Analog Integrated Circuit Design and Applications

Spring 2023

Nonlinearity and Mismatch

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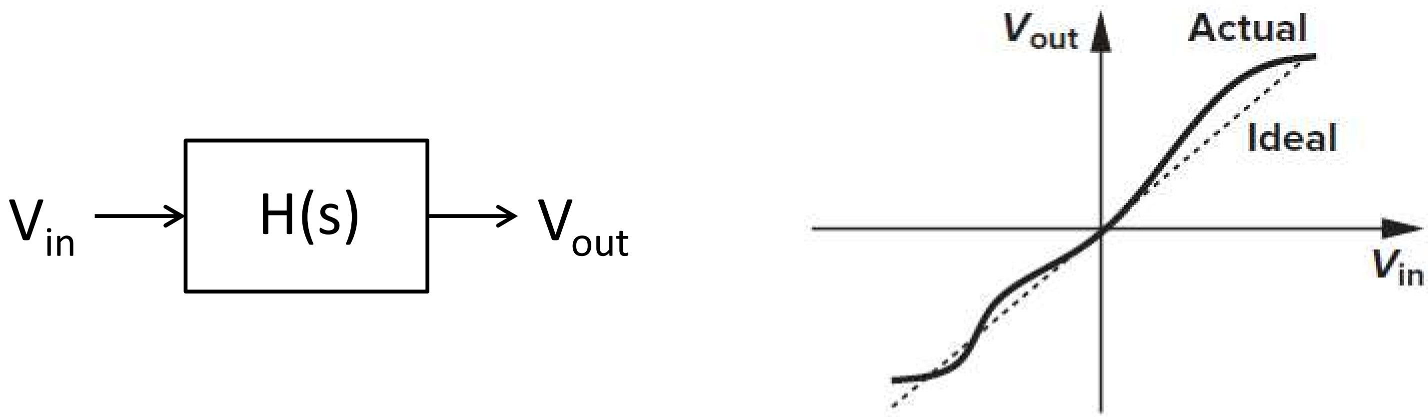
Refer to ch. 14, Razavi's Design of Analog CMOS ICs, 2nd ed.

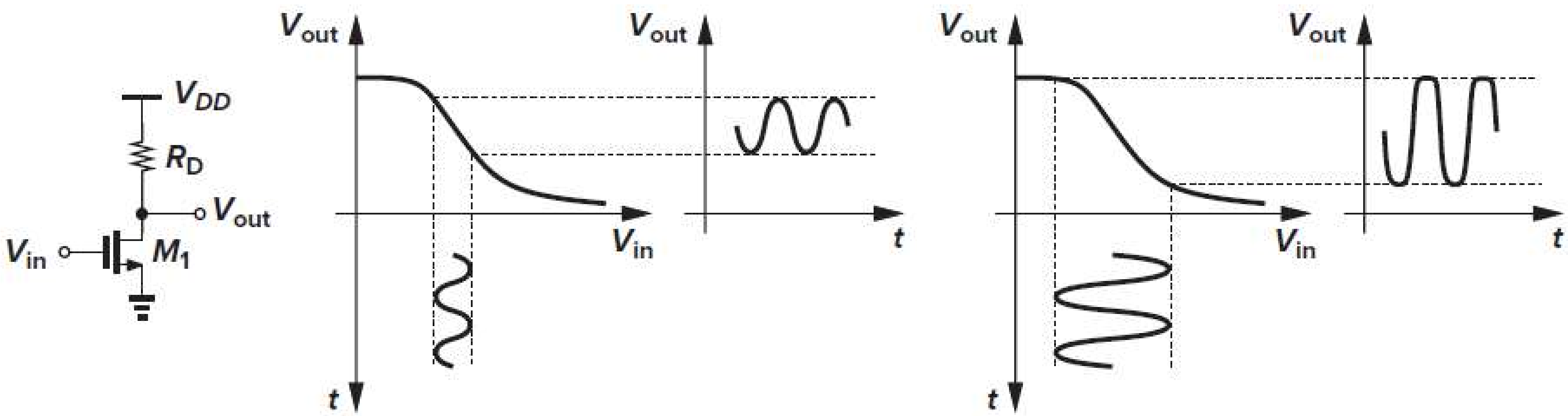
Outline

- Nonlinearity
  - THD: Total Harmonic Distortion
  - Linearization Techniques
- Mismatch
  - Random Mismatch
  - Systematic Mismatch
  - Input-Referred Offset
  - Current Mirror Deviation
  - THD Degradation
- Noise

Nonlinearity

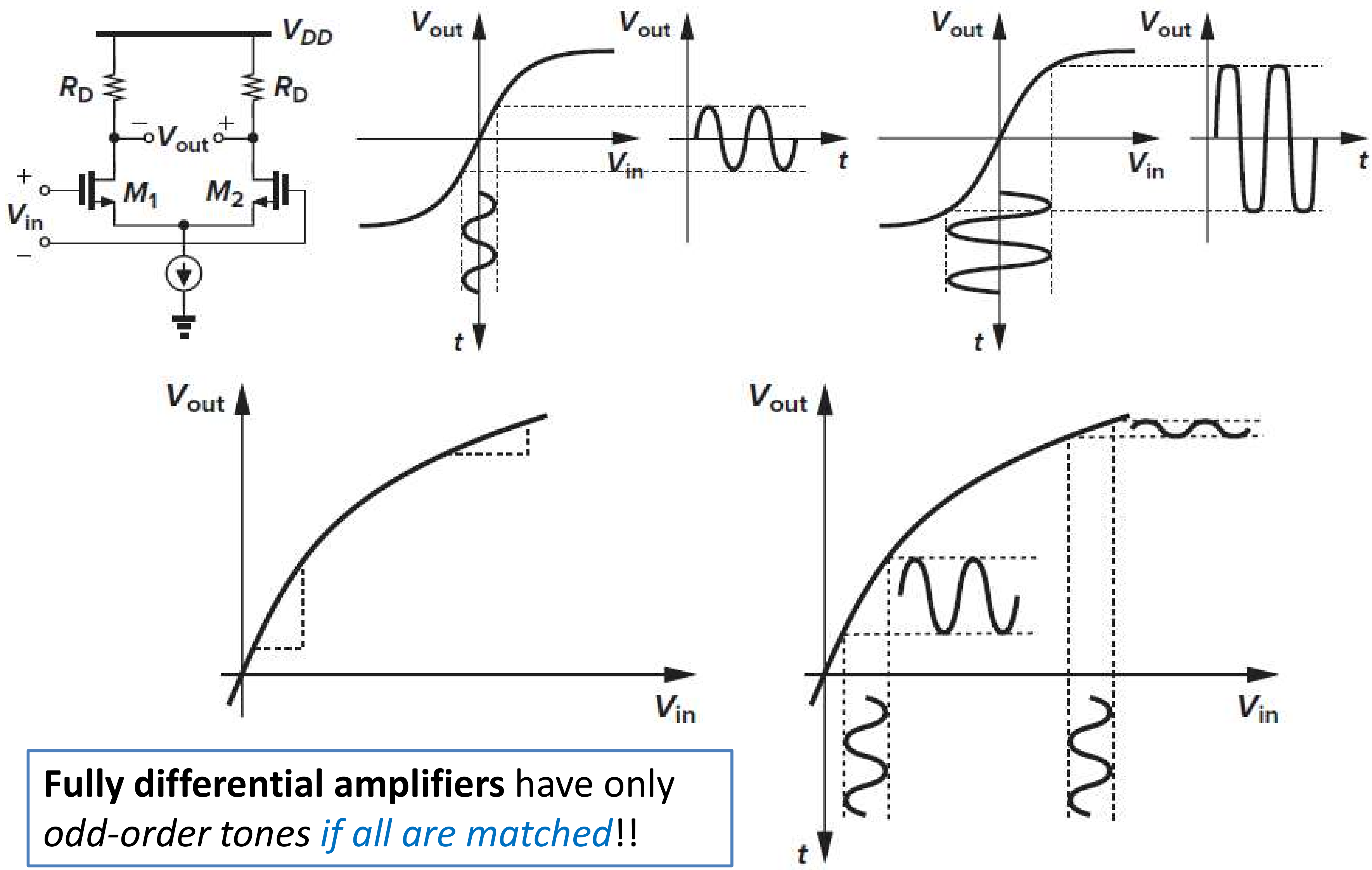


Single-ended amplifier



- Nonlinearity: the amplifier gain is not constant but varies with the input amplitude

Nonlinearity



Fully differential amplifiers have only *odd-order tones if all are matched*!!

Total Harmonic Distortion (THD)

$$\boxed{y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \cdots}$$

Input: $x(t) = A\cos\omega t$

$$y(t) = \alpha_1 A\cos\omega t + \alpha_2 A^2\cos^2\omega t + \alpha_3\cos^3\omega t + \cdots$$

$$= \underline{\alpha_1 A}\cos\omega t + \frac{\alpha_2 A^2}{2}[1+\cos(2\omega t)] + \underline{\frac{\alpha_3 A^3}{4}[3\cos\omega t}+\cos(3\omega t)] + \cdots$$

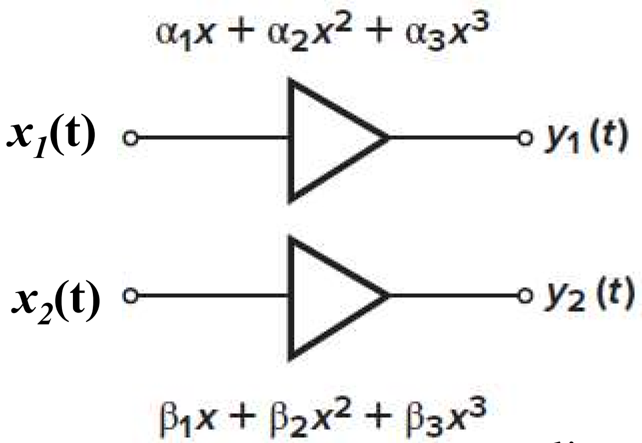
$$= \frac{\alpha_2 A^2}{2} + \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right)\cos\omega t + \frac{\alpha_2 A^2}{2}\cos 2\omega t + \frac{\alpha_3 A^3}{4}\cos 3\omega t$$

If considering only HD2 and HD3, THD = HD2+HD3 is

Single-ended: $$\boxed{\mathrm{THD} = \frac{(\alpha_2 A^2/2)^2 + (\alpha_3 A^3/4)^2}{(\alpha_1 A + 3\alpha_3 A^3/4)^2}}$$

Harmonic distortion is undesirable in most signal processing applications, including audio and video systems. High-quality audio products such as compact disc (CD) players require a THD of about 0.01% (−80 dB), and video products, about 0.1% (−60 dB).

