CHAPTER 16

Two-Port Networks

THE LEARNING GOALS FOR THIS CHAPTER ARE THAT STUDENTS SHOULD BE ABLE TO:

- Calculate the admittance, impedance, hybrid, and transmission parameter for two-port networks.
- Convert between admittance, impedance, hybrid, and transmission parameters.
- Describe the interconnection of two-port networks to form more complicated networks.

AN EXPERIMENT THAT HELPS STUDENTS DEVELOP AN UNDERSTANDING OF THE MODELS USED FOR TWO-PORT NETWORKS IS:

- Impedance Transmission Parameters: Impedance parameters for two-port networks for several circuits are determined analytically and from results obtained from PSpice. Measurements of the impedance parameters are performed on real circuits.

16.1 Admittance Parameters

We say that the linear network in **Fig. 16.1a** has a single *port*—that is, a single pair of terminals. The pair of terminals *A-B* that constitute this port could represent a single element (e.g., *R*, *L*, or *C*), or it could be some interconnection of these elements. The linear network in **Fig. 16.1b** is called a two-port. As a general rule, the terminals *A-B* represent the input port and the terminals *C-D* represent the output port.

In the two-port network shown in **Fig. 16.2**, it is customary to label the voltages and currents as shown; that is, the upper terminals are positive with respect to the lower terminals, the currents are into the two-port at the upper terminals and, because KCL must be satisfied at each port, the current is out of the two-port at the lower terminals. Since the network is linear and contains no independent sources, the principle of superposition can be applied to determine the current $\mathbf{I}_1$, which can be written as the sum of two components, one due to $\mathbf{V}_1$ and one due to $\mathbf{V}_2$. Using this principle, we can write

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2$$

where $\mathbf{y}_{11}$ and $\mathbf{y}_{12}$ are essentially constants of proportionality with units of siemens. In a similar manner $\mathbf{I}_2$ can be written as

$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2$$

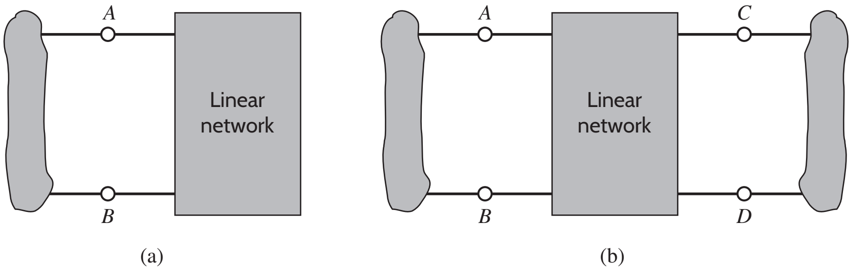


Figure 16.1
(a) Single-port network;
(b) two-port network.

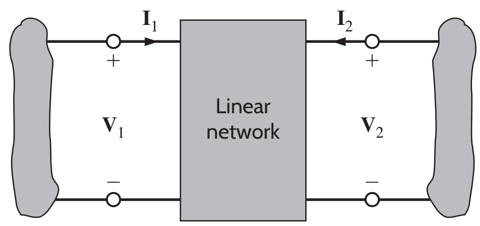


Figure 16.2
Generalized two-port network.

Therefore, the two equations that describe the two-port network are

$$\begin{aligned}\mathbf{I}_1 &= \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2\\ \mathbf{I}_2 &= \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2\end{aligned} \tag{16.1}$$

or in matrix form,

$$\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix} = \begin{bmatrix}\mathbf{y}_{11} & \mathbf{y}_{12}\\ \mathbf{y}_{21} & \mathbf{y}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix}$$

Note that subscript 1 refers to the input port and subscript 2 refers to the output port, and the equations describe what we will call the *Y parameters* for a network. If these parameters $\mathbf{y}_{11}$, $\mathbf{y}_{12}$, $\mathbf{y}_{21}$, and $\mathbf{y}_{22}$ are known, the input/output operation of the two-port is completely defined.

From Eq. (16.1), we can determine the *Y* parameters in the following manner. Note from the equations that $\mathbf{y}_{11}$ is equal to $\mathbf{I}_1$ divided by $\mathbf{V}_1$ with the output short-circuited (i.e., $\mathbf{V}_2 = 0$).

$$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2 = 0} \tag{16.2}$$

Since $\mathbf{y}_{11}$ is an admittance at the input measured in siemens with the output short-circuited, it is called the *short-circuit input admittance*. The equations indicate that the other *Y* parameters can be determined in a similar manner:

$$\begin{aligned}\mathbf{y}_{12} &= \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1 = 0}\\ \mathbf{y}_{21} &= \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2 = 0}\\ \mathbf{y}_{22} &= \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1 = 0}\end{aligned} \tag{16.3}$$

$\mathbf{y}_{12}$ and $\mathbf{y}_{21}$ are called the *short-circuit transfer admittances* and $\mathbf{y}_{22}$ is called the *short-circuit output admittance*. As a group, the *Y* parameters are referred to as the *short-circuit admittance parameters*. Note that by applying the preceding definitions, these parameters could be determined experimentally for a two-port network whose actual configuration is unknown.

EXAMPLE 16.1

We wish to determine the Y parameters for the two-port network shown in Fig. 16.3a. Once these parameters are known, we will determine the current in a 4-Ω load, which is connected to the output port when a 2-A current source is applied at the input port.

SOLUTION

From Fig. 16.3b, we note that

$$\mathbf{I}_1 = \mathbf{V}_1\left(\frac{1}{1} + \frac{1}{2}\right)$$

Therefore,

$$\mathbf{y}_{11} = \frac{3}{2}\,S$$

As shown in Fig. 16.3c,

$$\mathbf{I}_1 = -\frac{\mathbf{V}_2}{2}$$

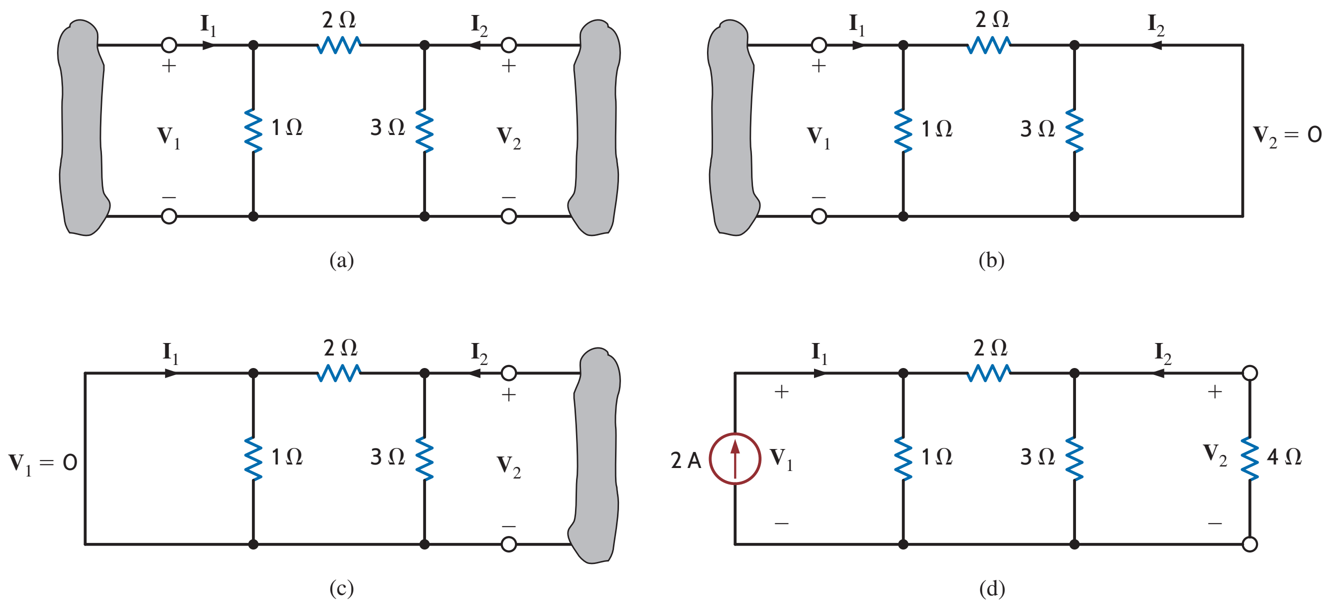


Figure 16.3
Networks employed in Example 16.1.

and hence,

$$\mathbf{y}_{12} = -\frac{1}{2}\,S$$

Also, $\mathbf{y}_{21}$ is computed from Fig. 16.3b using the equation

$$\mathbf{I}_2 = -\frac{\mathbf{V}_1}{2}$$

and therefore,

$$\mathbf{y}_{21} = -\frac{1}{2}\,S$$

Finally, $\mathbf{y}_{22}$ can be derived from Fig. 16.3c using

$$\mathbf{I}_2 = \mathbf{V}_2\left(\frac{1}{3} + \frac{1}{2}\right)$$

and

$$\mathbf{y}_{22} = \frac{5}{6}\,S$$

Therefore, the equations that describe the two-port itself are

$$\mathbf{I}_1 = \frac{3}{2}\mathbf{V}_1 - \frac{1}{2}\mathbf{V}_2 \qquad \mathbf{I}_2 = -\frac{1}{2}\mathbf{V}_1 + \frac{5}{6}\mathbf{V}_2$$

These equations can now be employed to determine the operation of the two-port for some given set of terminal conditions. The terminal conditions we will examine are shown in **Fig. 16.3d**. From this figure, we note that

$$\mathbf{I}_1 = 2\text{ A} \quad \text{and} \quad \mathbf{V}_2 = -4\mathbf{I}_2$$

Combining these with the preceding two-port equations yields

$$2 = \frac{3}{2}\mathbf{V}_1 - \frac{1}{2}\mathbf{V}_2$$
$$0 = -\frac{1}{2}\mathbf{V}_1 + \frac{13}{12}\mathbf{V}_2$$

or in matrix form

$$\begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{13}{12} \end{bmatrix}\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

Note carefully that these equations are simply the nodal equations for the network in Fig. 16.3d. Solving the equations, we obtain $\mathbf{V}_2 = 8/11$ V and therefore $\mathbf{I}_2 = -2/11$ A.

LEARNING ASSESSMENTS

E16.1 Find the Y parameters for the two-port network shown in Fig. E16.1.

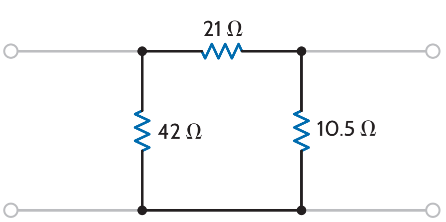


Figure E16.1

ANSWER:
$\mathbf{y}_{11} = \frac{1}{14}\,S$;
$\mathbf{y}_{12} = \mathbf{y}_{21} = -\frac{1}{21}\,S$; $\mathbf{y}_{22} = \frac{1}{7}\,S$.

