1. 10pts

To find where $x^4 - 2x^2 \le 0$, let $x^4 - 2x^2 = 0 \Rightarrow x^2(x^2-2) = 0 \Rightarrow x = 0$ or $x = \pm\sqrt{2}$. By the sign graph, $\underset{-\sqrt{2}}{++++++0}\;\underset{0}{--0}\;\underset{\sqrt{2}}{--0}+++++++$, we can see that $x^4 - 2x^2 \le 0$ on $\left[-\sqrt{2}, \sqrt{2}\right] \Rightarrow a = -\sqrt{2}$ and $b = \sqrt{2}$ minimize the integral.

2.

$$y = \int_2^{\sin x} \frac{dt}{\sqrt{1-t^2}},\ |x| < \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-\sin^2 x}}\left(\frac{d}{dx}(\sin x)\right) = \frac{1}{\sqrt{\cos^2 x}}(\cos x) = \frac{\cos x}{|\cos x|} = \frac{\cos x}{\cos x} = 1 \text{ since } |x| < \frac{\pi}{2}$$

3. ~0 5pts , 0~ 5pts

On $\left[-\frac{\pi}{4}, 0\right]$: The area of the rectangle bounded by the lines $y = \sqrt{2}$, $y = 0$, $\theta = 0$, and $\theta = -\frac{\pi}{4}$ is $\sqrt{2}\left(\frac{\pi}{4}\right) = \frac{\pi\sqrt{2}}{4}$. The area between the curve $y = \sec\theta\tan\theta$ and $y = 0$ is $-\int_{-\pi/4}^{0} \sec\theta\tan\theta\, d\theta = [-\sec\theta]_{-\pi/4}^{0} = (-\sec 0) - \left(-\sec\left(-\frac{\pi}{4}\right)\right) = \sqrt{2} - 1$. Therefore the area of the shaded region on $\left[-\frac{\pi}{4}, 0\right]$ is $\frac{\pi\sqrt{2}}{4} + \left(\sqrt{2} - 1\right)$.

On $\left[0, \frac{\pi}{4}\right]$: The area of the rectangle bounded by $\theta = \frac{\pi}{4}$, $\theta = 0$, $y = \sqrt{2}$, and $y = 0$ is $\sqrt{2}\left(\frac{\pi}{4}\right) = \frac{\pi\sqrt{2}}{4}$. The area under the curve $y = \sec\theta\tan\theta$ is $\int_0^{\pi/4} \sec\theta\tan\theta\, d\theta = [\sec\theta]_0^{\pi/4} = \sec\frac{\pi}{4} - \sec 0 = \sqrt{2} - 1$. Therefore the area of the shaded region on $\left[0, \frac{\pi}{4}\right]$ is $\frac{\pi\sqrt{2}}{4} - \left(\sqrt{2} - 1\right)$. Thus, the area of the total shaded region is $\left(\frac{\pi\sqrt{2}}{4} + \sqrt{2} - 1\right) + \left(\frac{\pi\sqrt{2}}{4} - \sqrt{2} + 1\right) = \frac{\pi\sqrt{2}}{2}$.

4. depends

Let $u = \cos\sqrt{\theta} \Rightarrow du = \left(-\sin\sqrt{\theta}\right)\left(\frac{1}{2\sqrt{\theta}}\right) d\theta \Rightarrow -2\, du = \frac{\sin\sqrt{\theta}}{\sqrt{\theta}}\, d\theta$

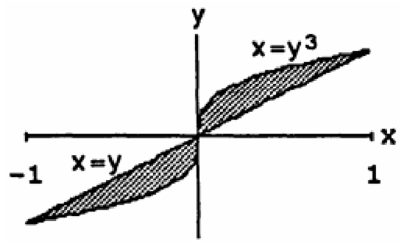
$$\int \frac{\sin\sqrt{\theta}}{\sqrt{\theta\cos^3\sqrt{\theta}}}\, d\theta = \int \frac{\sin\sqrt{\theta}}{\sqrt{\theta}\sqrt{\cos^3\sqrt{\theta}}}\, d\theta = \int \frac{-2\, du}{u^{3/2}} = -2\int u^{-3/2}\, du = -2\left(-2u^{-1/2}\right) + C = \frac{4}{\sqrt{u}} + C$$
$$= \frac{4}{\sqrt{\cos\sqrt{\theta}}} + C$$

5. depends

For the sketch given, $c = 0$, $d = 1$; $f(y) - g(y) = (12y^2 - 12y^3) - (2y^2 - 2y) = 10y^2 - 12y^3 + 2y$;

$$A = \int_0^1 (10y^2 - 12y^3 + 2y)\, dy = \int_0^1 10y^2\, dy - \int_0^1 12y^3\, dy + \int_0^1 2y\, dy = \left[\frac{10}{3}y^3\right]_0^1 - \left[\frac{12}{4}y^4\right]_0^1 + \left[\frac{2}{2}y^2\right]_0^1$$
$$= \left(\frac{10}{3} - 0\right) - (3 - 0) + (1 - 0) = \frac{4}{3}$$

