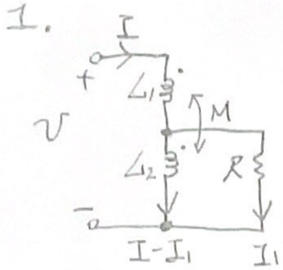
1.



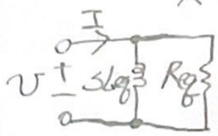
$\because M$ is maximum, $M = k\sqrt{L_1L_2}$

$\therefore k_{(max)} = 1 \Rightarrow M = \sqrt{L_1L_2}$

$$\begin{cases} V = sL_1I + s\sqrt{L_1L_2}(I-I_1) + sL_2(I-I_1) + s\sqrt{L_1L_2}\,I \\ sL_2(I-I_1) + s\sqrt{L_1L_2}\,I = RI_1 \end{cases}$$

$$\Rightarrow \begin{cases} V = (sL_1 + 2s\sqrt{L_1L_2} + sL_2)I - (s\sqrt{L_1L_2} + sL_2)I_1 \quad — ① \\ (s\sqrt{L_1L_2} + sL_2)I = (R + sL_2)I_1 \quad — ② \end{cases}$$

② $\rightarrow I_1 = \dfrac{sL_2 + s\sqrt{L_1L_2}}{R + sL_2}$ — ③ 代入①整理 $\rightarrow V = \dfrac{sR(L_1 + L_2 + 2\sqrt{L_1L_2})}{sL_2 + R}$ — ④



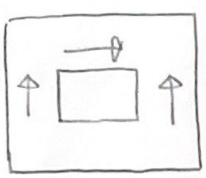
$V = \dfrac{sL_{eq}R_{eq}}{sL_{eq} + R_{eq}}$ — ⑤

$\because$ ④ = ⑤

$$\Rightarrow \begin{cases} sL_{eq} + R_{eq} = sL_2 + R \\ sL_{eq}R_{eq} = s(L_1 + L_2 + 2\sqrt{L_1L_2})R \end{cases}$$

$\Rightarrow L_1 = 0\,(H) \quad \therefore \begin{cases} R_{eq} = R\,(\Omega) \\ L_{eq} = L_2\,(H) \end{cases}$ #

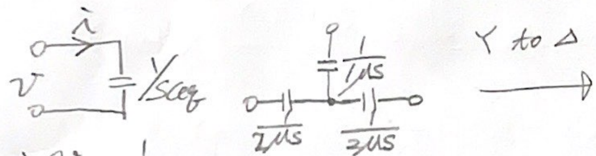
2.(a) $\rightarrow$: Induction magnetic field direction



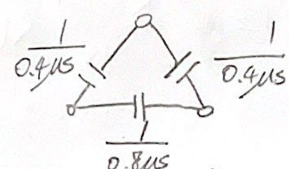
$\Rightarrow L_{ae} = (L_{ab} + 1 - 1) + (L_{dc} + 1 - 2) + (L_{fe} - 2 - 1)$

$= 1 + 1 - 1 + 3 + 1 - 2 + 4 - 2 - 1$

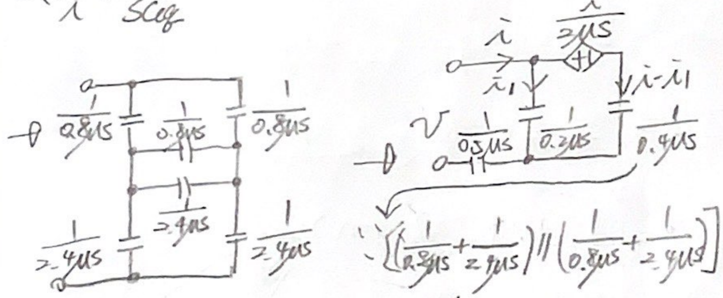
$= 4\,(H)$ #



$\therefore \dfrac{V}{i} = \dfrac{1}{sC_{eq}}$



$$\begin{cases} V = \dfrac{i_1}{0.3\mu s} + \dfrac{i}{0.5\mu s} \quad — ① \\ \dfrac{i_1}{0.3\mu s} = \dfrac{i}{2\mu s} + \dfrac{i - i_1}{0.4\mu s} \quad — ② \end{cases}$$



