Calculator is not allowed

112-2 Calculus Quiz 1
Chapter: 7-1~7-3 & 7-5~7-8
Date: 2024/04/01 10:20-12:10 Total: 100 pts

1. $f(x) = \frac{e^{4-x^2}}{x}$, find $(f^{-1})'(\overset{1/2}{\cancel{6}}) = ?$ (10 pts)

2. Find the following limits. (20 pts)

   $a. \lim\limits_{x\to 0}(sinx)^x$ (5 pts) $\qquad b. \lim\limits_{x\to\infty}\frac{\sin^{-1}x-\tan^{-1}x}{x^3}$ (5 pts)

   c. $\lim\limits_{x\to\frac{\pi}{2}}(cscx)^{tan^2x}$ (5 pts) $\qquad d \lim\limits_{x\to 0^+} x^{x^x}$ (5 pts)

3. $f'(0) = 4$, find $\frac{d}{dx}f(\frac{e^x-1}{e^x+1})\Big|_{x=0}$ (10 pts)

4. $y^{e^x} = x^{2^x}$, find $\frac{dy}{dx}$ (10 pts)

5. Evaluate the following integral: (10 pts)

   (a). $\int \frac{t-2}{t^2-6t+10}dt$ (5 pts) (b). $\int_{\sqrt{2}}^{2}\frac{sec^2(\sec^{-1}x)}{x\sqrt{x^2-1}}dx$ (5 pts)

6. $\frac{d}{dx}\left(\frac{(x^2+2)(x^2-2)}{(x^2+1)(x^2-1)}\right) = ?$ (Hint: Use Logarithmic Differentiation) (10 pts)

7. Evaluate the integral: $\int 2^{-x}\tanh(2^{1-x})\,dx = ?$ (10 pts)

8. Verify the integration formula: $\int x\coth^{-1}x\,dx = \frac{x^2-1}{2}\coth^{-1}x + \frac{x}{2} + C$

   (10 pts)

9. Order the following functions from slowest growing to fastest growing as $x \to \infty$. (10 pts)

   a. e^x b. x^x c. $(\ln x)^x$ d. $e^{\frac{x}{2}}$

(Questions on both sides of the paper)

Formula Table

1. $\displaystyle\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$ (Valid for $u^2 < a^2$)
2. $\displaystyle\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$ (Valid for all u)
3. $\displaystyle\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C$ (Valid for $|u| > a > 0$)

1. $\dfrac{d(\arcsin u)}{dx} = \dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}, \quad |u| < 1$
2. $\dfrac{d(\arccos u)}{dx} = -\dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}, \quad |u| < 1$
3. $\dfrac{d(\arctan u)}{dx} = \dfrac{1}{1+u^2}\dfrac{du}{dx}$
4. $\dfrac{d(\operatorname{arccot} u)}{dx} = -\dfrac{1}{1+u^2}\dfrac{du}{dx}$
5. $\dfrac{d(\operatorname{arcsec} u)}{dx} = \dfrac{1}{|u|\sqrt{u^2-1}}\dfrac{du}{dx}, \quad |u| > 1$
6. $\dfrac{d(\operatorname{arccsc} u)}{dx} = -\dfrac{1}{|u|\sqrt{u^2-1}}\dfrac{du}{dx}, \quad |u| > 1$

$$\sinh^{-1}x = \ln\left(x + \sqrt{x^2+1}\right), \quad -\infty < x < \infty$$
$$\cosh^{-1}x = \ln\left(x + \sqrt{x^2-1}\right), \quad x \geq 1$$
$$\tanh^{-1}x = \frac{1}{2}\ln\frac{1+x}{1-x}, \quad |x| < 1$$
$$\operatorname{sech}^{-1}x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right), \quad 0 < x \leq 1$$
$$\operatorname{csch}^{-1}x = \ln\left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|}\right), \quad x \neq 0$$
$$\coth^{-1}x = \frac{1}{2}\ln\frac{x+1}{x-1}, \quad |x| > 1$$

$$\frac{d(\sinh^{-1}u)}{dx} = \frac{1}{\sqrt{1+u^2}}\frac{du}{dx}$$
$$\frac{d(\cosh^{-1}u)}{dx} = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx}, \quad u > 1$$
$$\frac{d(\tanh^{-1}u)}{dx} = \frac{1}{1-u^2}\frac{du}{dx}, \quad |u| < 1$$
$$\frac{d(\coth^{-1}u)}{dx} = \frac{1}{1-u^2}\frac{du}{dx}, \quad |u| > 1$$
$$\frac{d(\operatorname{sech}^{-1}u)}{dx} = -\frac{1}{u\sqrt{1-u^2}}\frac{du}{dx}, \quad 0 < u < 1$$
$$\frac{d(\operatorname{csch}^{-1}u)}{dx} = -\frac{1}{|u|\sqrt{1+u^2}}\frac{du}{dx}, \quad u \neq 0$$

$$\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C, \quad a > 0$$
$$\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C, \quad u > a > 0$$
$$\int \frac{du}{a^2-u^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{u}{a}\right) + C, & u^2 < a^2 \\ \frac{1}{a}\coth^{-1}\left(\frac{u}{a}\right) + C, & u^2 > a^2 \end{cases}$$
$$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{u}{a}\right) + C, \quad 0 < u < a$$
$$\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\operatorname{csch}^{-1}\left|\frac{u}{a}\right| + C, \quad u \neq 0 \text{ and } a > 0$$