

Introduction to Analog Integrated Circuit Design

Fall 2023

Frequency Compensation and Stability

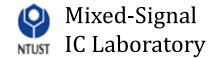
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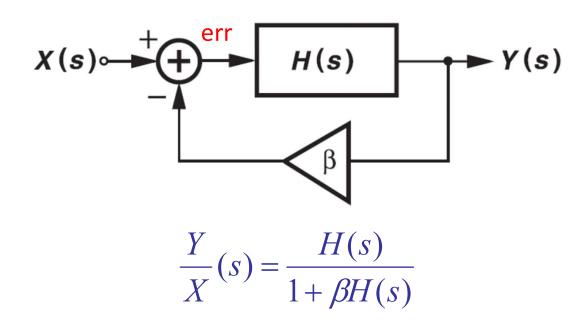
DECE, NTUST



Outline



- Generic Considerations
- Multiple Pole Systems
- Phase Margin
- Frequency Compensation
- Compensation of Two-Stage Opamps
- Slew-Rate of Two-Stage Opamps
- Other Compensation Skills



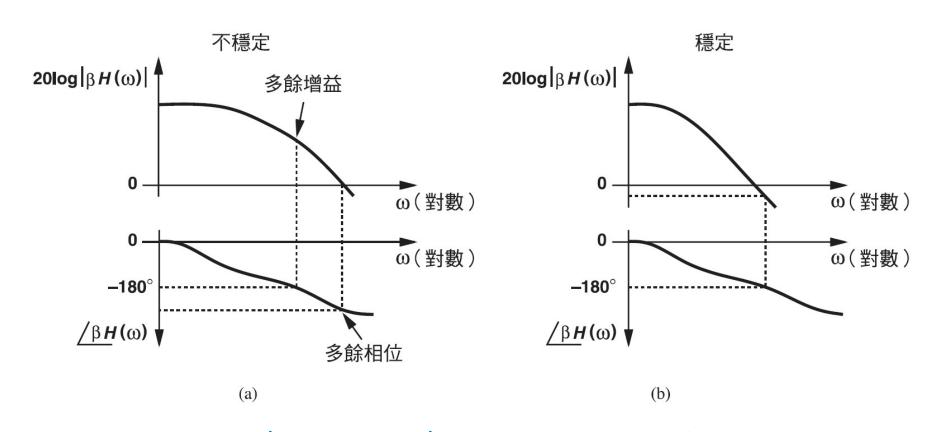
如果 $\beta H(s=j\omega_1)=-1$,增益將會趨近於無限大,而電路會放大自身所產生的雜訊直到其開始振盪為止。

$$\left| \beta H(j\omega_1) \right| = 1$$

(Barkhausen's criteria)

$$\angle \beta H(j\omega_1) = -180^\circ$$

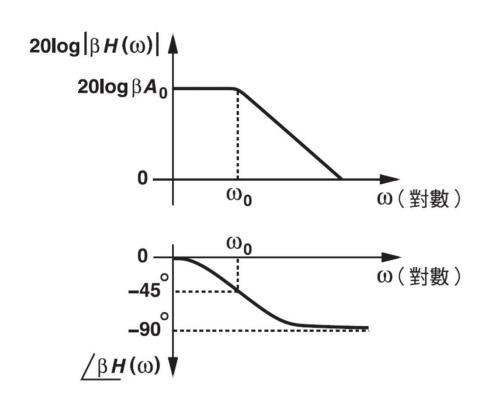
不穩定和穩定系統



不穩定系統和穩定系統迴路增益之波德圖。

'迴路增益=1'和'相位=180°'的頻率在穩定度中扮演著重要角色,而且分別被稱為增益交錯點(頻率)和相位交錯點(頻率)。

波德圖 (Bode Plot)

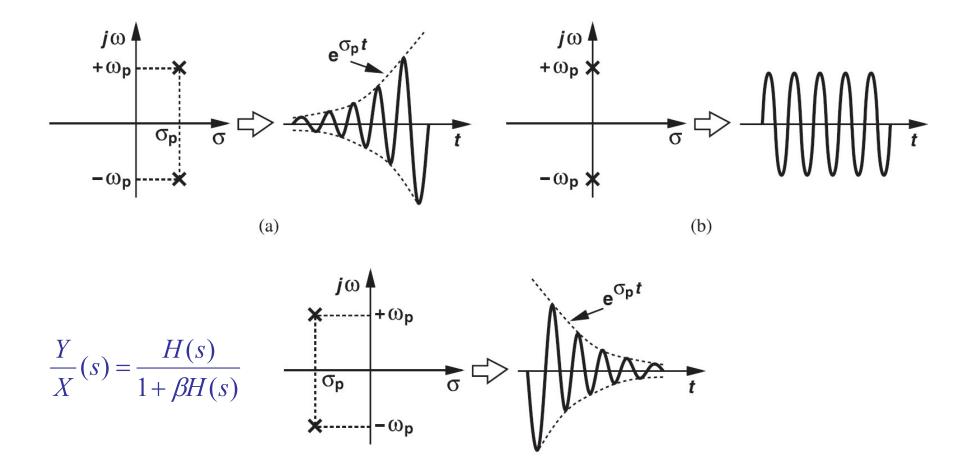


波德圖顯示了一個複數函數 大小和相位的漸近線特性, 乃是依據極點和零點的大小 而定。利用下列二個規則:

- (1)強度大小圖形的斜率在 每個零點頻率時改變 +20dB/dec,而在每個極點 頻率時改變-20dB/dec。
- (2)對一個左半平面極點(零點)頻率 ω_m 而言,相位大約在 $0.1\omega_m$ 時開始下降(上升),在 ω_m 時變化 -45° (+45°),且在 $10\omega_m$ 時變化 -90° (+90°)。

時域響應 (Time-Domain Response)



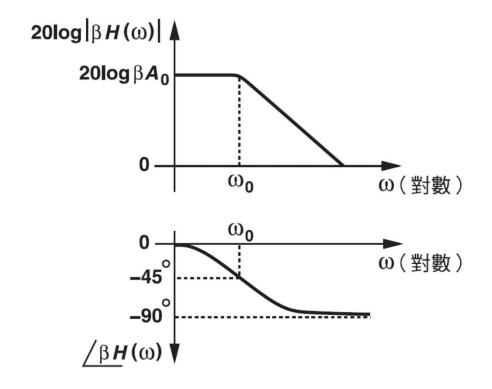


系統的時域響應 vs. 極點位置,(a)強度大小增加造成之不穩定狀態;(b)固定強度振盪造成之不穩定狀態;(c)穩定狀態。

(c)

單極點前授放大器之回授系統

$$H(s) = \frac{A_0}{1 + s/\omega_0}$$



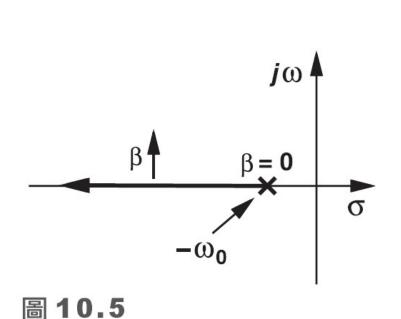
$$\frac{Y}{X}(s) = \frac{\frac{A_0}{1 + \beta A_0}}{1 + \frac{s}{\omega_0 (1 + \beta A_0)}}$$