$\because \left[\left(\dfrac{1}{0.8\mu s} + \dfrac{1}{2.4\mu s}\right) \parallel \left(\dfrac{1}{0.8\mu s} + \dfrac{1}{2.4\mu s}\right)\right] + \dfrac{1}{0.6\mu s}$

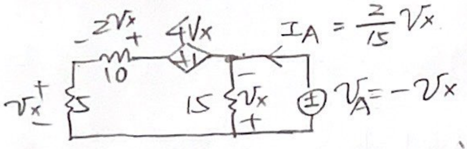
② $\rightarrow i_1 = \dfrac{2}{5}i$ — ③ 代入 ①

$\rightarrow \dfrac{V}{i} = \dfrac{1}{0.25\mu s} \quad \therefore C_{eq} = 0.25\mu(F)$ #

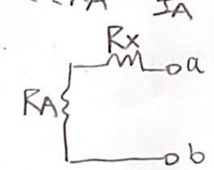
(b) $\because$ Parallel & critically system

$$\therefore \begin{cases} \alpha^2 - \omega_0^2 = 0 \\ \alpha = \dfrac{1}{2R_{ab}C_{eq}},\ \omega_0 = \dfrac{1}{\sqrt{L_{ae}C_{eq}}} \end{cases}$$

$\rightarrow R_{ab} = \pm\sqrt{\dfrac{L_{ae}}{4C_{eq}}} = \pm 2000$ (取正)



$\therefore R_A = \dfrac{V_A}{I_A} = -7.5\,(\Omega)$



$R_{ab} = 2000 = R_x + (-7.5)$

$\therefore R_x = 2007.5\,(\Omega)$ #

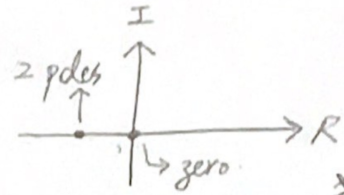
(c) $\omega_0 = \frac{1}{\sqrt{L_{ac}C_{eq}}} = 10^3\,(\text{rad/s})$ # , $Q = \omega_0 R_{ab} C_{eq} = 0.5$ (單位) #

$B\omega = 2\alpha = 2000\,(\text{rad/s})$ #

zero: $s = 0\,(\text{rad/s})$

pole: $s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -1000 \pm 0$ →

$= -1000$ or $-1000\,(\text{rad/s})$

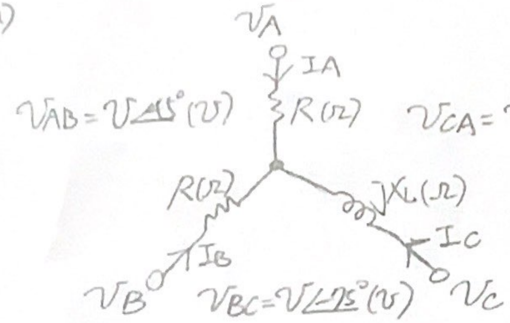


#

$\omega_H = \alpha + \sqrt{\alpha^2 + \omega_0^2} = 1000 + 1000\sqrt{2} = 2414.214\,(\text{rad/s})$ #

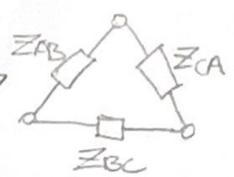
$\omega_L = -\alpha + \sqrt{\alpha^2 + \omega_0^2} = -1000 + 1000\sqrt{2} = 414.214\,(\text{rad/s})$ #

3.(a)



#

(b).

⇒ 

∵ $R = |jX_L|$ ∴ 令 $X_L = R$

→ $R \cdot R + R \cdot jR + R \cdot jR = R^2(1+j2)$

→ $Z_{AB} = \frac{R^2(1+j2)}{jR} = R(2-j1)\,(\Omega)$, $Z_{BC} = Z_{CA} = \frac{R^2(1+j2)}{R} = R(1+j2)\,(\Omega)$

$$\begin{cases} I_{AB} = \frac{V\angle 45^\circ}{R(2-j1)}\,(A) \\ I_{BC} = \frac{V\angle -75^\circ}{R(1+j2)}\,(A) \\ I_{CA} = \frac{V\angle 165^\circ}{R(1+j2)}\,(A) \end{cases} \rightarrow \begin{cases} I_A = I_{AB} - I_{CA} = 0.231\frac{V}{R}\angle -3.435^\circ\,(A) \\ I_B = I_{BC} - I_{AB} = 0.864\frac{V}{R}\angle -123.435^\circ\,(A) \\ I_C = I_{CA} - I_{BC} = 0.775\frac{V}{R}\angle 71.565^\circ\,(A) \end{cases}$$

