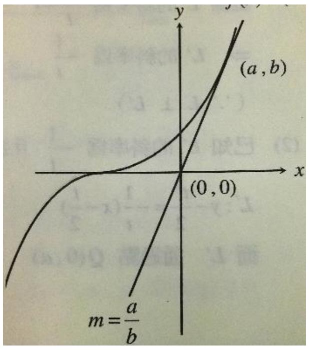
1.	**a.** 原式$=\lim_{x\to0}\left(\frac{x}{\tan x}\right).\left(x\sin\frac{1}{x}\right)=\lim_{x\to0}(\cos x)\lim_{x\to0}\left(\frac{x}{\sin x}\right).\lim_{x\to0}x\sin\frac{1}{x}=1\cdot0=0$ **b.** $\lim_{x\to0^+}\frac{\lvert x\rvert^3+x^3}{x}=\lim_{x\to0^+}\frac{x^3+x^3}{x}=\lim_{x\to0^+}2x^2=0$ $\lim_{x\to0^-}\frac{\lvert x\rvert^3+x^3}{x}=\lim_{x\to0^-}\frac{(-x)^3+x^3}{x}=\lim_{x\to0^-}\frac{0}{x}=0$ **c.** 原式 $=\lim_{x\to\infty}\frac{x^2+x+1-x^2}{\sqrt{x^2+x+1}+x}=\lim_{x\to\infty}\frac{x+1}{\sqrt{x^2+x+1}+x}=\frac{1}{2}$
2.	原式$=\lim_{x\to0}\frac{f(bx)}{x}=\lim_{u\to0}\frac{\frac{f(u)}{u}}{b}=b\lim_{u\to0}\frac{f(u)}{u}=bk$
3.	$g(x)=\frac{x^3+1}{x^2-x-2}=x+1+\frac{3x+3}{x^2-x-2}=x+1+\frac{3(x+1)}{(x-2)(x+1)}$ $\lim_{x\to2}g(x)=\infty$ 故$x=2$ 為垂直漸近線。 又 $\lim_{x\to\infty}\{g(x)-(x+1)\}=\lim_{x\to\infty}\frac{3(x+1)}{(x-2)(x+1)}=0$ $\therefore y=x+1$ 為一斜漸近線 $x=-1$，$g(x)$無定義，也非垂直漸近線 $\because\lim_{x\to-1^+}g(x)=-1$，$\lim_{x\to-1^-}g(x)=-1$
4.	28. Step 1: $\lvert mx-3m\rvert<c\Rightarrow -c<mx-3m<c\Rightarrow -c+3m<mx<c+3m\Rightarrow 3-\frac{c}{m}<x<3+\frac{c}{m}$ Step 2: $\lvert x-3\rvert<\delta\Rightarrow -\delta<x-3<\delta\Rightarrow -\delta+3<x<\delta+3.$ Then $-\delta+3=3-\frac{c}{m}\Rightarrow\delta=\frac{c}{m}$, or $\delta+3=3+\frac{c}{m}\Rightarrow\delta=\frac{c}{m}$. In either case, $\delta=\frac{c}{m}$.
5.	**a.** $f(x)=x^7+x^5+x+1,\ f(0)=1,\ f(-1)=-2$ By the Intermediate Value Thm, there must exist one ξ in $(-1,0)$ such that $f(\xi)=0$. **b.** Assume that $f(x)$ has two different real roots: ξ_1,ξ_2. Hence, $f(\xi_1)=f(\xi_2)=0$. By the Rolle's Thm, there must exist one $x\in(\xi_1,\xi_2)$ such that $f'(x)=0$. However, $f'(x)=7x^6+5x^4+1$ is never zero. Hence, $\xi_1\neq\xi_2$ is impossible. Therefore, $f(x)$ has exactly one real root.
