Introduction to Analog Integrated Circuits (112), DECE, NTUST

Homework 3 (Due date: 10/13)

HW3.1: (40 points)

Using small-signal parameters to find each amplifier's voltage gain and output resistance in Fig 3.1 (*channel length modulation* and *body effect* cannot be ignored).

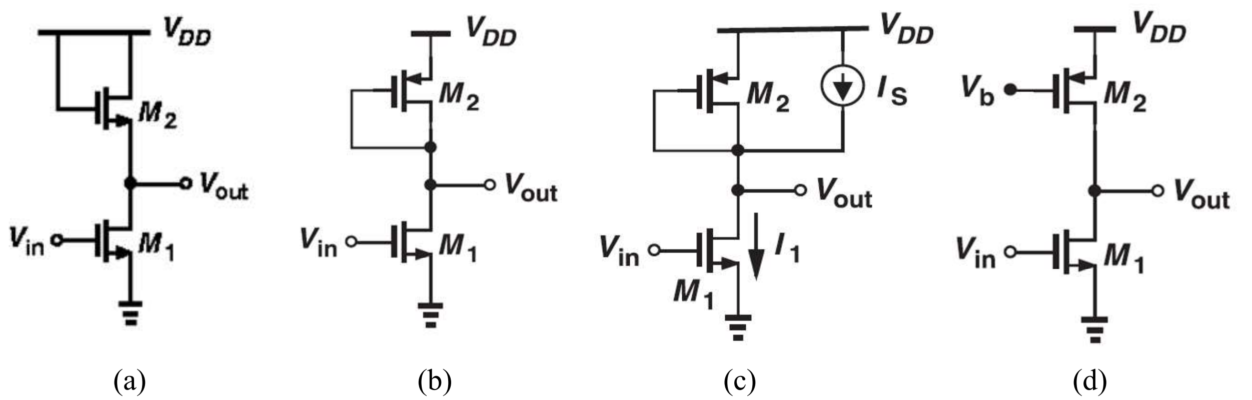


Fig. 3.1

HW3.2: (10 points)

Using small-signal parameters to derive the output resistance (Rout) in Fig. 3.2.

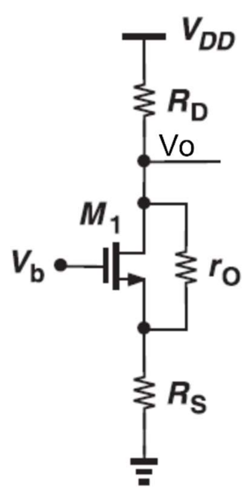


Fig. 3.2

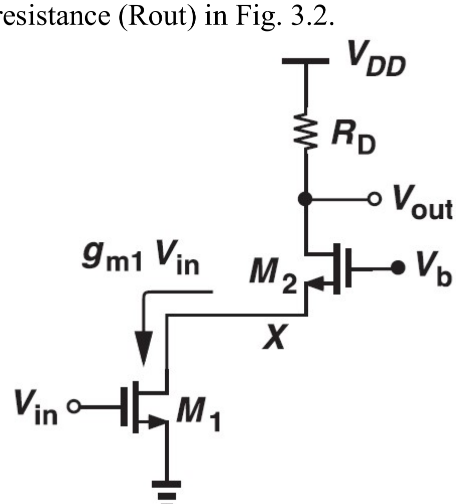


Fig. 3.3

HW3.3: (20 points)

Fig. 3.3 shows a common-gate circuit.

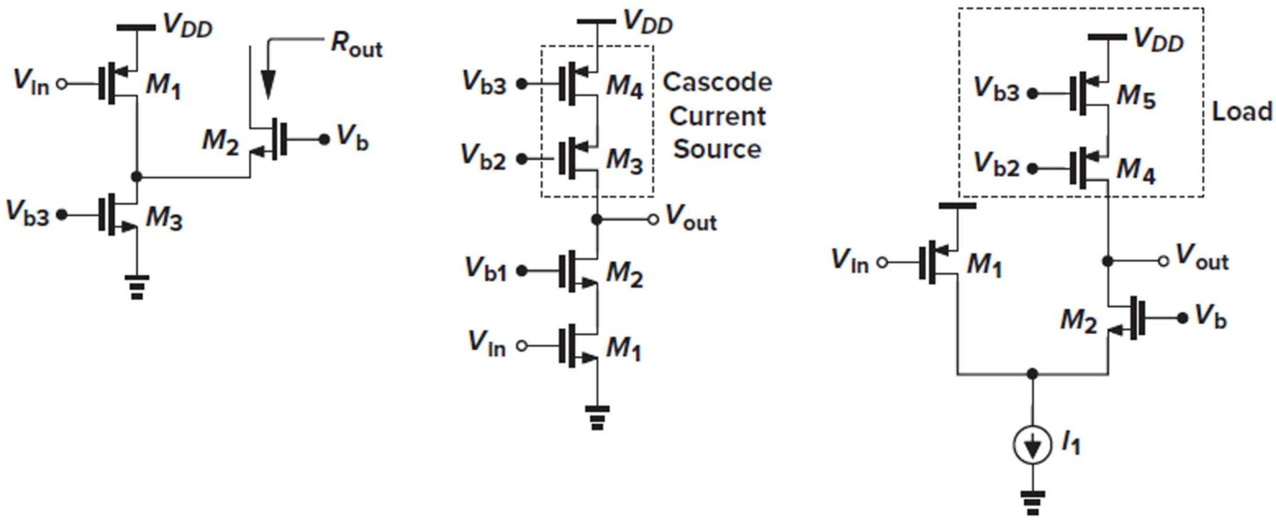
(a) If we define a minimum output voltage, Vout,min, how do you find out the valid input range of Vin?

