

Note: To get full points, you should write down the procedure **in detail**.

1. Let  $f(x, y) = xy e^{-xy^2}$ :
  - (a) (5 points) Find  $\nabla f$ .
  - (b) (5 points) Find the directional derivative of  $f$  at the point  $(1, 1)$  in the direction  $\vec{v} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}}$ .
  - (c) (5 points) In which direction does  $f$  change most rapidly at  $(1, 1)$ .
  - (d) (5 points) What is the maximum rate of change at  $(1, 1)$ .
2. (15 points) Find the area of the region in polar coordinates that is inside the curve  $r = 2 + 2\cos\theta$  and outside the curve  $r = 3$ .
3. Suppose  $\Gamma$  is the intersection curve of  $x - y - 1 = 0$  and  $y^2 - z^2 - 1 = 0$ 
  - (a) (10 points) Find the tangent line of  $\Gamma$  at  $P\left(\frac{9}{4}, \frac{5}{4}, -\frac{3}{4}\right)$ .
  - (b) (15 points) Find all the points on  $y^2 - z^2 - 1 = 0$  that lie closest from the origin  $(0, 0, 0)$ .
4. (10 points) find an equation for the plane tangent to the level surface  $f(x, y, z) = x^2 - y - 5z = 0$  at the point  $P_0(2, -1, 1)$ . Also, find parametric equations for the line that is normal to the surface at  $P_0$ .
5. (10 points) Evaluate the iterated integral  $\int_0^1 \int_{\sqrt{x}}^1 \cos(y^3) dy dx$ .
6. (10 points) Find the volume of the solid bounded by the planes  $z = x$ ,  $y = x$ ,  $x + y = 2$ , and  $z = 0$ .
7. (10 points) Let  $f(x, y) = 3x^2 - 2xy + y^2 - 8y$ . Find the location of local extreme and its values and saddle point(s) (if any). Please specify each point clearly if the point is local minimum, local maximum, or saddle point.
8. A parametric curve  $x = f(t) = 2t^2$ ,  $y = g(t) = t^3 - 4t$ .
  - (a) (5 points) This curve intersects itself at point  $P$ . Find the intersection point  $P$ .
  - (b) (5 points) Find the equation for the line tangent to the curve at the point  $Q(2, -3)$ .
  - (c) (10 points) At the point  $Q(2, -3)$ , is the curve concave upward or concave downward?