TAIWAN TECH National Taiwan University of Science and Technology

Introduction to Analog Integrated Circuit Design

Fall 2023

Frequency Response

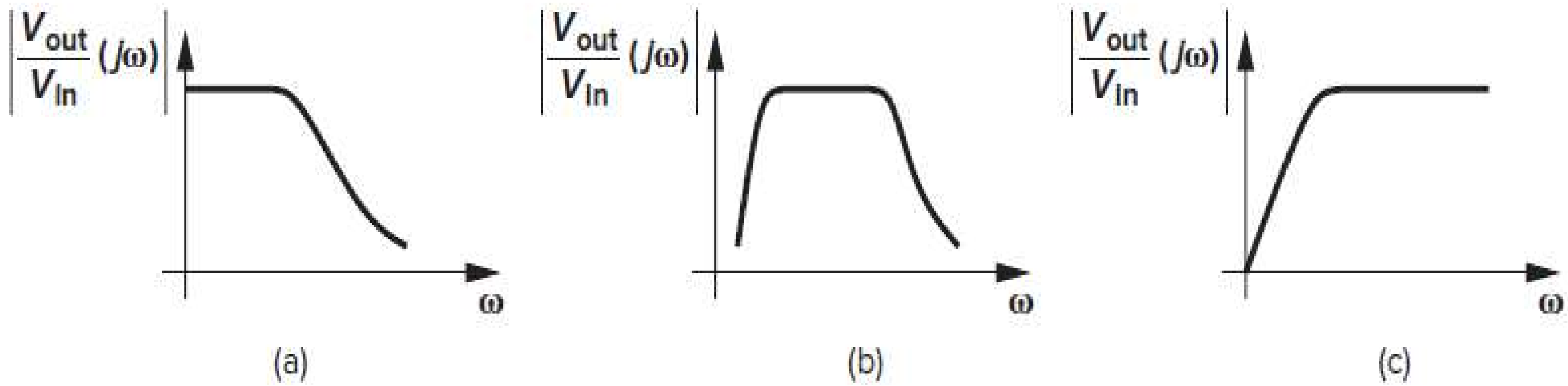
Yung-Hui Chung

MSIC Lab

DECE, NTUST

Mixed-Signal IC Laboratory NTUST

Frequency Response



Low-Pass Filter | Band-Pass Filter | High-Pass Filter

For MOSFETs, parasitic capacitors are existed between two nodes to construct poles or zeros

- Poles: decreased magnitude if higher frequency
- Zeros: increased magnitude if higher frequency

Miller Effect

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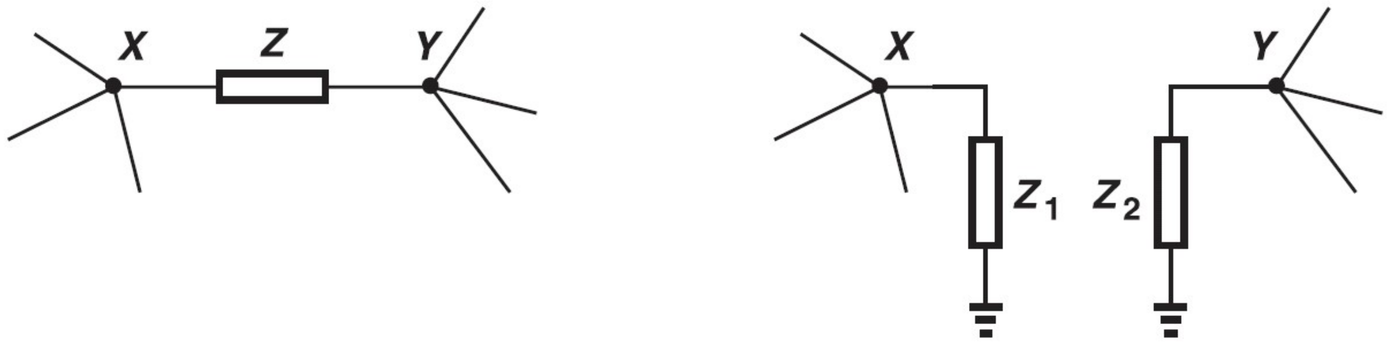
如果圖(a)之電路可以被轉換成圖(b)之電路，則 $Z_1 = Z/(1-A_v)$ 且 $Z_2 = Z/(1-A_v^{-1})$，其中 $A_v = V_Y/V_X$。



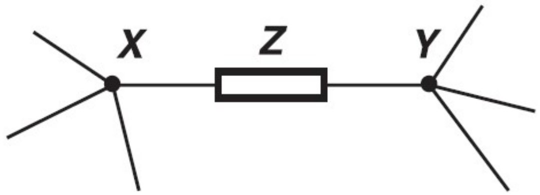
(a) (b)

證明：從 X 流經 Z 至 Y 之電流為 $(V_X - V_Y)/Z$，若這兩個電路要相等時，此電流必須與流經 Z_1 之電流相同。因此

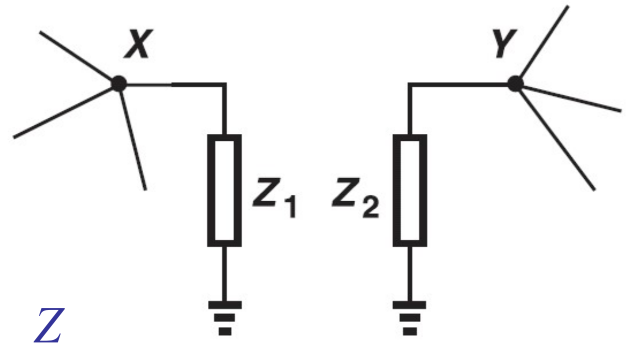
$$\frac{V_X - V_Y}{Z} = \frac{V_X}{Z_1} \Rightarrow Z_1 = \frac{Z}{1-\dfrac{V_Y}{V_X}} = \frac{Z}{1-A_V}$$

$$\frac{V_Y - V_X}{Z} = \frac{V_Y}{Z_2} \Rightarrow Z_2 = \frac{Z}{1-\dfrac{V_X}{V_Y}} = \frac{Z}{1-A_V^{-1}}$$

Miller Effect (Supplement)



(a)



(b)

$$Z_1 = \frac{Z}{1 - \frac{V_Y}{V_X}} = \frac{Z}{1 - A_V}$$

For capacitor in Miller path, Z = 1/sC

Z_1 = 1/sC(1-Av) => Z_1 = 1/s(C*(1-Av))

Since Av<0, Z_1 = 1/sCm => Cm = C*(1-Av), Cap is scaled up

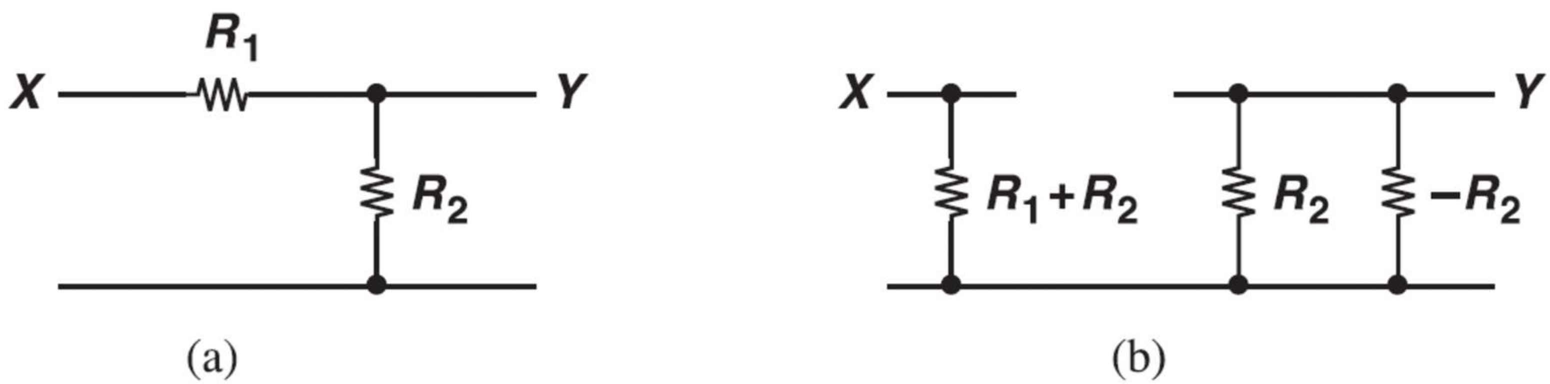
For resistor in Miller path, Z = R,

Z_1 = R/(1-Av)

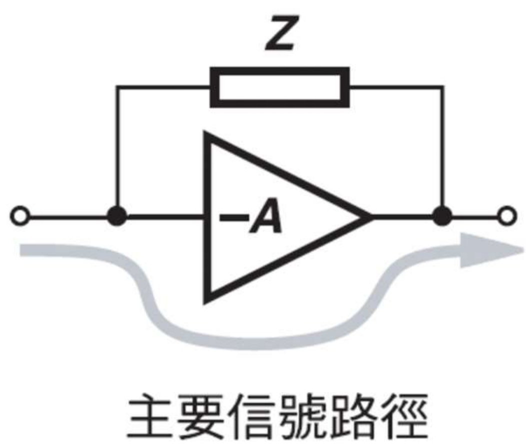
Since Av<0, Z_1 = Rm, Rm = R/(1-Av), Res is scaled down

Miller Effect

不適用米勒定律的例子



典型有效地應用米勒定律的例子



Miller Effect

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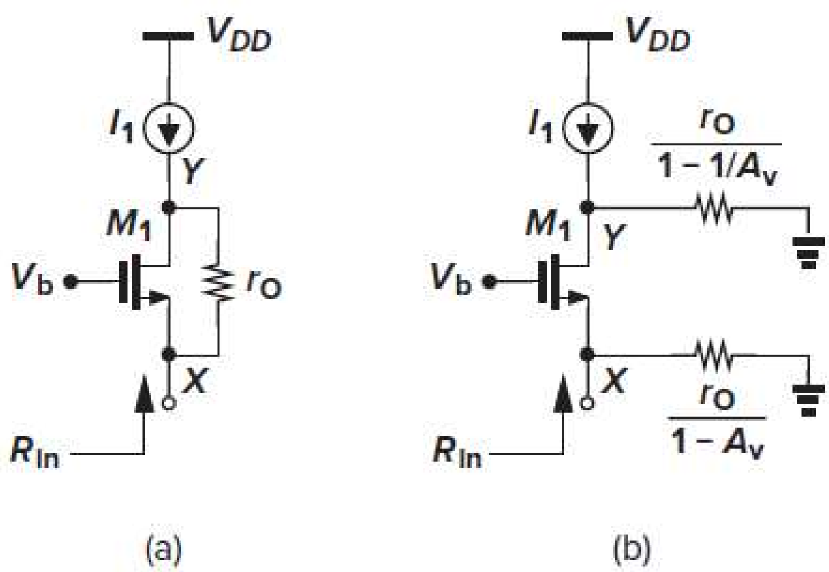
- 使用米勒效應去簡化電路將會遺漏轉移函數的零點 (zeros)。
- 如果應用在求得輸入 - 輸出轉移函數時，米勒定律不可同時用來計算輸出阻抗 (page 179, textbook)。
- 圖(b)之電路輸入阻抗為

