1. $[-2,6]$

$$\sum_{n=1}^{\infty}(-1)^n\cdot\frac{(x-2)^n}{4^n\cdot\sqrt{n}}$$

Root test:

$$\lim_{n\to\infty}\sqrt[n]{\left|\frac{(x-2)^n}{4^n\cdot\sqrt{n}}\right|}=\lim_{n\to\infty}\frac{|x-2|}{4\cdot 1}=\frac{|x-2|}{4}$$

⇒ The series $\boxed{\text{Converges Absolutely}}$ on the interval of:

$$\frac{|x-2|}{4}<1 \Rightarrow -4<x-2<4 \Rightarrow -2<x<6 \Rightarrow x:\boxed{(-2,6)}$$

when $x=-2$:

$$\sum_{n=1}^{\infty}(-1)^n\cdot\frac{(-4)^n}{4^n\cdot\sqrt{n}}=\sum_{n=1}^{\infty}\frac{\cancel{4^n}}{\cancel{4^n}\sqrt{n}} \Rightarrow \text{p-series, } p\le 1 \Rightarrow \text{divergence}$$

when $\boxed{x=6}$:

$$\sum_{n=1}^{\infty}(-1)^n\cdot\frac{\cancel{4^n}}{\cancel{4^n}\cdot\sqrt{n}}=\sum_{n=1}^{\infty}(-1)^n\cdot\boxed{\frac{1}{\sqrt{n}}}$$

① always positive
② eventually decreasing
③ approach to 0

⇒ $\boxed{\text{converges conditionally}}$

⇒ converge interval: $\boxed{(-2,6]}$ #1

2. $0+x+x^2$

$$f(x)=e^x\sin x$$

$$\sum_{n=0}^{\infty}a_n\cdot(x)^n=f(x)=a_0+a_1x+a_2x^2+\dots$$

$$f^n(x)=n!\cdot a_n+a_{n+1}x^{n+1}+\dots$$

$$f^n(0)=n!\cdot a_n \Rightarrow a_n=\frac{f^n(0)}{n!}$$

$$f(0)=0,\quad f'(0)=e^x\sin x+e^x\cos x\Big|_{x=0}=1$$

$$f''(0)=\cancel{e^x\sin x}+e^x\cos x+e^x\cos x+\cancel{e^x(-\sin x)}\Big|_{x=0}=2$$

⇒ The first three terms:

$$\frac{f(0)}{1}+\frac{f'(0)}{1}\cdot x+\frac{f''(0)}{2!}\cdot x^2=\boxed{0+x+x^2}$$ #2

3. $\sum_{n=0}^{\infty}(-1)^n\cdot\frac{e^2}{n!}(x+2)^n$

$$\sum_{n=0}^{\infty}a_n(x+2)^n=e^{-x}=f(x)$$

$$a_n=\frac{f^n(-2)}{n!}$$

$$f(-2)=e^2$$

$$f'(-2)=-e^{-x}\Big|_{-2}=-e^2$$

$$f''(-2)=e^{-x}\Big|_{-2}=e^2$$

~~f^{2n}~~

$$f^{(2n)}(-2)=e^2$$

$$f^{(2n+1)}(-2)=-e^2$$

$$\Rightarrow f^n(-2)=(-1)^n\cdot e^2$$

Taylor Series:

$$\Rightarrow\boxed{\sum_{n=0}^{\infty}(-1)^n\cdot\frac{e^2}{n!}\cdot(x+2)^n}$$ #3

4. $y=\pm\sqrt{x}$

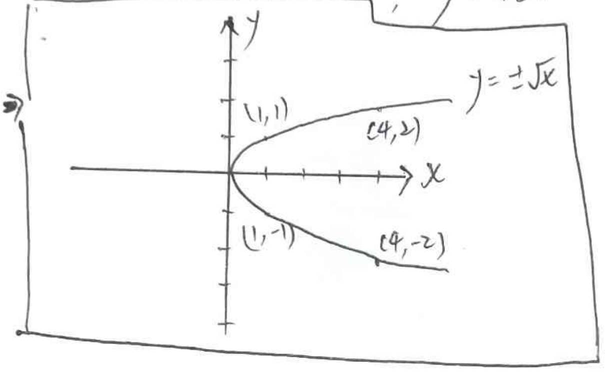
$x=(\sec^2 t)-1$; $y=\tan t$, $-\frac{\pi}{2}<t<\frac{\pi}{2}$