(b) Calculate the voltage gain and output resistance.

HW3.4: (30 points)

Calculate the voltage gain and output resistance of circuits in Fig. 3.4.

Note: Fig. 3.4(a) only needs to calculate the output resistance.

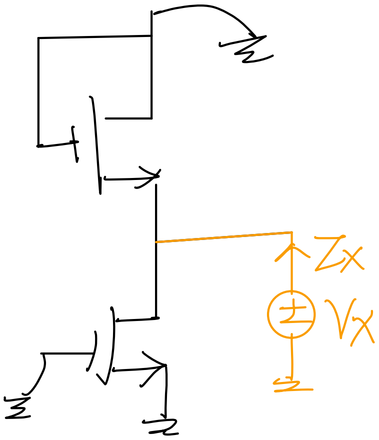


(a) (b) (c)

Fig. 3.4

HW3.1

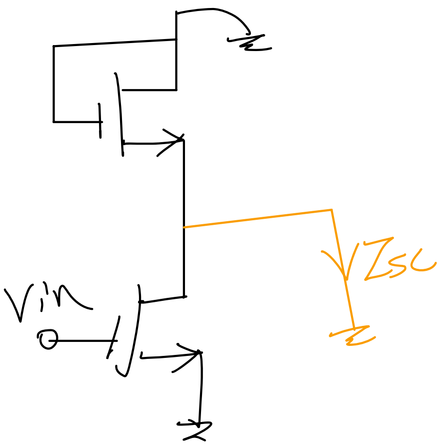
(a)



$$I_x = \left(g_{m2}+g_{m2b}+\frac{1}{r_{o1}}+\frac{1}{r_{o2}}\right)V_x$$

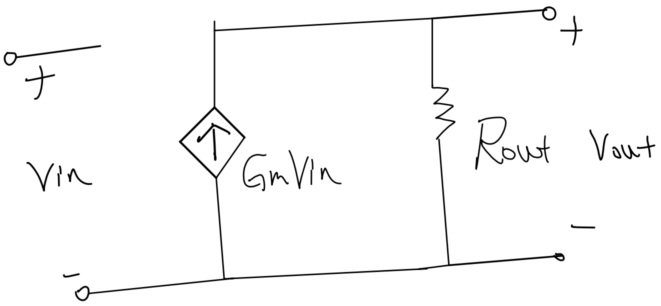
$$R_{out} = \frac{V_x}{I_x} = \frac{1}{\underbrace{g_{m2}+g_{m2b}}_{(1+\eta)g_{m2}}+\frac{1}{r_{o1}}+\frac{1}{r_{o2}}}$$

或 $R_{out} = \frac{1}{(1+\eta)g_{m2}} \,//\, r_{o1} \,//\, r_{o2}$ #



$$I_{sc} = -g_{m1}V_{in}$$

$$G_m = \frac{I_{sc}}{V_{in}} = -g_{m1}$$

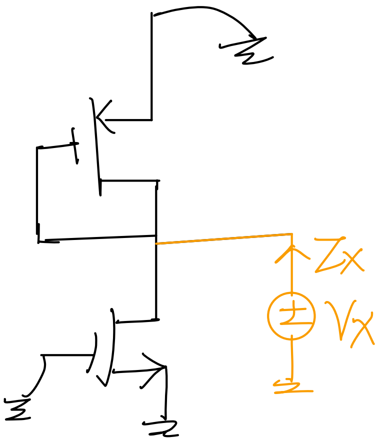


$A_v = G_m R_{out}$ #

$$A_v \simeq \frac{-g_{m1}}{(1+\eta)g_{m2}}$$

(b)

pmos $V_{SB} = 0$ No Body effect

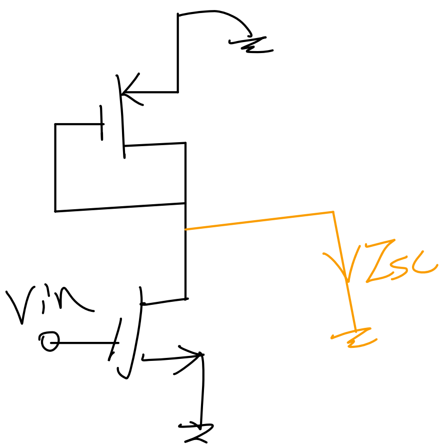


$$I_x = \left(g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}\right) V_x$$

$$R_{out} = \frac{V_x}{I_x} = \frac{1}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}}$$

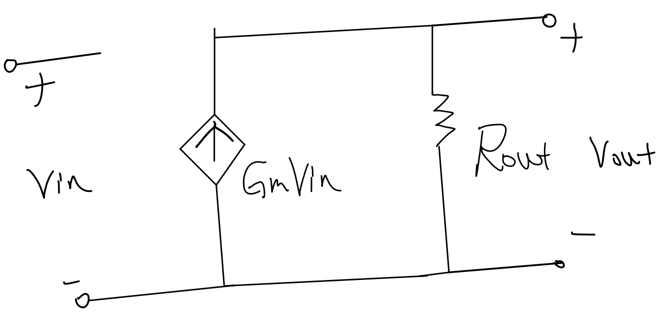
#

或 $R_{out} = \frac{1}{g_{m2}} \,||\, r_{o1} \,||\, r_{o2}$



$$I_{sc} = -g_{m1} V_{in}$$

$$G_m = \frac{I_{sc}}{V_{in}} = -g_{m1}$$



$A_v = G_m R_{out}$ #

$$A_v \simeq \frac{-g_{m1}}{g_{m2}}$$

少了 Body effect 的 factor

且在相同(W/L)下 pmos 的 g_{m2} 比 nmos 小

因此有比a题更大的 Gain

(c)

