

1. Find all horizontal asymptotes of  $y = \frac{1+3x-x^2}{x^2+5}$  (10%)  $\Rightarrow y = -1$
2. Let  $f(x) = \sqrt{x^2+3x} \cdot A = \lim_{x \rightarrow -\infty} \frac{f(x)}{x}$ , find  $\lim_{x \rightarrow -\infty} (f(x) - Ax)$  (10%)  $\Rightarrow A = -1$
3.  $f(x) = \sin x \cdot \sin 2x^2 \cdot \sin 3x^3 \cdot \sin 4x^4$ , find  $\lim_{x \rightarrow 0} \frac{f(x)}{x^{10}}$  (10%)  $\Rightarrow \frac{24}{5}$
4. Find A and B such that the function defined by  $f(x) = \begin{cases} Ax^2 + 5x - 9, & x < 1 \\ B, & x = 1 \\ (3-x)(A-2x), & x > 1 \end{cases}$  is continuous for all x. (10%)  $\Rightarrow A = 0, B = -4$
5.  $f(a) = 0$  and  $f'(a) = 3$ , find  $\lim_{x \rightarrow a} \frac{xf(a) - af(x)}{x-a}$  (10%)  $\Rightarrow -3a$
6.  $\frac{d}{dx} \left( \frac{10}{\sqrt{x}} - 9\sqrt[3]{x^5} + 17 \right) = ?$  (10%)  $\Rightarrow -5x^{-\frac{3}{2}} - 15x^{\frac{2}{3}}$
7.  $f(x) = \sqrt{2x + \sqrt{2x + \sqrt{2x + \dots}}}$ , find  $f'(x) = ?$  (10%)  $\Rightarrow f'(x) = \frac{1}{2} f(x)^{-\frac{1}{2}} \cdot f'(x)$
8. Use implicit differentiation to find  $\frac{dy}{dx}$ ,  $x^3 + y^3 = 18xy$  (5%)  $\Rightarrow \frac{dy}{dx} = \frac{6y-x^2}{y^2-6x}$
9. Determine all critical points  $f(x) = \frac{x^2}{x-2}$  (5%)  $\Rightarrow (0, 0), (4, 8)$
10. Find the value or values of c that satisfy the equation  $\frac{f(b)-f(a)}{b-a} = f'(c)$  in the conclusion of the Mean Value Theorem for the functions and intervals. (10%)  $\Rightarrow c = 1$
11. Find an open interval about c on which the inequality  $|f(x) - L| < \epsilon$  holds. Then give a value for  $\delta > 0$  such that for all x satisfying  $0 < |x - c| < \delta$  the inequality  $|f(x) - L| < \epsilon$  holds. (10%)  $\Rightarrow \delta = \frac{\epsilon}{m}$

12. Flying a kite A girl flies a kite at a height of 300 ft, the wind carrying the kite horizontally away from her at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 ft away from her? (10%)  $\frac{dz}{dt}$

13. Find the linearization of  $f(x) = x^9 + 8x^2 + 1$  at  $x = 1$ , and use it to estimate  $f(1.02)$ . (10%) 10.5

12.

$$x^2 + 300^2 = z^2$$

$$\frac{d}{dt} x^2 + 300^2 = \frac{d}{dt} z^2$$

$$\Rightarrow x \cdot \frac{dx}{dt} = z \cdot \frac{dz}{dt}$$

$\frac{dx}{dt} = 25 \text{ ft/sec}$

$z = 500 \text{ ft}$

$300 \text{ ft}$

$x$

girl

$$\frac{dz}{dt} = \frac{25x}{z} \Rightarrow \begin{cases} x = \sqrt{500^2 - 300^2} = 400 \\ z = 500 \end{cases} \Rightarrow \frac{dz}{dt} = \frac{25 \times 400}{500} = 20 \text{ m/s} \quad \#12$$

13.

$$y - y_0 = m(x - x_0)$$

$$\Rightarrow f(x) - f(a) = f'(a)(x - a)$$

$$\Rightarrow f(x) \approx L(x) = f(a) + f'(a)(x - a)$$

$a = 1$  let  $x = 1$ :

$$L(x) = (1 + 8 + 1) + (9 + 16)(x - 1)$$

$$= 10 + 25(x - 1)$$

$$\Rightarrow f(1.02) \approx L(1.02) = 10 + 25(1.02 - 1) = 10.5 \quad \#13$$