E16.2 If a 10-A source is connected to the input of the two-port network in Fig. E16.1, find the current in a 5-Ω resistor connected to the output port.

ANSWER:
$\mathbf{I}_2 = -4.29$ A.

E16.3 Find the Y parameters for the two-port network shown in Fig. E16.3.

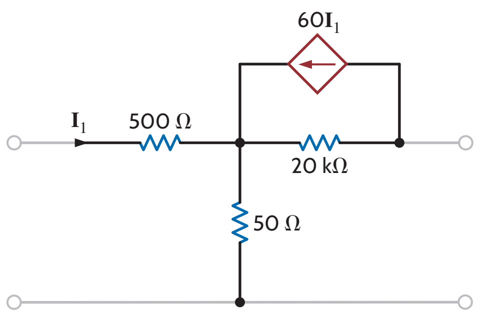


Figure E16.3

ANSWER:
$\mathbf{y}_{11} = 282.2\ \mu\text{S}$; $\mathbf{y}_{12} = -704\ \text{nS}$;
$\mathbf{y}_{21} = 16.9\ \text{mS}$; $\mathbf{y}_{22} = 7.74\ \mu\text{S}$.

16.2 Impedance Parameters

Once again, if we assume that the two-port network is a linear network that contains no independent sources, then by means of superposition we can write the input and output voltages as the sum of two components, one due to $\mathbf{I}_1$ and one due to $\mathbf{I}_2$:

$$\begin{aligned} \mathbf{V}_1 &= \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 &= \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \end{aligned} \qquad \textbf{16.4}$$

These equations, which describe the two-port network, can also be written in matrix form as

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} \qquad \textbf{16.5}$$

Like the Y parameters, these *Z parameters* can be derived as follows:

$$\begin{aligned} \mathbf{z}_{11} &= \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} \\ \mathbf{z}_{12} &= \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} \\ \mathbf{z}_{21} &= \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} \\ \mathbf{z}_{22} &= \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} \end{aligned} \qquad \textbf{16.6}$$

In the preceding equations, setting $\mathbf{I}_1$ or $\mathbf{I}_2 = 0$ is equivalent to open-circuiting the input or output port. Therefore, the Z parameters are called the *open-circuit impedance parameters*. $\mathbf{z}_{11}$ is called the *open-circuit input impedance*, $\mathbf{z}_{22}$ is called the *open-circuit output impedance*, and $\mathbf{z}_{12}$ and $\mathbf{z}_{21}$ are termed *open-circuit transfer impedances*.

EXAMPLE 16.2

We wish to find the Z parameters for the network in **Fig. 16.4a**. Once the parameters are known, we will use them to find the current in a 4-Ω resistor that is connected to the output terminals when a $12\underline{/0^\circ}$ −V source with an internal impedance of $1 + j0\ \Omega$ is connected to the input.

SOLUTION

From Fig. 16.4a, we note that

$$\begin{aligned} \mathbf{z}_{11} &= 2 - j4\ \Omega \\ \mathbf{z}_{12} &= -j4\ \Omega \\ \mathbf{z}_{21} &= -j4\ \Omega \\ \mathbf{z}_{22} &= -j4 + j2 = -j2\ \Omega \end{aligned}$$

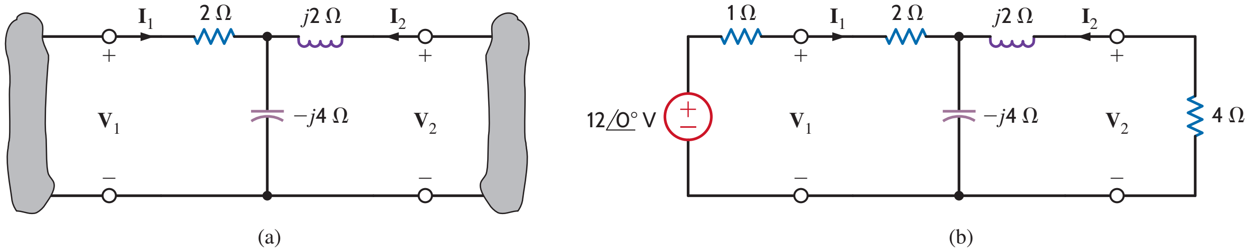


Figure 16.4

Circuits employed in Example 16.2.

The equations for the two-port network are, therefore,

$$\mathbf{V}_1 = (2 - j4)\mathbf{I}_1 - j4\mathbf{I}_2$$
$$\mathbf{V}_2 = -j4\mathbf{I}_1 - j2\mathbf{I}_2$$

The terminal conditions for the network shown in **Fig. 16.4b** are

$$\mathbf{V}_1 = 12\underline{/0^\circ} - (1)\mathbf{I}_1$$
$$\mathbf{V}_2 = -4\mathbf{I}_2$$

Combining these with the two-port equations yields

$$12\underline{/0^\circ} = (3 - j4)\mathbf{I}_1 - j4\mathbf{I}_2$$
$$0 = -j4\mathbf{I}_1 + (4 - j2)\mathbf{I}_2$$

It is interesting to note that these equations are the mesh equations for the network. If we solve the equations for $\mathbf{I}_2$, we obtain $\mathbf{I}_2 = 1.61\underline{/137.73^\circ}$ A, which is the current in the 4-Ω load.

LEARNING ASSESSMENTS

E16.4 Find the Z parameters for the network in Fig. E16.4. Then compute the current in a 4-Ω load if a $12\underline{/0^\circ}$ −V source is connected at the input port.

ANSWER:
$\mathbf{I}_2 = -0.73\underline{/0^\circ}$ A.

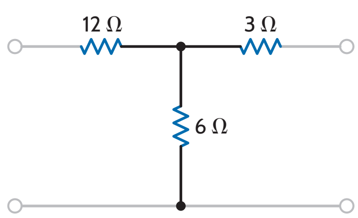


Figure E16.4

E16.5 Determine the Z parameters for the two-port network shown in Fig. E16.5.

ANSWER:
$\mathbf{z}_{11} = 15\ \Omega$; $\mathbf{z}_{12} = 5\ \Omega$;
$\mathbf{z}_{21} = 5 - j8\ \Omega$;
$\mathbf{z}_{22} = 5 - j4\ \Omega$.

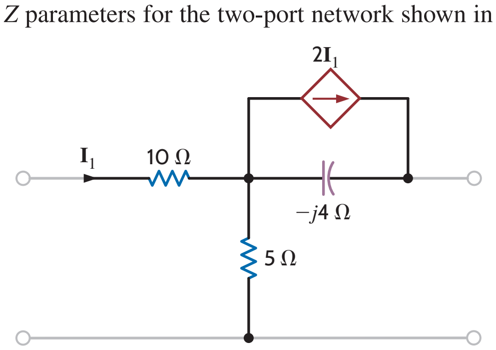


Figure E16.5

16.3 Hybrid Parameters

Under the assumptions used to develop the Y and Z parameters, we can obtain what are commonly called the *hybrid parameters*. In the pair of equations that define these parameters, $\mathbf{V}_1$ and $\mathbf{I}_2$ are the independent variables. Therefore, the two-port equations in terms of the hybrid parameters are

$$\begin{aligned}\mathbf{V}_1 &= \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 &= \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2\end{aligned} \qquad \textbf{16.7}$$

or in matrix form,

$$\begin{bmatrix}\mathbf{V}_1 \\ \mathbf{I}_2\end{bmatrix} = \begin{bmatrix}\mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{I}_1 \\ \mathbf{V}_2\end{bmatrix} \qquad \textbf{16.8}$$

These parameters are especially important in transistor circuit analysis. The parameters are determined via the following equations:

$$\begin{aligned}\mathbf{h}_{11} &= \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0} \\ \mathbf{h}_{12} &= \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0} \\ \mathbf{h}_{21} &= \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0} \\ \mathbf{h}_{22} &= \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}\end{aligned} \qquad \textbf{16.9}$$

The parameters $\mathbf{h}_{11}$, $\mathbf{h}_{12}$, $\mathbf{h}_{21}$, and $\mathbf{h}_{22}$ represent the *short-circuit input impedance*, the *open-circuit reverse voltage gain*, the *short-circuit forward current gain*, and the *open-circuit output admittance*, respectively. Because of this mix of parameters, they are called *hybrid parameters*. In transistor circuit analysis, the parameters $\mathbf{h}_{11}$, $\mathbf{h}_{12}$, $\mathbf{h}_{21}$, and $\mathbf{h}_{22}$ are normally labeled $\mathbf{h}_i$, $\mathbf{h}_r$, $\mathbf{h}_f$, and $\mathbf{h}_o$.

EXAMPLE 16.3

An equivalent circuit for the op-amp in Fig. 16.5a is shown in Fig. 16.5b. We will determine the hybrid parameters for this network.

SOLUTION

Parameter $\mathbf{h}_{11}$ is derived from Fig. 16.5c. With the output shorted, $\mathbf{h}_{11}$ is a function of only R_i, R_1, and R_2 and

$$\mathbf{h}_{11} = R_i + \frac{R_1R_2}{R_1 + R_2}$$

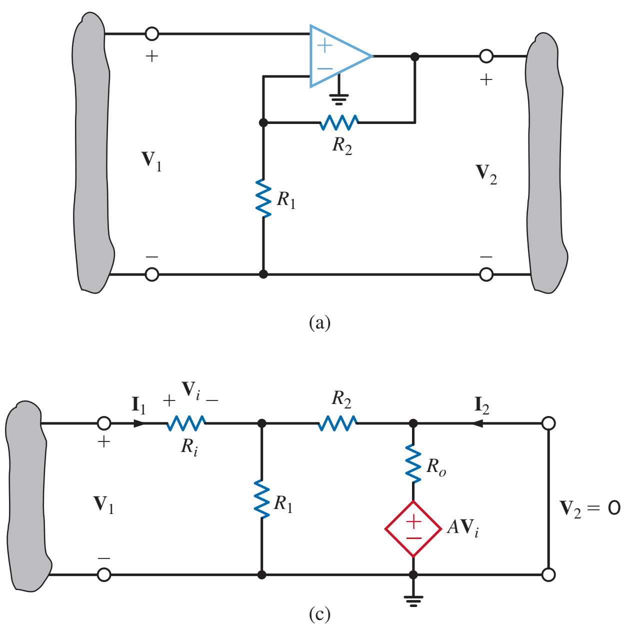


Figure 16.5

Circuits employed in Example 16.3.