$S = \frac{1}{2}\left(|I_A|^2 R + |I_B|^2 R + |I_C|^2 jR\right) = \frac{V^2}{R}\cdot(0.4 + j0.3)\,(VA)$ #

∵ $I_N = I_A + I_B + I_C = I_{AB} - I_{CA} + I_{BC} - I_{AB} + I_{CA} - I_{BC} = 0\,(A)$

∴ The neutral current is zero #.

(c) $S = V^2(0.4 + j0.3)$

⇒ $P.F_{old} = 0.8$, lagging

∴ $Q_{new} = \pm 0.4V^2 \tan[\cos^{-1}(0.98)]$

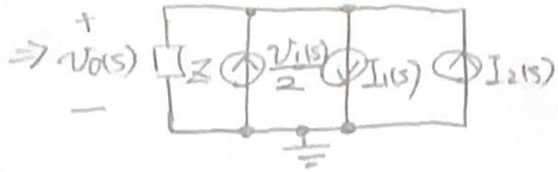
$= \pm 0.081V^2\,(VAR)$

∴ $Q_{need} = |Q_{new} - Q_{old}| = |\pm 0.081V^2 - 0.3V^2|$

$= 0.381V^2$ or $0.219V^2\,(VAR)$ #

Advantage: Improve voltage, Reduce line loss, Increase the life of equipment, Reduce elctricity bills ... etc #

4.



$Z = 2 // 2 // \frac{1}{s} = \frac{1}{s+1}$

$V_0(s) = \left[\frac{V_1(s)}{2} - I_1(s) + I_2(s)\right] \cdot Z$

令 $V_{1s}(t) = \sin[3(t - \frac{\pi}{6})]u(t-6\pi)$

$\mathcal{L}\{V_{1s}(t)\} = e^{-6\pi s}\mathcal{L}\{\sin[3(t+6\pi-\frac{\pi}{6})]\} = e^{-6\pi s}\mathcal{L}\{\overset{-\cos(3t)}{\cancel{\sin(3t-\frac{\pi}{2})}}\} = -e^{-6\pi s}\frac{s}{s^2+9}$

$\because V_1(t) = V_{1s}(4t) \quad \therefore \mathcal{L}\{V_1(t)\} = -\frac{1}{4}e^{-6\pi\cdot\frac{s}{4}}\frac{\frac{s}{4}}{(\frac{s}{4})^2+9} = -e^{-\frac{3}{2}\pi s}\frac{s}{s^2+144}$

$\mathcal{L}\{I_1(t)\} = \int_s^\infty \frac{4}{s_1^2+4^2}ds_1 = \theta\Big|_{\tan^{-1}(\frac{s}{4})}^{\frac{\pi}{2}} = \frac{\pi}{2} - \tan^{-1}(\frac{s}{4})$ （令 $s_1 = 4\tan\theta$ 用三角代换）

$\mathcal{L}\{I_2(t)\} = e^{-10s}\mathcal{L}\{\frac{t}{4}\sin(2t)u(t)\} = -\frac{e^{-10s}}{4}\cdot\frac{d}{ds}\left(\frac{2}{s^2+4}\right) = \frac{se^{-10s}}{(s^2+4)^2}$

$\therefore V_0(s) = \frac{-se^{-\frac{3}{2}\pi s}}{2(s+1)(s^2+144)} - \frac{\frac{\pi}{2}-\tan^{-1}(\frac{s}{4})}{s+1} + \frac{se^{-10s}}{(s+1)(s^2+4)^2} = A(s) - B(s) + C(s)$

$A(s) = e^{-\frac{3}{2}\pi s}\left(\frac{-1/290}{s+1} + \frac{-1}{290}\cdot\frac{s+12\cdot12}{s^2+144}\right)$

$\therefore \mathcal{L}^{-1}\{A(s)\} = \left\{\frac{-e^{\frac{3}{2}\pi}}{290}e^{-t} - \frac{1}{290}\{\cos[12(t-\frac{3}{2}\pi)] + 12\sin[12(t-\frac{3}{2}\pi)]\}\right\}u(t-\frac{3}{2}\pi)$

