Calculus (II) Quiz 4 Date: 2023/05/17

Total: 100 points

1. (**10** points) Investigate the convergence of the following sequences. Find the limit of each convergent sequence.
   (a) $a_n = \dfrac{n!}{3^n \cdot 7^n}$
   (b) $a_n = (3^n + 5^n)^{1/n}$

2. (**20** points) Find the Taylor series generated by f at $x = a$.
   (a) $f(x) = \ln(1+x), \quad a = 0$
   (b) $f(x) = e^x, \quad a = 2$

3. (**15** points) Find the area under one arch of the cycloid
   $$x = 5(t - \sin t), \quad y = 5(1 - \cos t).$$

4. (**20** points) A parametric curve $x = f(t) = 2t^2$, $y = g(t) = t^3 - 4t$.
   (a) Find the equation for the line tangent to the curve at the point $Q(2, -3)$.
   (b) At the point $Q(2, -3)$, is the curve concave upward or concave downward?

5. (**20** points) Find the area of the region that lies inside the curve $r = 3\cos\theta$ and outside the curve $r = 1 + \cos\theta$.

6. (**15** points) Find the lengths of the spiral $r = \theta^2$, $0 \le \theta \le \sqrt{5}$.