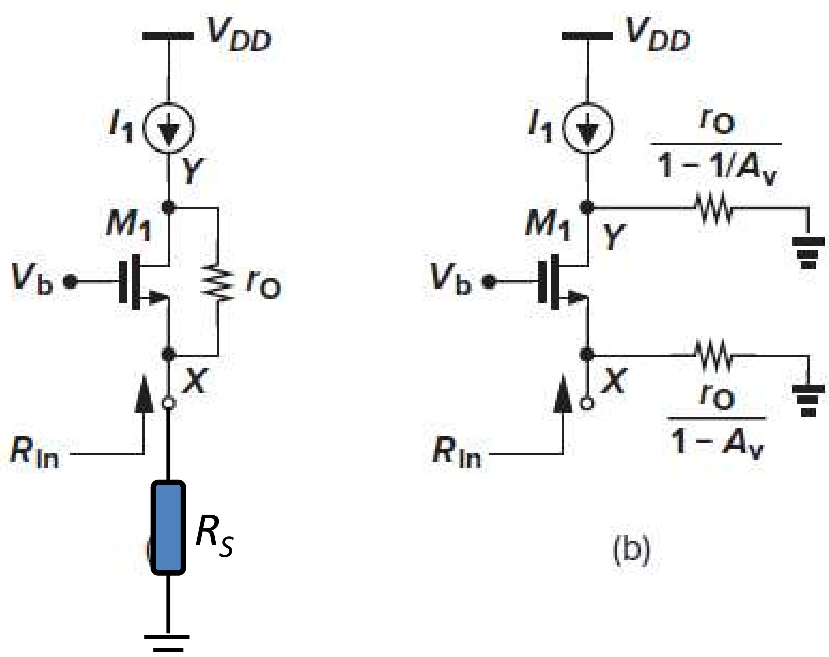


$$A_v = 1 + (g_m + g_{mb})r_O$$

$$\begin{aligned} R_{in} &= \frac{r_O}{1-[1+(g_m+g_{mb})r_O]} \Big\| \frac{1}{g_m+g_{mb}} \\ &= \frac{-1}{g_m+g_{mb}} \Big\| \frac{1}{g_m+g_{mb}} \\ &= \infty \end{aligned}$$

Miller Effect

- 使用米勒效應去簡化電路將會遺漏轉移函數的零點 (zeros)。
- 如果應用在求得輸入 - 輸出轉移函數時，米勒定律不可同時用來計算輸出阻抗 (page 179, textbook)。
- 圖(b)之電路輸出阻抗為



(b)

Applying Miller effect,

$$R_{out} = \frac{r_O}{1-1/A_v}$$

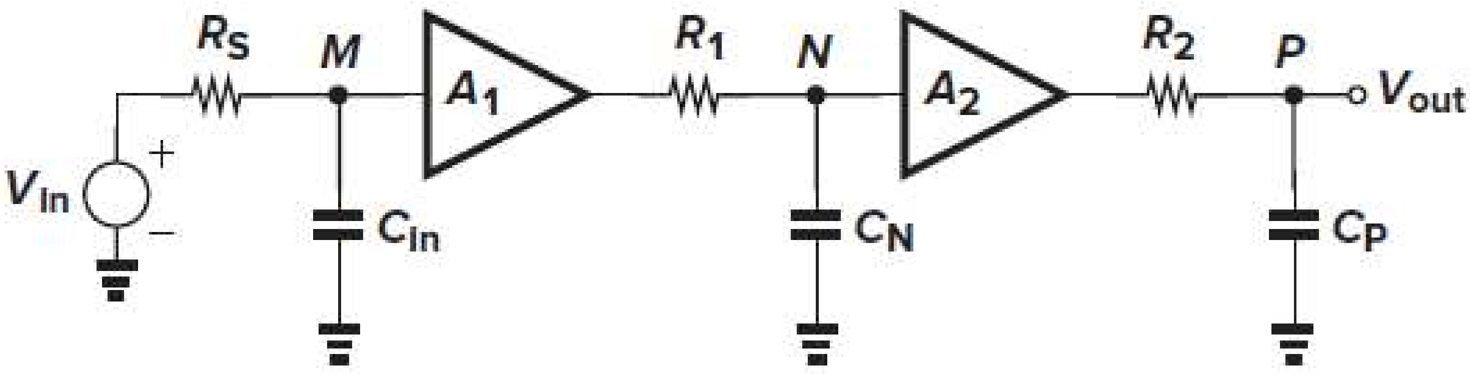
$$= \frac{r_O}{1-[1+(g_m+g_{mb})r_O]^{-1}}$$

$$= \cancel{\frac{1}{g_m+g_{mb}}} + r_O$$

But, in real,

$$R_{out} = r_{o1} + R_S + g_{m1} r_{o1} R_S$$

Association of Poles with Nodes

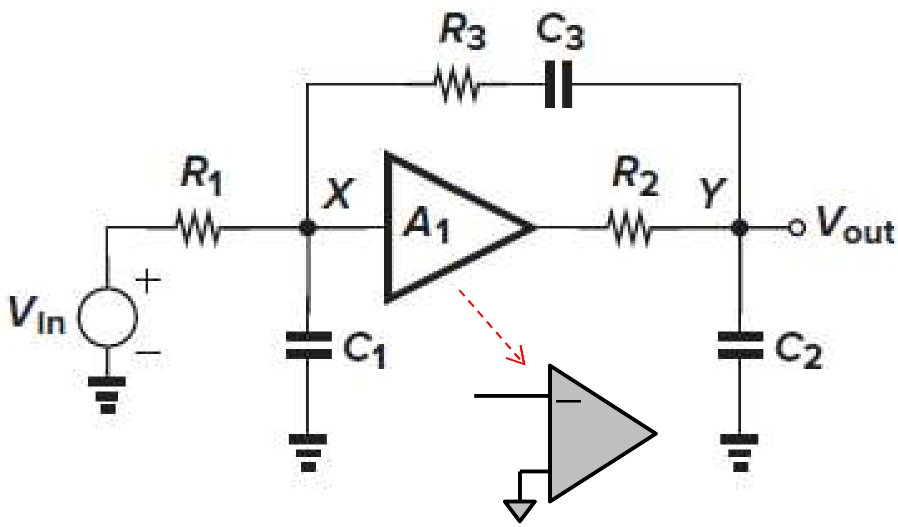


整體轉移函數 $$\frac{V_{out}}{V_{in}}(s) = \frac{A_1}{1+sR_S C_{in}} \cdot \frac{A_2}{1+sR_1 C_N} \cdot \frac{1}{1+sR_2 C_P}$$

This configuration is easy to analyze!!

But, does it worth?

Association of Poles with Nodes



$$V_X = -\frac{V_a}{A_1}$$

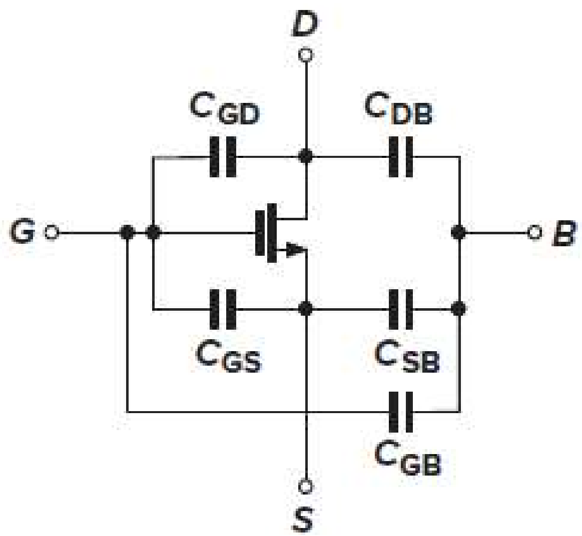
$$\frac{V_X - V_{in}}{R_1} + sC_1V_X + \frac{V_X - V_{out}}{Z_3} = 0$$

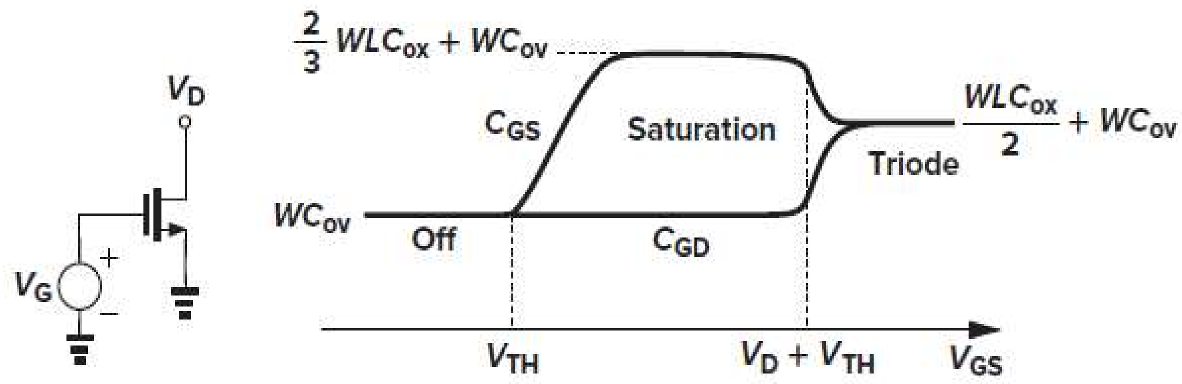
$$\frac{V_X - V_{out}}{Z_3} + sC_2V_{out} + \frac{V_{out} - V_a}{R_2} = 0$$

