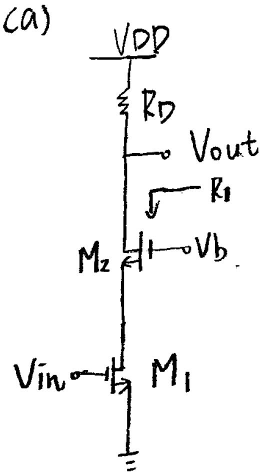
Q_1:

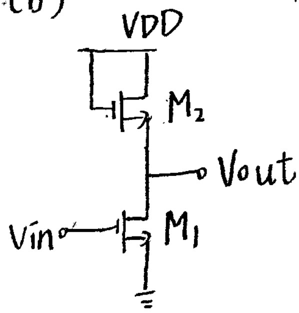
(a)



$$R_1 = g_{m1} r_{o1} r_{o2} + r_{o1} + r_{o2}$$

$$A_v = -g_{m1} \left(R_1 \, // \, R_D \right) \quad (10\%)$$

(b)

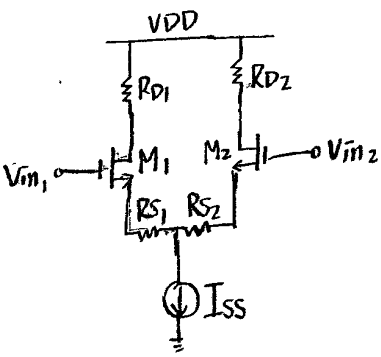


$$R_{out} = r_{o1} \, // \, r_{o2} \, // \left(\frac{1}{g_{m2}+g_{mb2}} \right)$$

$$A_v = -g_{m1} R_{out} = -g_{m1} \left[r_{o1} \, // \, r_{o2} \, // \left(\frac{1}{g_{m2}+g_{mb2}} \right) \right]$$

(10%)

Q_2:



(a) for $R_{S1} = R_{S2} = 0$ $\quad V_{in1} - V_{in2} = V_{in}$

$$V_{in1} - V_{in2} = V_{GS1} - V_{GS2}, \quad V_{GS} = \sqrt{\frac{2I_D}{k_n}} + V_{th}$$

$$\Rightarrow V_{in1} - V_{in2} = \sqrt{\frac{2I_{D1}}{k_{n1}}} - \sqrt{\frac{2I_{D2}}{k_{n2}}} \quad \text{if } V_{th1} = V_{th2}$$

$$(V_{in1} - V_{in2})^2 = \frac{2}{k_n} \left(I_{SS} - 2\sqrt{I_{D1} I_{D2}} \right) \quad \text{if } k_{n1} = k_{n2}$$

$$\frac{1}{2} k_n \cdot (V_{in1} - V_{in2})^2 - I_{SS} = -2\sqrt{I_{D1} I_{D2}}$$

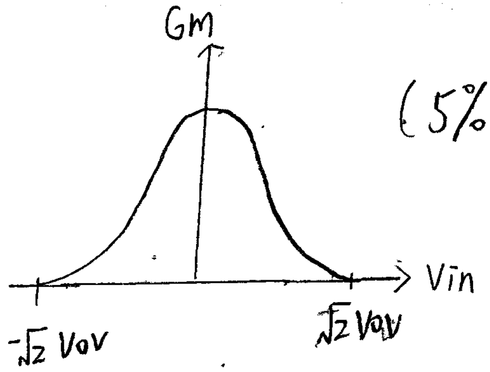
$$\left(\frac{1}{2} k_n V_{in}^2 - I_{SS} \right)^2 = 4 I_{D1} I_{D2}$$

and $4 I_{D1} I_{D2} = (I_{D1} + I_{D2})^2 - (I_{D1} - I_{D2})^2 = I_{SS}^2 - (I_{D1} - I_{D2})^2$

$$\Rightarrow (I_{D1} - I_{D2})^2 = -\frac{1}{4} k_n^2 V_{in}^4 + I_{SS} k_n V_{in}^2$$

$$I_{D1} - I_{D2} = \sqrt{k_n I_{SS}} \, V_{in} \sqrt{1 - \frac{k_n}{4 I_{SS}} V_{in}^2}$$

$$G_m = \frac{\partial I_D}{\partial V_{in}} = \frac{1}{2} k_n \cdot \frac{\frac{4I_{SS}}{k_n} - 2V_{in}^2}{\sqrt{\frac{4I_{SS}}{k_n} - V_{in}^2}}$$