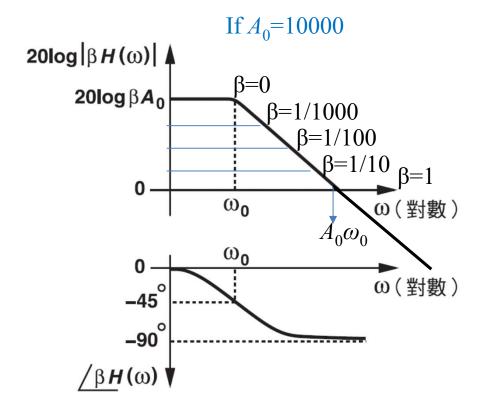
單極點系統之迴路增益波德圖。

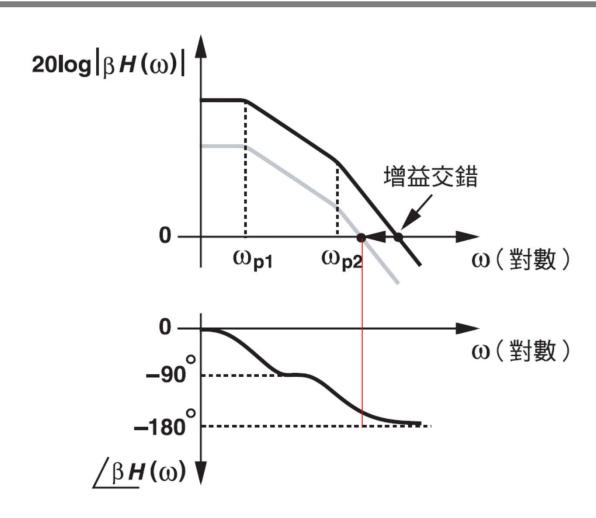
建立單極點系統之根軌跡(root locus)圖。

答:

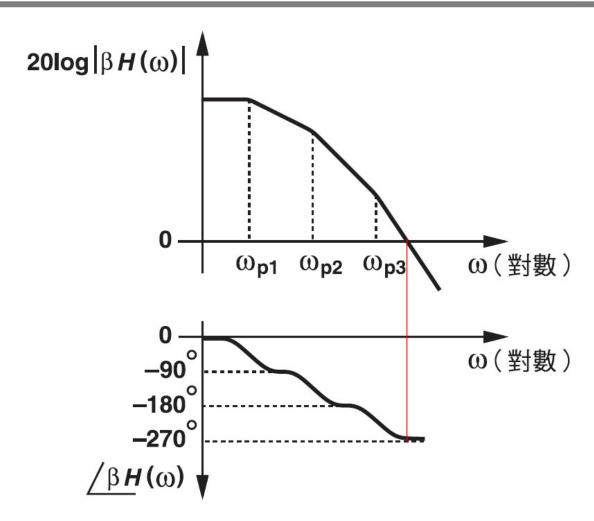
式(10.4)暗示了閉路迴路系統有一極點 $s_p = -\omega_0(1+\beta A_0)$,亦即一位於左半平面之實數極點,而當迴路增益增加時,此極點將遠離原點。





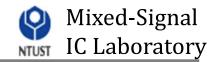


雙極點系統之迴路增益波德圖。



三極點系統之迴路增益波德圖。

例題 10.2



建立雙極點系統之根軌跡圖。=>基於回授之變動量

答:

將開路迴路轉移函數寫成:
$$H(s) = \frac{A_0}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$

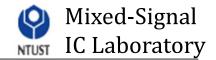
我們得到
$$\frac{Y}{X}(s) = \frac{A_0}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right) + \beta A_0} = \frac{A_0\omega_{p1}\omega_{p2}}{s^2 + (\omega_{p1} + \omega_{p2})s + (1 + \beta A_0)\omega_{p1}\omega_{p2}}$$

=> 閉路迴路極點為
$$S_{1,2} = \frac{-(\omega_{p1} + \omega_{p2}) \pm \sqrt{(\omega_{p1} + \omega_{p2})^2 - 4(1 + \beta A_0)\omega_{p1}\omega_{p2}}}{2}$$

如預期地,當 $\beta=0$, $s_{1,2}=-\omega_{p1}$, $-\omega_{p2}$ 。當 β 增加時,平方根項會降低,取其平方根為零,得到

$$\beta_1 = \frac{1}{A_0} \frac{(\omega_{p1} - \omega_{p2})^2}{4\omega_{p1}\omega_{p2}}$$

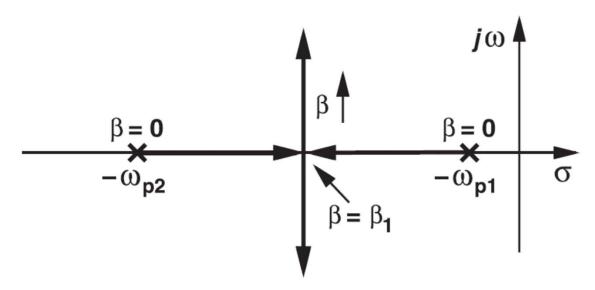
例題 10.2〈續〉



建立雙極點系統之根軌跡圖。=>基於回授之變動量,告訴我們甚麼?

答:

如圖10.7所示,極點在 $-\omega_{p1}$ 和 $-\omega_{p2}$ 開始($\beta=0$),隨著 β 逐漸增加,極點會互相靠近,當 $\beta=\beta_1$ 時會合為一點,而當 $\beta>\beta_1$ 時會變成複數。



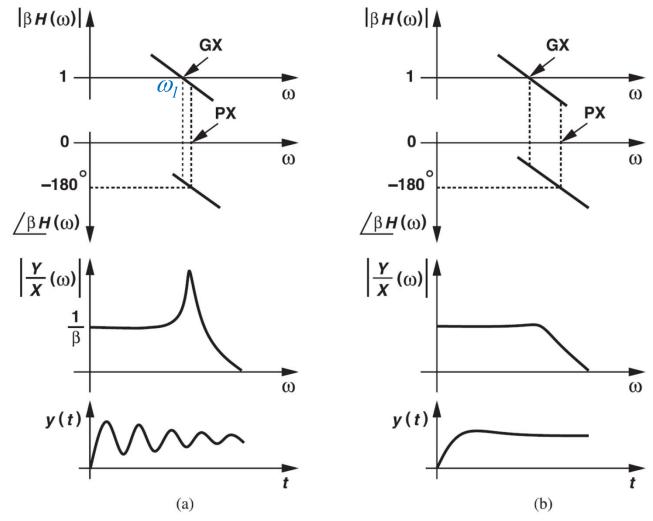
如預期地,當 $\beta = 0$, $s_{1,2} = -\omega_{p1}$, $-\omega_{p2}$ 。當 β 增加時,

平方根項會降低,取其平方根為零,得到

$$\beta_1 = \frac{1}{A_0} \frac{(\omega_{p1} - \omega_{p2})^2}{4\omega_{p1}\omega_{p2}}$$

相位安全邊限 (Phase Margin)





閉路迴路頻率和時間響應對於(a)在增益和相位交錯點間之小安全邊限和(b)之大安全邊限而言。

相位安全邊限 (Phase Margin)



$$\frac{Y}{X}(j\omega_{1}) = \frac{H(j\omega_{1})}{1 + \beta H(j\omega_{1})} \Rightarrow \omega_{1} \text{ means the } \text{freq at |gain|=1}
\Rightarrow 小安全邊限: ∠βH = -175°$$

$$= \frac{\frac{1}{\beta} \exp(-j175^{\circ})}{1 + \exp(-j175^{\circ})}$$

$$= \frac{1}{\beta} \cdot \frac{-0.9962 - j0.0872}{0.0038 - j0.0872}$$

$$\left| \frac{Y}{X} (j\omega_1) \right| = \frac{1}{\beta} \cdot \frac{1}{0.0872} \approx \frac{11.5}{\beta}$$

相位安全邊限定義為
$$PM = 180^{\circ} + \angle \beta H(\omega = \omega_1)$$

設計一雙極點系統使得 $|\beta H(\omega_{p2})|=1$ 且 $|\omega_{p1}|<<|\omega_{p2}|$ (圖10.10), 其相位安 全邊限為何?