6. A=2A2

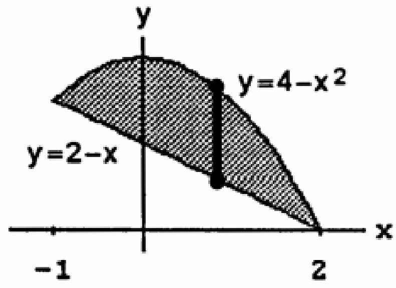


$A = A_1 + A_2$

Limits of integration: $x = y^3$ and $x = y \Rightarrow y = y^3$
$\Rightarrow y^3 - y = 0 \Rightarrow y(y-1)(y+1) = 0 \Rightarrow c_1 = -1, d_1 = 0$
and $c_2 = 0, d_2 = 1$; $f_1(y) - g_1(y) = y^3 - y$ and
$f_2(y) - g_2(y) = y - y^3 \Rightarrow$ by symmetry about the origin,
$A_1 + A_2 = 2A_2 \Rightarrow A = 2\int_0^1 (y - y^3)\, dy = 2\left[\frac{y^2}{2} - \frac{y^4}{4}\right]_0^1$
$= 2\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{1}{2}$

7. A^2-B^2 =>5 pts

$r(x) = 2 - x$ and $R(x) = 4 - x^2$



$$\Rightarrow V = \int_{-1}^{2} \pi \left([R(x)]^2 - [r(x)]^2\right) dx$$
$$= \pi \int_{-1}^{2} \left[(4-x^2)^2 - (2-x)^2\right] dx$$
$$= \pi \int_{-1}^{2} \left[(16 - 8x^2 + x^4) - (4 - 4x + x^2)\right] dx$$
$$= \pi \int_{-1}^{2} (12 + 4x - 9x^2 + x^4)\, dx$$
$$= \pi \left[12x + 2x^2 - 3x^3 + \tfrac{x^5}{5}\right]_{-1}^{2}$$
$$= \pi \left[\left(24 + 8 - 24 + \tfrac{32}{5}\right) - \left(-12 + 2 + 3 - \tfrac{1}{5}\right)\right] = \pi \left(15 + \tfrac{33}{5}\right) = \tfrac{108\pi}{5}$$

8. Formula > 5pts

For the sketch given, $a = 0$, $b = 3$;

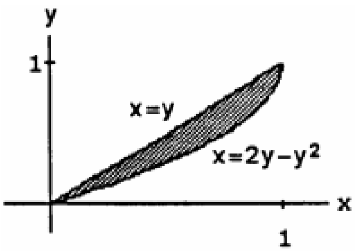
$$V = \int_a^b 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dx = \int_0^3 2\pi x \left(\frac{9x}{\sqrt{x^3+9}}\right) dx;$$

$$[u = x^3 + 9 \Rightarrow du = 3x^2\, dx \Rightarrow 3\, du = 9x^2\, dx;\ x = 0 \Rightarrow u = 9,\ x = 3 \Rightarrow u = 36]$$

$$\rightarrow V = 2\pi \int_9^{36} 3u^{-1/2}\, du = 6\pi \left[2u^{1/2}\right]_9^{36} = 12\pi\left(\sqrt{36} - \sqrt{9}\right) = 36\pi$$

9. Formula > 5pts

$c = 0$, $d = 1$;



$$V = \int_c^d 2\pi \left(\begin{smallmatrix}\text{shell}\\ \text{radius}\end{smallmatrix}\right)\left(\begin{smallmatrix}\text{shell}\\ \text{height}\end{smallmatrix}\right) dy = \int_0^1 2\pi y\,(2y - y^2 - y)\,dy$$
$$= 2\pi \int_0^1 y\,(y - y^2)\, dy = 2\pi \int_0^1 (y^2 - y^3)\, dy$$
$$= 2\pi \left[\tfrac{y^3}{3} - \tfrac{y^4}{4}\right]_0^1 = 2\pi\left(\tfrac{1}{3} - \tfrac{1}{4}\right) = \tfrac{\pi}{6}$$

10. (Dy/dx)^2 > 5pts

$$\frac{dy}{dx} = \sqrt{3x^4 - 1} \Rightarrow \left(\frac{dy}{dx}\right)^2 = 3x^4 - 1$$
$$\Rightarrow L = \int_{-2}^{-1} \sqrt{1 + (3x^4 - 1)}\, dx = \int_{-2}^{-1} \sqrt{3}\, x^2\, dx$$
$$= \sqrt{3}\left[\tfrac{x^3}{3}\right]_{-2}^{-1} = \tfrac{\sqrt{3}}{3}\left[-1 - (-2)^3\right] = \tfrac{\sqrt{3}}{3}(-1 + 8) = \tfrac{7\sqrt{3}}{3}$$