(5%)

for $G_m = 0$, assume $I_{SS} = I_{D1}$ $V_{in} = V_{in1}$

$$\frac{4I_{SS}}{k_n} - 2V_{in1}^2 = 0$$

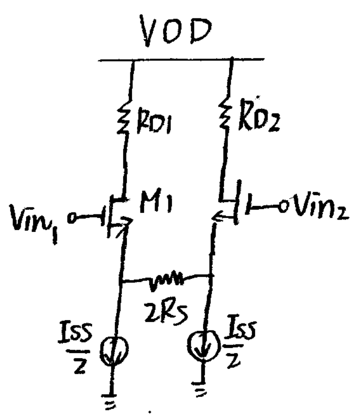
$$\frac{2I_{SS}}{k_n} = V_{in1}^2 \qquad V_{in1} = \sqrt{\frac{2I_{SS}}{k_n}} = \sqrt{2}\, V_{OV} \quad (5\%)$$

(b)

If $R_{S1} = R_{S2} = R_S \neq 0$, then the input linear region will increase from $\sqrt{2}\, V_{OV}$ to $\sqrt{2}\, V_{OV} + I_{SS} R_S$ and improve the nonlinear effect (4%)

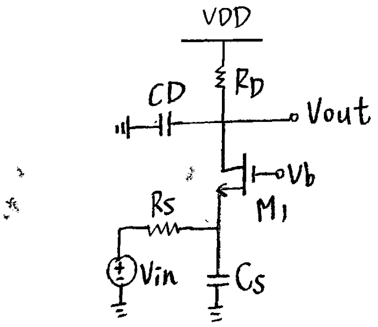
disadvantage: $G_m \downarrow$, VICM level $\nearrow$ (4%)

How to improve:



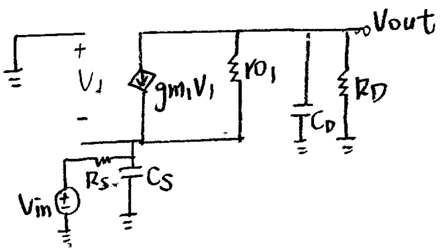
(2%)

Q3



$$g_m = g_{m1} + g_{mb1}$$

$$A_v(s) = \frac{V_{out}(s)}{V_{in}(s)} = A_0 \times \frac{1}{(1+\frac{s}{\omega_{in}})(1+\frac{s}{\omega_{out}})}$$



$$\begin{cases} \dfrac{V_{in}+V_1}{R_S} + V_1 sC_S + g_m V_1 + \dfrac{V_{out}+V_1}{r_{o1}} = 0 & \cdots\cdots ① \\ g_m V_1 + \dfrac{V_{out}+V_1}{r_{o1}} + V_{out}\left(\dfrac{1}{R_D /\!/ \frac{1}{sC_D}}\right) = 0 & \cdots\cdots ② \end{cases}$$

As the above ①,② we can get

$$A_v(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1+g_m r_{o1}}{s^2 C_S C_D R_S r_{o1} + s\left[C_S R_S (1+\frac{1}{R_D}) + C_D R_S (1+g_m r_{o1}) + C_D r_{o1}\right] + 1 + \frac{(1+g_m r_{o1})R_S+1}{R_D}}$$

DC gain ⇒ $s=0$ ⇒ $\dfrac{1+g_m r_{o1}}{1+\frac{(1+g_m r_{o1})R_S+R_S}{R_D}} = \dfrac{(1+g_m r_{o1})R_D}{R_D+R_S+(1+g_m r_{o1})R_S}$ (10%)

for $\lambda \neq 0$, $\omega_{p1} \approx \left[\dfrac{C_S R_S (R_D+1) + C_D R_D R_S (1+g_m r_{o1}) + C_D r_{o1} R_D}{R_D+R_S+(1+g_m r_{o1})R_S}\right]^{-1}$

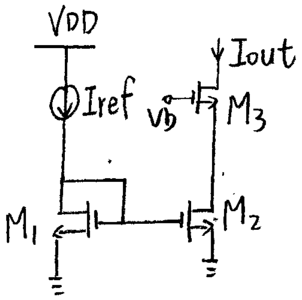
if $\omega_{p1} \gg \omega_{p2}$

$$\omega_{p2} = \frac{R_D+R_S+(1+g_m r_{o1})R_S}{\omega_{p1} \cdot C_S C_D R_S r_{o1} R_D}$$

