A Review on Prognostics Methods for Engineering Systems

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***Abstract*—Due to the advancements in sensing technologies and computational capabilities, system health assessment and prognostics have been extensively investigated in the literature. Industry has adopted and implemented many advanced system prognostic applications. This article reviews recent research advances and applications in prognostics modeling methods for engineering systems. The reviewed papers are classified into three major areas based on whether the physics of failure knowledge is incorporated for prognostics, i.e., the data-driven, physics-based, and hybrid prognostic methods. The technical merits and limitations of each prognostic method are discussed. This review also summarizes research and technological challenges in engineering system prognostics, and points out future research directions.**

***Index Terms*—Data-driven, hybrid method, physics-based, prognostics, prognostics and health management (PHM).**

I. Introduction

THE term "prognostic" is defined in Merriam–Webster dictionary as something that foretells. In prognostics and health management (PHM), prognostics is a process to predict future degradation and the remaining useful life (RUL) of the system based on the available degradation data. There are several definitions of interest for prognostics.

1) Prognostics is the process of estimating the remaining life of the component [1].
2) In ISO 13381-1, Section 5.1, the goal of prognostics is to provide the user with the capability to predict RUL with a satisfactory level of confidence [2].
3) Prognostics is the process of predicting the future reliability of a product of assessing the extent of derivation or degradation of the product from its expected norm operating conditions. It is the prediction of the future state of health based on current and historical health conditions [3].
4) Prognostics is to predict future damage/degradation and the RUL of in-service systems based on the measurement damage data [4].
5) Prognostics addresses the use of automated methods to detect, diagnose, and analyze the degradation of physical system performance, calculating the remaining life in acceptable operating state before failure or unacceptable degradation of performance occurs [5].

A degradation process involves multiple sources of uncertainties, which can all cause the inaccurate prediction of RUL. From the perspective of statistical uncertainty, the degradation process varies from unit to unit that is manufactured under the same process and condition. In a certain operation environment, there would be different degradation trajectories among units. For example, lithium-ion batteries tested under the same profile and environment would form different capacity degradation processes. Meanwhile, uncertainties from the operating condition and environmental condition, such as future loads and circumstance changes, exist in the degradation modeling. Modern prognostics approaches aim to effectively quantify these aforementioned uncertainties so that an accurate RUL prediction can be obtained. Advanced techniques, such as machine learning (ML) algorithms and artificial network, are used to model the degradation process. However, model errors still exist, which might be caused by misspecified models, missing failure modes, and unmodeled phenomena. It can be considered as biased understandings of the degradation process of interest. Comprehensive prognostics models intend to incorporate physical understandings of the degradation process so that model errors can be reduced. For example, the incorporation of gas-path models in the degradation modeling of turbine engines has been investigated [6]. However, physical understandings of the degradation mechanism are usually limited and incomplete for complex systems. Therefore, uncertainties from model errors can be rarely eliminated even with improved methods and investigations. Recently, uncertainties caused by measurement devices has attracted attentions since the measurement data are taken as the input of prognostics algorithm. Biased measurement/performance data might lead to total invalid prognostics. As a result, sensor failure, noises, sensor architecture, and signal fusion have been frequently investigated.

These uncertainties can be classified into four categories [7]–[9].

1) Input uncertainties are related to inherent variability for any process, such as initial state estimation, material property, geometric characteristics, manufacturing variability,

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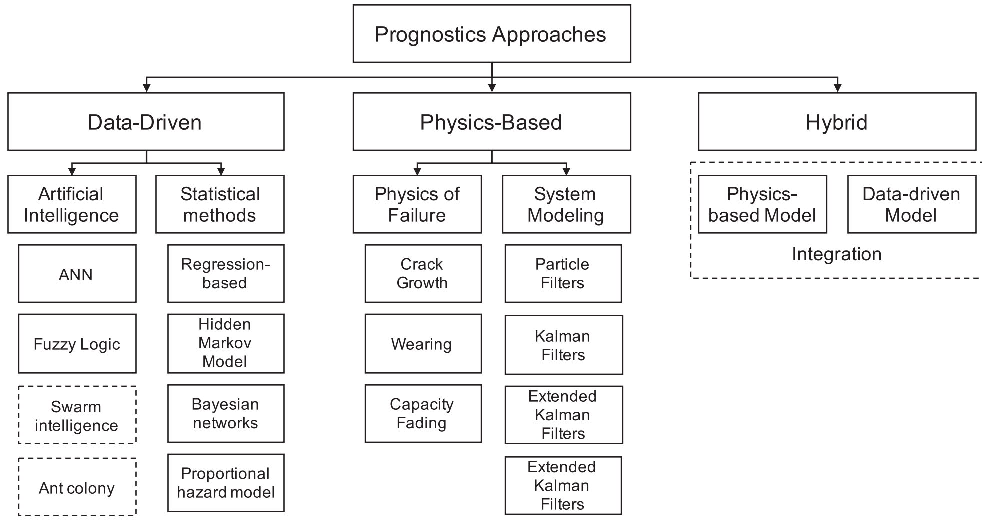

Fig. 1. Summary of prognostics approaches.

etc. These types of uncertainties are "born" with the degradation process and cannot be eliminated. Design of multiple experiment runs, such as repeated measurement design, is the most commonly used method to characterize these uncertainties.

2) Model uncertainties are related to model errors, such as misspecified methods, unexplained features, unmodeled phenomenon, etc. Advanced methods including data science and testing techniques have been developed to reduce these types of uncertainties. With the development of advanced sensing technique, large volumes of measurement data are available. The "big data" challenge might cause more model uncertainties.
3) Operational uncertainties are related to during operation and involving environmental conditions. These types of uncertainties are similar with model uncertainties, both of which stem from the limited understanding of the degradation process. Deeper investigation, especially under various operating and environmental conditions, can reduce these uncertainties.
4) Measurement uncertainties are related to uncertainties in measurement data, such as sensor noise, filter error, etc. Improved sensing techniques and advanced methods in data collection and processing can reduce these types of uncertainties.

Adequate prognostics methods are expected to handle all of these uncertainties. Specifically, prognostics models should have 1) capability in representing variety and veracity in data, 2) comprehensive physical basis so that model uncertainties can be highly reduced, and 3) robustness and computational effectiveness. This research is dedicated to providing comprehensive summary of prognostics modeling techniques so that readers can obtain a big picture of prognostics development and implementation.

Studies on prognostics have been growing fast due to its significance. Advanced modeling methods and experiments are implemented to reduce all aforementioned uncertainties. A vast amount of research has been done on the review of prognostics approaches and their applications, such as [5], [10]–[17]. Based on such review works, this article focuses on summarizing the most recent prognostics works considering data availability and the physical mechanisms of the applications. This article classifies these prognostics approaches into three categories, i.e., data-driven, physics-based, and hybrid approaches based on the usage of physical knowledge of the degradation mechanism (see Fig. 1). The rest of this article is organized as follows. Sections II, III, and IV review data-driven, physics-based, and hybrid methods. Section V concludes this article.

II. Data-Driven Prognostics

A. *Overview*

Data-driven approaches are used to identify the characteristics of the current damage state and to predict the future damage using available historical data (observed data) when rare physical understandings are available. The specialty of data-driven approaches is to derive models directly from the available data. Gouriveau *et al.* [18] took the analogy of a black box to describe data-driven prognostics. Data-driven approaches rely on the statistical characteristics [19]. Therefore, identification of these characteristics is the key process to develop data-driven methods. There are two major advantages of these methods with the first one being accessibility. These models can simply quantify the relationship between time and degradation states without or with rare domain knowledge of the system and so they come with a low implementation cost [20]. The development of computation techniques facilitates the implementation of data-driven methods. The other advantage is improved performance. With powerful ML techniques such as learning algorithms, data-driven approaches can largely reduce the model errors. Sometimes these models can broaden the cognition of a complex degradation process. However, it is clear that there are some disadvantages as well. These models are hard to interpret due to lacking system knowledge. Without physical understandings, it

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TABLE I
SUMMARY OF DATA-DRIVEN APPROACHES AND TYPICAL WORKS

Methods	Merits	Limitations
ANN [23]–[27]	Powerful approximation capability in modeling nonlinear and complex relationships	"Black-box" methods Require sufficient data
FL [28]–[32]	Good at condition classification	Lack of prediction capability
Statistical methods Regression-based [33]–[35]	Compelling capability in high dimension data	Instability and nontransparent
BN [36]–[40]	Advances in sparse data and causality relationship	Expensive computation
Gaussian process [41]–[43]	Advantages in modeling uncertainties Allow us to add prior knowledge to hyperparameters and covariances	Strong assumption
Random coefficients regression [44]–[46]	An appropriate covariance pattern model Flexible specification of the covariance structure in hierarchical data	Computation intensity
Proportional hazard rate [47], [48]	High model interpretation	Failure data/censored data are needed

is difficult to illustrate model parameters. Sufficient data are the most important ingredient and lack of training data holds back the implementation of these methods. Along with large amount of data, computational issues also exist. At last, data-driven methods are data-specific and so the obtained model may not be reused with another data application, especially in the case of heterogeneous and insufficient datasets with multiple sources of variations.

