TAIWAN TECH National Taiwan University of Science and Technology

Introduction to Analog Integrated Circuit Design

Fall 2023

Differential Amplifiers

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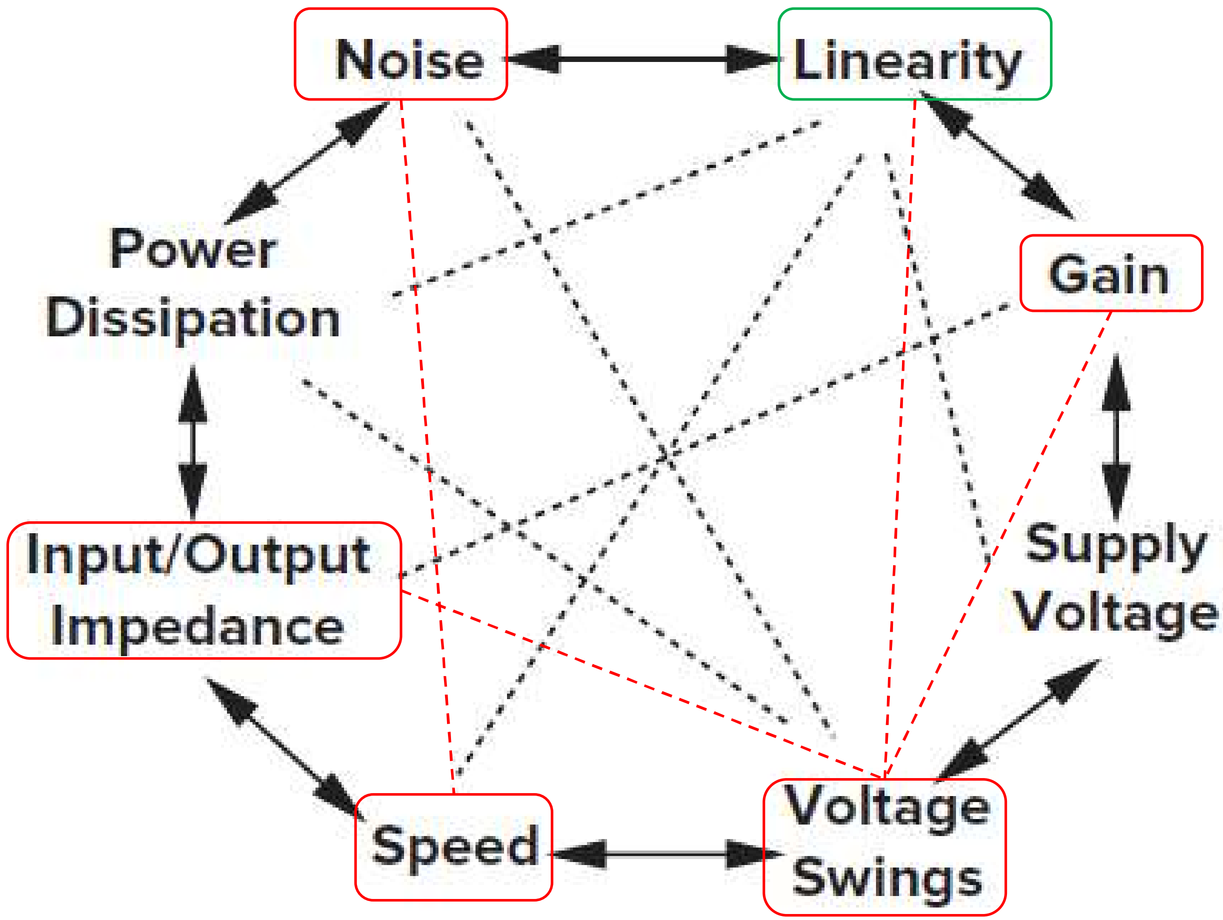
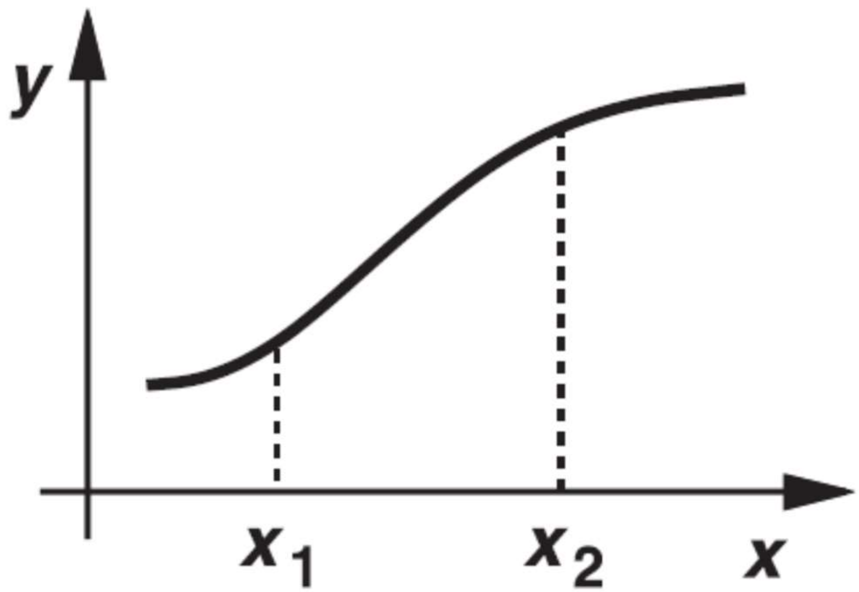
類比設計八邊形


Noise
Linearity
Power Dissipation
Gain
Input/Output Impedance
Supply Voltage
Speed
Voltage Swings

放大器之輸入－輸出特性



一個放大器之輸入－輸出特性通常為一非線性函數

$$y(t) \approx \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \cdots + \alpha_n x^n(t) \quad x_1 \le x \le x_2$$

x 的範圍夠小時 $y(t) \approx \alpha_0 + \alpha_1 x(t)$

Chapter 4
Differential Amplifiers

Differential output, $y = y_p - y_n$

Differential input, $x = x_p - x_n$

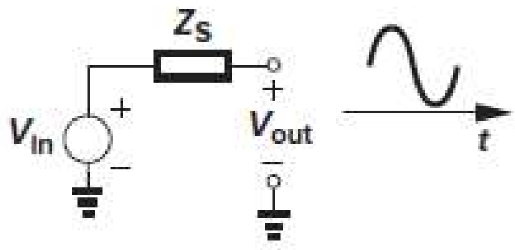
$y_p = a_0 + a_1(x) + a_2(x)^2 + a_3(x)^3 + a_4(x)^4 + a_5(x)^5$ +H.O.T.

$y_n = a_0 + a_1(-x) + a_2(-x)^2 + a_3(-x)^3 + a_4(-x)^4 + a_5(-x)^5$ +H.O.T.

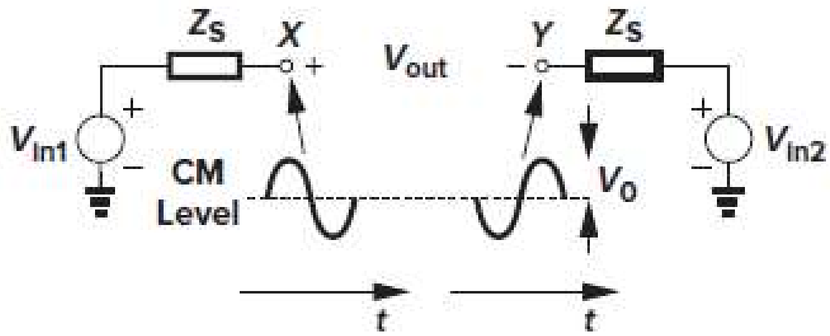
$y = y_p - y_n$ $=2a_1x + 2a_3x^3 + 2a_5x^5$ +H.O.T.(odd terms)

H.O.T. = High Order Terms

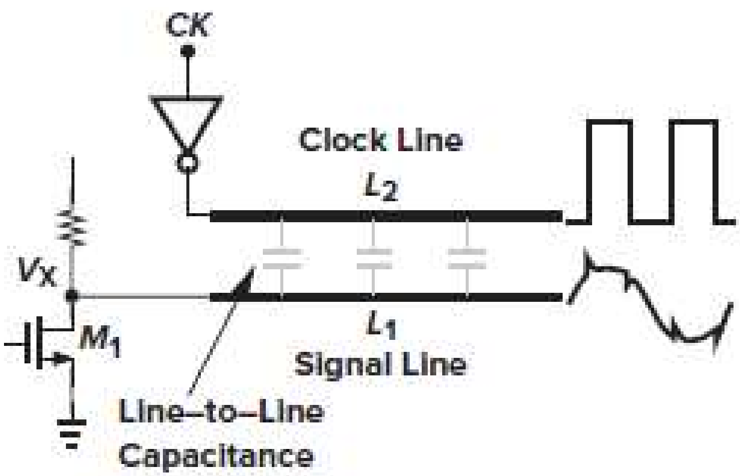
Single-Ended or Differential



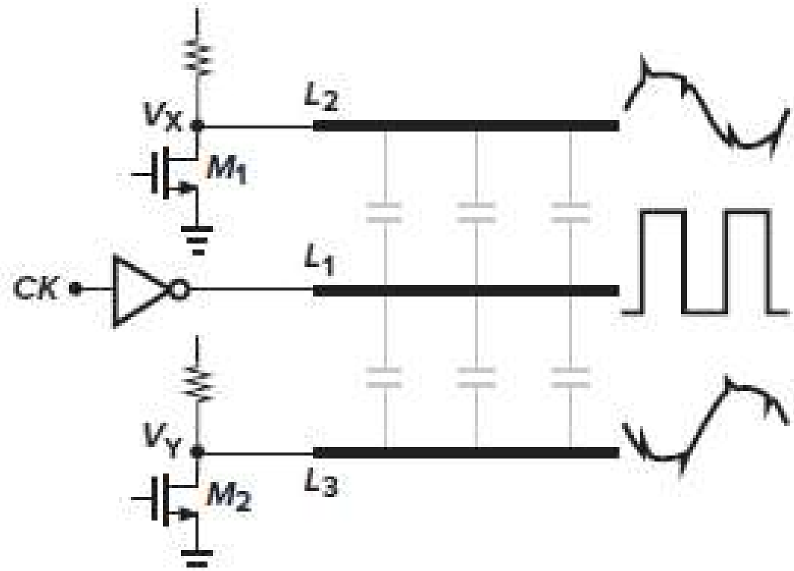
(a)



