Electromagnetics final exam

2023-06 ET3103301, ET3103302

1. (10%) Find the capacitance per unit length of two coaxial metal cylindrical tubes, of radii a and b, as shown in Fig.1.
   (15%)The coaxial cylindrical metal tubes (inner radius a, outer radius b) stands vertically in a tank of dielectric oil (susceptibility χ_e, mass density ρ). The inner one is maintained at potential $\mathcal{V}$, and the outer one is grounded, as shown in Fig.1-1. To what height (h) does the oil rise in the space between the tubes?

   Hint: $\vec{F}_v = F_g$, $F_g = mg = \rho\pi(b^2 - a^2)hg$

2. (10%) Two infinite insulated conducting plates maintained at potentials 0 and V_0 form a wedge-shaped configuration, as shown in Fig.2. Determine the potential distributions for the regions:
   1) $0 < \phi < a$
   2) $a < \phi < 2\pi$

3. (10%) A parallel-plate capacitor of width $\mathcal{W}$, length $\mathcal{L}$, and separation d has a solid dielectric slab of permittivity ε in the space between the plates as indicated in Fig.3. Determine:
   1) F_v when switch off (short) if the capacitor starts charging.
   2) The capacitance if the capacitor has been charged to a voltage V_0 while switch is on (open). (Use Gauss's law to solve).

   Note: 1) and 2) are individual questions.

4. (10%) A point charge Q is located at distances $a\ell$ and ℓ, respectively, from two grounded perpendicular conducting half-planes, as shown in Fig.4, where $a = (\frac{2\pi}{2+\pi})^{\frac{2}{3}} > 1$. After releasing the charge from rest, the charge will strike on the plane. Please determine (ignore radiation loss) the position where the charge will strike.

5. (15%) A 5V DC voltage applied to the ends of a 2km conducting wire with 1mm² cross section results in a current of 0.2A shown in Fig.5. Find (a) the conductivity of the wire, (b) the electric field intensity in the wire, (c) the power dissipated in the wire.

6. (10%) A metal bar of conductivity σ is bent to form a flat 90° sector of inner radius a, outer radius b, and thickness t shown in Fig.6. Find the resistance of the bar between the vertical curved surfaces at $\rho = a$ and $\rho = b$ if $b/a = 6/5$, $\sigma = 4\times10^7$ S/m and $t = 0.5$ cm.

7. (10%) A conducting sphere of radius a is surrounded by free space, shown in Fig.7. Initially, a charge density of ρ_{v0} is distributed uniformly throughout the sphere. Please derive the current density $\vec{J}$ of the sphere at t = 0 and t→∞. The dielectric constant and conductivity of the sphere are ε and σ, respectively.

8. (10%) If the magnetic flux density is given by $\vec{B} = \hat{a}_x 666x + \hat{a}_y 66y + \hat{a}_z 6cz$, find c .