小訊号分析結果呈(b)題

但 $I_2 = I_1 - I_S$，由 $g_m = \sqrt{2\beta I_D}$ 來看的話，會發現 g_{m2} 變小

若 $A_v \simeq \frac{-g_{m1}}{g_{m2}}$ 使得 A_v 變大

這邊假設 $I_S = 1 - x$ 且 $0 < x < 1$

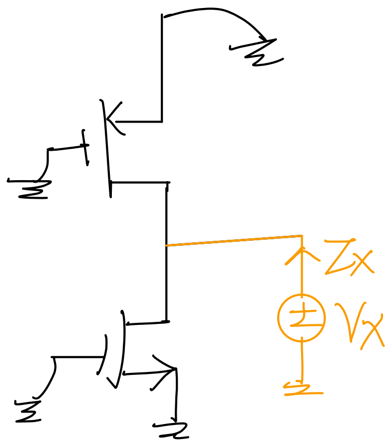
$$A_v \simeq \frac{1}{\sqrt{x}}\sqrt{\frac{\beta_1}{\beta_2}}$$

如果 $x = \frac{1}{4}$

則 $A_v = \frac{\sqrt{2\beta_1 I_1}}{\sqrt{2\beta_2 \frac{1}{4} I_1}} = 2\sqrt{\frac{\beta_1}{\beta_2}}$ 變為原本的兩倍

(D)