$\mathcal{L}^{-1}\{B(s)\} = e^{-t}\cdot\int_0^t \mathcal{L}^{-1}\{\frac{\pi}{2} - \tan^{-1}(\frac{s-1}{4})\}dt = e^{-t}\int_0^t \frac{1}{\tau}\mathcal{L}^{-1}\{\frac{d}{ds}[\tan^{-1}(\frac{s-1}{4}) - \frac{\pi}{2}]\}dt$

$= e^{-t}\int_0^t \frac{1}{\tau}\mathcal{L}^{-1}\{\frac{4}{(s-1)^2+16}\}dt = e^{-t}\int_0^t \frac{e^\tau\sin(4\tau)}{\tau}u(\tau)dt$

$\mathcal{L}^{-1}\{C(s)\} = \mathcal{L}^{-1}\left\{e^{-10s}\left[\frac{-1/25}{s+1} + \frac{(s-1)/25}{s^2+4} + \frac{s/5}{(s^2+4)^2} + \frac{4/5}{(s^2+4)^2}\right]\right\}$

$= \left\{\frac{-e^{10}}{25}e^{-t} + \frac{1}{25}\{\cos[2(t-10)] - \frac{1}{2}\sin[2(t-10)]\} + \frac{t-10}{20}\sin[2(t-10)] + \frac{1}{20}\sin[2(t-10)]\right.$

$\left. - \frac{t-10}{40}\cos[2(t-10)]\right\}u(t-10)$

$\therefore V_0(t) = \mathcal{L}^{-1}\{V_0(s)\} = \mathcal{L}^{-1}\{A(s)\} - \mathcal{L}^{-1}\{B(s)\} + \mathcal{L}^{-1}\{C(s)\}$

$= \left\{\frac{-e^{\frac{3}{2}\pi}}{290}e^{-t} - \frac{1}{290}\{\cos[12(t-\frac{3}{2}\pi)] + 12\sin[12(t-\frac{3}{2}\pi)]\}\right\}u(t-\frac{3}{2}\pi)$

$- e^{-t}\int_0^t \frac{e^\tau\sin(4\tau)}{\tau}u(\tau)dt + \left\{\frac{-e^{10}}{25}e^{-t} + \frac{1}{25}\{\cos[2(t-10)] - \frac{1}{2}\sin[2(t-10)]\}\right.$

$\left. + \frac{t-10}{20}\sin[2(t-10)] + \frac{1}{20}\sin[2(t-10)] - \frac{t-10}{40}\cos[2(t-10)]\right\}u(t-10)$ (V)

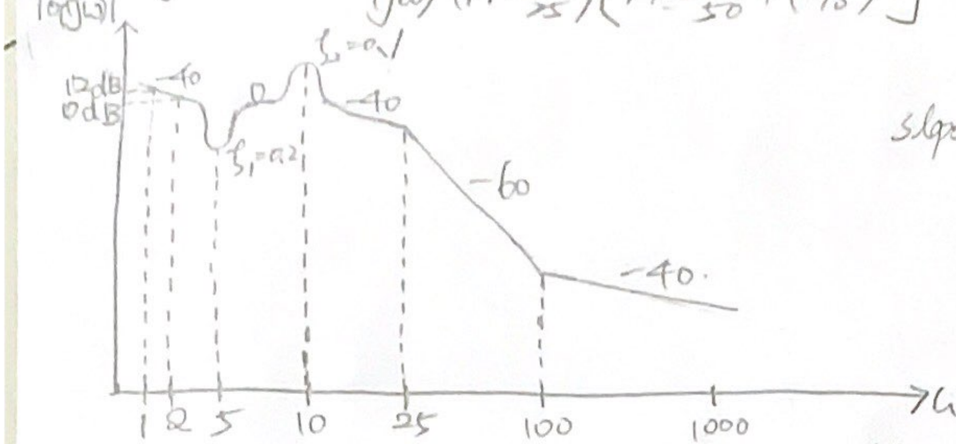
#

5 (a)

$$G(j\omega)=G(s)=\frac{-4\left(1+\frac{j\omega}{100}\right)\left[1+\frac{j\omega}{12.5}+\left(\frac{j\omega}{5}\right)^2\right]}{(j\omega)^2\left(1+\frac{j\omega}{25}\right)\left[1+\frac{j\omega}{50}+\left(\frac{j\omega}{10}\right)^2\right]}$$