答: Phase Margin (PM): 相位安全邊限

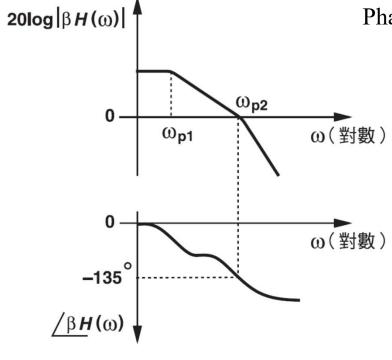
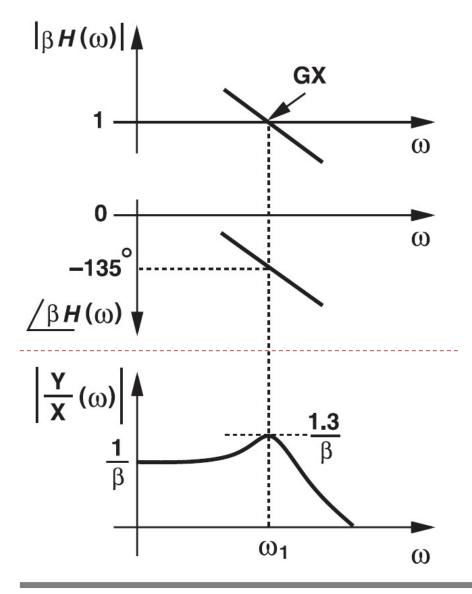


圖 10.10

因為在 $\omega=\omega_{p2}$ 時, $|\beta H(\omega_{p2})|=0$ dB。 $\angle\beta H$ 達到 -135° , $PM=45^{\circ}$ 。

Comments: In real control systems, $PM > 65^{\circ}$ is commonly used!!

45°相位安全邊限之閉路迴路



增益交錯頻率: Gain crossover frequency (freq at $|\beta H| = 1$)

相位交錯頻率: Phase crossover frequency (freq at $\angle \beta H = -180^{\circ}$)

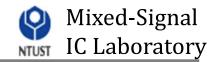
對 PM= 45° 而言,增益交錯頻率為 ω_1 且 $|\beta H(\omega_1)|=1$ 。At ω_1 , the closed-loop gain (Y/X) is

$$\frac{Y}{X} = \frac{H(j\omega_1)}{1+1\times\exp(-j135^\circ)}$$

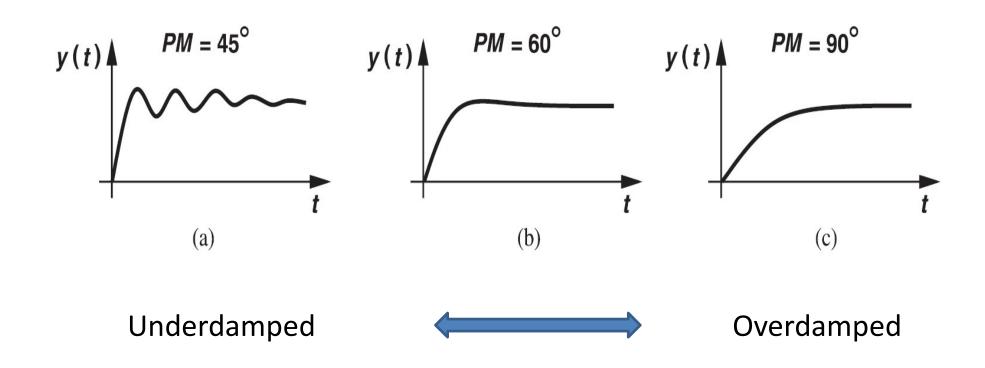
$$= \frac{H(j\omega_1)}{0.29-0.71j}$$

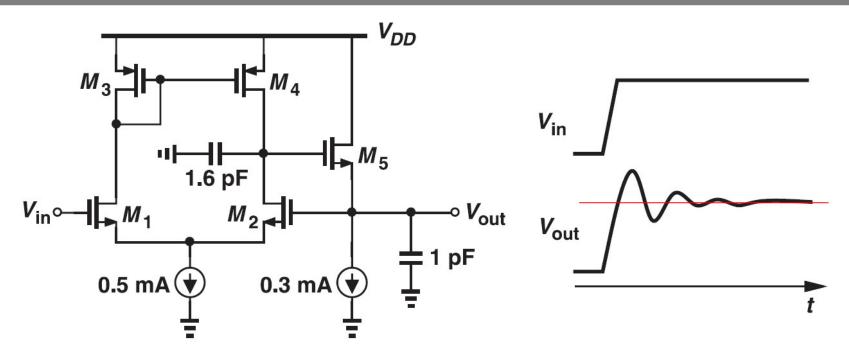
$$\left|\frac{Y}{X}\right| = \frac{1}{\beta} \cdot \frac{1}{|0.29-0.71j|} \approx \frac{1.3}{\beta}$$

不同相位安全邊限之時間響應

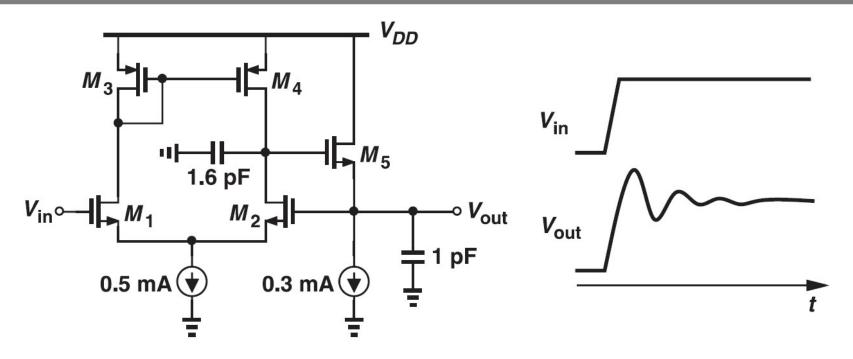


對相位安全邊限為45°、60°和 90°之時間響應。

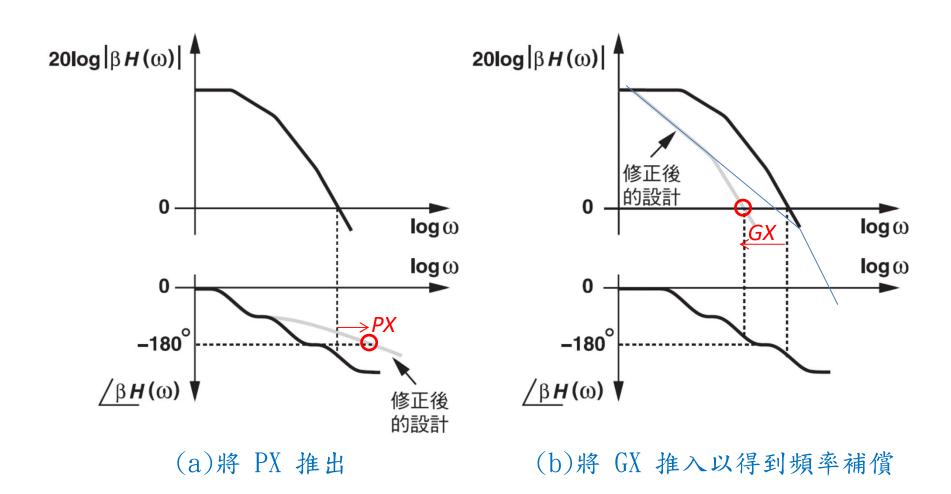




這是一個展現合理相位安全邊限但是 " 很差的安定特性" 的回授電路。 Why ??