=> To get its transfer function (T.F.) => V_{out}/V_{in}

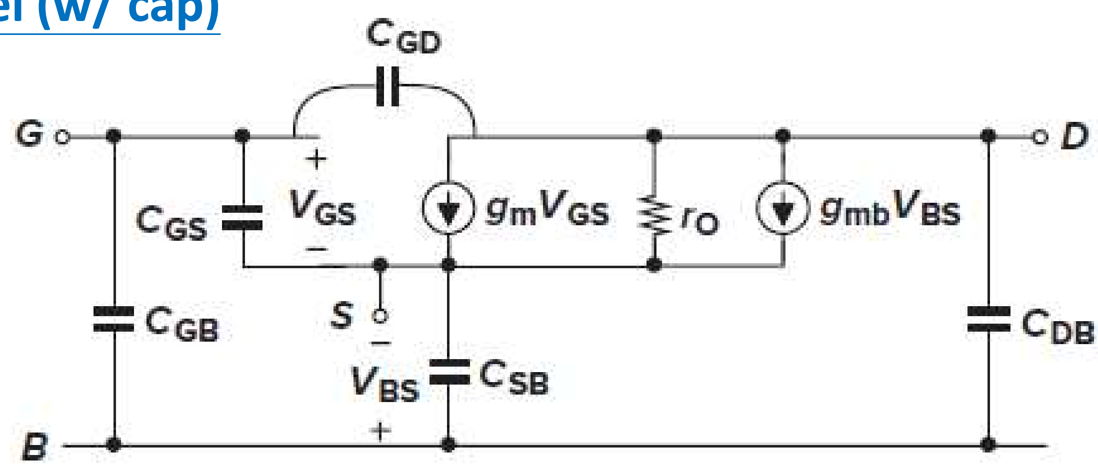
Review of MOSFET

Parasitic capacitors

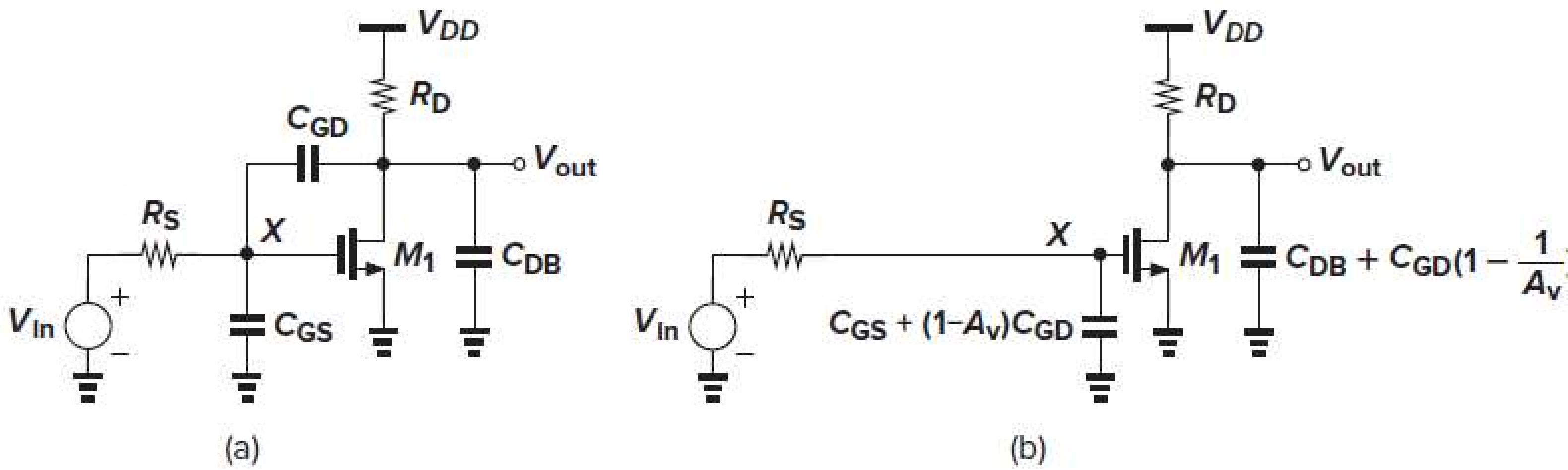




Small-signal model (w/ cap)



Common-Source Stage (C-S)



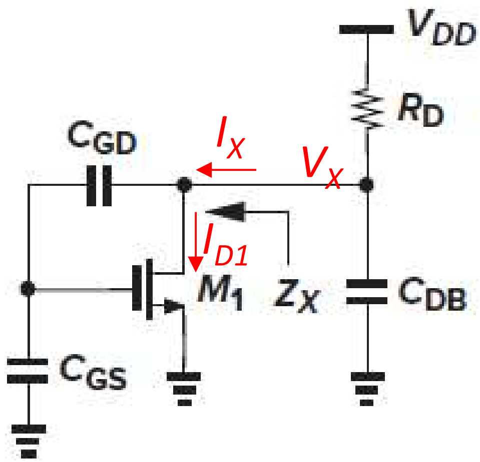
輸入節點：$C_{GS}+(1-A_v)C_{GD}$，其中 $A_v=-g_mR_D$

$$\omega_{in} = \frac{1}{R_S[C_{GS} + (1+g_mR_D)C_{GD}]}$$

輸出節點(?)：$C_{DB}+(1-A_v^{-1})C_{GD} \sim C_{DB}+C_{GD}$

$$\omega_{out} = \frac{1}{R_D(C_{DB} + C_{GD})}$$

T. F. of C-S Stage



$$V_X = \left(I_X - \underbrace{g_m \frac{C_{GD}}{C_{GS} + C_{GD}} V_X}_{I_{D1}} \right) \cdot \frac{1}{sC_{eq}}$$

$$C_{eq} = C_{GD}C_{GS}/(C_{GD} + C_{GS})$$

$$\Rightarrow Z_X = \frac{1}{C_{eq}s} \Big\| \underbrace{\left(\frac{C_{GD} + C_{GS}}{C_{GD}} \cdot \frac{1}{g_{m1}} \right)}_{R_X}$$

$$\Rightarrow \omega_{out} = \frac{1}{\left[R_D \Big\| \left(\frac{C_{GD} + C_{GS}}{C_{GD}} \cdot \frac{1}{g_{m1}} \right) \right] (C_{eq} + C_{DB})}$$

Thus is different from one shown on the last page

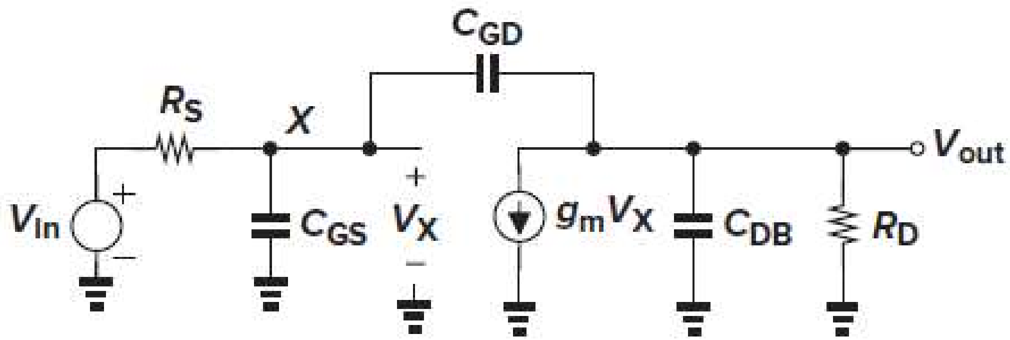
=> Miller effect cannot be applied at the output node

Transfer Function (T. F., ignored zero here)

$$\frac{V_{out}}{V_{in}}(s) = \frac{-g_m R_D}{\left(1 + \frac{s}{\omega_{in}}\right)\left(1 + \frac{s}{\omega_{out}}\right)}$$

Two poles

Direct Analysis of T. F.



Two poles
One zero

KCL at node X $$\frac{V_X - V_{in}}{R_S} + V_X C_{GS} s + (V_X - V_{out}) C_{GD} s = 0$$

KCL at node Vout $$(V_{out} - V_X) C_{GD} s + g_m V_X + V_{out}\left(\frac{1}{R_D} + C_{DB} s\right) = 0$$

$$V_X = -\frac{V_{out}\left(C_{GD}s + \frac{1}{R_D} + C_{DB}s\right)}{g_m - C_{GD}s}$$

$$-V_{out}\frac{[R_S^{-1} + (C_{GS} + C_{GD})s][R_D^{-1} + (C_{GD} + C_{DB})s]}{g_m - C_{GD}s} - V_{out} C_{GD} s = \frac{V_{in}}{R_S}$$

$$\boxed{\frac{V_{out}}{V_{in}}(s) = \frac{(C_{GD}s - g_m)R_D}{R_S R_D \xi s^2 + [R_S(1 + g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})]s + 1}}$$

where $\xi = C_{GS}C_{GD} + C_{GS}C_{DB} + C_{GD}C_{DB}$

A complex T. F., but…

Direct Analysis of T. F. (Supplement)

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$$\frac{V_{out}}{V_{in}}(s) = \frac{(C_{GD}s - g_m)R_D}{R_S R_D \xi s^2 + [R_S(1 + g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})]s + 1}$$

分子 => $-g_m R_D*(1-sC_{GD}/g_m)$

分母:

If $R_D(C_{GD}+C_{DB})$ is much smaller,

$\omega_{p1} = 1/[\, Rs(C_{GS}+C_{GD}(1+g_m R_D)\,]$

=> $\omega_{p2} = \ldots$

$$\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)$$

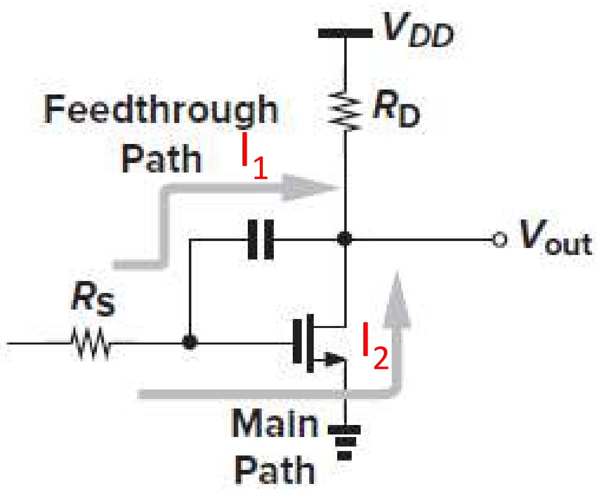
$$= 1 + \left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)s + \frac{s^2}{\omega_{p1}\omega_{p2}}$$

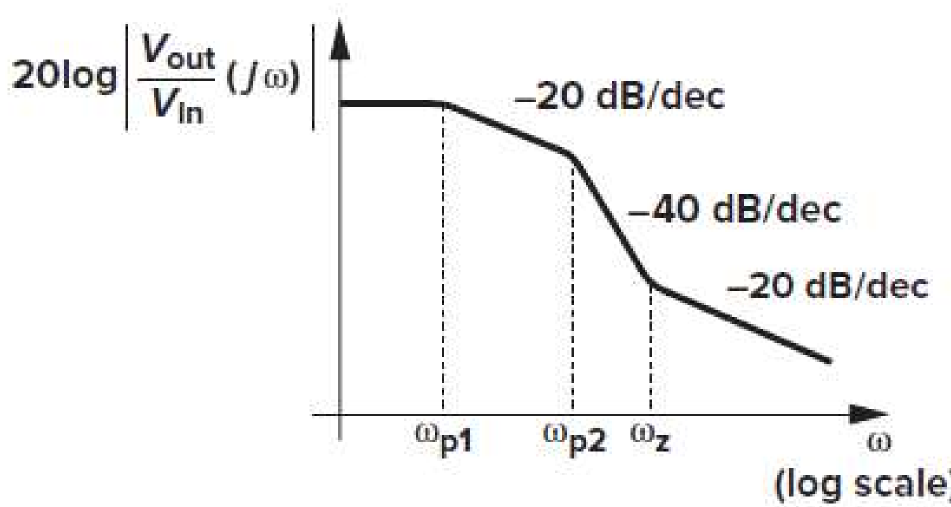
(If $\omega_{p1} << \omega_{p2}$)

