, respectively. $Mon_1(X)$ measures the absolute difference of the positive and negative derivatives of X. It changes from 0 to 1, and a higher score means a better performance in monotonicity.

The other metric was presented as [57,143]

$$\begin{cases} Mon_{2+}(X) = \frac{No.\ of\ d/dx>0}{K-1} + \frac{No.\ of\ d^2/d^2x>0}{K-2} \\ Mon_{2-}(X) = \frac{No.\ of\ d/dx<0}{K-1} + \frac{No.\ of\ d^2/d^2x<0}{K-2} \end{cases}, \tag{2}$$

where d^2/d^2x denotes the second-order derivatives of X; $Mon_{2+}(X)$ corresponds to the positive monotonicity and $Mon_{2-}(X)$ corresponds to the negative monotonicity.

The above two metrics are able to evaluate the monotonicity of HIs when the degradation trends of HIs are obvious. However, when the degradation trends are masked by random fluctuations, their performances will be reduced. In order to eliminate the interference from random fluctuations, some researchers [144] proposed a monotonicity metric by dividing a HI into several stages.

Table 1
Summary of the metrics for evaluating prognostic HIs.

Categories	Names	Refs.
Metrics depending on a single HI	• Monotonicity	Liu et al. [29], Javed et al. [45], Zhang el al. [47], Liao et al. [57,135], Yang et al. [142], Coble et al. [143], Camci et al. [144]
	• Robustness	Zhang et al. [47]
Metrics depending on a HI and time	• Trendability	Javed et al. [45], Zhang el al. [47], Niu et al. [78], Lei et al. [56], Carino et al. [72], Yang et al. [142], Li et al. [131], Hu et al. [145,146]
Metrics depending on a HI and stage sequence	• Identifiability	Liu et al. [60], Zurita et al. [64], Wang et al. [76], Yang et al. [142], Zhao et al. [147], Lin et al. [148]
Metrics depending on multiple HIs	• Consistency	Mosallum et al. [26,43], Liu et al. [60], Liu et al. [29]
Hybrid metrics	• Hybrid metrics	Zhang el al. [47], Liu et al. [60], Liu et al.[30]

$$Mon_3(X) = \frac{1}{S}\sum_{s=1}^{S} Sep_s, \quad Sep_s = \frac{a}{L} - \frac{\chi}{N_s}, \quad \chi = \begin{cases} 0 & \text{if } a/L \neq 1 \\ \beta & \text{if } a/L = 1 \end{cases}, \tag{3}$$

where S is the number of the stages; Sep_s represents the separability of the indicator values at the s th stage; L is the distance between the 25th and the 75th percentiles of the indicator values at the s th stage; a is the length of the non-overlapped portion with the neighbor segment; β is the number of measurements overlapping with the distribution in the neighbor segment and N_s is the number of measurements at the s th stage. This metric measures the monotonicity of a HI by comparing the distributions of consecutive stages. It is expected to eliminate the interference from random fluctuations to some extent. However, its stage number should be pre-determined artificially, which introduces interferences from human activities.

- Robustness

Due to the measurement noise, stochasticity of the degradation processes and the variation of operational conditions, random fluctuations are generally included in a HI curve, which may reduce the stability of the prediction results [62]. A suitable HI should be robust to these interferences and present a smooth degradation trend. This property is defined as robustness. Similar to the monotonicity, robustness is also an inherent property of a HI itself. Zhang et al. [47] proposed a metric for evaluating the robustness of HIs, which is denoted as

$$Rob(X) = \frac{1}{K}\sum_{k=1}^{K} \exp\left(-\left|\frac{x_k - x_k^T}{x_k}\right|\right), \tag{4}$$

where x_k is the indicator value of X at t_k, and x_k^T is the mean trend value of the HI at t_k which is generally acquired through smoothing methods.

3.2.2. Metrics depending on a HI and time

- Trendability

With the increase of the operating time, a component is more likely to degrade gradually. Therefore, the degradation trend of a HI is expected to present correlation with the operating time [142]. This property is named as trendability. Different from monotonicity and robustness, trendability is a correlation property between the HI and time. The correlation coefficient between the HI and time is generally used to measure the trendability [45,47,131].

$$Tre_1(X, T) = \frac{K\left(\sum_{k=1}^{K} x_k t_k\right) - \left(\sum_{k=1}^{K} x_k\right)\left(\sum_{k=1}^{K} t_k\right)}{\sqrt{\left[K\sum_{k=1}^{K} x_k^2 - \left(\sum_{k=1}^{K} x_k\right)^2\right]\left[K\sum_{k=1}^{K} t_k^2 - \left(\sum_{k=1}^{K} t_k\right)^2\right]}}, \tag{5}$$

where t_k is the k th value of the time and x_k is the HI value at t_k. $Tre_1(X, T)$ changes from -1 to 1. It approaches 1 or -1 when the HI has a strong positive or negative linear correlation with time.

The above metric is more sensitive to linear correlation than nonlinear correlation. However, machinery generally presents nonlinear degradation trends in real applications. To evaluate the trendability of the HI with nonlinear degradation trends, the Spearman coefficient, an enhanced version of the correlation coefficient, is used as the trendability metric in several publications [56,72].

$$Tre_2(\tilde{X}, \tilde{T}) = \frac{K\left(\sum_{k=1}^{K} \tilde{x}_k \tilde{t}_k\right) - \left(\sum_{k=1}^{K} \tilde{x}_k\right)\left(\sum_{k=1}^{K} \tilde{t}_k\right)}{\sqrt{\left[K\sum_{k=1}^{K} \tilde{x}_k^2 - \left(\sum_{k=1}^{K} \tilde{x}_k\right)^2\right]\left[K\sum_{k=1}^{K} \tilde{t}_k^2 - \left(\sum_{k=1}^{K} \tilde{t}_k\right)^2\right]}}, \tag{6}$$

where $\{\tilde{x}_k\}_{k=1:K}$ and $\{\tilde{t}_k\}_{k=1:K}$ are the rank sequence of the HI $\{x_k\}_{k=1:K}$ and time $\{t_k\}_{k=1:K}$, respectively. The Spearman coefficient transforms the nonlinear relationship between the HI and time into the linear relationship between the rank sequence of the HI and time. Therefore, it is able to evaluate the nonlinear relationship more effectively.

Besides, a similar trendability metric is given in [78], which uses the rank mutual information criterion [145,146] to measure the nonlinear correlation between the HI and the time.

3.2.3. Metrics depending on a HI and HS sequence

- Identifiability

Machinery generally experiences several different HSs during the whole lifetime. A suitable HI is expected to have the capability of identifying different HSs. This property is defined as identifiability. Different from trendability metrics which reflect the correlation between the HI and the time, identifiability measures the correlation between the HI and the stage sequence. In other words, trendability metrics are generally expressed as a function of the HI sequence and the time sequence. While identifiability metrics are expressed as a function of the HI sequence and the HS sequence. To acquire

the stage sequence, the state estimation problem is always transformed into a multi-stage classification problem. A consecutive degradation process is classified into several discrete HSs according to the severity of faults. And a series of labels are assigned to different HSs. The identifiability is measured by calculating the correlation coefficient between the HI and the HS labels [60,76,147].

$$Ide_1(X, C) = \frac{\sum_{k=1}^{K}(x_k - \bar{x})(c_k - \bar{c})}{\sqrt{\sum_{k=1}^{K}(x_k - \bar{x})^2(c_k - \bar{c})^2}},$$

(7)

where c_k is the HS label at t_k, $\bar{x}$ and $\bar{c}$ are the means of $\{x_k\}_{k=1:K}$ and $\{c_k\}_{k=1:K}$ respectively. Similar to $Tre_1(X, T)$, this metric is also more sensitive to linear correlation than nonlinear correlation.

In order to improve the sensitivity to nonlinear correlation, the Fisher's ratio is employed to measure the identifiability of HIs [142,148].

$$Ide_2(X, C) = \sum_{s=1}^{S}\sum_{d \neq s}^{S} \frac{(m_s - m_d)^2}{\sigma_s^2 + \sigma_d^2},$$

(8)

where S is the number of classes, and m_s and σ_s^2 are the mean and the variance of the s th class, respectively. A higher metric score means that the HI is more effective in identifying different HSs.

In $Ide_2(X, C)$ the identifiability is just estimated by calculating between-class scatters without considering the within-class scatters. The following metric [64] measures the identifiability of HIs by considering both between-class and within-class scatters.

$$Ide_3(X, C) = \frac{S_B(X, C)}{S_W(X, C)},$$

(9)

where $S_B(X, C) = \sum_{s=1}^{S} n_s(m_s - m)(m_s - m)^T$ is the between-class scatter matrix; $S_W(X, C) = \sum_{s=1}^{S}\sum_{x \in \Omega_s}(x - m_s)(x - m_s)^T$ is the within-class scatter matrix; n_s is the number of samples in the s th class; m_s is the mean of the indicators in the s th class; m is the mean of all indicators; and Ω_s represents the set of indicators in the s th class.