(b)

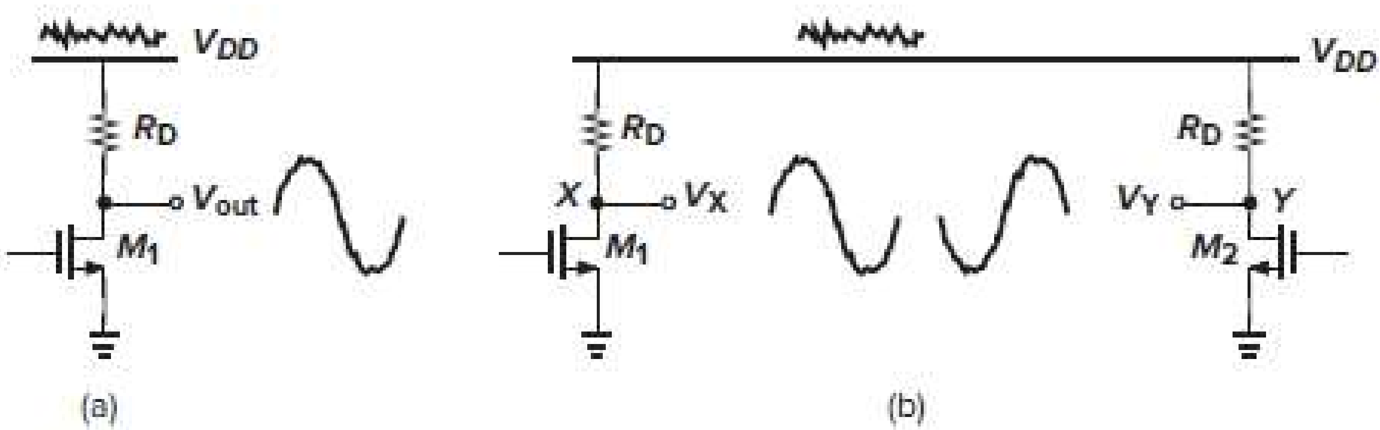


(a)

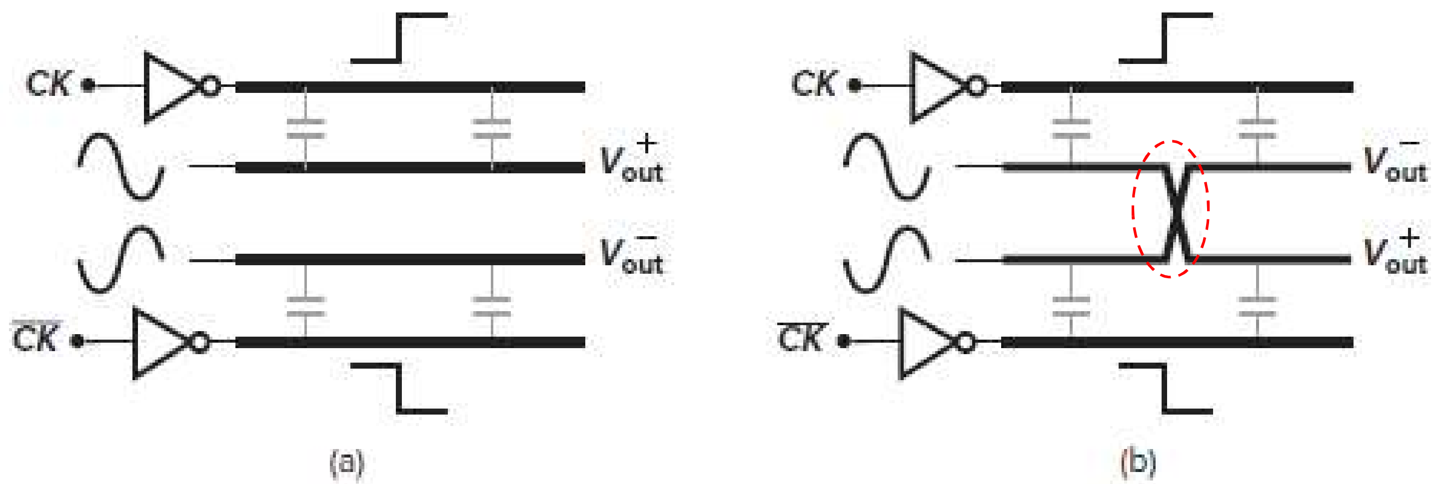


(b)

Effect of Coupling Noise



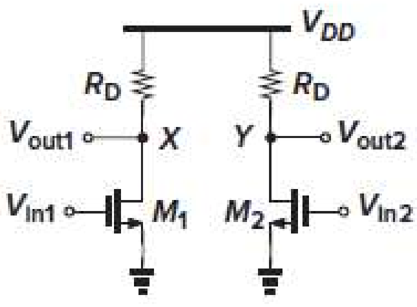
Supply gain: $A_{VD} = \Delta V_{out} / \Delta V_{DD}$



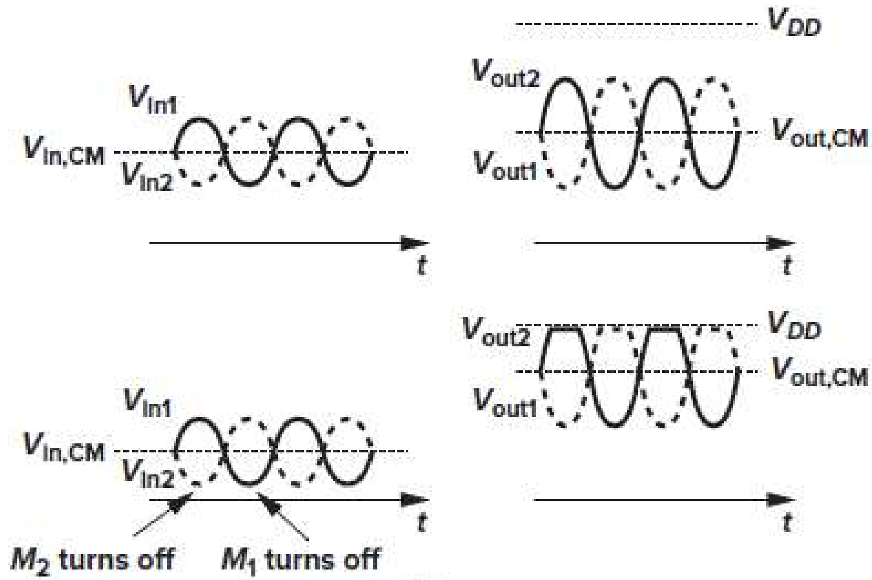
A layout trick to reduce the coupling

Differential Circuits

Pseudo-differential



(a)

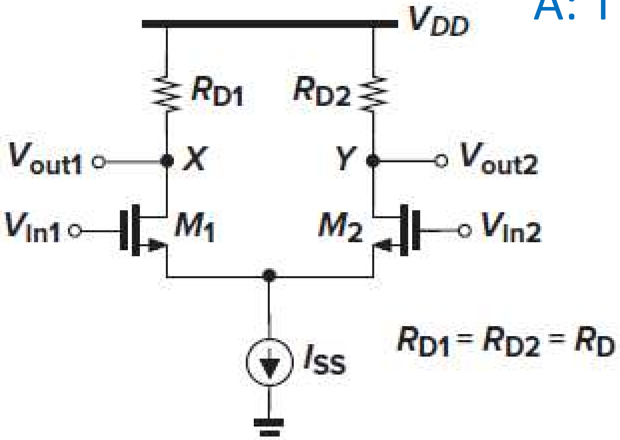


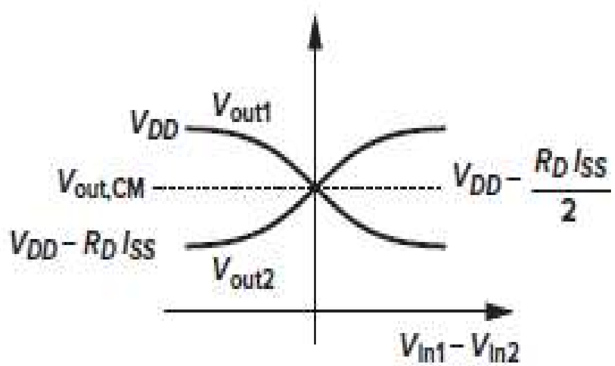
(b)

Differential pair

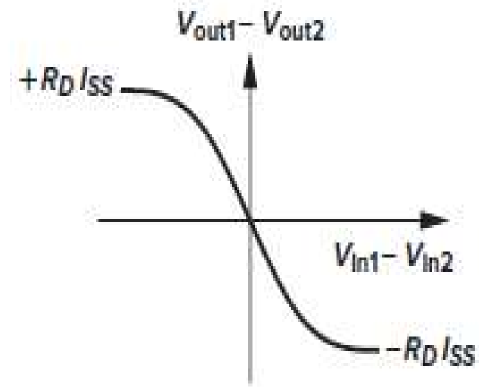
Q: What's the difference between these two cases?