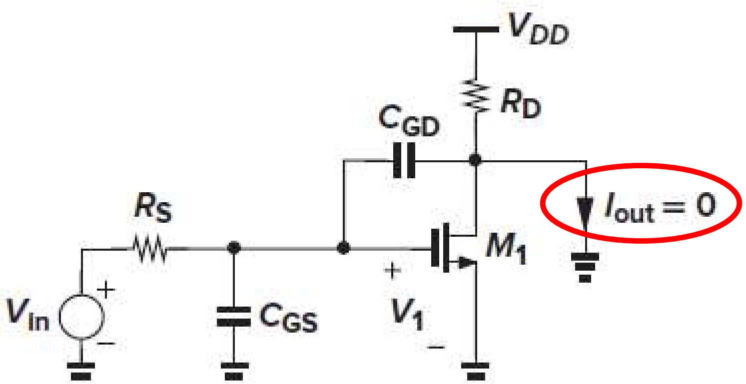
$$\sim 1 + \frac{s}{\omega_{p1}} + \frac{s^2}{\omega_{p1}\omega_{p2}}$$

Zero of C-S Stage

How to determine a zero? **By definition**







$I_{out} = I_1 + I_2 = 0$

By definition,

$sC_{GD}V_1 - g_mV_1 = 0$

…

Zin of C-S Stage

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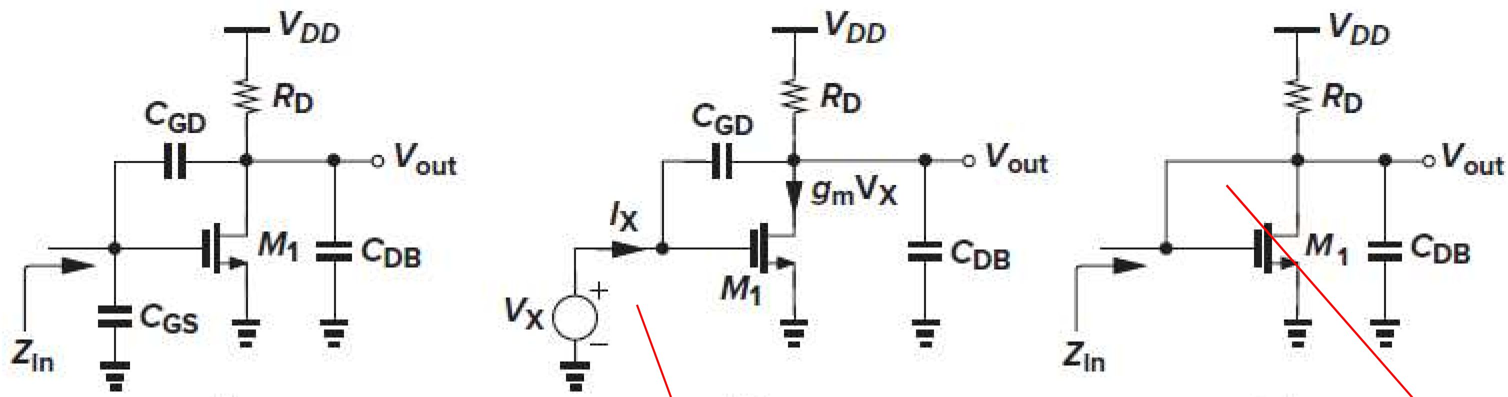


(a) (b) (c)

If large C_{GD} or at high-freq

Miller Effect

$$Z_{in} = \frac{1}{[C_{GS} + (1 + g_m R_D)C_{GD}]s}$$

A simplified watch, but be careful the assumption is true

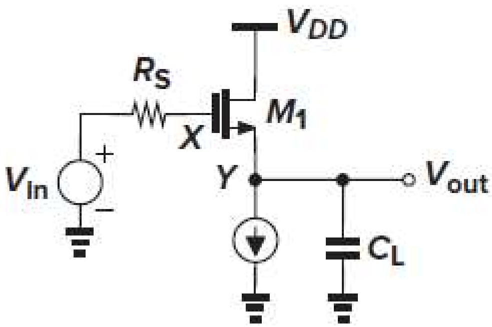
Compact Analysis (at low-freq.)

$$(I_X - g_m V_X)\frac{R_D}{1 + R_D C_{DB}s} + \frac{I_X}{C_{GD}s} = V_X$$

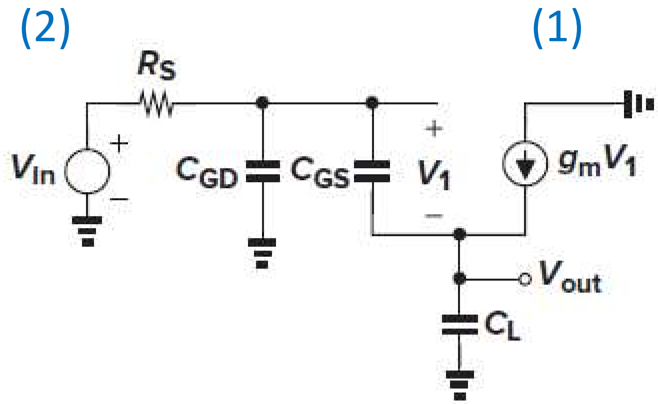
$$Z_X = \frac{V_X}{I_X} = \frac{1 + \cancel{R_D(C_{GD} + C_{DB})s}}{C_{GD}s(1 + g_m R_D + \cancel{R_D C_{DB}s})}$$

$$Z_{in} = 1/sC_{GS} \,//\, Z_X$$

Source Follower



(a)



(b)

$$\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)$$

$$= 1 + \left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)s + \frac{s^2}{\omega_{p1}\omega_{p2}}$$

(If $\omega_{p1} << \omega_{p2}$)

$$\sim 1 + \frac{s}{\omega_{p1}} + \frac{s^2}{\omega_{p1}\omega_{p2}}$$

(1) $V_1 C_{GS} s + g_m V_1 = V_{out} C_L s$

$$\Rightarrow V_1 = \frac{C_L s}{g_m + C_{GS} s} V_{out}$$

(2) $V_{in} = R_S[V_1 C_{GS} s + (V_1 + V_{out}) C_{GD} s] + V_1 + V_{out}$

$$\omega_{p1} \approx \frac{g_m}{g_m R_S C_{GD} + C_L + C_{GS}} = \frac{1}{R_S C_{GD} + \dfrac{C_L + C_{GS}}{g_m}}$$

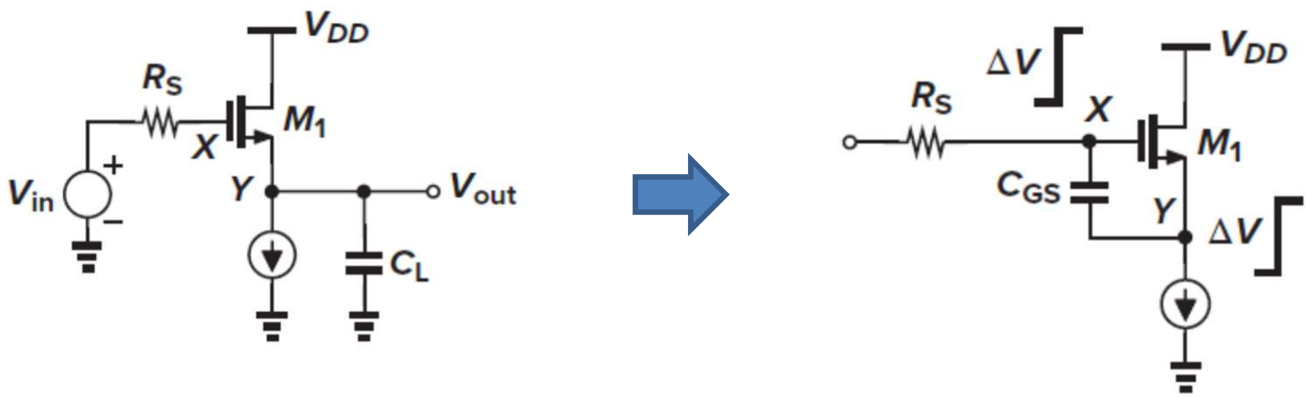
If $\omega_{p1} << \omega_{p2}$

$$\boxed{\frac{V_{out}}{V_{in}}(s) = \frac{g_m + C_{GS} s}{R_S(C_{GS} C_L + C_{GS} C_{GD} + C_{GD} C_L) s^2 + (g_m R_S C_{GD} + C_L + C_{GS}) s + g_m}}$$

=> What do you get from this equation?

Example 6.11

Examine the source follower transfer function if $C_L = 0$.



$$\frac{V_{out}}{V_{in}}(s) = \frac{g_m + C_{GS}s}{R_S(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L)s^2 + (g_m R_S C_{GD} + C_L + C_{GS})s + g_m}$$

$$\begin{aligned}\frac{V_{out}}{V_{in}} &= \frac{g_m + C_{GS}s}{R_S C_{GS} C_{GD} s^2 + (g_m R_S C_{GD} + C_{GS})s + g_m} \\ &= \frac{g_m + C_{GS}s}{(1 + R_S C_{GD} s)(g_m + C_{GS}s)} \\ &= \frac{1}{1 + R_S C_{GD} s}\end{aligned}$$

Zin of Source Follower

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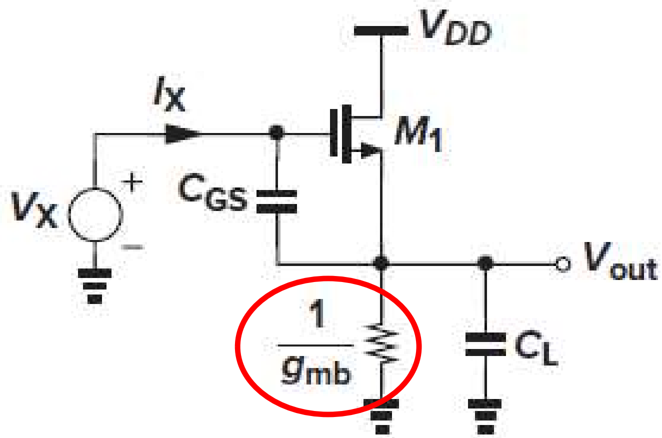


$$V_X = \frac{I_X}{C_{GS}s} + \left(I_X + \frac{g_m I_X}{C_{GS}s}\right)\left(\frac{1}{g_{mb}} \Big\| \frac{1}{C_L s}\right)$$

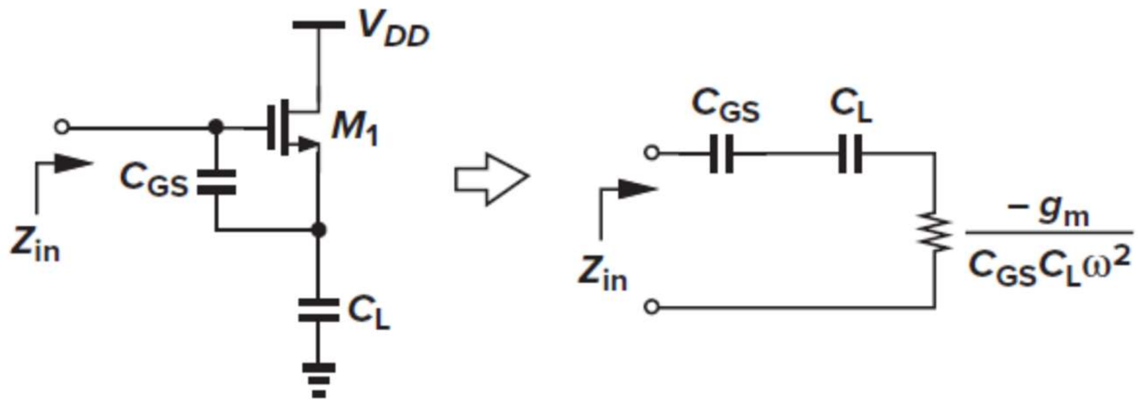
$$\boxed{Z_{in} = \frac{1}{C_{GS}s} + \left(1 + \frac{g_m}{C_{GS}s}\right)\frac{1}{g_{mb} + C_L s}}$$

$$Z_{in} = \frac{1}{sC_{GS}}\left(1 + \frac{g_m + SC_{GS}}{g_{mb} + SC_L}\right)$$

low frequencies, $g_{mb} \gg |C_L s|$ and $Z_{in} \approx \frac{1}{C_{GS}s}\left(1 + \frac{g_m}{g_{mb}}\right) + \frac{1}{g_{mb}}$

