1. (10%) Find the capacitance per unit length of two coaxial metal cylindrical tubes, of radii a and b, as shown in Fig.1.

(15%)The coaxial cylindrical metal tubes (inner radius a, outer radius b) stands vertically in a tank of dielectric oil (susceptibility χ_e, mass density ρ). The inner one is maintained at potential $\mathcal{V}$, and the outer one is grounded, as shown in Fig.1-1. To what height (h) does the oil rise in the space between the tubes?

Hint: $\vec{F}_v = F_g$,$F_g = mg = \rho\pi(b^2 - a^2)hg$

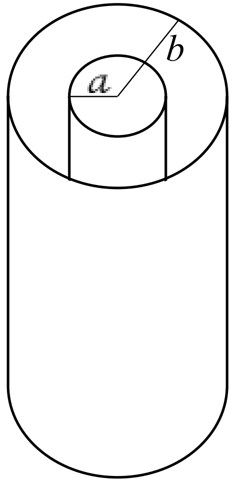


Fig.1

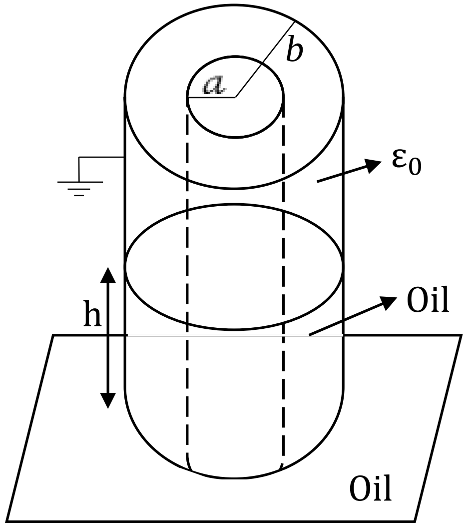


Fig.1-1

1. $\frac{C}{l} = \frac{2\pi\varepsilon_0}{ln\frac{b}{a}}$

2. Oil 所受之電力：$\vec{F}_V = \hat{a}_z \frac{1}{2} {V_0}^2 \frac{2\pi(\varepsilon-\varepsilon_0)}{ln\frac{b}{a}}$

Oil 所受之重力：$\vec{F}_g = mg = \rho V g = \rho\pi(b^2 - a^2)hg$

向上電力等於向下重力 → $F_V = F_g \rightarrow \frac{1}{2}{V_0}^2 \frac{2\pi(\varepsilon - \varepsilon_0)}{ln\frac{b}{a}} = \rho\pi(b^2 - a^2)hg$

$\varepsilon - \varepsilon_0 = \varepsilon_0\varepsilon_r - \varepsilon_0 = \varepsilon_0(1 + \chi_e) - \varepsilon_0 = \varepsilon_0\chi_e$

$\frac{1}{2}{V_0}^2 \frac{2\pi(\varepsilon_0\chi_e)}{ln\frac{b}{a}} = \rho\pi(b^2 - a^2)hg$, $h = \frac{\varepsilon_0\chi_e {V_0}^2}{\rho\pi(b^2 - a^2)g ln\frac{b}{a}}$

2. (10%) Two infinite insulated conducting plates maintained at potentials 0 and V_0 form a wedge-shaped configuration, as shown in Fig.2. Determine the potential distributions for the regions:

1) $0 < \phi < a$

2) $a < \phi < 2\pi$

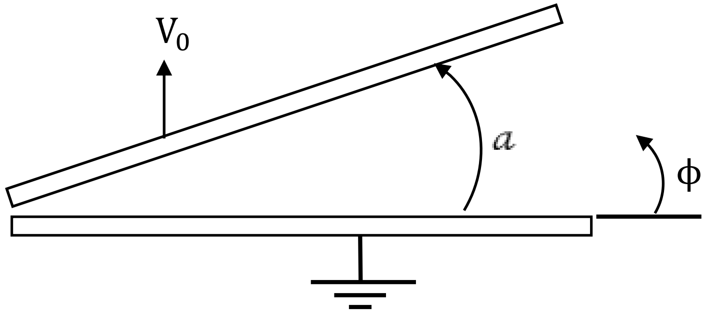


Fig.2

1. $0 < \phi < a$

$V(\phi)=C_1\phi+ C_2$...(1)

