

$$\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3} = \frac{\sin x}{\cos x} - \sin x = \frac{\sin x (1 - \cos x)}{\cos x}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1, \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}, \lim_{x \rightarrow 0} \frac{1}{\cos x} = 1$$

$$= \frac{1}{2} \cdot 1 = \frac{1}{2}$$

# 111-1 Calculus Midterm Exam

Date : 2022/10/26(Wed) 13:20-15:10 Total : 120 pts

Calculator is not allowed

1. Find the following limits (10 pts)

a.  $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$  (5 pts)

b.  $\lim_{x \rightarrow \infty} \left( x \sqrt{\frac{x-1}{x+1}} - x \right)$  (5 pts)

2. Find the derivative of following functions (20 pts)

a.  $f(x) = (x^2 + 1)(x + 5 + \frac{1}{x})$  (5 pts)

b.  $g(z) = \frac{(z-1)(z^2+z+1)}{z^3}$  (5 pts)

c.  $r(\theta) = \sin(\theta^2) \cos(2\theta)$  (5 pts)

d.  $y(t) = \left( 1 + \tan^4\left(\frac{t}{12}\right) \right)^3$  (5 pts)

3.  $f(4) = 0$  and  $f'(4) = 3$ ,  $\lim_{x \rightarrow 4} \frac{xf(4) - 4f(x)}{x-4} = ?$  (10 pts)

4.  $F(x) = f\left(\frac{x-1}{x+2}\right)$  and  $f'(y) = y^3$ , find  $\frac{dF(x)}{dx}$  (10 pts)

5.  $\lim_{x \rightarrow \infty} \left\{ \frac{x^2+1}{x+1} - ax - b \right\} = -2$ , find  $a + b = ?$  (10 pts)

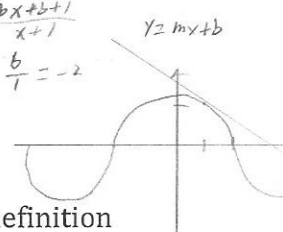
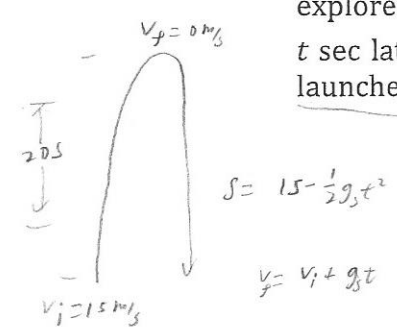
6. If  $f(x) = \begin{cases} 3x^2 - bx, & x \geq 1 \\ ax^2 + 2, & x < 1 \end{cases}$  and  $f'(1)$  exist, find  $a, b$  (10 pts)

7.  $f'(0) = 5$ , find  $\lim_{x \rightarrow 0} \frac{f(3x) - f(\sin x)}{x} = ?$  (10 pts)

8. Find the tangent line of  $f(x) = \cos x$  at point  $P\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$  using the definition of slope (Do not differential  $f(x)$  directly). (10 pts)

9.  $g(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ , is  $g(x)$  continuous at  $x=0$  or not? Explain and prove your answer. (10 pts)

10. Explorers on a small airless planet used a spring gun to launch a ball bearing vertically upward from the surface at a launch velocity of 15 m/sec. Because the acceleration of gravity at the planet's surface was  $g_s$  m/sec<sup>2</sup>, the explorers expected the ball bearing to reach a height of  $s = 15t - \frac{1}{2}g_s t^2$  m  $t$  sec later. The ball bearing reached its maximum height 20 sec after being launched. What was the value of  $g_s$ ? (8 pts)



11. The graphs in the accompanying figure show the position  $s$ , velocity  $v = \frac{ds}{dt}$ , and acceleration  $a = \frac{d^2s}{dt^2}$  of a body moving along a coordinate line as functions of time  $t$ . Which graph is which? Give reasons for your answers. (12 pts)