Distortion of Differential Amplifiers



$$y_1 - y_2 = (\alpha_1 x_1 - \beta_1 x_2) + (\alpha_2 x_1^2 - \beta_2 x_2^2) + (\alpha_3 x_1^3 - \beta_3 x_2^3)$$

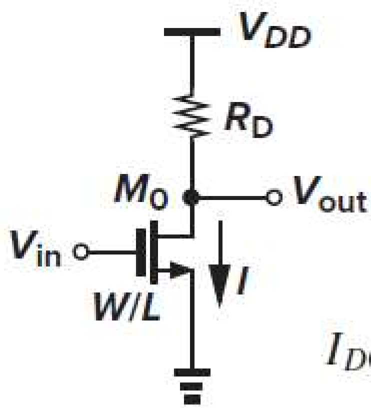
Assume $x_1 = x$ and $x_2 = -x$,

$$y_d = y_1 - y_2 = (\alpha_1 + \beta_1)x + (\alpha_2 - \beta_2)x^2 + (\alpha_3 + \beta_3)x^3$$

$$\text{if } \alpha_i = \beta_i,\ \forall i \ \Rightarrow\ \underline{y_d = 2\alpha_1 x + 2\alpha_3 x^3}$$

- Single-ended signals have tones of 2nd, 3rd, 4th, …
- Differential signals only have odd-order tones, ideally
- However, the **mismatch** causes even-order tones in differential signals
- In addition, **thermal gradient** also contributes mismatch

Harmonic Distortion: Single-ended



$$|A_v| \approx g_m R_D$$

$$= \mu_n C_{ox} \frac{W}{L} (\underline{V_{GS}} - V_{TH}) R_D$$

DC operating point

$$I_{D0} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH} + \underline{V_m\, cos\omega t})^2$$

Input signal

$$= \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 + \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) V_m \cos \omega t$$

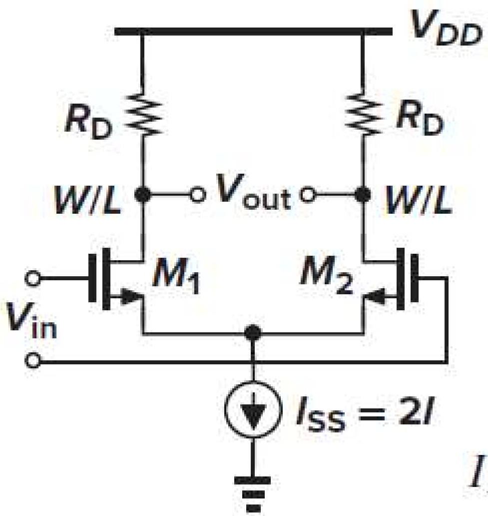
$$+ \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_m^2 \cos^2 \omega t$$

$$= I + \underbrace{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) V_m}_{A_F} \cos \omega t + \underbrace{\frac{1}{4} \mu_n C_{ox} \frac{W}{L} V_m^2}_{A_{HD2}} [1 + \cos(2\omega t)]$$

$$\boxed{\frac{A_{HD2}}{A_F} = \frac{V_m}{4(V_{GS} - V_{TH})}}$$

(A very simple version)

Harmonic Distortion: Differential Pair



$$I_{D1} - I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{in}\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - V_{in}^2}$$

(推導見下頁)

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{in}\sqrt{4(V_{GS}-V_{TH})^2 - V_{in}^2}$$

If $|V_{in}| \ll V_{GS} - V_{TH}$, then

$$I_{D1} - I_{D2} = \mu_n C_{ox}\frac{W}{L}V_{in}(V_{GS}-V_{TH})\sqrt{1 - \frac{V_{in}^2}{4(V_{GS}-V_{TH})^2}}$$

$$\approx \mu_n C_{ox}\frac{W}{L}V_{in}(V_{GS}-V_{TH})\left[1 - \frac{V_{in}^2}{8(V_{GS}-V_{TH})^2}\right]$$

$$= \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})\left[V_m \cos\omega t - \frac{V_m^3 \cos^3 \omega t}{8(V_{GS}-V_{TH})^2}\right]$$

$\cos^3 \omega t = [3\cos\omega t + \cos(3\omega t)]/4,$

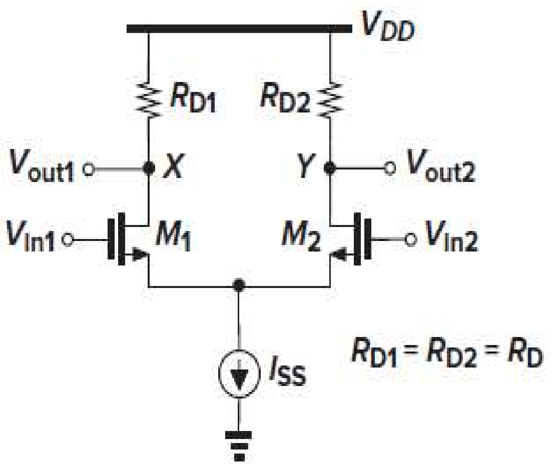
$$I_{D1} - I_{D2} = g_m\left[V_m - \frac{3V_m^3}{32(V_{GS}-V_{TH})^2}\right]\cos\omega t - g_m\frac{V_m^3\cos(3\omega t)}{32(V_{GS}-V_{TH})^2}$$

If $V_m \gg 3V_m^3/[8(V_{GS}-V_{TH})^2]$, then

$$\boxed{\frac{A_{HD3}}{A_F} \approx \frac{V_m^2}{32(V_{GS}-V_{TH})^2}}$$

Harmonic Distortion: Differential Pair

g_m vs ΔV_{in} 推導之補充



$$(V_{GS}-V_{TH})^2=\frac{I_D}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}$$

$$V_{GS}=\sqrt{\frac{2I_D}{\mu_n C_{ox}\frac{W}{L}}}+V_{TH}$$

$$V_{in1}-V_{in2}=\sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\frac{W}{L}}}-\sqrt{\frac{2I_{D2}}{\mu_n C_{ox}\frac{W}{L}}}$$

$$\underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}_{=K_n}(V_{in1}-V_{in2})^2-I_{SS}=-2\sqrt{I_{D1}I_{D2}}$$

$$\begin{cases} 4I_{D1}I_{D2}=I_{SS}^2-2I_{SS}K_n\Delta V_{in}^2+K_n^2\Delta V_{in}^4,\ \Delta V_{in}=V_{in1}-V_{in2} \\ 4I_{D1}I_{D2}=(I_{D1}+I_{D2})^2-(I_{D1}-I_{D2})^2=I_{SS}^2-(I_{D1}-I_{D2})^2 \end{cases}$$

$$\begin{aligned}(I_{D1}-I_{D2})^2&=I_{SS}^2-4I_{D1}I_{D2}\\&=-K_n^2\Delta V_{in}^4+2I_{SS}K_n\Delta V_{in}^2\end{aligned}$$

$$(I_{D1}-I_{D2})^2=-\frac{1}{4}\left(\mu_n C_{ox}\frac{W}{L}\right)^2(V_{in1}-V_{in2})^4+I_{SS}\mu_n C_{ox}\frac{W}{L}(V_{in1}-V_{in2})^2$$

$$I_{D1}-I_{D2}=\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1}-V_{in2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}-(V_{in1}-V_{in2})^2}$$

$$=\sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}}(V_{in1}-V_{in2})\sqrt{1-\frac{\mu_n C_{ox}(W/L)}{4I_{SS}}(V_{in1}-V_{in2})^2}$$

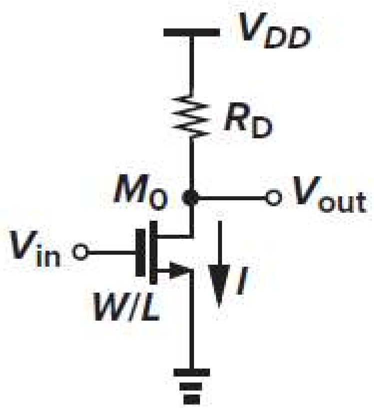
求 g_m

$$\boxed{\frac{\partial\Delta I_D}{\partial\Delta V_{in}}=\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\frac{\frac{4I_{SS}}{\mu_n C_{ox}W/L}-2\Delta V_{in}^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}W/L}-\Delta V_{in}^2}}}$$

Harmonic Distortion

if $V_m = 0.2(V_{GS} - V_{TH})$,

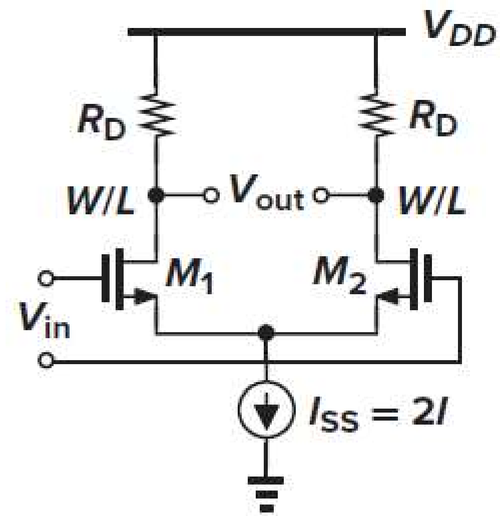
Single-ended



$$\frac{A_{HD2}}{A_F} = \frac{V_m}{4(V_{GS} - V_{TH})}$$

$= 5\%$

Fully differential

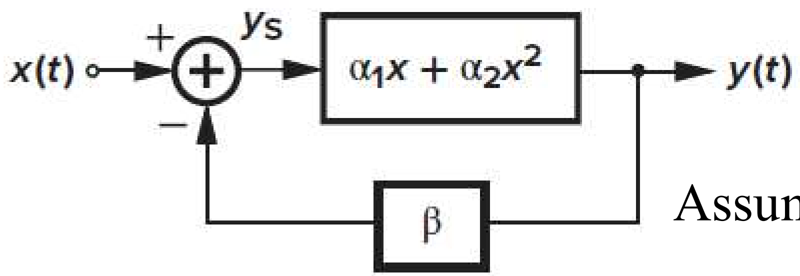


$$\frac{A_{HD3}}{A_F} \approx \frac{V_m^2}{32(V_{GS} - V_{TH})^2}$$

$= 0.125\%$

Keep in mind: How to maintain the advantages of fully differential amplifiers?