A: Their DC current are different!!



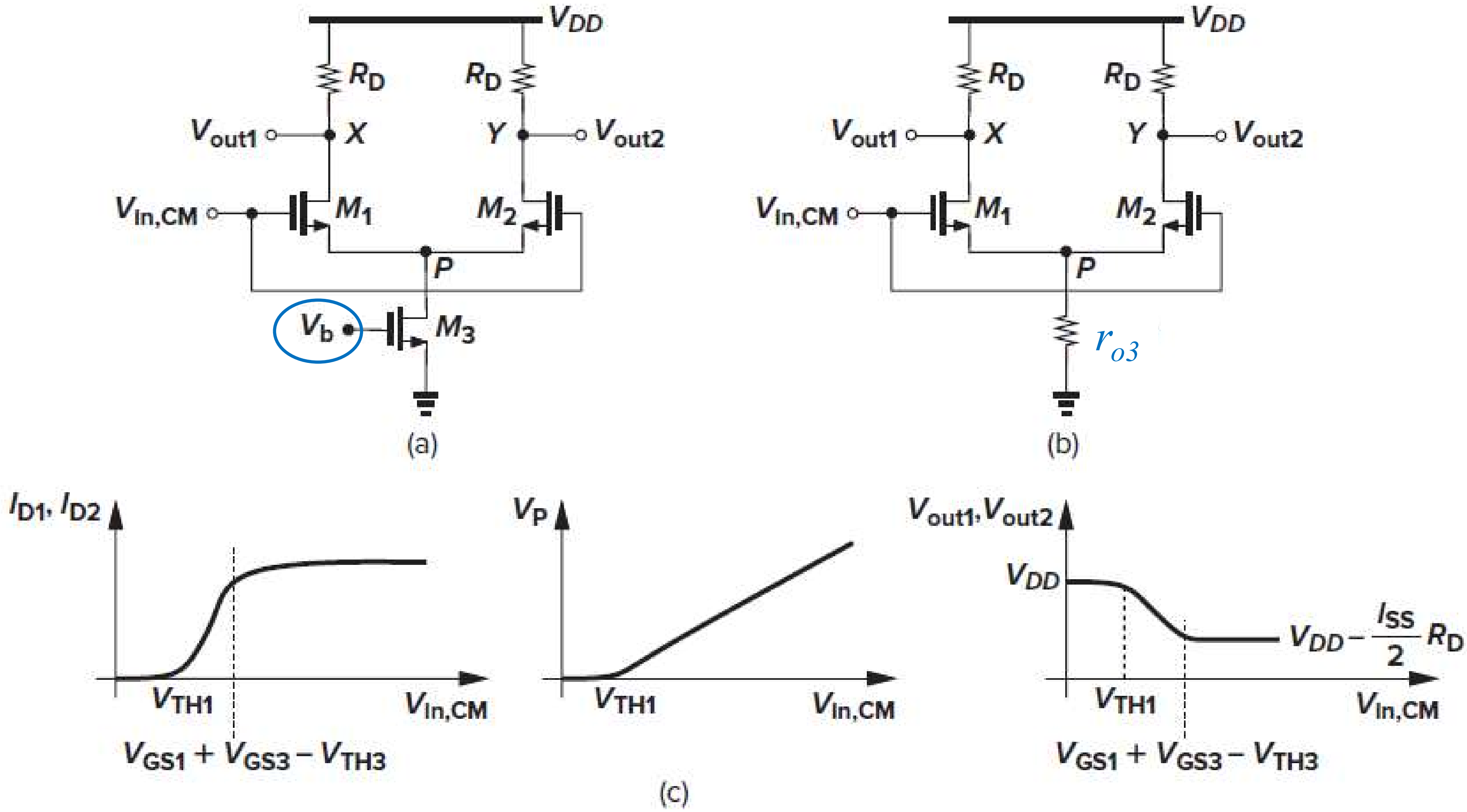


(a)



(b)

Differential Pair



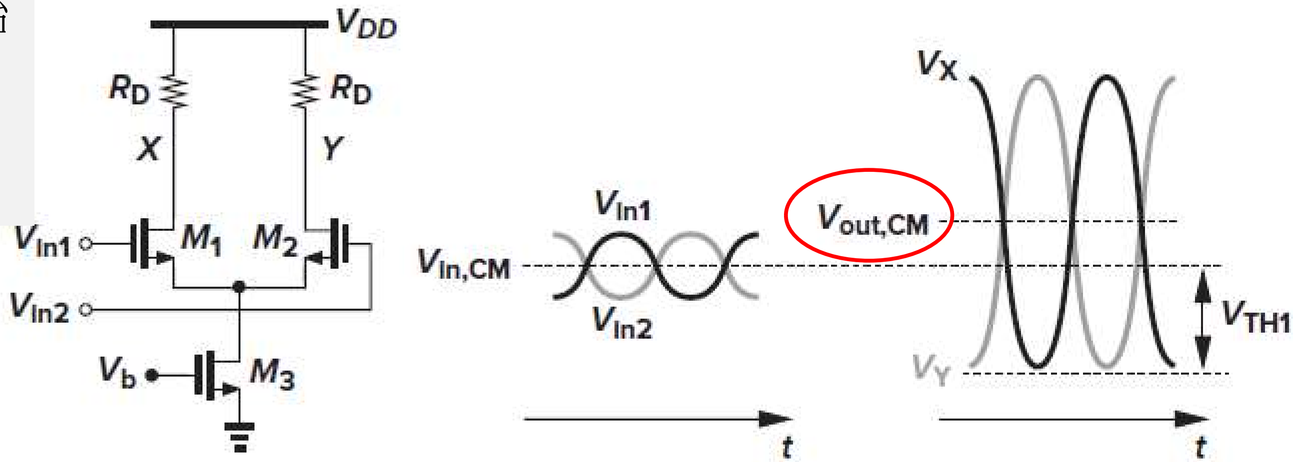
Input common-mode voltage ($V_{in,CM}$):

$$V_{GS1} + (V_{GS3} - V_{TH3}) \le V_{in,CM} \le \min\left[V_{DD} - R_D\frac{I_{SS}}{2} + V_{TH},\ V_{DD}\right]$$

先確認工作區!!

Differential Pair

從大訊號開始
→ 電流公式
→ g_m
→ g_m近似式



$$I_{D1} - I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1} - V_{in2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (V_{in1} - V_{in2})^2}$$

$$= \sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}}(V_{in1} - V_{in2})\sqrt{1 - \frac{\mu_n C_{ox}(W/L)}{4I_{SS}}(V_{in1} - V_{in2})^2}$$

Actually, it is an error term for approximation

$$(V_{in1} - V_{in2})^2 \ll 4I_{SS}/[\mu_n C_{ox}(W/L)]$$

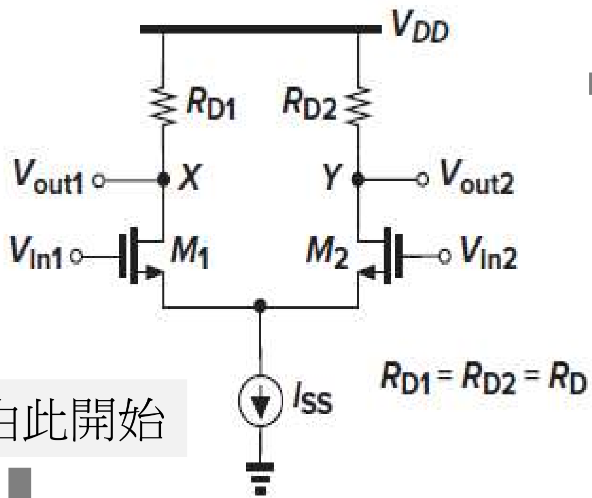
(see next page)

$$\boxed{\frac{\partial \Delta I_D}{\partial \Delta V_{in}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\frac{\frac{4I_{SS}}{\mu_n C_{ox} W/L} - 2\Delta V_{in}^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox} W/L} - \Delta V_{in}^2}}}$$

$$I_{D1} - I_{D2} = \sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}}(V_{in1} - V_{in2})$$

$$|A_v| = \sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}}R_D = g_m R_D$$

Differential Pair (Supplement)



