Total: 100 points

1. (**30** points) Find a cubic function $f(x) = ax^3 + bx^2 + cx + d$ that has a local maximum value of 27 at $x = -2$ and a local minimum value of 0 at $x = 1$.

2. (**20** points) Show that the functions $f(x) = x^3 + \dfrac{4}{x^2} + 7$ has exactly one zero in the interval $(-\infty, 0)$.

3. A function is given as $f(x) = \sqrt{3}x - 2\cos x, \quad 0 \le x \le 2\pi$.

   (a) (**12** points) Identify the coordinates of any local and absolute extreme points.

   (b) (**6** points) Specify on which interval(s), $f(x)$ is increasing, and on which interval(s), $f(x)$ is decreasing.

   (c) (**6** points) Identify the coordinates of inflection points.

   (d) (**6** points) Specify on which interval(s), $f(x)$ is concave up, and on which interval(s), $f(x)$ is concave down.

4. (**20** points) Verify that the function $f(x)$ satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

$$f(x) = \frac{1}{x} \quad \text{interval}: [1,3]$$