There are two major types of data-driven approaches, i.e., ML/artificial intelligence (AI) and statistical modeling. AI is the general field of intelligent-seeming algorithms. Machine learning, as a subfield of computer science and AI, aims to building systems that can learn from data, instead of explicitly programmed instructions while statistical modeling is a subfield of mathematics, which deals with finding relationship between variables to predict an outcome. In prognostics modeling, Pecht *et al.* [3], [16], [21] divided data-driven approaches into ML and statistical models. Peng *et al.* [5], [10], [11], [15], [22] classified data-driven prognostics into AI and statistical techniques. ML is considered as a subfield of AI, which uses historical data to automatically learn a model of the degradation process. Therefore, it is more reasonable to categorize data-driven approaches into AI and statistical techniques. Categories are not strictly separated and integrated methods are continuously emerging. As listed in Table I, AI prognostic approaches include ML regression, artificial neural network (ANN), fuzzy logic (FL), swarm intelligence (SI), ant colony, etc. Statistical approaches include Gaussian process regression, Bayesian network (BN), hidden Markov model (HMM), and proportional hazard rate. Merits and limitations of these data-driven approaches are also summarized (see Table I).

B. AI-Based Approaches

In the following sections, widely used or newly introduced AI-based approaches, such as ML-based regression, ANN, FL, and other AI methods (SI and ant colony), are reviewed, and issues and future research directions are discussed.

1) ML-Based Regression: ML is closely related to (and often overlaps with) computational statistics, which also focuses on prediction-making through the use of computers. Classification and regression based on ML are widely used in prognostics and diagnostics [3], [49]–[51]. Support vector machine (SVM) based models are the most popular ML methods [33], [52]–[55]. To our best knowledge, Corets and Vapnik [56] first proposed SVMs for data analysis and pattern recognition. SVM, which considers a structured risk, has a better generalization ability compared with conventional ML methods, such as ANNs. Better generalization ability means that this method can be applied to other fields more easily. Huang *et al.* [35] summarized SVMs-based estimation of RUL. Pham *et al.* [57] combined SVM and proportional hazard model. Relevance vector machine (RVM) is a Bayesian sparse kernel model that introduces a prior distribution over the model weights that are governed by a set of superparameters. Tipping is the one who denoted the RVM [58]. Compared with SVM, the most compelling feature of RVM is its computational efficiency for prediction because relative few vectors are used in the computation. RVM is widely used to learn the degradation model in the case of few data available. For example, Widodo and Yang [59] used RVM to predict survival probability of individual unit of machine component based on censored data. Liu *et al.* [60] investigated the degradation modeling with small size and low precision of prediction using RVM. RVM can be easily extended to multivariate RVM (MRVM) by introducing a one-to-many mapping function [61]. Lei [62] presented an MRVM-based algorithm to predict uncertainty and multiple fault features in a fault prognostic application. Considering that parsimonious solutions for regression and probabilistic classification can be obtained through RVM, Caesarendra *et al.* [52], Zio *et al.* [63], [64] combined RVM with other regression methods such as exponential and logistic regression. Wang *et al.* [65] proposed a conditional-three-parameter capacity degradation model where relevance vectors are used to find the representative training vectors containing the cycles of the relevance vectors. Considering the advantages of learning vector machines in dimension deduction, SVM/RVM-based regression models are still attracting interest. Many prognostics applications involving sensor data provide complex data structures such as imbalance classes. ML techniques such as support vector data description (SVDD) and isolation forests are useful in dealing with imbalance classes.

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Benkedjouh *et al.* [66] presented the application of SVDD, also called the one-class classification in detecting the outlier in the bearings based on vibration signals. Isolation forest, proposed by Liu *et al.* [67], is popular in abnormal detection and abnormal cycle estimation because of its advantages in outlier isolation and robustness with a small amount of abnormal data. Zhong *et al.* [68] used the isolation forest-based anomaly detection method to investigate gas path anomaly detection, where abnormal data are sparse or even unavailable compared with normal data. The monitoring data grouped by time series is taken as inputs as the isolation forest for anomaly detection and abnormal cycle estimation. Tree-based methods are also popular in ML application areas of PHM because of its outstanding capability in handling missing data, facilitating model interpretation, and lower data prepossessing requirement. However, such methods typically are not competitive with the best-supervised learning methods in terms of prediction accuracy. Ensemble regression methods like gradient boosting and random forest, which grow multiple trees, can dramatically improve the prediction accuracy. Patil *et al.* [69] employed ensemble regression techniques of gradient boosting and random forest in rolling element bearings RUL prediction with the vibration signals. Demonstrating the reliability of the proposed model is implemented through FEMTO bearing dataset provided by IEEE PHM 2012 Data Challenge. To improve the model accuracy and applicability, feature extraction prior to the ensemble tree methods is proposed in many works. For example, Xie *et al.* [70] used features extracted from the PHM15 Data Challenge of industrial plants fault detection, as the inputs ensemble decision tree classifier in failure detection.

2) ANN and Deep Learning: Considering the ability to incorporate numeric information from multiple and possibly disparate channels instantaneously, appropriately synthesized ANN can be used to implement accurate and fast online pattern recognition.

An ANN consists of an input layer of neurons or nodes, units, one or two hidden layers of neurons, and a final layer of output neurons. The ANN learns the unknown function by adjusting its weights with observations of input and output. With limited number of layers and neurons, ANNs have difficulties to represent complex features. Deep learning is featured by multiple hierarchical layers, which can transform the input data into more abstract representations. This article summarizes the evolution of neural networks in prognostics.

There are various neural network models, such as the feedforward neural network (FFNN), polynomial neural network, cascade correlation [71], Dynamic wavelet neural networks (DWNNs) [72], [73], and self-organizing map (SOM) [74]. In prognostics and RUL assessment, FFNN is the most widely used neural network. In FFNN, neurons are linked in an acyclic network. Weights of each neuron are obtained through supervised or unsupervised training. FFNNs have advantages in shortening computation time but have difficulties to represent complex hierarchical information. Wu *et al.* [75] employed a one-hidden-layer FFNN in bearing life percentage prediction using real-time conditional monitoring information and use Levenberg–Marquardt to train the neural network, where two moving average degradation signals and operation time in defective/worn condition are contained in the input layer and the predicted life percentage is the output of the neural network. Mahamad *et al.* [76] applied one-hidden-layer FFNN in bearing RUL prediction. They used the real-time and fitted measurements as inputs to predict the normalized life percentage and added the ANN model validation to avoid the overfitting issue. Assigning real-time/fitted measurements as inputs of ANN is used to handle unequally spaced inspections. Based on Wu and Mahamad's work, Tian [23] proposed a four-layer network where two hidden layers of neurons were used to improve the reliability of prediction results. With the same architecture, Wang *et al.* [77] investigated the application of FFNN with sigmoid hidden neurons in multidimensional mapping problems used in bearing PHM. In addition, ANN methods are often used with denoising techniques in signal processing for prognostics modeling. Qiu *et al.* [78]–[80] used the wavelet filter to denoise the vibration signals and the SOM techniques to detect errors and predict the RUL.

As a rapidly growing branch of ML, deep learning can well connect the "big" sensing data and system health monitoring. Deep learning allows computational models that are composed of multiple processing layers to learn representations of data with multiple levels of abstraction [81]. Autoencoder, recurrent neural networks (RNN), convolutional neural networks (CNNs), restricted Boltzmann machines (RBM), and deep believe network (DBN) are commonly used deep learning architecture. Zhao *et al.* [82] provided a survey of deep learning and its application to machine health monitoring. Four deep learning architectures, autoencodes, RBM, CNN, and RNN, and their applications are investigated. Focusing on autonomous ship PHM, Ellefsen *et al.* [83] also provided a review of deep learning in prognostics where deep learning architectures, autoencoder, CNN, DBN, and long short-term memory (LSTM). This article provides further investigation on RNNs and CNNs given their successes in prediction and classifications with large-scale and complex data structure.

