1.

$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right|<1 \Rightarrow \lim_{n\to\infty}\left|\frac{(3x-2)^{n+1}}{n+1}\cdot\frac{n}{(3x-2)^n}\right|<1 \Rightarrow |3x-2|\lim_{n\to\infty}\left(\frac{n}{n+1}\right)<1 \Rightarrow |3x-2|<1$
$\Rightarrow -1<3x-2<1 \Rightarrow \frac{1}{3}<x<1$; when $x=\frac{1}{3}$ we have $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ which is the alternating harmonic series and is conditionally convergent; when $x=1$ we have $\sum_{n=1}^{\infty}\frac{1}{n}$, the divergent harmonic series

(a) the radius is $\frac{1}{3}$; the interval of convergence is $\frac{1}{3}\le x<1$

(b) the interval of absolute convergence is $\frac{1}{3}<x<1$

(c) the series converges conditionally at $x=\frac{1}{3}$

2.

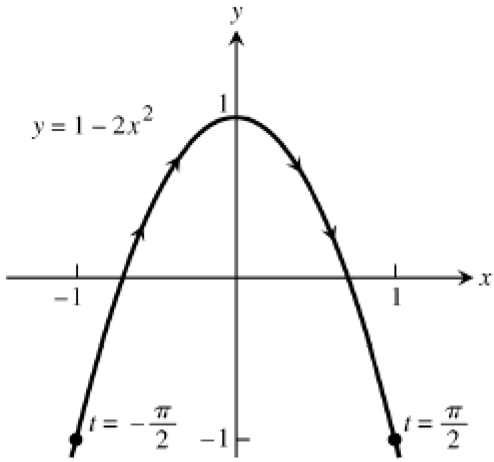
$f(x)=\sqrt{x}=x^{1/2}$, $f'(x)=\left(\frac{1}{2}\right)x^{-1/2}$, $f''(x)=\left(-\frac{1}{4}\right)x^{-3/2}$, $f'''(x)=\left(\frac{3}{8}\right)x^{-5/2}$; $f(4)=\sqrt{4}=2$,
$f'(4)=\left(\frac{1}{2}\right)4^{-1/2}=\frac{1}{4}$, $f''(4)=\left(-\frac{1}{4}\right)4^{-3/2}=-\frac{1}{32}$, $f'''(4)=\left(\frac{3}{8}\right)4^{-5/2}=\frac{3}{256} \Rightarrow P_0(x)=2$, $P_1(x)=2+\frac{1}{4}(x-4)$,
$P_2(x)=2+\frac{1}{4}(x-4)-\frac{1}{64}(x-4)^2$, $P_3(x)=2+\frac{1}{4}(x-4)-\frac{1}{64}(x-4)^2+\frac{1}{512}(x-4)^3$

3.

$$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!} \Rightarrow xe^x=x\left(\sum_{n=0}^{\infty}\frac{x^n}{n!}\right)=\sum_{n=0}^{\infty}\frac{x^{n+1}}{n!}=x+x^2+\frac{x^3}{2!}+\frac{x^4}{3!}+\frac{x^5}{4!}+\ldots$$

4.

$$x=\sin t,\ y=\cos 2t,\ -\frac{\pi}{2}\le t\le\frac{\pi}{2}$$
$$\Rightarrow y=\cos 2t=1-2\sin^2 t \Rightarrow y=1-2x^2$$



5.

$A=\int_0^1 x\,dy=\int_0^1(t-t^2)(-e^{-t})dt \quad \left[u=t-t^2 \Rightarrow du=(1-2t)dt;\ dv=(-e^{-t})dt \Rightarrow v=e^{-t}\right]$

$=e^{-t}(t-t^2)\Big|_0^1-\int_0^1 e^{-t}(1-2t)dt \quad \left[u=1-2t \Rightarrow du=-2dt;\ dv=e^{-t}dt \Rightarrow v=-e^{-t}\right]$

$=e^{-t}(t-t^2)\Big|_0^1-\left[-e^{-t}(1-2t)\Big|_0^1-\int_0^1 2e^{-t}dt\right]=\left[e^{-t}(t-t^2)+e^{-t}(1-2t)-2e^{-t}\right]\Big|_0^1$

$=(e^{-1}(0)+e^{-1}(-1)-2e^{-1})-(e^0(0)+e^0(1)-2e^0)=1-3e^{-1}=1-\frac{3}{e}$

6.

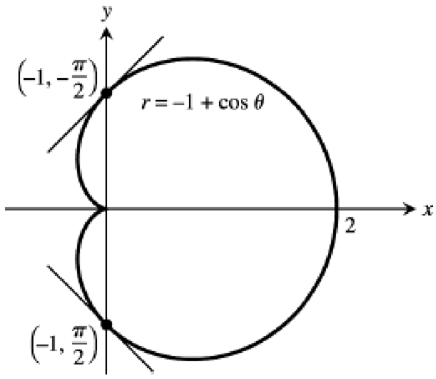
$$\frac{dx}{dt} = -\sin t \text{ and } \frac{dy}{dt} = \cos t \Rightarrow \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{(-\sin t)^2 + (\cos t)^2} = 1 \Rightarrow \text{Area} = \int 2\pi y \, ds$$
$$= \int_0^{2\pi} 2\pi(2 + \sin t)(1)dt = 2\pi\left[2t - \cos t\right]_0^{2\pi} = 2\pi[(4\pi - 1) - (0 - 1)] = 8\pi^2$$

7.