Fig. 16.5d is used to derive $\mathbf{h}_{12}$. Since $\mathbf{I}_1 = 0$, $\mathbf{V}_i = 0$ and the relationship between $\mathbf{V}_1$ and $\mathbf{V}_2$ is a simple voltage divider:

$$\mathbf{V}_1 = \frac{\mathbf{V}_2 R_1}{R_1 + R_2}$$

Therefore,

$$\mathbf{h}_{12} = \frac{R_1}{R_1 + R_2}$$

KVL and KCL can be applied to Fig. 16.5c to determine $\mathbf{h}_{21}$. The two equations that relate $\mathbf{I}_2$ to $\mathbf{I}_1$ are

$$\mathbf{V}_i = \mathbf{I}_1 R_i$$

$$\mathbf{I}_2 = \frac{-A\mathbf{V}_i}{R_o} - \frac{\mathbf{I}_1 R_1}{R_1 + R_2}$$

Therefore,

$$\mathbf{h}_{21} = -\left(\frac{AR_i}{R_o} + \frac{R_1}{R_1 + R_2}\right)$$

Finally, the relationship between $\mathbf{I}_2$ and $\mathbf{V}_2$ in Fig. 16.5d is

$$\frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{R_o(R_1 + R_2)}{R_o + R_1 + R_2}$$

and therefore,

$$\mathbf{h}_{22} = \frac{R_o + R_1 + R_2}{R_o(R_1 + R_2)}$$

The network equations are, therefore,

$$\mathbf{V}_1 = \left(R_i + \frac{R_1 R_2}{R_1 + R_2}\right)\mathbf{I}_1 + \frac{R_1}{R_1 + R_2}\mathbf{V}_2$$

$$\mathbf{I}_2 = -\left(\frac{AR_i}{R_o} + \frac{R_1}{R_1 + R_2}\right)\mathbf{I}_1 + \frac{R_o + R_1 + R_2}{R_o(R_1 + R_2)}\mathbf{V}_2$$

LEARNING ASSESSMENTS

E16.6 Find the hybrid parameters for the network shown in Fig. E16.4.

ANSWER: $\mathbf{h}_{11} = 14\ \Omega$; $\mathbf{h}_{12} = \frac{2}{3}$; $\mathbf{h}_{21} = -\frac{2}{3}$; $\mathbf{h}_{22} = \frac{1}{9}\ S$.

E16.7 If a 4-Ω load is connected to the output port of the network examined in Learning Assessment E16.6, determine the input impedance of the two-port with the load connected.

ANSWER: $\mathbf{Z}_i = 15.23\ \Omega$.

E16.8 Find the hybrid parameters for the two-port network shown in Fig. E16.3.

ANSWER: $\mathbf{h}_{11} = 3543.6\ \Omega$; $\mathbf{h}_{12} = 2.49 \times 10^{-3}$; $\mathbf{h}_{21} = 59.85$; $\mathbf{h}_{22} = 49.88\ \mu S$.

16.4 Transmission Parameters

The final parameters we will discuss are called the *transmission parameters*. They are defined by the equations

$$\begin{aligned} \mathbf{V}_1 &= \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2 \\ \mathbf{I}_1 &= \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2 \end{aligned} \tag{16.10}$$

or in matrix form,

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ -\mathbf{I}_2 \end{bmatrix} \tag{16.11}$$

These parameters are very useful in the analysis of circuits connected in cascade, as we will demonstrate later. The parameters are determined via the following equations:

$$\begin{aligned} \mathbf{A} &= \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} \\ \mathbf{B} &= \left.\frac{\mathbf{V}_1}{-\mathbf{I}_2}\right|_{\mathbf{V}_2=0} \\ \mathbf{C} &= \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} \\ \mathbf{D} &= \left.\frac{\mathbf{I}_1}{-\mathbf{I}_2}\right|_{\mathbf{V}_2=0} \end{aligned} \tag{16.12}$$

A, **B**, **C**, and **D** represent the *open-circuit voltage ratio*, the *negative short-circuit transfer impedance*, the *open-circuit transfer admittance*, and the *negative short-circuit current ratio*, respectively. For obvious reasons, the transmission parameters are commonly referred to as the *ABCD parameters*.

EXAMPLE 16.4

We will now determine the transmission parameters for the network in Fig. 16.6.

SOLUTION

Let us consider the relationship between the variables under the conditions stated in the parameters in Eq. (16.12). For example, with $\mathbf{I}_2 = 0$, $\mathbf{V}_2$ can be written as

$$\mathbf{V}_2 = \frac{\mathbf{V}_1}{1 + 1/j\omega}\left(\frac{1}{j\omega}\right)$$

or

$$\mathbf{A} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} = 1 + j\omega$$

Similarly, with $\mathbf{V}_2 = 0$, the relationship between $\mathbf{I}_2$ and $\mathbf{V}_1$ is

$$-\mathbf{I}_2 = \frac{\mathbf{V}_1}{1 + \dfrac{1/j\omega}{1 + 1/j\omega}}\left(\frac{1/j\omega}{1 + 1/j\omega}\right)$$

or

$$\mathbf{B} = \frac{\mathbf{V}_1}{-\mathbf{I}_2} = 2 + j\omega$$

In a similar manner, we can show that $\mathbf{C} = j\omega$ and $\mathbf{D} = 1 + j\omega$.

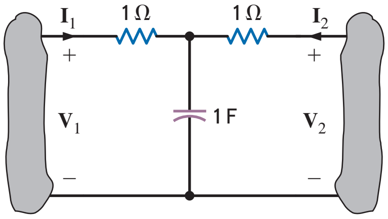


Figure 16.6 Circuit used in Example 16.4.

LEARNING ASSESSMENTS

E16.9 Compute the transmission parameters for the two-port network in Fig. E16.1.

ANSWER:
$\mathbf{A} = 3$; $\mathbf{B} = 21\ \Omega$;
$\mathbf{C} = \frac{1}{6}\ S$; $\mathbf{D} = \frac{3}{2}$.

E16.10 Find the transmission parameters for the two-port network shown in Fig. E16.5.

ANSWER:
$\mathbf{A} = 0.843 + j1.348$; $\mathbf{B} = 4.61 + j3.37\ \Omega$;
$\mathbf{C} = 0.056 + j0.09\ S$; $\mathbf{D} = 0.64 + j0.225$.

E16.11 Find $\mathbf{V}_s$ if $\mathbf{V}_2 = 220\underline{/0^\circ}$ V rms in the network shown in Fig. E16.11.

ANSWER:
$\mathbf{V}_s = 1015.9\underline{/-137.63^\circ}$ V rms.

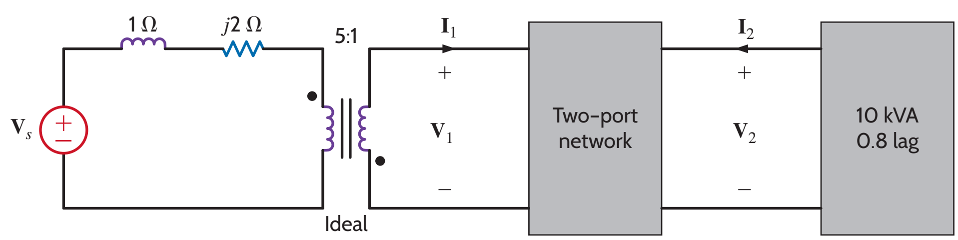


Figure E16.11

$$\begin{bmatrix}\mathbf{V}_1\\ \mathbf{I}_1\end{bmatrix} = \begin{bmatrix}0.333 + j0.333 & -(1.333 + j6)\\ j0.1667 & -(0.333 + j0.333)\end{bmatrix}\begin{bmatrix}\mathbf{V}_2\\ \mathbf{I}_2\end{bmatrix}$$

16.5 Parameter Conversions

If all the two-port parameters for a network exist, it is possible to relate one set of parameters to another since the parameters interrelate the variables $\mathbf{V}_1$, $\mathbf{I}_1$, $\mathbf{V}_2$, and $\mathbf{I}_2$.

Table 16.1 lists all the conversion formulas that relate one set of two-port parameters to another. Note that Δ_Z, Δ_Y, Δ_H, and Δ_T refer to the determinants of the matrices for the Z, Y, hybrid, and ABCD parameters, respectively. Therefore, given one set of parameters for a network, we can use Table 16.1 to find others.

TABLE 16.1 Two-port parameter conversion formulas

$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta_Y} & \frac{-y_{12}}{\Delta_Y} \\ \frac{-y_{21}}{\Delta_Y} & \frac{y_{11}}{\Delta_Y} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta_T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta_H}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$
$\begin{bmatrix} \frac{z_{22}}{\Delta_Z} & \frac{-z_{12}}{\Delta_Z} \\ \frac{-z_{21}}{\Delta_Z} & \frac{z_{11}}{\Delta_Z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & \frac{-\Delta_T}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_H}{h_{11}} \end{bmatrix}$
$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_Z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta_Y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta_H}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$
$\begin{bmatrix} \frac{\Delta_Z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_Y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta_T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$

LEARNING ASSESSMENT

E16.12 Determine the Y parameters for a two-port if the Z parameters are

$$\mathbf{Z} = \begin{bmatrix} 18 & 6 \\ 6 & 9 \end{bmatrix}$$

ANSWER: $\mathbf{y}_{11} = \frac{1}{14}\, S$; $\mathbf{y}_{12} = \mathbf{y}_{21} = -\frac{1}{21}\, S$; $\mathbf{y}_{22} = \frac{1}{7}\, S$.

16.6 Interconnection of Two-Ports

Interconnected two-port circuits are important because when designing complex systems it is generally much easier to design a number of simpler subsystems that can then be interconnected to form the complete system. If each subsystem is treated as a two-port network, the interconnection techniques described in this section provide some insight into the manner in which a total system may be analyzed and/or designed. Thus, we will now illustrate techniques for treating a network as a combination of subnetworks. We will, therefore, analyze a two-port network as an interconnection of simpler two-ports. Although two-ports can be interconnected in a variety of ways, we will treat only three types of connections: parallel, series, and cascade.

For the two-port interconnections to be valid, they must satisfy certain specific requirements that are outlined in the book *Network Analysis and Synthesis* by L. Weinberg (McGraw-Hill, 1962). The following examples will serve to illustrate the interconnection techniques.

In the parallel interconnection case, a two-port N is composed of two-ports N_a and N_b connected as shown in **Fig. 16.7**. *Provided that the terminal characteristics of the two networks N_a and N_b are not altered by the interconnection illustrated in the figure*, then the Y parameters for the total network are

$$\begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11a} + \mathbf{y}_{11b} & \mathbf{y}_{12a} + \mathbf{y}_{12b} \\ \mathbf{y}_{21a} + \mathbf{y}_{21b} & \mathbf{y}_{22a} + \mathbf{y}_{22b} \end{bmatrix} \qquad \textbf{16.13}$$

and hence to determine the Y parameters for the total network, we simply add the Y parameters of the two networks N_a and N_b.