$2\zeta_1\cdot\frac{1}{5}=\frac{1}{12.5}$, $\zeta_1=0.2$

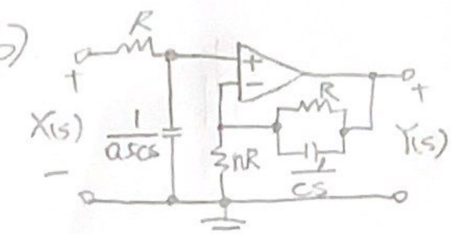
$2\zeta_2\frac{1}{10}=\frac{1}{50}$, $\zeta_2=0.1$



slope unit: dB/decade

#

(b)



$V_- = V_+ = X(s)\cdot\frac{1/0.5Cs}{R+1/0.5Cs} = X(s)\frac{1}{1+0.5RCs}$

$\frac{V_- - 0}{nR} = \frac{Y(s)-V_-}{R\,//\,1/Cs} \rightarrow Y(s) = V_-\cdot\frac{s+\frac{1}{\left(\frac{n}{n+1}\right)RC}}{s+\frac{1}{RC}}$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{1\cdot(1/0.5RC)\left[s+1/\left(\frac{n}{n+1}\right)RC\right]}{\left(s+1/RC\right)\left(s+1/0.5RC\right)}$$

$\therefore$ If $n=1$, this is a 1st order lowpass filter.

If $n\neq1$ & $n>0$, this is a 2nd order lowpass filter. #

(c)

$\left.\frac{Y(j\omega)}{X(j\omega)}\right|_{\omega=0} = 21 = \frac{n+1}{n} \rightarrow$ choose $n=0.05$ #

$\left.\frac{Y(j\omega)}{X(j\omega)}\right|_{\omega=50\cdot2\pi} = \frac{21}{\sqrt{2}}\angle45^\circ$, $\because \frac{1}{RC} < \frac{1}{0.5RC}$ & $\frac{1}{RC} < \frac{1}{\left(\frac{n}{n+1}\right)RC}$

$\therefore \frac{1}{RC}$ is first corner frequency

$\therefore \frac{1}{RC} = 2\pi\cdot50$ $\rightarrow$ choose $\begin{cases} R = 20k\,(\Omega) \\ C = 160n\,(F) \end{cases}$ #

(d) $X(s) = \int_0^\infty t^2\delta(t-2)e^{-st}\,dt = t^2e^{-st}\big|_{t=2} = 4e^{-2s}$

$G(s) = e^{-s}\mathcal{L}\{5u[(t+1)+1]\} = e^{-s}\mathcal{L}\{5u[t-(-2)]\} = \frac{5e^{-s}}{s}$ $\left(\begin{array}{l}\because (-2)<0\text{, 和 } u(t) \text{ 一樣} \\ \therefore \mathcal{L}\{u(t+2)\} = \mathcal{L}\{u(t)\} = \frac{1}{s}\end{array}\right)$

$R\cdot C = 1m\cdot10k = 10$, $n=1$

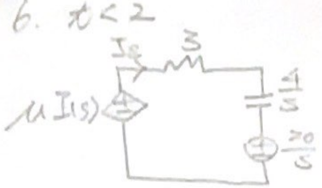
$\therefore Z(s) = 4e^{-2s}\cdot\frac{1}{s}\cdot\frac{1}{s+0.1}\cdot\frac{5e^{-s}}{s} = \frac{4e^{-3s}}{s(s+0.1)}$

$= 4e^{-3s}\cdot\left(\frac{10}{s}+\frac{-10}{s+0.1}\right)$

$\therefore Z(t) = \mathcal{L}^{-1}\{Z(s)\} = 40\left[1-e^{-0.1(t-3)}\right]u(t-3)\ (V)$

$\therefore Z(3) = 0\ (V)$ #

6. $t<2$



$$\mu I(s) = I_s\left(3+\frac{4}{s}\right) + \frac{20}{s}$$

$$I(s)\left(\mu-3-\frac{4}{s}\right) = \frac{20}{s}$$

$$\rightarrow I(s) = \frac{20}{(\mu-3)s-4} = \frac{20}{\mu-3}\cdot\frac{1}{s-\frac{4}{\mu-3}}$$

