102-02 Midterm Exam

(Date: 2014/04/22 14:30-16:20 110 minutes, total: 118 points)

. Evaluate

a. $\lim_{x\to 0}(\cot x - \frac{\pi}{x})$ (8 points)

b. $\lim_{x\to 0^+}\frac{\ln(\sin x)}{\ln(\tan x)}$ (8 points)

c. $\lim_{n\to\infty}\left(1-\frac{\pi}{n}\right)^n$ (8 points)

. Evaluate

a. $\int \frac{\sin^2 x\cos x}{1+\sin^2 x}\,dx$ (8 points)

b. $\int \frac{2x+2}{(x-1)(x^2+1)^2}\,dx$ (8 points)

c. $\int x(2^x)\,dx$ (8 points)

d. $\int \frac{dx}{(x-1)^2\sqrt{1+2x-x^2}}$ (8 points)

e. $\int_1^\infty \frac{1}{x^4+x^2}\,dx$ (8 points)

. Determine convergence or divergence for the following series and give the reason.

a. $\sum_{n=2}^\infty \frac{n}{(\ln n)^n}$ (6 points) root

b. $\sum_{n=1}^\infty \frac{\ln n}{n^2}$ (6 points)

c. $\sum_{n=1}^\infty \frac{n!}{n^n}$ (6 points) ratio

d. $\sum_{n=1}^\infty \frac{(-1)^{n+1}\sqrt{n}}{n+1}$ (6 points)

Determine the convergence interval of $\sum_{n=1}^\infty (-1)^n \frac{(x-2)^n}{4^n\sqrt{n}}$. (10 point

Find the Taylor series for $f(x)=\frac{1-e^{-x}}{x}$ at $x=0$.(Hint: find the T series for e^x first) (10 points)

If $f(x)=2x+\frac{4x^2}{2}+\frac{8x^3}{3}+\cdots+\frac{2^n x^n}{n}+\cdots$, and $x\in\left(-\frac{1}{2},\frac{1}{2}\right)$, wh $f(x)$? (10 points)