

111-2 Calculus Midterm (1/2)

Chapter : 7-2~7-3 + 7-5~7-7

Date : 2023/04/19 17:20-18:10 (50 minutes)

Total : 55 pts (50%)

1. Find the following limits

a. $\lim_{x \rightarrow 0} \frac{x - \tan^{-1} x}{\sin^{-1} x - x}$ (5 pts) 2

b. $\lim_{x \rightarrow 0^+} (1 + \frac{1}{x})^x$ (5 pts) 1

2. Evaluate the integral $\int \frac{\sin^2 x \cos x}{1 + \sin^2 x} dx$ (10 pts)

3. Find the following derivative of y respect to x using logarithmic differentiation

a. $y = (\ln x)^{\ln x}$ (5 pts)

b. $x = y^{xy}$ (5 pts)

4. Evaluate the integral $\int \frac{dx}{x\sqrt{x^6-4}}$ (hint: $u = \sqrt{x^6-4}$) (10 pts)

5. Evaluate the integral $\int_{-\ln 2}^0 \cosh^2(\frac{x}{2}) dx$ (5 pts)

6. Evaluate the integral $\int_{\ln(e-1)}^{\ln(e^2-1)} \frac{1}{1+e^x} dx$ (10 pts)

$$\begin{aligned} \int_{-\ln 2}^0 \cosh^2(\frac{x}{2}) dx &= \int_{-\ln 2}^0 \frac{e^{\frac{x}{2}} + e^{-\frac{x}{2}}}{2} dx \\ &= \frac{1}{2} \int_{-\ln 2}^0 (e^{\frac{x}{2}} + e^{-\frac{x}{2}}) dx \\ &= \frac{1}{2} [2e^{\frac{x}{2}} - 2e^{-\frac{x}{2}}]_{-\ln 2}^0 \\ &= \frac{1}{2} (2 - 2e^{-\ln 2}) = 1 - e^{-\ln 2} = 1 - \frac{1}{2} = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \int \frac{u^2}{1+u^2} du &= \int \frac{u^2 + 1 - 1}{1+u^2} du = \int \frac{u^2 + 1}{1+u^2} du - \int \frac{1}{1+u^2} du \\ &= \int \frac{u^2 + 1}{1+u^2} du - \int \frac{1}{1+u^2} du \\ &= \int 1 du - \int \frac{1}{1+u^2} du = u - \tan^{-1} u + C \\ &= \tan^{-1} u - \tan^{-1} u + C = C \end{aligned}$$

$$\begin{aligned} \int \frac{1}{x\sqrt{x^6-4}} dx &= \int \frac{1}{x\sqrt{x^6-4}} dx \\ u = \sqrt{x^6-4} \Rightarrow u^2 = x^6-4 \Rightarrow 2u du = 6x^5 dx \Rightarrow \frac{1}{3} du = x^5 dx \\ \int \frac{1}{x\sqrt{x^6-4}} dx = \int \frac{1}{x \cdot 3x^5} du = \frac{1}{3} \int \frac{1}{u^2} du = -\frac{1}{3u} + C = -\frac{1}{3\sqrt{x^6-4}} + C \end{aligned}$$

$$\begin{aligned} \int_{\ln(e-1)}^{\ln(e^2-1)} \frac{1}{1+e^x} dx &= \int_{\ln(e-1)}^{\ln(e^2-1)} \frac{1}{1+e^x} dx \\ u = e^x \Rightarrow du = e^x dx \Rightarrow \frac{1}{u} du = \frac{1}{1+u} du \\ \int_{\ln(e-1)}^{\ln(e^2-1)} \frac{1}{1+e^x} dx = \int_{e-1}^{e^2-1} \frac{1}{1+u} \frac{1}{u} du = \int_{e-1}^{e^2-1} \frac{1}{u(1+u)} du \\ = \int_{e-1}^{e^2-1} \left(\frac{1}{u} - \frac{1}{1+u} \right) du = \left[\ln u - \ln(1+u) \right]_{e-1}^{e^2-1} \\ = \ln(e^2-1) - \ln(e^2) - (\ln(e-1) - \ln(e)) = \ln(e^2-1) - \ln(e^2) - \ln(e-1) + \ln(e) \\ = \ln(e^2-1) - \ln(e^2) - \ln(e-1) + \ln(e) = \ln(e^2-1) - \ln(e^2) - \ln(e-1) + \ln(e) \end{aligned}$$

111-2 Calculus Midterm (2/2)

Chapter : 8-2~8-5 + 8-8

Date : 2023/04/26 17:20-18:10 (50 minutes)

Total : 55 pts (50%)

1. Evaluate the following integrals using Integral By Part (IBP)

a. $\int \ln(x^2 + 2x + 2)dx$ (10 pts)

b. $\int \sec^3 x dx$ (10 pts)

2. Evaluate the integral $\int \sin x \cos 3x \cos 5x dx$ (5 pts)

3. Evaluate the integral $\int \frac{x^2+5}{(x+1)(x^2-2x+3)} dx$ (10 pts)

4. Evaluate the integral $\int \frac{1}{(x^2+1)^2} dx$ (10 pts)

5. For what value of a that makes the improper integral $\int_{\sqrt{2}}^{\infty} (\frac{a}{\sqrt{x^2-1}} - \frac{x}{x^2+1}) dx$ converge? and what is the result of the improper integral? (10 pts)

$$\int \sin x \cos 3x \cos 5x dx$$

$$= \int \frac{1}{2} (\sin 4x + \sin 2x) \cdot \cos 5x dx$$

$$= \frac{1}{2} \int \sin 4x \cos 5x + \cos 5x \sin 2x dx$$

$$= \frac{1}{2} \int \frac{1}{2} (\sin 9x + \sin(-x)) + \frac{1}{2} (\sin 3x + \sin 7x) dx$$

$$= \frac{1}{4} \int \sin 9x - \sin x + \sin 3x + \sin 7x dx$$

$$= -\frac{1}{36} \cos 9x + \frac{1}{4} \cos x - \frac{1}{12} \cos 3x + \frac{1}{28} \cos 7x + C$$

$$\lim_{b \rightarrow \infty} \int_{\sqrt{2}}^b (\frac{a}{\sqrt{x^2-1}} - \frac{x}{x^2+1}) dx$$

$$\lim_{b \rightarrow \infty} (a \ln |\sqrt{x^2-1} + x| - \frac{1}{2} \ln |x^2+1|) \Big|_{\sqrt{2}}^b$$

$$\lim_{b \rightarrow \infty} a \ln |\sqrt{b^2-1} + b| - \frac{1}{2} \ln |b^2+1| - a \ln |1+\sqrt{2}| + \frac{1}{2} \ln 3$$

$$a \ln |\infty + \infty| - \frac{1}{2} \ln |\infty| - a \ln |1+\sqrt{2}| + \frac{1}{2} \ln 3$$

$$a \ln |\infty| - \ln |\infty| + \ln e^2 - a \ln |1+\sqrt{2}| + \frac{1}{2} \ln 3$$

$$\int \frac{dx}{\sqrt{x^2-1}} = \int \frac{x dx}{x^2+1}$$

$$x = \sec \theta$$

$$dx = \tan \theta \sec \theta d\theta$$

$$\int \frac{\tan \theta \sec \theta}{\tan \theta} d\theta$$

$$\frac{x}{\theta} \Big|_{\sqrt{2}}^x$$

$$a \int (\tan \theta + \sec \theta) d\theta = \frac{1}{2} \ln |x^2+1|$$

$$a \ln |\sqrt{x^2-1} + x| - \frac{1}{2} \ln |x^2+1|$$