At high frequencies, $g_{mb} \ll |C_L s|$ and $Z_{in} \approx \frac{1}{C_{GS}s} + \frac{1}{C_L s} + \frac{g_m}{C_{GS} C_L s^2}$

Negative Resistance



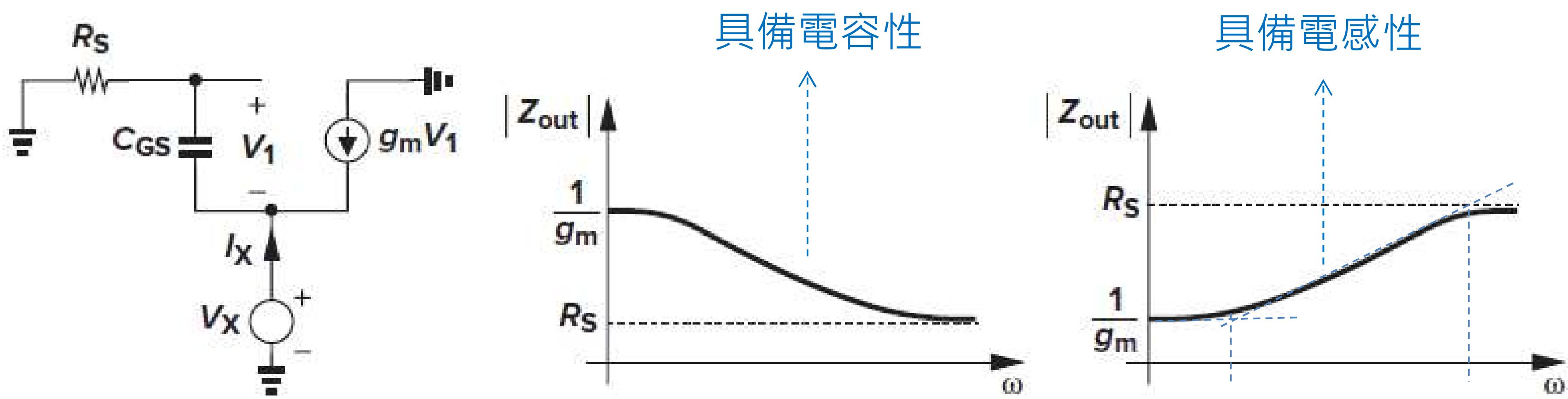
$$Z_{in} \approx \underbrace{\frac{1}{C_{GS}s} + \frac{1}{C_L s}} + \underbrace{\frac{g_m}{C_{GS}C_L s^2}}$$

C_L and C_{GS} are series-connected

$s = j\omega$

$$\frac{-g_m}{C_{GS}C_L\omega^2}$$

Zout of Source Follower



(a) (b) (c)

Q: Which one is right? It depends!!

- Zout (DC)=1/g_m
- Zout (HF) = R_S

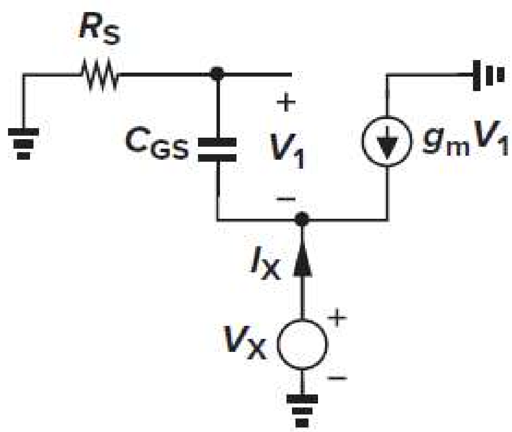
$$Z_{out} = \frac{V_X}{I_X}$$

$$= \frac{R_S C_{GS} s + 1}{g_m + C_{GS} s}$$

Pole is g_m/C_{GS}

Zero is $1/R_S C_{GS}$

Zout of Source Follower

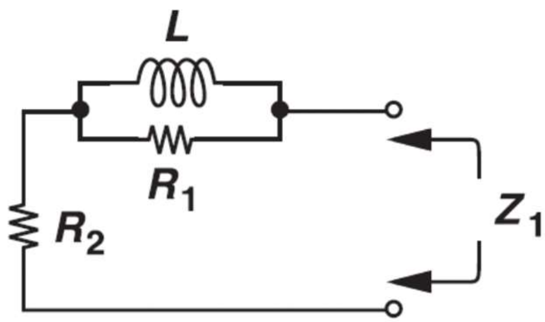


$$Z_{out} = \frac{V_X}{I_X} = \frac{R_S C_{GS} s + 1}{g_m + C_{GS} s}$$

$$Z_{out} - \frac{1}{g_m} = \frac{s C_{GS}\left(R_S - \frac{1}{g_m}\right)}{g_m + C_{GS} s}$$

$$\frac{1}{Z_{out} - \frac{1}{g_m}} = \frac{1}{R_S - \frac{1}{g_m}} + \frac{1}{s\frac{C_{GS}}{g_m}\left(R_S - \frac{1}{g_m}\right)}$$

電感性之分析



$$Z_{out} = Z_1 = sL//R_1 + R_2$$

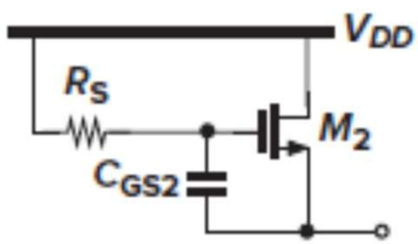
$$R_2 = 1/g_m$$

$$R_1 = R_S - 1/g_m$$

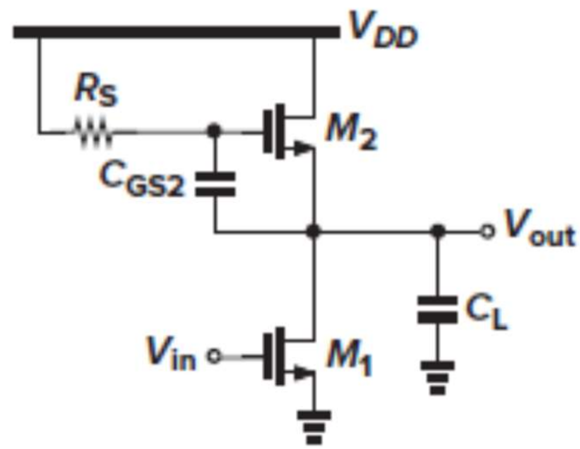
$$\boxed{L = \frac{C_{GS}}{g_m}\left(R_S - \frac{1}{g_m}\right)}$$

Example 6.14

Can we construct a (two-terminal) inductor from a source follower?



(a)



(b)

$$L = \frac{C_{GS2}}{g_{m2}}\left(R_S - \frac{1}{g_{m2}}\right)$$

$$R_1 = R_S - \frac{1}{g_{m2}}$$

$$R_2 = \frac{1}{g_{m2}}$$

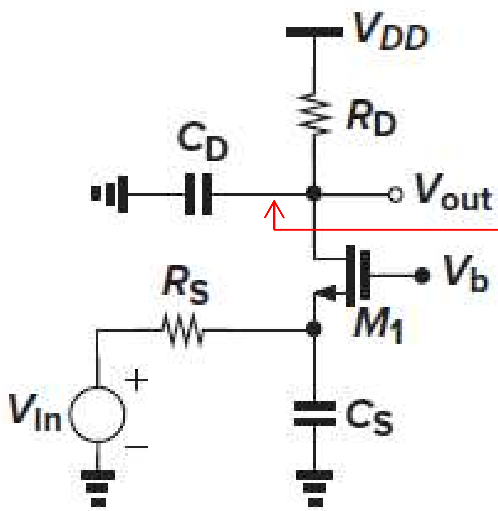
Yes, we can. Called an "active inductor," such a structure is shown in Fig. 6.30(a), providing an inductance of $(C_{GS2}/g_{m2})(R_S - 1/g_{m2})$. But the inductor is not ideal because it also incurs a parallel resistance equal to $R_1 = R_S = 1/g_{m2}$ and a series resistance equal to $1/g_{m2}$. Figure 6.30(b) depicts an application of active inductors: the inductance can partially cancel the load capacitance, C_L, at high frequencies, thus extending the *bandwidth*. However, the voltage headroom consumed by M_2 $(= V_{GS2})$ limits the gain. Also, C_{GD2}, which has been neglected in our analysis, limits the bandwidth enhancement.

Common-Gate Stage

$$C_D = C_L + C_{GD} + C_{DB}$$
$$C_S = C_{GS} + C_{SB}$$

C-G stage, $Z_{in} \approx \dfrac{Z_L + r_O}{1 + (g_m + g_{mb}) r_O}$

$$Z_{in} \approx \frac{Z_L}{(g_m + g_{mb}) r_O} + \frac{1}{g_m + g_{mb}}$$



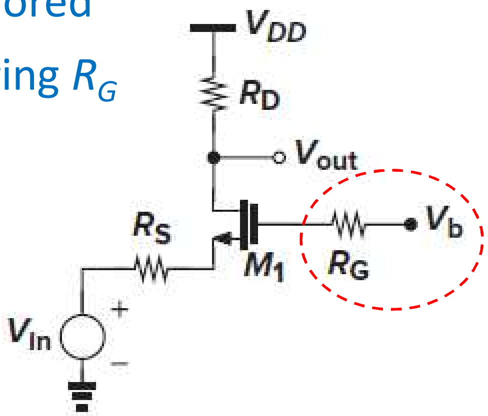
$Z_L = R_D \| [1/(C_D s)]$

the input and output nodes are "isolated" if channel-length modulation is neglected.

$$\frac{V_{out}}{V_{in}}(s) = \frac{(g_m + g_{mb}) R_D}{1 + (g_m + g_{mb}) R_S} \frac{1}{\left(1 + \dfrac{C_S}{g_m + g_{mb} + R_S^{-1}} s\right)(1 + R_D C_D s)}$$

