Engineering mathematics II   Midterm exam., 4/20/2021

This is an open-book test. The total score is 100 points. Please show your computations.

1.(10%) Consider the matrix

$$\underline{\underline{A}} = \begin{bmatrix} 1 & -3 & 0 & -2 \\ -2 & 3 & -1 & -1 \end{bmatrix}.$$

Assume that $\underline{\underline{A}}$ is the augmented matrix corresponding to a system of linear equations. Write down the system of linear equations corresponding to $\underline{\underline{A}}$, and then solve it.

2.(10%) Show that if $\underline{\underline{A}}$ is a 2×2 lower triangular matrix, then $\underline{\underline{A}}^n$ (i.e. product of n items of $\underline{\underline{A}}$) is also a 2×2 lower triangular matrix.

3. Consider the matrix

$$\underline{\underline{A}} = \begin{bmatrix} 2 & -3 \\ -7 & -11 \end{bmatrix}.$$

(a). **(5%)** Find $\underline{\underline{A}}^{-1}$ (i.e. the inverse of $\underline{\underline{A}}$).

(b). **(10%)** Express $\underline{\underline{A}}^{-1}$ as a product of elementary matrices.

4. Consider the matrix

$$\underline{\underline{A}} = \begin{bmatrix} 2 & -3 & 1 \\ -7 & 0 & -11 \\ 4 & 5 & 9 \end{bmatrix}.$$

(a). **(5%)** $\det(\underline{\underline{A}}) = ?$

(b). **(5%)** Find $\underline{\underline{A}}^{-1}$.

(c). **(5%)** $\det(\underline{\underline{A}}^{-1}) = ?$

5.(10%) Let V be the vector space of real functions whose first three derivatives exist. Find the values of α, β, and γ such that $W = \{f | f''' + 2\alpha f'' - 3\beta f' + 5f = \gamma\}$ is a subspace of V.

6. It is known that $\mathcal{P}_2 = \{a + bx + cx^2 | a, b, c \in \mathcal{R}\}$ (i.e. set of polynomials of degrees no greater than 2) with the usual polynomial addition and the usual number-to-polynomial multiplication is a vector space. Let us consider $S = \{1, 1 + x, 1 + x^2\}$ in $\mathcal{P}_2$.

(a). **(5%)** Show that S is linearly independent.

(b). **(5%)** Let us adopt S as an ordered basis for $\mathcal{P}_2$. Find the coordinate vector of $2 - x + 3x^2$ with respect to S.

7. Consider the matrix

$$
\underline{\underline{A}} = \begin{bmatrix}
11 & 30 & -8 & 153 & -1 & 0 & 21 \\
-51 & -50 & -52 & -353 & -3 & -64 & 75 \\
21 & 35 & 7 & 203 & 0 & 16 & -3 \\
121 & -33 & 275 & 231 & 21 & 28 & 11 \\
1 & 25 & -23 & 103 & -2 & -16 & 45
\end{bmatrix}.
$$

Its rref (reduced row echelon form) is

$$
\begin{bmatrix}
1 & 0 & 2 & 3 & 0 & 39.50 & -81.16 \\
0 & 1 & -1 & 4 & 0 & -23.24 & 48.61 \\
0 & 0 & 0 & 0 & 1 & -262.80 & 544.53 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}.
$$

(a). (4%) Find a basis for row-spsce($\underline{\underline{A}}$).

(b). (4%) Find a basis for column-spsce($\underline{\underline{A}}$).

(c). (4%) row-rank($\underline{\underline{A}}$) = ?

(d). (4%) column-rank($\underline{\underline{A}}$) = ?

(e). (4%) rank($\underline{\underline{A}}$) = ?

(f). (5%) Find a basis for null-spsce($\underline{\underline{A}}$).

(g). (5%) nullity($\underline{\underline{A}}$) = ?