Lana's Better CALC II Lecture Notes

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Preface

This is my attempt at making a more comprehensable set of lecture notes for the CALC II module. Due to its overall better legibility and structure, I've based the style of these lecture notes on that used in the LAG II lecture notes. Special thanks to me for spending a full ass day painstakingly reconstructing the LaTeX preamble used in the LAG II lecture notes, I hope I've done a good enough job and that these lecture notes are at least slightly better than the ones provided by our module. Basically, after I say anything just imagine it says "*From what I've been able to gather*" before it. Also this is my first time using LaTeX so hope it all looks good and up to code.

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Introduction and Overview

0.1 The Purpose of These Notes

$$f : \mathbb{R}^m \longrightarrow \mathbb{R}^n$$

0.2 A Brief Review of Calculus 1

0.3 A Brief Review of Linear Algebra and Geometry

0.4 The Four Types of Functions

In Calculus 1 and Calculus 2, our primary focus has been and will continue to be studying functions. The aim of this module is to expand the domain of what we learned previously in Calculus I to higher dimensions, and there are four distinct types of functions which we will consider to achieve this aim.

$$\begin{array}{r|l}
\mathbf{r} : \mathbb{R}^1 \longrightarrow \mathbb{R}^n & \text{(paths in } \mathbb{R}^n\text{)} \\
f : \mathbb{R}^m \longrightarrow \mathbb{R}^1 & \text{(scalar value functions on } \mathbb{R}^m\text{)} \\
\mathbf{v} : \mathbb{R}^n \longrightarrow \mathbb{R}^n & \text{(vector fields on } \mathbb{R}^n\text{)} \\
T : \mathbb{R}^m \longrightarrow \mathbb{R}^n & \text{(vector functions from } \mathbb{R}^m \text{ to } \mathbb{R}^n\text{)}
\end{array}$$

For each of these functions, we will explore how they are defined, how we can expand our definitions of derivatives and integrals to encompass them, and how these both relate to higher dimensional geometry. Moreover, we will see how calculus, with relation to these functions, gives rise to methods for computing useful geometric properties of objects in higher dimensional space.

Chapter 1

Paths and Parametric Equations

Let us assume that $n \in \mathbb{N}$ is a natural number such that $n > 1$. This chapter will cover functions

$$\mathbf{r} : \mathbb{R} \longrightarrow \mathbb{R}^n \tag{1.0.1}$$

mapping from $\mathbb{R}^1$ to $\mathbb{R}^n$, usually referred to as paths or, equivalently, as parametric equations

$$t \longmapsto (f(t), g(t)) \tag{1.0.2}$$

1.1 Paths and Curves

There is an important distinction to be made between paths and curves. While paths in $\mathbb{R}^n$ are functions, curves in $\mathbb{R}^n$ are instead geometric objects in n dimensional space.

Definition 1.1.1. Let $\mathbf{r} : \mathbb{R} \longrightarrow \mathbb{R}^n$ be a function. Then $\mathbf{r}$ is a **path** on $\mathbb{R}^n$

Definition 1.1.2. Let $C \subseteq \mathbb{R}^n$ be a subset of points in $\mathbb{R}^n$. If there exists some path $\mathbf{r} : \mathbb{R} \longrightarrow C \subseteq \mathbb{R}^n$ such that $\mathbf{r}$ is continuous, then C is a **curve**.

continuous subsets $C \subseteq \mathbb{R}^n$

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