

(a) $f'(x) = 3 - \frac{6}{\sqrt{x}} = \frac{3\sqrt{x}-6}{\sqrt{x}} \Rightarrow$ critical points at $x = 4$ and $x = 0$

(b) $f' = \left(\begin{array}{c} - - - \\ 0 \end{array} \middle| \begin{array}{c} + + + \\ 4 \end{array} \right) \Rightarrow$ increasing on $(4, \infty)$, decreasing on $(0, 4)$

1. (c) Local minimum at $x = 4$

(a) $f(x) = \sqrt{25 - x^2} \Rightarrow f'(x) = \frac{-x}{\sqrt{25-x^2}} \Rightarrow$ critical points at $x = 0$, $x = -5$, and $x = 5$

$\Rightarrow f' = \left(\begin{array}{c} + + + \\ -5 \end{array} \middle| \begin{array}{c} - - - \\ 5 \end{array} \right), f(-5) = 0, f(0) = 5, f(5) = 0 \Rightarrow$ local maximum is 5 at $x = 0$; local minimum of 0 at $x = -5$ and $x = 5$

2. (b) absolute maximum is 5 at $x = 0$; absolute minimum of 0 at $x = -5$ and $x = 5$

When $y = x^4 - 2x^2$, then $y' = 4x^3 - 4x = 4x(x+1)(x-1)$

and $y'' = 12x^2 - 4 = 12\left(x + \frac{1}{\sqrt{3}}\right)\left(x - \frac{1}{\sqrt{3}}\right)$. The curve

risers on $(-1, 0)$ and $(1, \infty)$ and falls on $(-\infty, -1)$ and $(0, 1)$.

At $x = \pm 1$ there are local minima and at $x = 0$ a local

maximum. The curve is concave up on $(-\infty, -\frac{1}{\sqrt{3}})$ and

$(\frac{1}{\sqrt{3}}, \infty)$ and concave down on $(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$. At $x = \pm\frac{1}{\sqrt{3}}$

3. there are points of inflection.

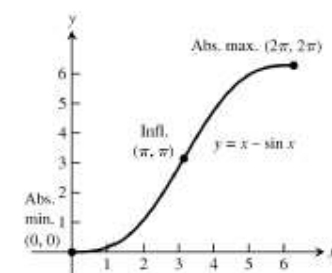
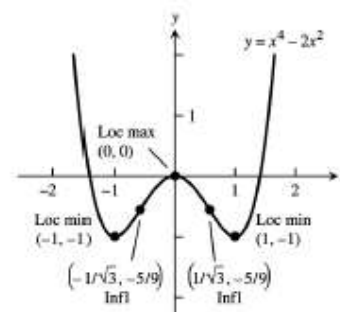
When $y = x - \sin x$, then $y' = 1 - \cos x$ and $y'' = \sin x$.

The curve rises on $(0, 2\pi)$. At $x = 0$ there is a local and

absolute minimum and at $x = 2\pi$ there is a local and absolute

maximum. The curve is concave up on $(0, \pi)$ and concave

down on $(\pi, 2\pi)$. At $x = \pi$ there is a point of inflection.



- 4.

With a volume of 1000 cm and $V = \pi r^2 h$, then $h = \frac{1000}{\pi r^2}$. The amount of aluminum used per can is

$A = 8r^2 + 2\pi rh = 8r^2 + \frac{2000}{r}$. Then $A'(r) = 16r - \frac{2000}{r^2} = 0 \Rightarrow \frac{8r^3 - 1000}{r^2} = 0 \Rightarrow$ the critical points are 0 and 5,

5. but $r = 0$ results in no can. Since $A''(r) = 16 + \frac{4000}{r^3} > 0$ we have a minimum at $r = 5 \Rightarrow h = \frac{40}{\pi}$ and $h:r = 8:\pi$.

$$\tan \theta = x$$

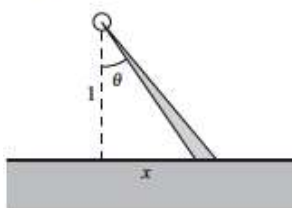
$$\frac{d\theta}{dt} = 3(2\pi) \text{ rad/min}$$

$$\sec^2 \theta \left(\frac{d\theta}{dt} \right) = \frac{dx}{dt}$$

$$\frac{dx}{dt} = (\tan^2 \theta + 1)(6\pi) = 6\pi(x^2 + 1)$$

$$\text{When } x = \frac{1}{2},$$

$$\frac{dx}{dt} = 6\pi \left(\frac{1}{4} + 1 \right) = \frac{15\pi}{2} \text{ km/min} = 450\pi \text{ km/h.}$$



6.

$$\int \frac{t\sqrt{t} + \sqrt{t}}{t^2} dt = \int \left(\frac{t^{1/2}}{t^2} + \frac{t^{1/2}}{t^2} \right) dt = \int (t^{-1/2} + t^{-3/2}) dt = \frac{t^{1/2}}{1/2} + \left(\frac{t^{-1/2}}{-1/2} \right) + C = 2\sqrt{t} - \frac{2}{\sqrt{t}} + C$$

7.

8. $a(t) = v'(t) = 20 \Rightarrow v(t) = 20t + C$; at $(0, 0)$ we have $C = 0 \Rightarrow v(t) = 20t$. When $t = 60$, then $v(60) = 20(60) = 1200 \frac{\text{m}}{\text{sec}}$.

9. A.

$$\Delta x = \frac{1-0}{2} = \frac{1}{2} \text{ and } x_i = i\Delta x = \frac{i}{2} \Rightarrow \text{a lower sum is } \sum_{i=0}^1 \left(\frac{i}{2} \right)^3 \cdot \frac{1}{2} = \frac{1}{2} \left(0^3 + \left(\frac{1}{2} \right)^3 \right) = \frac{1}{16}$$

B.

$$\Delta x = \frac{1-0}{2} = \frac{1}{2} \text{ and } x_i = i\Delta x = \frac{i}{2} \Rightarrow \text{an upper sum is } \sum_{i=1}^2 \left(\frac{i}{2} \right)^3 \cdot \frac{1}{2} = \frac{1}{2} \left(\left(\frac{1}{2} \right)^3 + 1^3 \right) = \frac{1}{2} \cdot \frac{9}{8} = \frac{9}{16}$$

$$(a) \sum_{k=1}^n \left(\frac{1}{n} + 2n \right) = \left(\frac{1}{n} + 2n \right) n = 1 + 2n^2 \quad (b) \sum_{k=1}^n \frac{c}{n} = \frac{c}{n} \cdot n = c$$

$$(c) \sum_{k=1}^n \frac{k}{n^2} = \frac{1}{n^2} \frac{n(n+1)}{2} = \frac{n+1}{2n}$$

10.

$$|x_1 - x_0| = |-1.6 - (-2)| = 0.4, |x_2 - x_1| = |-0.5 - (-1.6)| = 1.1, |x_3 - x_2| = |0 - (-0.5)| = 0.5,$$

11. $|x_4 - x_3| = |0.8 - 0| = 0.8$, and $|x_5 - x_4| = |1 - 0.8| = 0.2$; the largest is $\|P\| = 1.1$.