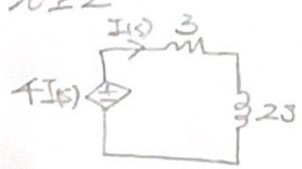
$\because$ The stable circuit means that the pole position must be on the left of the complex plane.

$$\therefore \frac{4}{\mu-3}<0$$

$$\rightarrow \mu-3<0 \rightarrow \mu<3$$

$\therefore$ $\mu<3$ makes the circuit stable for $t<2$. #

$t\geq 2$



$$4I(s) = I(s)\cdot(3+2s)$$

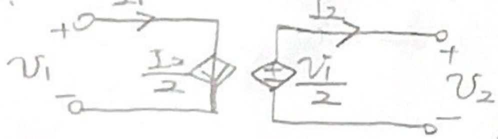
$$\rightarrow I(s)\cdot(1-2s) = 0$$

$$\rightarrow I(s) = 0$$

$$\therefore V_2(s) = 0\cdot 2s = 0$$

$$\rightarrow V_2(t) = \mathcal{L}^{-1}\{V_2(s)\} = 0\ (V)$$ #

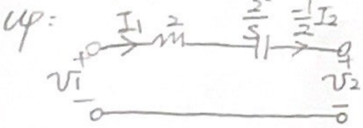
7.(a).



#

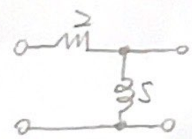
(b) Fig.6(a) is parallel of up and down.

up:



$$\begin{cases} I_1 = \dfrac{V_1}{2+\frac{4}{s}} - \dfrac{2V_2}{2+\frac{4}{s}} \\ I_2 = \dfrac{-2V_1}{2+\frac{4}{s}} + \dfrac{4V_2}{2+\frac{4}{s}} \end{cases} \quad \therefore Y_{up} = \begin{bmatrix} \frac{1}{2+\frac{4}{s}} & \frac{-2}{2+\frac{4}{s}} \\ \frac{-2}{2+\frac{4}{s}} & \frac{4}{2+\frac{4}{s}} \end{bmatrix} (S)$$

down:

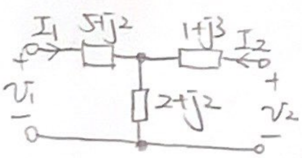


$$\therefore Y_{down} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{1}{2}+\frac{1}{s} \end{bmatrix} (S)$$

$$Y|_{\omega=2} = (Y_{up}+Y_{down})|_{\omega=2} = \begin{bmatrix} \frac{3+j}{4} & \frac{-2-j}{2} \\ \frac{-2-j}{2} & \frac{3+j}{2} \end{bmatrix} (S)$$

Parameter Conversion:

$$Y \text{ to } Z \rightarrow Z_Y = \begin{bmatrix} \frac{Y_{22}}{\Delta} & \frac{-Y_{12}}{\Delta} \\ \frac{-Y_{21}}{\Delta} & \frac{Y_{11}}{\Delta} \end{bmatrix} = \begin{bmatrix} 2+j4 & 1+j3 \\ 1+j3 & 1+j2 \end{bmatrix} (\Omega)$$

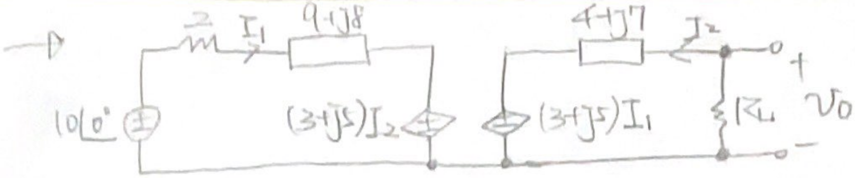


$$\begin{cases} V_1 = I_1(7+j4) + I_2(2+j2) \\ V_2 = I_1(2+j2) + I_2(3+j5) \end{cases} \rightarrow Z_1 = \begin{bmatrix} 7+j4 & 2+j2 \\ 2+j2 & 3+j5 \end{bmatrix} (\Omega)$$

$\because$ Z_1 & Z_Y is in series.

$$\therefore Z_{total} = Z_1 + Z_Y = \begin{bmatrix} 9+j8 & 3+j5 \\ 3+j5 & 4+j7 \end{bmatrix} (\Omega)$$