RNNs, capable of illustrating temporal behavior compared with FFNNs, are also widely used in prognostics modeling with sequential data [84]–[88]. RNNs are also powerful in computation and storage of large-scale PHM data because of two properties: 1) distributed hidden state allowing RNNs to store past information efficiently and 2) nonlinear dynamics allowing RNNs to update hidden states in complicated mapping ways. PHM Society Data Challenge attracts many applications of the RNN modeling. Two out three winners in PHM08 Data Challenge employed the RNN [89]. These data have three operating parameters (depth of cut, feed, and material type), and six sensors (acoustic emission sensor at two different positions, table and spindle; vibration sensor at two different positions, table and spindle; ac spindle motor current sensor and dc spindle motor current sensor). There are 16 cases with varying number of runs for each case. The number of runs was dependent on the degree of flank wear that was measured between runs. RNNs-based prognostics are also applied for the C-MAPSS datasets five datasets of which comes from a turbofan engine simulation model Commercial Modular Aero-Propulsion System Simulation (C-MAPSS) [90]. For example, Wang *et al.* [91] employed deep bidirectional LSTM neural network in C-MAPSS dataset. The BLSTM model leverages the same advantages discussed in the bidirectional RNN section, while also overcoming the vanishing gradient problem.

As an extension for RNNs, the LSTM technique captures long-term dependencies and provide more accurate predictions via a set of gates controlling when information enters the memory, when it is output, and when it is forgotten. Zheng *et al.* [93] applied a deep LSTM in PHM08 Challenge dataset. With N components in a system and p sensors of the same type of on each equipment, LSTM model takes each sequence of sensor measurements and learns how to model the whole sequence with respect to target RUL. Inputs are normalized signals. The output is calculated RUL. In PHM18 Data Challenge where the data challenge focuses on predicting time-to-failure (for each of three types of fault) at specific times of an ion mill etching tool, two of top three winners are LSTM modeling. TV *et al.* [94] chose the number of units h from the set $\{50, 100, 150, 200\}$ and used $L =$ two layers. Huang *et al.* [95] doubled the node number in two LSTM layers in the architecture proposed by Zheng *et al.* [93]. For other applications, such as lithium-ion battery data, LSTM method has great performance [92]. However, LSTM suffers from high complexity in hidden layer. For identical size of hidden layers, a typical LSTM has about four times more parameters than a simple RNN [96]. Various approaches are introduced to enhance the performance LSTM, such as S-LSTM, stacked LSTM, bidirectional LSTM, multidimensional LSTM, grid LSTM, etc.

Along with the LSTM, different architectures of RNNs are investigated for gradient vanishing and explosion, which include deep RNNs with multilayer perceptron, bidirectional RNN, recurrent CNNs, multidimensional RNNs, gated recurrent unit, peehole connections, memory networks, structurally constrained RNN, unitary RNNs, gated orthogonal recurrent unit, and hierarchical subsampling RNNs. Salehinejad *et al.* [97] provided a comprehensive summary of recent RNN methods, where readers can find more details of each aforementioned models.

Another deep learning architecture investigated in this article is CNNs in prognostics modeling, which are very popular models for machine vision applications. CNNs may consist of multiple convolutional layers, optionally with pooling layers in between, followed by fully connected perceptron layers. Typical CNNs learn through the use of convolutional layers to extract features using shared weights in each layer. The feature pooling layer (i.e., subsampling) generalizes the network by reducing the resolution of the dimensionality of intermediate representations (i.e., feature maps) as well as the sensitivity of the output to shifts and distortions. The extracted features, at the very last convolutional layer, are fed to fully connected perceptron model for dimensionality reduction of features and classification. Considering its strength in classification, CNN is a powerful tool in diagnosis [98]. Sateesh Babu *et al.* [99] are the first to use CNN to estimate RUL using multiple signals and a CNN is proposed. In the proposed neural networks, there are two convolutional layers and one full connection layer. Performance of the proposed neural network is compared with ML methods such as support vector regression and relevance vector regression. To take the temporal variation into accounts, Li *et al.* [100] used a four-convolutional-layer neural networks with a time window in C-MAPSS dataset for RUL prediction. Such time window is employed for sample preparation in order for better feature extraction. It is found that the four-layer CNN performs out neural network with one hidden layer, deep neural network with four hidden layers, RNN with five recurrent layers, and LSTM with five layers, and one fully connected layer. However, determining the time window length brings new issues and unbalanced signals might cause issue with the moving window scheme.

CNNs can also train large-scale neural networks, which provides paths to "big" data in prognostics modeling. There are several well-known multilayers of CNNs that could be employed in prognostics, such as AlexNet [101], VGG [102], GoogleNet [103], residual neural network [104], etc. However, training these multilayer neural networks is time consuming even on high computational performance devices like GPUs. Transfer learning using pretrained networks has been explored by several researchers. For example, Wang *et al.* [105] presented a universal bearing fault diagnosis model based on the pretrained AlexNet model, and only the last fully connected layer needs to be replaced, which could reduce extra time in establishing a new model. Zhang *et al.* [106] used transfer learning from LSTM in prediction RUL based on C-MAPSS datasets. Transfer learning methods can take advantages of existing data with which it can well deal with scenario with only small amount of data available. However, parameters in transfer learning models are learned from related domain, which might limit the applicability of such learning methods.

Other two deep learning models such as RBM and deep belief network (DBN) are also used to deal with PHM big data in RUL estimation. RBM is a two-layer neural network forming a bipartite graph consisting of visible and hidden units. Liao *et al.* [107], [108] discussed the application of Boltzmann machines in prognostics. Deutsch and He [109] used the restricted Boltzman machine for bearing RUL prediction and emphasize the computation efficiency with large scale PHM data. Ren *et al.* [110] discussed the application of deep learning-based prognostics in multivariate degradation processes and the challenges in big industrial data. Krishnan *et al.* [111] investigated the dimension reduction and proposed a two-step method of hierarchical dimension reduction and deep neural network. A DBN can be defined as a stack of RBMs in order to increase the representation of data. DBNs have been widely applied in prognostics modeling and RUL estimation [112]–[114]. Zhang *et al.* [115] used multiobjective DBNs to establish an ensemble model used for RUL estimation, where a multiobjective evolutionary algorithm is used to evolve multiple DBNs simultaneously. Integration of deep learning and other methods becomes a trend. Deutsch *et al.* [116] combined DBN with sliding windows and particle filter-based approaches to take advantage of both methods. To integrate advantages of various types of neural networks, stacked neural networks are proposed. To investigate the long-term dependencies, Zhao *et al.* [117] integrated CNN with the bidirectional LSTM mechanism where CNN is first used to process sensory data and the outputs of CNN are fed into the following bidirectional LSTMs.

