Final Exam. Calculus Et1601001, Et1601002, 2014/6/17

1 Find the slope of the tangent line to the polar curve $r=\cos(3\theta)$ at $\theta=\frac{3}{4}\pi$ (8%)

2 Find the area of the region that is inside the cardioid $r=2+2\cos\theta$ and outside the circle $r=3$ (10%)

3 Find the linear approximation (linearization) of the function $f(x,y)=\sqrt{20-x^2-7y^2}$ and use it to approximate $f(1.95,1.08)$ (10%)

4(a) If the electric potential at a point (x,y) in the $xy-$plane is $V(x,y)$, then the electric intensity vector at the point (x,y) is $\vec{E}=-\nabla V(x,y)$. Suppose that $V(x,y)=e^{-2x}\cos(2y)$. Find the electric intensity vector at $(\frac{\pi}{4},0)$ (8%)

(b) Let $r=\sqrt{x^2+y^2}$. Show that $\nabla r=\frac{\vec{r}}{r}$, where $\vec{r}=x\vec{i}+y\vec{j}$ (8%)

5 If $e^{xy}\cos(yz)-e^{yz}\sin(xz)-2=0$, find $\frac{\partial z}{\partial x},\frac{\partial z}{\partial y}$ (10%)

6 Find the tangent plane and the normal line to the ellipsoid surface $x^2+y^2+4z^2=12$ at the point (2,2,1) (10%)

7 Locate all relative extrema and saddle points(if any) of $f(x,y)=4xy-x^4-y^4$ (12%)

8 Find the shortest distance from the origin to the surface $xyz^2=2$ (10%)

9 Evaluate $\int_0^4\int_{\sqrt{y}}^2 e^{x^3}\,dxdy$ (10%) (Hint: change the order of integration)

10 Evaluate $\int_0^2\int_0^{\sqrt{2x-x^2}}\sqrt{x^2+y^2}\,dydx$ (10%) (Hint: change to polar coordinates)

11 Use cylindrical coordinates to find the volume of the solid enclosed by the paraboloid $z=x^2+y^2$ and the plane $z=9$ (10%)