$V_{gs2}=0$ 所以 $g_{m2}V_{gs2}=0$ 只剩下 r_o

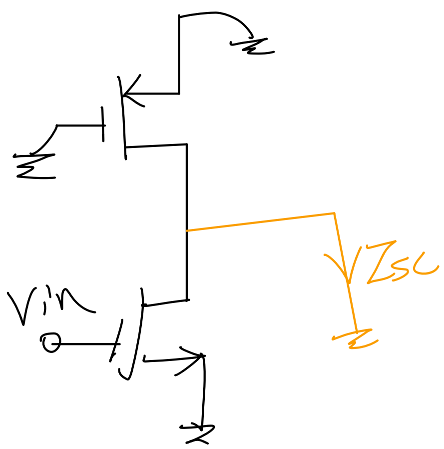


$$I_x = \left(\frac{1}{r_{o1}}+\frac{1}{r_{o2}}\right)$$

$$R_{out}=\frac{V_x}{I_x}=\frac{1}{\frac{1}{r_{o1}}+\frac{1}{r_{o2}}}$$

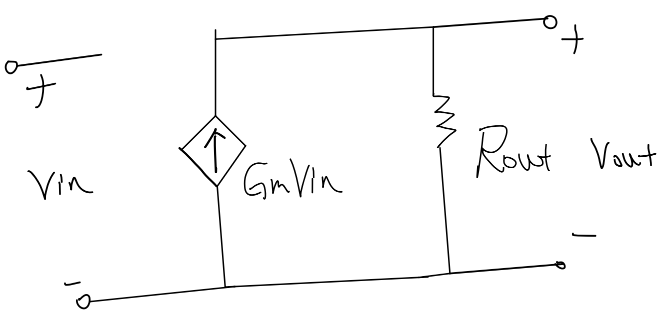
或 $R_{out}=r_{o1}//r_{o2}$

#



$$I_{sc}=-g_{m1}V_{in}$$

$$G_m=\frac{I_{sc}}{V_{in}}=-g_{m1}$$

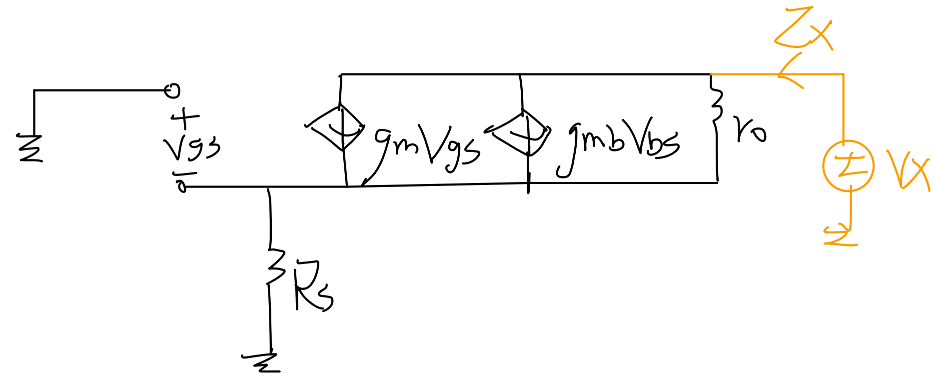


$A_v=G_mR_{out}$ #

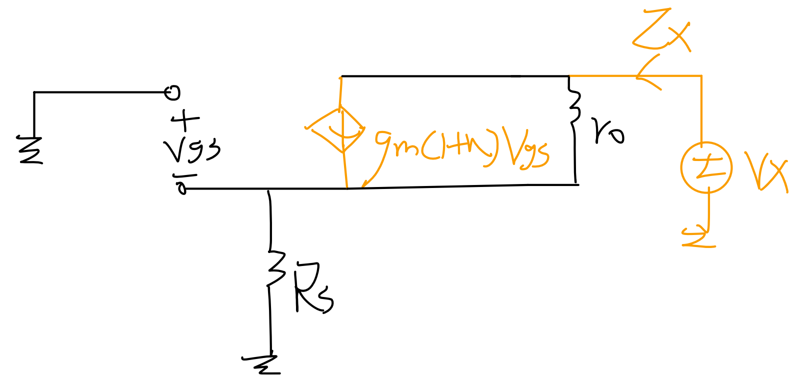
雖然有最大的 Gain

但 Vo 的直流準位難以被定義!!

HW3.2



$V_{bs} = V_{gs} \Rightarrow (g_m + g_{mb})V_{gs} = (1+\eta) g_m V_{gs}$



$V_{gs} = 0 - I_x R_s = -I_x R_s$

$V_x = I_x R_s + I_x[(1+\eta) g_m R_s + 1] r_o$

$R_{out}' = \frac{V_x}{I_x} = R_s + [(1+\eta) g_m R_s + 1] r_o$

或 $R_{out}' = r_o + [(1+\eta) g_m r_o + 1] R_s$

$R_{out} = R_D // R_{out}'$ #

HW3.3

(a)

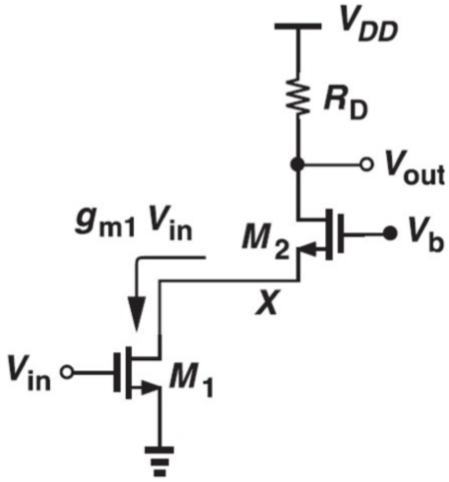


Fig. 3.3

$$V_{IN(max)} - V_{TH1} + V_{GS2} \le V_b \le V_{out(min)} + V_{TH2}$$

$$V_{IN(min)} \ge V_{TH1}$$

若 V_b 取 $V_{out(min)} + V_{TH2}$

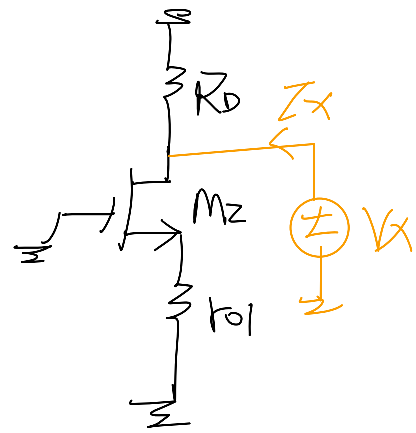
则 $V_{IN(max)} - V_{TH1} + V_{GS2} \le V_{out(min)} + V_{TH2}$

$$\Rightarrow V_{IN(max)} \le V_{out(min)} + V_{TH1} + \cancel{V_{TH2}} - (V_{ov2} + \cancel{V_{TH2}})$$

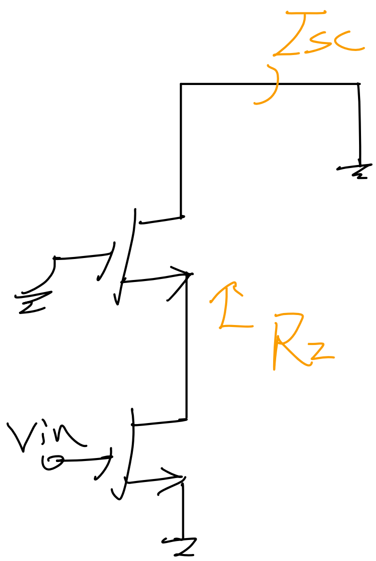
$$\Rightarrow V_{TH1} \le V_{IN} \le V_{out(min)} - V_{ov2} + V_{TH1}$$ #

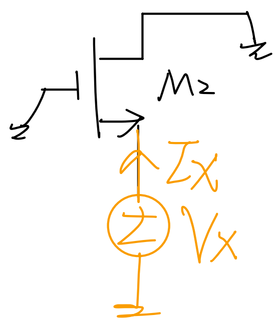
(b)

套用HW3.2的结果



$$R_{out} = R_D \,//\, \{ r_{o2} + [1+(g_{m2}+g_{m2b})r_{o2}]r_{o1} \}$$





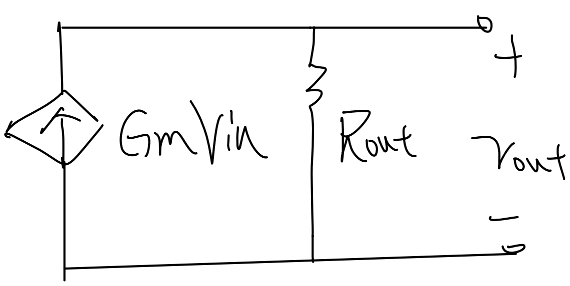
$$I_X = (g_{m2}+g_{m2b})V_X + \frac{V_X}{r_{o2}}$$

$$\Rightarrow R_2 = \frac{V_X}{I_X} = \frac{1}{g_{m2}+g_{m2b}+\frac{1}{r_{o2}}}$$