ANN-based prognostics is a powerful tool that can determine the nonlinear function of the system but it is usually a tough problem for system designers to fit domain knowledge to ANN in practical applications. However, the prognostic process itself is a "black box" for developers, which means it is very difficult or even impossible to have physical explanations of the networks outputs, and as ANN grows in size, training can become a complicated issue. For example, the number of layers of hidden layers that should be included as well as the number of processing nodes that should be used for each layer, and how to quantify weights, biases, and uncertainties in inputs and optimization process are confusing questions for model developers [5], [22], [43]. More importantly, the computation issue is raised when the size of the network grows. The development of GPU and cloud computing can accelerate the training of deep neural networks. Lym *et al.* from NVIDIA developed an analytical DeLTA, which can accurately model both arithmetic performance and traffic in the memory hierarchy [118]. It is clear that understanding uncertainty is important in prognostics modeling. The variation between units is one of the most topics in prognostics. To investigate such variation in deep learning a field known as Bayesian deep learning (BDL) with capability of modeling uncertainty was introduced. Kendall *et al.* [119], [120] discussed uncertainties in deep learning and a unified Bayesian deep learning framework, which allows us to learn mappings from input data to aleatoric uncertainty and compose these together with epistemic uncertainty approximations. In the age of Internet of Things (IoT) and Industry 4.0, massive real-time data are collected from complex systems. For example, imaging is one of the fastest growing technologies for condition monitoring and industrial asset management. Relative to most sensing techniques, industrial imaging devices are easier to use because they are noncontact and do not require permanent installation or fixturing. Image data also contain rich information about the object being monitored [121]. Moreover, challenges caused by high dimension and correlated sensing data in large-scale IoT-based systems will require more advanced modeling methods that can handle large-scale computation and complex causality analysis. Therefore, another important issue is to determine inputs in consideration of the increasing dimension of sensory data. The choice of inputs could significantly improve the computation efficiency. Edge computing, a distributed architecture, allows large-scale data processing and mobile computing near the source through decentralized processing power. Han *et al.* [122] did a survey on the convergence of edge computing and deep learning, where applications of edge computing and deep learning, edge computing frame enabling deep learning training and inference, and future trends are investigated. Park *et al.* [123] introduced a light-weight real-time fault detection system for edge computing used in industrial robots, which consists of two main parts front end monitors and back end fault detection trainer. The back end takes real-time data collected by the front end monitors and train fault detection model for anomaly detection. Advance devices with compatibility with various sensors and effective computing can facilitate the prognostics modeling for complex engineering prognostics. Prognostics algorithms in edge computing need to process online sensing data. Online prognostics algorithms are investigated in the field of PHM. For example, Mao *et al.* [124] proposed an online sequential prediction method for bearings fault diagnosis, where the principal curve and granulation division are introduced to simulate the flow distribution and overall distribution characteristics of fault data, respectively. The simulation methods are updated using bearing vibration signals arriving in sequence through the over sampling and under sampling process. Data processing is a critical step to build a robust model. Under IoT, digital signal process become more important given the scale and heterogeneity of sensing data. Rai and Upadhyay [125] reviewed the techniques of signal processing techniques, which can be categorized into three types, i.e., time domain analysis, frequency domain analysis, and time frequency domain analysis. The last one is the most popular method for the analysis of transient signals. Boufenar *et al.* [126] investigated the performance of time-frequency analysis methods in roller bearings condition monitoring, which include short time Fourier transform, Wigner–Ville distribution, and wavelets. In the wind turbine prognostics applications, Lin *et al.* [127] analyzed the acoustic emission signals using Hilbert–Huang transform (HHT) to detect elastic stress waves within structure failures. HHT can effectively separate the characteristic frequency components, which indicate the wind turbine bearing condition. Feature engineering is another practice in signal processing, where raw data are transformed into the features or attributes based on domain knowledge. Leather *et al.* [128] proposed an automatic feature generation method to find features that most improve the quality of the machine learned heuristic. The proposed method effectively improve the feature search process. In the proposed system includes data generation, feature search, and ML. In predictive analytics modeling, Bosch [129] investigated the automatic feature generation and selection techniques, where feature generation operators such as unary, binary, and group-by-then operators, and feature selection approaches such as Meta-features and background learner, are studied. The long run time is one of the challenges in feature generation algorithms.

3) Fuzzy Logic: FL is a method of reasoning that resembles human reasoning. The approach of FL imitates the way of decision making in humans that involves all intermediate possibilities between binary values. FL provides a formal mathematical framework for dealing with the vagueness of everyday reasoning. FL is a special set of Boolean logic with extensions to deal with imprecise information. As opposed to binary reasoning based on ordinary set theory, within the FL framework, measurement uncertainty and estimation imprecision can be properly accommodated. The main advantage of using FL is that it can account for uncertainty and nonlinear behavior in systems.

Byington *et al.* [130] used the FL to classify the system health states. The authors investigated F/A-18 stabilator electro-hydraulic servo valves provided by Boeing Phantom Works. Raw inputs include servo current, and ram position. Considering its strength in dealing with uncertainties, FL is usually used in combination with other methods such as ANN. For example, Ali *et al.* [131] explored the application of the simplified fuzzy

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adaptive resonance theory map (SFAM) neural network in rolling bearing with vibration signals. SFAM performs well in learning nonlinear time series during the fuzzy learning process in which a simple learning equation with a single user selectable parameter is used to construct layers in neural networks. Zio and Maio [132] predicted the RUL by implementing the fuzzy similarity analysis to the evolution data and reference data, which presents an effective prediction but also a concern on the choice of reference trajectories. Ramasso and Gouriveau [133] introduced an evolving real-time neuro-fuzzy system that forecasts observations in time and apply such system in C-MAPSS datasets. The concept of fuzzy neural networks (FNN) is proposed [134], which is a learning machine that finds the parameters of a fuzzy system (i.e., fuzzy sets and fuzzy rules) by exploiting approximation techniques from neural networks. There are several applications of FNN in prognostics and RUL prediction. For example, Li *et al.* [135] applied FNN in tool wear in drilling operation. Liang *et al.* [136] applied FNN method of condition maintenance (CM) in marine electric propulsion system. Fuzzy ML becomes an important subfield of ML. However, the recognition of FL inside of ML is still rather moderate in application [137]. FL is primarily used for the purpose of knowledge representation, and inference is mostly of a deductive nature. ML, on the other hand, is mainly concerned with inductive inference, namely, the induction of general, idealized models from specific, empirical data. In prognostics modeling, FL-based regression, classification, and clustering methods are used to deal with condition-based maintenance (CBM) data. Javed *et al.* [138] integrated the summation wavelet-extreme learning and substractive-maximum entropy fuzzy clustering to demonstrate the machine deterioration process. FL is also used in data fusion where heterogeneous data are classified through fuzzy methods [139]. The philosophy of FL can effectively explain the conditional uncertainties but the lack of prediction of time to failure limits its application [17], [140].

4) Other Intelligence Methods: SI is a field where algorithms are designed to solve problems in a way that is inspired by the behavior of real swarms or insect colonies [141]. Particle swarm optimization (PSO) is the most commonly used SI-based method. The concept of PSO is to use a number of particles that constitute a swarm moving around in the search space looking for the best solution and each particle "moves" according to the experience of other particles. PSO is often used to estimate parameters in prognostics modeling because of its strength of fast convergence compared with other optimization algorithms, such as genetic algorithm [142]–[145]. In [142], a modeling algorithm, namely, the PSO-based fuzzy regression approach, is proposed to generate fuzzy nonlinear regression models, which aim to address challenges in modeling including nonlinearity, small amount of experimental data in design phase, outliers, etc. Long *et al.* [144] used PSO algorithm to search the optimal autoregression model order in prognostics modeling for lithium-ion battery capacity. The authors do not discuss the model applicability in high-dimension sensing data. Similarly, using the lithium-ion battery capacity datasets, Hu *et al.* [145] used PSO to find the optimal combination of feature weights in k-nearest neural networks. Ant colony optimization (ACO) is another SI-based optimization method, which is inspired by the foraging behavior of real ants and used to solve discrete optimization problems. Similar with PSO, ACO is used with other prognostics modeling methods and mainly provides the optimal solutions for parameter estimation [146].

C. Statistical Approaches

Researchers have extensively explored another branch of data-driven methods, statistical data-driven methods. Comprehensive reviews on statistical methods have been provided. Kim *et al.* [4], Si *et al.* [13], and Si *et al.* [147] provided a comprehensive review of statistical approaches for prognostics and RUL prediction. There are four statistical models that are mainly used in prognostics modeling: regression-based model, Markov chain-based models, stochastic filter-based models, and HMMs. This article classifies stochastic filter-based methods as physics-based approaches because the state transition matrix incorporates physics understanding of degradation processes.