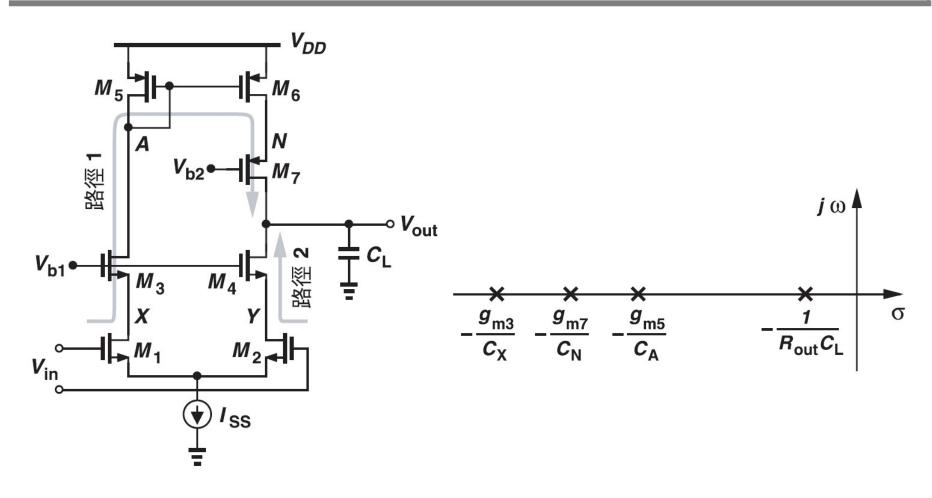


- 運算放大器通常必須被補償,也就是說其開迴路轉移函數必須被修正,使得閉迴路電路處於穩定狀態,且具有良好的時間響應特性。
- 需要頻率補償是因為 $|\beta H|$ 在 $\angle \beta H$ 達到 -180° 之前不會降至1以下。然後 我們假設穩定度可藉由下列二個方法完成:
 - (1)將總相位偏移最小化,因此可將相位交錯點向外推;
 - (2)減少增益,將增益交錯點向內推。



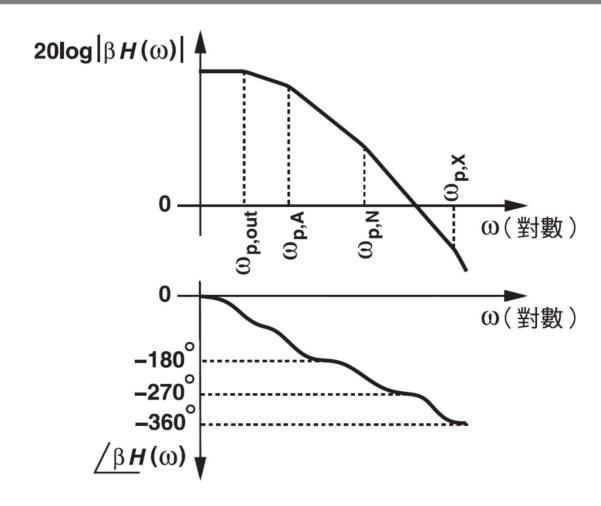
PX: Phase crossover frequency

GX: Gain crossover frequency

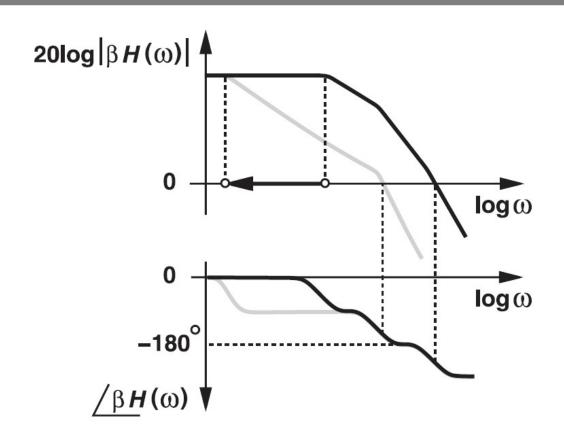


單端輸出伸縮運算放大器及其極點位置。

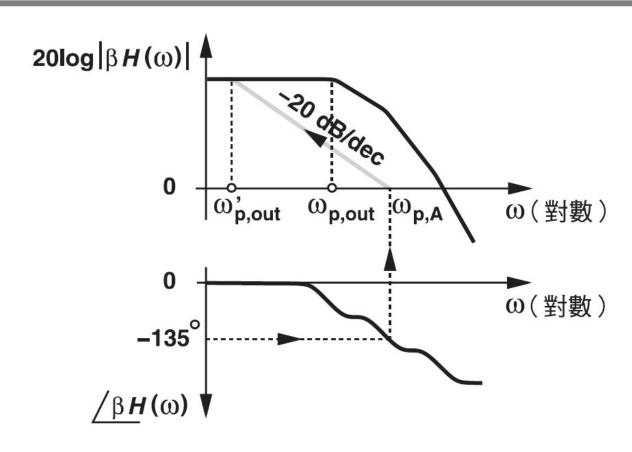
How to determine their contributions?



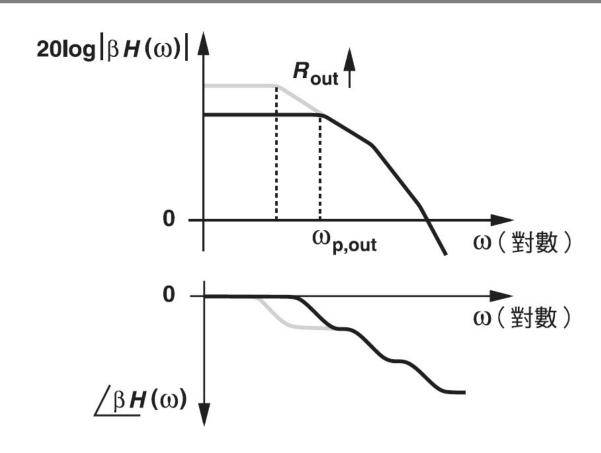
單端輸出伸縮運算放大器之迴路增益波德圖。



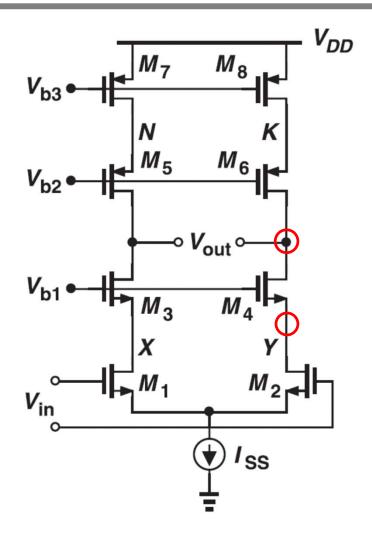
將主要極點往原點移動將會影響強度圖形(Amplitude plot)而不會影響相位圖形(Phase plot)中的重要部份。



在 45° 相位安全邊限下將主要極點往原點移動。

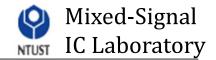


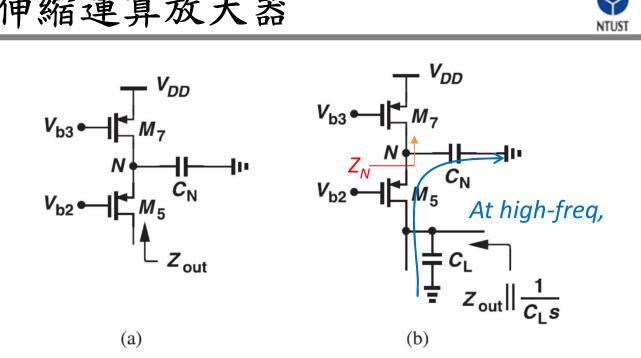
Larger Rout does not help the stability. Why?



