Linear System

Lecture 1: Continuous-Time Signal and System

Fall 2023

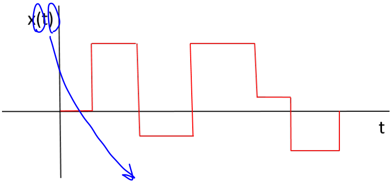
Introduction of Signal

What is signal ?

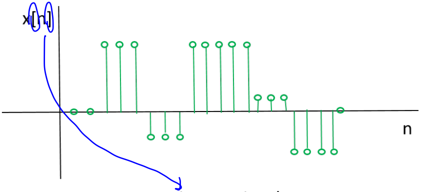
- A pattern conveys information by means of nature phenomenon.
- Examples of signal include:
  - **Electrical signals**: Voltages and currents in a circuit
  - **AM/FM Radio signals**: Acoustic pressure (sound) over time
  - **Video signals**: Intensity level of a pixel (camera, video) over time

Continuous and Discrete Signals

- **Continuous Time Signal:** Most signals are continuous time, such as voltage, current, etc.
  - Denote it as $x(t)$, and the time interval may be bounded (finite) or infinite.



Continuous v.s. Discrete Signals



Periodic and Aperiodic Signals

- **Aperiodic signals:** The signal is not periodic.

- **Periodic signals:** A signal is periodic if it repeats itself after a fixed period T.

  $$x(t) = x(t+T), \forall t. \text{ T is the period.}$$

  - Example: $\sin(t)$ is periodic.

Periodic and Aperiodic Signals

$x(t+mT) = x[t+(m-1)T+T] = x[t+(m-1)T] = ... = x(t)$, $m = 1, 2, 3, ...$ and mT is also the period.

- T is called **fundamental period**.
- $f = \dfrac{1}{T}$ is called **fundamental frequency**.

- Example:
  - $\sin(t)$ is periodic, and $\sin(t) = \sin(t + 2\pi) = ... = \sin(t + \boxed{m} \cdot 2\pi)$
  - Hence, 2π is fundamental period.

Periodic and Aperiodic Signals

If both $x_1(t)$ and $x_2(t)$ are periodic signals, is $x_1(t) + x_2(t)$ periodic ?

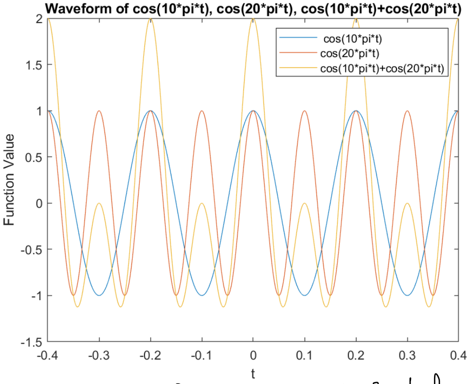
- If the periods of $x_1(t)$ and $x_2(t)$ are T_1 and T_2,
- The (fundamental) period of $x_1(t) + x_2(t)$ is the lowest common multiple (LCM) of $[T_1, T_2]$.
- The fundamental frequency is the greatest common divisor (GCD) of $(\frac{1}{T_1}, \frac{1}{T_2})$.

$$\underbrace{15\,Hz \quad 36\,Hz}_{3\,Hz}$$

$$3 \,\underline{|\, 15 \quad 36}$$
$$\quad 5 \quad 12$$

Periodic and Aperiodic Signals

Example: Find the fundamental Period of $\cos(10\pi t) + \cos(20\pi t)$



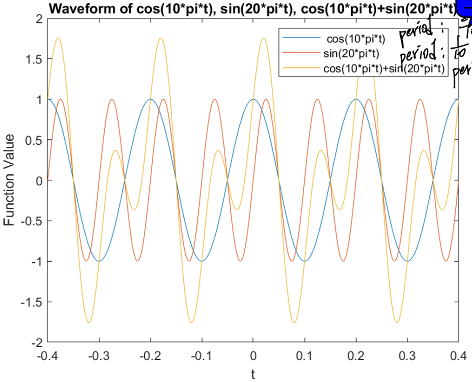
Period of $\cos(t)$: 2π sec $\Rightarrow$ Period of $\cos(2\pi t)$: $\frac{2\pi}{2\pi} = 1$ sec

- The fundamental period of $\cos(10\pi t) + \cos(20\pi t)$ is 0.2.

