1. $\sum_{n=1}^{\infty}(-1)^n \frac{(x-2)^n}{4^n\sqrt{n}}$ 收斂區間

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{(x-2)^{n+1}}{4^{n+1}\sqrt{n+1}}}{\frac{(x-2)^n}{4^n\sqrt{n}}}\right| = \frac{|x-2|}{4}\lim_{n\to\infty}\sqrt{\frac{n}{n+1}} = \frac{|x-2|}{4} < 1$$

$\Rightarrow -2 < x < 6$

$x = 6$ $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}}$ 收斂

$\Rightarrow$ 收斂區間 $-2 < x \leq 6$

$x = -2$ $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ 發散

2. $f(x) = e^x \sin x$ 取至第 3 項的 Maclaurin series.

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots$$

$$-\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \cdots$$

$$\longrightarrow e^x \sin x = \left(1 + x + \frac{1}{2!}x^2 + \cdots\right)\left(x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 \cdots\right)$$

$$= x + x^2 + \left(\frac{1}{2!} - \frac{1}{3!}\right)x^3 + \cdots$$

$$= x + x^2 + \frac{1}{3}x^3 + \cdots$$

3. $f(x)=e^{-x}$ 在 $x=-2$ Taylor series 展開式

$f'(x)=-e^{-x}$ $f''(x)=e^{-x}$ $\cdots$ $f^{(n)}(x)=(-1)^n e^{-x}$

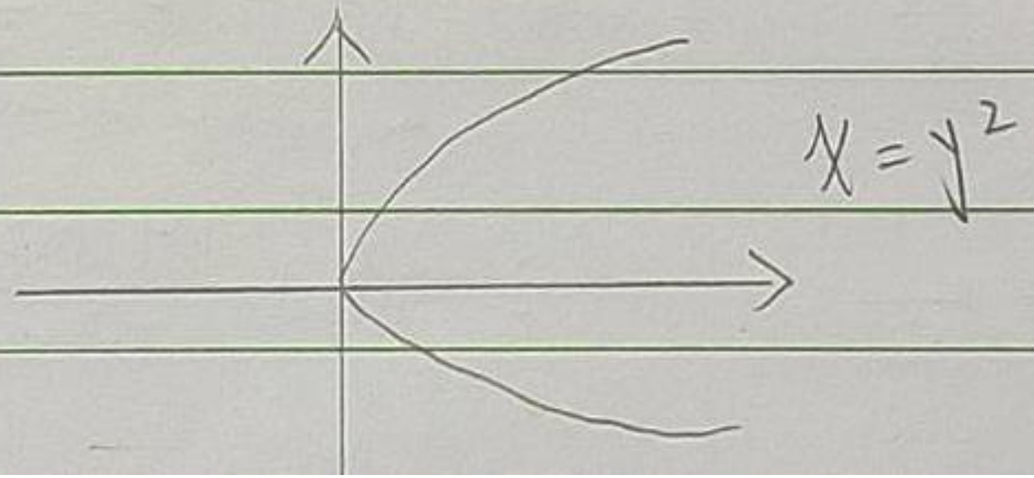
$\Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n e^2}{n!}(x+2)^n$

$\Rightarrow f(-2)=e^2$ $f'(-2)=-e^{-2}$ $\cdots\cdots$ $f^{(n)}(-2)=(-1)^n e^2$

4、

$x = \sec^2 t - 1 \quad y = \tan t \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$

$x = y^2$



5. 求 $x^{\frac{2}{3}}+y^{\frac{2}{3}}=1$ 的周長

令 $x=\cos^3 t \quad y=\sin^3 t \quad 0\le t\le 2\pi$

$$dL=\sqrt{(dx)^2+(dy)^2}=\sqrt{(x')^2+(y')^2}\,dt=\sqrt{[3\cos^2 t(-\sin t)]^2+(3\sin^2 t\cos t)^2}\,dt$$

$$=3\sqrt{\cos^4 t\sin^2 t+\sin^4 t\cos^2 t}\,dt=3\sqrt{\cos^2 t\sin^2 t(\cos^2 t+\sin^2 t)}\,dt=3\left|\sin t\cos t\right|dt \quad 0\le t\le 2\pi$$

$$L=\int dL=\int_0^{2\pi}3\left|\sin t\cos t\right|dt=4\int_0^{\frac{\pi}{2}}3\left|\sin t\cos t\right|dt=12\int_0^{\frac{\pi}{2}}\sin t\cos t\,dt$$

