Formal Prove I

Let's consider some curve **c** that fully lies on the surface of our implicit function **f**:

$$c\left(t\right) = \left(x\left(t\right), y\left(t\right), z\left(t\right)\right)$$
$$f\left(p\right) = 0$$

Because all points of the curve lies on the surface, we pass the curve inside our implicit function formula:

$$f\left(x\left(t\right), y\left(t\right), z\left(t\right)\right) = 0$$

Let's differentiate this equation with respect to **t**:

$$\frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt} = 0$$
$$\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) \cdot \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right) = 0$$

The first vector of the dot product is the gradient; second - the tangent to the curve.

$$\nabla f\left(p\right) \cdot c'\left(t\right) = 0$$

So, for all points of the surface the gradient in the point will be perpendicular to the tangent of the curve that lies on this surface. That proves that the gradient is proportional to the normal at the same