國立臺灣科技大學答案卷

National Taiwan University of Science and Technology Answer Sheet

評分 Score	教師簽章 Signature of Lecturer

姓名/Name 微文積分助教 學號/Student ID _____ 班級/Class _____

科目/Course title 微積分 日期/Date _____

記分欄

從此處開始寫起。試卷用紙務須節用，非經主試認可不得續用其他紙張作答。/Please write from here.

1. $f'(x)=\frac{x^2(x-1)}{x+2}$, $x\neq -2$

critical points: $x=0, 1, -2$

$f'(x)>0$ $f'(x)<0$ $f'(x)<0$ $f'(x)>0$

-2 0 1

increasing on $(-\infty,-2)$ and $(1,\infty)$

decreasing on $(-2,0)$ and $(0,1)$

2. $y=x^4-2x^2$

① Domain and Symmetries

$D=\{x \mid x\in \mathbb{R}\}$

$f(-x)=f(x) \Rightarrow$ even function

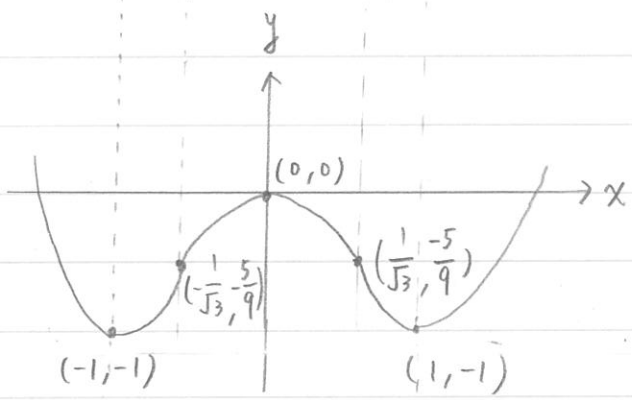
② $y'=4x^3-4x=4x(x+1)(x-1)$

$y''=12x^2-4=12\left(x^2-\frac{1}{3}\right)=12\left(x+\frac{1}{\sqrt{3}}\right)\left(x-\frac{1}{\sqrt{3}}\right)$

$f'(x)<0$ $f'(x)>0$ $f'(x)<0$ $f'(x)>0$

-1 0 1

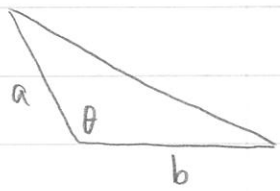
$f''(x)>0$ $-\frac{1}{\sqrt{3}}$ $f''(x)<0$ $\frac{1}{\sqrt{3}}$ $f''(x)>0$



③ Asymptotes

They don't exist.

3.



$A=\frac{1}{2}ab\sin\theta$

$\frac{d}{d\theta}A=\frac{d}{d\theta}(ab\sin\theta)=ab\cos\theta$ critical point: $\frac{\pi}{2}$

$A=\frac{1}{2}ab\sin\left(\frac{\pi}{2}\right)=\frac{1}{2}ab$

4. $f(x)=x^2+1$ over $[0,3]$

$\Delta x=\frac{3-0}{n}$

$c_k=0+\frac{3}{n}k$

$f(c_k)=f\left(\frac{3k}{n}\right)=\left(\frac{3k}{n}\right)^2+1=\frac{9k^2}{n^2}+1$

c_k

0 $0+\frac{3}{n}k$ 3

a b

$S_p=\sum_{k=1}^{n}f(c_k)\Delta x=\sum_{k=1}^{n}f\left(\frac{3}{n}k\right)\frac{3}{n}$

$=\sum_{k=1}^{n}\left(\frac{9k^2}{n^2}+1\right)\frac{3}{n}$

$=\sum_{k=1}^{n}\left(\frac{27k^2}{n^3}+\frac{3}{n}\right)$

$=\frac{27}{n^3}\sum_{k=1}^{n}k^2+\frac{3}{n}\sum_{k=1}^{n}1$

$=\frac{27}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}+\frac{3}{n}\cdot n=3+9=12$ ※

$=3+\frac{27n(n+1)(2n+1)}{6n^3}$

$=3+\frac{9(2n^3+3n^2+n)}{2n^3}$

$\lim_{n\to\infty}S_p$

$=\lim_{n\to\infty}\left(3+\frac{9(2n^3+3n^2+n)}{2n^3}\right)$

$=\lim_{n\to\infty}\left(3+\frac{18+\frac{27}{n}+\frac{9}{n^2}}{2}\right)$

5. $\lim_{\|P\|\to 0}(\sec c_k)\Delta x_k$, $P=\left[\frac{-\pi}{4},0\right]$

$\int_{-\frac{\pi}{4}}^{0}\sec x\,dx$

6. $\frac{d}{dx}\int_1^{\sin x}3t^2dt$. Let $u=\sin x$

$=\left(\frac{d}{du}\int_1^{u}3t^2dt\right)\cdot\frac{du}{dx}$

$=3(\sin x)^2\cdot\frac{d}{dx}\sin x$

$=3\sin^2 x\cos x$ ※

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7. $\int 3y\sqrt{7-3y^2}\,dy$　Let $u=7-3y^2$