3.2.4. Metrics depending on multiple HIs

• *Consistency*

Consistency describes the correlation among multiple HIs. There are two meanings in the consistency properties of HIs. First, for different HIs of a single unit, since they contain the information of the same degradation process, they are supposed to present some kinds of correlation with each other [26]. Second, for the same HIs of different units, given the same operational conditions and failure modes, the variance of their FTs should be minimal [29]. Aiming at the first meaning of the consistency, Liu et al. [60] calculated the correlation coefficient of two HIs to measure their consistency.

$$Con_1(X_1, X_2) = \frac{\sum_{k=1}^{K}(x_{1,k} - \bar{x}_1)(x_{2,k} - \bar{x}_2)}{\sqrt{\sum_{k=1}^{K}(x_{1,k} - \bar{x}_1)^2(x_{2,k} - \bar{x}_2)^2}}$$

(10)

where X_1 and X_2 are two HI vectors; $x_{1,k}$ and $x_{2,k}$ are the values of X_1 and X_2 at t_k, respectively; and $\bar{x}_1$ and $\bar{x}_2$ are the means of X_1 and X_2, respectively.

Mosallam et al. [26,43] proposed a consistency metric based on the pairwise symmetrical uncertainty.

$$Con_2(X_1, X_2) = \frac{2I(X_1, X_2)}{H(X_1) + H(X_2)},$$

(11)

where $I(X_1, X_2)$ is the mutual information of X_1 and X_2; $H(X_1)$ and $H(X_2)$ are the entropies of X_1 and X_2 respectively. The value of $Con_2(X_1, X_2)$ is normalized to the range of $[0, 1]$. A larger value means a higher similarity between two HIs.

A metric [143] proposed for the second meaning of the consistency is denoted as

$$Con_3(X) = \exp\left(\frac{-std(P_{EoL})}{mean|P_0 - P_{EoL}|}\right),$$

(12)

where P_{EoL} is a vector composed of the HI values at the end-of-life (EoL), and P_0 is a vector composed of the HI values at the initial time.

3.2.5. Hybrid metrics

The above five types of metrics evaluate the suitability of HIs from different aspects. However, one metric is not enough to select a suitable HI for the task of RUL prediction sometimes. Operators may hope to select HIs which hold various properties or make a trade-off among different properties. To handle this issue, multiple hybrid metrics have been developed.

Liu et al. [60] proposed a hybrid metric for selecting prognostic HIs from the properties of both identifiability and consistency.

$$HM_1(X) = \begin{cases} \arg\max_i\{Ide_1(X_i)\} & if\ m = 1 \\ \arg\max_{i \neq i_r}\{\alpha_1 Ide_1(X_i) - \alpha_2 \frac{1}{m-1}\sum_{r=1}^{m-1}|Con_1(X_i, X_{i_r})|\} & if\ m \geqslant 2 \end{cases}, \tag{13}$$

where $HM_1(X)$ outputs the descending order of the HIs according to their suitability; $m = 1, 2, \ldots, M$ is the index of the ranking order with M being the number of HIs; and α_1 and α_2 are weights of the identifiability metric and the consistency metric with $\alpha_i > 0$ and $\sum_{i=1}^2 \alpha_i = 1$.

Zhang et al. [47] formulated a hybrid metric by linearly weighting three metrics.

$$HM_2(X) = \alpha_1 Mon_1(X) + \alpha_2 Tre_1(X) + \alpha_3 Rob(X), \quad s.t. \quad \alpha_i > 0, \quad \sum_{i=1}^2 \alpha_i = 1. \tag{14}$$

Coble et al. [143] defined a hybrid metric as a weighted sum of three metrics.

$$HM_3(X) = \alpha_1 Mon_1(X) + \alpha_2 Mon_2(X) + \alpha_3 Con_3(X), \quad s.t. \quad \alpha_i > 0, \quad \sum_{i=1}^3 \alpha_i = 1. \tag{15}$$

Liu et al. [30] proposed a hybrid metric to evaluate the signal-to-noise ratio of a HI.

$$HM_4(X) = \frac{R^2}{\sigma^2 + \upsilon} \tag{16}$$

where $R = \sum_{i=1}^n (x_{i,k_i} - x_{i,1})/n$ is the range information of the HI; n is the number of training units; x_{i,k_i} and $x_{i,1}$ are the HI values of the ith unit at the EoL and the initial time, respectively; $\sigma^2 = \sum_{i=1}^n \boldsymbol{e}_i \prime \boldsymbol{e}_i/(n \cdot fd)$ is the variance of the model fitting errors; $\boldsymbol{e}_i$ represents the residual errors; fd is the total freedom degree of the model; and $\upsilon = \sum_{i=1}^n (x_{i,k_i} - \bar{x}_{.,k_i})^2/(n-1)$ is the variance of the FTs.

3.3. Epilog

This section focuses on the HI construction process of the machinery prognostics. First, publications related to machinery prognostic HIs are reviewed from two categories, i.e., PHIs and VHIs. Then, special requirements of HIs for prognostics are discussed, and commonly used metrics for prognostic HIs are described systematically. It is expected to provide helpful references and tutorial for researchers in the prognostic HI construction and evaluation.

4. HS division

HIs of machinery generally present varying degradation trends with the development of fault severity. The degradation processes of machinery should be divided into different HSs according to the varying trends of HIs before RUL prediction. The term "HS division" is similar to the widely accepted terms "fault detection" or "fault diagnosis" within the PHM community [149]. But their tasks are different from each other. Fault diagnosis is to identify the fault pattern and severity of machinery at a single time point. HS division, however, aims to divide the continuous degradation processes of machinery into different HSs according to the varying trends of HIs. To illustrate the necessity of HS division, three different types of degradation processes are shown in Fig. 7, which are from the milling dataset [96], the FEMTO bearing dataset [42] and the IMS bearing dataset [81] in Section 2, respectively. The flank wear of a milling tool [96] in Fig. 7(a) presents a gradually increasing trend during the whole lifetime, which can be described using a single degradation model. Therefore, there is no need to conduct the HS division process in such cases. The degradation trend of a rolling element bearing [42] in Fig. 7(b), however, performs two distinct stages, i.e., the healthy stage and the unhealthy stage. During the healthy stage, no fault occurs in the rolling element bearing and the RMS values present random fluctuation. During the unhealthy stage, RMS values increase with the deterioration of the bearing. Since there is no information about the degradation trend in the healthy stage, it is difficult and unnecessary to predict the RUL during this stage. The RUL prediction should be triggered from the start time of the unhealthy stage, which is defined as the first predicting time (FPT). In such cases, the task of HS division is to detect the incipient degradation of machinery and provide a suitable FPT for RUL prediction. In Fig. 7(c), multiple stages are observed in the degradation process of a double row bearing [81], including a healthy stage, a degradation stage and a critical stage. The RMS values are stable during the healthy stage and then experience an "increase-decrease-increase" trend during the degradation stage. With the aggregation of damage, the RMS values increase rapidly and reach the failure threshold in short time. In such complex cases, a single degradation model is unable to describe the time-varying degradation trends. Therefore, it is necessary to divide the HIs into multiple HSs according to the change of their degradation trends and assign different models or missions to each stage. In the following two subsections, the publications related to the two-stage division and multi-stage division are summarized, respectively.
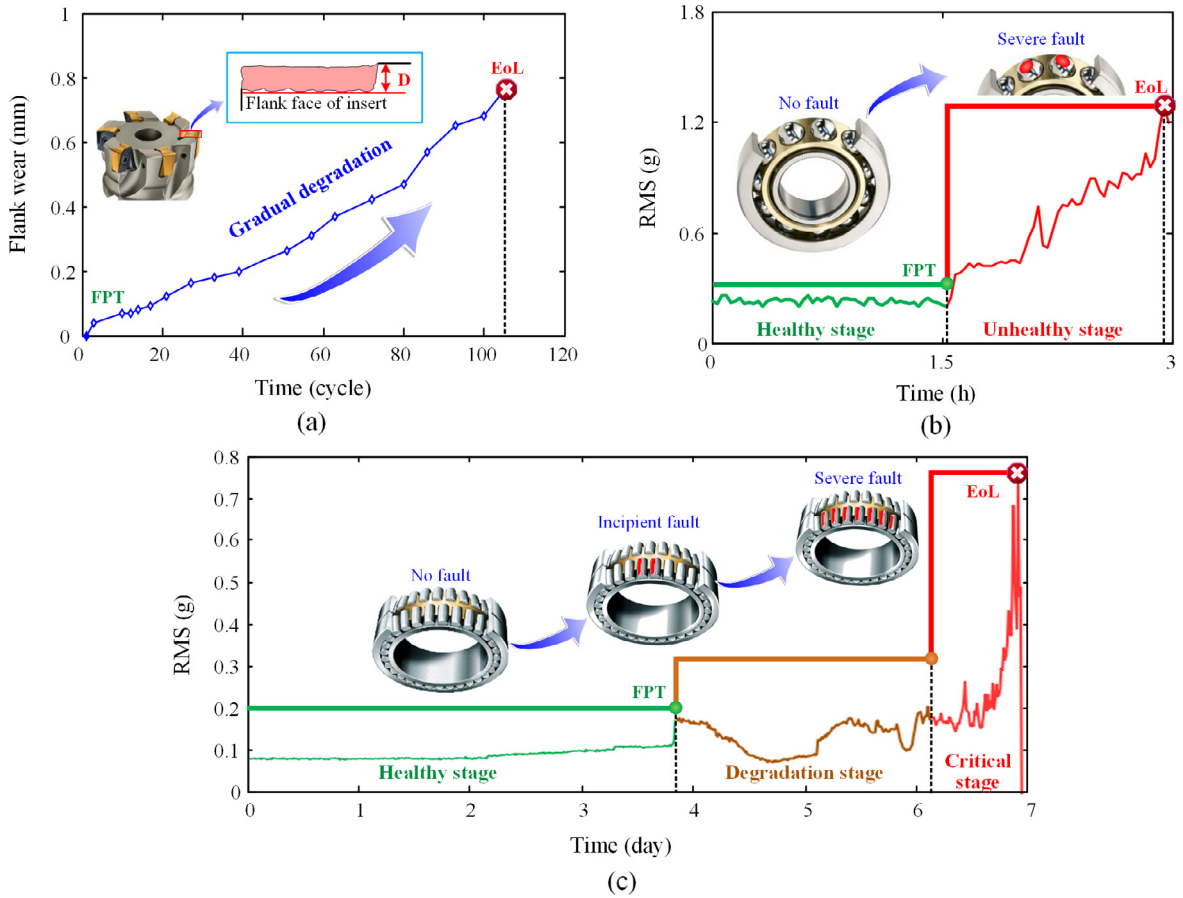
Fig. 7. Degradation processes with (a) one stage, (b) two stages and (c) multiple stages.

