

101-01 Calculus Midterm Exam (Date: 2012/11/20; 100 minutes, total: 106 points)

1. Find  $\lim_{t \rightarrow 2} \frac{\sqrt{1+\sqrt{2+t}}-\sqrt{3}}{t-2} = ?$  (8 points)

2. Find  $\lim_{x \rightarrow \infty} f(x)$ , when  $f(x) = \sqrt{x}(\sqrt{x+1} - \sqrt{x})$ ,  $x \geq 0$ . (8 points)

3. If  $f(x) = \sin x \cdot \sin(2x^2) \cdot \sin(3x^3) \cdot \sin(4x^4)$ ,  $\lim_{x \rightarrow 0} \frac{f(x)}{x^{10}} = ?$  (8 points)

4. Show that  $\lim_{x \rightarrow 2} \sqrt{2x} = 2$  by the  $\epsilon - \delta$  method. (10 points)

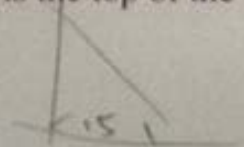
5. Suppose that  $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x > 0 \\ mx + b, & x \leq 0 \end{cases}$  is differentiable, find  $m$  and  $b$ . (8 points)

6. If  $f(x) = \sqrt{2x + \sqrt{2x + \sqrt{2x + \dots}}}$ , find  $f'(x) = ?$  (8 points)

7. Show that the tangent line to the curve  $x + 2x^2 - y - 5xy = 0$  at  $(0,0)$  is perpendicular to the one to the curve  $y^2 + 3xy + x^2 - x - y = 0$  at  $(0,0)$ . (10 points)

8. Write a formula that estimates the change that occurs in the lateral surface area of a right circular cone when the height changes from  $h_0$  to  $h_0 + dh$  and the radius does not change.  $S = \pi r \sqrt{r^2 + h^2}$  (Lateral surface area) (6 points)

9. A 25-m ladder is leaning against a wall when its base starts to slide away. By the time the base is 15m from the wall, the base is moving at the rate of 3m/sec. How fast is the top of the ladder sliding then? Down or Up? (10 points)



10. Show that the equation  $2x^7 = 1 - x$  has exactly one real solution. (8 points)
11. If  $f(x) = \frac{x}{(x-1)^2}$ , (12 points)
- Identify the intervals on which  $f(x)$  is increasing and decreasing.
  - Find the function's local maximum and local minimum.
  - Identify the intervals on which  $f(x)$  is concave up and concave down.
12. A manufacturer estimates that when  $x$  thousand units of a particular commodity are produced each week, the total cost will be  $C(x) = 0.4x^2 + 3x + 40$  thousand dollars, and all  $x$  units can be sold at a price of  $p(x) = 22.2 - 1.2x$  dollars per unit. (10 points)
- Determine the level of production that results in maximum profit. What is the maximum profit?
  - At what level of production is the average cost per unit  $A(x) = \frac{C(x)}{x}$  minimized?