$t = \frac{2\pi}{10\pi} = 0.2$ sec $t = \frac{2\pi}{20\pi} = 0.1$ sec

Periodic and Aperiodic Signals

Example: Find the fundamental Frequency of $\cos(10\pi t) + \sin(20\pi t)$:



period : $\frac{2\pi}{10\pi} = \frac{1}{5}$

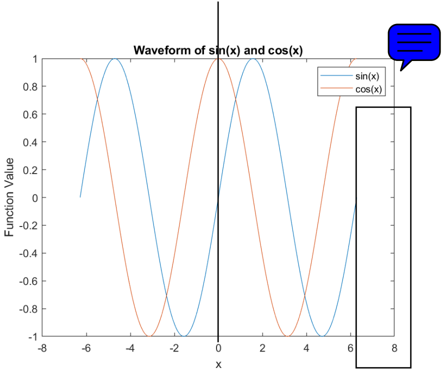
period : $\frac{1}{10}$

period : $\frac{1}{5}$ ⇒ freq : 5 Hz

- The fundamental frequency is 5.

Even and Odd Signals

- **Even signal**: A signal is even if $x(-t) = x(t),\ \forall t$ (i.e. it can be reflected in the y axis).
  - Example: $\cos(t)$
- **Odd signal**: A signal is odd if $x(-t) = -x(t),\ \forall t$.
  - Example: $\sin(t)$
- **Example:**



Even and Odd Signals

Assume that $x(t)$ is continuous signal and $x(t) = x_e(t) + x_o(t)$

- $x_e(t)$: The even part of the $x(t)$
- $x_o(t)$: The odd part of the $x(t)$

$$x(-t) = \underbrace{x_e(-t)}_{= x_e(t)} + \underbrace{x_o(-t)}_{-x_o(t)}$$

Then

Eliminate the odd part of the signal

1) $x_e(t) = \frac{1}{2}\{\underline{x(t) + x(-t)}\}$
2) $x_o(t) = \frac{1}{2}\{\underline{x(t) - x(-t)}\}$ vice versa of 1)
3) $x(-t) = x_e(t) - x_o(t)$

证明:

$$\begin{aligned} x(t) &= x_e(t) + x_o(t) \\ x(-t) &= \underbrace{x_e(-t)}_{= x_e(t)} + \underbrace{x_o(-t)}_{-x_o(t)} \\ -)\quad & \\ \hline \underbrace{x(t) - x(-t) = 0 + 2x_o(t)}& \end{aligned}$$

c.f. 1)

$$\begin{aligned} x(t) &= x_e(t) + x_o(t) \\ x(-t) &= \underbrace{x_e(-t)}_{x_e(t)} + \underbrace{x_o(-t)}_{-x_o(t)} \\ +)\quad & \\ \hline \underbrace{x(t) + x(-t) = 2x_e(t) + 0}& \end{aligned}$$

c.f. 2)

Exponential and Sinusoidal Signals

Exponential signal: $x(t) = Ae^{(\theta+\omega_0 t)}, A \in R, \omega_0 \in R$.

Sinusoidal signal: $x(t) = A\cos(\omega_0 t + \theta), \{t, A, \omega_0, \theta\} \in R$

Complex signal: $x(t) = Ae^{j\omega_0 t} = A\cos(\omega_0 t) + j(A\sin(\omega_0 t))$

- Signal is represented by two orthogonal vector space.
- Signal includes real and imaginary signal.
- Imaginary part is indicated by j in this course.

Exponential and Sinusoidal Signals

Example: $x(t) = e^{j(\omega_0 t+\theta)}$

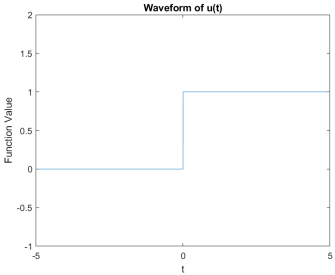
- $x(t) = \cos(\omega_0 t + \theta) + j\sin(\omega_0 t + \theta)$
- $\cos(\omega_0 t + \theta) = \text{Re}\{e^{j(\omega_0 t+\theta)}\}$
- $\sin(\omega_0 t + \theta) = \text{Im}\{e^{j(\omega_0 t+\theta)}\}$

- $x(t)$ is periodic signal with period: $T_0 = \dfrac{2\pi}{|\omega_0|}$
  - $\text{Re}\{e^{j(\omega_0 t+\theta)}\}$ and $\text{Im}\{e^{j(\omega_0 t+\theta)}\}$ are also periodic.