由此開始

$$(V_{GS} - V_{TH})^2 = \frac{I_D}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}$$

$$V_{GS} = \sqrt{\frac{2I_D}{\mu_n C_{ox}\frac{W}{L}}} + V_{TH}$$

$$V_{in1} - V_{in2} = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\frac{W}{L}}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}\frac{W}{L}}}$$

$$\underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}_{K_n}(V_{in1} - V_{in2})^2 - I_{SS} = -2\sqrt{I_{D1}I_{D2}}$$

$$\left\{\begin{aligned} &4I_{D1}I_{D2} = I_{SS}{}^2 - 2I_{SS}K_n\Delta V_{in}{}^2 + K_n{}^2\Delta V_{in}{}^4,\ \Delta V_{in} = V_{in1} - V_{in2} \\ &4I_{D1}I_{D2} = (I_{D1}+I_{D2})^2 - (I_{D1}-I_{D2})^2 = I_{SS}^2 - (I_{D1}-I_{D2})^2 \end{aligned}\right.$$

$$\begin{aligned}(I_{D1} - I_{D2})^2 &= I_{SS}{}^2 - 4I_{D1}I_{D2} \\ &= -K_n{}^2\Delta V_{in}{}^4 + 2I_{SS}K_n\Delta V_{in}{}^2\end{aligned}$$

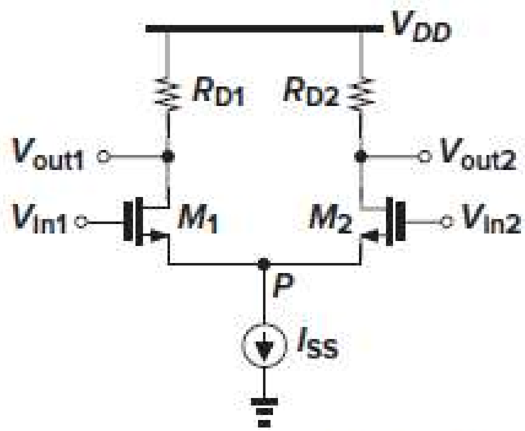
$$(I_{D1} - I_{D2})^2 = -\frac{1}{4}\left(\mu_n C_{ox}\frac{W}{L}\right)^2(V_{in1} - V_{in2})^4 + I_{SS}\mu_n C_{ox}\frac{W}{L}(V_{in1} - V_{in2})^2$$

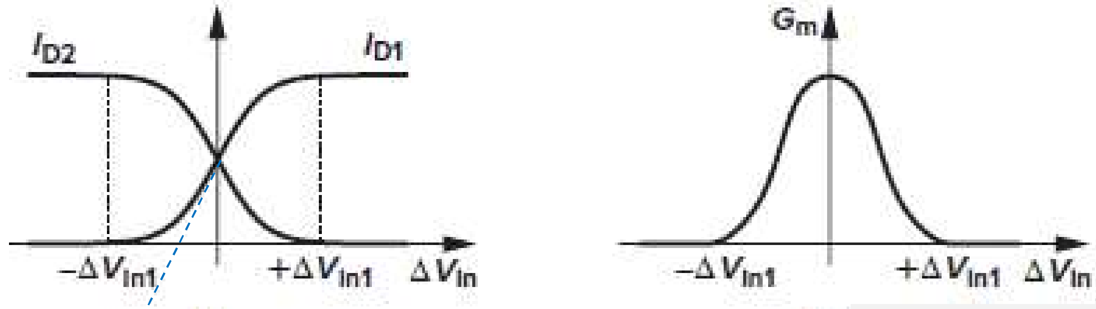
$$I_{D1} - I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1} - V_{in2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (V_{in1} - V_{in2})^2}$$

$$= \sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}}(V_{in1} - V_{in2})\sqrt{1 - \frac{\mu_n C_{ox}(W/L)}{4I_{SS}}(V_{in1} - V_{in2})^2}$$

$$\boxed{\frac{\partial \Delta I_D}{\partial \Delta V_{in}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\,\frac{\frac{4I_{SS}}{\mu_n C_{ox}W/L} - 2\Delta V_{in}^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}W/L} - \Delta V_{in}^2}}}$$

Differential Amplifier



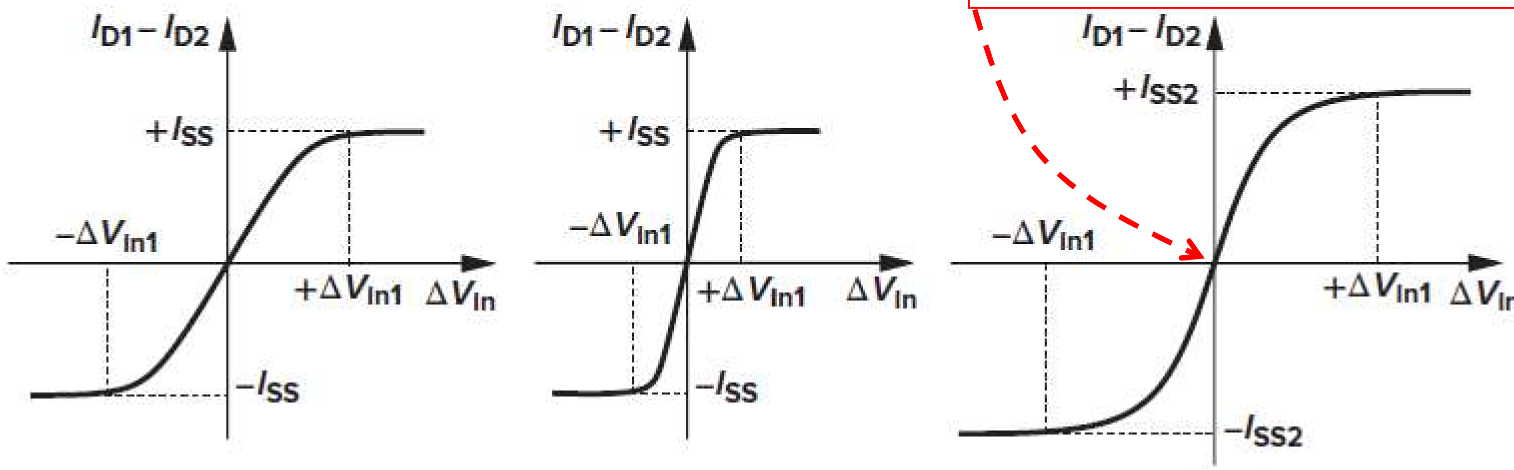


(a) (b)

HW3.1: ΔV_{in1}=?

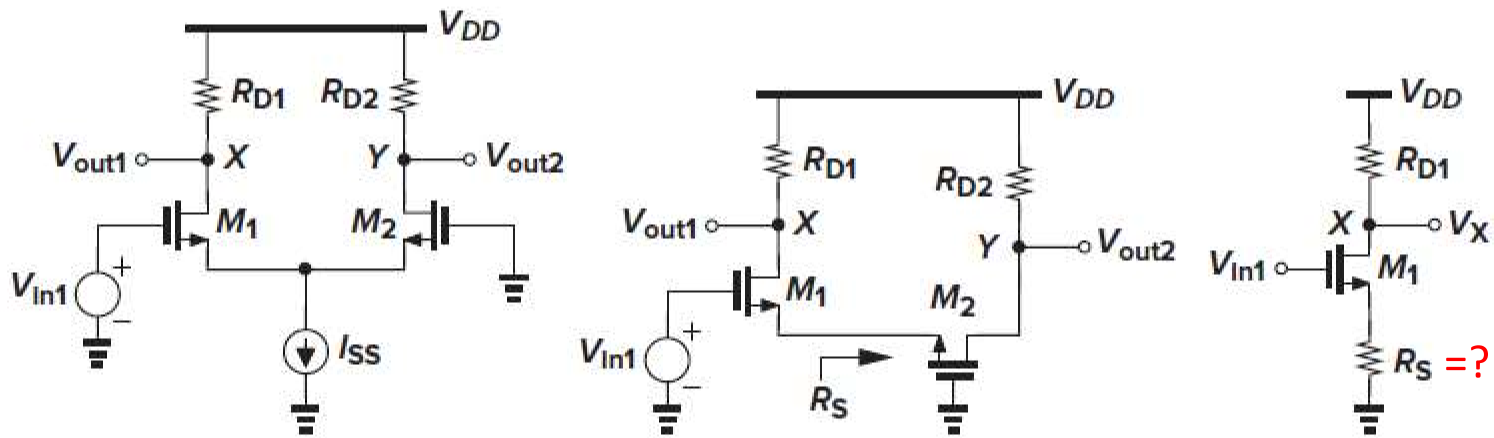
$$(V_{GS} - V_{TH})_{1,2} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

$$\boxed{I_{D1} - I_{D2} = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}} (V_{in1} - V_{in2})}$$



(a) (b) (W/L) ↑ (c) I_{SS} ↑

Small Signal Analysis



(b) (c)

Using superposition:
Find Vout by Vin1 firstly, then by Vin2.
Finally, add both results

$$\frac{V_X}{V_{in1}} = \frac{-R_D}{\dfrac{1}{g_{m1}} + \dfrac{1}{g_{m2}}}$$

$$\frac{V_Y}{V_{in1}} = \frac{R_D}{\dfrac{1}{g_{m2}} + \dfrac{1}{g_{m1}}}$$

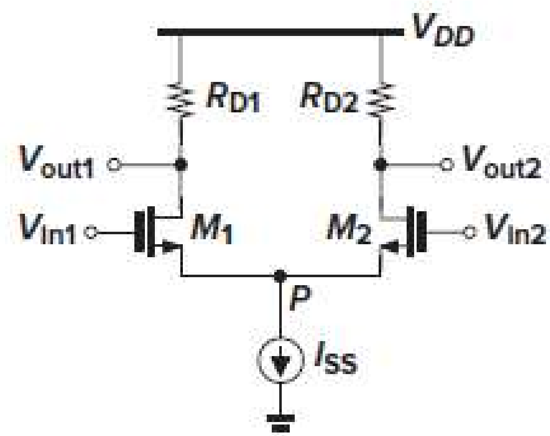
$$(V_X - V_Y)|_{\text{Due to } Vin1} = \frac{-2R_D}{\dfrac{1}{g_{m1}} + \dfrac{1}{g_{m2}}} V_{in1}$$

