

Probability: Multivariate Models

1.

$$\begin{aligned} Cov(X, Y) &= \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y] \\ &= \mathbb{E}[X^3] - \mathbb{E}[X]\mathbb{E}[X^2] \end{aligned}$$

Because $X \sim U(-1, 1)$, therefore $\mathbb{E}[X] = 0, \mathbb{E}[X^3] = 0$

$$\begin{aligned} Cov(X, Y) &= \mathbb{E}[X^3] - \mathbb{E}[X]\mathbb{E}[X^2] \\ &= 0 \end{aligned}$$

therefore

$$\begin{aligned} \rho(X, Y) &= \frac{Cov(X, Y)}{\sqrt{\mathbb{V}[X]\mathbb{V}[Y]}} \\ &= 0 \end{aligned}$$

Although X and Y are uncorrelated, it is definite that Y is dependent on X .
2.