B11102123

1. (**15** points) Evaluate the following integrals. (5 points for each)

   (a) $\int x\sqrt{2-5x}\,dx$ (b) $\int \frac{\cos(\pi/x)}{x^2}\,dx$ (c) $\int \frac{1}{\sqrt{x\sqrt{x}+x}}\,dx$

$u = 2-5x$, $du = -5dx$, $x = \frac{2-u}{5}$

$-\frac{1}{25}\left(-\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}\cdot 2\cdot u^{\frac{3}{2}}\right) + C$

$\frac{2}{125}u^{\frac{5}{2}} - \frac{4}{75}u^{\frac{3}{2}} + C$

$u = \frac{\pi}{x}$, $du = -\frac{\pi}{x^2}dx$, $-\frac{1}{\pi}\int \cos u\,du$, $-\frac{\sin u}{\pi} + C$

$\int \frac{1}{\sqrt{x}\cdot\sqrt{\sqrt{x}+1}}\,dx$, $\sqrt{x}+1 = u$, $du = \frac{1}{2\sqrt{x}}dx$, $dx = 2\sqrt{x}\,du$, $2\int \frac{1}{\sqrt{u}}\,du$, $2\cdot 2\cdot u^{\frac{1}{2}} + C$, $4u^{\frac{1}{2}} + C$

2. (**10** points) The radius of a circular disk is given as 24 cm with a maximum error in measurement of 0.2 cm. Use differentials to estimate maximum error and the relative error in the calculated area of the disk.

48, 9.6π

3. Let $f(x) = \int_0^{g(x)} \frac{1}{\sqrt{1+t^3}}\,dt$, where $g(x) = \int_0^{\cos x} \left[1+\sin(t^2)\right]\,dt$.

   (a) (**2** points) Find $g(\frac{\pi}{2})$

   (b) (**4** points) Find $g'(x)$

   (c) (**4** points) Find $f'(\frac{\pi}{2})$

4. (**10** points) Find the volume of the solid generated by revolving the plane region enclosed by $y = 2x - x^2$ and $y = 0$ about the line $x = 4$.

$x\cdot(2-x)$

5. (**10** points) Prove that the equation $3x + 1 - \sin x = 0$ has exactly one real solution.

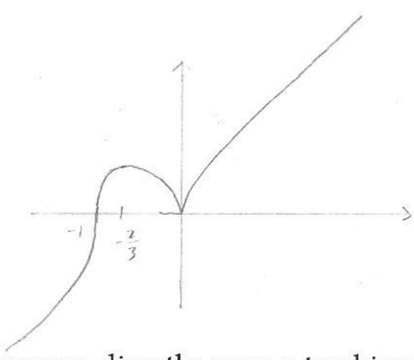
6. Let $f(x) = (x^3 + x^2)^{1/3}$

   (a) (**6** points) Find the intervals of increase or decrease.

   (b) (**6** points) Find the intervals of concavity.

   (c) (**2** points) Find the local maximum and minimum values.

   (d) (**1** points) Find the inflection points.



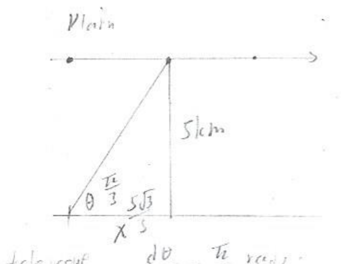
$f'(x) = \frac{1}{3}(x^3+x^2)^{-\frac{2}{3}}\cdot(3x^2+2x)$, $\frac{3x^2+2x}{3\cdot(x^3+x^2)^{\frac{2}{3}}}$, $\frac{x\cdot(3x+2)}{3\cdot\sqrt[3]{(x^3+x^2)^2}}$

$f'(x) = 0$, $x = -\frac{2}{3}$

$f'(x)$ not exist, $x = 0$ or $x = -1$

7. (**10** points) A plane flies horizontally at an altitude of 5 km and passes directly over a tracking telescope on the ground. When the angle of elevation is $\frac{\pi}{3}$, this angle is decreasing at a rate of $\frac{\pi}{6}$ rad/min. How fast is the plane traveling at that time?

(Questions on both sides of the paper.)