1) Regression-Based Models: Regression-based models are the most commonly used in prognostics modeling where the trend of degradation processes. The fundamental principle of these methods is that the health of the systems under study can be mapped by some key CM variables and, then, the RUL can be estimated by monitoring, trending, and predicting these CM variables with a predefined threshold D_F. Gaussian regression is one of the major regression-based models. Random coefficients regression models, such as Gaussian regression and mixed-effect models, can capture variations within subjects. A typical random-coefficient regression model $Y(t) = D(t; \varphi, \theta) + \epsilon(t)$. Lu and Meeker [148] first presented a general random coefficient nonlinear regression model to characterize the degradation path of multiple units. Random coefficient regression models can provide a PDF of the RUL given the failure threshold, but density function of RUL might not be closed-form for some cases. In most cases, a stepwise approximation or simulation has to be used for finding an approximated RUL [13]. Wang [149] summarized assumptions of the random coefficients model. That is, 1) the degradation state at various operating conditions can be observed at any time; 2) the monitored subject comes from a population, and each of which exhibits the same degradation process; and 3) the random effects across the population are known. Based on the assumption of these two papers, various works have been done, such as [25], [150]–[153]. Recently, multiple sources of variations are discussed. Si *et al.* [154] and Zhang *et al.* [155] investigated three resources of variability in degradation modeling. Guo *et al.* [45] proposed a mixed-effect model with moving windows to model the degradation of lithium-ion battery capacity based on the battery test data from CALCE [261]. Kontor *et al.* [46] introduced a mixed prior distribution in parameter estimation of mixed effects models. To deal with the complex data structure, Bayesian inference is discussed in Gaussian regression models. Rasmussen and Williams [156] investigated the Bayesian inference in ML and Gaussian regression. Based on Rasmussen's work, Baraldi *et al.* [157], Liu *et al.* [158], Richardson *et al.* [159] and Aye and Heyns [160] discussed advantages of Bayesian Gaussian process regression and its application in lithium-ion battery capacity prognostics. Zhang *et al.* [161] combined the stochastic degradation process modeling and remaining useful estimation with random effect modeling. As an extension of the Gaussian process modeling, functional data analysis (FDA) becomes a powerful tool in repeated measurement due to its strength in dimension reduction and modeling with sparse and missing data [162]–[166]. In FDA, a Gaussian process can be written as $Y_i(t) = \mu(t) + \sum_{k=1}^{\infty} \eta_{ik}\phi_k(t) + \epsilon(t)$ where $\phi_k(t)$ are basis functions and $\epsilon(t)$ is the error function. The choice of basis functions depends on characteristics of data. B-splines and Fourier basis functions are the two most commonly used. In the future, wavelet basis function will become a powerful tool to deal with large-scale degradation data due to its high transforming efficiency [73]. The concept of functional component principle (FPCA) applied in this research combine the functional data and principal component analysis (PCA) and can significantly reduce the dimension of the covariance matrix, and the covariances between measurements can be modeled through kernel functions. Cheng *et al.* [167] applied FPCA in lithium-ion battery capacity degradation, and Bayesian inference is used to determine scores. Guo *et al.* [168] focused on multiple sources of variations analysis in the degradation process using FPCA. The field of prognostics has overlaps with chemometrics modeling. Guo *et al.* developed a series of analytic approaches in the field of canonical variate analysis (CVA) that aims to maximize the correlation between two sets of variables [169]–[172]. CVA is more adapted than PCA to represent sensory data but CVA has the risk of computing instability with highly correlated sensory attributes [173]. Li *et al.* [174] use CVA-based state space model for performance estimation, where the new observation and residual are computed through matrix transfer operations in CVA. The PCA inspires a joint modeling-based prognostics where the mean function and covariance matrix are modeled separately. To our best knowledge, Sklar [175] is the first who proposed the idea of joint modeling. Pinheiro *et al.* [176], [177] provided a basic structure of joint modeling method. Smith and Kohn, Chen and Dunsonm and Zhang and Lang [178]–[180] extended the joint modeling method through reparameterizing covariance in various methods. Covariance matrix reparameterization becomes the core issue of the joint modeling methods. Guo and Li [181] proposed a mean-covariance decomposition-based prognostics to model the lithium-ion battery, and a set of trigonometric function was used to parameterize the covariance matrix. Another increasing area in prognostics is the spatio-temporal modeling in surface quality and crack analysis where data are collected across time as well as space. With more sensing signals are available thanks to advanced sensing techniques, spatial and temporal data can provide a comprehensive assessment of the system/component degradation state over spatial and temporal factors. Generalized spatio-temporal models can be used to model the degradation process of complex systems and correlations of subsystems/components can be modeled through the generalized spatial process model. Liu *et al.* [182] provided a comprehensive discussion of statistical modeling for spatio-temporal degradation data. They characterized the degradation at any spatial location and time as an additive superposition of two stochastic components: a dynamic spatial degradation generation process, and a spatio-temporal degradation propagation process. That is, the basic model of the spatio-temporal is $Y(s,t) = G_\Delta(s,t) + Z(s,t)$ where G_Δ is a spatial process that represents the amount of degradation occurring at location s over the time interval $(t-\Delta, t]$ and $Z(s,t)$ is the propagation of degradation over space and time. Oumouni and Schoefs [183] modeled the spatial variability and heterogeneity of the degradation process through Gamma and Gaussian processes. Density-based methods, which models the data according to the density of the objects, such as local outlier factor (LOF), can detect many degradation patterns, especially when data are multivariate and degradation is characterized by shift to a lower density space. Diez-Olivan *et al.* [184] employ the LOF method to quantify the distance from the event to the centroid through measuring the degree of isolation of a point with respect to its neighbors. The local density is also considered when detecting anomalies in monitoring diesel engine parameters including environmental pressure, temperature, fuel flow, exhaust temperature, cooling air temperature, alternator frequency, etc. Zhao *et al.* [185] used LOF to avoid the false alarm in the condition detection of solar photovoltaic arrays and simplify the data processing and modeling training. In wind turbine applications, many regression-based modeling techniques have been investigated. It generally involves a process of building a model of key output parameter based on sensor data measurements collected during stable/normal operations. Hameed *et al.* [186] did a comprehensive review on algorithms of condition monitoring and fault detection of wind turbines, which investigates algorithms in vibration, oil, thermography, physical conditions, strain measurement, acoustic signal, electrical effects, and rotor performance. Schlechtingen *et al.* [187], [188] proposed a system for wind turbine condition monitoring using adaptive neuro-fuzzy interference system, where data from the standard supervisory control and data acquisition (SCADA) system is used to monitor the wind turbine condition through the normal behavior models. The work is based on continuously measured wind turbine SCADA data from 18 turbines of the 2 MW class covering a period of 30 months. The fuzzy theory is used to analyze the prediction errors in pattern modeling.

In the future, regression-based models are still major fields of prognostics modeling especially when large scales of degradation data are available. Robust algorithms that can handle the complex data structure and high-dimension data are appropriate.

2) HMM and Hidden Semi-Markov Model (HSMM): HMM is a powerful tool for prognostics, especially when direct observed degradation state data are not available. HMM is composed of two stochastic processes: a hidden Markov chain and an observable process. The hidden Markov chain $(Z_n, n \geq 0)$ is considered as the real degradation state, while the observable process $(Y_n, n \geq 0)$ is the observed signals. The relationship between Z_n and Y_n is described through a conditional probability $P(Y_n|Z_n = i)$. HMMs can characterize the variations in degradation processes and reveal the hidden state and changing process well. In prognostics modeling, Bunks and McCarthy [189] applied an HMM-based method in the CBM of Westland helicopter state-dependent. Various works have been done based on Bunks's work, such as [190]–[192]. Baruah and Chinnam

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

[193] presented a novel method for employing an HMM model to carry out both diagnostic as well as prognostic activities for metal cutting tools. Medjaher *et al.* [194] proposed a mixture Gaussian HMM to represent the evolution of the component's health condition by hidden states by using temporal or frequency features extracted from the raw signals provided by the sensors. Le *et al.* [195] discussed the application of the HMM in multiple failure modes. However, the parameter estimation and transition matrix specification in HMM are still challenging. The limits of HMM are mainly lying in three aspects: 1) the core assumption of HMMs, a given state at time t only depending on the state at time $t-1$, is unsound in practical engineering applications; 2) the Markov property, i.e., independence of past history, limits its power in prediction; and 3) it is difficult to obtain the distribution of RUL in a closed-form due to the fact that only mean and variance of RUL can be estimated.

HSMMs as a generalization of HMMs are defined by allowing the unobserved state process to be a semi-Markov chain. In HSMMs, each state has variable duration and a number of observations being produced while in the state. This makes HSMM suitable for use in a wider range of applications [196]. Peng *et al.* proposed a series of HSMMs-based diagnosis and prognosis method [5], [197]–[200]. They presented an HSMM-based framework and methodology for both diagnostics and prognostics. Su and Shen [201] used the multihidden semi-Markov model to improve the effectiveness and accuracy. Similar with HMMs, the parameter estimation is one of the challenges in HSMM. Cartella *et al.* [202] selected the optimal model (number of hidden states, during distribution family, and the number of Gaussian mixture) through AIC. Zhu and Liu [203] applied the HSMM in online tool wear monitoring and determined duration parameter and RUL basis ML methods. Xiao *et al.* [204] proposed a modified duration-dependent HSMM where the health state transition probabilities and the observation probabilities are both defined not only as state-dependent like traditional HSMM does, but also as duration dependent. One of the main drawbacks is, as with HMMs, HSMM-based methods can only conduct mean and its variance of RUL estimation, and cannot provide a PDF of the RUL. Moreover, concerns on being limited to discrete state space and lack of capability to model long temporal dependence, RNNs are considered as substitute of HMMs, which can capture long-range time dependencies, overcoming the major limitation of Markov models [205]. As in Markov models, any state in a traditional RNN depends only on the current input as well as on the state of the network at the previous time step.

