B11102123

111-2 Calculus Midterm (1/2)

Chapter : 7-2~7-3 + 7-5~7-7

Date : 2023/04/19 17:20-18:10 (50 minutes)

Total : 55 pts (50%)

1. Find the following limits

   a. $\lim\limits_{x\to 0} \frac{x-\tan^{-1}x}{\sin^{-1}x-x}$ (5 pts)

   b. $\lim\limits_{x\to 0^+} (1+\frac{1}{x})^x$ (5 pts)

2. Evaluate the integral $\int \frac{\sin^2 x\cdot \cos x}{1+\sin^2 x}dx$ (10 pts)

3. Find the following derivative of y respect to x using logarithmic differentiation

   a. $y = (\ln x)^{\ln x}$ (5 pts)

   b. $x = y^{xy}$ (5 pts)

4. Evaluate the integral $\int \frac{dx}{x\sqrt{x^6-4}}$ $(hint: u=\sqrt{x^6-4})$ (10 pts)

5. Evaluate the integral $\int_{-ln2}^{0} cosh^2\left(\frac{x}{2}\right)dx$ (5 pts)

6. Evaluate the integral $\int_{\ln(e-1)}^{\ln(e^2-1)} \frac{1}{1+e^x}dx$ (10 pts)

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111-2 Calculus Midterm (2/2)

Chapter : 8-2~8-5 + 8-8

Date : 2023/04/26 17:20-18:10 (50 minutes)

Total : 55 pts (50%)

1. Evaluate the following integrals using Integral By Part (IBP)
   a. $\int \ln(x^2+2x+2)dx$ (10 pts)
   b. $\int sec^3_{\uparrow} dx$ (10 pts)
   x

2. Evaluate the integral $\int sinxcos3xcos5xdx$ (5 pts)

3. Evaluate the integral $\int \frac{x^2+5}{(x+1)(x^2-2x+3)}dx$ (10 pts)

4. Evaluate the integral $\int \frac{1}{(x^2+1)^2}dx$ (10 pts)

5. For what value of a that makes the improper integral $\int_{\sqrt{2}}^{\infty}(\frac{a}{\sqrt{x^2-1}} - \frac{x}{x^2+1})dx$ converge? and what is the result of the improper integral? (10 pts)

$\int \sin x \cos 3x \cos 5x \, dx$

$= \int \frac{1}{2}(\sin 4x + \sin(-2x)) \cdot \cos 5x \, dx$

$= \frac{1}{2}\int \sin 4x \cos 5x + \cos 5x \sin(-2x) \, dx$

$= \frac{1}{2}\int \frac{1}{2}(\sin 9x + \sin(-x)) + \frac{1}{2}(\sin 3x + \sin(-7x)) \, dx$

$= \frac{1}{4}\int \sin 9x - \sin x + \sin 3x - \sin 7x \, dx$

$= -\frac{1}{36}\cos 9x + \frac{1}{4}\cos x - \frac{1}{12}\cos 3x + \frac{1}{28}\cos x + c$

$\lim_{b\to\infty} \int_{\sqrt{2}}^{b} (\frac{a}{\sqrt{x^2-1}} \quad \frac{x}{x^2+1}) dx$

$\lim_{b\to\infty} (a\ln|\sqrt{x^2-1}+x| - \frac{1}{2}\ln|x^2+1|) \Big|_{\sqrt{2}}^{b}$

$\lim_{b\to\infty} a\ln|\sqrt{b^2-1}+b| - \frac{1}{2}\ln|b^2+1| - a\ln|1+\sqrt{2}| + \frac{1}{2}\ln 3$

$a\ln|\infty+\infty| - \frac{1}{2}\ln|\infty| \quad a\ln|1+\sqrt{2}| + \frac{1}{2}\ln 3$

$a\ln|\infty| - \ln|\infty| + \ln e^2 - a\ln|1+\sqrt{2}| + \frac{1}{2}\ln 3$

$a\int \frac{dx}{\sqrt{x^2-1}} - \int \frac{x\,dx}{x^2+1}$

$x = \sec\theta$

$dx = \tan\theta\sec\theta \, d\theta$

$a\int \frac{\tan\theta\sec\theta \, d\theta}{\tan\theta}$

$a\ln|\tan\theta+\sec\theta| \quad \frac{1}{2}\ln|x^2+1|$

$a\ln|\sqrt{x^2-1}+x| - \frac{1}{2}\ln|x^2+1|$