Step and Pulse Signals

Step signal

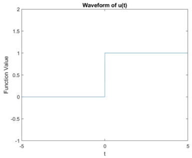
- $\mathsf{u}(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$



Step and Pulse Signals

Continuous time pulse signal

- $\delta(t) = \begin{cases} \sim\infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$
  - $\underline{\delta(t) = \frac{d}{dt}\mathsf{u}(t)}$



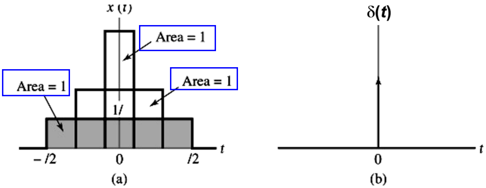


Figure: The evaluation of delta signal from step signal.

Step and Pulse Signals

Continuous time pulse signals

Assume $x(t)$ is cont. at $t = 0$, $\int_{t_2}^{t_1} x(t)\delta(t)dt = x(0), t_2 < 0 < t_1$

- $\delta(0) \to \infty$
- $\delta(t) = 0, t \neq 0$
- $\underline{\int_{-\infty}^{\infty} \delta(t)dt = 1}$
- $\delta(t) = \delta(-t)$, δ is even signal
- $\delta(t) = \frac{d}{dt}\mathsf{u}(t)$
- $\mathsf{u}(t) = \int_{-\infty}^{t} \delta(\tau)d\tau = \int_{0}^{\infty} \delta(t - v)dv$, setting $\tau = t - v$

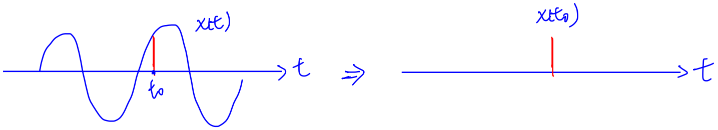
$d\tau = -dv$

Step and Pulse Signals

#sampling
#sample

Them.

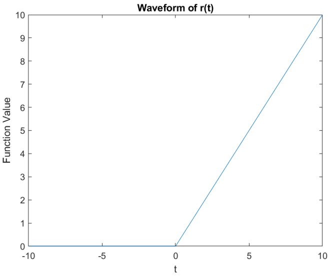
- $\int_{t_2}^{t_1} x(t)\delta(t-t_0)dt = \begin{cases} x(t_0), & t_2 < t_0 < t_1 \\ 0, & \text{otherwise} \end{cases}$
- $x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$ ⇒ sampling



Ramp Signals

Neither Energy Signal, nor Power Signal

- $r(t) = tu(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$



$E(x) = \int_{-\infty}^{\infty} [t \cdot u(t)]^2 dt$

$\because u(t) = 0, \ t < 0$

$\int_{0}^{\infty} [t u(t)]^2 dt$

$= \int_0^\infty t^2 dt = \frac{\infty}{3}$

$P(T) = \lim_{T \to \infty} \frac{\int_{-T}^{T} [t u(t)]^2 dt}{2T}$

$\because u(t) = 0, \ t < 0$

$= \lim_{T \to \infty} \frac{\int_{0}^{T} [t \cdot u(t)]^2 dt}{2T}$

$= \lim_{T \to \infty} \frac{\frac{T^{\cancel{3}\,2}}{3}}{2\cancel{T}}$

$= \lim_{T \to \infty} \frac{T^2}{6} = \infty$

Square Signals, rect(t), $\Pi(t)$

此課程中常用

τ：有信號的時間

Assume $A \in R$,

- $x(t) = A \text{ rect}(\frac{t}{\tau}) = A\ \Pi(\frac{t}{\tau}) = \begin{cases} A, & \boxed{\frac{-\tau}{2}} \leq t \leq \boxed{\frac{\tau}{2}} \\ 0, & \text{otherwise} \end{cases}$
- $A \text{ rect}(\frac{t}{\tau}) = A \text{ u}(t + \frac{\tau}{2}) - A \text{ u}(t - \frac{\tau}{2})$