或 $$R_2 = \frac{r_{o2}}{1+(g_{m2}+g_{m2b})r_{o2}}$$

$$I_{sc} = -g_{m1}V_{in}\frac{r_{o1}}{r_{o1}+R_2} = -g_{m1}\frac{r_{o1}[1+(g_{m2}+g_{m2b})r_{o2}]}{r_{o1}+r_{o2}+(g_{m2}+g_{m2b})r_{o2}r_{o1}}v_{in}$$

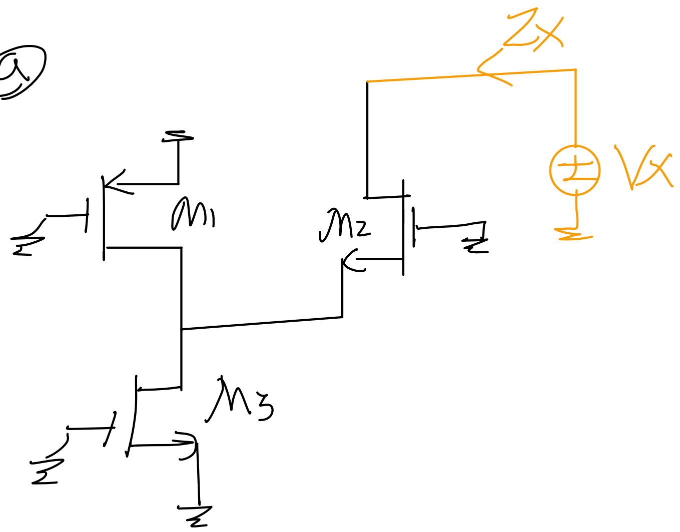
$$G_m = \frac{I_{sc}}{V_{in}} = -g_{m1}\frac{r_{o1}}{r_{o1}+R_2} = -g_{m1}\frac{r_{o1}[1+(g_{m2}+g_{m2b})r_{o2}]}{r_{o1}+r_{o2}+(g_{m2}+g_{m2b})r_{o2}r_{o1}} \overset{(r_{o1}\gg R_2)}{\simeq} -g_{m1}$$



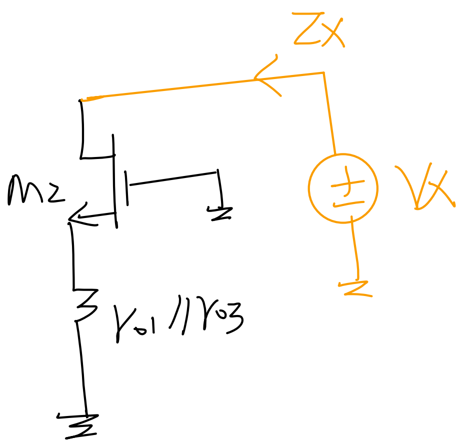
$$A_v = G_m R_{out} \quad \#$$

HW3.4

(a)



M1及M3 $g_m V_{gs} = 0$ 只剩 r_o



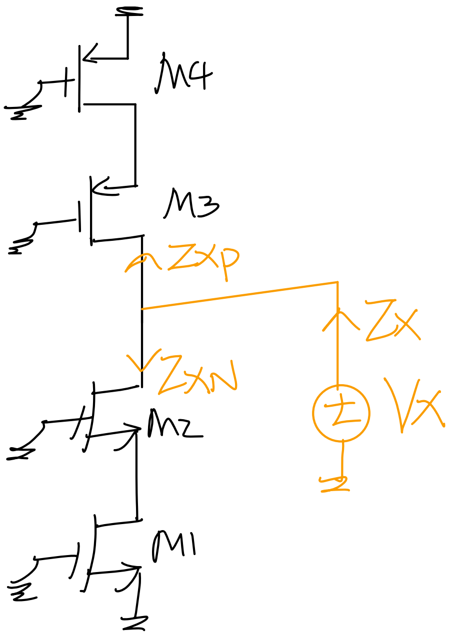
將 $r_{o1} // r_{o3}$ 當作 R_s，即可套用HW3.2的結果

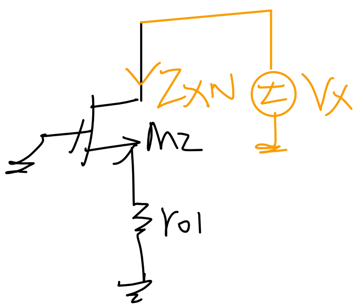
$R_{out} = r_{o2} + [1 + (g_{m2} + g_{m2b}) r_{o2}](r_{o1} // r_{o3})$ #

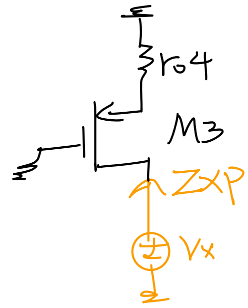
(b)

