107-01 政修小考②

1. $y=f(x)=\sqrt{25-x^2}$

$$f'(x)=\frac{-x}{\sqrt{25-x^2}}=-x(25-x^2)^{-\frac{1}{2}}$$

$x=0 \Rightarrow f'(x)=0$

$x=\pm5 \Rightarrow f'(x)$ is undefined

$\therefore$ critical point, $x=0$, $x=5$, $x=-5$

x	-5	$(-5,0)$	0	$(0,5)$	5
$f(x)$	0		5		0
$f'(x)$	$+$	$+$	0	$-$	$-$
$f''(x)$	$-$	$-$	$-$	$-$	$-$
		↗	相對極大	↘	

(A)

$f(-5)=0$, $f(5)=0$ > local minimum is 0 at $x=5$ and $x=-5$

$f(0)=5$ → local maximum is 5 at $x=0$

(B)

absolute maximum is 5 at $x=0$

absolute minimum is 0 at $x=5$ and $x=-5$

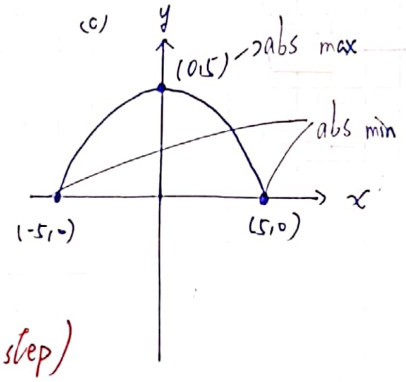
$$f''(x)=-(25-x^2)^{-\frac{1}{2}}+(-x)(\tfrac{-1}{2})(25-x^2)^{-\frac{3}{2}}(-2x)$$

$$=\frac{-1}{\sqrt{25-x^2}}+\frac{-x^2}{\sqrt{(25-x^2)^3}}$$

$$=\frac{x^2-25-x^2}{\sqrt{(25-x^2)^3}}$$

$$=\frac{-25}{\sqrt{(25-x^2)^3}}$$

(C)



2. $y=\frac{x^4+1}{x^2}=x^2+x^{-2}$ (sketch by all step)

(1) Domain: $(-\infty,0)\cup(0,\infty)$

(2) $$y'=2x-2x^{-3}=2x-\frac{2}{x^3}=\frac{2x^4-2}{x^3}=\frac{2(x^2+1)(x+1)(x-1)}{x^3}$$

$$y''=2+6x^{-4}=2+\frac{6}{x^4}=\frac{2x^4+6}{x^4}$$

(3) $x=1$, $x=-1$, $\Rightarrow y'=0$

$x=0 \Rightarrow y'$ is undefined

$\therefore$ critical point, $x=0$, $x=1$, $x=-1$

$\because f(1)=0$ and $f''(1)>0$

$\Rightarrow f(x)$ has local minimum at $x=1$

$f(-1)=0$ and $f''(-1)>0$

$\Rightarrow f(x)$ has local minimum at $x=-1$

(4) by the sign graph

$$f'=(-\mid+\mid-\mid+)$$

$-\infty \quad -1 \quad 0 \quad 1 \quad +\infty$

$f'(x)<0$ when x is on $(-\infty,-1)$ ⇒ decreasing

$f'(x)>0$ when x is on $(-1,0)$ ⇒ increasing

$f'(x)<0$ when x is on $(0,1)$ ⇒ decreasing

$f(x)>0$ when x is on $(1,\infty)$ ⇒ increasing

(5) by the sign graph

$$f''=(+\mid+\mid+\mid+)$$

$-\infty \quad -1 \quad 0 \quad 1 \quad +\infty$

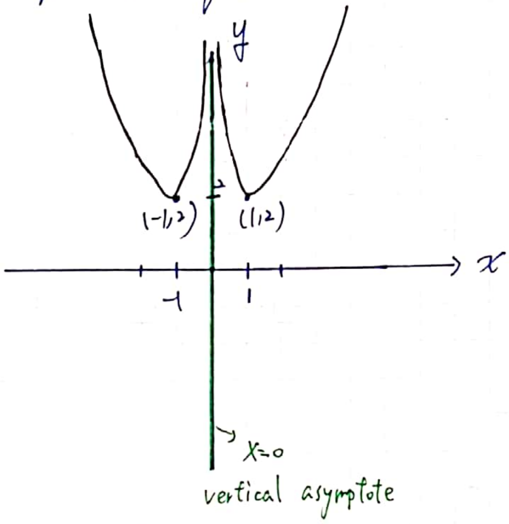
there is no point of inflection

(6)