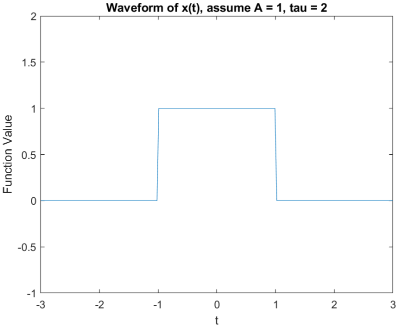


Figure: x(t) when $A = 1, \tau = 2$

Triangular Signals, tri(t), $\Lambda(t)$

Assume $A \in R$,

- $$x(t) = A\ \text{tri}(\frac{t}{\tau}) = A\ \Lambda(\frac{t}{\tau}) = \begin{cases} -\frac{A}{\tau}(t-\tau), & 0 \le t \le \tau \\ \frac{A}{\tau}(t+\tau), & -\tau \le t \le 0 \\ 0, & \text{otherwise} \end{cases}$$

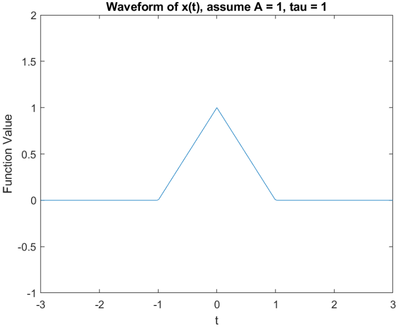
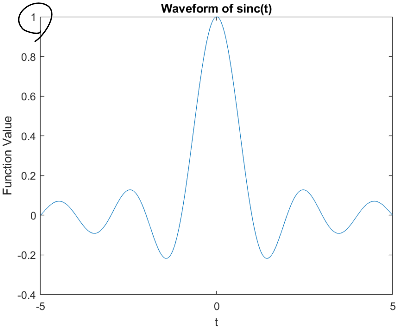


Figure: x(t) when $A = 1, \tau = 1$

sinc(t) Signal

$$\lim_{t\to 0}\frac{\sin(\pi t)}{\pi t} = \lim_{t\to 0}\frac{\cos(\pi t)\cdot\pi}{\pi} = 1$$

- $\operatorname{sinc}(t) = \begin{cases} \dfrac{\sin(\pi t)}{\pi t}, & t \in R, t \neq 0 \\ 1, & t = 0 \end{cases}$



Energy and Power Signal

Example: Assume $x(t)$ is the voltage or current when the resistance $R = 1\Omega$,

- Instantaneous power: $P(t) = |x(t)|^2 \geq 0$

  All time
- $\boxed{\text{Total}}$ energy: $E_T = \lim_{\boxed{T \to \infty}} \int_{\boxed{-T}}^{\boxed{T}} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt \geq 0$
- Partial energy: $E = \int_T |x(t)|^2 dt \geq 0$
  - The energy between (T_1, T_2) interval:
    $$E = \int_{T_1}^{T_2} |x(t)|^2 dt = \int_{T_1}^{T_2} [x_{Re}^2(t) + X_{Im}^2(t)] dt$$
- Average power: $P_{av} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \geq 0$
  - The average power between (T_1, T_2) interval:
    $$P_{av} = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} |x(t)|^2 dt$$

Energy and Power Signal

Is $x(t)$	Energy Signal	&&	Power Signal	
	O		O	?
	O		X	at the same time?
	X		O	
	X		X	

Energy and Power Signal

- **Energy signal**: $x(t)$ is Energy Signal, if $0 < E_T < \infty$.
- **Power signal**: $x(t)$ is Power Signal, if $0 < P_{av} < \infty$.

Energy and Power Signal

Square Signals, P.18

Example: Given $x(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ 0, & \underline{\text{o.w.}} \end{cases}$, is $x(t)$ Energy or Power Signal?