4.1. Two-stage division

The simplest strategy for two-stage division is to identify whether the HI exceeds a constant alarm threshold. Wang et al. [150] detected the initial point of a defect for bearings when their RMS values exceeded a pre-specified alarm threshold. Ginart et al. [151] proposed an alarm setting mechanism based on the longest time constant of machine and statistical properties of the candidate baseline for fault detection of machinery. Niu et al. [130,152] applied this mechanism to the health condition monitoring of rolling element bearings. Wang et al. [71] used the 3σ interval as the alarm threshold of the Mahalanobis distance to determine the FPT. Qian et al. [124] determined the alarm threshold using the Chebyshev inequality function and detected the fault occurrence with the help of the AR model. Jin et al. [67] transformed the HI values into Gaussian distributed data using Box-Cox transformation and calculated the 3σ interval as the alarm threshold. Zhang et al. [153] used a pre-specified confidence threshold to confirm the occurrence of incipient faults. Ajami et al. [154] gave a fault alarm when the Hotelling T^2 statistic and squared prediction error exceeded a certain threshold. Yu et al. [86] extracted HIs using the locality preserving projection approach and calculated their confidence bounds as the alarm thresholds using the kernel density estimation approach. HIs may perform random fluctuations because of the interference of random noise. In these cases, the strategy of a HI exceeding a constant alarm threshold may yield a false alarm. To avoid this problem, a trigger mechanism is generally added into the two-stage division strategy. Li et al. [62] proposed a continuous trigger mechanism to reduce the interference of random noise. Li et al. [155] detected fault occurrence of gearboxes using a probability trigger mechanism, i.e., more than a given percentage of HI values exceed an alarm threshold. Yin et al. [156] presented a robust data-driven fault detection scheme for wind turbines with robust residual generators constructed from available process data as well as suitable decision logic. Shakya et al. [157] used an algorithm based on Chebyshev inequality to determine an alarm threshold and identify time of fault occurrence once more than five consecutive data points exceed the threshold. Georgoulas et al. [158] performed the anomaly detection by combining the decisions of three different anomaly detectors through a majority voting scheme. To further improve the accuracy of two-stage division, a strategy based on adaptive alarm thresholds is developed, whose alarm thresholds are updated according to the time-varying degradation processes. Alkan et al. [159] proposed a variance sensitive adaptive alarm threshold based on PCA to detect the fault of electromechanical systems. Hu et al.

[160] employed the adaptive Gaussian threshold model and one-class support vector machine (SVM) to realize health monitoring of turbo-pumps online and offline, respectively. They also used the relevance vector machine (RVM) to establish an adaptive alarm threshold for bearings running at changing speeds [115]. Besides the above publications, there are still some other studies related to two-stage division. Fink et al. [161] treated the RUL prediction as a two-stage classification task by predicting whether the machinery is healthy or failed after a defined time interval. Schlechtingen et al. [162] compared three different model-based approaches for two-stage division of wind turbines, i.e., a regression based model and two artificial neural network (ANN) based models.

4.2. Multi-stage division

Two-stage division is only applicable in cases where the degradation trends of machinery in the unhealthy stage are consistent and can be expressed using a single degradation model. However, the degradation trends of machinery may change due to the variations of fault patterns or operational conditions. It is difficult to describe the degradation processes using a signal degradation model. Under this circumstance, the unhealthy stage should be further divided into different stages according to the various degradation trends. Some researchers divided the degradation processes into multiple stages with the analysis of change points in HIs or spectra. Kimotho et al. [61] divided the degradation processes of bearings into five stages by analyzing the changes of frequency amplitudes in the power spectral density. Sutrisno et al. [63] segmented the degradation processes of bearings into several stages based on the anomaly detection of the frequency spectra. Hu et al. [163] divided the degradation processes of generator bearings into four stages based on the change points of confidence levels. Hong et al. [50] segmented the degradation processes of bearing into four stages according to the change rate of HIs. Some publications applied clustering algorithms, such as *K*-nearest neighbor (*K*NN) [27], fuzzy *c*-means [35,60] and *K*-means [164], into the multi-stage division of machinery. The multi-stage degradation processes can also be described using discrete state transition models such as HMMs [28,32,33,65,89,165–168] and dynamic state space models [169]. AI classifiers are applied to multi-stage division as well, such as various ANNs [34,66,90,170], SVM [53,171], neural fuzzy (NF) systems [64,97].

4.3. Epilog

This section discusses the HS division process of machinery prognostics. The concept and the necessity of HS division are illustrated at first. Then, some advice is given in the selection of two-stage and the multi-stage division strategies. Finally, recent publications on this topic are summarized following the two different categories, i.e., two-stage division and multi-stage division. This section provides a comprehensive and systematic review related to the HS division of machinery prognostics for the first time, aiming to provide guidance for operators in industrial applications as well as inspirations for researchers in academic studies.

5. RUL prediction

The RUL of machinery is defined as "the length from the current time to the end of the useful life" [10], which is expressed as $l_k = t_{EoL} - t_k$, where t_{EoL} is the EoL, t_k is the current time and l_k is the RUL at t_k. Most publications [62,172–174] also define the RUL as the time left before the health states of machinery cross a FT. In these cases, the RUL is also expressed using

$$l_k = \inf(l : x(l + t_k) \geqslant \gamma), \tag{17}$$

where $\inf(\cdot)$ represents the inferior limit of a variable; $x(l + t_k)$ is the health state at $l + t_k$ with $l \geqslant 0$ and γ is the FT. It should be noticed that, the FT should be described using a probability distribution instead of a constant line, since it is influenced by various variability resources. However, in most publications, the FT is generally simplified as a constant line, because a constant line is not only easier to acquire in real applications, but also simplifies the RUL prediction process.

The major task of RUL prediction is to forecast the time left before the machinery loses its operation ability based on the condition monitoring information. It is the last technical process as well as the ultimate goal of machinery prognostics. Similar to the HI construction discussed in Section 3, there are still two major issues related to the RUL prediction. 1) How to predict the RUL based on the condition monitoring information? 2) How to measure the prediction accuracy of different approaches? These two issues will be discussed in the following two subsections, respectively.

5.1. RUL prediction approaches

As shown in Table 2, several papers [1,7–11,174] have reviewed common RUL prediction approaches and classified them into various categories. It is seen that these papers cover various categories with different emphases. In addition, their classification standards and naming rules are different from each other, which may confuse readers in the classification of RUL prediction approaches. For example, the stochastic model-based approaches are divided into statistical approaches in [1], data-driven models/approaches in [7,9,11] and conditional probability models in [8]. To unify the classification names, we analyze and compare the meanings and coverage of different categories in the seven papers of Table 2 first. Then, we

Table 2
Review papers and standards related to RUL prediction approaches.

Refs.	Up to year	Categories of RUL prediction approaches
Jardine et al. [1]	2005	Statistical approaches, model-based approaches and AI approaches
Heng et al. [7]	2008	Traditional reliability approaches, physics-based models, data-driven models and integrated approaches
Sikorska et al. [8]	2009	Knowledge-based models, aggregate reliability functions, physical models, statistical models, conditional probability models and ANN
Lee et al. [9]	2010	Model based approaches, data-driven approaches and hybrid prognostic approaches
Si et al. [10]	2011	Statistical data driven approaches
Kan et al. [11]	2015	Model-based methods, data-driven models and combination models
ISO 13381-1 [174]	2015	Heuristic models, statistical models, physics-based models, data-driven models and hybrid models

reorganize the RUL prediction approaches into the following four categories according to their basic techniques and methodologies, i.e., physics model-based approaches, statistical model-based approaches, AI approaches and hybrid approaches. Fig. 8 shows a pie chart of publications related to the four categories covered by this review. Although this review paper cannot cover all publications, the statistical result still works in showing the approximate proportion of each category. In the following subsections, these four categories will be summarized in order.