Likewise, if the two-port N is composed of the series connection of N_a and N_b, as shown in **Fig. 16.8**, then *once again, as long as the terminal characteristics of the two*

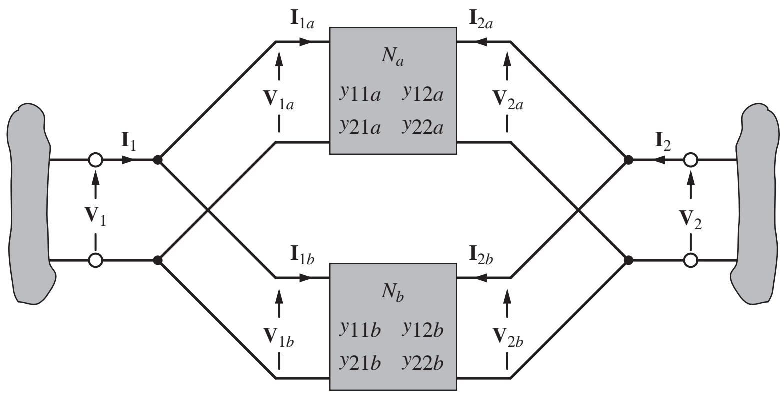


Figure 16.7 Parallel interconnection of two-ports.

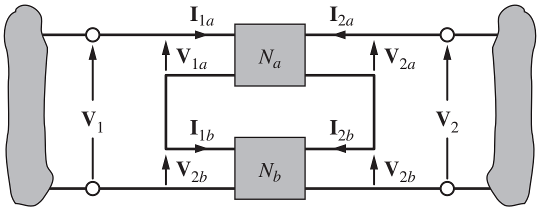


Figure 16.8 Series interconnection of two-ports.

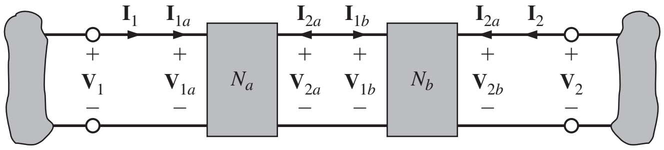


Figure 16.9
Cascade interconnection of networks.

networks N_a and N_b are not altered by the series interconnection, the Z parameters for the total network are

$$\begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11a} + \mathbf{z}_{11b} & \mathbf{z}_{12a} + \mathbf{z}_{12b} \\ \mathbf{z}_{21a} + \mathbf{z}_{21b} & \mathbf{z}_{22a} + \mathbf{z}_{22b} \end{bmatrix} \qquad \textbf{16.14}$$

Therefore, the Z parameters for the total network are equal to the sum of the Z parameters for the networks N_a and N_b.

Finally, if a two-port N is composed of a cascade interconnection of N_a and N_b, as shown in **Fig. 16.9**, the equations for the total network are

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_a & \mathbf{B}_a \\ \mathbf{C}_a & \mathbf{D}_a \end{bmatrix} \begin{bmatrix} \mathbf{A}_b & \mathbf{B}_b \\ \mathbf{C}_b & \mathbf{D}_b \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ -\mathbf{I}_2 \end{bmatrix} \qquad \textbf{16.15}$$

Hence, the transmission parameters for the total network are derived by matrix multiplication as indicated previously. The order of the matrix multiplication is important and is performed in the order in which the networks are interconnected.

The cascade interconnection is very useful. Many large systems can be conveniently modeled as the cascade interconnection of a number of stages. For example, the very weak signal picked up by a radio antenna is passed through a number of successive stages of amplification—each of which can be modeled as a two-port subnetwork. In addition, in contrast to the other interconnection schemes, no restrictions are placed on the parameters of N_a and N_b in obtaining the parameters of the two-port resulting from their interconnection.

EXAMPLE 16.5

We wish to determine the Y parameters for the network shown in **Fig. 16.10a** by considering it to be a parallel combination of two networks as shown in **Fig. 16.10b**. The capacitive network will be referred to as N_a, and the resistive network will be referred to as N_b.

SOLUTION

The Y parameters for N_a are

$$\mathbf{y}_{11a} = j\frac{1}{2}S \qquad \mathbf{y}_{12a} = -j\frac{1}{2}S$$
$$\mathbf{y}_{21a} = -j\frac{1}{2}S \quad \mathbf{y}_{22a} = j\frac{1}{2}S$$

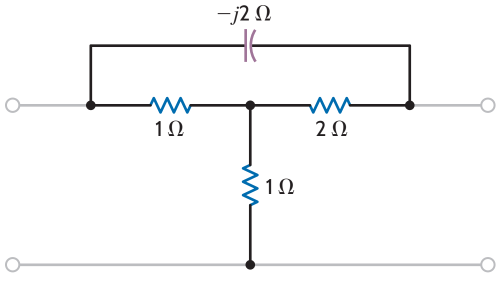
and the Y parameters for N_b are

$$\mathbf{y}_{11b} = \frac{3}{5}S \qquad \mathbf{y}_{12b} = -\frac{1}{5}S$$
$$\mathbf{y}_{21b} = -\frac{1}{5}S \quad \mathbf{y}_{22b} = \frac{2}{5}S$$

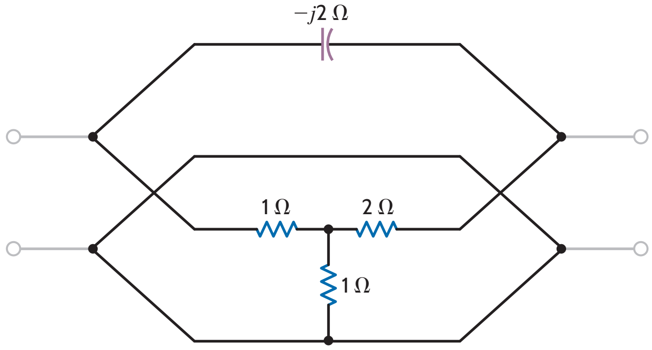
Hence, the Y parameters for the network in Fig. 16.10 are

$$\mathbf{y}_{11} = \frac{3}{5} + j\frac{1}{2}S \qquad \mathbf{y}_{12} = -\left(\frac{1}{5} + j\frac{1}{2}\right)S$$
$$\mathbf{y}_{21} = -\left(\frac{1}{5} + j\frac{1}{2}\right)S \quad \mathbf{y}_{22} = \frac{2}{5} + j\frac{1}{2}S$$

To gain an appreciation for the simplicity of this approach, you need only try to find the Y parameters for the network in Fig. 16.10a directly.



(a)



(b)

Figure 16.10

Network composed of the parallel combination of two subnetworks.

EXAMPLE 16.6

Let us determine the Z parameters for the network shown in Fig. 16.10a. The circuit is redrawn in **Fig. 16.11**, illustrating a series interconnection. The upper network will be referred to as N_a and the lower network as N_b.

SOLUTION

The Z parameters for N_a are

$$\mathbf{z}_{11a} = \frac{2-2j}{3-2j}\,\Omega \qquad \mathbf{z}_{12a} = \frac{2}{3-2j}\,\Omega$$

$$\mathbf{z}_{21a} = \frac{2}{3-2j}\,\Omega \qquad \mathbf{z}_{22a} = \frac{2-4j}{3-2j}\,\Omega$$

and the Z parameters for N_b are

$$\mathbf{z}_{11b} = \mathbf{z}_{12b} = \mathbf{z}_{21b} = \mathbf{z}_{22b} = 1\,\Omega$$

Hence, the Z parameters for the total network are

$$\mathbf{z}_{11} = \frac{5-4j}{3-2j}\,\Omega \qquad \mathbf{z}_{12} = \frac{5-2j}{3-2j}\,\Omega$$

$$\mathbf{z}_{21} = \frac{5-2j}{3-2j}\,\Omega \qquad \mathbf{z}_{22} = \frac{5-6j}{3-2j}\,\Omega$$

We could easily check these results against those obtained in Example 16.5 by applying the conversion formulas in Table 16.1.

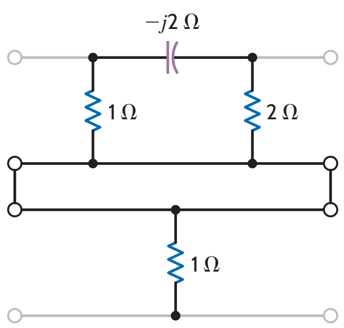


Figure 16.11

Network in Fig. 16.10a redrawn as a series interconnection of two networks.

EXAMPLE 16.7

Let us derive the two-port parameters of the network in Fig. 16.12 by considering it to be a cascade connection of two networks as shown in Fig. 16.6.

SOLUTION

The ABCD parameters for the identical T networks were calculated in Example 16.4 to be

$$\mathbf{A} = 1 + j\omega \qquad \mathbf{B} = 2 + j\omega$$
$$\mathbf{C} = j\omega \qquad \mathbf{D} = 1 + j\omega$$

Therefore, the transmission parameters for the total network are

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} 1 + j\omega & 2 + j\omega \\ j\omega & 1 + j\omega \end{bmatrix} \begin{bmatrix} 1 + j\omega & 2 + j\omega \\ j\omega & 1 + j\omega \end{bmatrix}$$

Performing the matrix multiplication, we obtain

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} 1 + 4j\omega - 2\omega^2 & 4 + 6j\omega - 2\omega^2 \\ 2j\omega - 2\omega^2 & 1 + 4j\omega - 2\omega^2 \end{bmatrix}$$

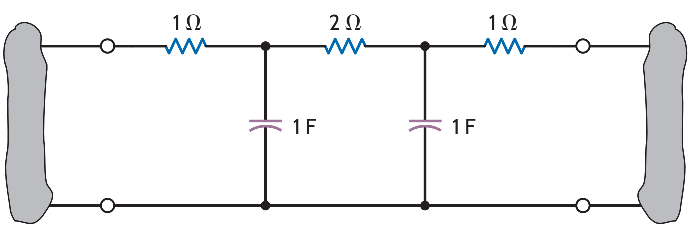


Figure 16.12
Circuit used in Example 16.7.

EXAMPLE 16.8

Fig. 16.13 is a per-phase model used in the analysis of three-phase high-voltage transmission lines. As a general rule in these systems, the voltage and current at the receiving end are known, and it is the conditions at the sending end that we wish to find. The transmission parameters perfectly fit this scenario. Thus, we will find the transmission parameters for a reasonable transmission line model and, then, given the receiving-end voltages, power, and power factor, we will find the receiving-end current, sending-end voltage and current, and the transmission efficiency. Finally, we will plot the efficiency versus the power factor.

