1. (**15** points) Evaluate the following integrals. (5 points for each)

(a) $\displaystyle\int_0^{\pi/2} \cos x \sin(\sin x)\, dx$ (b) $\displaystyle\int_9^{64} \frac{1}{\sqrt{x}\left(\sqrt{1+\sqrt{x}}\right)}\, dx$ (c) $\displaystyle\int_0^1 x^3 \left(1+9x^4\right)^{-3/2} dx$

2. (**10** points) Find the areas of the region bounded by $y = \sin x$ and $y = \sin^2 x$, between $x = 0$ and $x = \pi/2$.

3. (**10** points) Air is pumped into a spherical balloon so that to volume increases at a rate of 100 (cm^3/s). How fast is the radius of the balloon increasing when diameter is 50 (cm)?

4. (**10** points) Find the length of the arc of the curve $x^2 = (y-4)^3$ from point $P(1,5)$ to point $Q(8,8)$.

5. (**10** points) Let $0 < a < b$. Use the mean value theorem to show that

$$\sqrt{b} - \sqrt{a} < \frac{b-a}{2\sqrt{a}}.$$

(Hint): Use the function $f(x) = \sqrt{x}$.

6. A function is defined as

$$f(x) = \int_1^{x^2} \frac{1}{\sqrt{1+t^2}}\, dt.$$

(a) (**2** points) Find $f(1)$.
(b) (**3** points) Find $f'(x)$.
(c) (**5** points) Find the linearization of $f(x)$ at $x = 1$.
(d) (**5** points) At which x the function $f(x)$ has a minimum value.

7. (**10** points) Find the dimensions of the circular cylinder of **greatest** volume that can be inscribed in a cone of base radius R and height H if the base of the cylinder lies in the base of the cone. Please express the radius and height of the cylinder in terms of R and H.

8. (**10** points) Find the volume of the solid generated by revolving the region between the x-axis and the curve $y = x^2 - 2x$ about the line $y = 2$.

9. (**10** points) Find the exact area of the surface obtained by rotating the curve $x = \dfrac{1}{3}\left(y^2+2\right)^{3/2}$, $1 \le y \le 2$ about the x-axis.

10. Let $f(x) = x\sqrt{2-x^2}$.

(a) (**4** points) Find the domain of the function $f(x)$.
(b) (**6** points) Find the intervals of increase and decrease.
(c) (**4** points) Find the intervals of concavity.
(d) (**4** points) Find the local maximum and minimum values.
(e) (**2** points) Find the inflection points.