

Note

1. Allowed Tools: Calculators
2. Leaving the classroom during the exam period will be considered as submitting the exam.
3. Every calculation process yields a score.
4. The answer ought to be presented in decimal format, not as a fraction.

Name : \_\_\_\_\_ SID : \_\_\_\_\_

9. May. 2024

1. (40%) After deriving  $Z_{in}(s)$  in Fig. 1, find (a)  $\omega_0$ ; (b)  $\omega_1, \omega_2$ ; (c)  $Q$ ; (d) BW.
2. (50%) Find the **magnitude characteristic** of the Bode plot for the transfer functions as follows: (a)  $H_a(s) = \frac{s+1}{s-1}$ , (b)  $H_b(s) = \frac{s-1}{s+1}$ , (c)  $H_c(s) = \frac{s(s+100)}{(s+1)(s^2+30s+200)}$ .
3. (10%) Explain the differences of **magnitude and phase plots** between  $H_a(s)$  and  $H_b(s)$  in problem 2.

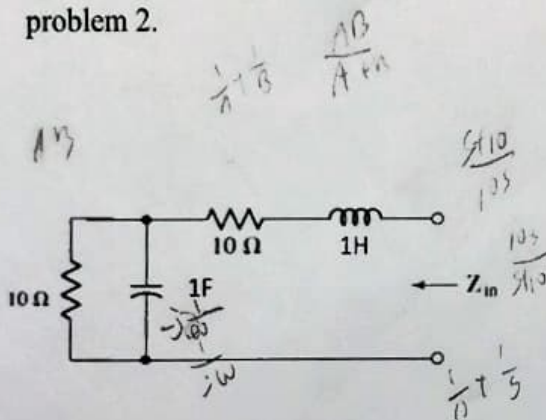


Fig. 1

Handwritten calculations and notes:

$Z_{in} = 10 \parallel \frac{1}{j\omega} + 10 + j\omega$

$Z_{in} = \frac{10 \cdot \frac{1}{j\omega}}{10 + \frac{1}{j\omega}} + 10 + j\omega$

$Z_{in} = \frac{10}{1 + j\omega} + 10 + j\omega$

$Z_{in} = \frac{10 + (10 + j\omega)(1 + j\omega)}{1 + j\omega}$

$Z_{in} = \frac{10 + 10 + j\omega + j\omega + \omega^2}{1 + j\omega}$

$Z_{in} = \frac{20 + j2\omega + \omega^2}{1 + j\omega}$

$Z_{in} = \frac{\omega^2 + j2\omega + 20}{1 + j\omega}$

$\omega_0 = \sqrt{20} = 4.47 \text{ rad/s}$

$\omega_1 = 0, \omega_2 = 4.47$

$Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{4.47}{4.47} = 1$

BW =  $\omega_2 - \omega_1 = 4.47$

For problem 2:

(a)  $H_a(s) = \frac{s+1}{s-1}$

(b)  $H_b(s) = \frac{s-1}{s+1}$

(c)  $H_c(s) = \frac{s(s+100)}{(s+1)(s^2+30s+200)}$

For problem 3:

Magnitude plots show the gain of the system, while phase plots show the phase shift. For  $H_a(s)$  and  $H_b(s)$ , the magnitude plots are identical, but the phase plots are mirror images of each other.