5.1.1. Physics model-based approaches

Physics model-based approaches describe degradation processes of machinery through building mathematical models on the basis of the failure mechanisms or the first principle of damage [175]. The parameters of the physics models are correlated to the material properties and stress levels, which are generally identified by using specific experiments, finite element analysis or other suitable techniques. As shown in Fig. 8, physics model-based approaches only cover 10% of the publications. The Paris-Erdogan (PE) model is one of the most widely used physics models in the RUL prediction of machinery. It was first proposed in [176] to describe the crack growth. Then, many versions [95,139,177–189] were developed and applied to the field of machinery prognostics. Wang et al. [190] and Lei et al. [56] transformed the PE model into an empirical model for RUL prediction of machinery. Liao [135] and Sun et al. [191] enhanced the PE model into a state space model. Besides the PE model and its variants, there are still some other physics models in the RUL prediction of machinery. Oppenheimer et al. [192] used the Forman crack growth law [193] to predict the RUL of cracked rotor shafts. Baraldi et al. [194,195] and Hu et al. [196] employed the Norton law to describe the creep evolution of turbines and predicted the RUL with the combination of Kalman filtering (KF) and particle filtering (PF). Chan et al. [197] developed a physical time-dependent crack growth model and applied it to the life prediction of turbo-propulsion systems. El-Tawil et al. [198] introduced the stochastic description into a nonlinear damage law to predict the RUL of pipeline tubes. The physics model-based approaches are able to provide accurate estimation of RUL if the physics model is developed with complete understanding of the failure mechanisms and effective estimation of model parameters. For some complex mechanical systems, however, it is difficult to understand the physics of damage, which restricts the application of these approaches.
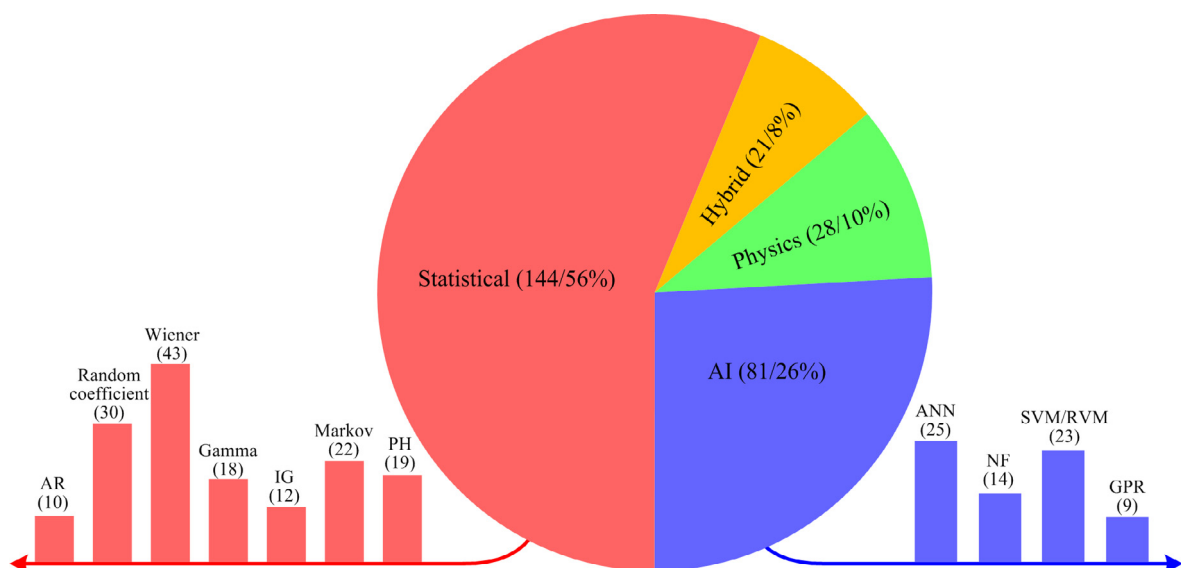
Fig. 8. Pie chart of publications related to the four categories of RUL prediction approaches.

5.1.2. Statistical model-based approaches

Statistical model-based approaches, also named as empirical model-based approaches, estimate the RUL of machinery by establishing statistical models based on empirical knowledge, and generally present the RUL prediction result as a conditional PDF depending on observations [10]. In these approaches, RUL prediction models are constructed by fitting available observations into random coefficient models or stochastic process models under a probabilistic method, without relying on any physics or principles. Random variances are generally introduced into model parameters to describe the uncertainties caused by different kinds of variability sources, such as the temporal variability, unit-to-unit variability and measurement variability [69]. Therefore, the statistical model-based approaches are effective in describing the uncertainty of the degradation process and its influence on RUL prediction. This category has become the most popular one among the four categories as shown in Fig. 8. There have been some reviews [10,199] summarizing the statistical model-based approaches systematically. To avoid repetition with them, this paper focuses on the advancements of the statistical model-based approaches in recent years.

5.1.2.1. AR models. AR models assume that the future state value of machine is a linear function of past observations and random errors [8]. Qian et al. [124] utilized the AR model to predict the degradation processes of bearings. Barraza-Barraza et al. [200] constructed three AR models with exogenous variables to predict the RUL of aluminum plates with fatigue crack. Escobet et al. [201] used an AR model to predict the RUL of a conveyor belt system. Some researchers [202,203] also combined AR models with the PF algorithm to predict the RUL of machinery. An enhanced version of AR models, i.e., the AR moving average model was also employed into the RUL prediction of machinery in [203–206]. The major advantage of these approaches is the simplicity of their calculation. The disadvantage is that their performance highly depends on the trend information of historical observations, which may lead to inaccurate forecasts as time goes on.

5.1.2.2. Random coefficient models. Random coefficient models describe the stochasticity of degradation processes by adding random coefficients into degradation models, which are generally assumed to be normally distributed. Lu et al. [207] and Meeker et al. [208] used a nonlinear mixed-effects model to describe the machinery degradation processes and predicted the RUL PDF through Monte Carlo simulation. Gebraeel et al. [172] proposed an exponential model with random error terms and estimated the model parameters using a Bayesian approach. Other variants of this model were developed in [23,129,209–215]. Zhou et al. [216] constructed a random coefficient model based on the amplitudes of HIs after axis transformation. Wang et al. [150,217,218] and Carr et al. [219,220] described the RUL distribution using a general random coefficient model. Park et al. [221] used a nonlinear random-coefficient model to describe the degradation processes and used the maximum likelihood estimation algorithm to estimate the model parameters. Coble et al. [102] developed a prognostic method by incorporating the prior belief into random coefficient models and predicted the RULs of machining tools. A random coefficient model constructed with the sum of two exponential functions was applied in [67,119]. Some researchers [106,222–227] combined random coefficient models with Bayesian filtering algorithms for RUL prediction of machinery. Random coefficient models are able to provide a PDF of RUL by including the variation of random coefficients into the prediction result. But the assumption of Gaussian distribution for the random coefficients may restrict their applications. In addition, it is unable to describe the temporal variability in RUL prediction [228].

5.1.2.3. Wiener process models. Wiener process models are generally presented as a drift term plus a diffusion term following Browning motion. They are a kind of the most commonly used stochastic process models. Doksum et al. [229] applied Wiener process models to the RUL prediction of variable-stress accelerated degradation tests. Whitmore et al. [230] proposed a time-scale transformation strategy to transform the time-varying degradation drift of Wiener process models into a constant degradation drift. Tseng et al. [231,232] scheduled optimal burn-in policies based on the RUL prediction of Wiener process models. Park et al. [233–235] developed several Wiener process models based on a generalized cumulative damage approach. Some researchers [70,114,236–241] developed a linear Wiener process model for the RUL prediction of machinery. Gebraeel et al. [172] proposed an exponential model with Browning motion errors. This model was further studied and improved in [29,62,118,173,242–245]. Si et al. [246] proposed an age-dependent nonlinear model and derived a closed-form RUL PDF. This model was developed in [69,71,112,247–256]. Li et al. [257] and Zhang et al. [116] constructed general age- and state-dependent models for machinery RUL prediction. Bian et al. [258] developed Wiener process models by considering the interactions between different components and time-varying operational conditions. Fang et al. [259] proposed a RUL prediction approach based on a Wiener process model in cases of missing data. Paroissin et al. [260] established a randomly delayed Wiener process model considering the randomness of the degradation initiation time. Compared with random coefficient models, Wiener process models are able to describe the temporal variability of the degradation processes. And they are effective in modeling the non-monotonic processes by assuming the random noise following a Brownian motion. However, Wiener process models are based on the assumption of Markov property, i.e., the future state just depends on the current state and is independent of the past behavior. But this assumption does not always work in real applications. In addition, the analytical solutions of RUL PDF are difficult to be derived for state-dependent Wiener process models. In such cases, numerical or analytical solutions are generally used to approximate the RUL PDF [257].