- $E_T = \int_0^1 1 dt = 1$
- $P_{av} = \lim_{T \to \infty} \frac{E_T}{2T} = 0$ Not a power signal. See P.22
- $x(t)$ is Energy Signal.

Energy and Power Signal

Example: Given $P_{av} = 1$, is it Energy or Power Signal?

- $\lim_{T \to \infty} \frac{E_T}{2T} = 1$
- $E_T = \infty$ Not a energy signal.
  See P.22
- It is Power Signal.

Energy and Power Signal

Example: Assume $x(t) = A\cos(2\pi f_0 t + \theta) = A\cos(\omega_0 t + \theta)$

1) Find the instant. power of x(t)
2) Find the total energy of x(t) $\int_{-\infty}^{\infty} |x(t)|^2 dt$
3) Find the partial energy of x(t) within the N periods.
4) Find the average power of x(t)

3) let $\theta = 0 \Rightarrow x(t) = A\cos(2\pi f_0 t) \Rightarrow T_0 = \frac{1}{f} \Rightarrow NT_0 = \frac{N}{f}$

$\int_0^{\frac{N}{f}} |A\cos(2\pi f_0 t)|^2 dt = A^2 \int_0^{\frac{N}{f}} |\cos(2\pi f_0 t)|^2 dt$

$= A^2 \int_0^{\frac{N}{f}} \cos^2(2\pi f_0 t) dt$

$\because \cos^2(x)$ always positive

$= \frac{A^2}{2} \int_0^{\frac{N}{f}} \cos(4\pi f_0 t) + 1 \, dt$

$C^2 - S^2 = C2 \Rightarrow 2C^2 - 1 = C2$ $(1 - C^2)$

$\Rightarrow C^2 = \frac{C2 + 1}{2}$

Energy and Power Signal

$= \frac{A^2}{2}\left(\frac{1}{4\pi f_0}\cancel{\int_0^{4\pi N}\cos(u)\,du} + \frac{N}{f}\right) = \boxed{\frac{A^2 N}{2f}}$

為什麼這裡不用考慮電阻?

1) Instant. power, $P(t)$

$$P(t) = |x(t)|^2 = A^2\cos^2(2\pi f_0 t + \theta)$$

2) Total energy, E_T

$$E_T = \int_{-\infty}^{\infty} |x(t)|^2 = \infty$$

3) Partial energy of N periods

$$E = \int^{NT_0} |x(t)|^2 dt = \frac{A^2 T_0}{2} N, T_0 = \frac{1}{f_0}$$

4) Average power, P_{av}

$$P_{av} = \frac{1}{T_0}\int_{T_0} |x(t)|^2 dt = \frac{1}{T_0}\frac{A^2 T_0}{2} = \frac{A^2}{2}, T_0 = \frac{1}{f_0}$$

Based on 4), $x(t)$ is power signal.

4) $\lim_{T\to\infty} \frac{\int_{-T}^{T}|A\cos(2\pi f_0 t+\theta)|^2 dt}{2T}$

let $\theta = 0$:

$\lim_{T\to\infty} \frac{A^2\int_{-T}^{T}|\cos(2\pi f_0 t)|^2 dt}{2T}$

$= A^2 \lim_{T\to\infty} \frac{\int_{-T}^{T}\cos^2(2\pi f_0 t)\,dt}{2T}$ ∵ $\cos^2(t)$ always ≥ 0

$c^2 - s^2 = c2$

$\Rightarrow c^2 - 1 + c^2 = c2 \Rightarrow c^2 = \frac{c2+1}{2}$

$= A^2 \lim_{T\to\infty} \frac{\frac{1}{2}\int_{-T}^{T}\cos(4\pi f_0 t)+1\,dt}{2T}$

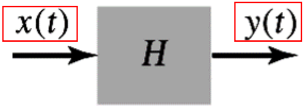
$= \frac{A^2}{2} \lim_{T\to\infty} \frac{\left(\overset{0}{\cancel{\int_{-T}^{T}\cos(4\pi f_0 t)\,dt}} + 2T\right)}{2T}$

$= \boxed{\frac{A^2}{2}}$

Introduction of System

System: A setup converts a series of input to a series of output.