$$=6\int_0^{\frac{\pi}{2}}\sin 2t\,dt=-(3\cos 2t)\Big|_0^{\frac{\pi}{2}}$$

$$=6$$

6. $r = \dfrac{4}{2\cos\theta - \sin\theta}$

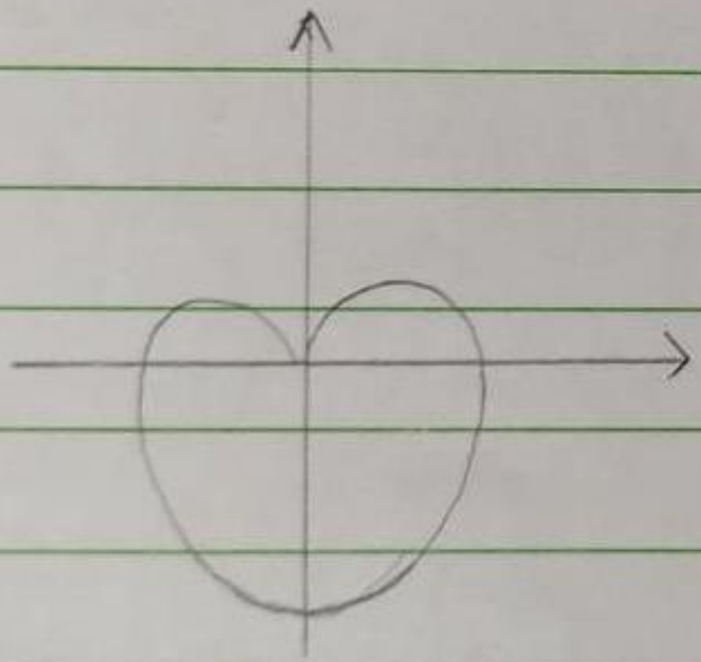
$\Rightarrow r(2\cos\theta - \sin\theta) = 4$

$2r\cos\theta - r\sin\theta = 4$

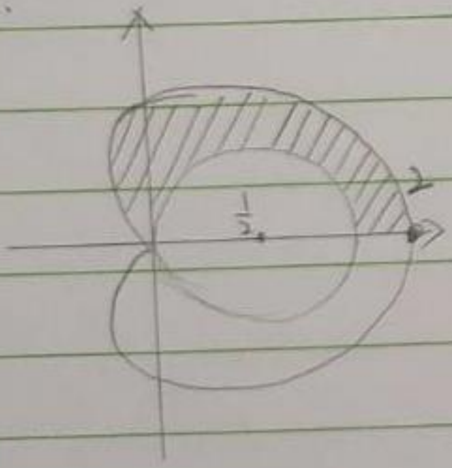
$\Downarrow$

$2x - y = 4$ or $y = 2x - 4$

7. $r = 1 - \sin\theta$



8.




斜線面積：$\frac{1}{2}\int_0^{\pi}(1+\cos\theta)^2\,d\theta-\frac{1}{2}\int_0^{\frac{\pi}{2}}\cos^2\theta\,d\theta$

$=\frac{1}{2}\int_0^{\pi}1+2\cos\theta+\cos^2\theta\,d\theta-\frac{1}{2}\left(\frac{1}{2}\theta+\frac{1}{4}\sin2\theta\Big|_0^{\frac{\pi}{2}}\right)$

$=\frac{1}{2}\int_0^{\pi}1+2\cos\theta+\frac{1+\cos2\theta}{2})\,d\theta=\frac{1}{2}\left(\frac{3}{2}\theta+2\sin\theta+\frac{1}{4}\sin2\theta\right)\Big|_0^{\pi}-\frac{\pi}{8}$

$=\frac{5}{8}\pi$

9. 求 domain, range $f(x,y) = \dfrac{1}{\sqrt{16-x^2-y^2}}$

$16-x^2-y^2 > 0$

$16-x^2-y^2 \to 0$, $f(x,y) \to \infty$

$x^2+y^2 < 16$

$16-x^2-y^2 \to 16$, $f(x,y) \to 14$

$\Rightarrow$ domain $(x,y) \in R : x^2+y^2 < 16$

range $[\frac{1}{4}, \infty)$

10 $\lim\limits_{(x,y)\to(0,0)} \frac{xy+y^3}{x^2+y^2}$ 令 $y=kx$

$$\lim_{x\to 0}\frac{x\cdot kx+k^3x^3}{x^2+k^2x^2}=\lim_{x\to 0}\frac{kx^2+k^3x^3}{(1+k^2)x^2}=\lim_{x\to 0}\left(\frac{k}{1+k^2}+\frac{k^3x}{1+k^2}\right)$$

$$=\frac{k}{1+k^2}$$

隨 k 改變，故極限不存在