6.	(1) 設曲線上一點 (a,b) 的切線方程式通過 (0,0) 如右圖所示  則 $\begin{cases}b=(a+1)^3\\m=f'(a)=\frac{b}{a}\end{cases}$ (2)$f'(a)=\frac{d}{dx}f(x)\vert_{x=a}=3(x+1)^2\vert_{x=a}=3(a+1)^2=\frac{b}{a}$ $\because b=3a(a+1)^2\ and\ b=(a+1)^3$ $\therefore(a+1)=0\ or\ 3a=a+1\Rightarrow a=-1\ or\ a=\frac{1}{2}$ (3) $a=-1$-> (a, b)= (-1, 0) ->曲線上一點$(-1,0)$且經過(0,0)的切線方程式為$y=0$ $a=\frac{1}{2}$-> (a, b)= $\left(\frac{1}{2},\frac{27}{8}\right)$->曲線上一點$\left(\frac{1}{2},\frac{27}{8}\right)$ 且切線斜率 $m=\frac{b}{a}=\frac{27}{4}$ 的切線方程式 ： $y-$

	$\frac{27}{8}=\frac{27}{4}\left(x-\frac{1}{2}\right)\Rightarrow y=\frac{27}{4}x$
7.	視$y=y(x)$，對原式之 x 微分得 $6(x^2+y^3)^5(2x+3y^2y')=3x^2-2yy'$ 解$\frac{dy}{dx}$得$y'=\frac{3x^2-12x(x^2+y^3)^5}{18y^2(x^2+y^3)^5+2y}$
8.	$Let\ \ f(x)=\sqrt[3]{x}$ $f'(x)=\frac{1}{3}x^{-\frac{2}{3}}$ $f(x_0+dx)\approx f(x_0)+f'(x_0)dx$ when $x_0=8$ and $dx=0.2$ $$\sqrt[3]{8.2}\approx f(8)+f'(8)*0.2=\sqrt[3]{8}+\frac{1}{3\sqrt[3]{8^2}}(0.2)=2+\frac{1}{60}\approx 2+0.0167=2.0167$$
9.	$$f'(0)=\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0}\frac{\frac{x(1+x)\dots(n+x)}{(1-x)\dots(n-x)}-0}{x}=\lim_{x\to 0}\frac{(1+x)\dots(n+x)}{(1-x)\dots(n-x)}=1$$
10.	(a) $V=xyz\ \Rightarrow\ \frac{dV}{dt}=yz\frac{dx}{dt}+xz\frac{dy}{dt}+xy\frac{dz}{dt}\ \Rightarrow\ \frac{dV}{dt}\Big\|_{(4,3,2)}=(3)(2)(1)+(4)(2)(-2)+(4)(3)(1)=2\ m^3/sec$ (b) $S=2xy+2xz+2yz\ \Rightarrow\ \frac{dS}{dt}=(2y+2z)\frac{dx}{dt}+(2x+2z)\frac{dy}{dt}+(2x+2y)\frac{dz}{dt}$ $\Rightarrow\ \frac{dS}{dt}\Big\|_{(4,3,2)}=(10)(1)+(12)(-2)+(14)(1)=0\ m^2/sec$ (c) $\ell=\sqrt{x^2+y^2+z^2}=(x^2+y^2+z^2)^{1/2}\Rightarrow\frac{d\ell}{dt}=\frac{x}{\sqrt{x^2+y^2+z^2}}\frac{dx}{dt}+\frac{y}{\sqrt{x^2+y^2+z^2}}\frac{dy}{dt}+\frac{z}{\sqrt{x^2+y^2+z^2}}\frac{dz}{dt}$ $\Rightarrow\ \frac{d\ell}{dt}\Big\|_{(4,3,2)}=\left(\frac{4}{\sqrt{29}}\right)(1)+\left(\frac{3}{\sqrt{29}}\right)(-2)+\left(\frac{2}{\sqrt{29}}\right)(1)=0\ m/sec$
11.	Its volume is $V=x^2h$ $S=x^2+4xh=108\Rightarrow h=\frac{108-x^2}{4x}$ Then $V=x^2h=x^2\left(\frac{108-x^2}{4x}\right)=27x-\frac{x^3}{4}$ Let $V'=27-\frac{3}{4}x^2=0\Rightarrow x=\pm 6\ \Rightarrow\ (\because x>0)\ we\ only\ take\ x=6$ (critical point) We also consider two end points: x=0 and $6\sqrt[2]{3}\ (\ by\ 27x-\frac{x^3}{4}\geq 0)$ V at x=6 has a maximum value $\Rightarrow h=3\ inches$ So the dimensions that produce a box with a maximum volume are $x=6\ inches\ and\ h=3\ inches.$