If M_1=M_2 and R_{D1}=R_{D2}

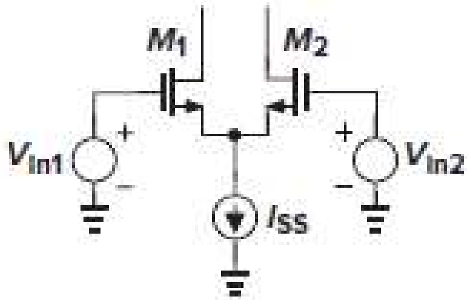
$$(V_X - V_Y)|_{\text{Due to } Vin1} = -g_m R_D V_{in1}$$

$$(V_X - V_Y)|_{\text{Due to } Vin2} = g_m R_D V_{in2}$$

$$\boxed{\frac{(V_X - V_Y)_{tot}}{V_{in1} - V_{in2}} = -g_m R_D}$$

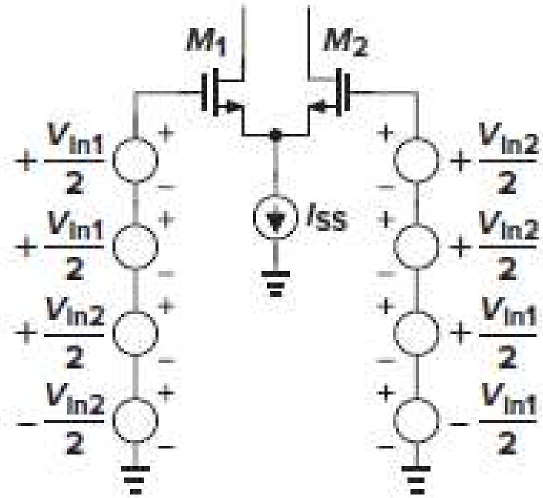


Differential Amplifier

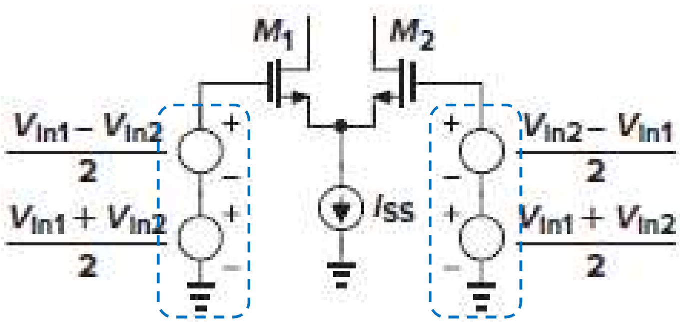


Here we introduce two kinds of input:

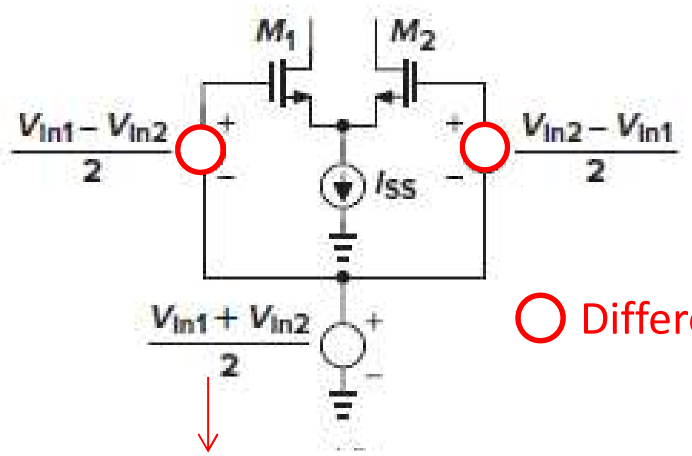
1. Common-mode input (large signal)
2. Differential input (small signal)



(b)



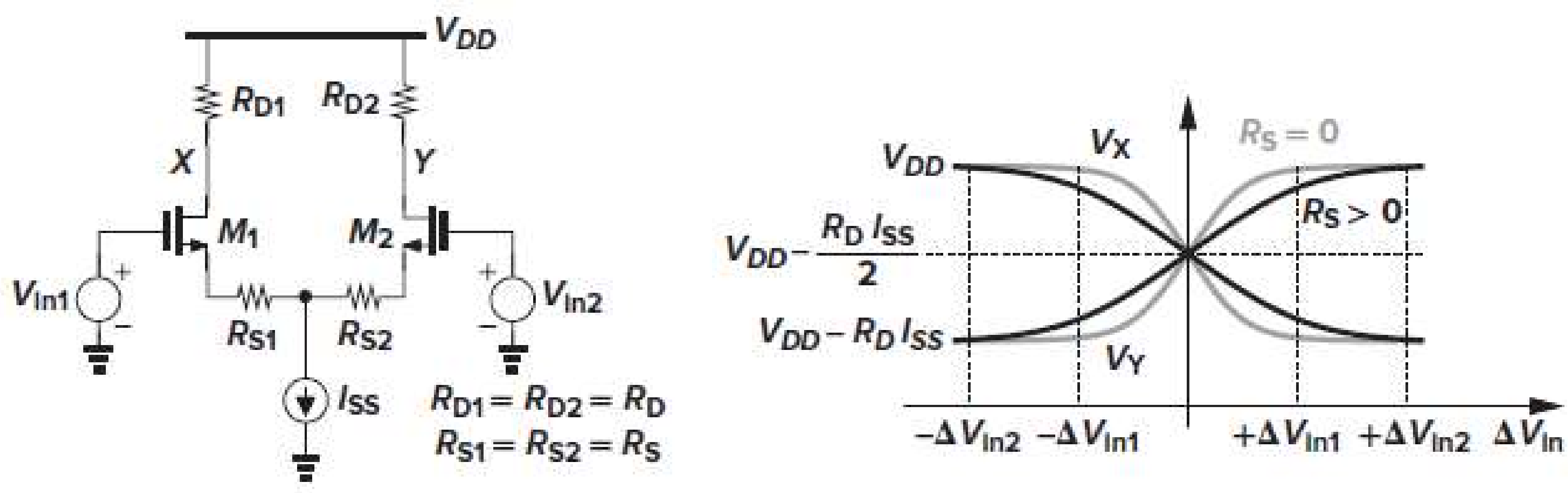
(c)



Differential input

Common-mode voltage

Degenerated Differential Pair



(a) (b)

$$|A_v| = \frac{R_D}{\frac{1}{g_m} + R_S}$$

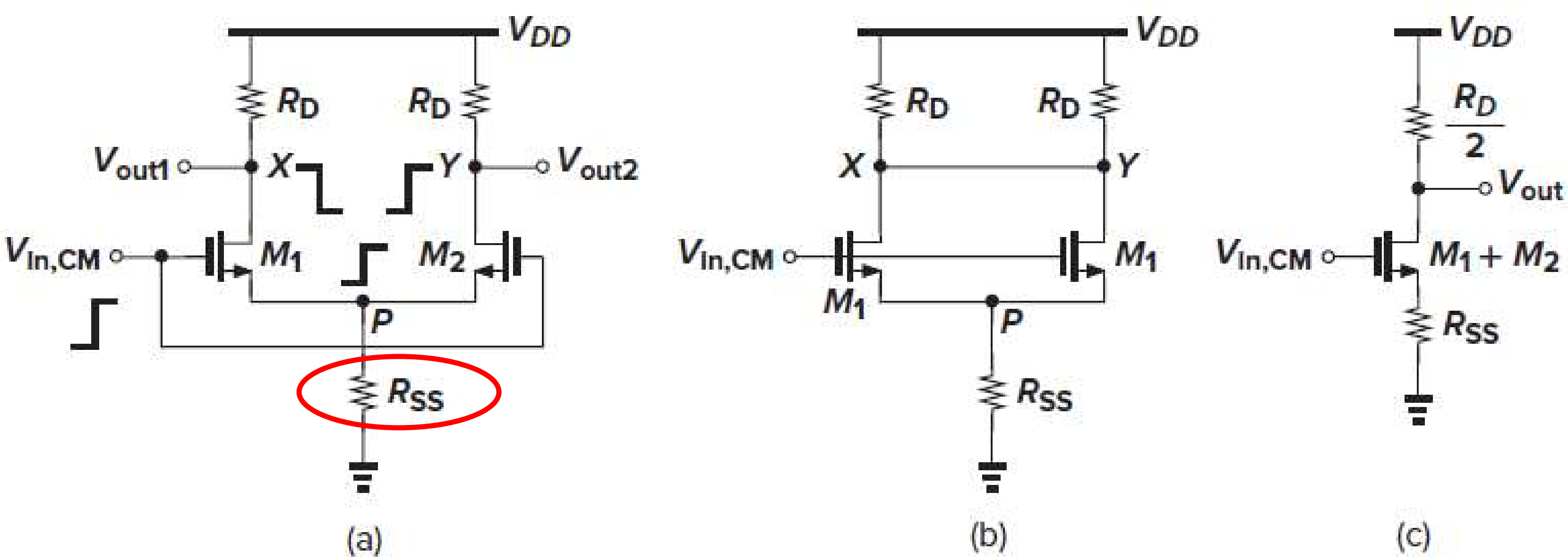
$$|A_V| = \frac{g_m}{1 + g_m R_S} R_D$$

What do we get from the degenerated differential pair?