- **Continuous time system**: Input and output signals are continuous time signals.
  - $y(t) = H\{x(t)\}$



Continuous Time Systems

Classifications

1) Linear and Nonlinear System
2) Time Varying and Time Invariant System
3) Memory and Memoryless System
4) Causal and Noncausal System
5) Invertibility and Inverse System
6) Stable and Unstable System

Linear and Nonlinear System

Given two input signals $x_1(t)$ and $x_2(t)$ as well as two output signals $y_1(t)$ and $y_2(t)$, **the system is linear:**

1) If the input signal is $x_1(t) + x_2(t)$, the output signal is $y_1(t) + y_2(t)$.
   - This is called ' **additivity (Superposition property)**'.
2) If the input signal is $\alpha x_1(t), \alpha \in R$, the output signal $\alpha y_1(t)$.
   - This is called '**Homogeneity**'.

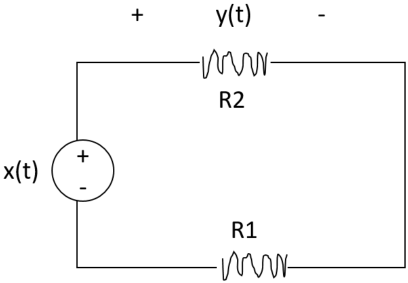
Linear System

Properties

1) If the input signal is $\alpha x_1(t) + \beta x_2(t)$, the output signal is $\alpha y_1(t) + \beta y_2(t)$.
2) It the input signal $x_1(t) = 0$, the output signal $y_1(t) = 0$.
   - **If $x_1(t) = 0$ but $y_1(t) \neq 0$, the system is nonlinear.**
   - A good way to judge if the system is linear or not.

Linear System

Example: Is this system Linear or Not ?



Linear System

1) $y(t) = \frac{R_2}{R_1 + R_2} x(t)$
2) Assume the input signal is $x_1(t)$ and $x_2(t)$, and the output signal is $y_1(t)$ and $y_2(t)$
3) $y_1(t) = \frac{R_2}{R_1 + R_2} x_1(t)$, $y_2(t) = \frac{R_2}{R_1 + R_2} x_2(t)$
4) If the input signal is $x(t) = \alpha x_1(t) + \beta x_2(t), \alpha, \beta \in R$
5) The output signal
$$y(t) = \frac{R_2}{R_1 + R_2} x(t) = \frac{R_2}{R_1 + R_2} (\alpha x_1(t) + \beta x_2(t))$$
$$= \alpha \frac{R_2}{R_1 + R_2} x_1(t) + \beta \frac{R_2}{R_1 + R_2} x_2(t) = \alpha y_1(t) + \beta y_2(t).$$

Therefore, it is a **linear system**.

Time Varying and Time Invariant System

Assume the input and output signals are $x(t)$ and $y(t)$,

Time Invariant System:

- If we delay t_0 to input $x(t)$ to the system, the output of $y(t)$ is also delayed t_0.

Time Varying System:

- If we delay t_0 to input $x(t)$ to the system, the output of $y(t)$ is **NOT** delayed t_0.

Time Varying and Invariant System

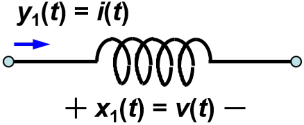
Example: Given $x(t) = \cos(t)$, Is $y(t) = \cos(t)/R, R = 1$ time varying or invariant ?

- Because $y(t - t_0) = \cos(t - t_0)$, the system is time invariant.

Time Varying and Invariant System

Example: Is inductor (i.e., L is the inductance) time varying or invariant ?

- $y_1(t) = \frac{1}{L}\int_{-\infty}^{t} x_1(\tau)d\tau$



Time Varying and Invariant System

Example Is inductor (i.e., L is the inductance) time varying or invariant ?