M1、M4 $g_m V_{gs} = 0$ 只剩下 r_o

又可以套用 HW3、2 的結果



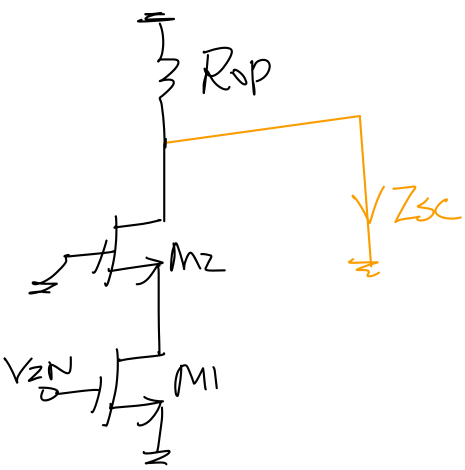




$$R_{oN} = \frac{V_x}{Z_{xN}} = r_{o2} + [1+(g_{m2}+g_{m2b})r_{o2}]\,r_{o1}$$

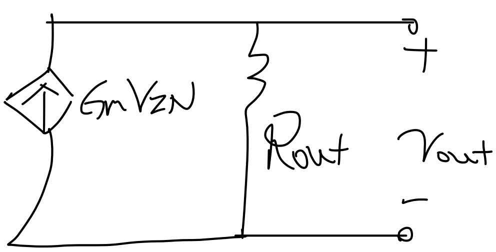
$$R_{op} = \frac{V_x}{Z_{xp}} = r_{o3} + [1+(g_{m3}+g_{m3b})r_{o3}]\,r_{o4}$$

$$R_{out} = R_{oN} \,//\, R_{op} \quad \#$$



$$Z_{sc} = -g_{m1}V_{zN} \times \frac{r_{o1}}{r_{o1} + \frac{r_{o2}}{1+(g_{m2}+g_{m2b})r_{o2}}} \simeq -g_{m1}V_{zN}$$

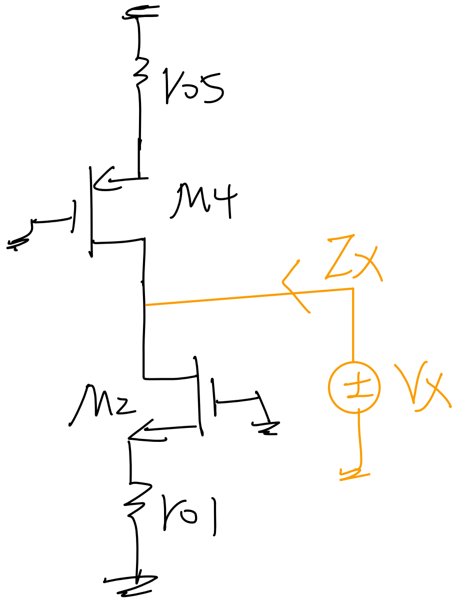
$$G_m = \frac{Z_{sc}}{V_{in}} \simeq -g_{m1}$$



$$A_v = G_m R_{out} \quad \#$$

(c)

M1 及 M5 $g_m V_{gs} = 0$ 剩 r_o

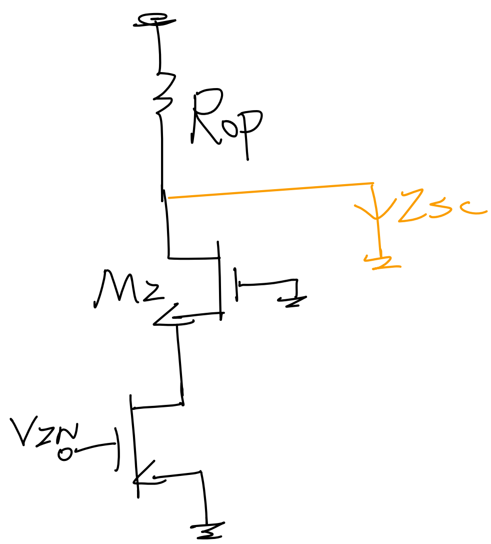


套用HW3-3 (b)

$$R_{op} = r_{o4} + [1 + (g_{m4} + g_{m4b}) r_{o4}] r_{o5}$$

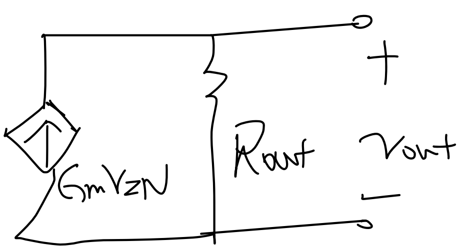
$$R_{on} = r_{o2} + [1 + (g_{m2} + g_{m2b}) r_{o2}] r_{o1}$$

$$R_{out} = R_{on} \,//\, R_{op} \quad \#$$



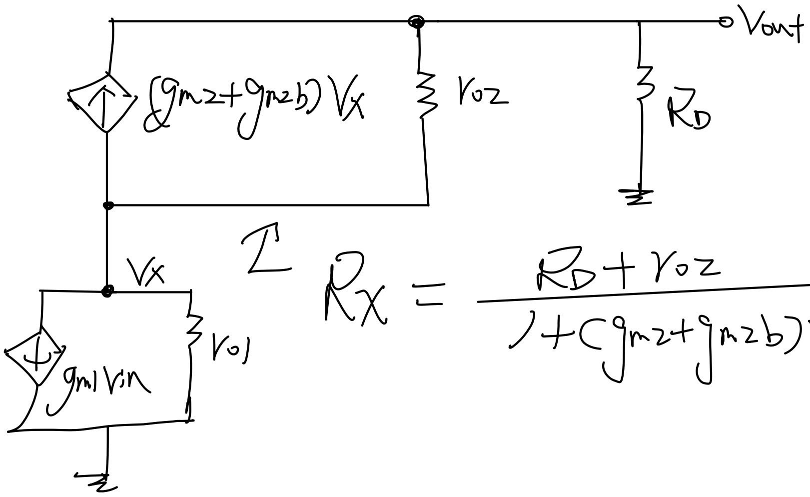
$$I_{sc} = -g_{m1} V_{IN} \times \frac{r_{o1}}{r_{o1} + \frac{r_{o2}}{1 + (g_{m2} + g_{m2b}) r_{o2}}} \simeq -g_{m1} V_{IN}$$