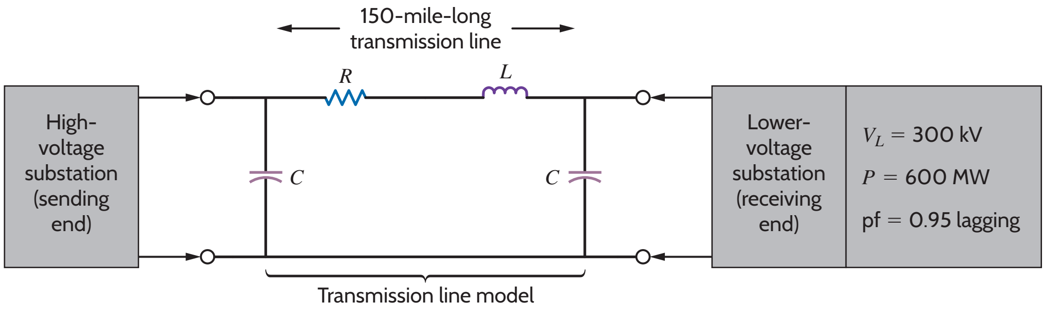
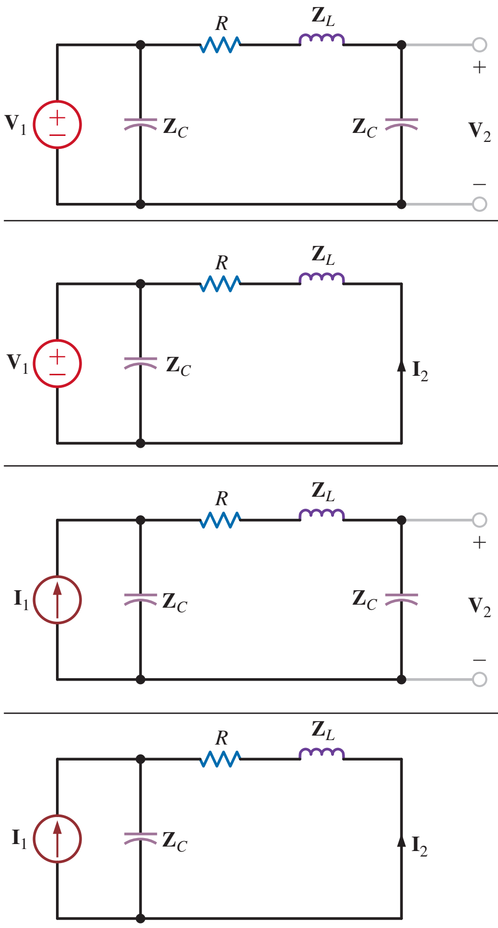


Figure 16.13
A π-circuit model for power transmission lines.



$$\left.\frac{\mathbf{V}_2}{\mathbf{V}_1}\right|_{\mathbf{I}_2=0} = \frac{\mathbf{Z}_C}{\mathbf{Z}_C + \mathbf{Z}_L + R}$$

$$\mathbf{A} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} = \frac{\mathbf{Z}_C + \mathbf{Z}_L + R}{\mathbf{Z}_C} = 0.9590\angle 0.27^\circ$$

$$\left.\frac{-\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} = \frac{1}{\mathbf{Z}_L + R}$$

$$\mathbf{B} = \left.\frac{\mathbf{V}_1}{-\mathbf{I}_2}\right|_{\mathbf{V}_2=0} = \mathbf{Z}_L + R = 100.00\angle 84.84^\circ\ \Omega$$

$$\left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} = \frac{\mathbf{Z}_C^2}{2\mathbf{Z}_C + \mathbf{Z}_L + R}$$

$$\mathbf{C} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} = \frac{2\mathbf{Z}_C + \mathbf{Z}_L + R}{\mathbf{Z}_C^2} = 975.10\angle 90.13^\circ\ \mu\text{S}$$

$$\left.\frac{-\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0} = \frac{\mathbf{Z}_C}{\mathbf{Z}_C + \mathbf{Z}_L + R}$$

$$\mathbf{D} = \left.\frac{\mathbf{I}_1}{-\mathbf{I}_2}\right|_{\mathbf{V}_2=0} = \frac{\mathbf{Z}_C + \mathbf{Z}_L + R}{\mathbf{Z}_C} = 0.950\angle 0.27^\circ$$

Figure 16.14

Equivalent circuits used to determine the transmission parameters.

SOLUTION

Given a 150-mile-long transmission line, reasonable values for the π-circuit elements of the transmission line model are $C = 1.326\ \mu\text{F}$, $R = 9.0\ \Omega$, and $L = 264.18$ mH. The transmission parameters can be easily found using the circuits in **Fig. 16.14**. At 60 Hz, the transmission parameters are

$$\mathbf{A} = 0.9590\angle 0.27^\circ \qquad \mathbf{C} = 975.10\angle 90.13^\circ\ \mu\text{S}$$
$$\mathbf{B} = 100.00\angle 84.84^\circ\ \Omega \qquad \mathbf{D} = 0.9590\angle 0.27^\circ$$

To use the transmission parameters, we must know the receiving-end current, $\mathbf{I}_2$. Using standard three-phase circuit analysis outlined in Chapter 11, we find the line current to be

$$\mathbf{I}_2 = -\frac{600\angle \cos^{-1}(\text{pf})}{\sqrt{3}(300)(\text{pf})} = -1.215\angle -18.19^\circ\ \text{kA}$$

where the line-to-neutral (i.e., phase) voltage at the receiving end, $\mathbf{V}_2$, is assumed to have zero phase. Now we can use the transmission parameters to determine the sending-end voltage and power. Since the line-to-neutral voltage at the receiving end is $300/\sqrt{3} = 173.21$ kV, the results are

$$\mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2 = (0.9590\angle 0.27^\circ)(173.21\angle 0^\circ) + (100.00\angle 84.84^\circ)(1.215\angle -18.19^\circ) = 241.92\angle 27.67^\circ\ \text{kV}$$

$$\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2 = (975.10 \times 10^{-6}\underline{/90.13°})(173.21\underline{/0°}) + (0.9590\underline{/0.27°})(1.215\underline{/-18.19°}) = 1.12\underline{/-9.71°} \text{ kA}$$

At the sending end, the power factor and power are

$$\text{pf} = \cos(27.67 - (-9.71)) = \cos(37.38) = 0.80 \text{ lagging}$$
$$P_1 = 3V_1I_1(\text{pf}) = (3)(241.92)(1.12)(0.80) = 650.28 \text{ MW}$$

Finally, the transmission efficiency is

$$\eta = \frac{P_2}{P_1} = \frac{600}{650.28} = 92.3\%$$

This entire analysis can be easily programmed into an Excel spreadsheet. A plot of the transmission efficiency versus power factor at the receiving end is shown in **Fig. 16.15**. We see that as the power factor decreases, the transmission efficiency drops, which increases the cost of production for the power utility. This is precisely why utilities encourage industrial customers to operate as close to unity power factor as possible.

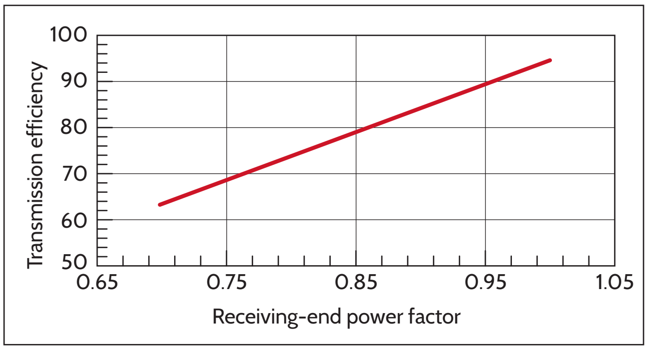


Figure 16.15
The results of an Excel simulation showing the effect of the receiving-end power factor on the transmission efficiency. Because the Excel simulation used more significant digits, slight differences exist between the values in the plot and those in the text.

SUMMARY

- Four of the most common parameters used to describe a two-port network are the admittance, impedance, hybrid, and transmission parameters.
- If all the two-port parameters for a network exist, a set of conversion formulas can be used to relate one set of two-port parameters to another.
- When interconnecting two-ports, the Y parameters are added for a parallel connection, the Z parameters are added for a series connection, and the transmission parameters in matrix form are multiplied together for a cascade connection.

PROBLEMS

16.1 Given the two networks in Fig. P16.1, find the Y parameters for the circuit in Fig. P16.1 (a) and the Z parameters for the circuit in Fig. P16.1 (b). Calculate (a) y_{11}, (b) y_{12}, (c) y_{21}, (d) y_{22}, (e) z_{11}, (f) z_{12}, (g) z_{21}, (h) z_{22} if $\mathbf{Z}_L = 30\ \Omega$.

