ENGINEERING MATHEMATICS (II) – FINAL

Winter 2021

ID: ______________________ Name: ____________________

Note: Provide clear derivations or explanations of your answers. You will not get the credits if only the final results are given.

PROBLEM 1 (15 pts)

Consider a linear transformation T given by

$$T \ : \mathcal{R}^4 \to \mathbf{P}_3$$

where

$$T(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}) = (a_1 + a_3) + 3a_2 x + (2a_2 - a_4)x^2$$

and $\mathbf{P}_3$ denotes the polynomials of degrees less than 3.

(a) (10 pts) Determine the image and kernel of T, and their dimensions.

(b) (5 pts) Is T onto? Is T one-to-one? Justify your answer.

PROBLEM 2 (20 pts)

Find the standard matrix representation of the following linear transformation from $\mathcal{R}^2$ to $\mathcal{R}^2$:

Reflects each vector about the line $x = y$, doubles the length, and then rotates it 30^0 in the counterclockwise direction.

<u>PROBLEM 3</u> (20 pts)

Transform a basis $\left\{ \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \right\}$ for $\mathcal{R}^3$ into an orthonormal basis.

PROBLEM 4 (15 pts)

Suppose a 3×3 matrix $\mathbf{A}$ has eigenvalues -1,2,3. If

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 0 \\ -1 & \eta & 4 \\ 0 & 1 & 2 \end{bmatrix}$$

(a) (5 pts) Determine η.

(b) (5 pts) Is $(2\mathbf{A} + \mathbf{I})^{-1}$ diagonalizable? Explainable your answer.

(c) (5 pts) Determine the determinant of $\mathbf{A}^2 - 2\mathbf{I}$.

PROBLEM 5 (20 pts)

Consider a bottle which contains two types of particles, type X and type Y. Assume that each hour 30% of the type X particle will becomes type Y particle, while 10% of the type Y particle will become type X particle. Suppose that the total number of particle remains the same. Determine

$$\frac{\text{number of particle Y}}{\text{number of particle X}}$$

after a long run. You can assume any arbitrary initial condition.

<u>PROBLEM 6</u> (10 pts)

Determine the distance from a point (w, z) in $\mathcal{R}^2$ to a line $y = ax$. **Hint**: Determine a normal vector to this line first.