$V(0)=0$...(2)

$0= C_1{*}0 + C_2$...(3)

$C_2=0$...(4)

BC : $V(a)= V_0$... (5)

$V_0 = C_1 a + C_2$... (6)

(4)&(6)... $V_0 = C_1 a$...(7)

$C_1=\frac{V_0}{a}$...(8)

$V(\phi)= \frac{V_0}{a}\phi$

2. $a < \phi < 2\pi$

$V(\phi)=C_1\phi+ C_2$...(9)

BC : $V(a) = V(0)$...(10)

$V(0) = C_1{*}a + C_2$...(11)

$V(2\pi) = 0$...(12)

$0 = C_1 2\pi + C_2$...(13)

(11)&(12)&(13)... $C_1= -\frac{V_0}{2\pi - a}$...(14)

$C_2= V_0- C_1 a = V_0 - (-\frac{V_0}{2\pi - a})a$

$= V_0+ \frac{V_0}{2\pi - a} a$...(15)

$V(\phi)= -\frac{V_0}{2\pi - a}\phi+ V_0+ \frac{V_0}{2\pi - a} a$

$= V_0 - \frac{V_0}{2\pi - a}(\phi - a)$

3. (10%) A parallel-plate capacitor of width $\mathcal{W}$, length $\mathcal{L}$, and separation d has a solid dielectric slab of permittivity ε in the space between the plates as indicated in Fig.3. Determine:

1)F_v when switch off (short) if the capacitor starts charging.

2)The capacitance if the capacitor has been charged to a voltage V_0 while switch is on (open). (Use Gauss's law to solve).

Note: 1) and 2) are individual questions.

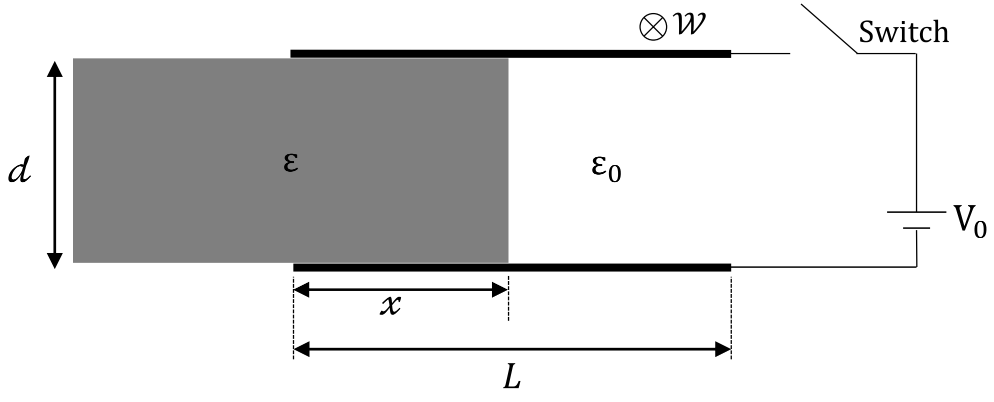


Fig.3

1) $F_v = \frac{dW_e}{dx}\hat{a}_x$, $W_e = \frac{1}{2}\frac{\varepsilon W x+\varepsilon_0 W(L-x)}{d}V_0^2$, $F_v = \frac{1}{2}V_0^2\frac{(\varepsilon-\varepsilon_0)w}{d}\hat{a}_x$

2) $\oint_s \vec{D}\cdot\vec{dS} = Q_{in} \rightarrow D_1S_1 + D_2S_2 = Q$

$\varepsilon EWx + \varepsilon_0 EW(L-x) = Q$ ◦ $E = \dfrac{Q}{\varepsilon Wx + \varepsilon_0 W(L-x)}$

