

Total: 100 points

1. (10 points) Find the domain of the following functions

(a) $f(x) = \frac{x+3}{4-\sqrt{x^2-9}}$

(b) $f(x) = \sqrt{2-\sqrt{x}}$

2. (10 points) Let $f(x)$ be a function with domain \mathbb{R} .

(a) Show that $f_e(x)$ is an even function if $f_e(x) = f(x) + f(-x)$

(b) Show that $f_o(x)$ is an odd function if $f_o(x) = f(x) - f(-x)$

3. (10 points) Find the limit $\lim_{x \rightarrow c} \frac{\sin(x-c)}{x^2-c^2}$ where $c \neq 0$.

4. (10 points) Find the limit $\lim_{\theta \rightarrow 1} \sec(\theta \sec^2 \theta - \tan^2 \theta - 1)$.

5. (20 points) For what value of b is

$$f(x) = \begin{cases} \frac{x-b}{b+1} & x \leq 0 \\ x^2 + b & x > 0 \end{cases}$$

continuous at every x .

6. (20 points) Let $f(x)$ be defined as

$$f(x) = \frac{\sqrt{x^2+4}}{x}$$

Determine the domain of the function, and use various limits to find the asymptotes and the ranges.

7. (20 points) Use $\epsilon - \delta$ definition to prove that

$$\lim_{x \rightarrow 0^-} \frac{x}{|x|} = -1$$