3) BN-Based Models: BNs are the combination of probability and graph theory. A BN is a directed acyclic probabilistic graph that is constituted with nodes and directed lines. There are four key components in BN: 1) a set of nodes $\{X_1, X_2, \ldots, X_n\}$ representing variables/components; 2) directed arcs between X_i and X_j $(i, j \in \{1, \ldots, n\})$ and $i \neq j$, with $X_i \rightarrow X_j$ indicating that X_i is a direct cause of X_j; 3) directed acyclic graph; and 4) conditional probability distribution of X_i given all its direct causes. As an extension of BN, dynamic BNs (DBN), a tool generalizing the HMMs and stochastic filters, represents the state of the world as a set of variables, and model the probabilistic dependencies of the variables within and between time steps. Dong and Yang [206] investigated a DBN-based prognosis method to predict RUL for drill-bits. Based on Dong's work, Hu *et al.* [207] applied DBN to illustrate the potential multiple failure modes and corresponding root causes. Tobon-Mejia *et al.* [208] employed DBN to estimate the conditional probability distribution in HMMs. Bartram *et al.* [209], [210] summarized a generic procedure of DBN application in prognostics. HMMs could be considered as a subclass of DBN [211]. In prognostics, HMMs and DBN are often used together where DBN is used to represent HMMs [194]. Codetta-Raiteri and Portinale [212] proposed a framework for the fault detection, identification, and recovery process based on DBN. Xiao *et al.* [213] discussed the multistep-ahead prediction using DBN-based methods. In consideration of emerging IoT-based complex systems, DBNs have potentials in areas of causal effects and system degradation state assessment. Based on the sensing data, correlations of subsystems/components can be found through DBN learning.

4) Hazard Rate and Proportional Hazard Rate-Based Models: Cox's proportional hazard model is the most reported method in survival analysis [214]. A proportional hazard rate model is composed of a baseline hazard function and a function covariate. That is, $h(t|z(t)) = h_0(t)c(\beta z(t))$ where β is a vector of regression coefficients, $z(t)$ is the covariate vector, and $h_0(t)$ is the baseline hazard rate. The number of application of the proportional hazard model is increasing recently. Si *et al.* [13] gave an excellent review of the proportional hazard model. The choice of the baseline hazard rate $h_0(t)$ can be nonparametric or parametric, and Weibull distribution is the most commonly used [47], [48]. Zhang *et al.* [215] used a mixture Weibull proportional hazard model to predict the failure of a mechanical system with multiple failure modes. The proportional hazard rate model is often used to estimate the survival function. Tran *et al.* [216] explored a three-stage prognostics model where dynamic system behaviors are identified first, then the proportional hazard model is generated to estimate the survival function, and finally SVM is used to forecast the RUL. Royston and Altman [217] investigated the external validation of the Cox's prognostics model. Wang *et al.* [77] proposed a three-phase prognostics algorithm where the time-series analysis of vibration signals is done through neural networks, and the RUL is assessed through the proportional hazard rate model. The most important advantage of the proportional hazard model is that covariate information can be easily integrated with a baseline hazard function. However, proportional hazard models need event data such as failures and censored data especially given that it is required that the baseline hazard failure function and regression coefficients are estimated at the same time, which is unfeasible in applications.

D. Summary of Data-Driven Prognostics

Data-driven prognostics, such as ANN and regression-based models, can be easily used to analyze degradation processes and RUL assessment. In practice, combinations of multiple data-driven techniques are used to deal with complex data structures

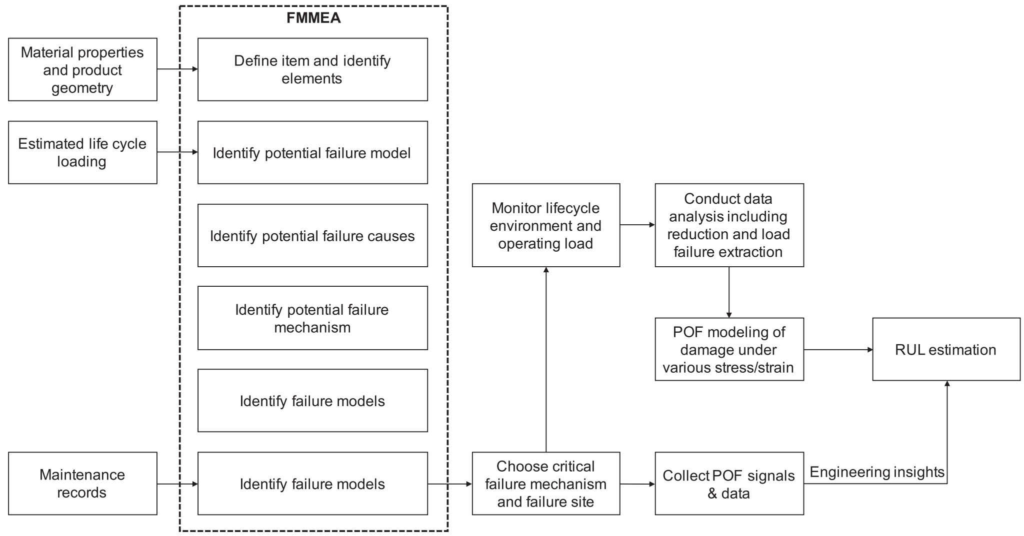

Fig. 2. Procedure of physics-based prognostics (adapted from [3]).

and to improve the predictive performance. However, there are a few areas of concern.

1) *Model uncertainties:* Due to lack of the physical understanding, the choice of data-driven models comes with the risk of undetected phenomenon. Take the regression-based models, for example, covariates of regression models might not carry physical meaning. Hybrid methods are used to reduce this type of issue.

2) *Input and measurement uncertainties:* Uncertainties resulting from bias and noise in data are important issues regardless of prognostics methods, which have a significant effect on the prediction performance. The bias cannot be handled with data-driven approaches because there are no parameters related to it, which is one of the drawbacks of data-driven approaches. Prior knowledge of the degradation process used in Bayesian inference can be used to "calibrate" the information in data.

III. Physics-Based Approaches

A. Overview

Physics-based or model-based prognostics is to incorporate the physics of failure (PoF) and to quantify characteristics of the degradation process under various loads and operation conditions [218]–[221]. These approaches can be categorized as PoF approach and system modeling approaches. Generally, PoF models are system/component specific, such as crack-growth model, rotation machine model, wearing model, gas-path model, and electrochemical model. To obtain these PoF models, several steps of failure modes, mechanisms, and effects analysis (FMMEA), feature extraction, and RUL estimation are needed (see Fig. 2). Matthew *et al.* [222] presented an FMMEA-centered prognostics framework. This approach is effective and descriptive due to incorporating the physical understanding of the degradation process. System modeling approaches assume the system can be described by a model that can illustrate the stochastic behavior of the system over degradation. For example, state-space model can transform a physical system into a set of input, output, and state variable related to the first-order differential equations. Fan *et al.* [223] applied FMMEA in high-power white light-emitting diode lights, where overstress (Catastrophic) failure or wear-out (gradual) failure mechanisms are considered as the major root causes of failures and, then, physics-based models of these two mechanisms are used for prediction. Oh *et al.* [224] provided a review of developments in condition monitoring and prognostics for insulated gate bipolar transistor modules and summarize failure mode, root causes, stresses, and mechanisms for PoF models, such as rainflow counting and Miner's rule. One of the advantages of the physics-based prognostics is to successfully incorporate the physical understanding of the system, which eases the interpretation of parameters in prognostics models. More importantly, with physical understanding, model errors caused by unexplained features or unmodeled phenomena can be significantly reduced. Therefore, PoF can be used to improve the accuracy of prognostics modeling. More importantly, PoF is a critical part of system level prognostics modeling for the dependencies between subsystems and components. Wang *et al.* [225] presented the PoF analysis of reliability-critical components to provide a basis for system level design. In their application of power electronics, PoF focuses on critical components under critical stress conditions. Among other components, switching devices and capacitors are two of the most vulnerable components in terms of failure level. Based on the physical understanding on the system and critical components, lifetime prediction based on thermal-voltage is discussed. They also proposed a generic frame of reliability prediction toolbox where prediction methods are based on [226]. Cookah *et al.* [227] proposed a combined probabilistic PoF-based model for pitting and corrosion-fatigue degradation mechanisms, where the degradation is modeled as a function of physical and critical environmental stress including amplitude and frequency loads, the concentration of corrosive chemical agents. The degradation rate is based on factors of