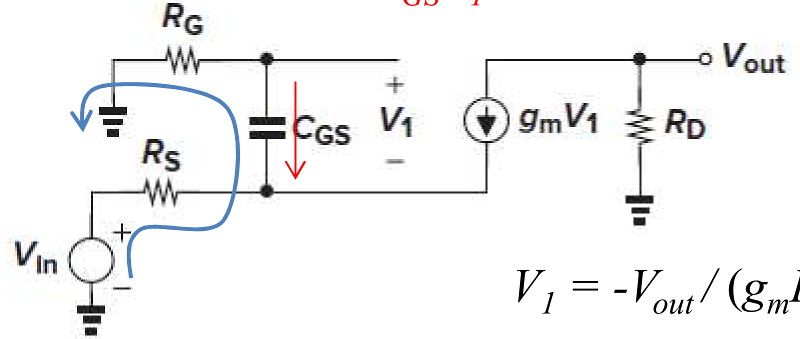
T.-F. of Common-Gate (C_{GS} only)

- g_{mb} is ignored
- Considering R_G



(a)

$$i = sC_{GS}V_1$$



(b)

$$V_1 = -V_{out}/(g_mR_D)$$

$$V_{in} - (sC_{GS} + g_m)\frac{V_{out}}{g_mR_D}R_S - \frac{V_{out}}{g_mR_D} - C_{GS}s\frac{V_{out}}{g_mR_D}R_G = 0$$

It follows that

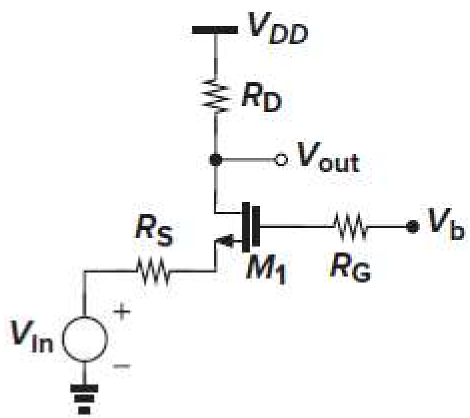
$$\frac{V_{out}}{V_{in}} = \frac{g_mR_D}{(R_G + R_S)C_{GS}s + 1 + g_mR_S}$$

yielding a pole at

$$\omega_p = \frac{1 + g_mR_S}{(R_G + R_S)C_{GS}}$$

(a *higher frequency*)

Q: Why mention R_G?



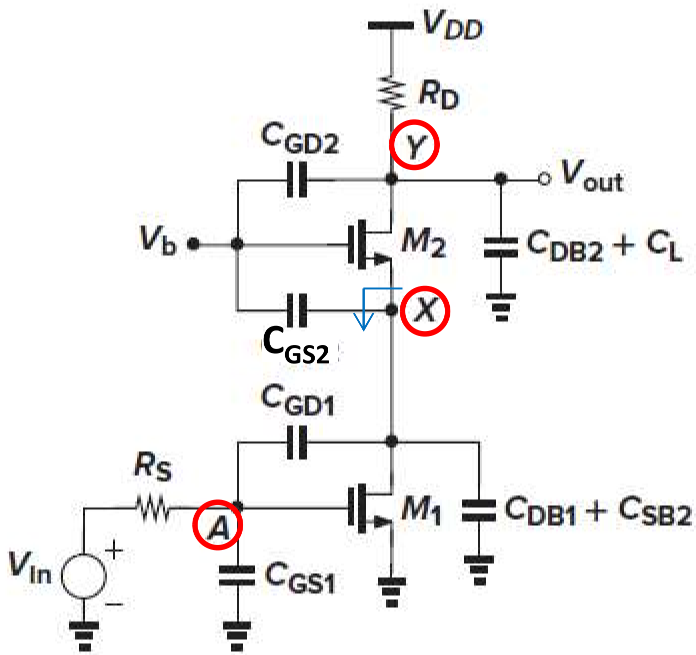
C_{GS} only

$$\frac{V_{out}}{V_{in}} = \frac{g_m R_D}{(R_G + R_S)C_{GS}s + 1 + g_m R_S}$$

$$\omega_p = \frac{1 + g_m R_S}{(R_G + R_S)C_{GS}}$$

R_G directly adds to R_S in this case, lowering the pole magnitude

Cascode Stage



$$\omega_{p,A} = \frac{1}{R_S\left[C_{GS1} + \underline{\left(1 + \frac{g_{m1}}{g_{m2} + g_{mb2}}\right) C_{GD1}}\right]}$$

$$\omega_{p,X} = \frac{g_{m2} + g_{mb2}}{\underbrace{2C_{GD1} + C_{DB1} + C_{SB2} + C_{GS2}}_{\rightarrow C_X}}$$

$$\omega_{p,Y} = \frac{1}{R_D(C_{DB2} + \underline{C_L} + C_{GD2})}$$

$$Z_{out} = (1 + g_{m2}r_{O2})Z_X + r_{O2}$$

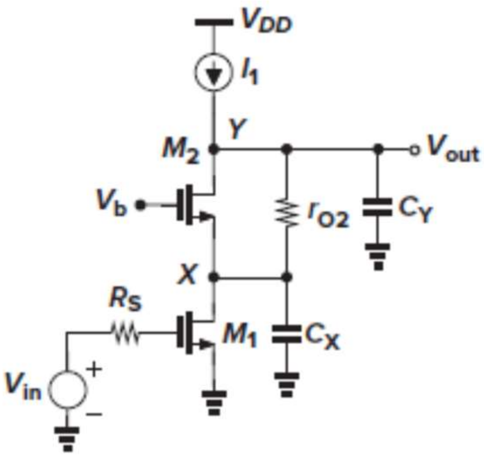
$$Z_X = r_{O1}||(\underline{C_X}s)^{-1}$$

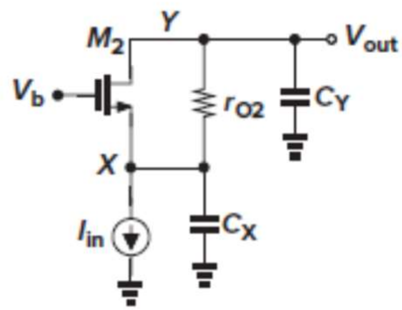
Why "cascode"?

$\Rightarrow$ $\omega_{p,A}$ is higher

$\Rightarrow$ What is the dominant pole?

Example 6.16





$$-r_{O2}\left[(V_{out}C_Y s + I_{in})\frac{g_{m2}}{C_X s} + V_{out}C_Y s\right] - (V_{out}C_Y s + I_{in})\frac{1}{C_X s} = V_{out}$$

That is

$$\frac{V_{out}}{I_{in}} = -\frac{g_{m2}r_{O2}+1}{C_X s}\cdot\frac{1}{1+(1+g_{m2}r_{O2})\dfrac{C_Y}{C_X}+C_Y r_{O2}s}$$

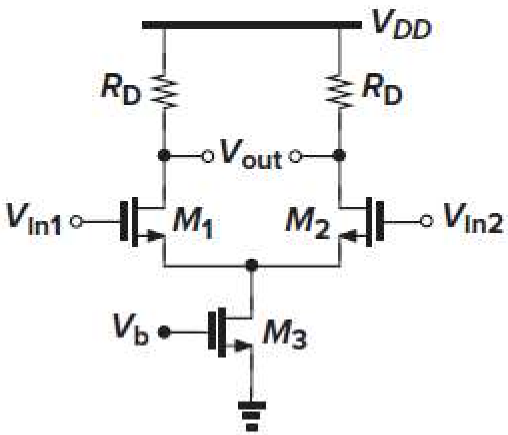
which, for $g_{m2}r_{O2} \gg 1$ and $g_{m2}r_{O2}C_Y/C_X \gg 1$ (i.e., $C_Y > C_X$), reduces to

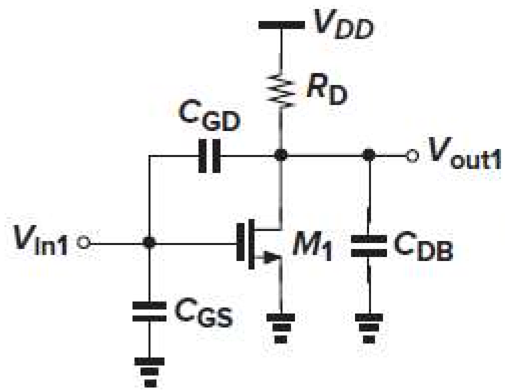
$$\frac{V_{out}}{I_{in}} \approx -\frac{g_{m2}}{C_X s}\frac{1}{\dfrac{C_Y}{C_X}g_{m2}+C_Y s}$$

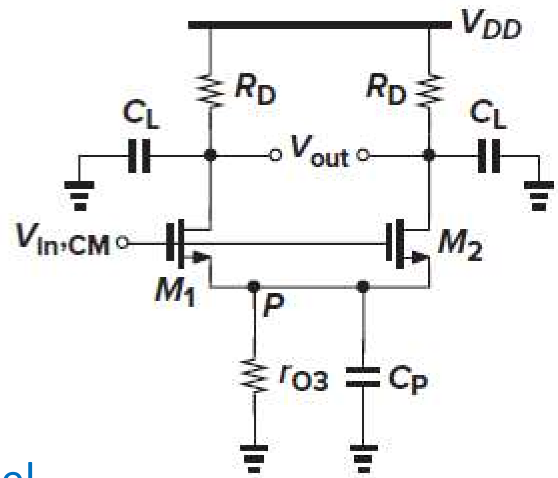
and hence

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1}g_{m2}}{C_Y C_X s}\frac{1}{g_{m2}/C_X+s}$$

Differential Pair: CMRR







(c)

2. Common-mode model

$$A_{v,CM} = -\frac{\Delta g_m \left[R_D \middle\| \left(\frac{1}{C_L s} \right) \right]}{(g_{m1} + g_{m2}) \left[r_{O3} \middle\| \left(\frac{1}{C_P s} \right) \right] + 1}$$

1. Differential model

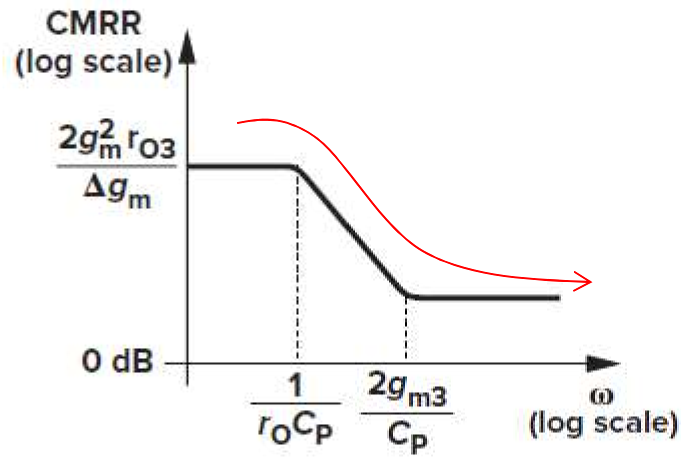
$$A_{DM} = -g_m \left(R_D \,||\, Z_L \right)$$

