102-01 Calculus Midterm Exam (Date: 2013/11/19 110 minutes, total: 110 points)

1. Show that $\lim_{x\to 3}\sqrt{3x}=3$ by the ε-δ method. (10 points)

2. **a.** Find $\lim_{x\to 0}\frac{x}{\sqrt[3]{8+x}-\sqrt[3]{8+x^3}}$. (6 points) **b.** Find $\lim_{x\to 0}\frac{[x+1]+|x|}{x}$. (6 points)

3. **a.** Find $\lim_{x\to n}[[x]-x]$. (6 points) **b.** Find $\lim_{x\to\infty}\frac{x+\cos x}{x+1}$. (6 points)

4. Find the asymptotes of $f(x)=\frac{2x^3}{x^2-1}$. (8 points)

5. Assume $f(a)=0$ and $f'(a)=3$, find $\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$. (8 points)

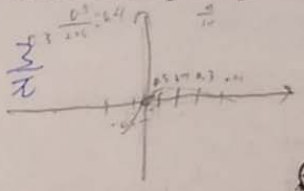
6. Find $f'(0)$, if $f(x)=(x^2+\sqrt{x+\sqrt[3]{x+1}})^{\pi}$. (8 points)

7. Find the equation of the tangent lines to the graph of $f(x)=(x+1)^3$, where the tangent lines pass through the origin (0,0). (10 points)

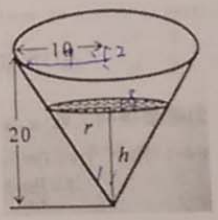
8. Show that $x^7+x^5+x+1=0$ has exactly one real root. (6 points)

9. Find the approximation of $\sqrt{100.013}$ by linearization. (6 points)

10. Water runs into a conical tank at the rate of 3 m^3/min. A tank stands point down and has height of 20 m and a base radius 10 m. How fast is the water level rising when the water is 2 m deep? (8 points)(hint: $V=\frac{1}{3}\pi r^2 h$ for a conical tank when the base radius is r and the height is h).



11. If $f(x)=\frac{x}{x^2+1}$. (12 points)



a. Identify the intervals on which $f(x)$ is increasing and decreasing.

b. Find the function's local maximum and local minimum.

c. Identify the intervals on which $f(x)$ is concave up and concave down.

12. As the figure shows, a cylinder is inscribed (內接) in a cone whose height is 120 cm and radius is 50 cm, please find the height and the radius of the inscribed cylinder so that the volume of the cylinder has a maximum value. (10 points)