Introduction to Analog Integrated Circuits (112), DECE, NTUST

Homework 5 (Due date: 11/14)

HW5.1: (20 points)

Using a long-channel model, **prove** that, in strong inversion, the transistor M_R behaves like a resistor ($R_{on,R}$) with its resistance,

$$R_{on,R} = \frac{(W/L)_C}{(W/L)_R} \frac{1}{g_{m,C}}$$

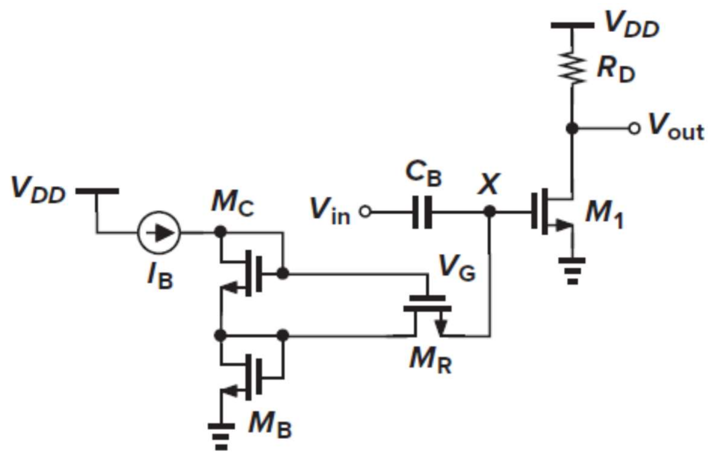

Fig. 5.1

HW5.2: (30 points)

The circuit of Fig. 5.2 is designed with $(W/L)_{1,0} = 8/2$, $(W/L)_{3,2} = 4*8/2$, and $I_{REF} = 100\ \mu$A. Assume $\mu_n C_{ox}$=800μA/V^2, VDD=3V and γ=0. V_{TH}=0.5V

(a) Determine V_X and the acceptable range of V_b.

(b) Estimate the deviation of I_{out} from 400 μA if the drain voltage of M_3 is higher than V_X by 1 V, if λ=0.1V^{-1}.

(c) How to design V_b to have a minimum drain voltage of M_3?

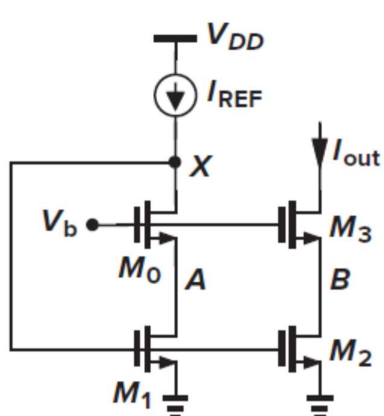

Fig. 5.2

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HW5.3 (30 points)

In the circuit shown in Fig. 5.3, a source follower using a *wide* transistor and a small bias current is inserted in series with the gate of M_3 to bias M_2 at the edge of saturation. Assuming M_0–M_3 are identical and $\lambda \neq 0$, estimate the mismatch between I_{out} and I_{REF} if

(a) $\gamma = 0$ (20 points), and

(b) $\gamma \neq 0$ (10 points).

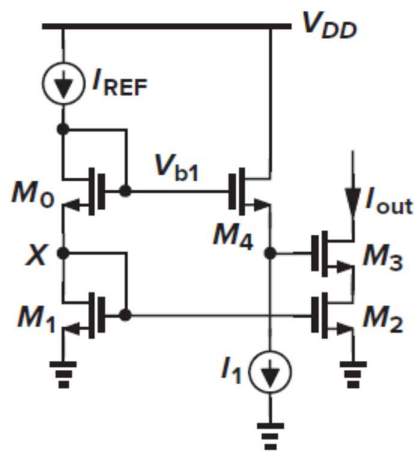


Fig. 5.3

HW5.4: (20 points)

The circuit shown in Fig. 5.4 exhibits a *negative* input inductance. Calculate the input impedance of the circuit and identify the inductive component.