此組態避免了映射極點,故顯示了於較大頻寬情況下的穩定度。

全差動伸縮運算放大器

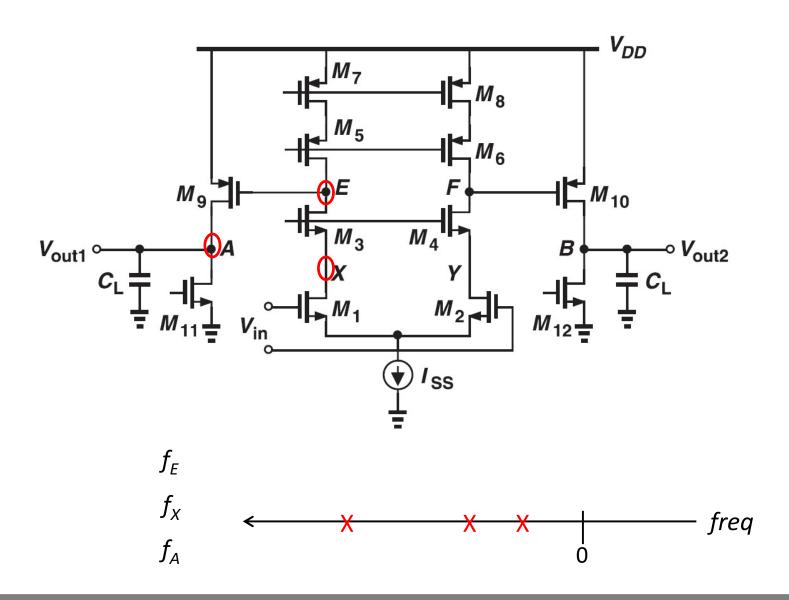


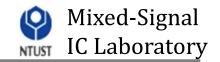


疊接電流源內在節點之元件電容的效應。

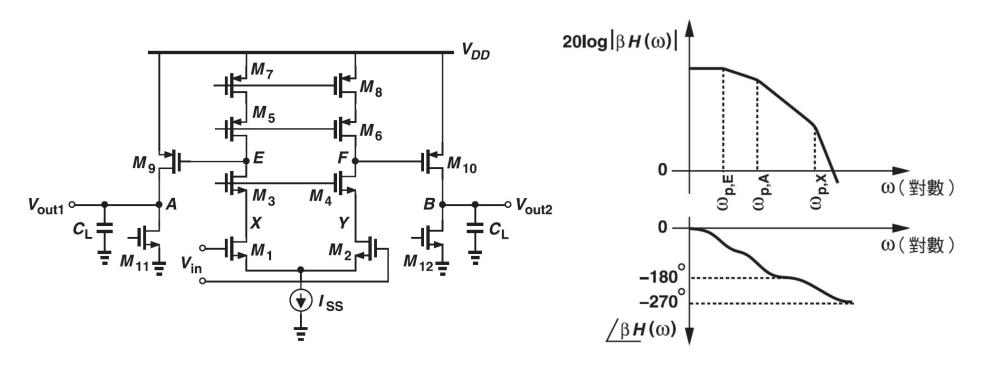
$$Z_{out} = (1 + g_{m5}r_{O5})Z_N + r_{O5} \approx (1 + g_{m5}r_{O5})\frac{r_{O7}}{r_{O7}C_Ns + 1}$$

$$Z_{out} \parallel \frac{1}{C_L s} = \frac{(1 + g_{m5} r_{O5}) \frac{r_{O7}}{r_{O7} s C_N + 1} \cdot \frac{1}{C_L s}}{(1 + g_{m5} r_{O5}) \frac{r_{O7}}{r_{O7} s C_N + 1} + \frac{1}{C_L s}} = \frac{(1 + g_{m5} r_{O5}) r_{O7}}{[(1 + g_{m5} r_{O5}) r_{O7} C_L + r_{O7} C_N] s + 1}$$



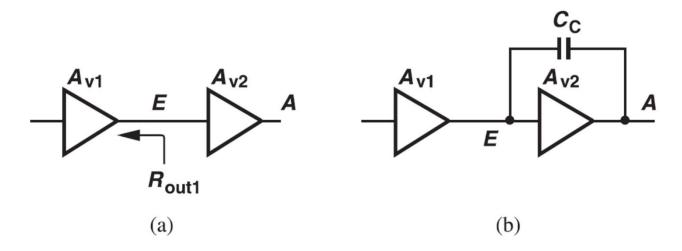


雙級運算放大器之迴路增益波德圖。



雙級運算放大器之米勒補償

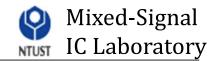


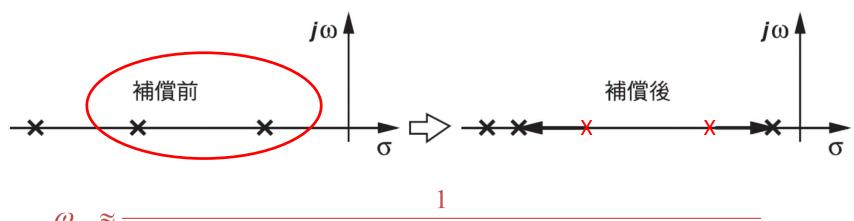


圖中第一級展現了一高輸出阻抗而第二級則提供了小增益, 提供了適合米勒放大電容的環境。

此概念是在節點E產生一大電容為 $(1+A_{v2})C_C$,將對應極點推至 $R_{outl}^{-1}[C_E+(1+A_{v2})C_C]^{-1}$,其中 C_E 象徵了在加入 C_C 之前節點E之電容。所以一低頻極點可利用一小電容來完成,此技巧稱為「米勒補償」。