$V = Ed$, $V = \dfrac{Q}{\varepsilon Wx + \varepsilon_0 W(L-x)}d$ ◦

$C = \dfrac{Q}{V}$, $C = \dfrac{\varepsilon Wx + \varepsilon_0 W(L-x)}{d}$

4. (10%) A point charge Q is located at distances $a\ell$ and ℓ, respectively, from two grounded perpendicular conducting half-planes, as shown in Fig.4, where $a=(\frac{2\pi}{2+\pi})^{\frac{2}{3}} > 1$ (After releasing the charge from rest, the charge will strike on the plane. Please determine (ignore radiation loss) the position where the charge will strike.

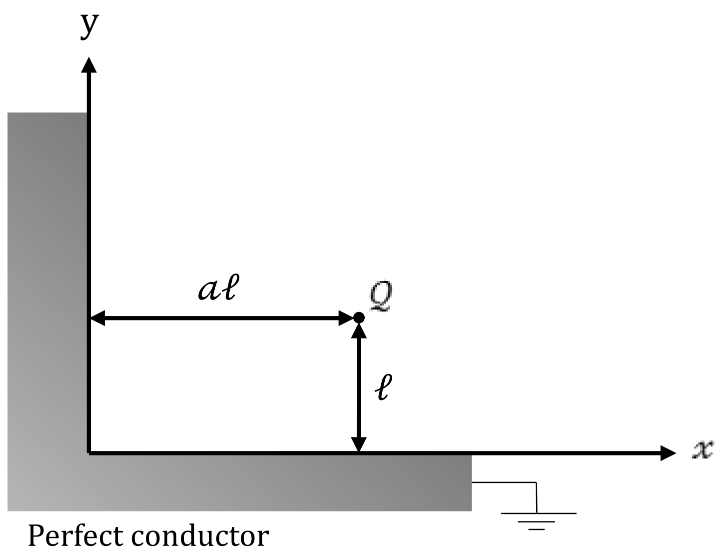
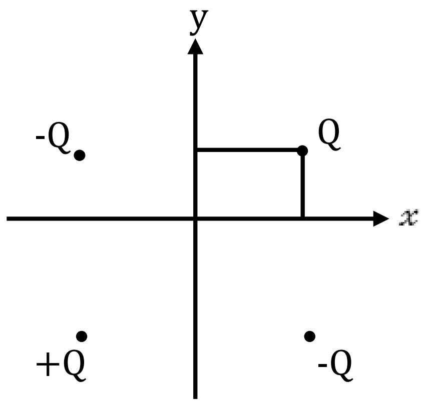
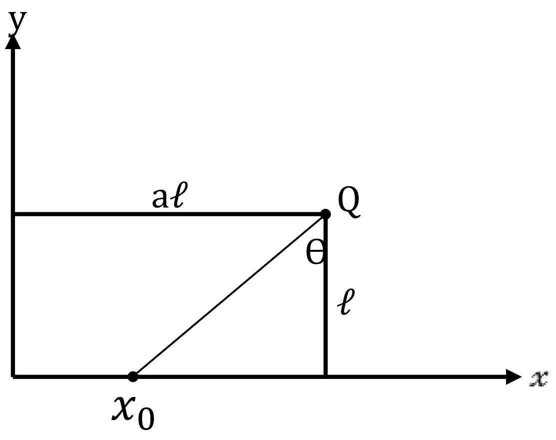


Fig.4





$\vec{F}=\frac{Qq\vec{R}}{4\pi\varepsilon_0 R^3}$, a = $(\frac{2\pi}{2+\pi})^{\frac{2}{3}}$=1.143

第一象限Q之受力, $\vec{F}=\frac{Q^2}{4\pi\varepsilon_0}[\frac{-[2a\ell,0]}{8a^3\ell^3}+\frac{[2a\ell,2\ell]}{(4a^2\ell^2+4\ell^2)^{3/2}}-\frac{[0,2\ell]}{8\ell^3}]$