$$\lim_{x\to0^+}f(x)=+\infty\ ;\ \lim_{x\to0^-}f(x)=+\infty$$

$\therefore x=0$ is a vertical asymptote of $f(x)$

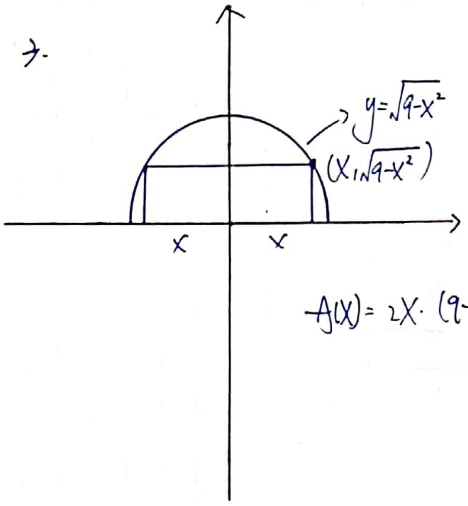
8. plot the key point and sketch the curve.



$f(1) = \frac{1+1}{1} = 2$

$f(-1) = \frac{1+1}{1} = 2$

7.



$A(X) = 2X\cdot(9-X^2)^{\frac{1}{2}}$

$A'(X) = 2\cdot\left((9-X^2)^{\frac{1}{2}} + X\cdot\frac{1}{\cancel{2}}(9-X^2)^{-\frac{1}{2}}\cdot(-\cancel{2}X)\right)$

$= 2\cdot\left(\sqrt{9-X^2} + \frac{-X^2}{\sqrt{9-X^2}}\right)$

$= 2\cdot\frac{9-2X^2}{\sqrt{9-X^2}}$

$= \frac{4(\frac{9}{2}-X^2)}{\sqrt{9-X^2}} = \frac{4(X+\frac{3}{\sqrt{2}})(X-\frac{3}{\sqrt{2}})}{\sqrt{9-X^2}}$

$A'(X)=0 \Rightarrow X=\frac{3}{\sqrt{2}}$ or $\frac{-3}{\sqrt{2}}$ (負不合)

Area $= 2\cdot\frac{3}{\sqrt{2}}\cdot\sqrt{9-\frac{9}{2}} = 3\cancel{\sqrt{2}}\cdot\frac{3}{\cancel{\sqrt{2}}} = 9$

A: $A = 9$ unit^2
$w = 3\sqrt{2}$ unit
$h = \frac{3}{\sqrt{2}}$ unit

$3\sqrt{2}$ unit by $\frac{3}{\sqrt{2}}$ unit

4. $\int x^{\sqrt{2}-1}dx$

$= \frac{1}{\sqrt{2}\cancel{-1}\cancel{+1}}x^{\sqrt{2}\cancel{-1}\cancel{+1}} + C$

$= \frac{1}{\sqrt{2}}x^{\sqrt{2}} + C$ #

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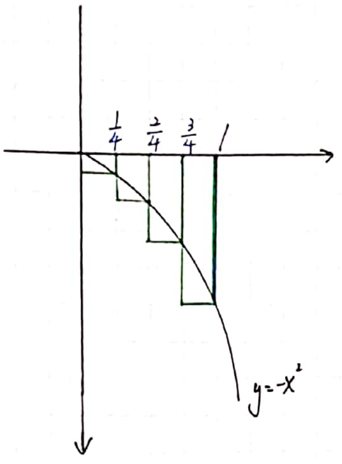
5.