for $\lambda = 0$ $\quad \omega_{p1} = \dfrac{1+g_m R_S}{R_S C_S}$

$$\omega_{p2} = \frac{1}{R_D C_D} \qquad (10\%)$$

Q4

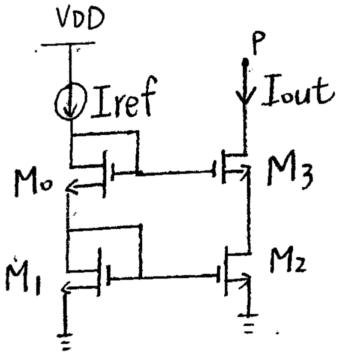


(a)

$$\frac{I_{out}}{I_{ref}} = \frac{\cancel{\frac{1}{2}\mu_n C_{ox}}\left(\frac{W}{L}\right)_2 \cancel{(V_{GS_2}-V_{th_1})^2}(1+\lambda V_{DS_2})}{\cancel{\frac{1}{2}\mu_n C_{ox}}\left(\frac{W}{L}\right)_1 \cancel{(V_{GS_1}-V_{th_2})^2}(1+\lambda V_{DS_1})}$$

$$= \frac{\left(\frac{W}{L}\right)_2 (1+\lambda V_{DS_2})}{\left(\frac{W}{L}\right)_1 (1+\lambda V_{DS_1})} \quad (10\%)$$

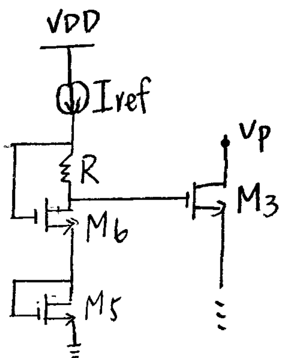
(b)



The gate voltage of M_3 is $V_{gs_1} + V_{gs_0} = 2V_{ov} + 2V_{th}$

the output swing (V_p) ↓ (5%)

How to improve



The gate voltage of M_3 is $V_{gs5} + V_{gs6} - I_{ref}R$

$= 2V_{ov} + 2V_{th} - I_{ref} \cdot R$

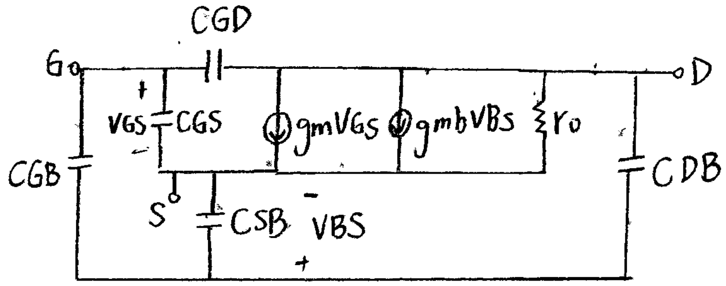
If $I_{ref} \cdot R = V_{th}$

$V_{G,M_3} = 2V_{ov} + V_{th}$

the output swing (V_p) ↗ (5%)

Q5

(a)

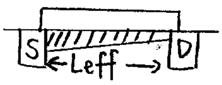


(10%)

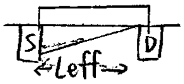
(b)

channel length modulation

⇒ Mos通道實際長度會隨著閘極和汲極間的電位差降低而逐漸減少，當mos在飽合區時，Leff長度縮小，電流就會相對上升



$V_{DS} < V_{GS} - V_{th}$



$V_{DS} > V_{GS} - V_{th}$

$$\frac{1}{L_{eff}} = \frac{1}{L-\Delta L} = \frac{1}{L(1-\frac{\Delta L}{L})}$$

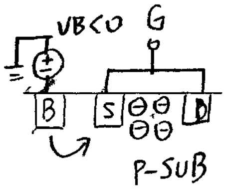
$$= \left(\frac{1+\frac{\Delta L}{L}}{L}\right) \quad \frac{\Delta L}{L} = 1+\lambda V_{DS}$$

$$I = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_{GS}-V_t)^2(1+\underbrace{\lambda V_{DS}}_{\text{修正項}})$$ (3%)

設計上當 I_D 在飽合區時，VDS 增減會對 ro 造成變化，在公式上有非線性變化 (2%)

body effect

⇒ 當 VB<0 時，body 端電壓會吸引更多電洞到基板，產生更多的負電荷到 gate 端下方，gate 端需施加更大的正電壓來形成 inversion layer



(3%)

設計上當 VBS≠0 時，則 $V_{th} = V_{th0} + \gamma\left(\sqrt{2\phi_f - V_{BS}} - \sqrt{|2\phi_f|}\right)$ (2%)