- $y_1(t) = \frac{1}{L}\int_{-\infty}^{t} x_1(\tau)d\tau$

input is delayed t_0

1) Let $x_2(t) = x_1(t - t_0)$, thus the output $y_2(t)$ is
   - $y_2(t) = \frac{1}{L}\int_{-\infty}^{t} x_1(\tau - t_0)d\tau$

$u = (\tau - t_0)$, $du = d\tau$; $\frac{1}{L}\int_{-\infty}^{(t)-t_0} x_1(u)\, du$

2) Let $y_1(t - t_0)$ be the output of the inductor with t_0 time shift,
   - $y_1(t - t_0) = \frac{1}{L}\int_{-\infty}^{t-t_0} x_1(\tau)d\tau$

$u = \tau + t_0$, $du = d\tau$; $\frac{1}{L}\int_{-\infty}^{(t+t_0)-t_0} x_1(u - t_0)\, du$

$\because u = \tau + t_0$ $\therefore \tau = u - t_0$

3) Set $\tau^{'} = \tau - t_0$,
   - $y_2(t) = \frac{1}{L}\int_{-\infty}^{t-t_0} x_1(\tau^{'})d\tau^{'}$

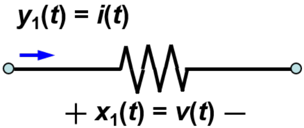
$= \frac{1}{L}\int_{-\infty}^{t} x_1(u - t_0)\, du$

4) Hence, the inductor is time invariant.

Time Varying and Invariant System

Example: Is thermistor time varying or invariant ?

- $y_1(t) = x_1(t)/R(t)$, $R(t)$ is resistance.



Time Varying and Invariant System

Example: Is thermistor time varying or invariant ?

- $\underbrace{y_1(t)}_{\text{Output}} = \underbrace{x_1(t)}_{\text{Input}}/R(t)$, $R(t)$ is resistance.

1) Let $y_2(t)$ is the system output of $x_1(t-t_0)$, then
   - $$y_2(t) = \frac{x_1(t-t_0)}{R(t)}$$
2) Let $y_1(t-t_0)$ be the output of the thermistor with t_0 time shift,
   - $$y_1(t-t_0) = \frac{x_1(t-t_0)}{R(t-t_0)}$$
3) Since $R(t) \neq R(t-t_0)$, $y_1(t-t_0) \neq y_2(t), t_0 \neq 0$
4) The thermistor is time variant.

Memory and Memoryless System

Given the input signal $x(t)$, and output signal $y(t)$,

Memoryless System:

- If $y(t_0), t_0 \in R$ is only related to $x(t_0)$ and not related to $x(t'), t' \neq t_0$.

Memory System:

- ~~If $y(t_0)$ is related to not only $x(t_0)$ but also $x(t'), t' \neq t_0$.~~

Memory and Memoryless System

Example: Is it a memory system ?

1) Resistor: $i(t) = \frac{1}{R} v(t)$
   - Memoryless
2) Inductor: $i(t) = \frac{1}{L} \int_{-\infty}^{t} v(\tau) d\tau$
   - Memory

Causal and Noncausal System

Given an output signal $y(t)$,

Causal System:

- If $y(t_0)$ is only related to $x(t), t \boxed{\leq} t_0$.
  - Only related to the past input signal.

Noncausal System:

- If $y(t_0)$ is related to $x(t), t > t_0$. e.g. $y(t) = \int_t^{\infty} x(\tau)\, d\tau$
  - Related to the future input signal.

NOTE:

1) A causal system must be capable of operating in real time.
2) All memoryless systems are also Causal !

Causal and Noncausal Signal

Example: Given $x(t)$ is input and $y(t)$ is the corresponding system output, check if the system is causal or not.

1) $y(t) = x(t + t_0), t_0 > 0$
   - The system is '**Noncausal**' because $y(t)$ is related to the signal t_0 behind.

2) $y(t) = x(t - t_0), t_0 > 0$
   - The system is '**Causal**' because $y(t)$ is related to the signal t_0 ahead.

Invertibility and Inverse System

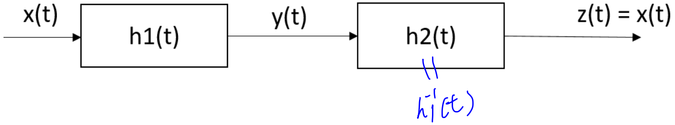
Assume two input signals, $x_1(t), x_2(t)$ and two corresponding output signals, $y_1(t), y_2(t)$,

One to one system:

- If $x_1(t) \neq x_2(t)$, $y_1(t) \neq y_2(t)$.