(b) right-hand endpoint

$c_k = \frac{k}{4}$; $\Delta x_k = \frac{1}{4}$

$\Rightarrow \sum_{k=1}^{4} f(c_k)\,\Delta x_k$

$= \sum_{k=1}^{4} \left(-\frac{k^2}{16}\right)\cdot\frac{1}{4}$

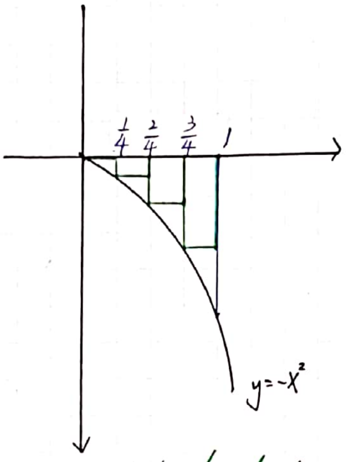
$= \frac{-1}{64}\cdot\sum_{k=1}^{4} k^2$

$= \frac{-1}{64}\cdot\frac{{}^{2}\cancel{4}\cdot 5\cdot\cancel{9}^{3}}{\cancel{6}}$

$= \frac{-30}{64}$

$= \underline{\frac{-15}{32}}$ ※ ↘

(d) <u>lower sum</u> ※



(a) left-hand endpoint

$c_k = \frac{k-1}{4}$, $\Delta x_k = \frac{1}{4}$

$\Rightarrow \sum_{k=1}^{4} f(c_k)\,\Delta x_k$

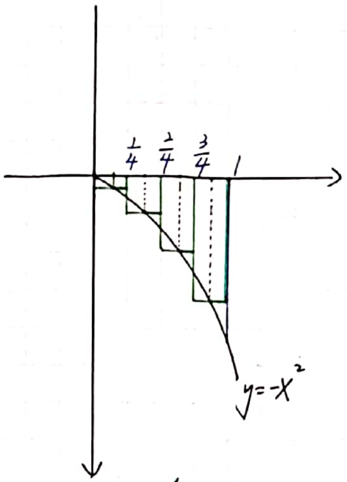
$= \sum_{k=1}^{4} \left(-\frac{k^2-2k+1}{16}\right)\cdot\frac{1}{4}$

$= \frac{-1}{64}\sum_{k=1}^{4} k^2-2k+1$

$= \frac{-1}{64}\cdot\left[\frac{4\cdot5\cdot9}{6}-2\cdot\frac{4\cdot5}{2}+4\right]$

$= \frac{-1}{64}\cdot(30-20+4)$

$= \underline{\frac{-7}{32}}$ ※



(c) midpoint

$c_k = \frac{2k-1}{8}$, $\Delta x_k = \frac{1}{4}$

$\Rightarrow \sum_{k=1}^{4} f(c_k)\cdot\Delta x_k$

$= \sum_{k=1}^{4} \left(-\frac{4k^2-4k+1}{64}\right)\frac{1}{4}$

$= \frac{1}{-64\cdot4}\cdot\sum_{k=1}^{4} 4k^2-4k+1$

$= \frac{1}{-64\cdot4}\cdot\left(4\cdot\frac{4\cdot5\cdot9}{6}-4\cdot\frac{4\cdot5}{2}+4\right)$

$= \frac{-1}{64\cdot4}\cdot(120-40+4)$

$= \frac{-84}{64\cdot4}$

$= \underline{\frac{-21}{64}}$ ※

6. minimize the value of $\int_a^b (x^4-2x^2)\,dx$

To find where $x^4-2x^2 \le 0$

let $x^4-2x^2=0$, $x^2(x^2-2)=0$, $x^2(x+\sqrt{2})(x-\sqrt{2})=0$

$x=0, \pm\sqrt{2}$

by the sign graph

$f = (+ \underset{-\sqrt{2}}{|} - \underset{0}{|} - \underset{\sqrt{2}}{|} +)$ If $-\sqrt{2}<x<\sqrt{2}$, then $x^4-2x^2<0$