Feedback on THD



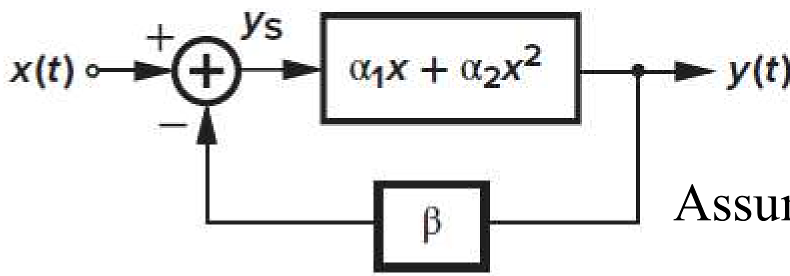
Assume $y(t)=a\cdot cos\omega t + b\cdot cos2\omega t$

$$
\begin{aligned}
y_S &= x(t) - \beta y(t) \\
&= V_m \cos\omega t - \beta(a\cos\omega t + b\cos 2\omega t) \\
&= (V_m - \beta a)\cos\omega t - \beta b\cos 2\omega t
\end{aligned}
$$

$$
\begin{aligned}
y(t) &= \alpha_1[(V_m - \beta a)\cos\omega t - \beta b\cos 2\omega t] \\
&\quad + \alpha_2[(V_m - \beta a)\cos\omega t - \beta b\cos 2\omega t]^2 \\
&= [\alpha_1(V_m - \beta a) - \alpha_2(V_m - \beta a)\beta b]\cos\omega t \\
&\quad + \left[-\alpha_1\beta b + \frac{\alpha_2(V_m - \beta a)^2}{2}\right]\cos 2\omega t + \cdots
\end{aligned}
$$

三角函數

Feedback on THD



Assume $y(t)=a\cdot cos\omega t + b\cdot cos2\omega t$

$$y(t) = a\cos(\omega t) + b\cos(2\omega t)$$

$$a = (\alpha_1 - \alpha_2\beta b)(V_m - \beta a)$$

$$b = -\alpha_1\beta b + \frac{\alpha_2(V_m - \beta a)^2}{2}$$

$$a \approx \alpha_1(V_m - \beta a)$$ If α_2 and b are small

$$a = \frac{\alpha_1}{1+\beta\alpha_1}V_m$$

To calculate *b*, we write

$$V_m - \beta a \approx \frac{a}{\alpha_1}$$

$$b = -\alpha_1\beta b + \frac{1}{2}\alpha_2\left(\frac{a}{\alpha_1}\right)^2$$

$$b(1+\alpha_1\beta) = \frac{\alpha_2}{2}\left(\frac{a}{\alpha_1}\right)^2$$

$$= \frac{\alpha_2}{2\alpha_1^2}\frac{\alpha_1^2}{(1+\beta\alpha_1)^2}V_m^2$$

$$b = \frac{\alpha_2 V_m^2}{2}\frac{1}{(1+\beta\alpha_1)^3}$$

$$\frac{b}{a} = \frac{\alpha_2 V_m}{2}\frac{1}{\alpha_1}\frac{1}{(1+\beta\alpha_1)^2}$$

Improved by feedback

Linearization Technique (1)

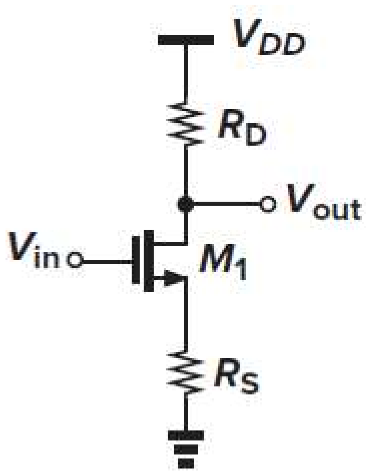
A common-source stage biased at a current I_1 experiences an input voltage swing that varies the drain current from $0.75I_1$ to $1.25I_1$. Calculate the variation of the small-signal voltage gain (a) with no degeneration and (b) with degeneration such that $g_m R_S = 2$, where g_m denotes the transconductance at $I_D = I_1$.

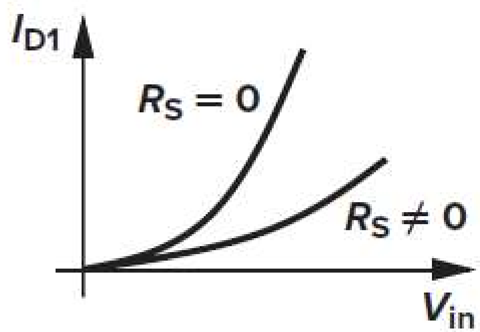
[Solution]

Assuming square-law behavior, we have $g_m \propto \sqrt{I_D}$. For the case of no degeneration,

$$\frac{g_{m,high}}{g_{m,low}} = \sqrt{\frac{1.25}{0.75}}$$

With $g_m R_S = 2$, (actually, too small)





$$G_m = \frac{g_m}{1 + g_m R_S}$$

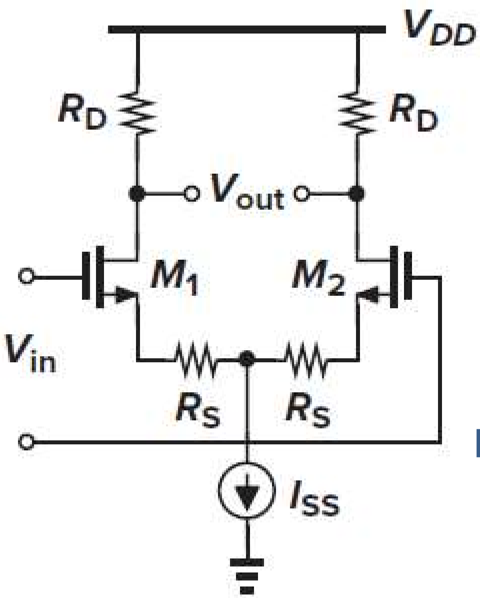
$$\frac{G_{m,high}}{G_{m,low}} = \frac{\dfrac{\sqrt{1.25}g_m}{1+\sqrt{1.25}g_m R_S}}{\dfrac{\sqrt{0.75}g_m}{1+\sqrt{0.75}g_m R_S}}$$

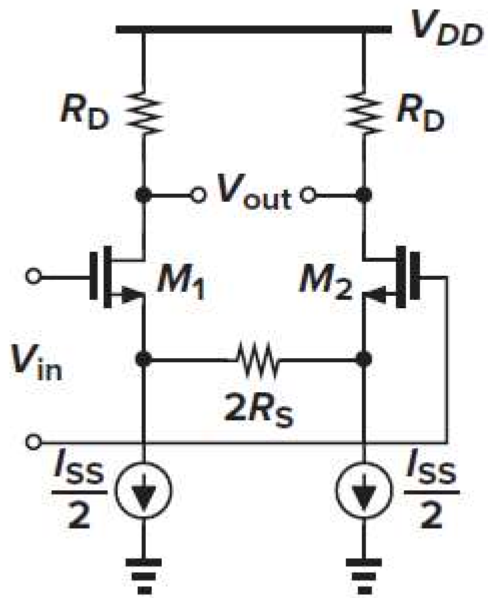
$$= \sqrt{\frac{1.25}{0.75}} \cdot \frac{1+2\sqrt{0.75}}{1+2\sqrt{1.25}}$$