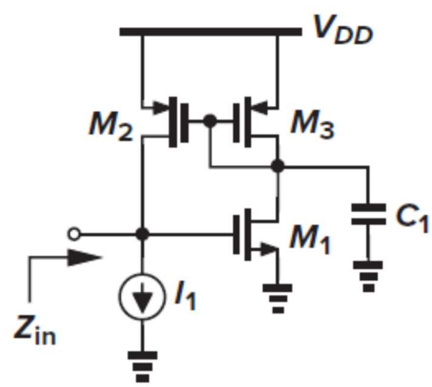


Fig. 5.4

HW5.1

由於 M_R 的 $I_D=0$ 使得 $V_{DS}=0$

因此 $V_{GSC}=V_{GSR} \Rightarrow V_{ovC}=V_{ovR}$

將 M_R 操作於 Deep triode Region $\Rightarrow R_{ON,R}=\frac{1}{g_{mR}}$

$$\frac{g_{mC}}{g_{mR}}=\frac{\mu C_{ox}(W/L)_C V_{ovC}}{\mu C_{ox}(W/L)_R V_{ovR}}=\frac{(W/L)_C}{(W/L)_R}$$

$$\Rightarrow R_{ON,R}=\frac{(W/L)_C}{(W/L)_R}\frac{1}{g_{mC}} \quad \#$$

HW5.2

ⓐ

Assume $\beta_i=\mu C_{ox}(W/L)_i$

$$V_X=V_{GS1}=\sqrt{\frac{2I_{REF}}{\beta_1}}+V_{TH1}=0.25+0.5=0.75V \quad \#$$

$$V_{ov1}+V_{GS0}\le V_b\le V_X+V_{TH0}$$

$$\Rightarrow 0.25+0.75\le V_b\le 0.75+0.5$$

$$\Rightarrow 1V\le V_b\le 1.25V \quad \#$$

ⓑ 〈法一〉

Assume $V_b = V_{b(min)} = 1V$

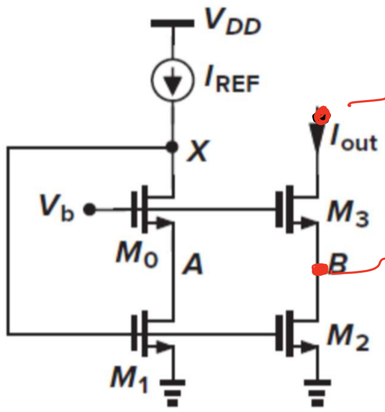


Fig. 5.2

$\rightarrow V_X + 1 = 1.75V$

$\rightarrow \Delta V_{DS2}$

$$V_{DS1} = V_b - V_{GS0} = 0.25V$$

$$V_{DS2} = V_{DS1} + \Delta V_{DS2}$$

$$V_{DS3} = V_X - V_{DS2} = 1.5 - \Delta V_{DS2}$$

$$V_{OV3} = V_b - V_{DS2} - V_{TH3} = 0.25 - \Delta V_{DS2}$$

$$I_{out} = \frac{\beta_2}{2} V_{OV2}^2 (1 + \lambda V_{DS2})$$

$$I_{out} = \frac{\beta_3}{2} V_{OV3}^2 (1 + \lambda V_{DS3})$$

$$\begin{cases} I_{out} = \frac{\beta_2}{2} \times 0.25^2 \times [1 + \lambda(0.25 + \Delta V_{DS2})] & -① \\ I_{out} = \frac{\beta_3}{2} \times (0.25 - \Delta V_{DS2})^2 [1 + \lambda(1.5 - \Delta V_{DS2})] & -② \end{cases}$$

由 ① = ② 推得

$$\Rightarrow -0.1 \Delta V_{DS2}^3 + 1.1 \Delta V_{DS2}^2 - 0.5875 \Delta V_{DS2} + 7.8125m = 0$$

$$\Rightarrow \Delta V_{DS2} = 10.437V,\ 0.5484V,\ \underline{0.01364V}\ \checkmark$$

$$I_{out} = 400u \times \frac{1 + \lambda(V_{DS1} + \Delta V_{DS2})}{1 + \lambda V_{DS1}} \simeq 400.531uA \ \#$$

<法二>

$$R_{out} \simeq g_{m3} r_{o3} r_{o2}$$

$$\Delta V_{DS2} = \Delta V \times \frac{r_{o2}}{R_{out}} = \frac{\Delta V}{g_{m3} r_{o3}}$$

$$I_{out} = 4 \times I_{REF}(1 + \lambda \Delta V_{DS2})$$

$$g_{m3} r_{o3} = \frac{2I_D}{V_{ov}} \times \frac{1}{\lambda I_D} = \frac{2}{\lambda V_{ov3}} \simeq \frac{2}{\lambda V_{ov0}} = 80$$