5.1.2.4. Gamma process models. Gamma process models assume that the increments of degradation processes at disjoint time intervals are independent random variables with a gamma distribution. Noortwijk et al. [261] overviewed the devel-

opment of gamma processes in stochastic degradation modeling. Kuniewski et al. [262] assumed that the degradation process started at a random time following a non-homogeneous Poisson process and then developed following a gamma process. Bagdonavicius et al. [263] described the degradation processes using a gamma process model and considered the influence of covariates and traumatic events on degradation. Lawless et al. [264] constructed a tractable gamma process model by introducing the effects of random degradation rates. Pandey et al. [228] compared the random coefficient model with the gamma process model and concluded that the gamma process model is more versatile for RUL prediction. Park et al. [233–235] developed a gamma process model based on a general cumulative damage approach. Chakraborty et al. [265] improved the random coefficient model in [172] into a gamma process model by assuming the model parameter as a highly skewed gamma distribution. Tsai et al. [266–268] studied two misspecification problems of a gamma process when it was wrongly assumed as a Wiener process and random effects were not taken into consideration. Other publications related to RUL prediction based on gamma processes are in [269–273]. Similar to Wiener process models, Gamma process models are able to describe the temporal variability of degradation processes. Nevertheless, they also have the following shortcomings. First, gamma process models are also restricted to the assumption of Markov property. Second, noise in gamma process models must follow a gamma distribution, which means that they are only effective in describing the monotonic processes.

5.1.2.5. Inverse Gaussian process models. Inverse Gaussian (IG) process models assume that the degradation processes of machinery have independent increments following an IG distribution. Wang et al. [274] first discussed the application of IG in the degradation process description. Based on the above study, Ye [275,276] and his co-authors [277] have conducted lots of work in this aspect. Qin et al. [278] and Zhang et al. [279] applied the IG process model to the growth prediction of the corrosion defect depth of underground energy pipelines. Peng [280] improved the IG process model in [274] by including an inverse normal-gamma mixture. Liu et al. [281] proposed a degradation modeling approach for a system with multiple degradation patterns based on IG processes. Peng et al. [282] developed a general Bayesian framework for degradation analysis of IG process models. Giner et al. [283] developed software for the probability distribution calculation based on the IG process model. Pan et al. [284] proposed a RUL prediction approach based on the IG process model with random effects. The major superiority of IG process models is that different kinds of random effects are able to be incorporated into them. This superiority increases the flexibility of IG process models in describing various degradation processes. However, IG process models have similar properties to gamma process models, thus they are still restricted to Markov property and monotonic processes.

5.1.2.6. Markov models. Markov models assume that the degradation processes of machinery transform within a finite state space following the principle of the Markov property [285]. The Markov models were first introduced into the field of RUL prediction by Kharoufeh et al. [285]. After that, Kharoufeh et al. [286,287] enriched the basic theory of Markov models in the application of RUL prediction. Kurt et al. [288] formulated an infinite-horizon Markov decision process model to optimize maintenance schedules. Giorgio et al. [289] established degradation models for the wear of cylinder liners by combining a Markov process and a gamma process. Liu et al. [290] proposed a non-homogeneous continuous time Markov model to predict the RUL of multi-state systems and optimize the replacement policy based on the prediction result. Markov models describe the degradation processes of machinery with the assumption that the health states can be observed directly. However, the health states are generally impossible to be observed. To describe the degradation processes of hidden health states, the HMM was applied to the machinery prognostics [28,32,165,166,291–294]. To improve the flexibility of HMMs for representing complicated state transition processes, the hidden semi-Markov model was also applied in the field of machinery prognostics [37,167,295–300]. Since the health conditions of machinery can generally be divided into several HSs, Markov models are suitable in dealing with the multi-stage transition processes. However, all Markov models are based on the assumption of Markov property, which may lead to inconsistence against real cases. In addition, the transition probabilities among different HSs are often estimated by large numbers of training units, which may be difficult to be acquired in real applications.

5.1.2.7. Proportional hazards models. Proportional hazards (PH) models were first proposed by Cox in [301], which assumed that the hazard rate of a system was composed of two multiplicative factors, i.e., a baseline hazard function and a covariate function. Makis et al. [302,303] used a PH model to describe the failure rate of a system, and established replacement rules by minimizing the long-run expected average cost per unit time. Kumar et al. [304] gave an excellent review about the development of PH models. Jardine et al. [305] developed condition monitoring software named as EXAKT based on PH models, and applied it to the condition monitoring of different kinds of machinery [306,307]. Banjevic et al. [308] predicted the RUL of machinery with the combination of a condition monitoring process and a Weibull PH model. Liao et al. [92] employed a PH model-based approach to predict the RUL of bearings. Elsayed et al. [309] designed an optimum multiple-stress accelerated life testing plan based on PH models. Other RUL prediction approaches developed from PH models were presented in [76,310–316]. PH models integrate the information from both the event data and condition monitoring data [7]. Thus, they are expected to achieve more accurate prediction results when enough event data and condition monitoring data are available. However, it is difficult to capture these two kinds of data simultaneously. Furthermore, the covariate functions of PH models need to be described using other stochastic process models, such as Markov models, which adds burden to the computational procedure.

5.1.3. AI approaches

AI approaches attempt to learn the machinery degradation patterns using AI techniques from available observations instead of building physics models or statistical models. They are capable of dealing with prognostic issues of complex mechanical systems whose degradation processes are difficult to be interrelated by physics models or statistical models. Therefore, they are attracting more and more attentions in the field of machinery prognostics. According to the statistical result in Fig. 8, this category has the second largest amount of publications behind the statistical model-based approaches. The results of AI approaches are hard to be explained because of the lack of transparency, thus these techniques are always named as "black boxes". The commonly used AI techniques in the field of machinery prognostics include ANNs, NF systems, SVM/RVM, *K*NN, Gaussian process regression (GPR), etc. The following contents of this subsection will review their advancements, respectively.

5.1.3.1. ANN.
ANNs mimic the working process of human brains which connect lots of nodes in a complex layer structure. They are the most commonly used AI techniques in the field of machinery RUL prediction. Among them, Feed-forward neural networks (FFNNs) are the most popular ones. Most publications [25,91,105,117,132,142,317–321] used a FFNN to learn the relationship between the HIs and the lifetime. Sbarufatti et al. [322] combined FFNNs with sequential Monte-Carlo sampling to predict the RUL of fatigue cracks. Pan et al. [73] and Xiao et al. [79] used a FFNN to conduct multi-step ahead prediction for the bearing health states. Wang et al. [76] used a three-layer FFNN to predict the future HIs, and input the predicted HIs into a PH model to estimate the hazard rate and survival probability. Recurrent neural networks (RNNs) are also widely used in the RUL prediction because of its ability in dealing with explicit time-series data. Zemouri et al. [38,323] proposed a recurrent radial basis function network and used it to predict the RUL of machinery. Malhi et al. [113] proposed a competitive learning-based approach to revise the training technique of RNNs aiming at improving the long-term prediction accuracy. Heimes et al. [17] proposed a prognostic approach based on an RNN which was trained using time gradient calculations and extended KF. Peng et al. [19] enhanced the RNN by replacing the hidden layer using a large sparse reservoir and developed a new RUL prediction approach. Liu et al. [324] proposed an enhanced RNN for RUL prediction by improving RNNs' memory property. Besides the above two commonly used ANNs, there are some other ANNs [45,73,325,326] in the field of machinery RUL prediction. ANNs are able to learn complex non-linear relationships by training the multi-layer networks. Therefore, they are expected to have a good performance in the RUL prediction of complex systems. However, they still have their own limitations. Besides the low transparency, ANNs generally require large numbers of high-quality training data, which are difficult to capture in industrial applications. In addition, their structures and parameters are generally initialized randomly or specified manually, which reduces their generalization ability among different cases.

5.1.3.2. NF systems.
NF systems are fuzzy logic systems whose inference structures are determined by expertise and membership functions are optimized by ANNs [327]. The NF-based time-series forecasting approach was proposed by Jang et al. [328]. Wang et al. [329] adopted the NF system to develop an online prognostic approach for different kinds of gear faults. Wang [330,331] further improved this approach from several aspects to improve its forecasting performance. Liu et al. [332] proposed a multi-step predictor based on a weighted recurrent NF system. Zhao et al. [333] verified the effectiveness of NF systems in predicting the machinery health states using bearing degradation data. Tran et al. [334] proposed a multi-step ahead prediction approach with the combination of NF systems and regression trees. Chen et al. [335,336] constructed a high-order state space model using an adaptive NF inference system and applied this approach to machinery RUL prediction. EI-Koujok et al. [22] proposed a prognostic method based on NF systems by applying the parsimony principle to balance the complexity and accuracy capability. Ishibashi et al. [20] established a genetic fuzzy rule-based prognostic system with the membership functions tuned using the genetic algorithm. Zurita et al. [64] predicted the future HI values by means of an adaptive NF inference system and estimated the health states based on the predicted HI values. Hussain et al. [337] employed the adaptive NF inference system to predict the degradation processes of wind turbine gearboxes. NF systems take advantage of both the expert knowledge and the intelligent ANNs, thus being expected to be competitive candidates for machinery RUL prediction. However, they still need lots of high-quality training data. More research work should be conducted in the RUL prediction based on NF systems when available training data are limited.