$$= 0.84\sqrt{\frac{1.25}{0.75}}$$

*Gain variation: $(G_{m,high}-G_m)/G_m$ & $(G_{m,low}-G_m)/G_m$

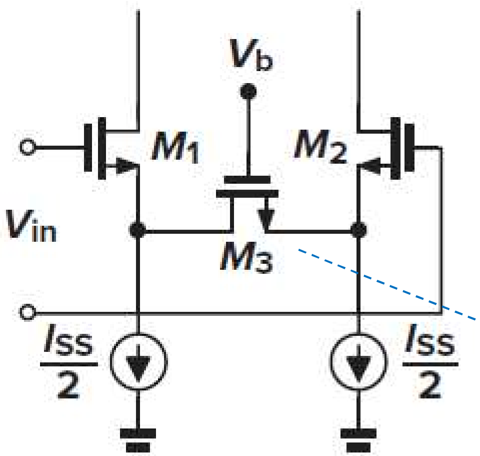
SCP Degeneration





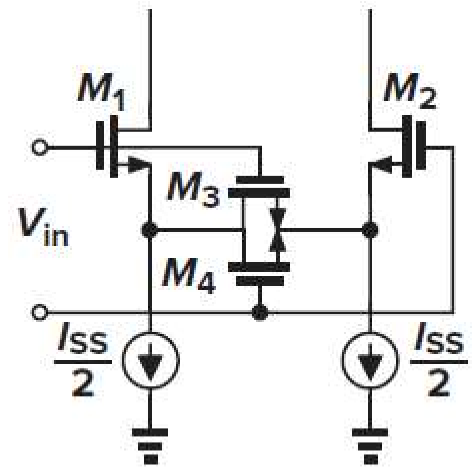
SCP: Source Couple Pair

$$G_m = \frac{g_m}{1 + g_m R_S}$$

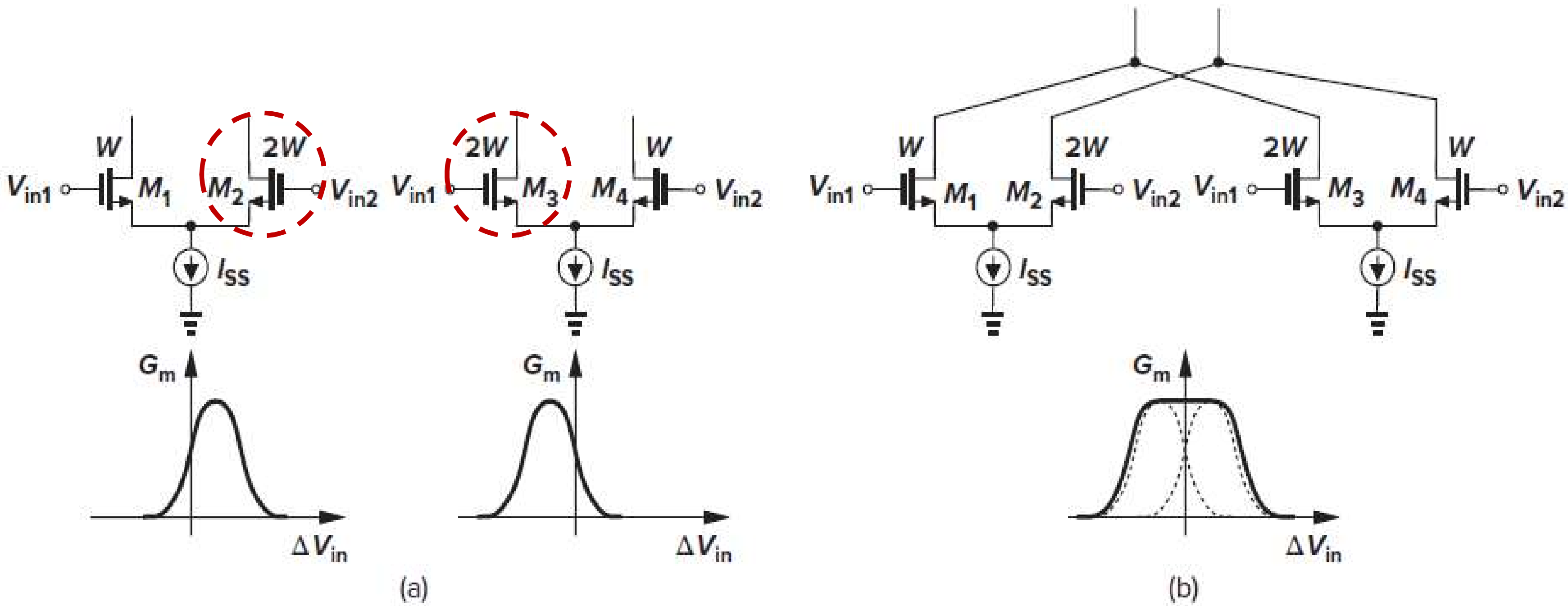


MOSFET operating in the deep triode region

Vb is applied as an input–tracking control



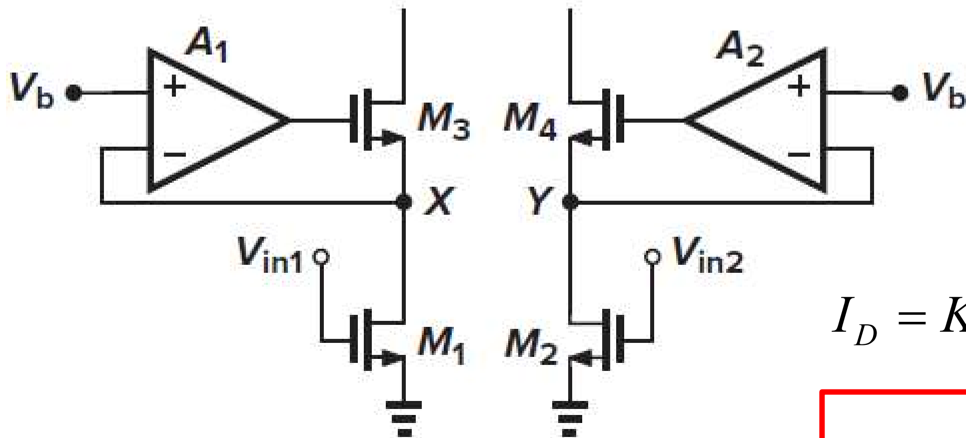
Linearization Technique (2)



- Shift g_m by changing the input pair into unbalance conditions: left-shift and right-shift
- The composite final g_m has a flat shape if the optimal design is achieved
- Issue is the g_m flatness is PVT-sensitive

Linearization Technique (3)

Differential pair using input devices operating in the triode region



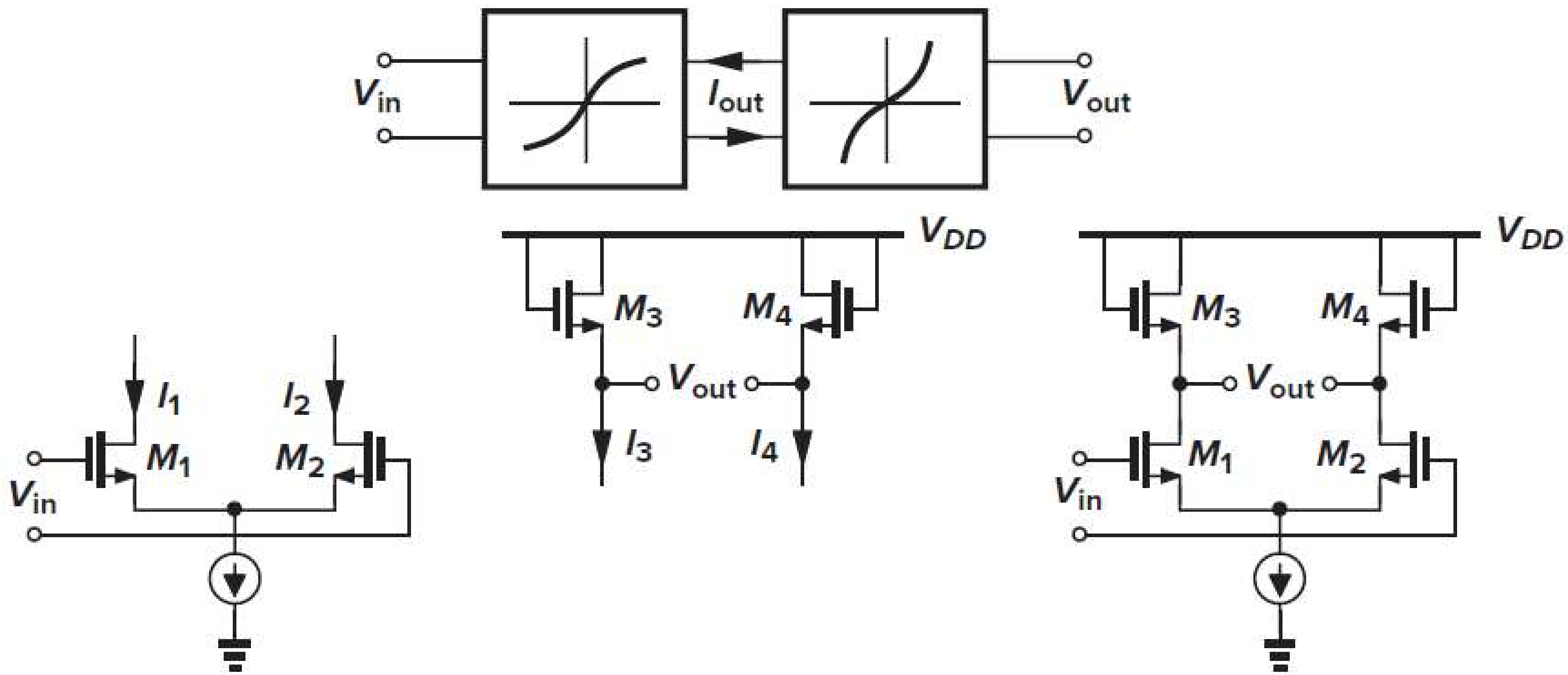
$$I_D = K'_N\left(\frac{W}{L}\right)\left[2\left(V_{GS}-V_{TH}\right)V_{DS}-V_{DS}{}^2\right]$$

$$\boxed{g_m = \frac{\partial I_D}{\partial V_{GS}} = 2K'_N\left(\frac{W}{L}\right)V_{DS}}$$

Issues:

- V_{DS} must be low enough to ensure that each input transistor remains in the triode region
- The input common-mode level must be tightly controlled, and it must track V_b so as to define I_{D1} and I_{D2}
- Additional noise contribution from M3, M4, and two amplifiers
- g_m may be too small since V_{DS} is small

Linearization Technique (4)



(a) (c)

Q1: Can you directly find out the voltage gain?