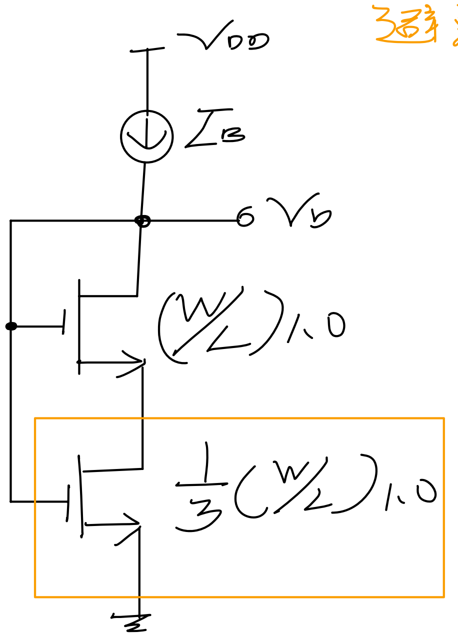
$$\Delta V_{DS2} = \frac{1V}{80} = 0.0125V$$

$$\Rightarrow I_{out} = 4 \times 100u \times (1 + 0.1 \times 0.0125) \simeq 400.5uA$$ #

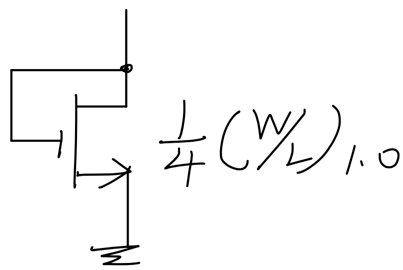
(c)

設計 $V_b = 2V_{ov} + V_{TH}$ 則可得到 $V_{D3(min)}$

實際上最好保留額外的margin
避免因為Body effect使M_2跑進triode



可等效為
$\Rightarrow$



這顆的 $\frac{1}{3}(W/L)_{1.0}$，一樣是利用等效長度串聯的方式來去實現
並且利用相同W和L的單位電晶體

HW5.3

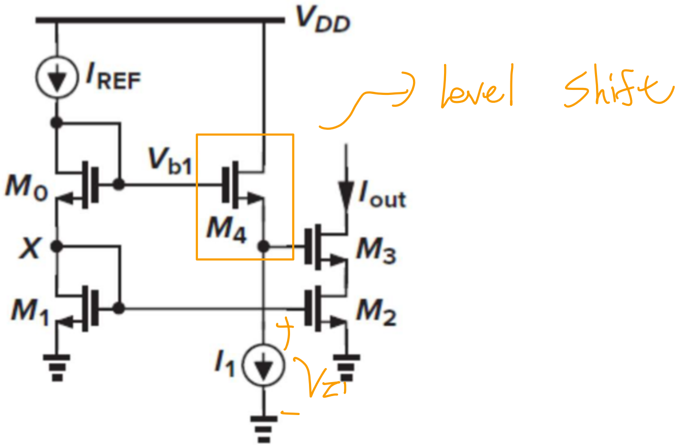


Fig. 5.3

由於 $M_{0\sim3}$ size 皆相同
因此 V_{ov} 相同

ⓐ

$\gamma = 0 \Rightarrow$ All $V_{TH(z)} = V_{th0}$

$V_{b1} = V_{GS1} + V_{GS0} = 2V_{ov} + V_{TH0} + V_{TH1} = 2V_{ov} + 2V_{th0}$

$V_{DS2} = V_{b1} - V_{GS4} - V_{GS3}$

M_4 using wide transistor 意謂著 V_{ov} 極小

也就是說 $V_{GS4} = V_{ov4} + V_{TH4} \simeq V_{TH4} = V_{th0}$

$\Rightarrow V_{DS2} \simeq V_{ov}$

$$\frac{I_{out}}{I_{REF}} = \frac{1+\lambda V_{ov}}{1+\lambda(V_{ov}+V_{th0})} \quad \#$$

