1.	提示：分母、分子同乘上 $\sqrt{1+\sqrt{2+t}}+\sqrt{3}$ 詳解：原式 $=\lim_{t\to 2}\frac{\left(\sqrt{1+\sqrt{2+t}}-\sqrt{3}\right)\left(\sqrt{1+\sqrt{2+t}}+\sqrt{3}\right)}{(t-2)\left(\sqrt{1+\sqrt{2+t}}+\sqrt{3}\right)}$ $=\lim_{t\to 2}\frac{1+\sqrt{2+t}-3}{(t-2)\left(\sqrt{1+\sqrt{2+t}}+\sqrt{3}\right)}$ $=\lim_{t\to 2}\frac{\sqrt{2+t}-2}{(t-2)(\sqrt{1+\sqrt{2+t}}+\sqrt{3})}$ 再對分母分子同乘 $\sqrt{2+t}+2$ $=\lim_{t\to 2}\frac{(\sqrt{2+t}-2)(\sqrt{2+t}+2)}{(t-2)\left(\sqrt{1+\sqrt{2+t}}+\sqrt{3}\right)\left(\sqrt{2+t}+2\right)}$ $=\lim_{t\to 2}\frac{2+t-4}{(t-2)(\sqrt{1+\sqrt{2+t}}+\sqrt{3})(\sqrt{2+t}+2)}$ $=\lim_{t\to 2}\frac{1}{(\sqrt{1+\sqrt{2+t}}+\sqrt{3})(\sqrt{2+t}+2)}$ $=\frac{1}{(\sqrt{1+\sqrt{2+2}}+\sqrt{3})(\sqrt{2+2}+2)}$ $=\frac{1}{8\sqrt{3}}$ $=\frac{\sqrt{3}}{24}$
2.	詳解： $\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\sqrt{x}\left(\sqrt{x+1}-\sqrt{x}\right)=\lim_{x\to\infty}\frac{\sqrt{x}(\sqrt{x+1}-\sqrt{x})(\sqrt{x+1}+\sqrt{x})}{\sqrt{x+1}+\sqrt{x}}$ $=\lim_{x\to\infty}\frac{\sqrt{x}(x+1-x)}{\sqrt{x+1}+\sqrt{x}}=\frac{\sqrt{x}}{\sqrt{x}(\sqrt{1+\frac{1}{x}}+1)}=\frac{1}{2}$
3.	詳解： $\lim_{x\to 0}\frac{\sin x\cdot\sin 2x^2\cdot\sin 3x^3\cdot\sin 4x^4}{x^{10}}$ $=\lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{\sin 2x^2}{2x^2}\cdot\frac{\sin 3x^3}{3x^3}\cdot\frac{\sin 4x^4}{4x^4}\cdot 2\cdot 3\cdot 4$ $=1\cdot 1\cdot 1\cdot 1\cdot 2\cdot 3\cdot 4$ $=24\quad(\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1)$
4.	詳解：$\lim_{x\to 2}\sqrt{2x}=2$ (1). $\forall\varepsilon>0\ (0<\varepsilon<2),\exists\delta>0,\text{when } \lvert x-2\rvert<\delta,\left\lvert\sqrt{2x}-2\right\rvert<\varepsilon$ (2). $\left\lvert\sqrt{2x}-2\right\rvert<\varepsilon\Rightarrow -\varepsilon<\sqrt{2x}-2<\varepsilon\Rightarrow 2-\varepsilon<\sqrt{2x}<2+\varepsilon$ $\Rightarrow\frac{(2-\varepsilon)^2}{2}<x<\frac{(2+\varepsilon)^2}{2}$