米勒補償所造成之極點分離



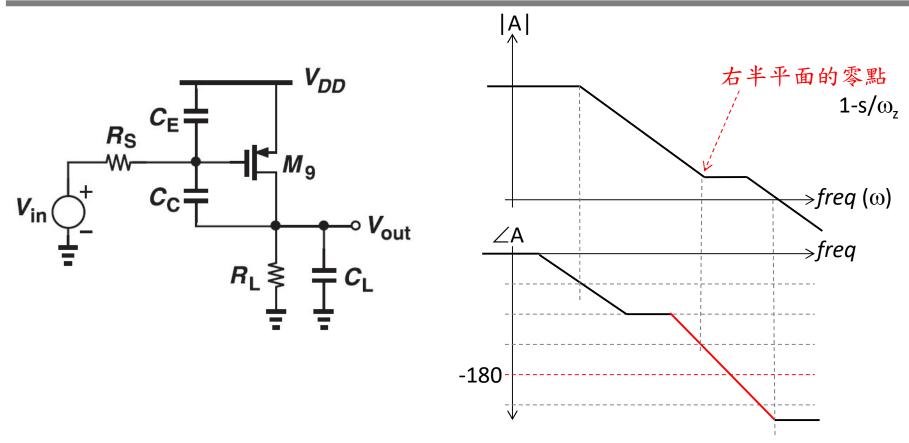


$$\omega_{p1} \approx \frac{1}{R_{out1}[(1+g_{m9}R_L)(C_C + C_{GD9}) + C_E] + R_L(C_C + C_{GD9} + C_L)}$$

$$\approx \frac{1}{R_{out1}(1+g_{m9}R_L)C_C}$$

$$\omega_{p2} \approx \frac{R_{S}[(1+g_{m9}R_{L})(C_{C}+C_{GD9})+C_{K}]+R_{L}(C_{C}+C_{GD9}+C_{L})}{R_{S}R_{L}[(C_{C}+C_{GD9})C_{E}+(C_{C}+C_{GD9})C_{L}+C_{E}C_{L}]}$$

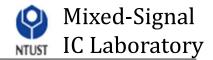
$$\approx \frac{1+g_{m9}R_{L}}{R_{L}(C_{L}+C_{E})} = \frac{g_{m9}}{C_{L}+C_{E}}$$



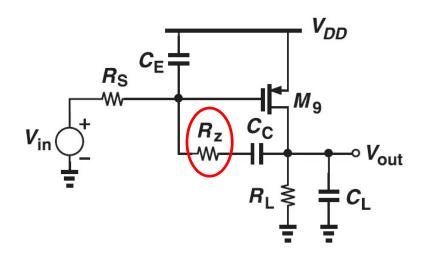
雙級運算放大器之簡易電路及右半平面零點的效應。

當極點位於左半平面時,在右半平面的零點會產生相位偏移,因此將相位交錯點推向原點。從近似波德圖來看,零點會減緩強度的下降,因此將增益交錯點推離原點,所以穩定度會變差。

右半平面零點的移動



加入 R_Z 以移走右半平面零點。=> How to get R_Z ?



Find zero:

$$-g_{m9}V_{in}+V_{in}/(R_z+1/sC_C)=0, s=\omega_z$$

$$\omega_z \approx \frac{1}{C_C(\varphi_{m0}^{-1}-R_z)}$$

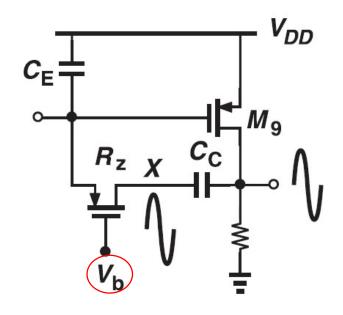
將零點推往左半平面以消除第二極點 (Why?)

$$\frac{1}{C_C(g_{m9}^{-1} - R_Z)} = \frac{-g_{m9}}{C_L + C_E}$$

$$R_Z = \frac{C_L + C_E + C_C}{g_{m9}C_C} \approx \frac{C_L + C_C}{g_{m9}C_C}$$

But, Rz $\propto 1/g_{m9}$?

使用MOSFET來實現 $R_Z \Longrightarrow Why?$



•將零點推往左半平面以消除第二極點是難以做到的,假如" C_L is defined by customer"

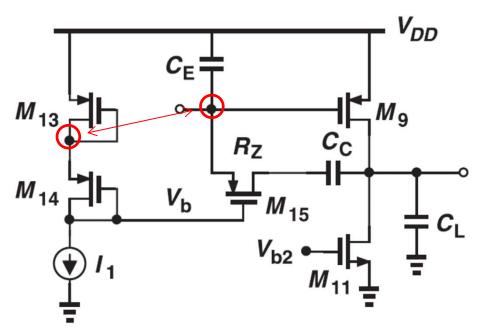
$$R_Z = \frac{C_L + C_E + C_C}{g_{m9}C_C} \approx \frac{C_L + C_C}{g_{m9}C_C}$$

- • $R_z \propto 1/g_{m9}$? => 使用MOSFET來實現 R_Z
- \bullet 一般來說,電晶體 (R_z) 運作於三極管區
- •當輸出電壓透過 C_C 耦合至節點X時, R_z 可能會改變很多,所以大信號安定響應可能會變差

Q: Vb 如何產生?

在適當溫度和製程下產生Vb





$$g_{m} = \sqrt{2K_{n}I_{D}}$$

$$= K_{n} \left(V_{gs} - V_{th}\right)$$

$$\propto \left(\frac{W}{L}\right) \left(V_{gs} - V_{th}\right)$$

$$R_Z = (\mu_p C_{ox}(W/L)(V_{gs}-V_{TH}))^{-1}$$

 $\sim g_{m15}^{-1}$

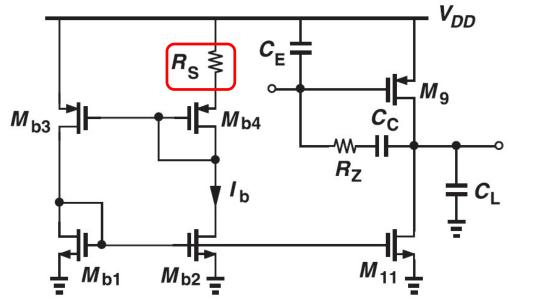
當極點-零點抵消現象發生時,

$$R_Z = g_{m15}^{-1} = g_{m14}^{-1} \frac{(W/L)_{14}}{(W/L)_{15}} = g_{m9}^{-1} \left(1 + \frac{C_L}{C_C}\right)$$

$$(W/L)_{15} = \sqrt{(W/L)_{14}(W/L)_9} \sqrt{\frac{I_{D9}}{I_{D14}}} \frac{C_C}{C_C + C_L}$$

相對於 R_s 定義 g_{mg} 的方法

解決電阻變異的另一種作法:利用電路產生 R_z 與 g_{m9} 之關聯!!



$$R_Z \approx \frac{C_L + C_C}{g_{m9}C_C}$$

$$\propto R_S \left(\frac{C_L + C_C}{C_C}\right)$$

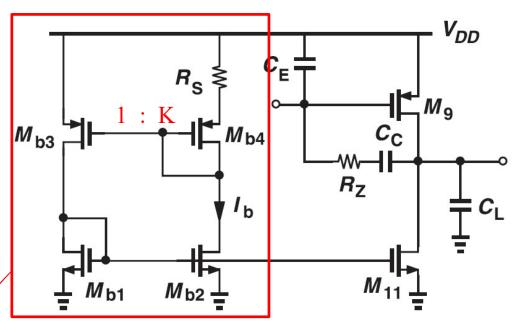
使用一簡單電阻 R_z 且對於和 R_z 相當匹配之電阻 R_S 。

$$I_b \propto R_S^{-2}$$
 (see next page)

