Final Exam. Calculus (101-1) ET1601001, 1601002, 2013/1/22

1 Find the following limits: (a) $\lim\limits_{x\to 0^+}(\frac{1}{x}-\frac{1}{\ln(x+1)})$ (8%) (b) $\lim\limits_{x\to+\infty}(\frac{2x-1}{2x+1})^x$ (8%)

2 (a) $y=(10^x+\ln x)^{e^x}$. Find $\frac{dy}{dx}$ (8%) lim $(x+1)^{-1}$ $=-(x+1)^{-2}$ 1-

(b) An electric charge Q is distributed uniformly along a line of length $2a$, lying along the y-axis. A point charge q lies on the x-axis, at a distance x from the origin. If $F=-q\frac{dV}{dx}$, $V(x)=\frac{1}{4\pi\varepsilon_0}\frac{Q}{2a}\ln\frac{\sqrt{a^2+x^2}+a}{\sqrt{a^2+x^2}-a}$, ε_0 is a constant, find $F(x)$ (8%)

3 Evaluate the following integrals: (a) $\int_1^4\frac{dx}{\sqrt{x}+x}$ (8%) (b) $\int_{\pi/6}^{\pi/4}\frac{\csc x}{\sqrt{\ln(\csc x+\cot x)}}dx$ (8%)

45° 30°

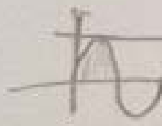
(c) $\int_e^{e^4}\frac{dx}{x\sqrt{\log_2 x}}$ (8%) $\int\frac{\sqrt{\ln 2}}{x\sqrt{\ln x}}$ $\int_1^4\frac{}{\sqrt{x}(1+\sqrt{x})}$ $u=\ln(c$ $du=\frac{}{\csc x+c}$

4 $f(x)=\int_1^{\sqrt{\ln x}}\frac{1}{t^2+3^t}dt$, $x>1$. Find $(f^{-1})'(0)$ (8%)

$\sqrt{y}=2-\sqrt{x}$ $y=4+x-4\sqrt{x}$

5 Find the area of the region bounded by $\sqrt{x}+\sqrt{y}=2$, $y=x^2$ and x-axis (8%)

6 The base of a solid is the region bounded by the graphs of $y=\sin x$, $y=1$ and $x=0$. The cross-sections perpendicular to the x-axis are rectangles of perimeter(周長)8. Find the volume of the solid. (10%)



7 Find the volume of the solid by revolving the region bounded by $y=2x-x^2$, $y=$ around $x=1$ (10%)

8 Find the length of the arc $9y^2=4x^3$ from $(0,0)$ to $(3,2\sqrt{3})$ (10%) $x=1$

9 Find the area of the surface obtained by revolving the curve $y=\frac{1}{2\sqrt{2}}x\sqrt{1-x^2}$ on $[0,1]$ about x-axis (10%)

$2a\sqrt{\ }$

$2a(x^2+a^2)^{\frac{1}{2}}$

$a(x^2+a^2)^{-\frac{1}{2}}\cdot 2x$

$=2ax\frac{1}{\sqrt{x^2+a^2}}$