	$\therefore$ take $\delta = \min\left\{2-\frac{(2-\varepsilon)^2}{2}, \frac{(2+\varepsilon)^2}{2}-2\right\} = \min\left\{2\varepsilon-\frac{\varepsilon^2}{2}, 2\varepsilon+\frac{\varepsilon^2}{2}\right\} = 2\varepsilon-\frac{\varepsilon^2}{2}$ $\therefore\ \forall\varepsilon>0\ (0<\varepsilon<2), \exists\delta = 2\varepsilon-\frac{\varepsilon^2}{2}$, when $\lvert x-2\rvert<\delta$ $-\left(2\varepsilon-\frac{\varepsilon^2}{2}\right) < x-2 < 2\varepsilon-\frac{\varepsilon^2}{2} < 2\varepsilon+\frac{\varepsilon^2}{2} \Rightarrow 2-2\varepsilon+\frac{\varepsilon^2}{2} < x < 2+2\varepsilon+\frac{\varepsilon^2}{2}$ $\Rightarrow 4-4\varepsilon+\varepsilon^2 < 2x < 4+4\varepsilon+\varepsilon^2 \Rightarrow \sqrt{(2-\varepsilon)^2} < \sqrt{2x} < \sqrt{(2+\varepsilon)^2}$ $(2-\varepsilon)-2 < \sqrt{2x}-2 < (2+\varepsilon)-2$ $\therefore \left\lvert\sqrt{2x}-2\right\rvert < \varepsilon$
5.	詳解：Test for continuity $\lim_{x\to0^+} f(x) = \lim_{x\to0^+}\left(x^2\sin\frac{1}{x}\right) = 0$ $\lim_{x\to0^-} f(x) = \lim_{x\to0^+}(mx+b) = b$ differentiable $\rightarrow$ continuous, so that b $= 0$. Test for differentiability $\lim_{x\to0^+}\frac{f(x)-f(0)}{x-0} = \lim_{x\to0^+}\frac{x^2\sin\frac{1}{x}-0}{x} = \lim_{x\to0^+}\left(x\sin\frac{1}{x}\right) = 0$ $\lim_{x\to0^-}\frac{f(x)-f(0)}{x-0} = \lim_{x\to0^-}\frac{mx-0}{x} = \lim_{x\to0^-}(m) = m$ differentiable $\rightarrow$ the limit exists, so that m $= 0$.
6.	詳解： (1). $f(x) = \sqrt{2x+\sqrt{2x+\sqrt{2x+\sqrt{\ldots\ldots}}}} = \sqrt{2x+f(x)} \Rightarrow f^2(x) = 2x+f(x)$ (2). $\frac{d}{dx}f^2(x) = \frac{d}{dx}\left(2x+f(x)\right) \Rightarrow 2f(x)f'(x) = 2+f'(x) \Rightarrow f'(x) = \frac{2}{2f(x)-1}$
7.	詳解：(1). 曲線 1：$x+2x^2-y-5xy=0$ (0,0)代入上式滿足 $\therefore$ (0,0)為曲線 1 上一點 且 $\frac{d}{dx}(x+2x^2-y-5xy) = \frac{d}{dx}0$ $\Rightarrow 1+4x-\frac{dy}{dx}-5y-5x\frac{dy}{dx} = 0 \quad \therefore \frac{dy}{dx} = \frac{1+4x-5y}{5x+1}$ $\therefore$ 切線斜率$m_1 = \left.\frac{dy}{dx}\right\rvert_{x=0,y=0} = \left.\frac{1+4x-5y}{5x+1}\right\rvert_{x=0,y=0} = 1$ (2). 曲線 2：$y^2+3xy+x^2-x-y=0$ (0,0)代入上式滿足 $\therefore$ (0,0)為曲線 2 上一點 且 $\frac{d}{dx}(y^2+3xy+x^2-x-y) = \frac{d}{dx}0$ $\Rightarrow 2y\frac{dy}{dx}+3y+3x\frac{dy}{dx}+2x-1-\frac{dy}{dx} = 0 \quad \therefore \frac{dy}{dx} = \frac{3y+2x-1}{1-3x-2y}$

$\therefore$ 切線斜率$m_2 = \left.\dfrac{dy}{dx}\right|_{x=0,y=0} = \left.\dfrac{3y+2x-1}{1-3x-2y}\right|_{x=0,y=0} = -1$

(3). 由 (1)、(2) 結果可知 (0,0) 為兩曲線之共同交點

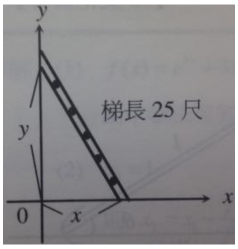
且$m_1 \cdot m_2 = -1$ ，故得証。

8. 詳解：

$S = \pi r\sqrt{r^2+h^2}$, r constant $\Rightarrow dS = \pi r \cdot \frac{1}{2}(r^2+h^2)^{-1/2}2h\,dh = \frac{\pi r h}{\sqrt{r^2+h^2}}dh$. Height changes from h_0 to h_0+dh