$$g_{m9} \propto \sqrt{I_{D9}} \propto \sqrt{I_{D11}} \propto R_S^{-1}$$

相對於 R_s 定義 g_{mg} 的方法





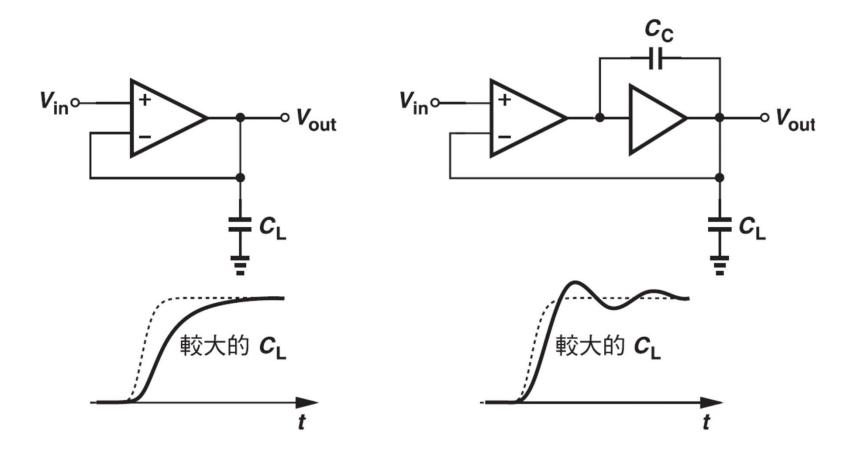
Neglecting body effect,

$$V_{SG3} = V_{SG4} + I_b R_S$$

$$\sqrt{\frac{2I_{b}}{K_{P}(W/L)_{3}}} + |V_{TH3}| = \sqrt{\frac{2I_{b}}{K_{P}(W/L)_{4}}} + |V_{TH4}| + I_{b}R_{S}$$

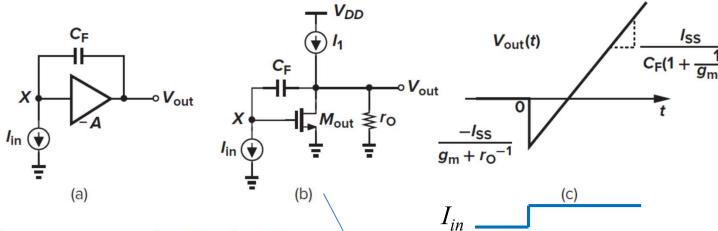
$$\sqrt{\frac{2I_b}{K_P(W/L)_3}} \left(1 - \frac{1}{\sqrt{K}}\right) = I_b R_S \qquad \blacksquare \qquad I_b = \frac{2}{K_P(W/L)_3} \left(1 - \frac{1}{\sqrt{K}}\right)$$

$$I_{b} = \frac{2}{K_{P} (W/L)_{3}} \left(1 - \frac{1}{\sqrt{K}}\right)^{2} \frac{1}{R_{S}^{2}}$$



增加負載電容對單級和雙級運算放大器步級響應的影響。 Why?

放大器之迴轉現象



where I_{in} is a current step given by $I_{SS}u(t)$



$$V_{out}(t) \approx \frac{I_{SS}}{C_F}t$$

$$V_{out}/r_O + g_m V_X + I_{in} = I_1$$

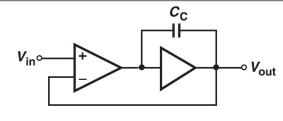
$$C_F d(V_{out} - V_X)/dt = I_{in}$$

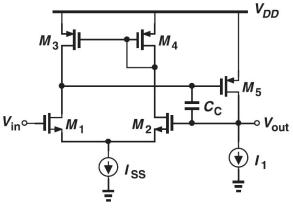
$$C_F \left(1 + \frac{1}{g_m r_O} \right) \frac{dV_{out}}{dt} = I_{in} - \frac{C_F}{g_m} \frac{dI_{in}}{dt}$$

$$V_{out}(t) = \frac{I_{SS}}{C_F(1 + \frac{1}{g_m r_O})} tu(t) - \frac{I_{SS}}{g_m + \frac{1}{r_O}} u(t)$$

雙級運算放大器之迴轉現象



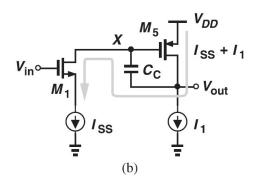


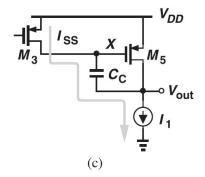


For (b), if M5 cannot provide $l_{SS}+l_1$, Vx will drop significantly to reduce l_{SS}

For (c), $I_1 = I_{SS} + I_{D5}$. If $I_1 < I_{SS}$, M3 will enter the triode region

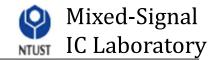
=> This makes two-stage opamp consumes more power to maintain the slew issue

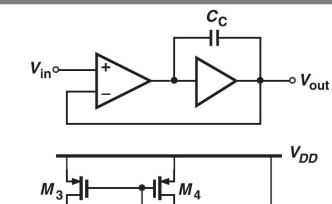




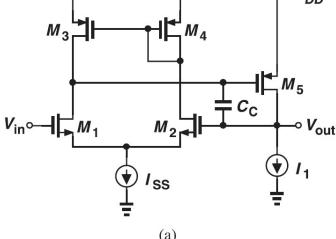
- (a)簡單雙級運算放大器;(b)在正迴轉期間之簡化電路;
- (c)在負迴轉期間之簡化電路。

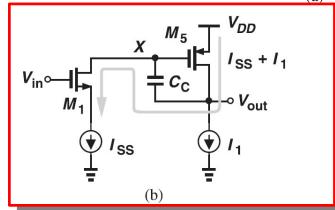
雙級運算放大器之正迴轉現象

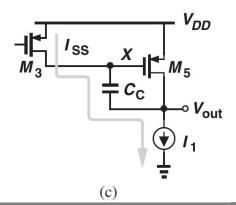




For (b), if M5 cannot provide $I_{SS}+I_1$, Vx will drop significantly to reduce I_{SS}





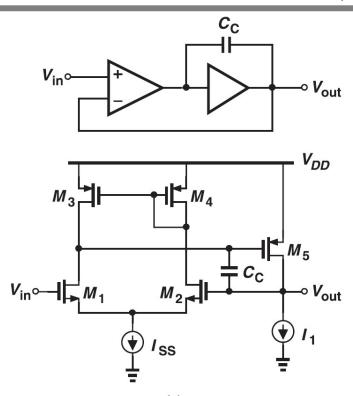


Yung-Hui Chung

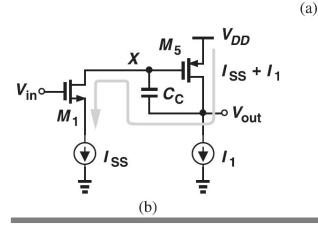
Frequency Compensation and Stability

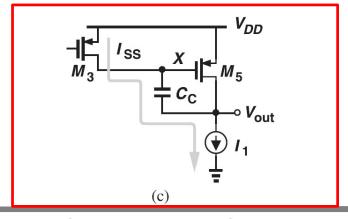
雙級運算放大器之負迴轉現象





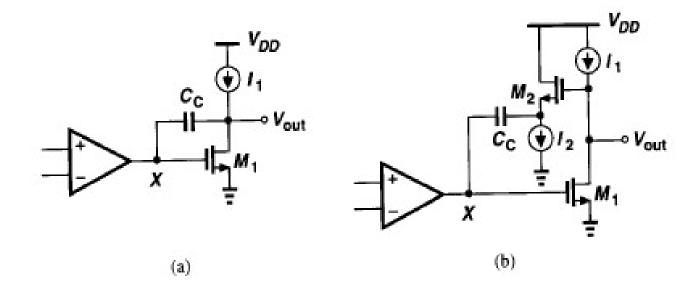
For (c), $I_1=I_{SS}+I_{D5}$. If $I_1<I_{SS}$, M3 will enter the triode region