5.1.3.3. SVM/RVM.
SVM is a kind of AI techniques based on the statistical learning theory proposed by Vapnik [338]. Different kinds of SVM have been applied to the RUL prediction of machinery, such as the least square-SVM [87,130], one-class SVM [72] and multi-class SVM [61,65]. Widodo et al. [339] trained a SVM model using both the censored and the complete data, and predicted the survival probability of machinery. Tran et al. [340] integrated a SVM-based RUL prediction module into an intelligent condition-based maintenance platform. Support vector regression (SVR) is the common application form of SVM in the field of prognostics [128]. Benkedjouh et al. [75,128] used the SVR to map the HIs into nonlinear regressions, and then fitted the obtained regressions into power models for RUL prediction of machinery. Liu et al. [341,342] developed a modified probabilistic SVR to predict the degradation processes of nuclear power plant components. Fumeo et al. [74] developed an online SVR model for the RUL prediction of bearings by optimizing the trade-off between the accuracy and the computing efficiency. Some other SVR-based prediction approaches were also developed in [39,44,53,60,343]. Despite the widespread applications of SVM, they also suffer from some limitations. The major one is that they only provide point prediction rather than probabilistic prediction [344]. To tackle the limitations of SVM, Tipping [345] formulated the RVM, which has the same functional form as SVM but provides a full predictive distribution. Recently, RVM has been introduced

into the area of machinery prognostics [24,93]. Compared with ANNs, SVM and RVM are superior to deal with the issues of small sample sizes. Thus, they may be more suitable for the issues of RUL prediction where only limited measurements are available. However, the performance of SVM and RVM is highly dependent on the selected kernel functions. And standard methods of choosing kernel functions have not been established. Moreover, parameter estimation is still a challenge for SVM and RVM.

5.1.3.4. GPR. GPR is an AI technique which implements Gaussian processes for regression purposes [346]. Gaussian processes are cumulative damage processes of random variables with joint multivariate Gaussian distributions. Rasmussen [347] introduced theoretical details and the flexibility of the GPR in non-linear regression. Goebel et al. [348] and Saha et al. [349] applied the GPR to RUL prediction. Hong et al. [49] applied the GPR with three different covariance functions for RUL prediction of bearings. Huber et al. [350] developed an online recursive strategy for GPR models to reduce the computational and memory demands in cases of large datasets. Liu et al. [351] improved long-term prediction performance of GPR by combining two covariance functions to capture the actual trends of both global degradation and local regeneration. Aye et al. [352] predicted the degradation trends of rolling element bearings using an integrated GPR model. In contrast to the above AI techniques, GPR has high adaptability and is suitable for dealing with the RUL prediction issue of high-dimension and small-size datasets [11]. The major drawback of GPR is that it generally has heavy computational demand.

5.1.4. Hybrid approaches

As mentioned before, all the three categories have their own limitations in the RUL prediction. A hybrid approach attempts to integrate advantages of different approaches through their integration. As shown in Fig. 8, this category covers the least amount of publications among the four categories. Some papers [335,336,353–358] used different methods to construct a degradation model and combined it with PF for RUL prediction. Some researchers [18,40,322,359] predicted the RULs using several prediction approaches and made a final decision through some kinds of fusion strategies. The AI techniques were often combined with random coefficient models in hybrid approaches [23,117,360,361]. Several hybrid approaches [33,89] were developed by combining NF systems with HMMs. Zemouri et al. [38] combined ANNs with AR models to develop a hybrid approach. Sankavaram et al. [362] established a hybrid integrated diagnosis and prognosis framework, and applied it to the prognostics of automotive and on-board electronic systems. Wang et al. [24] employed the similarity-based approach to predict the RUL of machinery with the cooperation of sparse learning of RVM for HIs. More detailed information about the hybrid RUL prediction approaches was provided in [363].

5.2. Metrics for RUL prediction

Establishing uniform metrics is significant for the comparison of RUL prediction approaches. Various RUL prediction metrics have been proposed according to different requirements of researchers and operators. These metrics evaluate the RUL prediction results from different aspects. It is hard to identify which one is better than others. As shown in Table 3, these metrics can be classified into three categories according to the different independent variables of their functions: 1) metrics depending on ground truth RULs, 2) metrics depending on run-to-failure data, and 3) metrics depending on available measurements. Different available information is required in the calculation processes of these three categories. Therefore, readers are suggested to select suitable ones from these candidates according to both their requirement and available information in practice. These metrics are summarized and listed as follows for convenient selection of researchers.

Table 3
Summary of the metrics for RUL prediction.

Categories	Names	Refs.
Metrics depending on ground truth RULs	• RMSE	Yang et al. [142]
	• CI	Yang et al. [142]
	• Prediction horizon	Saxena et al. [364]
	• α-λ accuracy	Saxena et al. [364]
	• Relative accuracy	Saxena et al. [364]
	• CRA	Saxena et al. [364]
	• Convergence	Saxena et al. [364]
	• ETA	PHM 2008 [14], IEEE PHM 2012 [41]
Metrics depending on run-to-failure data	• Predictability	Javed et al. [35]
	• Mean prediction error, Std	Zemouri et al. [38]
	• Overall average bias	Zemouri et al.[38]
	• Overall average variability	Zemouri et al.[38]
	• Reproducibility	Zemouri et al.[38]
Metrics depending on available measurements	• Online RMSE	Hu et al. [196]
	• Online coverage	Hu et al. [196]
	• Online width	Hu et al. [196]

5.2.1. Metrics depending on ground truth RULs

(1) Root mean square error

Root mean square error (RMSE) [142] is defined as the root mean square of the RUL prediction errors during the time interval from t_{FPT} to t_{EoL}, which is denoted as

$$RMSE = \sqrt{\frac{1}{EoL - FPT} \sum_{k=FPT}^{EoL} (l_{t_k} - l_{t_k}^*)^2}, \tag{18}$$

where FPT and EoL are the time indexes of the FPT and EoL respectively, and l_{t_k} and $l_{t_k}^*$ are the predicted RUL and the ground truth RUL at t_k, respectively. A higher RMSE score means a larger average prediction error.

(2) Confidence interval

The confidence interval (CI) [142] metric is defined as an interval that contains a specified percentage of the predicted RULs at a time index.

$$Pr\left(l_{t_k} - \frac{CI_{t_k}}{2} \leqslant l_{t_k}^i \leqslant l_{t_k} + \frac{CI_{t_k}}{2}\right) = \xi, \tag{19}$$

where $Pr(\cdot)$ is the probability density function; ξ is the pre-specified percentage which is generally selected as 95%; and $l_{t_k}^i$ is the predicted RUL of the instance i at t_k. In real application, a narrow CI is preferred since it means that the RUL prediction results are more stable and concentrated, which provides a more credible guidance for maintenance decision.

(3) Prediction horizon

Prediction horizon [364] is defined as the difference between the time when the prediction results first satisfy a specified performance criterion and the EoL, which is expressed as follows

$$PH = t_{EoL} - t_{i_{\alpha\beta}}, \tag{20}$$

where $i_{\alpha\beta} = \min\{k | (k \in \Omega) \wedge (\pi(p(l_{t_k}))|_{-\alpha}^{+\alpha}) \geqslant \beta\}$ is the first time index when predictions satisfy β -criterion for a given α; Ω is the set of all time indexes; $p(l_{t_k})$ is the predicted RUL PDF at t_k; t_{EoL} is the predicted EoL; $\pi(p_f(l_{t_k}))|_{-\alpha}^{+\alpha} = \int_{\alpha^-}^{\alpha^+} p(l_{t_k}) d(l_{t_k})$, with $\alpha^- = l_{t_k}^* - \alpha t_{EoL}$ and $\alpha^+ = l_{t_k}^* + \alpha t_{EoL}$ and $l_{t_k}^*$ is the ground truth RUL at t_k. The prediction horizon metric provides a time length when the prediction results of an approach are within specified limits around the ground truth RUL. It is obvious that a longer prediction horizon means that more prediction results have desired credibility. Therefore, an approach with a longer prediction horizon would be better than others.

(4) α-λ accuracy

$\alpha - \lambda$ accuracy [364] is defined as a binary metric that evaluates whether the prediction results fall in specified α-bounds at particular time indexes.

$$\alpha\text{-}\lambda \text{ accuracy} = \begin{cases} 1 & \text{if } \pi(p(l_{t_\lambda}))|_{-\alpha}^{+\alpha} \geqslant \beta \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

where λ is the time window modifier such that $t_\lambda = t_{FPT} + \lambda(t_{EoL} - t_{FPT})$ with t_{FPT} representing the FPT; $\pi(p(l_{t_\lambda}))|_{-\alpha}^{+\alpha} \geqslant \beta$ means the probability mass of the prediction PDF within the α-bands, i.e., $\int_{\alpha^-}^{\alpha^+} p(l_{t_\lambda}) dt$, is not less than the minimum acceptable probability β, with $\alpha^- = l_{t_\lambda}^* - \alpha l_{t_\lambda}$; $\alpha^+ = l_{t_\lambda}^* + \alpha l_{t_\lambda}$ and l_{t_λ} is the predicted RUL at t_λ. Taking $\alpha = 0.1$ and $\lambda = 0.5$ as an example, this metric measures whether the prediction result falls within 10% accuracy of the ground truth RUL at the halfway of the prediction period $t_{FPT} + 0.5(t_{EoL} - t_{FPT})$. If the output is 1, it implies that the desired prediction accuracy is satisfied at a specific time.

