Calculator is not allowed

111-1 Calculus Midterm Exam

Date : 2022/10/26(Wed) 13:20-15:10 Total : 120 pts

1. Find the following limits (10 pts)

   a. $\lim_{x\to 0}\frac{tanx-sinx}{x^3}$ (5 pts) b. $\lim_{x\to\infty}\left(x\sqrt{\frac{x-1}{x+1}}-x\right)$ (5 pts)

2. Find the derivative of following functions (20 pts)

   a. $f(x)=(x^2+1)(x+5+\frac{1}{x})$ (5 pts)

   b. $g(z)=\frac{(z-1)(z^2+z+1)}{z^3}$ (5 pts)

   c. $r(\theta)=\sin(\theta^2)\cos(2\theta)$ (5 pts)

   d. $y(t)=\left(1+\tan^4(\frac{t}{12})\right)^3$ (5 pts)

3. $f(4)=0$ and $f'(4)=3$, $\lim_{x\to 4}\frac{xf(4)-4f(x)}{x-4}$ = ? (10 pts)

4. $F(x)=f\left(\frac{x-1}{x+2}\right)$ and $f'(y)=y^3$, find $\frac{dF(x)}{dx}$ (10 pts)

5. $\lim_{x\to\infty}\{\frac{x^2+1}{x+1}-ax-b\}=-2$, find $a+b$ = ? (10 pts)

6. If $f(x)=\begin{cases}3x^2-bx, & x\geq 1\\ ax^2+2, & x<1\end{cases}$ and $f'(1)$ exist, find a , b (10 pts)

7. $f'(0)=5$, find $\lim_{x\to 0}\frac{f(3x)-f(sinx)}{x}$ = ? (10 pts)

8. Find the tangent line of $f(x)=\cos x$ at point $P(\frac{\pi}{4},\frac{\sqrt{2}}{2})$ using the definition of slope (Do not differential $f(x)$ directly). (10 pts)

9. $g(x)=\begin{cases}xsin\frac{1}{x}, & x\neq 0\\ 1, & x=0\end{cases}$, is $g(x)$ continuous at x = 0 or not ? Explain and prove your answer. (10 pts)

10. Explorers on a small airless planet used a spring gun to launch a ball bearing vertically upward from the surface at a launch velocity of 15 m/sec. Because the acceleration of gravity at the planet's surface was g_s m/sec^2, the explorers expected the ball bearing to reach a height of $s=15t-(\frac{1}{2})g_st^2$ m t sec later. The ball bearing reached its maximum height 20 sec after being launched. What was the value of g_s? (8 pts)

(Questions on both sides of the paper)

11. The graphs in the accompanying figure show the position s, velocity $v = \frac{ds}{dt}$, and acceleration $a = \frac{d^2s}{dt^2}$ of a body moving along a coordinate line as functions of time t. Which graph is which? Give reasons for your answers. (12 pts)