$r^2 = 4r\sin\theta \Rightarrow x^2 + y^2 = 4y \Rightarrow x^2 + y^2 - 4y + 4 = 4 \Rightarrow x^2 + (y-2)^2 = 4$, circle with center $C = (0, 2)$ and radius 2

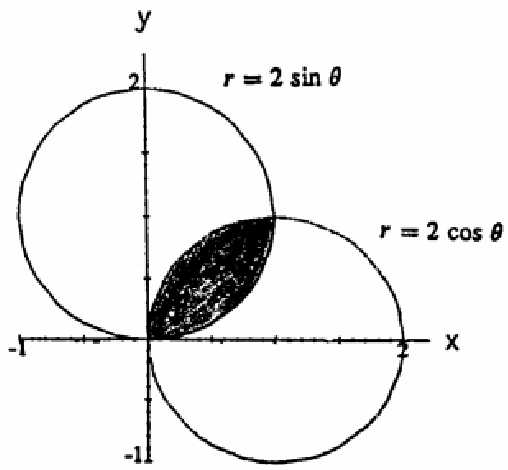
8.

$\theta = \frac{\pi}{2} \Rightarrow r = -1 \Rightarrow \left(-1, \frac{\pi}{2}\right)$, and $\theta = -\frac{\pi}{2} \Rightarrow r = -1$
$\Rightarrow \left(-1, -\frac{\pi}{2}\right)$; $r' = \frac{dr}{d\theta} = -\sin\theta$; Slope $= \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$
$= \frac{-\sin^2\theta + r\cos\theta}{-\sin\theta\cos\theta - r\sin\theta} \Rightarrow$ Slope at $\left(-1, \frac{\pi}{2}\right)$ is
$\frac{-\sin^2\left(\frac{\pi}{2}\right) + (-1)\cos\frac{\pi}{2}}{-\sin\frac{\pi}{2}\cos\frac{\pi}{2} - (-1)\sin\frac{\pi}{2}} = -1$; Slope at $\left(-1, -\frac{\pi}{2}\right)$ is
$\frac{-\sin^2\left(-\frac{\pi}{2}\right) + (-1)\cos\left(-\frac{\pi}{2}\right)}{-\sin\left(-\frac{\pi}{2}\right)\cos\left(-\frac{\pi}{2}\right) - (-1)\sin\left(-\frac{\pi}{2}\right)} = 1$



9.

$$r = 2\cos\theta \text{ and } r = 2\sin\theta \Rightarrow 2\cos\theta = 2\sin\theta$$
$$\Rightarrow \cos\theta = \sin\theta \Rightarrow \theta = \frac{\pi}{4}\text{; therefore}$$
$$A = 2\int_0^{\pi/4} \frac{1}{2}(2\sin\theta)^2 \, d\theta = \int_0^{\pi/4} 4\sin^2\theta \, d\theta$$
$$= \int_0^{\pi/4} 4\left(\frac{1-\cos 2\theta}{2}\right) d\theta = \int_0^{\pi/4} (2 - 2\cos 2\theta) \, d\theta$$
$$= \left[2\theta - \sin 2\theta\right]_0^{\pi/4} = \frac{\pi}{2} - 1$$



10.

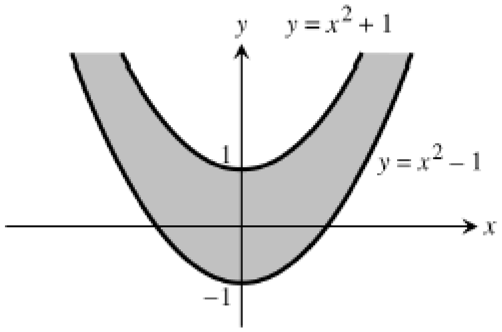
$r = \sqrt{1 + \sin 2\theta}\,, 0 \le \theta \le \pi\sqrt{2} \Rightarrow \frac{dr}{d\theta} = \frac{1}{2}(1 + \sin 2\theta)^{-1/2}(2 \cos 2\theta) = (\cos 2\theta)(1 + \sin 2\theta)^{-1/2}$; therefore

$$\text{Length} = \int_0^{\pi\sqrt{2}} \sqrt{(1 + \sin 2\theta) + \frac{\cos^2 2\theta}{(1 + \sin 2\theta)}}\, d\theta = \int_0^{\pi\sqrt{2}} \sqrt{\frac{1 + 2\sin 2\theta + \sin^2 2\theta + \cos^2 2\theta}{1 + \sin 2\theta}}\, d\theta$$

$$= \int_0^{\pi\sqrt{2}} \sqrt{\frac{2 + 2\sin 2\theta}{1 + \sin 2\theta}}\, d\theta = \int_0^{\pi\sqrt{2}} \sqrt{2}\, d\theta = \left[\sqrt{2}\,\theta\right]_0^{\pi\sqrt{2}} = 2\pi$$

11.

Domain: all points (x, y) satisfying
$x^2 - 1 \le y \le x^2 + 1$



12.

$$\lim_{\substack{(x,y) \to (0,0) \\ \text{along } y = kx^2}} \frac{x^4 - y^2}{x^4 + y^2} = \lim_{x \to 0} \frac{x^4 - (kx^2)^2}{x^4 + (kx^2)^2} = \lim_{x \to 0} \frac{x^4 - k^2x^4}{x^4 + k^2x^4} = \frac{1 - k^2}{1 + k^2} \Rightarrow \text{different limits for different values of k}$$