(5) Relative accuracy

Relative accuracy (RA) [364] is a metric defined based on the relative error of the RUL prediction result at a specific time.

$$RA_\lambda = 1 - \frac{|l_{t_\lambda}^* - l_{t_\lambda}|}{l_{t_\lambda}^*}, \tag{22}$$

where $l_{t_\lambda}^*$ and l_{t_λ} are the ground truth RUL and the predicted RUL at t_λ respectively, with $t_\lambda = t_{FPT} + \lambda(t_{EoL} - t_{FPT})$.

Compared with the $\alpha - \lambda$ accuracy, RA measures the prediction results more quantitatively instead of determining whether the prediction results fall in a given accuracy level. The perfect score for relative accuracy is 1, which means that the predicted RUL is equal to the ground truth RUL.

(6) *Cumulative relative accuracy*

It should be noticed that both the $\alpha - \lambda$ accuracy and RA measure the prediction accuracy at a single time point. To give a more comprehensive measure about the RUL prediction results during a period of lifetime, a cumulative RA (CRA) metric is developed by aggregating the relative accuracy values at specific time instances.

$$CRA_\lambda = \frac{1}{|\Omega_\lambda|} \sum_{k \in \Omega_\lambda} w(l_{t_k}) RA_\lambda, \tag{23}$$

where $w(l_{t_k})$ is a weight function depending on the predicted RUL; Ω_λ is the set of all time indexes from t_{FPT} to t_λ; and $|\Omega_\lambda|$ is the cardinality of the set.

(7) *Convergence*

Convergence [364] is defined as the distance between the origin and the centroid of the area under the curve of a metric M_k, such as the relative accuracy.

$$C_M = \sqrt{(x_c - t_{FPT})^2 + y_c^2}, \tag{24}$$

where (x_c, y_c) denotes the centroid of a non-negative metric M_k between t_{FPT} and t_{EoL}, which are calculated using

$$x_c = \frac{\sum_{k=FPT}^{EoL-1}(t_{k+1}^2 - t_k^2) M_k}{2 \sum_{k=FPT}^{EoL-1}(t_{k+1} - t_k) M_k}, \tag{25}$$

$$y_c = \frac{\sum_{k=FPT}^{EoL-1}(t_{k+1}^2 - t_k^2) M_k^2}{2 \sum_{k=FPT}^{EoL-1}(t_{k+1} - t_k) M_k}. \tag{26}$$

The convergence metric measures the convergence speed of a certain metric. A lower metric score means a faster convergence speed, which is expected in real applications, since we hope the prediction results approach the ground truth RUL as soon as possible during the RUL prediction process.

(8) *Exponential transformed accuracy*

In order to distinguish the different hazard severity caused by the underestimate and overestimate of RUL, two RUL prediction metrics based on exponential transformation were developed for the prognostic challenge of PHM 2008 [14] and IEEE PHM 2012 [41], respectively. These metrics are named as exponential transformed accuracy (ETA). They both assign a larger deduction to the overestimated results than the underestimated results, since overestimation may lead to more severe damage than underestimation. Their formulas are expressed as

$$ETA_1 = \begin{cases} \exp\left(-\frac{ER}{10}\right) - 1 & \text{if } ER < 0 \\ \exp\left(\frac{ER}{13}\right) - 1 & \text{if } ER \geqslant 0 \end{cases} \tag{27}$$

$$ETA_2 = \begin{cases} \exp\left(-\ln(0.5)\frac{RE}{5}\right) & \text{if } RE \leqslant 0 \\ \exp\left(\ln(0.5)\frac{RE}{20}\right) & \text{if } RE > 0 \end{cases} \tag{28}$$

with $ER = l - l^*$ and $RE = 100(l^* - l)/l^*$. l^* and l are the ground truth RUL and predicted RUL, respectively.

5.2.2. Metrics depending on run-to-failure data

(1) *Predictability*

A similar prediction metric based on the exponential transformation is named as predictability [35] which is defined as the ability of an approach to predict a series of future HI values at a specific horizon which satisfies a desired performance limit. Different from the ETA metric, predictability measures the prediction accuracy from the perspective of degradation trend prediction instead of the RUL prediction.

$$Pred(X|H,L) = \exp\left(-|\ln(0.5)\frac{MFE_X^H}{L}|\right), \tag{29}$$

where H is a specific horizon; X is the predicted HI values at the specific horizon; L is a desired performance limit; and MFE_X^H is the mean forecast error between the prediction values X and the actual values X^*.

$$MFE_X^H = \frac{1}{H}\sum_{h=1}^{H}(X_h - X_h^*) \tag{30}$$

(2) *Mean prediction error and standard deviation*

Some other metrics based on predicted HI values were presented in [38]. Suppose that there are M different prediction models acquired from distinct training processes, and each model is employed to predict the HI values H-step ahead. The mean prediction error and the standard deviation of the prediction results are formulated as follows:

$$E(m) = \frac{1}{H}\sum_{h=1}^{H}(X_h^m - X_h^*), \quad Std(m) = \sqrt{\frac{1}{H}\sum_{h=1}^{H}(X_h^m - X_h^*)^2}, \tag{31}$$

where H is the amount of prediction steps, and X_h^m and X_h^* represent the predicted HI value of the m th model and the actual value in the h th step, respectively. On the basis of these two metrics, the following three metrics [38] are developed.

(3) *Overall average bias*

Overall average bias is defined as the mean of the absolute mean prediction errors of the M models.

$$OAB = \frac{1}{M}\sum_{m=1}^{M}|E(m)| \tag{32}$$

This metric measures how close the predicted HI values of the M models are to the actual values, and the perfect score is 0.

(4) *Overall average variability*

Overall average variability is defined as the mean of the standard deviations of the M models.

$$OAV = \frac{1}{M}\sum_{m=1}^{M}Std(m) \tag{33}$$

This metric evaluates how closely the predicted HI values of the M models are clustered together, and the perfect score is 0.

(5) *Reproducibility*

Reproducibility is defined as a mean distance between all prediction results of M prediction models.

$$Rep = \sqrt{\frac{2}{M(M-1)}\sum_{i=1}^{M}\sum_{j=i+1}^{M}(d_{i,j})^2}, \tag{34}$$

where $d_{i,j}$ is the Euclidian distance between the i th and j th prediction models with $(d_{i,j})^2 = (E(j) - E(i))^2 + (Std(j) - Std(i))^2$. This metric measures the similarity of prediction results of all the M prediction models, and the perfect score is 0.

5.2.3. Metrics depending on available measurements

The above metrics evaluate the performance of prediction approaches based on ground truth RULs or run-to-failure data. All of them belong to an offline assessment strategy. Considering the online monitoring cases, where only HIs $X_{1:t_k}$ before and at the current lifetime t_k are observed, the offline strategy does not work in such cases. Three prediction metrics were specially designed in [196] for the online monitoring cases. Suppose that l_{t_k} is the predicted RUL at t_k. $win_t = [t, t + l_{t_k}]$ is a time window with the fixed time length l_{t_k}, and $t = t_s, t_{s+1}, \ldots t_k - l_{t_k}$ with t_s being the initial time of evaluation. The FT at t is defined as the estimated state value at $t + l_{t_k}$ based on $X_{1:t_k}$, i.e., $FT_t = \hat{x}_{t+l_{t_k}|X_{1:t_k}}$. The RUL PDF estimated based on $X_{1:t}$ and FT_t is denoted as $p(l_t|X_{1:t}, FT_t)$. The three online prediction metrics are described as follows.

(1) *Online root mean square error*

$$ORMSE_{t_k} = \sqrt{\frac{\sum_{t=t_s}^{t_k-l_{t_k}} win_t a^2}{t_k - t_s - l_{t_k} + 1}} \tag{35}$$

$$win_t a = \int |l_t - l_{t_k}| p(l_t|X_{1:t}, FT_t) d(l_t) \tag{36}$$

Online RMSE measures the error between the predicted RULs and the expected ones, and a lower score means a higher prediction accuracy.

(2) *Online coverage*

$$OC_{t_k} = \frac{1}{t_k - t_s - l_{t_k} + 1} \sum_{t=t_s}^{t_k-l_{t_k}} win_t c \tag{37}$$

$$win_t c = \begin{cases} 1 & C_{\inf}(p(l_t|X_{1:t}, FT_t)) < l_{t_k} < C_{\sup}(p(l_t|X_{1:t}, FT_t)) \\ 0 & \text{otherwise} \end{cases} \tag{38}$$

where $C_{\inf}(p(l_t|X_{1:t}, FT_t))$ and $C_{\sup}(p(l_t|X_{1:t}, FT_t))$ are the lower and upper bounds of the 90% CI of $p(l_t|X_{1:t}, FT_t)$. Online coverage measures the probability of the expected RUL belonging to the 90% CI of the predicted RUL PDF, with a range of $[0, 1]$. A higher score implies a more credible prediction result.

(3) *Online width*

$$OW_{t_k} = \frac{1}{t_k - t_s - l_{t_k} + 1} \sum_{t=t_s}^{t_k-l_{t_k}} win_t w \tag{39}$$

$$win_t w = C_{\sup}(p(l_t|X_{1:t}, FT_t)) - C_{\inf}(p(l_t|X_{1:t}, FT_t)) \tag{40}$$

Online width measures the mean width of 90% CI of the predicted RUL PDF during the time window. A lower score is expected since it may provide a more concentrated RUL prediction result.