$$G_m = \frac{I_{sc}}{V_{in}} \simeq -g_{m1}$$



$$A_v = G_m R_{out}$$

<補充> HW4.2



$$R_X = \frac{R_D + r_{o2}}{1+(g_{m2}+g_{mb2})r_{o2}}$$

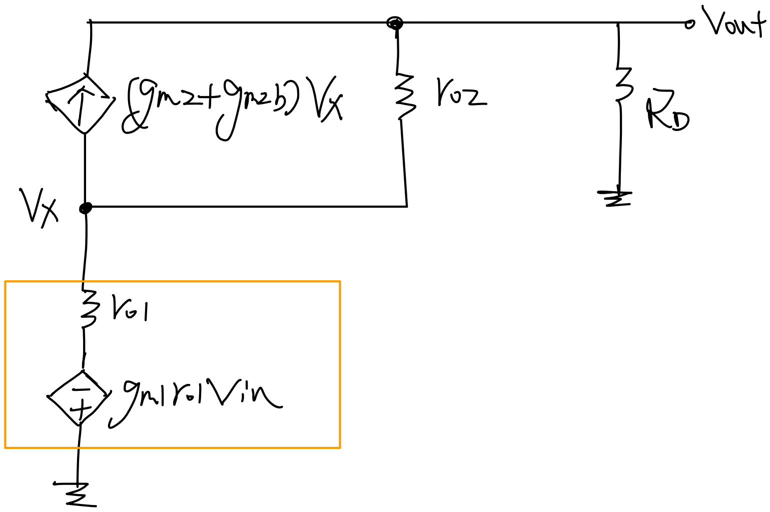
$$\frac{v_o}{v_{in}} = \frac{v_o}{v_x} \times \frac{v_x}{v_{in}}$$

$$\frac{v_x}{v_{in}} = -g_{m1}r_{o1}$$ (X)

這是錯的!!

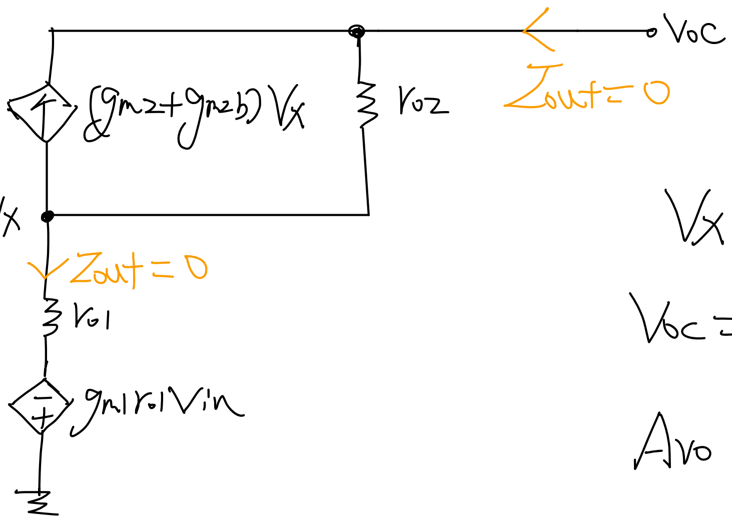
要考慮r_{o1}與R_X的分流!!

這是HW3.3 有多位同學犯的錯誤，下次不要再寫錯了！



可将M1换作等效电压源，不就变回单纯的CG放大器

利用电压放大器模型做等效!!

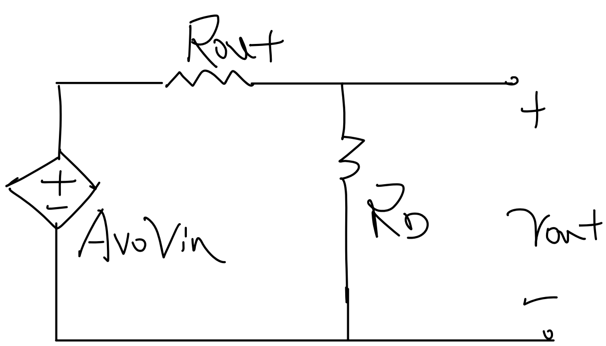


$$V_x = -g_{m1} r_{o1} V_{in}$$

$$V_{oc} = -g_{m1} r_{o1} V_{in} - (g_{m2}+g_{m2b}) r_{o2} V_x$$

$$A_{vo} = \frac{V_{oc}}{v_{in}} = -g_{m1} r_{o1} [(g_{m2}+g_{m2b}) r_{o2} + 1]$$

$$R_{out} = r_{o1} + r_{o2} + (g_{m2}+g_{m2b}) r_{o2} r_{o1}$$



$$\frac{v_{out}}{v_{in}} = A_{vo} \frac{R_D}{R_{out}+R_D} \quad \#$$

補充〈轉導法〉

Lemma In a linear circuit, the voltage gain is equal to $-G_m R_{out}$, where G_m denotes the transconductance of the circuit when the output is shorted to ground [Fig. 3.32(b)] and R_{out} represents the output resistance of the circuit when the input voltage is set to zero [Fig. 3.32(c)].