Q2: How does it achieve the linearization?

$$\Delta I = I_1 - I_2 = g_{m1,2}\Delta V_{in} \qquad \Delta V_{out} = \frac{\Delta I}{g_{m3,4}} \qquad \Delta V_{out} = \frac{g_{m1,2}}{g_{m3,4}}\Delta V_{in}$$

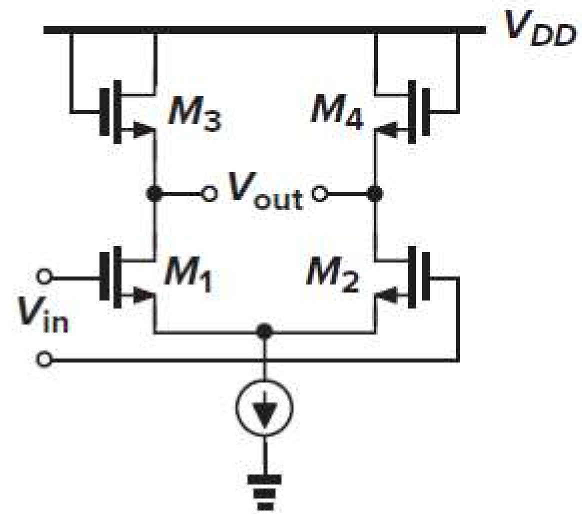
Linearization Technique (4)

$$V_{in1} - V_{in2} = V_{GS1} - V_{GS2}$$

$$= \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_{1,2}}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_{1,2}}}$$

$$V_{out} = V_{GS3} - V_{GS4}$$

$$= \sqrt{\frac{2I_3}{\mu_n C_{ox}\left(\frac{W}{L}\right)_{3,4}}} - \sqrt{\frac{2I_4}{\mu_n C_{ox}\left(\frac{W}{L}\right)_{3,4}}}$$

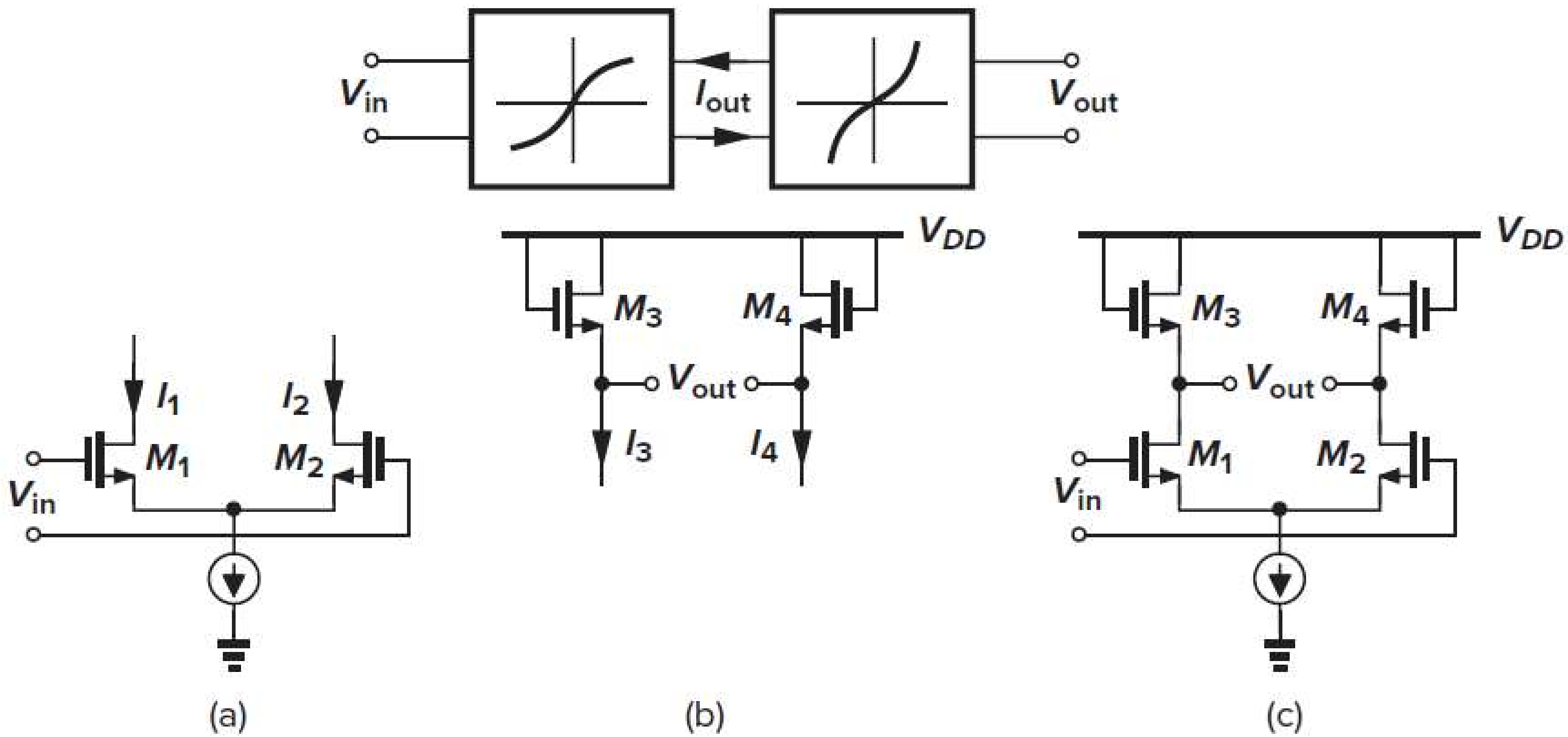


$$V_{out} = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_{3,4}}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_{3,4}}}$$

$$= \frac{1}{\sqrt{\left(\frac{W}{L}\right)_{3,4}}}(V_{in1} - V_{in2})\ sqrt\left(\frac{W}{L}\right)_{1,2}$$

$$A_v = \sqrt{\frac{\left(\frac{W}{L}\right)_{1,2}}{\left(\frac{W}{L}\right)_{3,4}}}$$

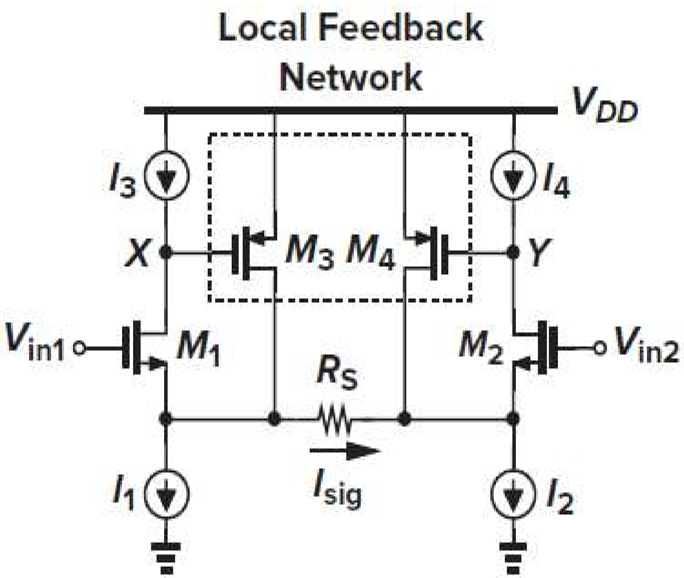
Linearization Technique (4)

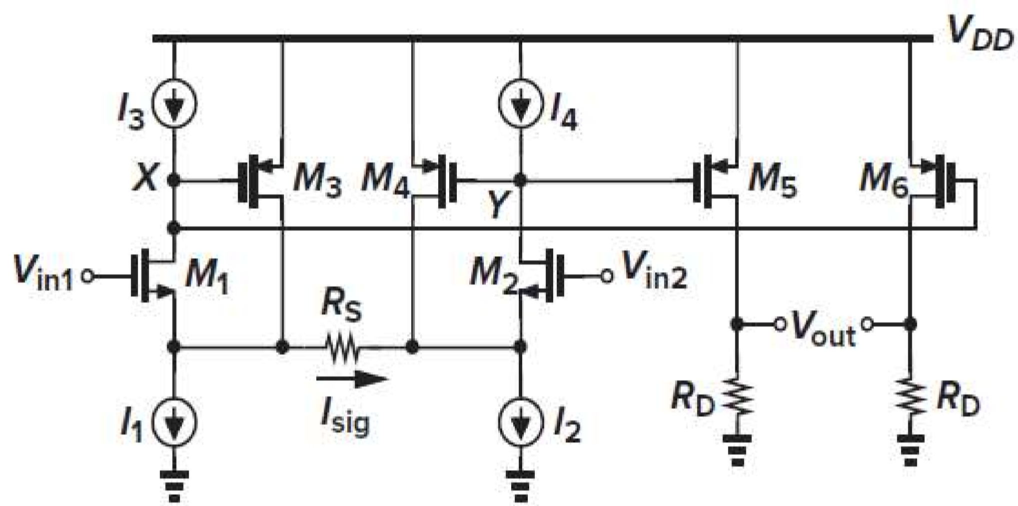


- In practice, **body effect** and other nonidealities in short-channel devices will further give rise to nonlinearity in this circuit.
- Furthermore, as the differential input level increases, driving *M1 or M2* into the triode region, the linearization is no longer hold and the gain drops sharply