$\vec{F}=\frac{Q^2}{4\pi\varepsilon_0\ell^2}[-0.1098,-0.1786]$

$\frac{F_x}{F_y}=tan\theta=\frac{0.1098}{0.1786}$, $\theta=31.58°$

$x_0=a\ell-\ell tan\theta$, $x_0=0.5283\ell$

碰撞點座標 : (x,y)=(0.5283ℓ,0)

5. (15%) A 5V DC voltage applied to the ends of a 2km conducting wire with 1mm² cross section results in a current of 0.2A shown in Fig.5. Find (a) the conductivity of the wire, (b) the electric field intensity in the wire, (c) the power dissipated in the wire.

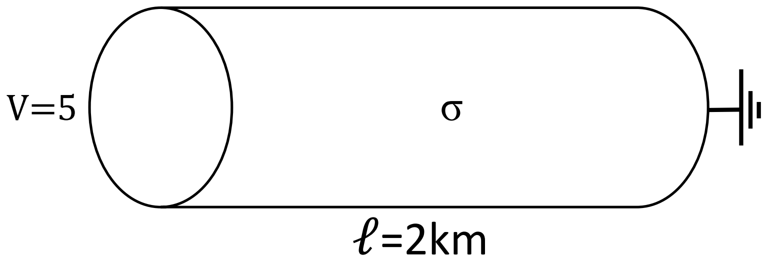


Fig.5

1. $R = \frac{V}{I} = \frac{l}{\sigma S} \rightarrow \frac{5}{0.2} = \frac{2\times 10^3}{\sigma \times (10^{-3})^2}, \sigma = 8 \times 10^7 S/m$

2. $E = \frac{V}{d} = \frac{5}{2\times 10^3} \rightarrow E = 2.5 \times 10^{-3}\ V/m$

3. $P_\sigma = \sigma E^2 V \rightarrow P_\sigma = 8 \times 10^7 \times (2.5 \times 10^{-3})^2 \times (10^{-3})^2 \times 2 \times 10^3$

   $P_\sigma = 1W$

6. (10%) A metal bar of conductivity σ is bent to form a flat 90° sector of inner radius a, outer radius b, and thickness t shown in Fig.6. Find the resistance of the bar between the vertical curved surfaces at $\rho = a$ and $\rho = b$ if $b/a = 6/5$, $\sigma = 4\times10^7\,\mathrm{S/m}$ and $t = 0.5$ cm.