(1) DC Gain is reduced

(2) Input range is increased

Differential Pair with CM Input



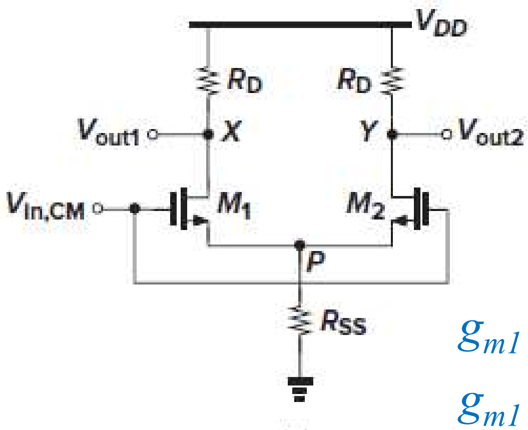
$$A_{v,CM} = \frac{V_{out,CM}}{V_{in,CM}} = -\frac{(2g_m)}{1+(2g_m)R_{SS}}\frac{R_D}{2} = -\frac{g_m}{1+2g_m R_{SS}}R_D$$

$A_{v,\mathrm{CM}}$: The gain that common-mode input variation is translated into the output common mode (Vi,cm => Vo,cm)

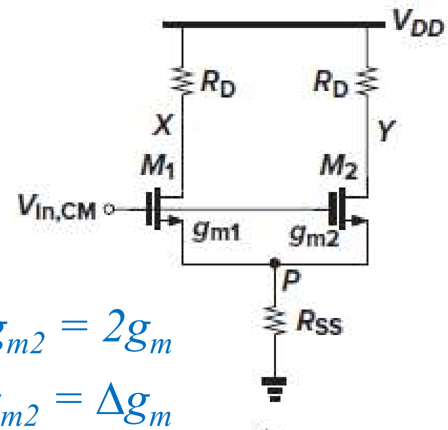
$A_{\mathrm{CM\text{-}DM}}$: The gain that common-mode input variation is translated into the differential output (Vi,cm => Vo,diff)

CMRR (M1≠M2)

Small signal analysis



(a)



(b)

$$g_{m1} + g_{m2} = 2g_m$$

$$g_{m1} - g_{m2} = \Delta g_m$$

1

$$(g_{m1} + g_{m2})(V_{in,CM} - V_P)R_{SS} = V_P$$

$$V_P = \frac{(g_{m1} + g_{m2})R_{SS}}{(g_{m1} + g_{m2})R_{SS} + 1} V_{in,CM}$$

$$V_X = -g_{m1}(V_{in,CM} - V_P)R_D$$

$$= \frac{-g_{m1}}{(g_{m1} + g_{m2})R_{SS} + 1} R_D V_{in,CM}$$

$$V_Y = -g_{m2}(V_{in,CM} - V_P)R_D$$

$$= \frac{-g_{m2}}{(g_{m1} + g_{m2})R_{SS} + 1} R_D V_{in,CM}$$

$$V_X - V_Y = -\frac{g_{m1} - g_{m2}}{(g_{m1} + g_{m2})R_{SS} + 1} R_D V_{in,CM}$$

$$A_{CM-DM} = -\frac{\Delta g_m R_D}{(g_{m1} + g_{m2})R_{SS} + 1}$$

2

$$|A_{DM}| = \frac{R_D}{2} \frac{g_{m1} + g_{m2} + 4g_{m1}g_{m2}R_{SS}}{1 + (g_{m1} + g_{m2})R_{SS}}$$

$$\approx g_m R_D$$

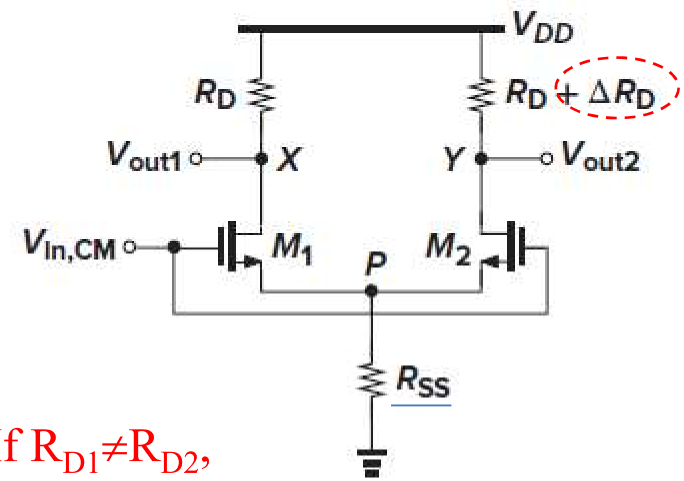
$$\text{CMRR} = \left| \frac{A_{DM}}{A_{CM-DM}} \right|$$

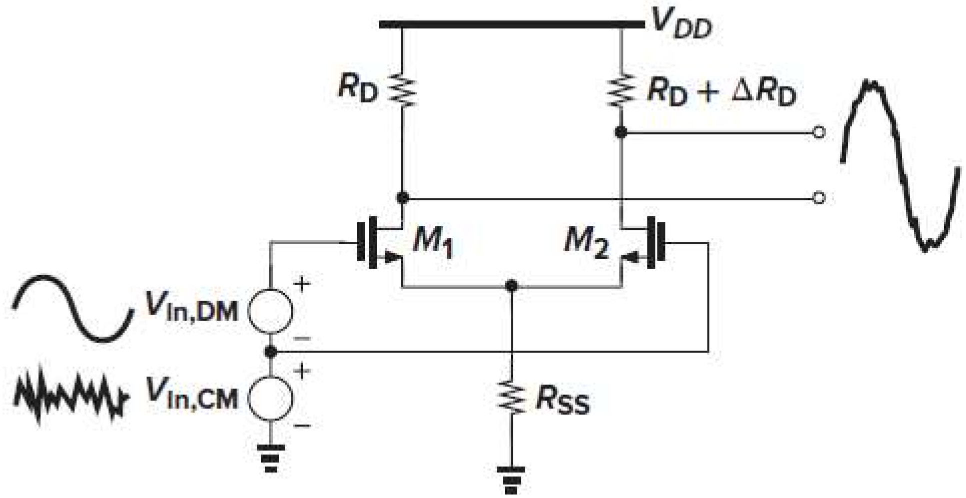
$$\text{CMRR} = \frac{g_{m1} + g_{m2} + 4g_{m1}g_{m2}R_{SS}}{2\Delta g_m}$$

$$\approx \frac{g_m}{\Delta g_m}(1 + 2g_m R_{SS})$$

CMRR ($R_{D1} \neq R_{D2}$)

Small signal analysis





If $R_{D1} \neq R_{D2}$,

$$\Delta V_P = \frac{R_{SS}}{R_{SS} + \dfrac{1}{2g_m}} \Delta V_{in,CM}$$

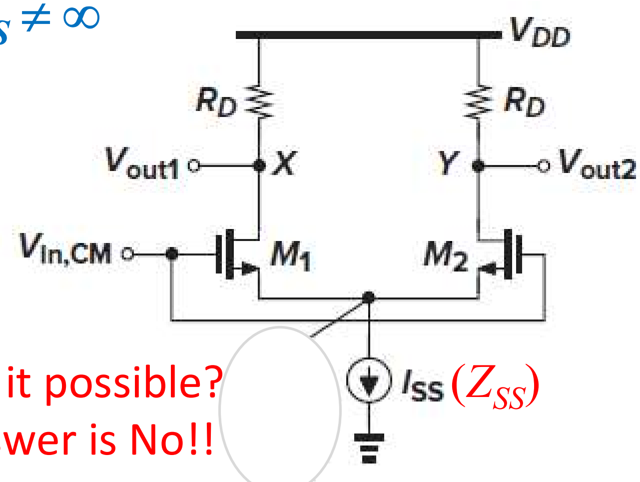
$$\Delta V_X = -\Delta V_{in,CM} \frac{g_m}{1 + 2g_m R_{SS}} R_D$$

$$\Delta V_Y = -\Delta V_{in,CM} \frac{g_m}{1 + 2g_m R_{SS}} (R_D + \Delta R_D)$$

$$A_{CM-DM} = \frac{V_{out2} - V_{out1}}{\Delta V_{in,CM}} = -\frac{g_m \Delta R_D}{1 + 2g_m R_{SS}}$$

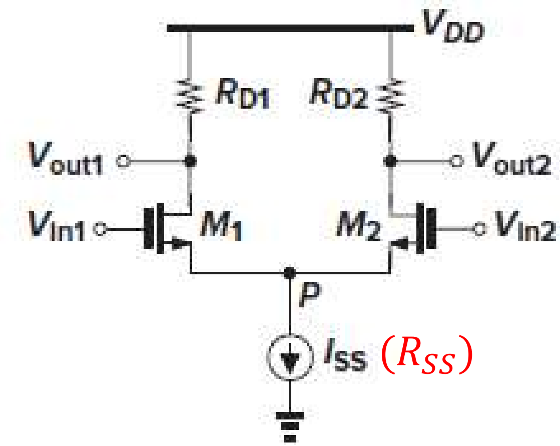
$$CMRR = \left| \frac{A_{DM}}{A_{CM-DM}} \right| = \frac{R_D}{\Delta R_D}(1 + 2g_m R_{SS})$$

If g_m or R_D has mismatch, common-mode input variation will cause diff. output change **if $R_{SS} \neq \infty$**



Q: $R_{SS} = \infty$, is it possible?
If Z_{SS}, the answer is No!!