5.3. Epilog

This section focuses on the last process as well as the goal of prognostics, i.e., RUL prediction. In the first subsection, RUL prediction approaches commonly used in literature are organized into four different categories based on the analysis and conclusion of various review papers. Then, these categories are introduced systematically including their definitions, common approaches, merits and shortcomings. In the second subsection, the metrics for RUL prediction are reviewed and displayed in terms of three categories according to their different independent variables. There is still an Achilles's heel in machinery RUL prediction which is not discussed in the above two subsections, i.e., how to handle the uncertainty characterization in long-term prediction. The common strategy is to handle this issue by using Bayesian filtering algorithms. To highlight this issue, a summary about the Bayesian filtering algorithms is given at the end of this section.

Bayesian filtering algorithms characterize the future uncertainty of machinery degradation processes by updating the state probabilistic estimation according to real-time monitoring information. In Bayesian filtering algorithms, the degradation processes of machinery are always described using state space models, which are constructed based on the physics models, stochastic process models or AI techniques mentioned above. The KF algorithm [194,239,251,365,366] is one of the commonly used Bayesian filtering algorithms in machinery RUL prediction. But it is only effective in dealing with linear Gaussian problems. Therefore, some of its enhanced versions are also employed into the area of machinery RUL prediction, such as extended KF [46,67,220,249], unscented KF [222] and switching KF [169]. The PF algorithm is a Monte Carlo-based Bayesian filtering algorithm particularly used for nonlinear and/or non-Gaussian problems. It was first introduced into the field of machinery prognostics by Orchard et al. [226]. After that, PF has achieved remarkable development in real-time RUL prediction, because it provides a reasonable and theoretically accepted manner for characterizing future uncertainty in real time. More publications related to PF-based RUL prediction approaches are as follows [36,56,62,69,131,139,153,179–181,185,189–191,195,196,202,203,205,214,222–227,322,335,336,354,355,357]. And a systematic review about PF-based prognostics is provided in [367].

6. Conclusions and future challenges

This paper has reviewed the state of the art of machinery prognostics following the four processes of the prognostic program, namely data acquisition, HI construction, HS division and RUL prediction. In the data acquisition section, four prognos-

tic datasets are described in detail, including the data sources, properties and applications. The HI construction section summarizes HI construction approaches in existing publications and gives a list of the metrics for evaluating prognostic HIs. The HS division section reviews the related publications by dividing them into two categories, i.e., two-stage division and multistage division. In the RUL prediction section, RUL prediction approaches are addressed following four categories, and common RUL prediction metrics are also explained. It should be mentioned that the literature on this subject is massive and diverse. A review on all the literature is impossible and omission of some papers would be inevitable. In addition, lots of non-English published papers related to machinery prognostics are not covered by this review due to the limitation of language proficiency.

It is also noticed that some commercial corporations take enormous efforts to advance the technology of machinery prognostics, and acquire dramatic achievement recently. For example, General Electric develops an industrial cloud-based platform called Predix. Pratt & Whitney constructs a service platform for commercial engines, which is named as EngineWise. Rolls Royce uses a system named as Engine Health Management to track the health state of thousands of engines operating worldwide. Alstom launches a new predictive maintenance tool, HealthHub, to examine the health state of trains. Several PHM systems for wind turbines have also been constructed, such as the WindCon system of SKF in Sweden and the VestasOnLine system of Vestas in Denmark. There are also some commercial platforms for PHM of rotating equipment, such as the Watchdog developed by the center of IMS, University of Cincinnati in the United States and the 3DSignals created by an Israeli company.

Although lots of advancements have been achieved in the field of machinery prognostics, there are still several aspects which need to be further investigated. The last but not the least task of this paper is to list the challenges and opportunities in this field, which is expected to point out the development directions and give some suggestions for researchers.

(1) *Challenges in data acquisition*

● *How to deal with the issue of RUL prediction where limited data are available?*

With the development of special requirement and flexible manufacturing, mechanical systems are becoming more complicated and distinctive. As a result, data acquisition of machinery prognostics evolves into an extreme situation, i.e., few or even no event data is available especially for newly commissioned equipment. Physics model-based approaches may perform their advantages in dealing with this issue, since all the other approaches need some available event data to train or estimate model parameters. However, it is generally hard to understand the first principle and establish a suitable physics model for a complex mechanical system. Under this circumstance, it is necessary to make full use of information from different sources, such as physics knowledge of similar equipment, subjective expertise from designers and manufacturers, and censored event data or censored condition monitoring data.

● *How to deal with the issues of RUL prediction under the big-data situation?*

With the increase of the population of monitored machines and installed sensors, data acquisition of machinery prognostics turns into another extreme situation, i.e., big-data situation [368]. The big-data situation provides more opportunities as well as great challenges for both diagnostics and prognostics, such as how to find out useful information from abundant data resources quickly. There have been some publications in the field of intelligent diagnostics of machinery with the application of big-data techniques [369]. Unfortunately, few papers have been published related to the machinery prognostics under big-data situation. More research work should be conducted in this field.

● *How to predict the RUL with the help of measured data from laboratory environment?*

From Section 2 it is seen that there are more run-to-failure datasets captured from laboratory environment than those acquired from industrial fields. So it may be reasonable to predict the RUL of industrial equipment with the help of measured data from laboratory environment. The transfer learning may be helpful for the RUL prediction in industrial fields. The key idea of transfer learning [370] is to transfer the knowledge gained from the original domain to the target domain. Therefore, with the help of transfer learning methods, the knowledge learned from the laboratory data may help to improve the accuracy of RUL prediction in industrial fields.

(2) *Challenges in HI construction*

● *How to reveal the relationship between the HIs and the damage levels?*

The relationship between HIs and damage levels, such as the crack length and wear area, is generally difficult to be revealed. One reason is that the damage levels of in-service equipment are difficult to be measured directly without special designed experiments and advanced observing techniques in micro perspective. Another reason is that, even though the damage levels are able to be measured, the relationship between the HIs and the damage levels is too complicated to be

explained using a simple expression. This problem may be handled with the help of dynamic model analysis [371] or phenomenological model analysis [372] of a mechanical system.

- *How to fuse multi-sensor data which contain heterogeneous information?*

Since the data from different sensors or locations may present various sensitivities for a large scale mechanical system, it is important to develop effective methodologies to fuse multi-stream sensor information for machinery prognostics. Initial research related to this topic has appeared in [29,373,374]. There are still various aspects needed to be further studied in this topic, such as selecting location for sensors, sparse representation of data and information fusion of multi-dimension data.

- *How to determine a FT for a newly constructed HI?*

Nowadays, the widely used methods for FT determination are based on some ISO standards, such as the ISO/7919 and ISO/10816 series, or some standards specially designed for certain industries, such as VDI/3834 for wind turbines. These standards only determine FTs for some original PHIs, such as RMS and peaks of vibrations signals. However, there is no standard to determine a FT for a newly constructed HI, especially those who have no definite physical meanings. Javed et al. [35] provided a novel idea of dynamic FT setting for VHIs, which may inspire further investigations on this topic.

(3) *Challenges in HS division*

- *How to define different HSs in industrial applications?*

The accuracy of HS division in literature is generally evaluated subjectively without verification by the actual degradation stages of machinery. It has the similar reason with the first challenge in HI construction, that the actual degradation stages of machinery are hard to be monitored without special designed experiments and advanced observing techniques in micro perspective.How to distinguish the changes caused by the fault development and other factors?
Through the analysis in Section 4 it is concluded that the HSs are generally segmented based on the changes of signal characteristics or the trends of HIs. In reality, the change caused by the fault development is always obscure especially during the interim period between two stages. It is possible to be submerged by other factors, such as the time-varying operational conditions and environment interferences. Therefore, new HS division approaches should be established to separate the changes caused by the fault development from other factors.

- *How to take advantage of the HS division results to guide the RUL prediction process?*

As discussed in Section 4, HS division actually plays a service role for RUL prediction. Two-stage division provides a FPT for RUL prediction. Multi-stage division may serve the RUL prediction process through the following three strategies. First, a RUL prediction issue is transformed into a multi-stage classification issue through the multi-stage division, therefore various multi-class classification techniques can be applied to the area of RUL prediction [164,171]. Second, different tasks are assigned into different stages after HS division, such as condition monitoring in the normal stage, one-step prediction in the slight degradation stage, RUL prediction in the severe degradation stage and shutdown in the failure stage [50]. Third, due to the variation of the degradation trends in different stages, multi-model prediction is expected to perform better than a single prognostic model.

(4) *Challenges in RUL prediction*

- *How to predict the RUL of a single component involving multiple faults?*

It is a common phenomenon to discover multiple faults in a single component in industrial applications. However, this phenomenon is often ignored in academic research for simplifying predicting approaches.

- *How to predict the machinery RUL at a system level?*

A mechanical system must be composed of multiple components. The defect of one component may spread to others because of frequent connection between different components. Therefore, it is a significant task to analyze the fault interaction among different components for the RUL prediction of machinery at a system level.

- *How to manage the uncertainties in RUL prediction?*

The RUL of machinery is difficult to predict mainly due to uncertainties caused by the inherent degradation processes and time-varying operational conditions. Effective estimation and handling of the uncertainties is the basis of accurate RUL prediction [69,251]. There have been some publications in this aspect [114,210,271]. More research work is still needed.

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