112-1 Calculus Quiz 1

Chapter : 1-2~2-6

Date : 2023/10/11 13:20-15:10

Total : 110 pts

1. Find the domain and range of the following f ◦ g functions. (10 pts)
   $f(x) = \sqrt{x}$, $g(x) = 3 - 2x - x^2$

2. Determine if the following functions are an odd or even function. (10 pts)
   a. $f(x) = x|x|$ (5 pts)
   b. $g(x) = \frac{\tan x + \sin x + x^2 \sin x}{x^3}$ (5 pts)

3. Find the following limits ($[x]$: $Gauss\ function\ of\ x$) (40 pts)
   a. $\lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2}$ (8 pts) b. $\lim_{h\to 0} \frac{\sqrt{1+h+h^2}-1}{\sqrt{1+h}-\sqrt{1-h}}$ (8 pts)
   c. $\lim_{x\to 0^-} \frac{[x+1]+|x|}{x}$ (8 pts) d. $\lim_{x\to 0} \frac{\tan x - \sin x}{x^3}$ (8 pts)
   e. $\lim_{n\to+\infty} \sqrt{n}(\sqrt{n+1} - \sqrt{n})$ (8 pts)

4. Prove $\lim_{x\to 1}(5x - 3) = 2$ by the precise definition of the limit. (10 pts)

5. Show that $f(x) = \frac{x^2+x-6}{x^2-4}$ has a continuous extension to $x = 2$ and find the extension. (10 pts)

6. Show that the equation $x^3 - 15x + 1 = 0$ has three solutions in the interval [-4,4] (Hint: Intermediate Value Theorem) (10 pts)

7. Discuss the continuity of $f(\theta) = [\theta]\sin\theta$ at the interval $-1 \le \theta \le 1$. ([x]:Gauss function of x) (10 pts)

8. Find the horizontal and vertical asymptotes of $f(x) = \frac{x^2-x}{x^2-2x-3}$. (10 pts)