=> Body effect can be solved by deep-nwell nMOSTs

Linearization Technique (5)





$$V_{in1} = V_{cm} + V_{in}/2$$

$$V_{in2} = V_{cm} - V_{in}/2$$

$$V_{in} = V_{GS1} + I_{sig}R_S - V_{GS2}$$

$$= I_{sig}R_S, \text{ if } \underline{V_{GS1} = V_{GS2}}$$

where $\underline{I_3 = I_4}$, $I_1 = I_3 + I_{M3}$; $I_2 = I_4 + I_{M4}$

$$V_{out} = (I_6 - I_5)R_D$$

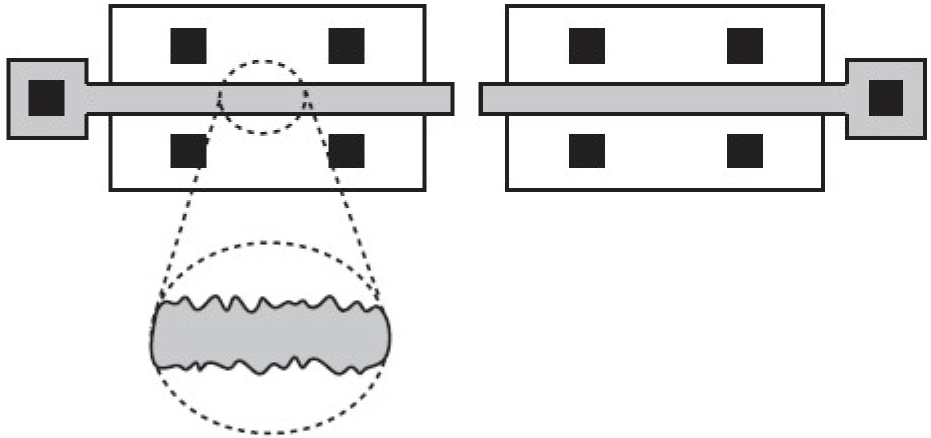
$$I_5 = I_{M4} \text{ and } I_6 = I_{M3}$$

$$V_{out} = (I_{M3} - I_{M4})R_D$$

$$= -I_{sig}R_D = \underline{-\frac{R_D}{R_S}}V_{in}$$

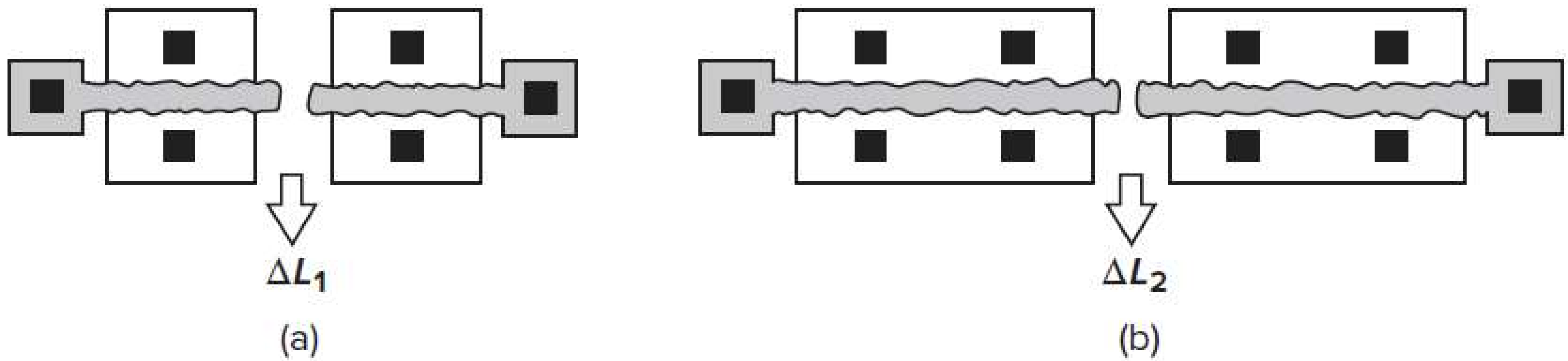
這與課本不同

Mismatch



- Random mismatches due to microscopic variations in device dimensions （來自晶圓廠，在晶片製造上的非理想因素）
- Study of mismatch consists of two steps:
  - (1) To identify and formulate the mechanisms that lead to mismatch between devices, but it is not an easy work. Generally, it depends on the Foundry's process quality control
  - (2) To analyze the effect of device mismatches upon the performance of circuits

Mismatch

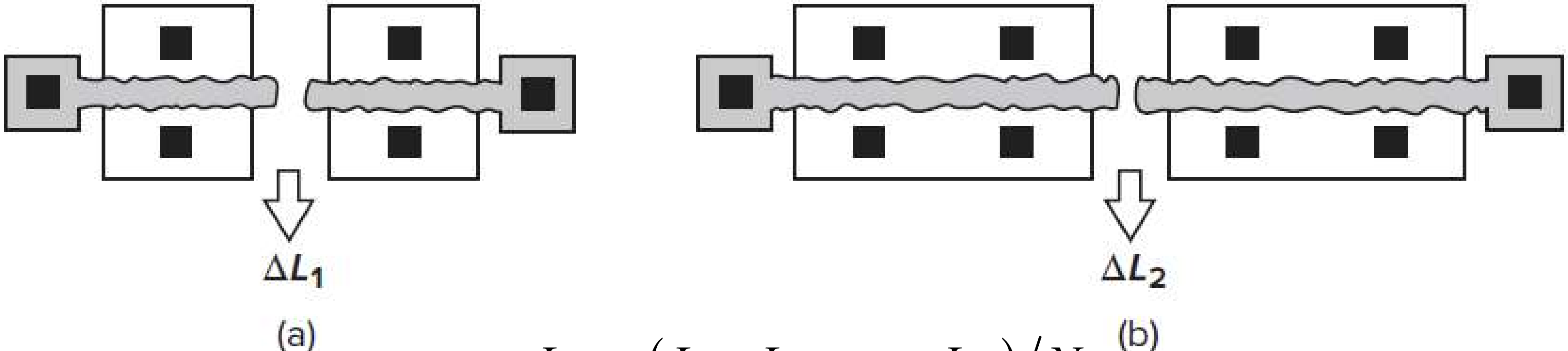


$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2$$

Mismatch issue on
- Current mirrors
- Bias voltages

- For two nominally-identical transistors
- Mismatches of $\mu_n C_{ox}$, W, L, and V_{TH} result in mismatches between drain currents (for a given V_{GS}) or gate-source voltages (for a given drain current)
- Intuitively, larger devices exhibit smaller mismatches

Mismatch - Concept



考量通道長度(*L*)的變異:

$L_i = L_0 + \Delta L_i$

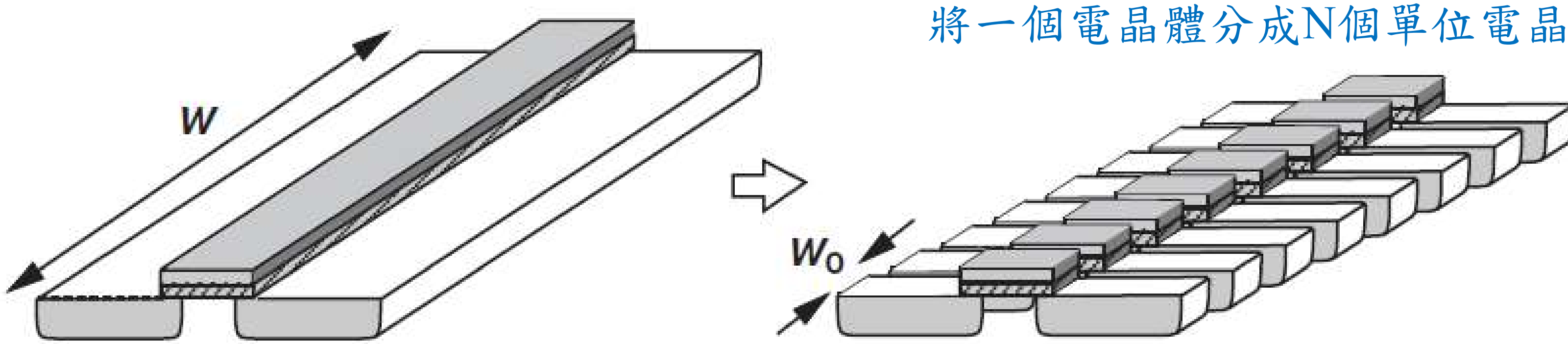
ΔL_i is a random variable

$$L_{eq} \approx \left(L_1 + L_2 + \cdots + L_N\right)/N$$

$$\Delta L_{eq} \approx \frac{\sqrt{\Delta L_1^{\,2} + \Delta L_2^{\,2} + \cdots + \Delta L_N^{\,2}}}{N} = \frac{\sqrt{N \Delta L_0^{\,2}}}{N} = \frac{\Delta L_0}{\sqrt{N}}$$

where ΔL_0 is the statistical variation of the length for a transistor with the length of L_0

將一個電晶體分成N個單位電晶體



Random Variables

Assume x_i is a zero mean random variable which is independent to each other

$E\left[x_i\right]=0$, and $E\left[x_i^2\right]=\sigma^2$, $\forall i$

Assume a function $y=x_1+x_2+x_3+x_4$,

$$E\left[y\right]=E\left[x_1\right]+E\left[x_2\right]+E\left[x_3\right]+E\left[x_4\right]=0$$

$$\sigma_y^{\ 2}=E\left[y^2\right]=E\left[x_1^{\ 2}\right]+E\left[x_2^{\ 2}\right]+E\left[x_3^{\ 2}\right]+E\left[x_4^{\ 2}\right]=4\sigma^2$$