#



求 V_{oc} ±

$$\begin{cases} 10\angle 0^\circ = (11+j8)I_1 + (3+j5)I_2 \\ I_2 = 0 \text{ 代入上式} \end{cases}$$

$\rightarrow I_1 = 0.735\angle -36.027^\circ$ (A)

$\therefore V_{oc} = (3+j5)I_1 = 4.287\angle 23.009^\circ$ (V)

求 I_{sc} ↓

$$\begin{cases} 10 = (11+j8)I_1 + (3+j5)I_2 \\ (3+j5)I_1 + (4+j7)I_2 = 0 \end{cases}$$

$\rightarrow I_2 = \frac{\Delta I_2}{\Delta} = 0.737\angle 151.935^\circ$ (A)

$\therefore I_{sc} = -I_2 = 0.737\angle -28.065^\circ$ (A)

$\therefore Z_{th} = \frac{V_{oc}}{I_{sc}} = 5.817\angle 51.074^\circ$ (Ω) $\rightarrow R_L = |Z_{th}|$ is $\rightarrow P_{max}$.

$$\therefore P_{max} = \frac{1}{2}\cdot\left|\frac{V_{oc}}{Z_{th}+R_L}\right|^2 \times R_L = 0.485\ (W)$$ #

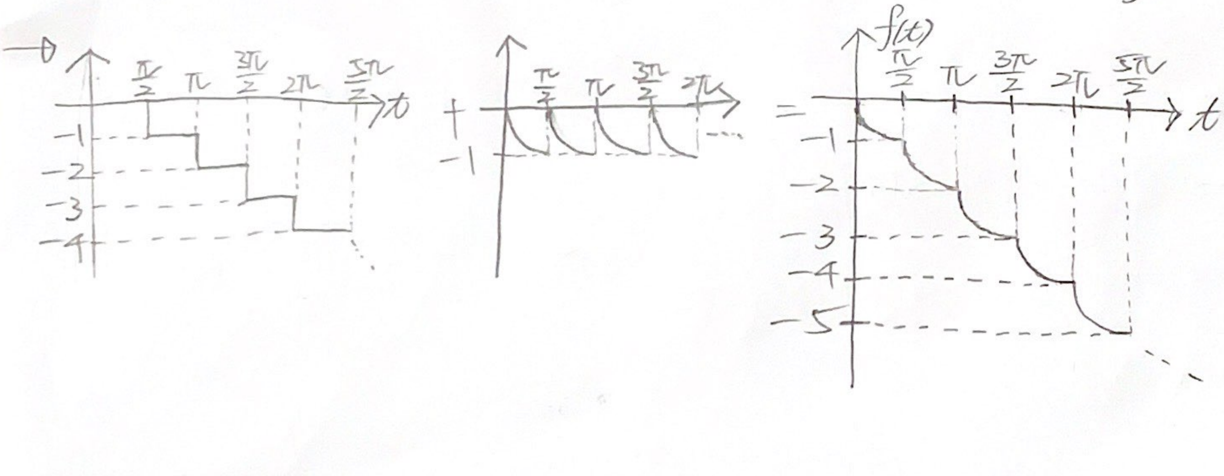
8. $s = j\omega$

$$\therefore F(j\omega) = F(s) = \frac{1}{s(1-e^{\frac{\pi}{2}s})} + \frac{se^{-\frac{\pi}{2}s}-1}{(s^2+1)(1-e^{-\frac{\pi}{2}s})}$$

$$= \frac{-e^{-\frac{\pi}{2}s}}{s}\cdot\frac{1}{1-e^{-\frac{\pi}{2}s}} + \frac{se^{-\frac{\pi}{2}s}-1}{s^2+1}\cdot\frac{1}{1-e^{-\frac{\pi}{2}s}}$$

$$= \left(\frac{-e^{-\frac{\pi}{2}s}}{s} + \frac{se^{-\frac{\pi}{2}s}-1}{s^2+1}\right)\left(1 + e^{-\frac{\pi}{2}s} + e^{-\frac{\pi}{4}s} + e^{-\frac{\pi}{6}s} + \cdots\right)$$

$\therefore \mathcal{L}^{-1}\{F(s)\} = f(t) = [-t\,u(t-\frac{\pi}{2})] + \{\sin t\,u(t-\frac{\pi}{2}) - \sin t\,u(t)\}$, $0 \le t \le \frac{\pi}{2}$, $f(t) = f(t+\frac{\pi}{2})$



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