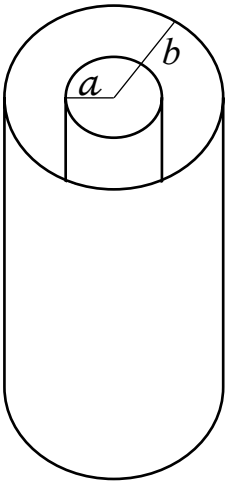

Fig.1

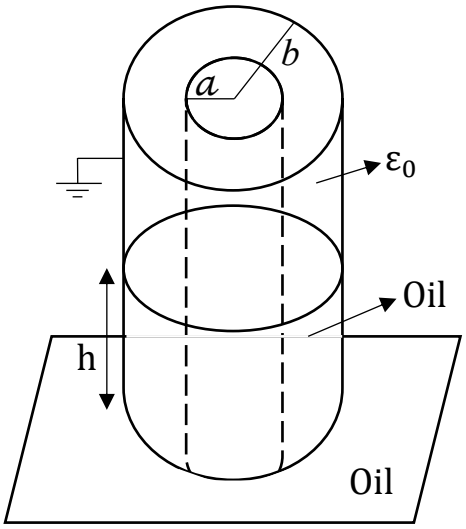

Fig.1-1

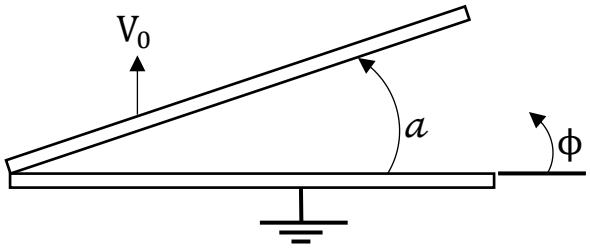

Fig.2

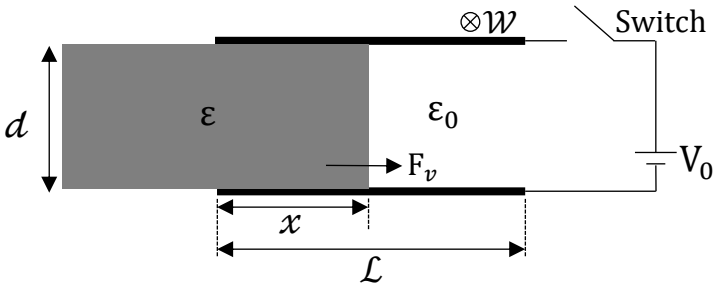

Fig.3

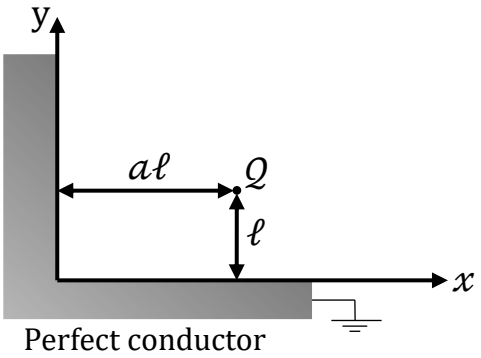

Fig.4

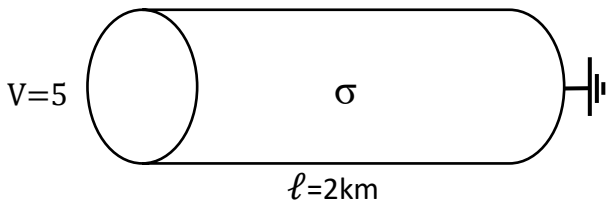

Fig.5

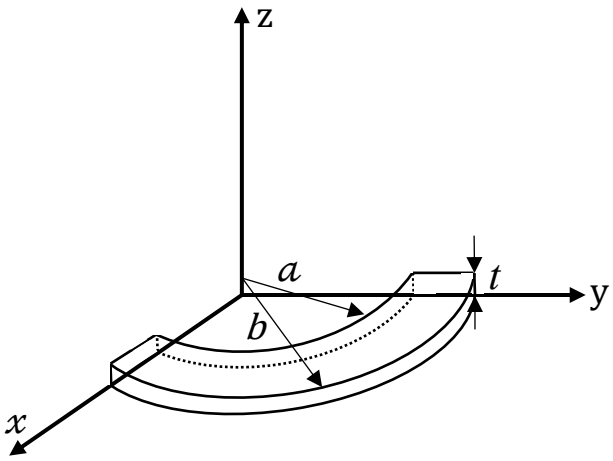

Fig.6

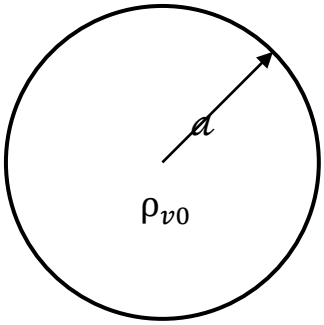

Fig.7