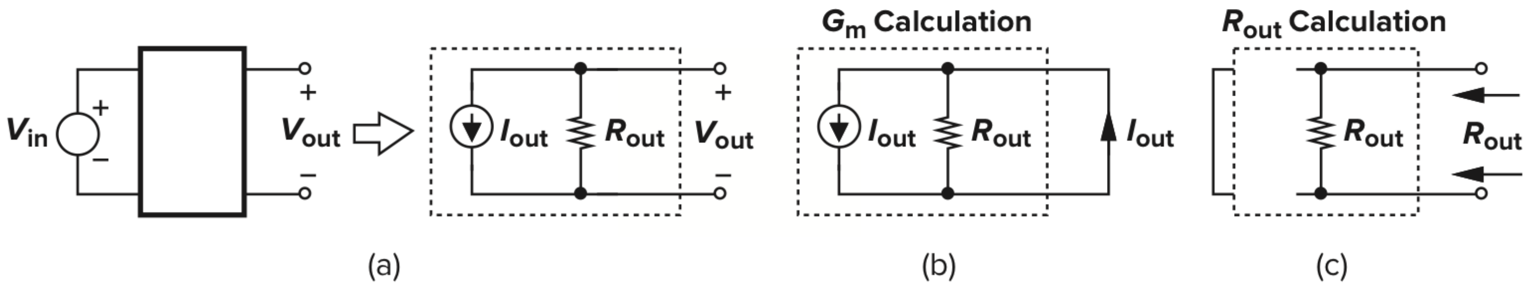


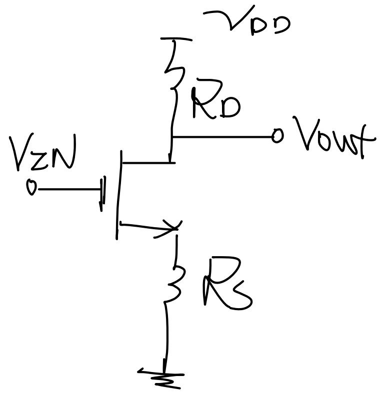
Figure 3.32 (a) Norton equivalent of a linear circuit; (b) G_m calculation; and (c) R_{out} calculation.

The lemma can be proved by noting that the output voltage in Fig. 3.32(a) is equal to $-I_{out}R_{out}$, and I_{out} can be obtained by measuring the short-circuit current at the output. Defining $G_m = I_{out}/V_{in}$, we have $V_{out} = -G_m V_{in} R_{out}$. This lemma proves useful if G_m and R_{out} can be determined by inspection. Note the direction of I_{out}.

使用轉導法可以將較為複雜的電路，簡化的比較容易分析，是一種還不錯的電路分析方式，至少我自己是蠻喜歡用的，但也不一定要使用轉導放大器模型，其他四種放大器的模型也可以拿來做等效哦。
同學們可以自己練習看看，如何使用轉導法來做分析！！

CS :

$$G_m = \frac{g_m r_o}{R_S + [1 + (g_m + g_{mb}) R_S] r_o}$$

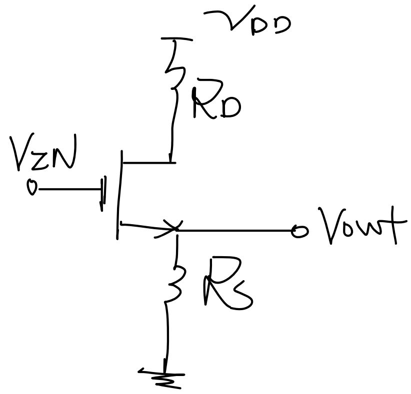


$$R_{out} = R_S + [1 + (g_m + g_{mb}) R_S] r_o$$

$$A_v = -G_m (R_{out} // R_D)$$

CD :

$$G_m = g_m \frac{r_o}{R_D + r_o}$$

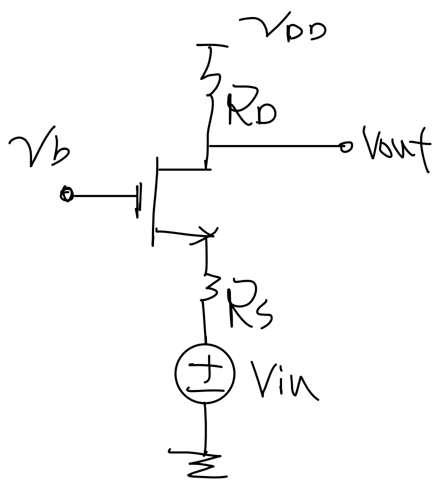


$$R_{out} = \frac{R_D + r_o}{1 + (g_m + g_{mb}) r_o}$$

$$A_v = G_m (R_{out} // R_S)$$

CG :

$$G_m = \frac{1 + (g_m + g_{mb}) r_o}{R_S + [1 + (g_m + g_{mb}) R_S] r_o}$$



$$R_{out} = R_S + [1 + (g_m + g_{mb}) R_S] r_o$$

$$A_v = G_m (R_D // R_{out})$$