$\Rightarrow dS = \frac{\pi r h_0 (dh)}{\sqrt{r^2+h_0^2}}$

9. 詳解：(1). 此處定義向右速度為正，向左為負，向上為正，向下為負。



(2). 梯長 25 公尺，則由圖可知$x^2+y^2=25^2$

當 $x=15$ ，$\frac{dx}{dt}=3$ 求 $\frac{dy}{dt}=?$

(3). $x^2+y^2=25^2 \quad \Rightarrow \quad \frac{d}{dt}(x^2+y^2)=\frac{d}{dt}25^2$

$\Rightarrow \quad 2x\dfrac{dx}{dt}+2y\dfrac{dy}{dt}=0 \quad \Rightarrow \quad \dfrac{dy}{dt}=-\dfrac{x}{y}\dfrac{dx}{dt}$

已知 $x=15 \quad \Rightarrow \quad 15^2+y^2=25^2 \quad \Rightarrow \quad y=20$ 且 已知 $\frac{dx}{dt}=3$

$\therefore \left.\dfrac{dy}{dt}\right|_{x=15} = -\dfrac{15}{20}\cdot 3 = -\dfrac{9}{4}$ 公尺/分 (向下滑動)

10. 詳解：$f(x)=2x^7+x-1$ ， $f(0)=-1<0$ ， $f(1)=2>0$

在 $(0,1)$ 內存在 ξ 使得 $f(\xi)=0$

設 $f(x)$ 有兩相異實根 ξ_1,ξ_2 ，且 $\xi_1<\xi_2$ ，則有 $f(\xi_1)=f(\xi_2)=0$

由 Rolle 定理知存在 $x\in(\xi_1,\xi_2)$ 使得 $f'(x)=0$

$f'(x)=14x^6+1\neq 0$，矛盾，故 $f(x)$ 恰有一實根。

若 $\xi_1=\xi_2=\xi$ 為重根，則 $f(\xi_1)=f(\xi_2)=0 \Rightarrow f'(\xi)=0$

但 $f'(x)\neq 0$ ，故 $\xi_1\neq\xi_2$ ，故得証，恰有一根

11. 詳解：

(1). $f(x)=\frac{x}{(x-1)^2}$， $x\neq 1$

$\Rightarrow f'(x)=\dfrac{1\cdot(x-1)^2-x\cdot 2(x-1)}{(x-1)^4}=-\dfrac{x+1}{(x-1)^3}$

$\Rightarrow f''(x)=\dfrac{1\cdot(x-1)^3-x\cdot 2(x-1)}{(x-1)^4}=\dfrac{2(x+2)}{(x-1)^4}$

(2). 列表:

x		-2		-1		1	
$f'(x)$			$-$	0	$+$	×	$-$
$f''(x)$	$-$	0		$+$		×	$+$
圖形						×	

	$f(x)$ 說明	$-\frac{2}{9}$ 反曲點	$-\frac{1}{4}$ 極小	$\times$
12.	詳解：			

a. 利潤 = 總售價 − 總成本 = 單價 × 銷售數量 − 總成本

$\Rightarrow f(x) = p(x) \cdot x - C(x) = (22.2 - 1.2x)x - (0.4x^2 + 3x + 40)$

$= -1.6x^2 + 19.2x - 40$ 欲求最大

令 $f'(x) = -3.2x + 19.2 = 0 \quad \Rightarrow \quad x = 6$ 且 $f''(6) = -3.2 < 0$

當 $x = 6$ (仟)時，有最大利潤 $f(6) = 17.6$ (仟元)

b. $A(x) = \frac{C(x)}{x} = \frac{0.4x^2+3x+40}{x}$

$$\Rightarrow A'(x) = \frac{(0.8x + 3)x - (0.4x^2 + 3x + 40) \cdot 1}{x^2} = \frac{0.4x^2 - 40}{x^2} = \frac{0.4(x^2 - 100)}{x^2}$$

令 $A'(x) = 0 \quad \Rightarrow \quad x = 10$ (-10 不合)

且 $A'(10^-) < 0$ ， $A'(10^+) > 0$

$\therefore$ 當 $x = 10$ (仟)時，有最小單位成本 $A(10) = 11$