加入一源極隨耦器的補償技巧

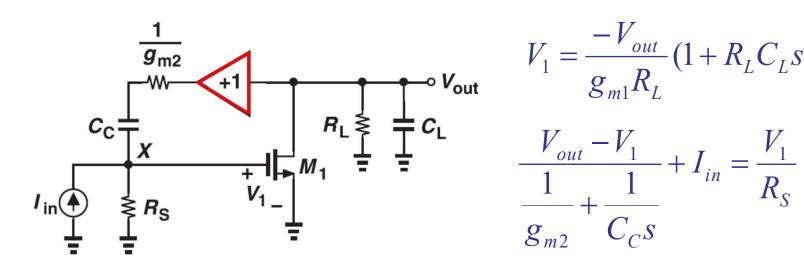




- (a)由 C_C 所產生之右半平面零點之雙級運算放大器;
- (b)加入一源極隨耦器以移走此一右半平面零點。

Q: What do we lose if using a source follower as (b)?

加入一源極隨耦器的簡化等效電路



$$V_1 = \frac{-V_{out}}{g_{m1}R_L}(1 + R_L C_L s)$$

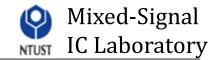
$$\frac{V_{out} - V_1}{\frac{1}{g_{m2}} + \frac{1}{C_C s}} + I_{in} = \frac{V_1}{R_S}$$

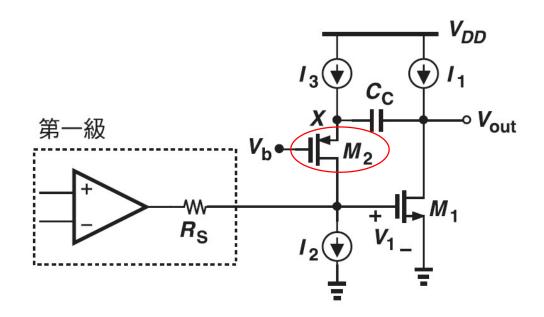
$$\frac{V_{out}}{I_{in}} = \frac{-g_{m1}R_LR_S(g_{m2} + C_C s)}{R_LC_LC_C(1 + g_{m2}R_S)s^2 + [(1 + g_{m1}g_{m2}R_LR_S)C_C + g_{m2}R_LC_L]s + g_{m2}}$$

$$\omega_{p1} \approx \frac{g_{m2}}{g_{m1}g_{m2}R_LR_SC_C} \approx \frac{1}{g_{m1}R_LR_SC_C}$$

$$\omega_{p2} \approx \frac{g_{m1}g_{m2}R_LR_SC_C}{R_LC_LC_Cg_{m2}R_S} \approx \frac{g_{m1}}{C_L}$$

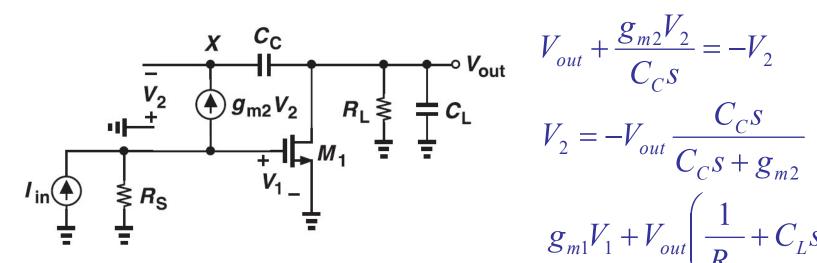
使用共閘極組態之補償技巧





使用補償電容來隔離主動回授組態之直流位準和輸出位準是較為理想的。

共閘極組態的簡化等效電路



$$V_{out} + \frac{g_{m2} V_2}{C_C s} = -V_2$$

$$V_2 = -V_{out} \frac{C_C s}{C_C s + g_{m2}}$$

$$g_{m1} V_1 + V_{out} \left(\frac{1}{R_L} + C_L s\right) = g_{m2} V_2$$

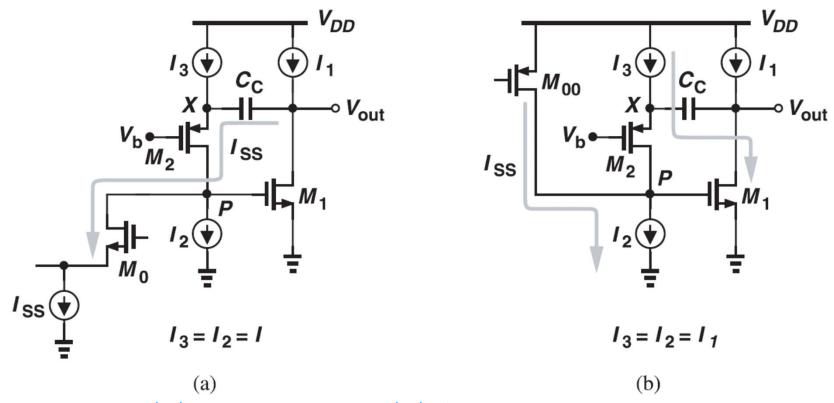
$$\frac{V_{out}}{I_{in}} = \frac{-g_{m1}R_{S}R_{L}(g_{m2} + sC_{C})}{R_{L}C_{L}C_{C}s^{2} + [(1 + g_{m1}R_{S})g_{m2}R_{L}C_{C} + C_{C} + g_{m2}R_{L}C_{L}]s + g_{m2}}$$

$$\omega_{p1} \approx \frac{1}{g_{m1}R_{L}R_{S}C_{C}}$$

$$\omega_{p2} \approx \frac{g_{m2}R_{S}g_{m1}}{C_{L}} \quad \text{(originally is } \frac{g_{m1}}{C_{L}} \text{ Why?}$$

共閘極組態補償電路之迴轉特性

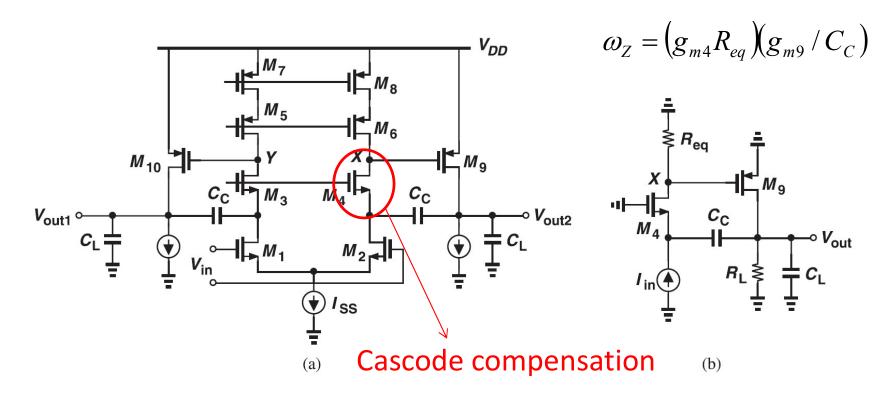




在(a)正迴轉期間和(b)負迴轉期間之電路。

Positive slewing, $I_1 \ge I_{SS} + I_{D1}$

Negative slewing, $I_2 \ge I_{SS} + I_{D2}$



(a)補償雙級運算放大器的另一個方法;(b)(a)中之簡化等效電路。