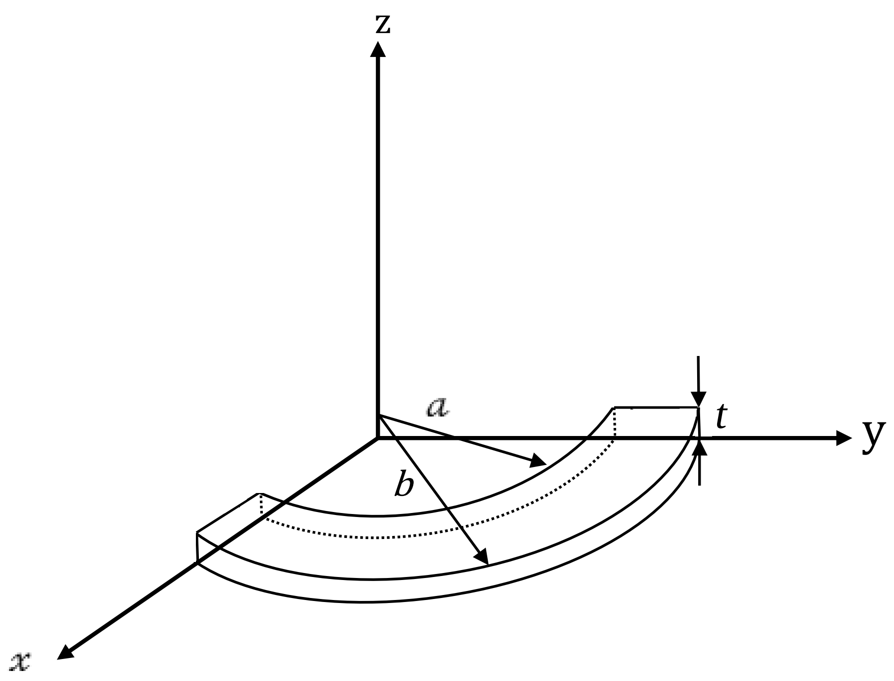


Fig.6

$$C = \frac{\varepsilon\pi/2}{ln\frac{b}{a}}t$$

$$\mathrm{RC} = \frac{\varepsilon}{\sigma}$$

$$R = \frac{1}{\sigma}\frac{2ln\frac{b}{a}}{\pi t}$$

代入 : $R = \frac{1}{4*10^7}\frac{2*\ln(\frac{6}{5})}{\pi*0.5*10^{-2}} = 5.803 * 10^{-7}\Omega$

7. (10%) A conducting sphere of radius a is surrounded by free space, shown in Fig.7. Initially, a charge density of ρ_{v0} is distributed uniformly throughout the sphere. Please derive the current density $\vec{J}$ of the sphere at t = 0 and t→∞. The dielectric constant and conductivity of the sphere are ε and σ, respectively.

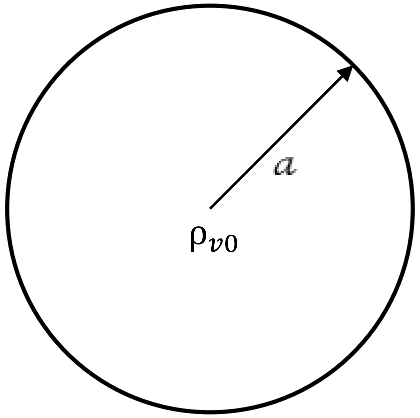


Fig.7

$\vec{J} \leftarrow \rho_{V0}$ ， $\nabla \cdot \vec{J} = -\frac{\partial \rho_V}{\partial t}$ (1), $\rho_V = \rho_{V0} e^{\frac{-t}{\tau}}$ (2)。(2)代入(1)可得：

$$\nabla \cdot \vec{J} = \frac{1}{\tau}\rho_{V0} e^{\frac{-t}{\tau}} \rightarrow \vec{J} = \frac{1}{\tau}\rho_{V0} e^{\frac{-t}{\tau}} \frac{\vec{R}}{3}, \tau = \frac{\varepsilon}{\sigma}$$

1. $t = 0 \rightarrow \vec{J} = \frac{1}{\tau}\rho_{V0}\frac{\vec{R}}{3}$, $\tau = \frac{\varepsilon}{\sigma}$

2. $t = \infty \rightarrow \vec{J} = 0$

電流密度向量 $\vec{J}$的每個分量（ $\vec{J}$x、 $\vec{J}$y、 $\vec{J}$z ）都與位置向量 $\vec{R}$相關。在解析推導的過程中，當計算 $\vec{J}$的分量對位置向量 $\vec{R}$的偏導數時，這個結果中的 $\vec{R}$/3 是因為我們在計算 $\vec{J}$x 分量對 $\vec{R}$ 的偏導數時，必須考慮到 $\vec{R}$ 是一個矢量，並將其分解成 x、y、z 三個分量。而 $\vec{R}$/3 表示位置向量 $\vec{R}$的每個分量都除以 3，這是由於在推導過程中的數學運算所得到的結果。

8. (10%) If the magnetic flux density is given by $\vec{B} = \hat{a}_x 666\text{x} + \hat{a}_y 66\text{y} + \hat{a}_z 6\text{cz}$, find c .

$$\nabla \cdot B = 0 \ , \ \frac{\partial}{\partial x}(666x) + \frac{\partial}{\partial y}(66y) + \frac{\partial}{\partial z}(6cz) = 0$$

666+66+6c=0 , c = -122