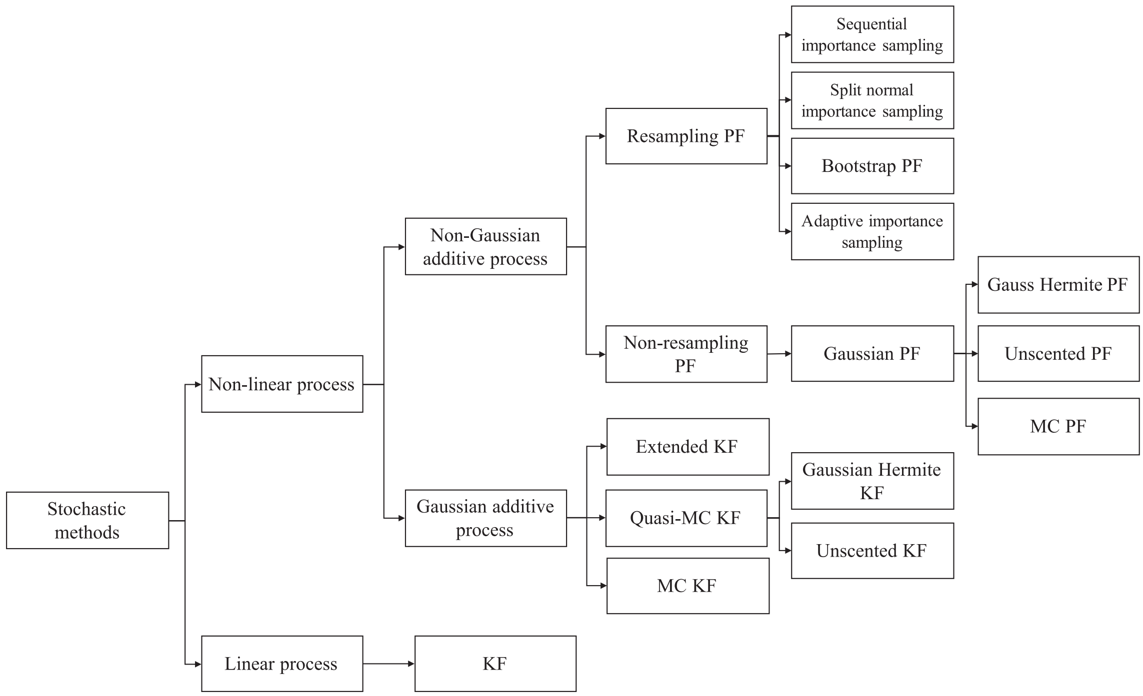

Fig. 3. Choices of stochastic filters.

TABLE II
SUMMARY OF PHYSICS-BASED APPROACHES

Models	Application
Paris's law crack	[229]–[232]
Stiffness-based rule	[237], [238]
Electronic	[239], [240]
Fatigue	[241]

piping material, characteristics of the fluid, process conditions, geometry, and location.

The limitation of physics-based approaches mainly lies in its high implementation cost. Physics-based models are considered as the most expensive approaches among prognostics approaches [20]. To obtain high-fidelity physical models, sufficient experiments are required. Intensive computation might cause model fail. Physics-based models are usually system specific, and then the reusability is limited. Table II summarizes the most commonly used physics-based approaches.

1) Physics of Failure: PoF has been investigated in many industries, such as rotation machinery, electronic, wearing, turbine, and aerospace system. For example, Cubillo *et al.* [241] summarized common failure modes, degradation mechanisms, and degradation models of rotating machinery. Breteler *et al.* [242] proposed a method considering the actual loads acting on the system by taking into account how the component will fail or in other words the PoF. Stringer *et al.* [243] presented physics-based models as a key component of prognostic and diagnostic algorithms of health monitoring systems. The benefit of physics-based models is the ability to incorporate the physical understanding of the system degradation and improve the accuracy of prognostics modeling. Physical laws, such as crack growth, fatigue, and wearing, can provide a "long-term" prediction under various loads. Statistical filters used to estimate the state space are considered as an integrated method that combines physics-based and data-driven prognostics. Physics-based models, such Paris' law, are usually deterministic functions, which are limited in analyzing variabilities of the degradation process.

2) Stochastic Filter-Based Models: Stochastic filters are the most commonly used physics-based prognostics. A state-space model for the unobserved condition and observed process related variables is established to model the conditional residual time [244]. A common form of filter-based model is $x_t = f(x_{t-1}, u_t, w_t) \leftrightarrow p(x_t|x_{t-1})$ and $y_t = h(x_t, v_t) \leftrightarrow p(y_t|x_t)$, where u_t is the input of the system and w_t and v_t are noises. There are three commonly used stochastic filters to estimate the state—Kalman filter (KF) [extended KF (EKF) or unscented KF (UKF)], histogram, and particle filters. The choice of filters depends on the characteristics of system. Sikorska *et al.* [245] summarized the scenarios with appropriate filters, see Fig. 3.

1) *Particle filter-based models:* The objective of a particle filter can be defined as a sequential estimation of the distribution of the state, which includes three components: the filter distribution $p(x_t|y_{1:t})$, the prediction distribution $p(x_t|y_{1:t-1})$, and the smoothing distribution $p(x_t|y_{1:T})t < T$. There are two assumptions of particle filters: Monte Carlo assumption and importation sampling assumption. A general scheme for particle filters is composed by initialization, prediction, update, and resampling. Particle filter-based models attract more interest in prognostics modeling. An *et al.* [246] provided a tutorial for particle filter-based prognostics in MATLAB. Jouin *et al.*

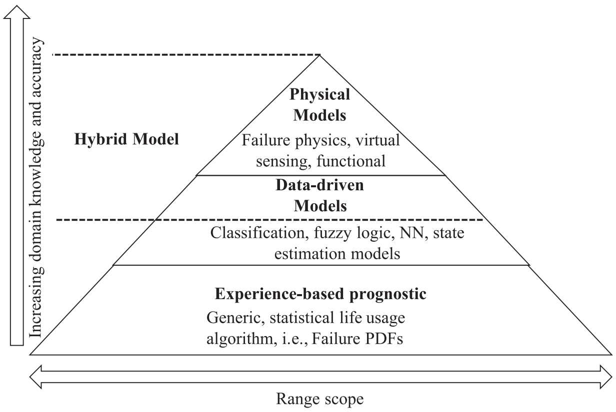

Fig. 4. Application of various prognostics approaches.

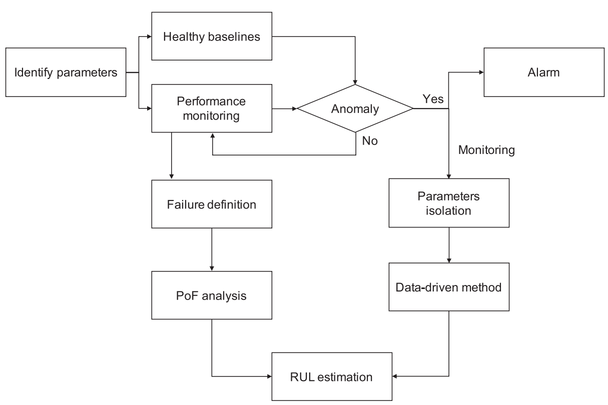

Fig. 5. Procedure of hybrid approaches (adapted from [3]).

[247] reviewed the particle filter-based prognostics and discussed the future directions. They reviewed 46 published works and provided details of practical issues of particle filter-based methods. Recently researchers are focusing on the explanation of state-space models. Kantas *et al.* [248] on particle methods for parameter estimation in state-space models.

2) *KF-based models:* A KF is an optimal estimator, i.e., infers parameters of interest from indirect, inaccurate, and uncertain observations. It is recursive so that new measurements can be processed as they arrive. Due to their advantages in optimality, online real-time processing, and natural interpretation, KF-based models are widely used [238], [239], [249]–[255]. There are two types of KFs: EKF and UKF. EKF is an extension of KF for a nonlinear application. By using partial derivatives and Taylor series expansion, EKF linearizes the “Predict” and “Update” functions for current estimates. After linearization, the remaining process resembles that when using a traditional KF. Many works applied EKF-based models in engineering practice [256]–[258]. Xu and Chen [259] proposed a state-space model where the parameter vector is treated as the hidden state, and the state-transition model is used to track the evolution of the parameter vector as the battery ages.

UKF is proposed to solve the flaw in EKF. EKF can be viewed as providing “first-order” approximations to the optimal terms.

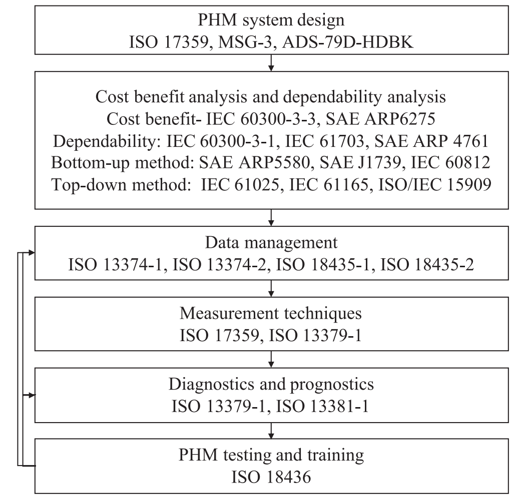

Fig. 6. PHM development and implementation standards [273].