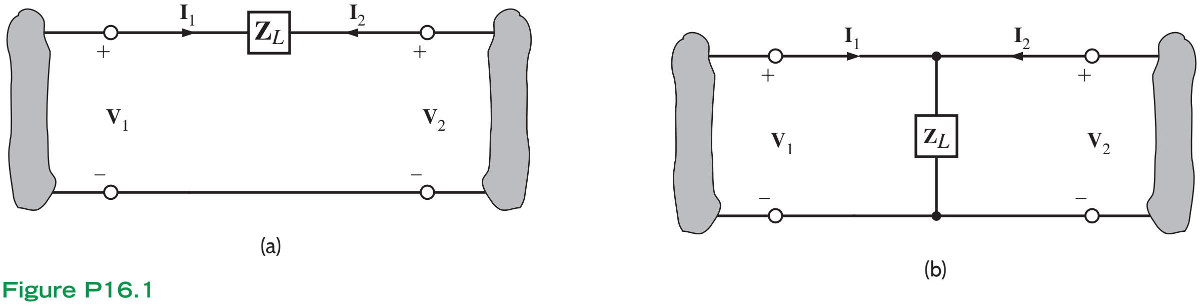


Figure P16.1

16.2 Find the Y parameters for the two-port network shown in Fig. P16.2.

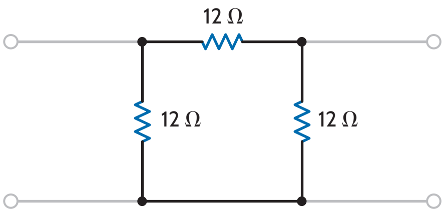


Figure P16.2

16.3 Find the Y parameters ((a) y_{11}, (b) y_{12}, (c) y_{21}, (d) y_{22}) for the two-port network in Fig. P16.3.

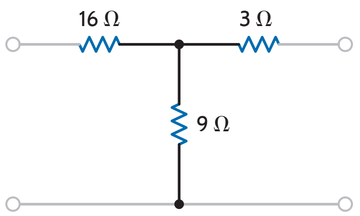


Figure P16.3

16.4 Determine the Y parameters for the two-port network shown in Fig. P16.4.

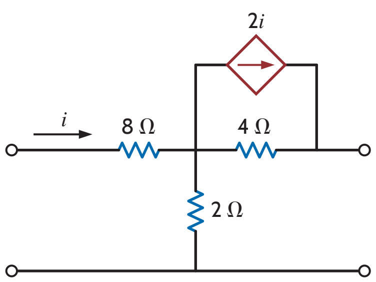


Figure P16.4

16.5 Determine the admittance parameters for the network shown in Fig. P16.5.

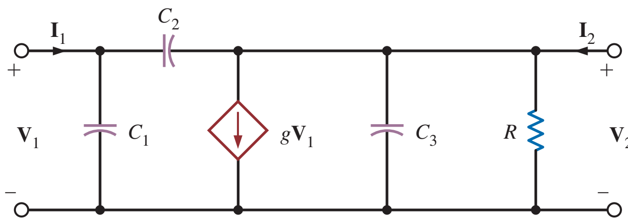


Figure P16.5

16.6 Find the Y parameters for the two-port network in Fig. P16.6.

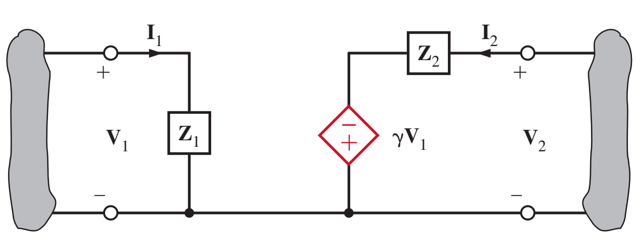


Figure P16.6

16.7 Find the Z parameters for the two-port network in Fig. P16.7.

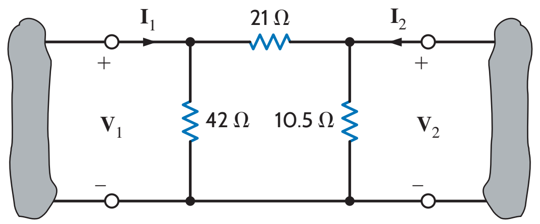


Figure P16.7

16.8 Find the Z parameters for the two-port network shown in Fig. P16.8 and determine the voltage gain of the entire circuit with a 4-kΩ load attached to the output.

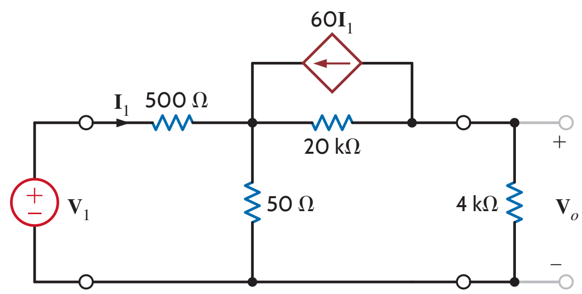


Figure P16.8

16.9 Determine a two-port network that is represented by the following Z parameters.

$$[Z] = \begin{bmatrix} 6 + j3 & 5 - j2 \\ 5 - j2 & 8 - j \end{bmatrix}$$

16.10 Determine the input impedance of the network shown in Fig. P16.10 in terms of the Z parameters and the load impedance Z.

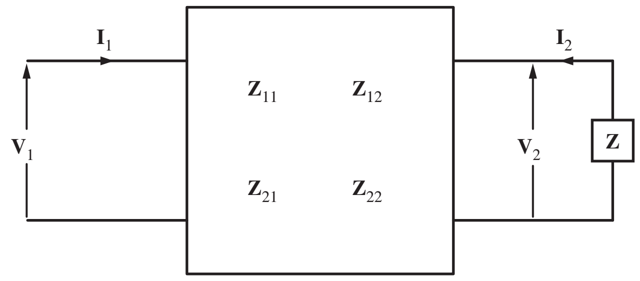


Figure P16.10

16.11 Find the voltage gain of the two-port network in Fig. P16.10 if a 12-kΩ load is connected to the output port.

16.12 Find the input impedance of the network in Fig. P16.10.

16.13 Find the Y parameters for the network in Fig. P16.13.

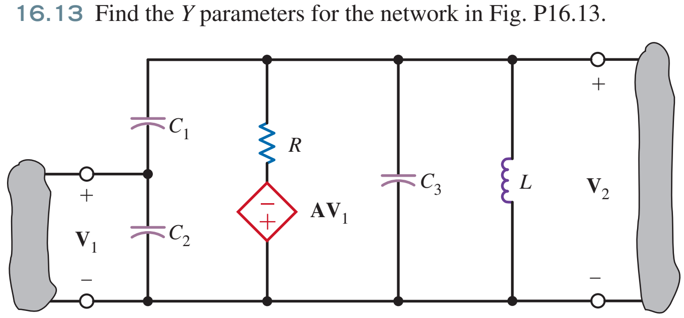


Figure P16.13

16.14 Find the Z parameters for the network in Fig. P16.14.

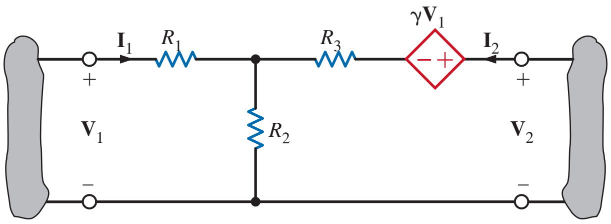


Figure P16.14

16.15 Find the Z parameters of the two-port network in Fig. P16.15.

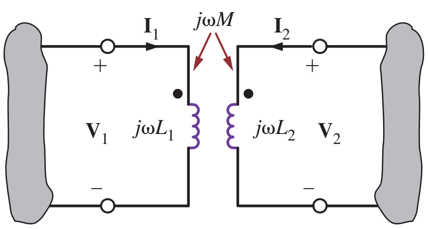


Figure P16.15

16.16 Given the network in Fig. P16.16, (a) find the Z parameters for the transformer, (b) write the terminal equation at each end of the two-port, and (c) use the information obtained to find $\mathbf{V}_2$.

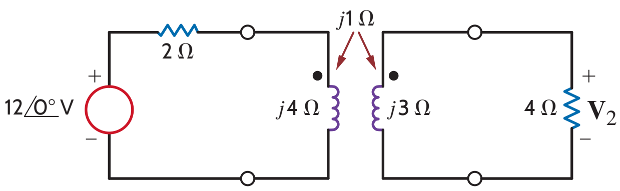


Figure P16.16

16.17 Find the Z parameters for the network in Fig. P16.17.

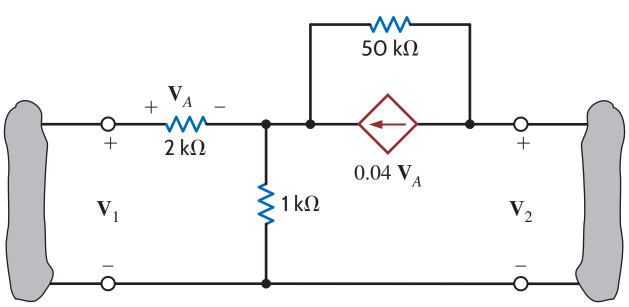


Figure P16.17

16.18 Calculate I_1 and I_2 in the circuit.

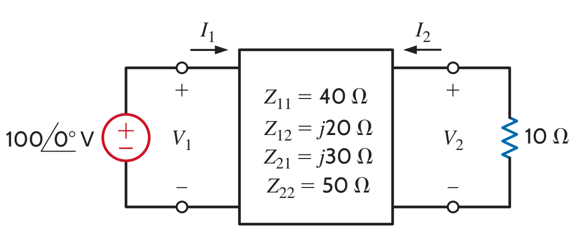


Figure P16.18

16.19 Find the Z parameters for the two-port network shown in Fig. P16.19.

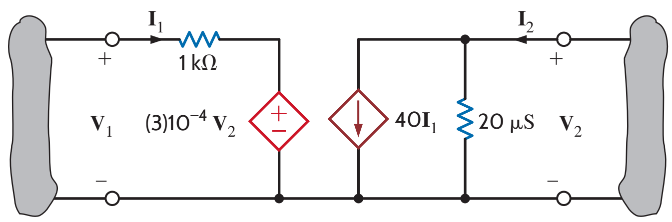


Figure P16.19

16.20 Find [Z] of a two-port network if

$$[Z] = \begin{bmatrix} 10 & 1.5 \\ 2 & 4 \end{bmatrix}$$

16.21 Determine the Z parameters for the two-port network in Fig. P16.21.

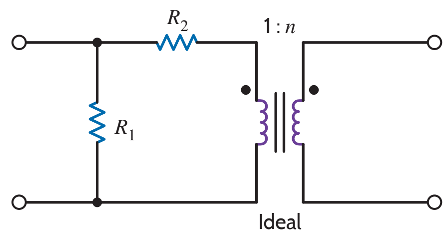


Figure P16.21

16.22 Determine (a) R_1, (b) R_2, (c) R_3 in the circuit diagram shown in Fig. P16.22 that has the following Y parameters.

$$[\mathbf{Y}] = \begin{bmatrix} \frac{15}{56} & -\frac{1}{8} \\ -\frac{1}{8} & \frac{18}{80} \end{bmatrix}$$

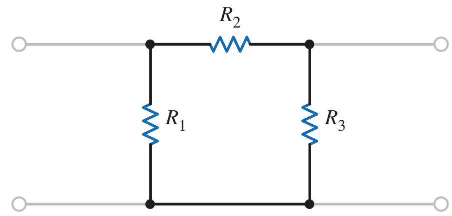


Figure P16.22

16.23 Determine $\mathbf{Z}_1$ ((a) real part and (b) imaginary part), $\mathbf{Z}_2$ ((c) real part and (d) imaginary part), $\mathbf{Z}_3$ ((e) real part and (f) imaginary part) in the circuit diagram shown in Fig. P16.23 that has the following Z parameters (all in ohms):

$$[\mathbf{Z}] = \begin{bmatrix} 6 - j1 & 4 - j7 \\ 4 - j7 & 7 + j4 \end{bmatrix}$$

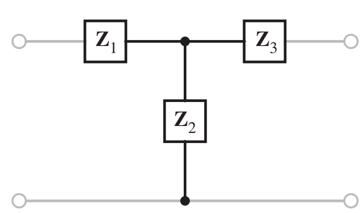


Figure P16.23

16.24 Compute the hybrid parameters for the network in Fig. E16.21.

16.25 Compute the hybrid parameters ((a) h_{11}, (b) h_{12}, (c) h_{21}, (d) h_{22}) for the network in Fig. P16.25.

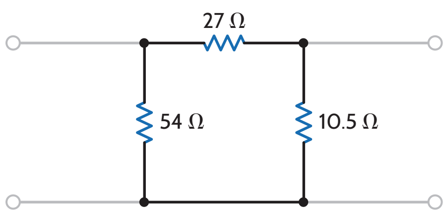


Figure P16.25

16.26 Find the hybrid parameters for the network in Fig. P16.23.

16.27 Consider the network in Fig. P16.27. The two-port network is a hybrid model for a basic transistor. Determine the voltage gain of the entire network, $\mathbf{V}_2/\mathbf{V}_s$, if a source $\mathbf{V}_s$ with internal resistance R_1 is applied at the input to the two-port network and a load R_L is connected at the output port.