$\frac{d\theta}{dt} = \frac{\pi}{6}$ rad/min

$\frac{5}{x} = \tan\theta$

$x = \frac{5}{\tan\theta}$ $\Rightarrow$ $\frac{dx}{dt} = \frac{0 - 5\sec^2\theta\,\frac{d\theta}{dt}}{(\tan\theta)^2}$

$= -5\,\frac{\sec^2\theta}{\tan^2\theta}\cdot\frac{d\theta}{dt}$

$= -5\cdot\frac{1}{\cos^2\theta\tan^2\theta}\cdot\frac{d\theta}{dt}$

$\frac{dx}{dt} = V$

$V = -5\cdot\frac{1}{\cos^2\frac{\pi}{3}\cdot\tan^2\frac{\pi}{3}}\cdot -\frac{\pi}{6}$

$= \frac{5}{6}\pi\cdot\frac{1}{\frac{1}{4}\cdot 3}$

$= \frac{5}{6}\pi\cdot\frac{4}{3}$

$= \frac{10}{9}\pi$ km/min

$\frac{2}{3}y^{-\frac{1}{3}}\frac{dy}{dx} = -\frac{2}{3}x^{-\frac{1}{3}}$ $\frac{dy}{dx} = -(\frac{x}{y})^{-\frac{1}{3}} = -(\frac{y}{x})^{\frac{1}{3}}$

8. A curve $C: x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ on xy-plane. There is a point $P(\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4})$ on the curve C.

   (a) (**5** points) Find the lines that are tangent to the curve C at the point P.

   (b) (**5** points) Find the lines that are normal to the curve C at the point P.

   (c) (**10** points) Find the arc length of the curve C from the point $P(\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4})$ to the point $Q(1, 0)$ on the curve C.

$y^{\frac{2}{3}} = 1 - x^{\frac{2}{3}}$ $y = (1-x^{\frac{2}{3}})^{\frac{3}{2}}$ $\frac{dy}{dx} = \frac{3}{2}(1-x^{\frac{2}{3}})^{\frac{1}{2}} \cdot -\frac{2}{3}x^{-\frac{1}{3}} = -\frac{(1-x^{\frac{2}{3}})^{\frac{1}{2}}}{\sqrt[3]{x}}$ $\int_{\frac{\sqrt{2}}{4}}^{1}\sqrt{1+(\frac{dy}{dx})^2}\,dx = \int_{\frac{\sqrt{2}}{4}}^{1}\sqrt{1+\frac{1-x^{\frac{2}{3}}}{x^{\frac{2}{3}}}} = \int_{\frac{\sqrt{2}}{4}}^{1}\sqrt{\frac{1}{x^{\frac{2}{3}}}}\,dx$

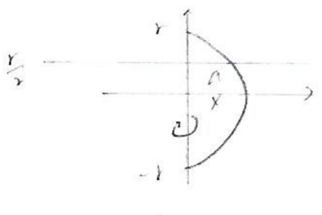
9. (**10** points) If a resistor of R ohms is connected across a battery of E volts with internal resistance r ohms, then the power (in watts) in the external resistor is

$$P = \frac{E^2R}{(R+r)^2}$$

$\frac{E^2}{4r}$

If E and r are fixed but R varies, what is the maximum value of the power?

10. (**10** points) A curve $x = \sqrt{r^2 - y^2}$, $0 \le y \le r/2$ is rotated about y-axis. Please find the area of the resulting surface.



$x = (r^2 - y^2)^{\frac{1}{2}}$

$\frac{dx}{dy} = \frac{1}{2}(r^2-y^2)^{-\frac{1}{2}} \cdot -2y = -y \cdot \frac{1}{\sqrt{r^2-y^2}}$

$ds = \sqrt{1+\frac{y^2}{r^2-y^2}}\,dy$

$A = \int 2\pi r\,ds$

$= \int 2\pi x\,ds$

$= 2\pi\sqrt{r^2-y^2}\cdot\sqrt{1+\frac{y^2}{r^2-y^2}}\,dy$

$= 2\pi\int_0^{\frac{r}{2}}\sqrt{r^2-y^2+y^2}\,dy$

$= 2\pi\int_0^{\frac{r}{2}} r\,dy$

$= 2\pi r y\Big|_0^{\frac{r}{2}}$

$= 2\pi \cdot r \cdot \frac{r}{2}$

$= \pi r^2$