Inverse System:

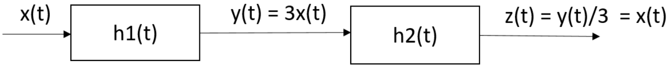
- If $h_1(t)$ is inverse system, there exists an unique system, $h_2(t)$.
- If $h(t)$ is the cascaded by $h_1(t)$ and $h_2(t)$, the output signal of $h(t)$ is the same as the input signal.



Inverse System

Example: Is $y(t) = 3x(t)$ an inverse system ? If yes, please find its inverse system.

- Assume $x_1(t) \neq x_2(t)$, $3x_1(t) \neq 3x_2(t) \longrightarrow y_1(t) \neq y_2(t)$
  - It is a **one to one** system.



- To obtain the inverse system, we swap $x(t)$ and $y(t)$.
  - Thus, $x(t) = 3y(t) \longrightarrow y(t) = \dfrac{1}{3}x(t)$
- Therefore, the inverse system, $z(t) = \dfrac{1}{3}y(t)$, is obtained.

Inverse System

Example: Is $y(t) = \cos(x(t))$ an inverse system ? If yes, please find its inverse system.

- Assume $x_2(t) = x_1(t) + 2k\pi, k \in Z$,
- Thus, $\cos(x_2(t)) = \cos[x_1(t) + 2k\pi] = \cos(x_1(t))$.
- $y_1(t) = y_2(t)$, and the system is **NOT one to one**.

=> inverse system not existed

Inverse System

Example: Is $y(t) = x(t + t_0)$ an inverse system ? If yes, please find its inverse system.

- Assume $x_1(t) \neq x_2(t)$,
- Thus, $x_1(t + t_0) \neq x_2(t + t_0) \longrightarrow y_1(t) \neq y_2(t)$.
- Therefore, it is a **one to one system**.

- To obtain the inverse system, we swap $x(t)$ and $y(t)$.
- Thus, $x(t) = y(t + t_0)$
- Set $t^{'} = t + t_0$, $x(t' - t_0) = y(t') \longrightarrow y(t) = x(t - t_0)$
- Therefore, the inverse system is
- $z(t) = y(t - t_0)$, is obtained.

$y(t) = x(t+t_0)$

let $u = t+t_0$

$\Rightarrow y(u-t_0) = x(u)$

$\Rightarrow x(t) = y(t-t_0)$

$\Rightarrow$ The inverse system is:

$y(t) = x(t-t_0)$

Inverse System

Example: Is $y(t) = x^2(t)$ an inverse system ? If yes, please find its inverse system.

Let

$x_1(t) = t$

$x_2(t) = -t$ $\Rightarrow x_1(t) \neq x_2(t)$

$\Rightarrow$ $y_1(t) = x_1^2(t) = t^2$

$y_2(t) = x_2^2(t) = (-t)^2 = t^2$ $\Rightarrow y_1(t) = y_2(t) \Rightarrow$ not one to one

$\Rightarrow$ the inverse system doesn't exist.

- Since both $x(t)$ and $-x(t)$ produce the same output $y(t)$, the system is **not one-to-one**.
- The system is not invertible.

Stable and Unstable System

1) If $|x(t)| < B < \infty, \forall t$, $x(t)$ is called **Bounded Signal**.
2) If input signal, $x(t)$, is bounded and the output signal, $y(t)$, is also bounded, it is called **Bounded Input Bounded Output, BIBO**.
   - This system is a '**Stable System**'.
   - $|x(t)| < B_1 < \infty \longrightarrow |y(t)| < B_2 < \infty$

Stable and Unstable System

Example: Is the system stable or not ?

1) $y(t) = \cos(x(t))$
   - It is stable because $|y(t)| = |\cos(x(t))| \leq 1 = B_2 < \infty$, if $|x(t)| < B_1 < \infty$

2) $y(t) = tx(t)$
   - It is not stable because $y(t) \to \infty, t \to \infty$, if $x(t) = 1$

3) $y(t) = e^{x(t)}$

   bound input
   - Assume $|x(t)| < B_1 < \infty$, $|y(t)| = |e^{x(t)}| \leq e^{B_1} = B_2 < \infty$
   - Hence, BIBO. It is stable.

Reference See P.28 to recall

- "Signal and Systems" , 2nd Edition, by S. Haykin and B. D. Van Veen Wiley