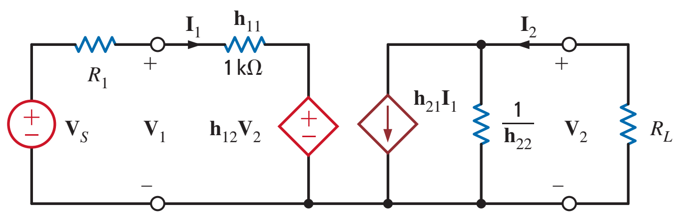


Figure P16.27

16.28 Find the hybrid parameters of the two-port network.

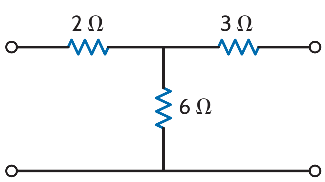


Figure P16.28

16.29 Determine the hybrid parameters for the network in Fig. P16.29. $R_1 = 30\ \Omega$, $R_2 = 21\ \Omega$, $C = 170$ mF, $\alpha = 0.8$, $\omega = 100$ radians per second.

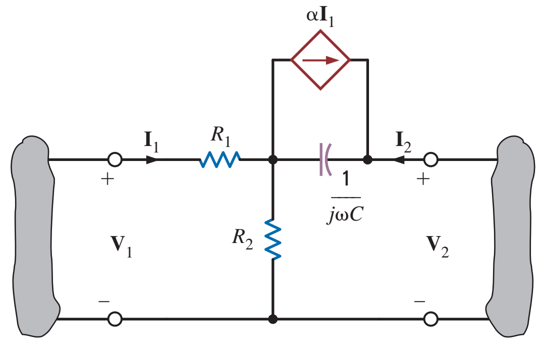


Figure P16.29

16.30 Find the ABCD parameters for the networks in Fig. P16.1.

16.31 Find the transmission parameters for the network in Fig. P16.31.

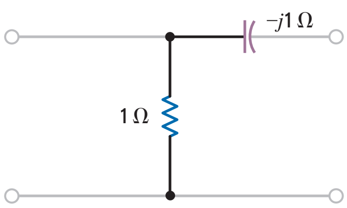


Figure P16.31

16.32 Given the network in Fig. P16.32, find the transmission parameters for the two-port network and then find $\mathbf{I}_o$ using the terminal conditions.

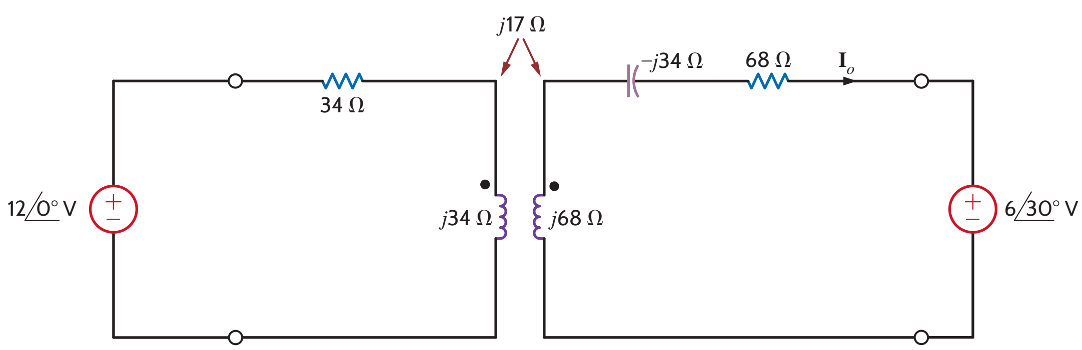


Figure P16.32

Enter arguments from 0 to 2π.

16.33 Find the voltage gain $\mathbf{V}_2/\mathbf{V}_1$ for the network in Fig. P16.33 using the ABCD parameters.

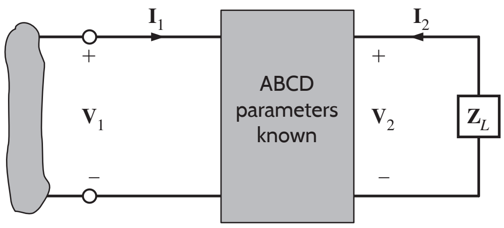


Figure P16.33

16.34 Find the input admittance of the two-port in Fig. P16.34 if the Y parameters are $\mathbf{y}_{11} = 17\ S$, $\mathbf{y}_{12} = -16\ S$, $\mathbf{y}_{21} = -16\ S$, $\mathbf{y}_{22} = 17\ S$ and the load $\mathbf{Y}_L$ is $8\ S$.

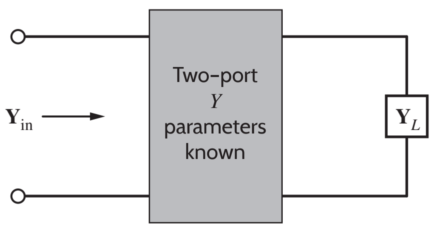


Figure P16.34

16.35 Find the voltage gain $\mathbf{V}_2/\mathbf{V}_1$ for the network in Fig. P16.35 using the Z parameters.

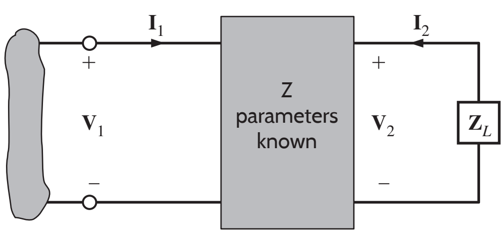


Figure P16.35

16.36 Draw the circuit diagram (with all passive elements in ohms) for a network that has the following Y parameters:

$$[\mathbf{Y}] = \begin{bmatrix} \frac{5}{11} & -\frac{2}{11} \\ -\frac{2}{11} & \frac{3}{11} \end{bmatrix}$$

16.37 Draw the circuit diagram for a network that has the following Z parameters:

$$[\mathbf{Z}] = \begin{bmatrix} 6 + j4 & 4 + j6 \\ 4 + j6 & 10 + j6 \end{bmatrix}$$

16.38 Following are the hybrid parameters for a network:

$$\begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} = \begin{bmatrix} \frac{11}{5} & \frac{2}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

Determine the Y parameters for the network.

16.39 If the Y parameters for a network are known to be

$$\begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} = \begin{bmatrix} \frac{2}{11} & -\frac{3}{11} \\ -\frac{3}{11} & \frac{3}{11} \end{bmatrix}$$

find the Z parameters ((a) z_{11}, (b) z_{12}, (c) z_{21}, (d) z_{22}).

16.40 Find the Z parameters ((a) z_{11}, (b) z_{12}, (c) z_{21}, (d) z_{22}) if $\mathbf{a} = 3$, $\mathbf{b} = 5$, $\mathbf{c} = 4\ S$, $\mathbf{d} = 9$.

16.41 Find the hybrid parameters in terms of the Z parameters.

16.42 Find the transmission parameters for the two-port in Fig. P16.42.

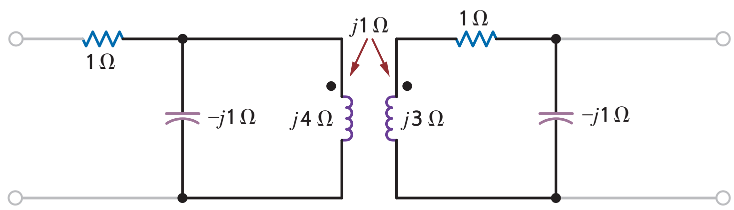


Figure P16.42

16.43 Find the transmission parameters for the two-port in Fig. P16.43 and then use the terminal conditions to compute $\mathbf{I}_o$.

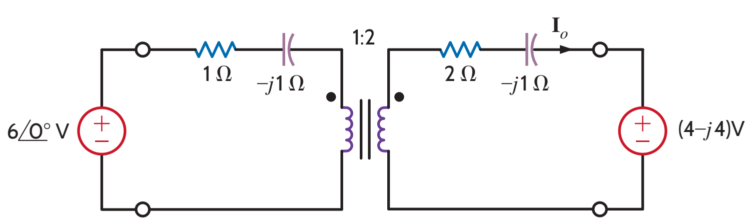


Figure P16.43

16.44 Find the Y parameters for the network in Fig. P16.44.

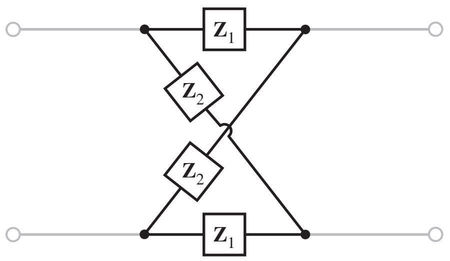


Figure P16.44

16.45 Find the Y parameters of the two port in Fig. P16.45.

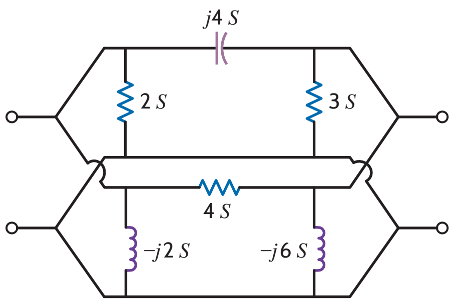


Figure P16.45

16.46 Determine the Y parameters ((a) y_{11}, (b) y_{12}, (c) y_{21}, (d) y_{22}) for the network shown in Fig. P16.46.

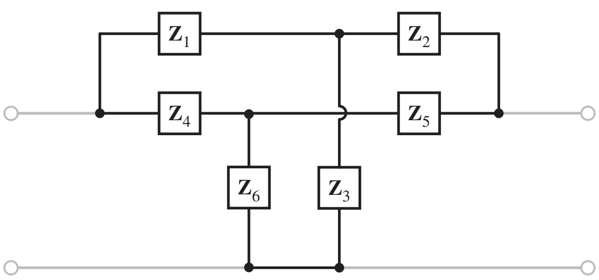


Figure P16.46

16.47 Find the Y parameters of the two-port network in Fig. P16.47. Find the input admittance of the network when the capacitor is connected to the output port.

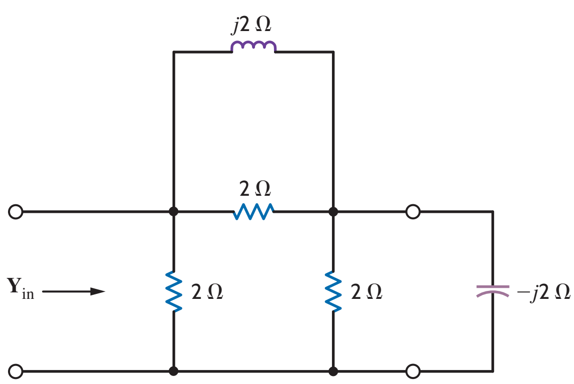


Figure P16.47

16.48 The Y parameters of a network are

$$[\boldsymbol{Y}] = \begin{bmatrix} -0.5 & -0.2 \\ -0.2 & 0.4 \end{bmatrix}$$

Determine the Z parameters of the network.