These approximations, however, can introduce significant errors in the true posterior mean and covariance of the transformed (Gaussian) random variable, which may lead to suboptimal performance and sometimes divergence of the filter [260]. In UKF, the state distribution is again represented by a Gaussian random variable but is now specified using a minimal set of carefully chosen sample points. These sample points completely capture the true mean and covariance of the Gaussian random variable, and when propagated through the true nonlinear system, capture the posterior mean and covariance accurately to the third order. Due to its advantages in prediction accuracy, many works have been exploring applications of UKF in prognostics. He *et al.* [261], Xiong *et al.* [262], Zhang and Pisu [250], and Miao *et al.* [263] used an unscented Kalman filtering-based method to self-adjust the model parameters and provide state of charge estimation of lithium-ion batteries. Zheng and Fang [264] integrated UKF and relevance vector regression to estimate RUL of lithium-ion batteries where the RVM model is employed as a nonlinear time-series prediction model to predict the UKF future residuals which otherwise remain zero during the prediction period.

B. Summary of Physics-Based Approaches

The characteristic of system/component specific limits the application of physics-based models. However, the physical understanding from physics-based models can be used to improve the interpretation and accuracy of prognostics studies. The incorporation of system knowledge in multiple degradation trajectories is a challenging task since there might be multiple signals from subsystems/components at the same measurement factor. Moreover, most of the works focus on single input and output in the space-state function. Within complex systems, the health state of the system is determined by states of subsystems/components. The case of multiple correlated inputs remains a challenge.

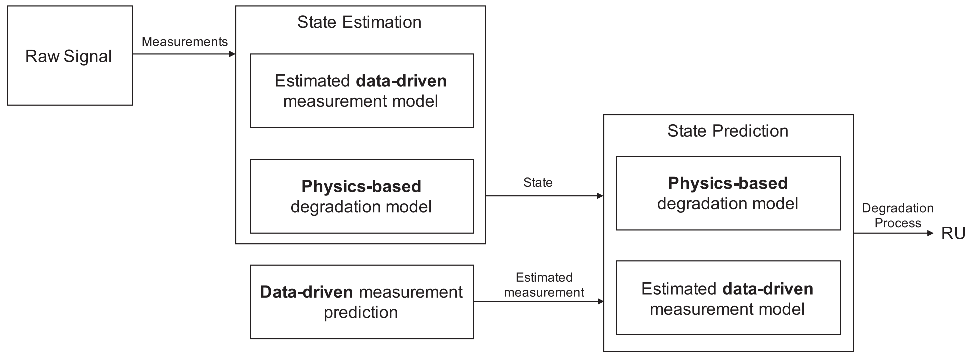

Fig. 7. Framework of hybrid approaches (adapted from [279]).

IV. Hybrid Approaches

Hybrid approaches are the integration of both data-driven and physics-based prognostics. It is an intuitive idea to leverage the strength of both aforementioned data-driven and physics-based prognostics to improve the prediction performance. Hybrid approaches, on one side, can broaden the usage of physics-based models and also improve the interpretation of data-driven models. Fig. 4, adapted from [20], illustrates the application range and the prediction accuracy of data-driven and physics-based models. Hybrid models "inherit" the broad application range of data-driven approaches and high fidelity from physics-based approaches. Extensive work on hybrid approaches has been done. For the implementation procedure, Kumar *et al.* [265] and Cheng and Pecht [266] proposed nine steps for fusing prognostics method and emphasized the importance of understanding PoF (see Fig. 5). Based on the major objective of diagnostics, Sankavaram *et al.* [267] designed a six-step integrated diagnostic and prognostic process to implement the hybrid approaches. These frameworks can be used for any system but not the only way. Five key steps required for a hybrid method include data acquisition, feature extraction, diagnostics, degradation modeling, and RUL prediction. Diagnostics is to identify failure modes and degradation mechanism, which can be used to provide the physical understanding of prognostics modeling. In system-level prognostics, hybrid methods have their advantages in modeling the structural and stochastic dependencies between subsystems and components. For example, Aizpurua *et al.* proposed a complete list of works focusing on prognostics-based dynamic reliability and corresponding maintenance [268]–[271], where the Boolean-Driven Markov Process (BDMP) is used to express a system dependability prediction model that are developed by designers and analysts of the system. Based on the BDMP, operational data, component degradation equations, and other sensing data are taken as the prediction model inputs of RUL and reliability. Choo *et al.* [272] investigated the PHM of smart manufacturing systems where the adaptive multiscale PHM integrates lower-level PHM information. The hierarchical Markov decision process is used to describe the system stochastic behaviors. For smart manufacturing systems, Vogl *et al.* [273] summarized standards for PHM techniques within manufacturing operations, which provide thorough picture of PHM system development and implementation in operation, see Fig. 6. Daigle *et al.* [274] used state-space equations to construct a distributed solution to system level prognostics where the system model is decomposed into independent submodels and local prognostics submodels are formed based on the local prognostics information. The decomposition process and the initiation of state-space equation are based on physical understanding. In [275], Daigle *et al.* applied the distributed approach in national airspace systems. Similarly, Khorasgani *et al.* [276] extended particle filtering based on stochastic simulation approach estimate component degradation rates and predicted system state based on a stochastic process. Bian and Gebraeel [277] and Assaf [278] investigated the stochastic dependencies of multiple components through stochastic differential equations, which is developed based on the physical understanding of the system.

Based on methods of degradation modeling and RUL prediction, Liao and Kottig [279] classified the hybrid approaches into four categories, see Fig. 7.

1) Use a data-driven model to infer a prognostics model and use a physics-based model to predict RUL, such as [266], [280], [281].
2) Use a data-driven model to replace the system model of physics-based model, such as [282].
3) Use a data-driven model to prediction future degradation state and use physics-based for RUL prediction, such as [283].
4) Use both data-driven and physics-based model for RUL prediction and fuse their results, such as [284]–[286].

The first three categories can be understood as data-driven or physics-based model serve as priori for each other, i.e., these two methods work in series. The forth category is to launch data-driven and physics-based models simultaneously and fuse their results, i.e., two integrated methods work in parallel.

Hybrid models are widely used for prognostics due to their strength inheriting from both data-driven and physics-based models. However, existing hybrid methods used in repeated measurement data, mainly regarding the development of data-driven techniques and physical understanding, are rarely incorporated. In the future, we will develop adequate hybrid prognostics models that can effectively incorporate both physical understanding and sensing data information. To maintain the model accuracy for the purpose of prediction, the incorporation of the physical understanding is done through covariate identification and functional forms selection for the models.

V. Conclusion

This article aimed to review and summarize the emerging prognostics modeling methods. The reviewed models were classified into data-driven, physics-based, and hybrid approaches based on the extent of physics knowledge usage in the degradation processes modeling. Advantages and disadvantages of models reported in the literature were critically discussed. We summarized challenges in literature and summarized future directions in prognostics modeling.

1) Prognostics of complex systems becomes one of the main future directions. Current prognostics modeling mainly focus on single components. For a complex system, multidimensional degradation data of subsystem/components is available. Data fusion of multidimensional degradation data of complex systems is one of the challenges. Complex correlation analysis among subsystems/components makes it challenging at the system-level prognostics. Probabilistic graphical models is a powerful tool to analyze the conditional dependence structure between random variables.
2) Robust models based on the complex data structure are needed. Sensing data can be available in forms of complex data structures, such as few data and unbalanced or missing data. Bayesian inference attracts growing attention. For the case of few data or no data available, physics-based models can provide prior knowledge for understanding the degradation process. However, how to validate the expertise judgment remains a challenge. The major challenge in prognostics modeling over unbalanced/missing data is to incorporate all information of multiple subjects.
3) Online monitoring and prognostics is an emerging trend. Prognostics modeling with online data is expected to be adaptive. When recovery or maintenance actions are taken, the system degradation state estimation scheme based on the health state of subsystems/components should be updated automatically. Moreover, storage and analysis of large-scale degradation datasets can be very challenging. Cloud-based storage and analysis will be one of the major trends in reliability and maintenance modeling and practices.
4) External environmental variables, such as temperature, vibration, and stress level, determine the degradation process. This becomes a complicated issue since those variables will impact the observed degradation state, which, in turn, will influence the RUL estimation.

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