3. CMRR calculation:

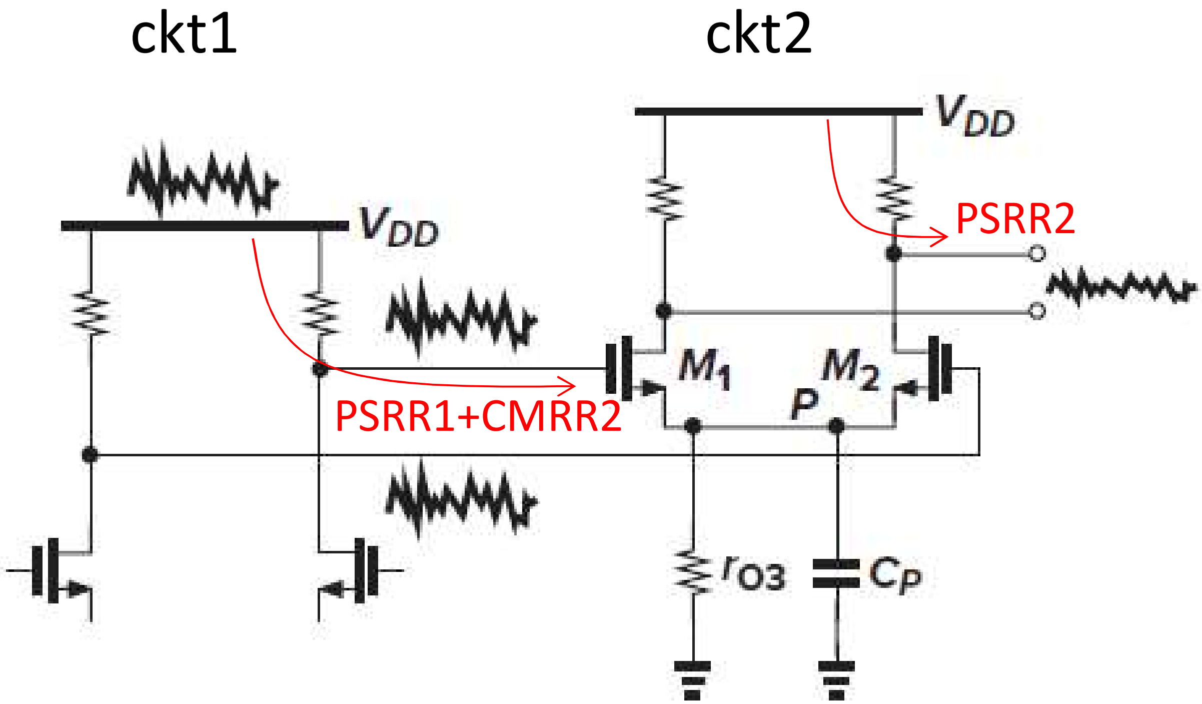
$$\text{CMRR} \approx \frac{g_m}{\Delta g_m} \left[1 + 2 g_m \left(r_{O3} || \frac{1}{C_P s} \right) \right]$$

$$\approx \frac{g_m}{\Delta g_m} \frac{r_{O3} C_P s + 1 + 2 g_m r_{O3}}{r_{O3} C_P s + 1}$$

where $g_m = (g_{m1} + g_{m2})/2$



Differential Pair: PSRR

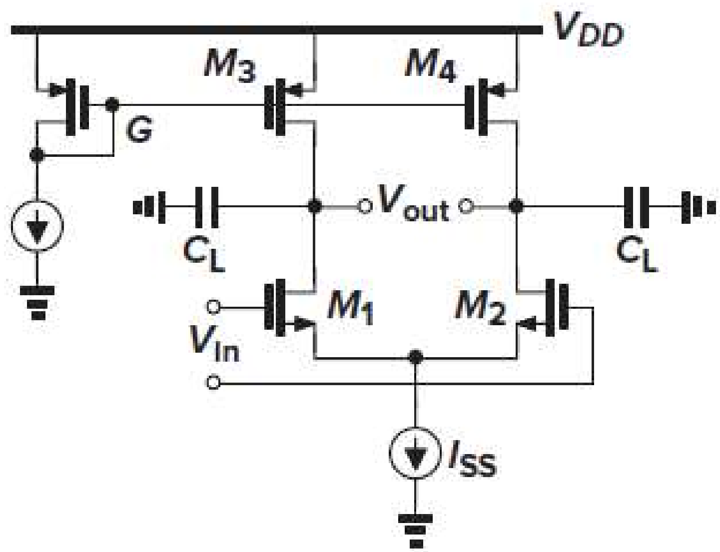


Two kinds of supply errors for ckt1 and ckt2

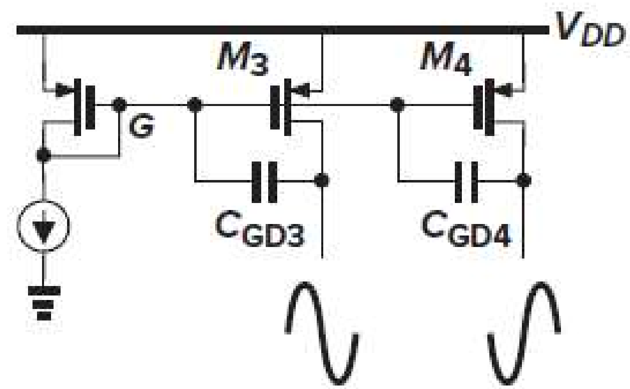
(1) **PSRR1 + CMRR2** (Supply noise to input common-mode ($V_{i,CM}$); then $V_{i,CM}$ affects diff. output by $A_{CM\text{-}DM}$)

(2) **PSRR2** (Supply noise directly affects the diff. output)

Differential Pair with high-Z loads



(a)

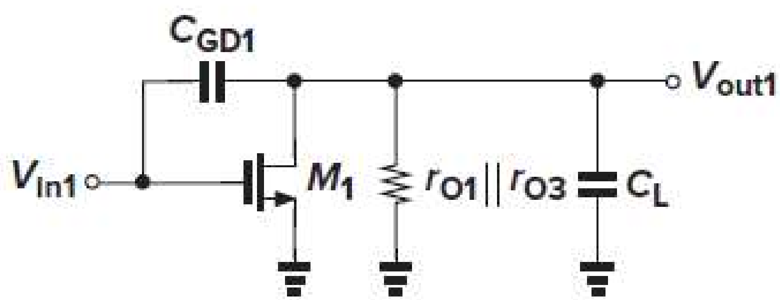


To find all poles and zeros, check the signal path

Hint:

This fully-differential amplifier has something to be concerned, the output common-mode voltage (Vocm)

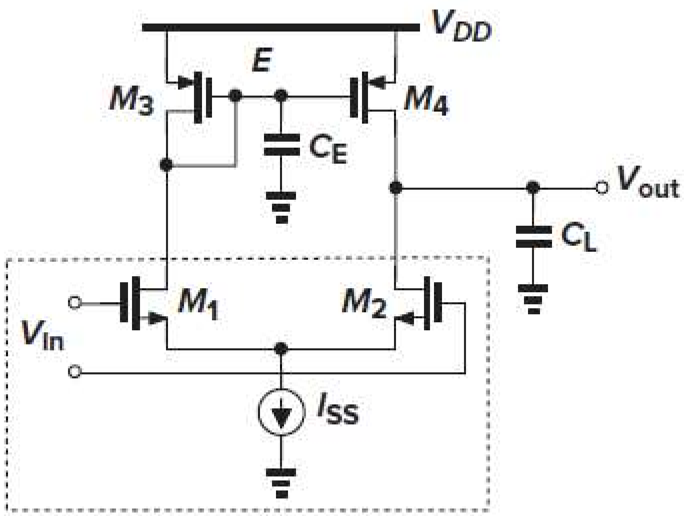
In fact, we need a CMFB (common-mode feedback) loop to maintain this (we discuss this in later lectures)!!



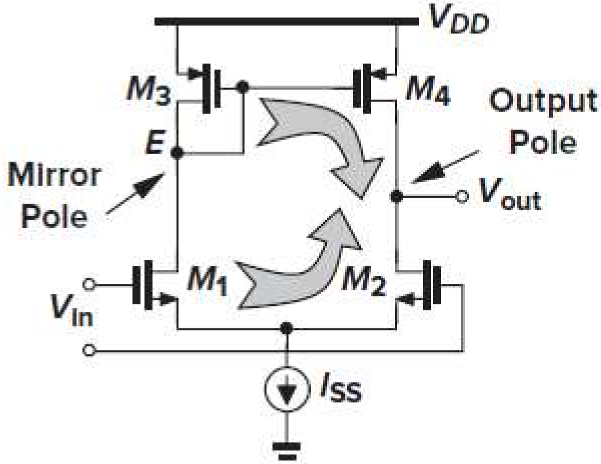
(c)

A simple **single-pole** circuit

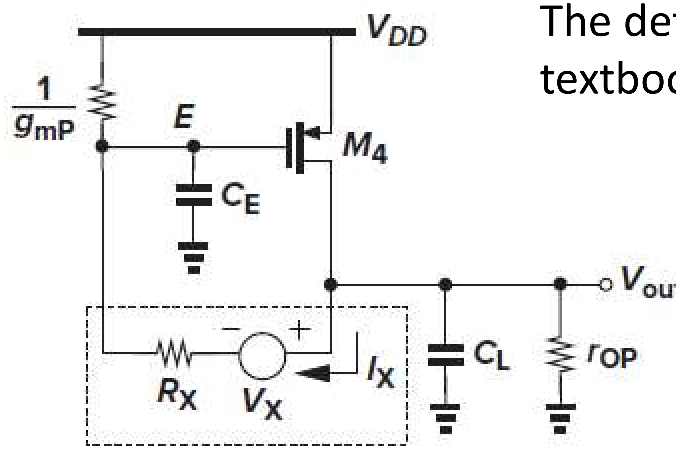
Differential Pair with active loads



(a)



