Engineering mathematics I

Midterm exam., 10/26/2023

This is an open-book test. The total score is 110 points. Please show your computations.

$\frac{21}{-3\times 2}$

1. Consider the first-order differential equation:

$$\frac{dy}{dx} = x^2 \cdot e^{-x} \cdot y^4 .$$

$48+4^{?}+8$

$\frac{3-1?+8}{5\times 2}$

Notice that this is a separable differential equation.

(a). (10%) Find the general solution to the differential equation.
<P.S.> You can refer to some table of integrations, if necessary.

(b). (5%) In addition to the differential equation, let us also impose the initial condition: $y(0) = -1$. Find the solution to this initial-value problem.

$\frac{96}{5\times 3}$ 32

2. Consider the differential equation:

$$\frac{dy}{dx} + \sin(x) \cdot y = e^{\cos(x)} .$$

Notice that this is a first-order linear differential equation.

$3s^2+3s+7s+17$

(a). (5%) Find the integration factor for this differential equation.

(b). (10%) Find the general solution to the differential equation.

(c). (5%) In addition to the differential equation, let us also impose the initial condition: $y(0) = 1$. Find the solution to the initial-value problem.

$\cancel{2xy}^2$

3. Consider the first-order differential equation:

$$(2x + y^2) + 2xy \cdot \frac{dy}{dx} = 0 .$$

$2x+y^2 \ \partial x$

x^2+xy^2

(a). (5%) Show that this is an exact differential equation.

$2xy\,\partial y$ xy^2

(b). (10%) Solve the differential equation.

(c). (5%) Continued from the preceeding subproblem, if it is further known that $y(1) = 2$, then $y(2) = ?$

$-c+17=$

4. Consider the initial-value problem:

$48+4^{?}+8$

$$y'' - 3y' - 4y = e^{-x} , \text{ and } \quad y(0) = 3 , \ y'(0) = -2 .$$

96

(a). (5%) Find the homogeneous solutions to the differential equation in this problem.

$\frac{96}{25}$

(b). (5%) Find a particular solution to the differential equation in this problem.

(c). (10%) Please solve this problem with the Laplace-transform method.

$(s+1)$

$(s-4)(s+1)$

$$y'' + \frac{1}{2x} y' - \frac{3}{2x^2} y = 0$$

$P \qquad Q$

5. Consider the second-order differential equation:

$$2x^2 \cdot \frac{d^2y}{dx^2} + x \cdot \frac{dy}{dx} - 3y = 0 \ .$$

(a). (5%) Show that x^{-1} is a solution to the differential equation.

(b). (10%) By applying the method of reduction of order, find another solution to the differential equation that is linearly independent with x^{-1}.

6. (10%) A linear time-invariant (LTI) system is described by the differential equation below, where $x(t)$ denotes the input signal, and $y(t)$ denotes the output signal:

$$4y^{(5)}(t) + 3y^{(4)}(t) - y'''(t) + y''(t) - 2y'(t) - 3y(t) = x(t) \ .$$

Find the transfer function of this system.

7. (10%) A linear time-invariant system (LTI) is described by the differential equation below, where $x(t)$ denotes the input signal, and $y(t)$ denotes the output signal:

$$y''(t) - 2y'(t) + 3y(t) = x(t) \ .$$

Find the impulse response of this system.

$x^{-1} \quad x^{\frac{3}{2}}$

$$t' + \frac{-2x^{-1} + \frac{1}{2x}x^{-1}}{x^{-1}} \qquad -x^{-1} \quad \frac{3}{2}x^{\frac{1}{2}}$$

$$-2 + \frac{1}{2x} \qquad \frac{3}{2}x^{-\frac{1}{2}} + x^{\frac{1}{2}}$$

$$\frac{-3}{2x}$$

$4s^5 Y.$

$1 \pm \sqrt{2}$

$s - (1+\sqrt{2}i) \quad s - (1-\sqrt{2}i)$

$$\frac{1}{s^2-2s+3} \qquad \frac{\sqrt{2}}{(s-1)^2+2} \quad \frac{s+1}{s^2+2} = \frac{s}{s^2-2s+3}$$

$$\frac{1}{(s-1)^2+2} \qquad s^3+2s = s^3 - 2s^2 + 3s + s^2 - 2s + 3$$

$-2s^2$

s^3