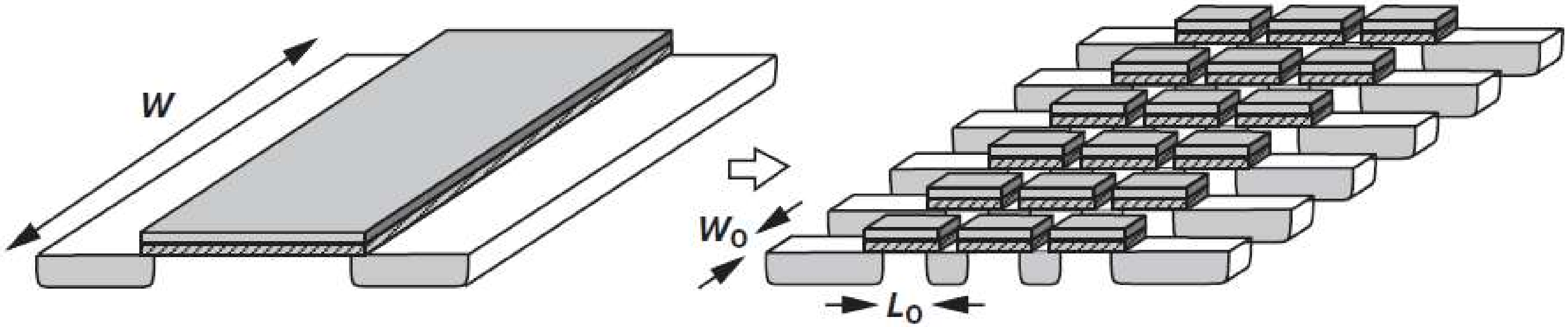
$$\Rightarrow \sigma_y=2\sigma$$

Considering two new random variables,

$$y_1=\sum_{i=1}^{N}x_i \Rightarrow \sigma_{y_1}=\sqrt{N}\sigma$$

$$y_2=\frac{1}{N}\sum_{i=1}^{N}x_i \Rightarrow \sigma_{y_2}^{\ 2}=\frac{1}{N^2}\sum_{i=1}^{N}E\left[x_i^{\ 2}\right]=\frac{\sigma^2}{N} \Rightarrow \sigma_{y_2}=\frac{\sigma}{\sqrt{N}}$$

Mismatch

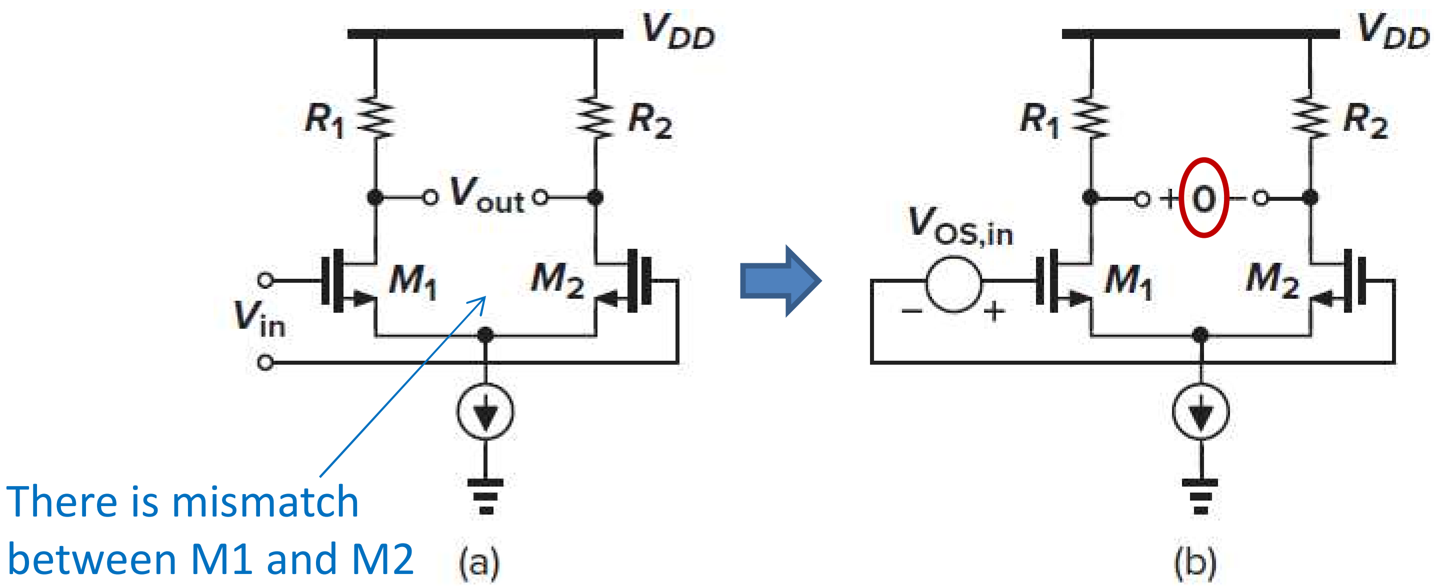


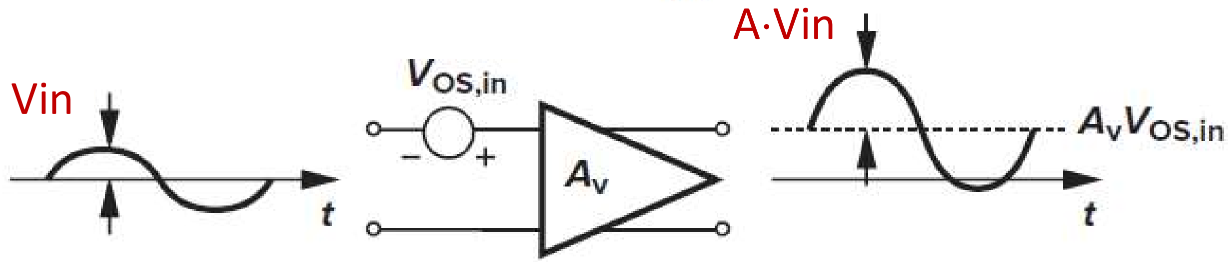
$$\Delta V_{TH} = \frac{A_{V_{TH}}}{\sqrt{WL}}$$

$$\frac{\Delta\beta}{\beta} = \Delta\left(\mu C_{ox}\frac{W}{L}\right)\Big/\beta = \frac{A_{\beta}}{\sqrt{WL}}$$

- We postulate that $\beta=\mu C_{ox}(W/L)$ and V_{TH} suffer from less mismatch if the device area increases (larger transistor dimension)
- For given W_0 *and* L_0, as the number of unit transistors increases, β and V_{TH} experience greater averaging, leading to smaller mismatch between two large transistors

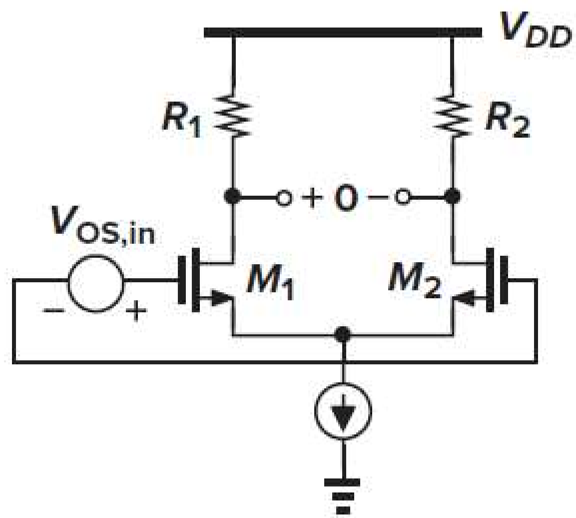
Mismatch of Input Pair





- **DC Offset** (V_{OS})
  - Two conditions: (1) Vin=0 but Vout≠0; (2) Vin≠0 but Vout=0
  - Figure (b) presents the input-referred offset voltage ($V_{OS,in}$), caused by the device mismatch

Input-Referred Offset



$V_{TH1} = V_{TH}$, $V_{TH2} = V_{TH} + \Delta V_{TH}$

$\left(\frac{W}{L}\right)_1 = \frac{W}{L}$, $\left(\frac{W}{L}\right)_2 = \frac{W}{L} + \Delta\left(\frac{W}{L}\right)$

$R_1 = R_D$, $R_2 = R_D + \Delta R$

Assume $\lambda = \gamma = 0$, $\Delta(\mu_n C_{ox}) = 0$

$V_{out} = 0$, $I_{D1} R_1 = I_{D2} R_2$

$I_{D1} = I_D$, $I_{D2} = I_D + \Delta I_D$

$$V_{OS,in} = V_{GS1} - V_{GS2}$$

$$= \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}} + V_{TH1} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}} - V_{TH2}$$

$$= \sqrt{\frac{2}{\mu_n C_{ox}}}\left[\sqrt{\frac{I_D}{W/L}} - \sqrt{\frac{I_D + \Delta I_D}{W/L + \Delta(W/L)}}\right] - \Delta V_{TH}$$

$$= \sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}}\left[1 - \sqrt{\frac{1 + \Delta I_D/I_D}{1 + \Delta(W/L)/(W/L)}}\right] - \Delta V_{TH}$$

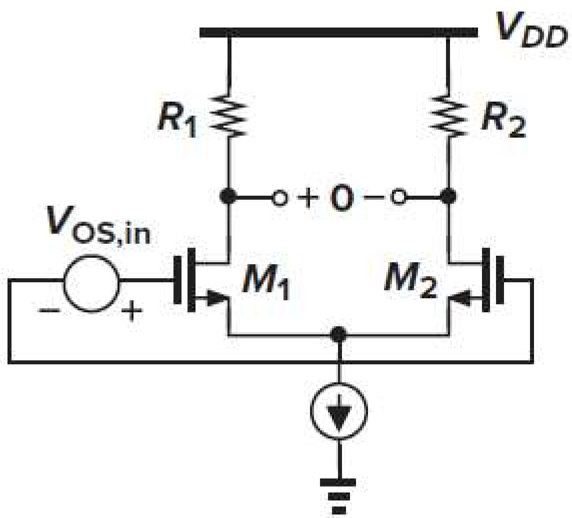
Assume $\Delta I_D/I_D$ and $\Delta(W/L)/(W/L) \ll 1$,

for $\varepsilon \ll 1$, $\sqrt{1+\varepsilon} \approx 1 + \varepsilon/2$

$$V_{OS,in} = \sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}}\left\{1 - \left(1 + \frac{\Delta I_D}{2I_D}\right)\left(1 - \frac{\Delta(W/L)}{2(W/L)}\right)\right\} - \Delta V_{TH}$$

$$V_{OS,in} = \sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}}\left[\frac{-\Delta I_D}{2I_D} + \frac{\Delta(W/L)}{2(W/L)}\right] - \Delta V_{TH}$$

