Note: Don't use the calculator. To get full points, you should write down the procedure **in detail**.

1. (30 points) Let $f(x,y) = x^2e^{-y}$. (6 points for each)
   (a) Find the gradient of f.
   (b) Find the directional derivative of f at the point $P(-2,0)$ in the direction toward the point $Q(2,-3)$.
   (c) Find the maximum increasing rate of change of f at the point $P(-2,0)$. Which is the direction of the maximum increasing rate of change?
   (d) Find the tangent plane of $z = f(x,y)$ at the point $(-2,0,4)$.
   (e) Let $z = f(x,y)$ and $x = u^2 - v^2, \quad y = 2uv$. Find $\frac{\partial z}{\partial v}\Big|_{(u,v)=(0,\sqrt{2})}$.

2. (10 points) Find the radius of convergence and interval of convergence of the series.
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{n \cdot 4^n}$
   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n(x-2)^n}{(n+2)!}$

3. Please answer the following questions.
   (a) (3 points) Find the Maclaurin series expansion for $\ln(1+x)$ for $|x| < 1$.
   (b) (5 points) Please utilize the result of (a) to find the Maclaurin series expansion of $f(x) = \ln(1 + 4x + 3x^2)$. Write out the general terms.
   (c) (2 points) What is the radius of convergence for the result of (b).

4. (20 points) Evaluate the integrals: (10 points for each)
   (a) $\displaystyle\int_0^2 \int_{x^2}^4 \frac{x^5}{\sqrt{x^6+y^3}}\, dy\, dx.$
   (b) $\displaystyle\int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} (2x+y)\, dx\, dy.$

5. A parametric curve $x = 3t - t^3, y = 3t^2$.
   (a) (4 points) Show that the curve intersects itself at the point $(0,9)$
   (b) (6 points) Find the length of the **loop** of the curve.

6. (10 points) Find all the local maxima, local minima, and saddle point(s) of the function $f(x,y) = (x^2+y^2)e^{-x}$.

7. (10 points) Find the maximum and minimum values of the function $f(x,y,z) = x+y-z$ over the sphere $x^2+y^2+z^2 = 1$.

8. Let C_1 be the curve $(x^2+y^2)^2 = 2a^2xy$ and C_2 be the curve $x^2+y^2 = \dfrac{a^2}{2}$ where $a > 0$
   (a) (6 points) Find polar equations for the curves C_1 and C_2.
   (b) (4 points) Find all points of intersection of C_1 and C_2.
   (c) (10 points) Find the area of the region that lies inside C_1 and C_2.