# Finals Problem Set No. 1

Digital Signals Processing

**General Instructions.** Solve for the following problems numerically using manual solutions. Submissions must be in PDF format only and should be digitally encoded using MS Word or LaTeX only. Additional figures must be generated using Python code.

**Part 1**. [200 pts] Compute for the Energy and Power of the following signals in terms of per unit use the proper formula whether the signal is continuous or discrete.

a. $\sin[n]$

$E_{pu} = |\sin[n]|^2$

$= |\sin[n]|@\sin[n]|$

$P_{pu} = \frac{1}{2\pi}(|sin[n]|@|sin[n]|)$

$= 784.613$ - Discrete

b. $\cos(t)$

$E_{pu} = \int |cos(\pi)|^2 dt$

$= \frac{1}{2}(cos(\pi))^2$

$= \int \frac{1 + cos(2\pi)}{2} dt$

$= \frac{1}{2}( + \frac{1}{2}(\sin(2\pi))$

$= \frac{\pi}{2} + \frac{\sin(2\pi)}{4}$

$P_{pu} = \frac{1}{2\pi}[\frac{\pi}{2} + \frac{sin(2\pi)}{4}$

Continuous

c. $2[\cos(t) + isin(t)]$

$E_{pu} = \int 2[|\cos(\pi)||^2 + |\sin(\pi)|^2]$

$= 2|cos^2(\pi)| = 2cos^2(\pi)$

$= |isin(\pi)|^2 = |i|^2|\sin(\pi)|^2 = sin^2(\pi)$

$= \frac{1}{2}\left(\frac{\pi + \sin(2\pi)}{2}\right) + \left(\frac{\pi + \cos(2\pi)}{2}\right)$

$= \frac{\pi+\sin(2\pi)}{4} + \frac{\pi+\cos(2\pi)}{4}$

$P_{pu} = \frac{1}{2\pi}[\frac{\pi + \sin(2\pi)}{4} + \frac{\pi + \cos(2\pi)}{4}$

Continuous

d. $y[n] = \frac{2}{\sqrt{\pi}} e^{-in}$

$$E_{pu} = \sum \left| \frac{2}{\sqrt{\pi}} - e^{in} \right|^2$$

$$= \left| \frac{2}{\sqrt{\pi}} - e^{in} \right| @ | \frac{2}{\sqrt{\pi}} - e^{in} |$$

$$P_{pu} = \frac{1}{2\pi}(\left| \frac{2}{\sqrt{\pi}} - e^{in} \right| @ \left| \frac{2}{\sqrt{\pi}} - e^{in} \right|)$$

= 3574.341 Discrete

**Part 2**. [100 pts] Prove the equality of the following trigonometric functions using Euler identities. Further prove the equality in Python. Furthermore, plot the left-hand side and right-hand side of the equation and their Euler forms as additional proof.

$$sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta))$$

$$= sin^2(\emptyset) = (\sin(\emptyset))^2 = \left(\frac{e^{i\emptyset} - e^{i\emptyset}}{2i}\right)^2$$

$$= \frac{1}{2}\left(\frac{e^{i\emptyset} - e^{i\emptyset}}{2i}\right) * \frac{1}{2}\left(\frac{e^{i\emptyset} - e^{i\emptyset}}{2i}\right)$$

$$= \frac{1}{4}\left(\frac{e^{2i\emptyset} - 2 + e^{-21\emptyset}}{-1}\right)$$

$$= -\frac{1}{4}\left[\left(\cos(2\emptyset) + isin(2\emptyset)\right) - 2 + \left(\cos(2\emptyset) - isin(2\emptyset)\right)\right]$$

$$= -\frac{1}{4}(-4 + 2\cos(2\emptyset))$$

$$= \frac{1}{2} - \frac{1}{2}(\cos(\emptyset))$$

$$= \frac{1 - \cos(2\emptyset)}{2}$$

**Part 3**. [100 pts] Prove the equality of the following trigonometric functions using Euler identities. Further prove the equality in Python (NumPy).

$$e^{i\pi} + 1 = 0$$

$$e^{i\emptyset} = \cos(\emptyset) + isin(\emptyset)$$

$$e^{i\pi} = \cos(\pi) + isin(\pi)$$

$$\cos(\pi) = \ -1 \sin(\pi) = 0$$

$$e^{i\pi} = -1 + i(0)$$

$$e^{i\pi} = -1$$

$$-1 + 1 = 0$$