$\Rightarrow$ <u>$a=-\sqrt{2}$ and $b=\sqrt{2}$ minimize the integral</u> ※

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7. linearization of $g(x)=3+\int_1^{x^2}\sec(t-1)dt$ at $x=-1$

$g'(x)=\frac{d}{dx}\left(3+\int_1^{x^2}\sec(t-1)\,dt\right)$

$=\frac{d}{dx}\int_1^{x^2}\sec(t-1)\,dt$, 令 $u=x^2$

$=\frac{d}{du}\int_1^{u}\sec(t-1)\,dt\times\frac{du}{dx}$

$=\sec(u-1)\times\frac{d}{dx}(x^2)$

$=\sec(x^2-1)\times 2x$

$g'(-1)=\sec(1-1)\times 2x$

$=1\cdot 2\cdot(-1)$

$=-2$

$g(-1)=3$

∴ linearization at $x=-1$

$\Rightarrow L(x)=g(-1)+g'(-1)\cdot(x-(-1))$

$=3-2(x+1)$

$=3-2x-2$

$=-2x+1$ #

8.

$\int\frac{1}{x^3}\sqrt{\frac{x^2-1}{x^2}}\,dx=?$

$=\int\frac{1}{x^3}\sqrt{1-\frac{1}{x^2}}\,dx$ 令 $u=1-\frac{1}{x^2}=1-x^{-2}$

$du=2x^{-3}dx$

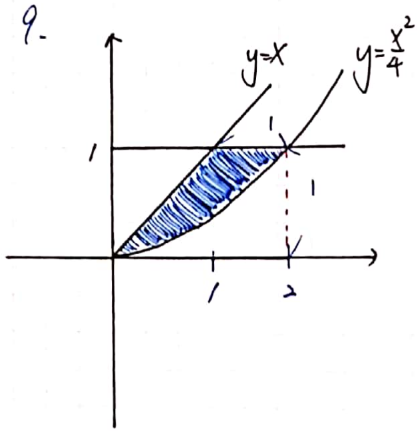
$\frac{1}{2}du=\frac{1}{x^3}dx$

$=\frac{1}{2}\int\sqrt{u}\,du$

$=\cancel{\frac{1}{2}}\cdot\frac{2}{3}u^{\frac{3}{2}}+c$

$=\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}+c$ #

9.



$A=\frac{(1+2)\times 1}{2}-\int_0^2\left(\frac{x^2}{4}-0\right)dx$

$=\frac{3}{2}-\frac{1}{4}\cdot\frac{1}{3}x^3\Big|_0^2$

$=\frac{3}{2}-\frac{\cancel{8}^{2}}{\cancel{12}_{3}}$

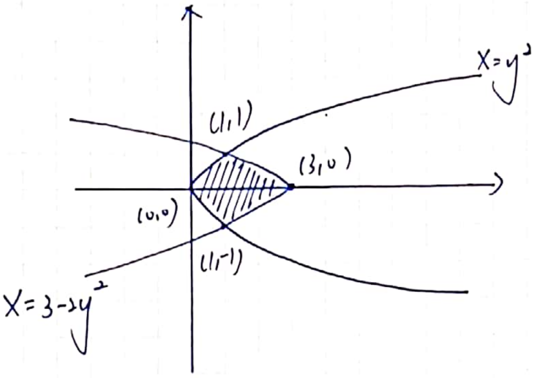
$=\frac{9-4}{6}$

$=\frac{5}{6}$ #

10.

Find the are of regions enclosed by the lines and curves.

$x-y^2=0$、 $x+2y^2=3$ $y^2+2y^2=3$, $y=\pm1$



$$\text{area} = \int_{-1}^{1} (3-2y^2) - y^2\,dy$$

$$= \int_{-1}^{1} (3-3y^2)\,dy$$

$$= 2\int_{0}^{1} 3-3y^2\,dy$$

$$= 2\cdot\left[(3y-y^3)\Big|_0^1\right]$$

$$= 2\cdot(3-1)$$

$$= \underline{4}\ ※$$