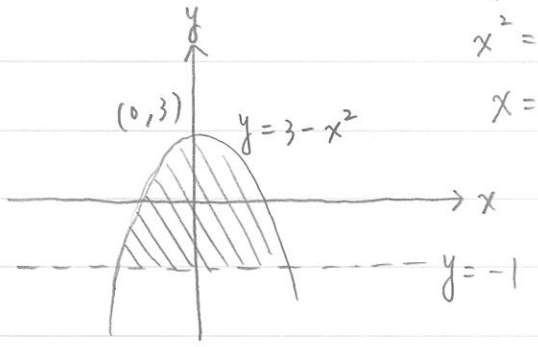
$du=-6y\,dy$

$=\int \sqrt{u}\cdot\left(-\frac{1}{2}du\right)$　$\Rightarrow \frac{-1}{2}du=3y\,dy$

$=-\frac{1}{3}u^{\frac{3}{2}}+C$

$=-\frac{1}{3}(7-3y^2)^{\frac{3}{2}}+C$ ※

8. $y=3-x^2$, $y=-1$



$y=3-x^2$

$x^2=3-y$

$x=\pm\sqrt{3-y}$

$A=\int_{-1}^{3}\left[\sqrt{3-y}-(-\sqrt{3-y})\right]dy$

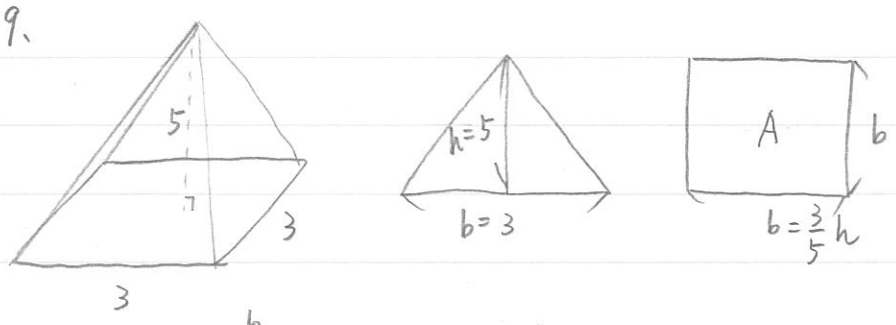
$=\int_{-1}^{3}2\sqrt{3-y}\,dy$　Let $u=3-y$

$du=-dy$

$=\int_{4}^{0}2\sqrt{u}\,(-du)$　$y=-1\Rightarrow u=4$; $y=3\Rightarrow u=0$

$=-\frac{4}{3}u^{\frac{3}{2}}\Big|_4^0=\frac{4}{3}\cdot 4^{\frac{3}{2}}=\frac{4}{3}\cdot 2^3=\frac{32}{3}$ ※

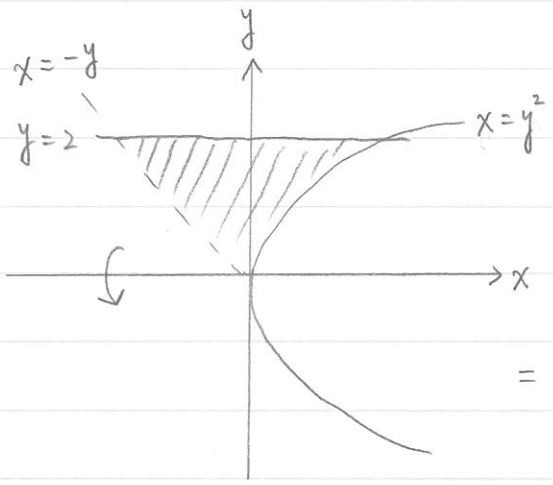
9.



$V=\int_a^b A(x)\,dx=\int_0^5\left(\frac{3}{5}\right)^2h^2\,dh$

$=\frac{9}{25}\cdot\frac{1}{3}h^3\Big|_0^5=\frac{3}{25}\cdot 5^3=15$ #

10.



$V=2\pi rh$

$V=\int_0^2 2\pi y\left(y^2-(-y)\right)dy$

$=\int_0^2 2\pi(y^3+y^2)\,dy$

$=2\pi\left(\frac{1}{4}y^4+\frac{1}{3}y^3\right)\Big|_0^2$

$=2\pi\left(4+\frac{8}{3}\right)=\frac{40}{3}\pi$ ※