Input-Referred Offset



$$\Delta I_D/I_D \approx -\Delta R_D/R_D$$

$$V_{OS,in} = \frac{1}{2}\sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}}\left[\frac{\Delta R_D}{R_D} + \frac{\Delta(W/L)}{(W/L)}\right] - \Delta V_{TH}$$

$$= \frac{V_{GS}-V_{TH}}{2}\left[\frac{\Delta R_D}{R_D} + \frac{\Delta(W/L)}{(W/L)}\right] - \Delta V_{TH}$$

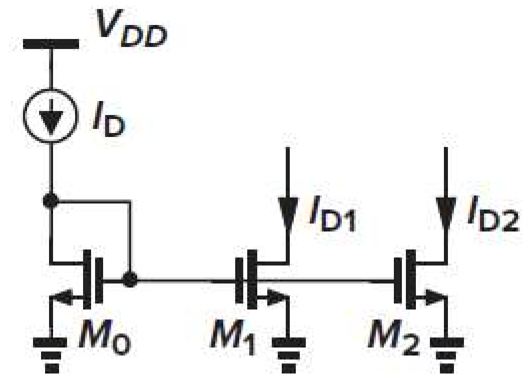
=> $V_{OS,in}$ is also a random variable

$$V_{OS,in}^2 = \left(\frac{V_{GS}-V_{TH}}{2}\right)^2\left[\left(\frac{\Delta R_D}{R_D}\right)^2 + \left(\frac{\Delta(W/L)}{(W/L)}\right)^2\right] + \Delta V_{TH}^2$$

$$\sigma_{OS,in}^2 = \left(\frac{V_{GS}-V_{TH}}{2}\right)^2\left[\sigma\left(\frac{\Delta R_D}{R_D}\right)^2 + \sigma\left(\frac{\Delta(W/L)}{(W/L)}\right)^2\right] + \sigma_{V_{TH}}^2$$

Q: From this result, to get a small offset, how to design V_{GS}-V_{TH}?

Mismatch of Current Mirror



Recall $y = f\left(x_1, x_2, \ldots\right)$

$$\Delta y = \frac{\partial f}{\partial x_1}\Delta x_1 + \frac{\partial f}{\partial x_2}\Delta x_2 + \cdots$$

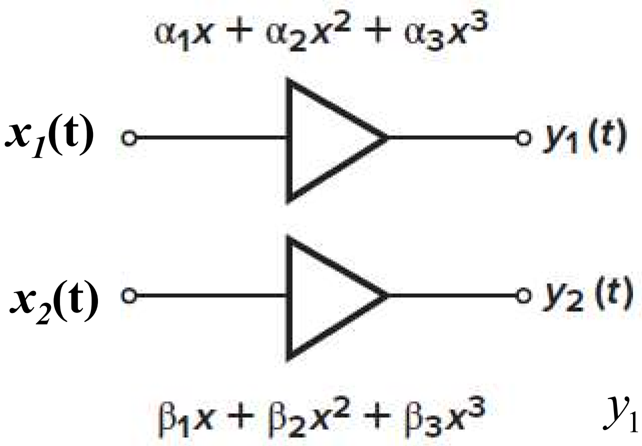
$$\Delta I_D = \frac{\partial I_D}{\partial\left(W/L\right)}\Delta\left(\frac{W}{L}\right) + \frac{\partial I_D}{\partial\left(V_{GS} - V_{TH}\right)}\Delta\left(V_{GS} - V_{TH}\right)$$

$$\Delta I_D = \frac{1}{2}\mu_n C_{ox}\left(V_{GS} - V_{TH}\right)^2 \Delta\left(\frac{W}{L}\right) - \mu_n C_{ox}\left(\frac{W}{L}\right)\left(V_{GS} - V_{TH}\right)\Delta V_{TH}$$

$$\frac{\Delta I_D}{I_D} = \frac{\Delta\left(W/L\right)}{W/L} - \underline{\left(\frac{V_{GS} - V_{TH}}{2}\right)^{-1}}\Delta V_{TH}$$

- Different from $V_{OS,in}$, the current mismatch is inversely proportional to gate-overdrive voltage (V_{GS}-V_{TH})

Even-Order Distortion: Mismatch



$$y_1 - y_2 = \left(\alpha_1 x_1 - \beta_1 x_2\right) + \left(\alpha_2 x_1^{\,2} - \beta_2 x_2^{\,2}\right) + \left(\alpha_3 x_1^{\,3} - \beta_3 x_2^{\,3}\right)$$

Assume $x_1 = x$ and $x_2 = -x$,

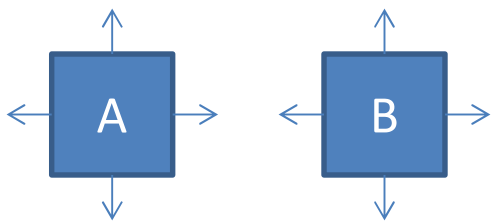
if $\alpha_i \neq \beta_i, \forall i$

$$y_d = y_1 - y_2 = \left(\alpha_1 + \beta_1\right)x + \underline{\left(\alpha_2 - \beta_2\right)x^2} + \left(\alpha_3 + \beta_3\right)x^3$$

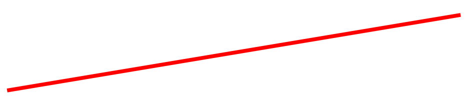
- Single-ended signals have tones of 2nd, 3rd, 4th, …
- Fully differential circuits ideally have only odd-order tones
- However, **mismatch** causes even-order tones in differential signals
- In addition, the (thermal) gradient also contributes mismatch

Gradient: Systematic Issue

Random mismatch

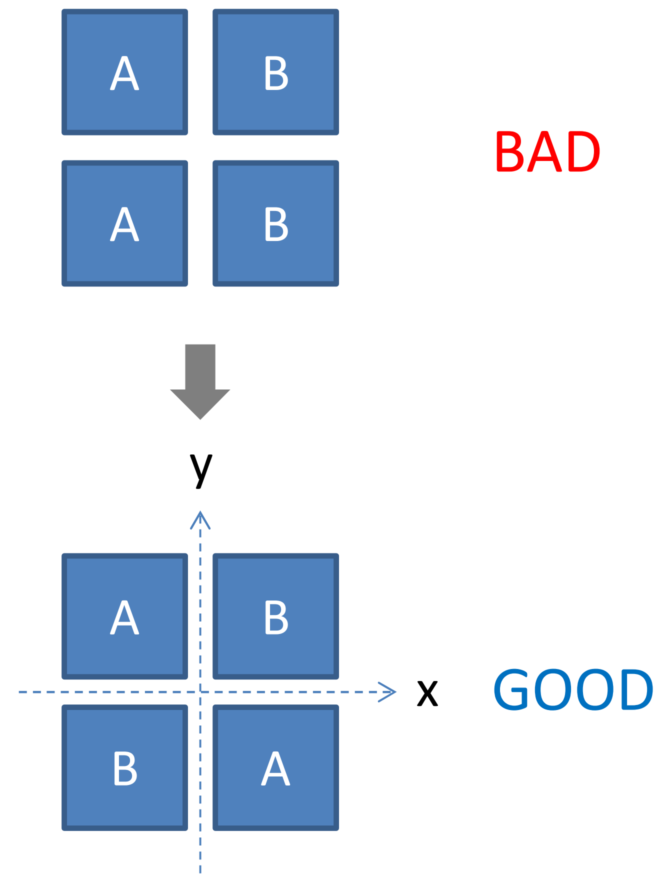


Thermal gradient



Gradient is a deterministic error which causes additional mismatch errors

Layout pattern for gradient



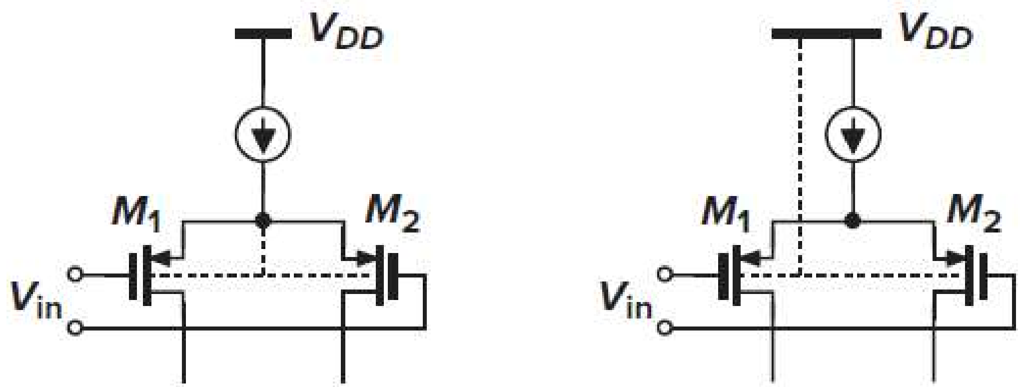
The solution is **common-centroid** layout patterns

CMRR Considerations

$$A_{CM-DM} = \frac{\Delta V_{OS,out}}{\Delta V_{CM,in}},\ A_{DM} = \frac{\Delta V_{OS,out}}{\Delta V_{OS,in}}$$

$$CMRR = \frac{A_{DM}}{A_{CM-DM}} = \frac{A_{DM}}{\frac{\Delta V_{OS,out}}{\Delta V_{CM,in}}} = \frac{\Delta V_{CM,in}}{\frac{\Delta V_{OS,out}}{A_{DM}}} = \frac{\Delta V_{CM,in}}{\Delta V_{OS,in}}$$

Less mismatch, higher CMRR



(a) (b)

In Fig. (a), body effect is eliminated and the threshold voltages of *M1 and M2* are independent of the input CM level. *But, large layout area is introduced!!*

In Fig. (b), if they suffer from mismatches in their body effect coefficients, the input offset voltage varies with the input CM level