1102 Calculus Midterm Exam Date:2022/05/26 Total:100

1. Find the interval of convergence: $\sum_{n=1}^{\infty}(-1)^n \frac{(x-2)^n}{4^n \cdot \sqrt{n}}$ (10%)

2. Find the first three terms of the Maclaurin series: $f(x) = e^x sinx$ (10%)

3. Find the Taylor series generated by $f(x) = e^{-x}$ at $x = -2$. (10%)

4. Find a Cartesian equation and graph the Cartesian equation.

   $x = (sec^2 t) - 1, y = tant, \frac{-\pi}{2} < t < \frac{\pi}{2}$ (10%)

5. Find the perimeter(周長) of $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ (10%)

6. Replace the following polar equation by equivalent Cartesian equations.

   $r = \frac{4}{2cos\theta - sin\theta}$ (10%)

7. Graph the curve $r = 1 - sin\theta$ in the Cartesian xy-plane. (10%)

8. Find the area of top half of the region inside the cardioid $r = 1 + cos\theta$ and outside the circle $r = cos\theta$. (10%)

9. Find the function's domain and range. $f(x, y) = \frac{1}{\sqrt{16 - x^2 - y^2}}$ (10%)

10. Let $f(x, y) = \frac{xy + y^3}{x^2 + y^2}$. Does $\lim_{(x,y)\to(0,0)} f(x, y)$ exist? (hint: two path test)

    (10%)