ⓑ $\gamma \neq 0$

M_0、M_3、M_4 需考慮 Body effect

$$V_{TH0} = V_{th0} + \gamma\left(\sqrt{2\phi_f + V_{GS1}} - \sqrt{2\phi_f}\right)$$

$$V_{TH3} = V_{th0} + \gamma\left(\sqrt{2\phi_f + V_{DS2}} - \sqrt{2\phi_f}\right)$$

$$V_{TH4} = V_{th0} + \gamma\left(\sqrt{2\phi_f + V_{Z1}} - \sqrt{2\phi_f}\right)$$

$$V_{b1} = 2V_{ov} + V_{TH0} + V_{TH1}$$

$$V_{DS2} \simeq V_{b1} - V_{GS3} - V_{TH4}$$

$$= 2V_{ov} + V_{TH0} + V_{TH1} - (V_{ov} + V_{TH3}) - V_{TH4}$$

$$= V_{ov} + V_{TH1} + V_{TH3} - V_{TH4}$$

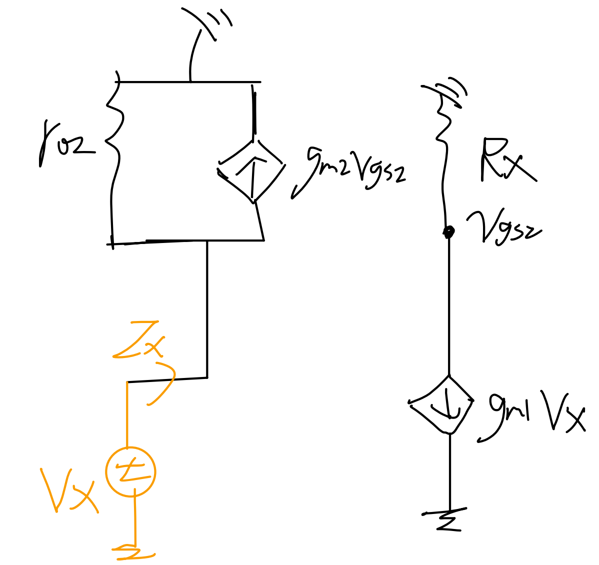
$$V_{DS1} = V_{ov} + V_{TH1}$$

$$\frac{I_{out}}{I_{REF}} = \frac{1+\lambda(V_{ov} + V_{TH1} + V_{TH3} - V_{TH4})}{1+\lambda(V_{ov} + V_{TH1})} \quad \#$$

OR

$$\frac{I_{out}}{I_{REF}} = \frac{1+\lambda\left[V_{ov} + V_{th0} + \gamma\left(\sqrt{2\phi_f + V_{DS2}} - \sqrt{2\phi_f + V_{Z1}}\right)\right]}{1+\lambda(V_{ov} + V_{th0})} \quad \#$$

HW5.4



$$R_x = \frac{1}{g_{m3}} \parallel r_{o3} \parallel r_{o1} \parallel \frac{1}{sC_1}$$

$$\simeq \frac{1}{g_{m3}} \parallel \frac{1}{sC_1} = \frac{1}{g_{m3}+sC_1}$$

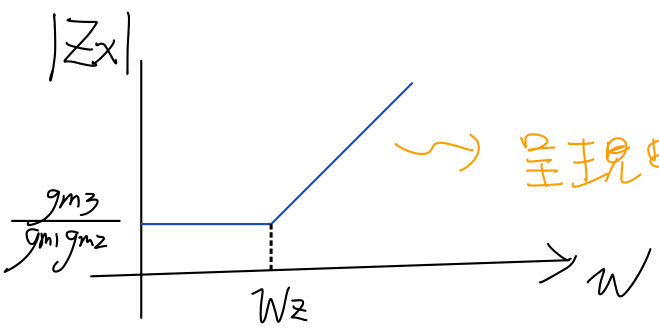
$$V_{gs2} = -g_{m1}V_xR_x$$

$$I_x = \frac{V_x}{r_{o2}} + g_{m2}V_{gs2}$$

$$\Rightarrow I_x = V_x\left(\frac{1}{r_{o2}} - g_{m1}g_{m2}R_x\right)$$

$$\Rightarrow \frac{V_x}{I_x} = r_{o2} \parallel \frac{-1}{g_{m1}g_{m2}R_x} = r_{o2} \parallel \frac{-(sC_1+g_{m3})}{g_{m1}g_{m2}}$$

$$\Rightarrow \frac{V_x}{I_x} = r_{o2} \parallel -\left(\frac{sC_1}{g_{m1}g_{m2}} + \frac{g_{m3}}{g_{m1}g_{m2}}\right) \simeq -\left(\frac{sC_1}{g_{m1}g_{m2}} + \frac{g_{m3}}{g_{m1}g_{m2}}\right)$$



$$W_z = \frac{g_{m3}}{C_1}$$

⟶ 呈現电感性，阻抗随w上升而上升