$x=|\tan^2 t|$

$\boxed{y=}\ \tan t = \pm\sqrt{\tan^2 t} = \boxed{\pm\sqrt{x}}$ #4

when $t=-\frac{\pi}{2}$, $x=+\infty$, ~~$y=+\infty$~~ ($y=-\infty$

when $t=\frac{\pi}{2}$, $x=+\infty$, $y=+\infty$



#4

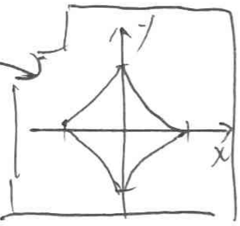
$x+y=1$ $y=1-x$

5. ~~#4~~ 6

$x^{\frac{2}{3}}+y^{\frac{2}{3}}=1$ ~~$\Rightarrow y^{\frac{2}{3}}=-x^{\frac{2}{3}}+1 \Rightarrow y=\pm(-x^{\frac{2}{3}}+1)^{\frac{3}{2}}$~~

~~let t=x~~

$\Rightarrow (x^{\frac{1}{3}})^2+(y^{\frac{1}{3}})^2=1 \Rightarrow$ if $x=0,\ y=\pm1$
$x=\pm1,\ y=0$



$S=\int ds=\int\sqrt{dx^2+dy^2}=\int\sqrt{1+(\frac{dx}{dy})^2}\,dx$

$\Rightarrow \frac{1}{4}S=\int_0^1\sqrt{1+(\frac{dy}{dx})^2}\,dx$

∵ 第一象限 ~~[illegible]~~ ∴ $y=(1-x^{\frac{2}{3}})^{\frac{3}{2}}$

$\frac{dy}{dx}=\frac{3}{2}(1-x^{\frac{2}{3}})^{\frac{1}{2}}\cdot(-\frac{2}{3}x^{-\frac{1}{3}})=-x^{-\frac{1}{3}}(1-x^{\frac{2}{3}})^{\frac{1}{2}}$

$\sqrt{1+(\frac{dy}{dx})^2}=\sqrt{x^{-\frac{2}{3}}(1-x^{\frac{2}{3}})+1}=\sqrt{x^{-\frac{2}{3}}-1+1}=$ ~~$x^{-\frac{1}{3}}\cdot\sqrt{1-x^{\frac{2}{3}}}$~~ $x^{-\frac{1}{3}}$

$\Rightarrow \frac{1}{4}S\cdot4=$ ~~$4\cdot\int_0^1\sqrt{x^{\frac{2}{3}}+1}\,dx \quad u=x^{\frac{2}{3}}+1$~~ $=4\cdot\int_0^1 x^{-\frac{1}{3}}\cdot$ ~~$\sqrt{1-x^{\frac{2}{3}}}\,dx$~~ dx

~~$u=1-x^{\frac{2}{3}}$~~
~~$du=-\frac{2}{3}x^{-\frac{1}{3}}dx$~~ ~~$4\cdot\int_1^{0}-\frac{3}{2}u^{\frac{1}{2}}du=-6\cdot\frac{2}{3}[u^{\frac{3}{2}}]_1^0=\boxed{4}$~~ #5 $\frac{3}{2}\cdot4[x^{\frac{2}{3}}]_0^1=\boxed{6}$ #5

6. $y=2x-4$

$r=\frac{4}{2\cos\theta-\sin\theta} \Rightarrow 2r\cos\theta-r\sin\theta=4 \Rightarrow 2x-y=4 \Rightarrow \boxed{y=2x-4}$ #6

7. $r = \cancel{4} = 1 - \sin\theta$

$x = \cos\theta - \sin\theta\cos\theta$

If (r, θ) on the graph :

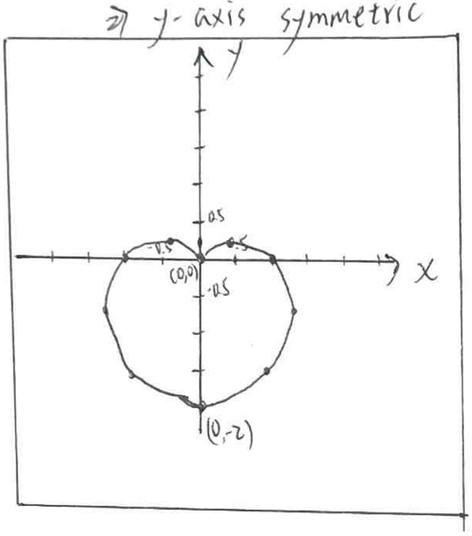
$(\cancel{r}, -\theta)$ not on the graph

⇒ no x-axis symmetric

$(r, \pi - \theta)$ on the graph:

⇒ y-axis symmetric

θ	r	~~$x = r\cos\theta$~~	~~$y = r\sin\theta = \sin\theta - \sin^2\theta$~~
$-\frac{\pi}{2}$	2	~~0~~	~~-2~~
$-\frac{\pi}{3}$	$1 + \frac{\sqrt{3}}{2}$ ↗0.866	+0.98	
$-\frac{\pi}{4}$	$1 + \frac{\sqrt{2}}{2}$ ↗0.707		
$-\frac{\pi}{6}$	$\frac{3}{2}$		
0	1		
$\frac{\pi}{6}$	$\frac{1}{2}$		
$\frac{\pi}{4}$	$1 - \frac{\sqrt{2}}{2}$		
$-\frac{\pi}{3}$	$1 - \frac{\sqrt{3}}{2}$		
$\frac{\pi}{2}$	0		



8. $\frac{1}{2}\pi$ ok

$\cancel{x}$ $r = 1 + \cos\theta$

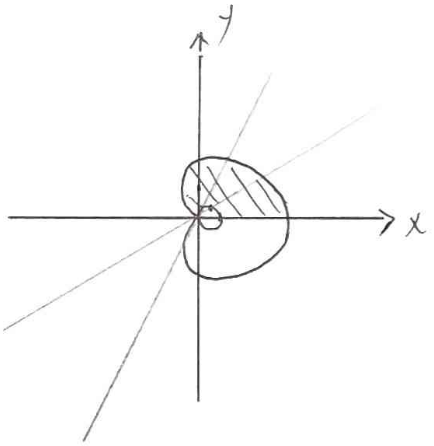
If (r, θ) on the graph:

$(r, -\theta)$ on the graph

⇒ x-axis symmetric

$(r, \pi - \theta)$ not on the graph

⇒ no y-axis symmetric



$r = \cos\theta$

~~$r^2 = r\cos\theta \Rightarrow x^2 + y^2 = x$~~

If (r, θ) on the graph

$(r, -\theta)$ on the graph

⇒ x-axis symmetric

$(r, \pi - \theta)$ not on the graph

⇒ y-axis not symmetric

θ	r
0	1
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$
$\frac{\pi}{2}$	0
$\frac{2}{3}\pi$	$-\frac{1}{2}$
$\frac{3}{4}\pi$	$-\frac{\sqrt{2}}{2}$
$\frac{5}{6}\pi$	$-\frac{\sqrt{3}}{2}$
π	-1

$$\text{Area} = \left[\frac{1}{2}\int_0^{\pi}(1+\cos\theta)^2\,d\theta\right] - \left(\left(\frac{1}{2}\right)^2\pi\right)\frac{1}{2}$$

$$= \left[\frac{1}{2}\int_0^{\pi} 1 + 2\cos\theta + \overset{\frac{1+\cos 2\theta}{2}}{\cancel{\cos^2\theta}}\,d\theta\right] - \left(\frac{1}{2}\right)^2\pi = \frac{1}{2}\left[\theta + 2\sin\theta + \frac{1}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right)\right]_0^{\pi} - \left(\overset{\frac{1}{8}\pi}{\cancel{\frac{1}{4}}}\right)\pi$$

$$= \frac{1}{2}\left[\overset{\frac{3}{2}\pi}{\cancel{\left(\pi + 0 + \frac{1}{2}(\pi + 0)\right)}} - \overset{0}{\cancel{\left(0 + 0 + \frac{1}{2}(0 + 0)\right)}}\right] - \overset{\frac{1}{8}\pi}{\cancel{\frac{1}{4}\pi}} = \boxed{\cancel{\frac{1}{2}\pi}}\ \frac{5}{8}\pi$$

#8

9. $\boxed{\begin{array}{l}\text{Domain: } x^2+y^2 < 16 \\ \text{Range: } [\cancel{0}\,\tfrac{1}{4}, +\infty)\end{array}}$ #9. $\because \underbrace{\sqrt{16-x^2-y^2}}_{>0}$

10. No.

$$f(x,y) = \frac{xy+y^3}{x^2+y^2}$$

$$\lim_{\substack{(x,y)\to(0,0)\\ \text{along } x=0}} f(x,y) = \cancel{\lim_{(x,y)\to(0,0)}} = \lim_{y\to 0}\frac{y^3}{y^2} = \lim_{y\to 0}\frac{6y}{2} = 0$$

$$\lim_{\substack{(x,y)\to(0,0)\\ \text{along } x=y}} f(x,y) = \cancel{\lim_{x\to 0} x^2 +} = \lim_{y\to 0}\frac{y^2+y^3}{2y^2} = \lim_{y\to 0}\frac{2+6y}{4} = \frac{1}{2}$$

$$\because \lim_{\substack{(x,y)\to(0,0)\\ \text{along } x=0}} f(x,y) = 0 \neq \lim_{\substack{(x,y)\to(0,0)\\ \text{along } x=y}} f(x,y) = \frac{1}{2}$$

$$\therefore \boxed{\lim_{(x,y)\to(0,0)} f(x,y) \text{ does not exist}}$$

#10