The detail is in the textbook, page 202



$$V_X = g_{mN} r_{ON} V_{in} \qquad R_X = 2r_{ON}$$

we have assumed that $1/g_{mP} \ll r_{OP}$

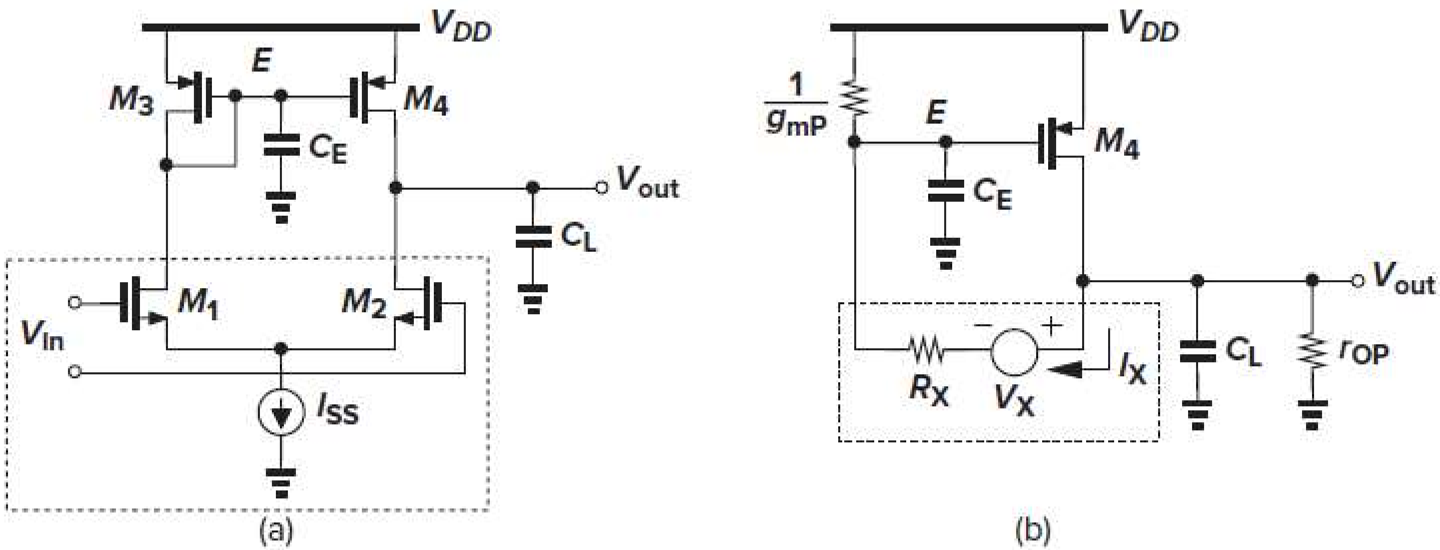
$$I_X = -g_{mN} V_{in}/2$$

$$V_E = (V_{out} - V_X)\frac{\dfrac{1}{C_E s + g_{mP}}}{\dfrac{1}{C_E s + g_{mP}} + R_X}$$

$$V_{out} = (-g_{mp} V_E - I_X)(r_{op} || 1/sC_L) = \ldots$$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{g_{mN} r_{ON}(2g_{mP} + C_E s) r_{OP}}{2r_{OP} r_{ON} C_E C_L s^2 + [(2r_{ON} + r_{OP})C_E + r_{OP}(1 + 2g_{mP} r_{ON})C_L]s + 2g_{mP}(r_{ON} + r_{OP})}}$$

Differential Pair with active loads



$$\frac{V_{out}}{V_{in}} = \frac{g_{mN}r_{ON}(2g_{mP}+C_E s)r_{OP}}{2r_{OP}r_{ON}C_E C_L s^2 + [(2r_{ON}+r_{OP})C_E + r_{OP}(1+2g_{mP}r_{ON})C_L]s + 2g_{mP}(r_{ON}+r_{OP})}$$

$$\frac{V_{out}}{V_{in}} = \frac{2g_{mP}g_{mN}r_{OP}r_{ON}}{2g_{mP}(r_{OP}+r_{ON})}\frac{1+s/(2g_{mP}/C_E)}{1+as+bs^2}$$

$$\frac{V_{out}}{V_{in}} = g_{mN}(r_{OP}||r_{ON})\frac{1+s/\omega_z}{(1+s/\omega_{p1})(1+s/\omega_{p2})}$$

$$\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)$$
$$= 1+\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)s+\frac{s^2}{\omega_{p1}\omega_{p2}}$$

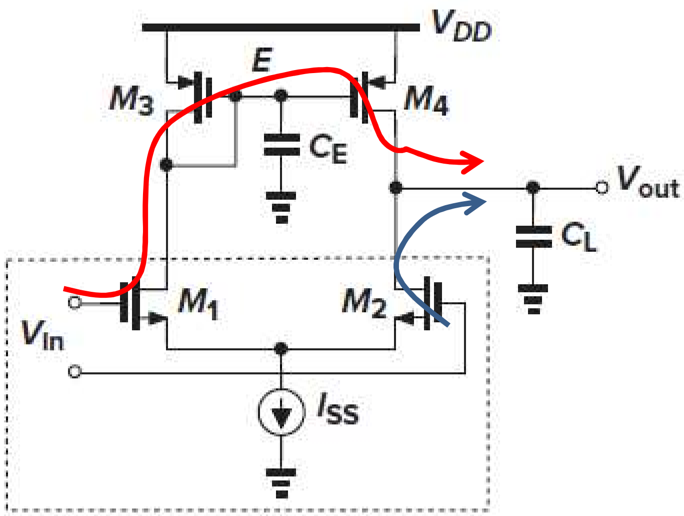
(If $\omega_{p1} << \omega_{p2}$)

$$\sim 1+\frac{s}{\omega_{p1}}+\frac{s^2}{\omega_{p1}\omega_{p2}}$$

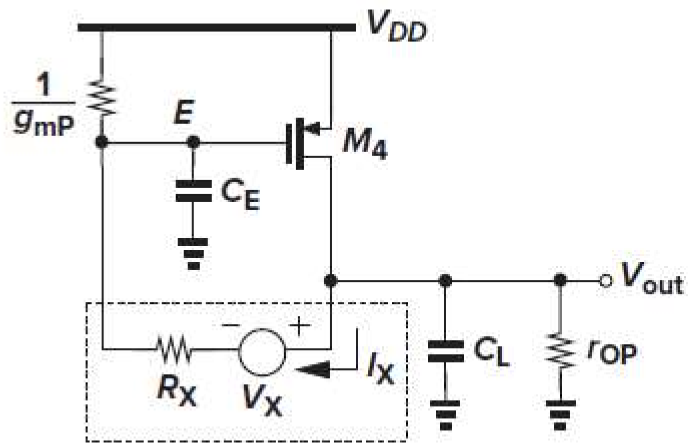
Differential Pair with active loads

slow

fast



(a)



(b)

Another circuit analysis method:

$$\frac{V_{out}}{V_{in}} = \frac{A_0}{1+s/\omega_{p1}}\left(\frac{1}{1+s/\omega_{p2}}+1\right)$$

$$= \frac{A_0(2+s/\omega_{p2})}{(1+s/\omega_{p1})(1+s/\omega_{p2})}$$

$$A_0 = \frac{g_{mN}\left(r_{OP} \,||\, r_{ON}\right)}{2}$$

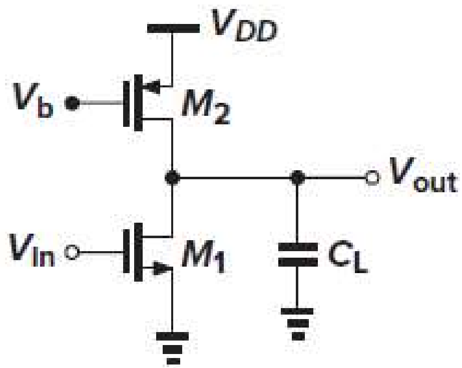
$$\omega_{p1} \approx \frac{2g_{mP}(r_{ON}+r_{OP})}{(2r_{ON}+r_{OP})C_E + r_{OP}(1+2g_{mP}r_{ON})C_L}$$

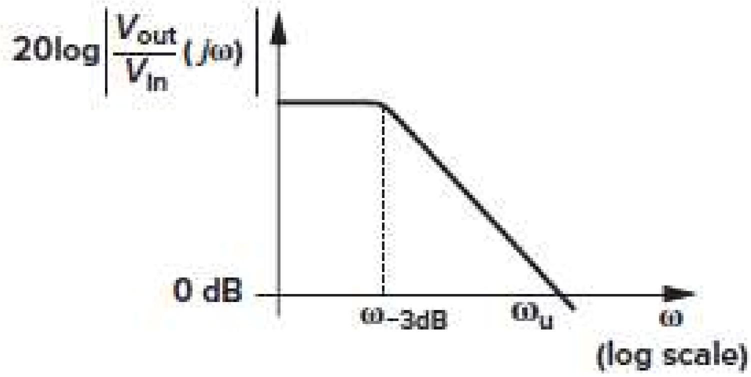
$$\approx \frac{1}{(r_{ON}\|r_{OP})C_L}$$

$$\omega_{p2} \approx \frac{g_{mP}}{C_E} \qquad \omega_z = \frac{2g_{mp}}{C_E} = 2\omega_{p2}$$

Gain-Bandwidth Trade-Offs

One-Pole Circuits





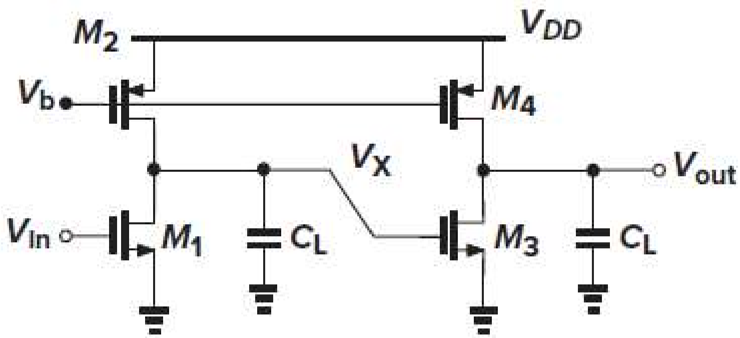
$$\begin{aligned}\mathrm{GBW} &= A_0\omega_p \\ &= g_{m1}(r_{O1}||r_{O2})\frac{1}{2\pi(r_{O1}||r_{O2})C_L} \\ &= \frac{g_{m1}}{2\pi C_L}\end{aligned}$$

$$\frac{A_0}{\sqrt{1+(\frac{\omega_u}{\omega_p})^2}} = 1$$

$$\text{if } A_0^2 \gg 1 \quad \omega_u = \sqrt{A_0^2-1}\,\omega_p \approx A_0\omega_p$$

Gain-Bandwidth Trade-Offs

Multi-Pole Circuits

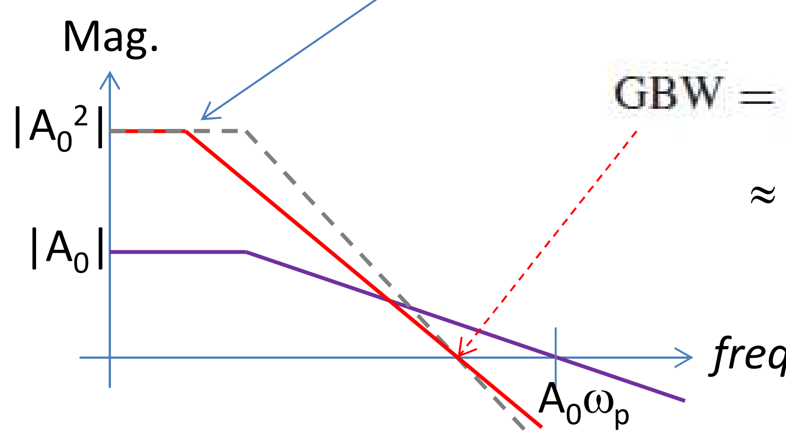


$$\frac{A_0^2}{1+\frac{\omega_{-3dB}^2}{\omega_p^2}} = \frac{A_0^2}{\sqrt{2}}$$

$$\omega_{-3dB} = \sqrt{\sqrt{2}-1}\,\omega_p$$

$$\approx 0.64\omega_p$$

$$\frac{V_{out}}{V_{in}} = \frac{A_0^2}{(1+\frac{s}{\omega_p})^2}$$



$$\mathrm{GBW} = \sqrt{\sqrt{2}-1}\,A_0\omega_p$$

$\approx 0.64 A_0\omega_p$

N single pole circuits series-connected, $\omega_{-3dB} = \sqrt{\sqrt[N]{2}-1}\,\omega_p$