

Total: 100 points

1. Let $f(x) = \frac{\sqrt{\pi x}(x-7)}{|x-7|}$

(a) (20 points) Find $\lim_{x \rightarrow 7^+} \frac{\sqrt{\pi x}(x-7)}{|x-7|}$

(b) (20 points) Find $\lim_{x \rightarrow 7^-} \frac{\sqrt{\pi x}(x-7)}{|x-7|}$

2. Assume $f(x) = x^4 \cos\left(\frac{2}{x}\right)$.

(a) (20 points) Show that $-x^4 \leq x^4 \cos\left(\frac{2}{x}\right) \leq x^4$

(b) (20 points) Find $\lim_{x \rightarrow 0} x^4 \cos\left(\frac{2}{x}\right)$

3. (20 points) Use $\epsilon - \delta$ definition to prove the following limit.

$$\lim_{x \rightarrow 2} (2 - 3x) = -4$$