16.49 Find the transmission parameters of the network in Fig. E16.4 by considering the circuit to be a cascade interconnection of three two-port networks as shown in Fig. P16.49.

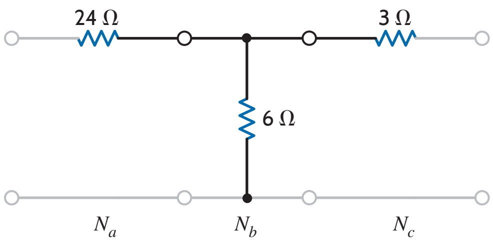


Figure P16.49

16.50 Find the transmission parameters of the circuit.

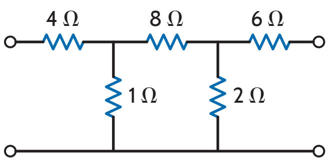


Figure P16.50

16.51 Find the transmission parameters for the two-port network.

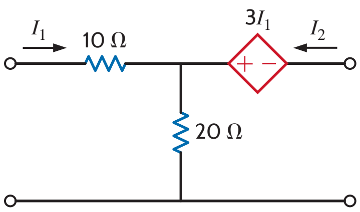


Figure P16.51

16.52 Find the ABCD parameters for the circuit in Fig. P16.52.

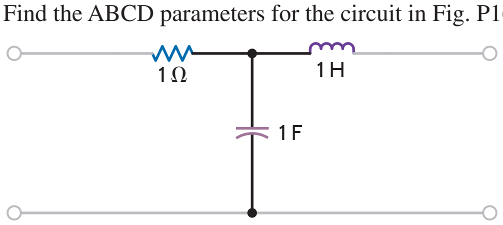


Figure P16.52

16.53 Determine the output voltage $\mathbf{V}_o$ in the network in Fig. P16.53 if the Z parameters for the two-port are

$$\mathbf{Z} = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$$

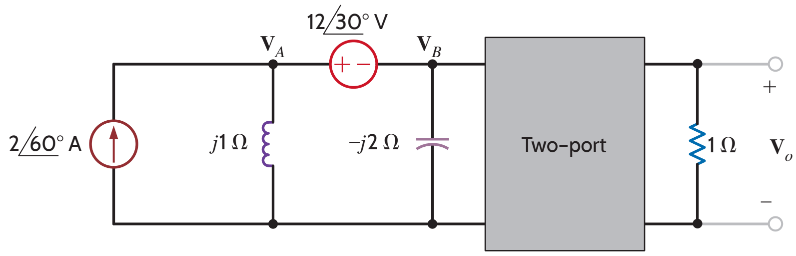


Figure P16.53

16.54 Find the Z parameters for the two-port network in Fig. P16.54 and then determine $\mathbf{I}_o$ for the specified terminal conditions.

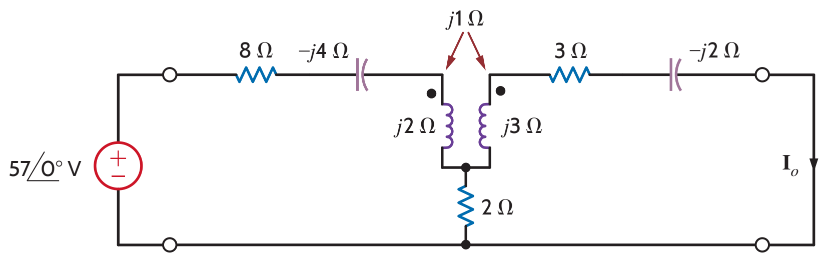


Figure P16.54

Enter arguments from 0 to 2π.

16.55 Determine the output voltage $\mathbf{V}_o$ in the network in Fig. P16.55 if the Z parameters for the two-port are

$$\mathbf{Z} = \begin{bmatrix} 120 & 80 \\ 80 & 120 \end{bmatrix}$$

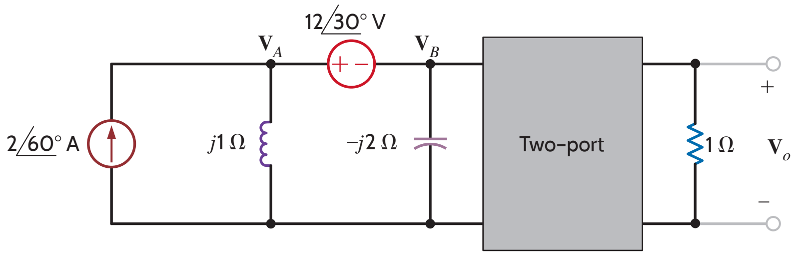


Figure P16.55

Enter arguments from 0 to 2π.

16.56 Find the transmission parameters of the two-port in Fig. 16.56 and then use the terminal conditions to compute $\mathbf{I}_o$.

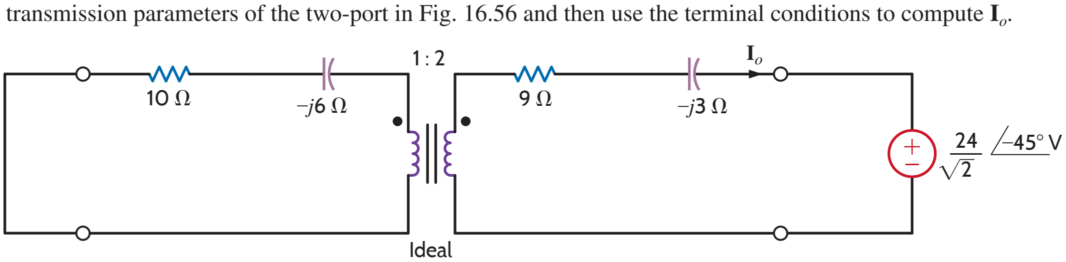


Figure P16.56

16.57 The h parameters of a circuit are given as

$h_{11} = 600\ \Omega$, $h_{12} = 10^{-3}$, $h_{21} = 120$, $h_{22} = 2 \times 10^{-6}\ S$

Draw a circuit model of the device including the value of each element.

TYPICAL PROBLEMS FOUND ON THE FE EXAM

16PFE-1 A two-port network is known to have the following parameters:

$$y_{11} = \frac{1}{14} S \quad y_{12} = y_{21} = -\frac{1}{21} S \quad y_{22} = \frac{1}{7} S$$

If a 2-A current source is connected to the input terminals as shown in Fig. 16PFE-1, find the voltage across this current source.

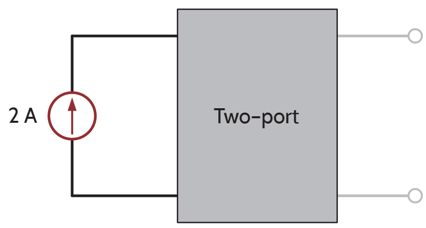


Figure 16PFE-1

a. 36 V **c.** 24 V

b. 12 V **d.** 6 V

16PFE-2 Find the Thévenin equivalent resistance at the output terminals of the network in Fig. 16PFE-1.

a. 3 Ω **c.** 12 Ω

b. 9 Ω **d.** 6 Ω

16PFE-3 Find the Y parameters for the two-port network shown in Fig. 16PFE-3.

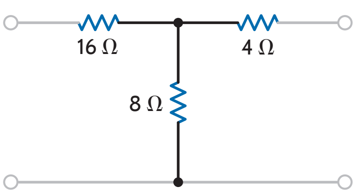


Figure 16PFE-3

a. $y_{11} = \frac{5}{32} S$, $y_{21} = y_{12} = -\frac{5}{14} S$, $y_{22} = \frac{9}{14} S$

b. $y_{11} = \frac{7}{48} S$, $y_{21} = y_{12} = -\frac{3}{16} S$, $y_{22} = \frac{7}{16} S$

c. $y_{11} = \frac{3}{25} S$, $y_{21} = y_{12} = -\frac{1}{15} S$, $y_{22} = \frac{4}{15} S$

d. $y_{11} = \frac{3}{56} S$, $y_{21} = y_{12} = -\frac{1}{28} S$, $y_{22} = \frac{3}{28} S$

16PFE-4 Find the Z parameters of the network shown in Fig. 16PFE-4.

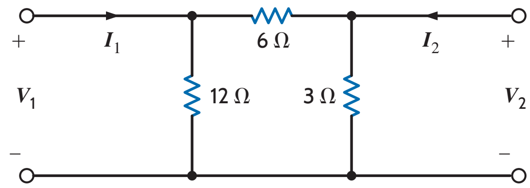


Figure 16PFE-4

a. $z_{11} = \frac{19}{3}\,\Omega$, $z_{12} = \frac{5}{3}\,\Omega$, $z_{22} = \frac{7}{3}\,\Omega$

b. $z_{11} = \frac{22}{5}\,\Omega$, $z_{12} = \frac{7}{5}\,\Omega$, $z_{22} = \frac{9}{5}\,\Omega$

c. $z_{11} = \frac{36}{7}\,\Omega$, $z_{12} = \frac{12}{7}\,\Omega$, $z_{22} = \frac{18}{7}\,\Omega$

d. $z_{11} = \frac{27}{6}\,\Omega$, $z_{12} = \frac{7}{6}\,\Omega$, $z_{22} = \frac{13}{6}\,\Omega$

16PFE-5 Calculate the hybrid parameters of the network in Fig. 16PFE-5.

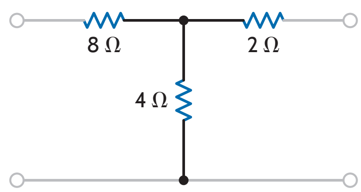


Figure 16PFE-5

a. $h_{11} = \frac{28}{3}\,\Omega$, $h_{21} = -\frac{2}{3}$, $h_{12} = \frac{2}{3}$, $h_{22} = \frac{1}{6} S$

b. $h_{11} = \frac{16}{5}\,\Omega$, $h_{21} = -\frac{1}{5}$, $h_{12} = \frac{1}{5}$, $h_{22} = \frac{3}{10} S$

c. $h_{11} = \frac{19}{4}\,\Omega$, $h_{21} = -\frac{3}{4}$, $h_{12} = \frac{3}{4}$, $h_{22} = \frac{5}{8} S$

d. $h_{11} = \frac{32}{9}\,\Omega$, $h_{21} = -\frac{2}{9}$, $h_{12} = \frac{2}{9}$, $h_{22} = \frac{1}{18} S$