CMRR by Mismatch



$$\text{CMRR} = \frac{A_{DM}}{A_{CM-DM}}$$

$R_{D1} \neq R_{D2}$

$$\text{CMRR} = \frac{R_D}{\Delta R_D}(1 + 2g_m R_{SS})$$

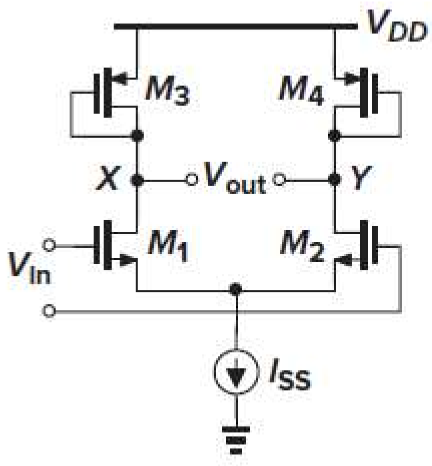
$g_{m1} \neq g_{m2}$

$$\text{CMRR} = \frac{g_m}{\Delta g_m}(1 + 2g_m R_{SS})$$

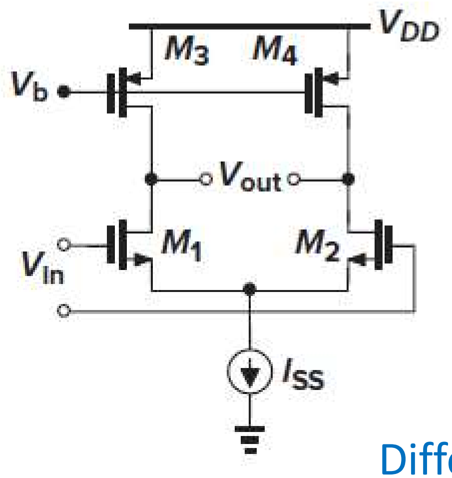
$$\text{CMRR} = \left(\frac{g_m}{\Delta g_m} + \frac{R_D}{\Delta R_D}\right)(1 + 2g_m R_{SS})$$

Q: How to improve CMRR?

Differential Amplifiers



(a)



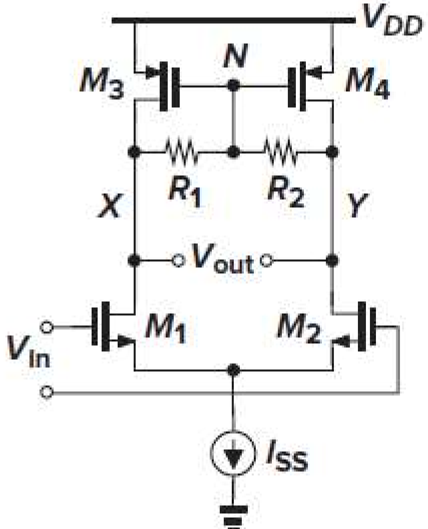
(b)

Differential DC gain:

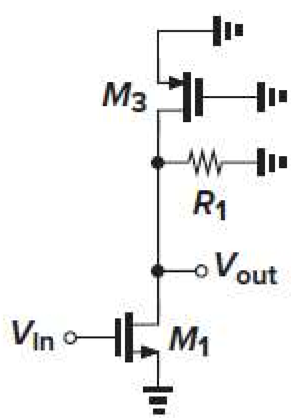
(a)
$$A_V = \frac{V_Y - V_X}{V_{in}} \sim \frac{g_{m1,2}}{g_{m3,4}}$$

(b)
$$A_V = \frac{V_Y - V_X}{V_{in}} \sim g_{m1,2}\left(r_{o3,4}||r_{o1,2}\right)$$

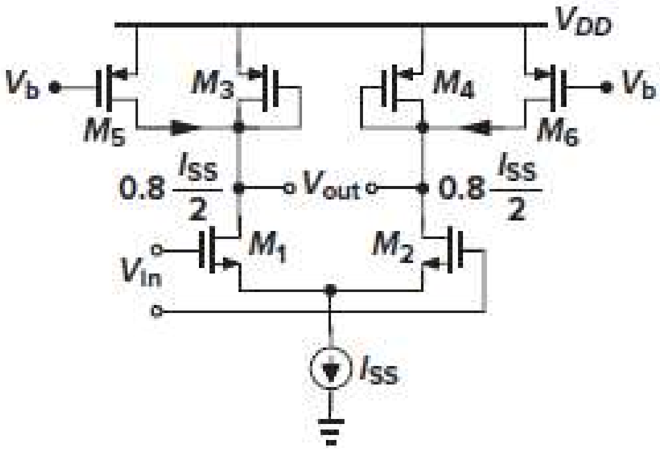
(c)
$$A_V = \frac{V_Y - V_X}{V_{in}} \sim g_{m1,2}\left(R_{1,2}||r_{o3,4}||r_{o1,2}\right)$$



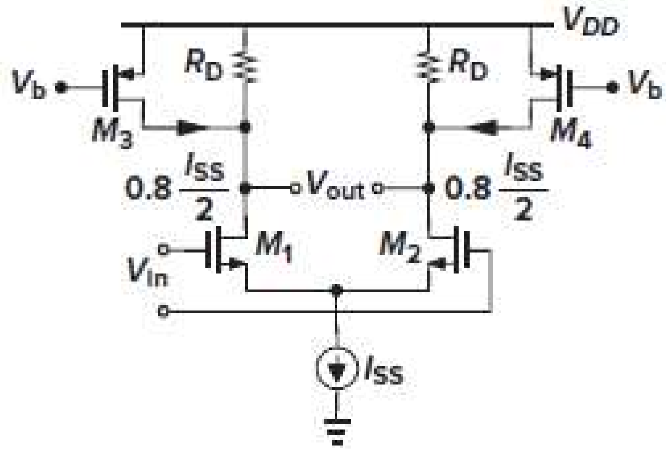
(c)



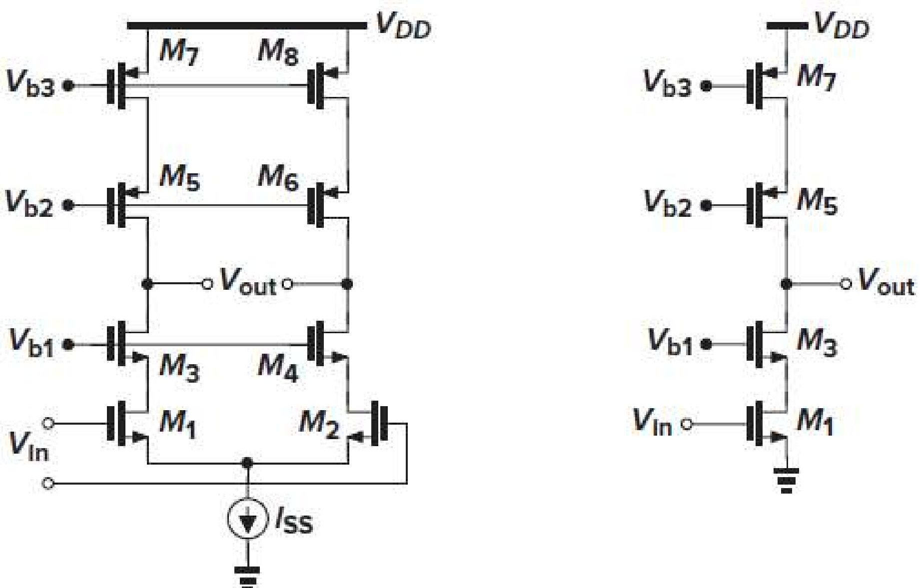
Differential Amplifiers



Amp (a)



Amp (b)



Amp (c)

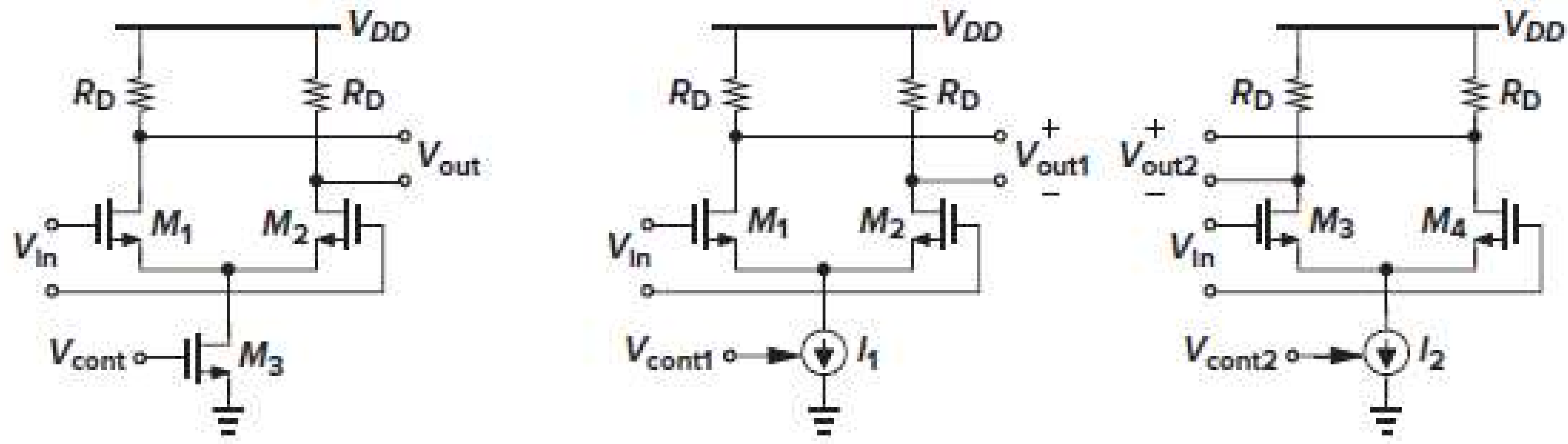
Two DC design parameters:

- Differential gain
- Output resistance

Appendix:

- Voltage Gain Amplifier
- Gilbert Cell

Variable Gain Amplifier (VGA)



(a)

(b)

$$A_V = \frac{V_{out}}{V_{in}} \sim g_{m1,2} R_D$$

$$g_{m1,2} = \sqrt{2K_{1,2} I_{D1,2}}$$

$$I_{D1,2} = \frac{I_3}{2} = \beta_3 (V_{cont} - V_{TH3})^2$$

$$g_{m1,2} = \sqrt{2K_{1,2}\beta_3}(\underbrace{V_{cont} - V_{TH3}}_{\Delta V_{cont}})$$

$$V_{out1} = g_{m1,2} R_D V_{in}$$

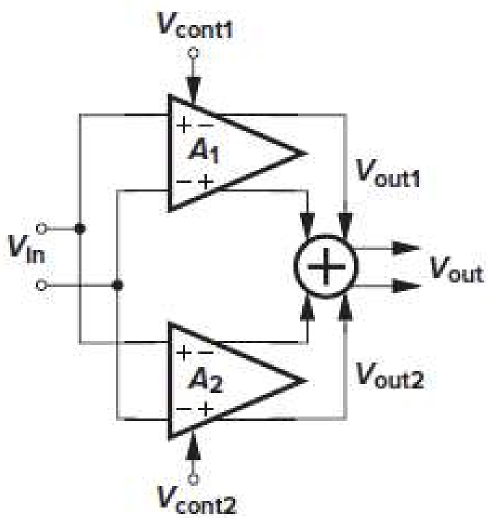
$$= R_D \sqrt{2K_{1,2}\beta_3}\, \Delta V_{cont1} V_{in}$$

$$\underline{V_{out1} = G\, \Delta V_{cont1} V_{in}}$$

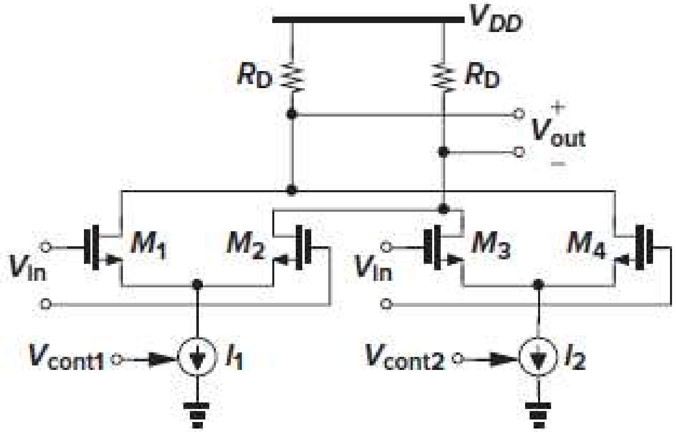
Two observations:

- Gain can be adjusted
- Multiplication of two analog signals

Gilbert Cell (1)

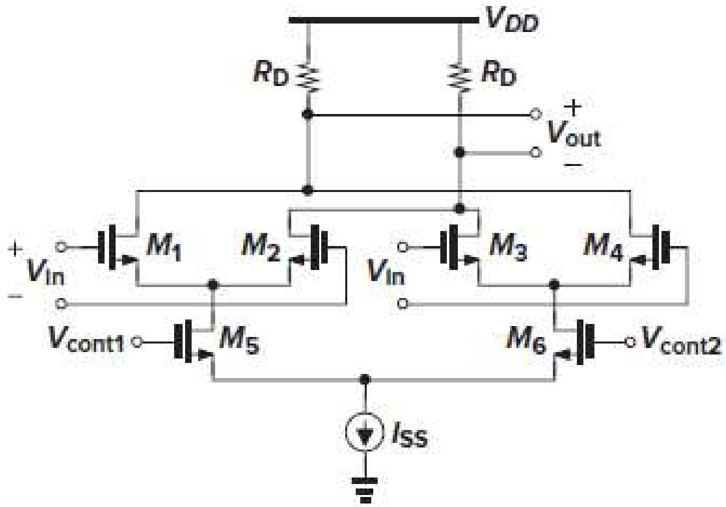


(a)

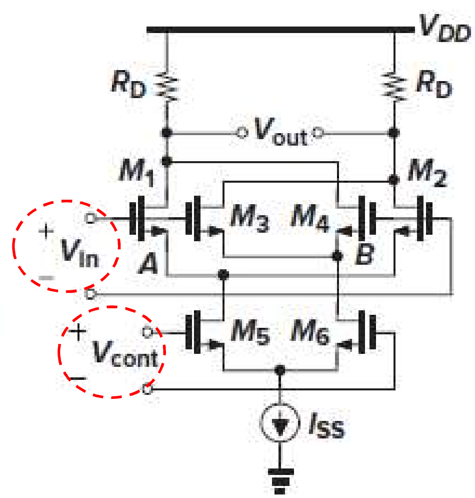


(b)

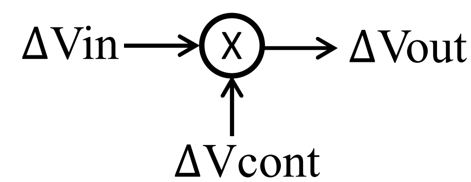
$$\Delta V_{out} = G_1\,\Delta V_{cont1}\Delta V_{in} + G_2\,\Delta V_{cont2}\Delta V_{in}$$



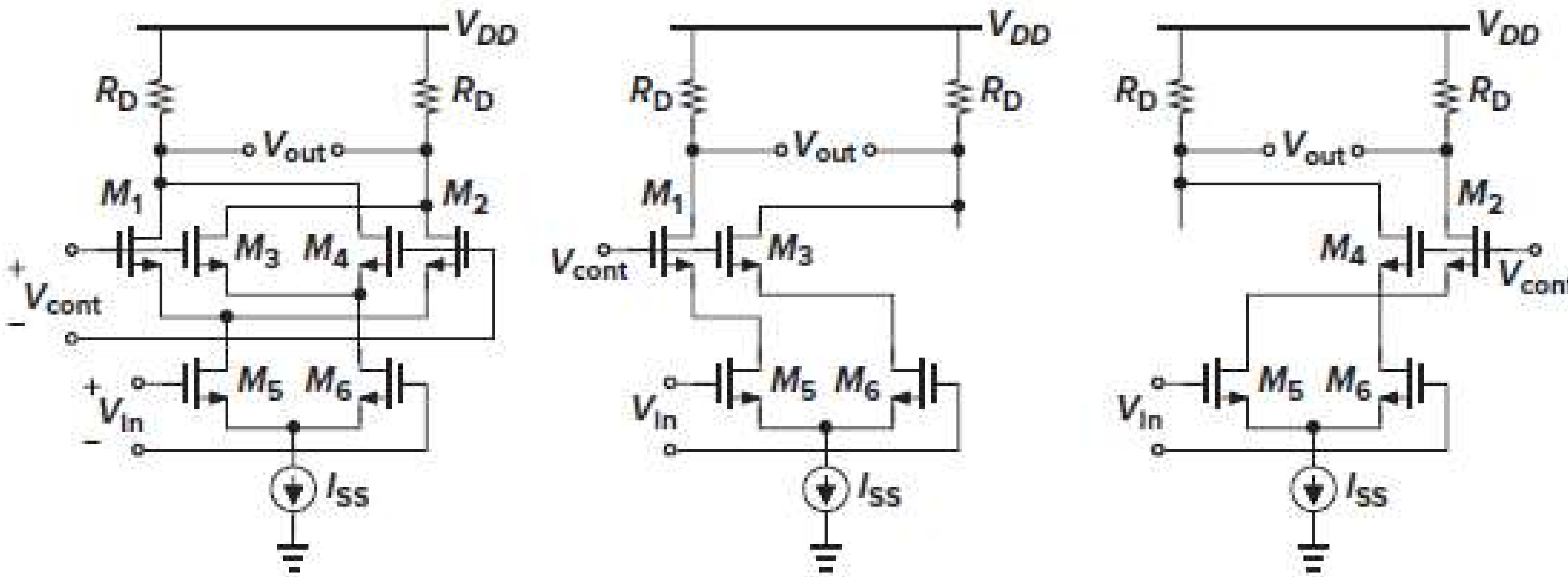
(c)



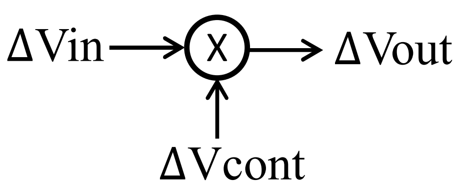
(d)



Gilbert Cell (2)



(a) (b) (c)



Vcont is much larger

Vout ~ $+G$Vin

Vcont is much smaller

Vout ~ $-\,G$Vin