國立臺灣科技大學答案卷

National Taiwan University of Science and Technology Answer Sheet

評 分 Score	教師簽章 Signature of Lecturer

姓名/Name ______ 學號/Student ID ______ 班級/Class ______

科目/Course title 微積分 ______ 日期/Date ______

記分欄

從此處開始寫起。試卷用紙務須節用，非經主試認可不得續用其他紙張作答。/Please write from here.

1、

(a) $\lim\limits_{h\to 0}\frac{\sqrt{5h+4}-2}{h}$

$=\lim\limits_{h\to 0}\frac{(\sqrt{5h+4}-2)(\sqrt{5h+4}+2)}{h(\sqrt{5h+4}+2)}$

$=\lim\limits_{h\to 0}\frac{(\sqrt{5h+4})^2-2^2}{h(\sqrt{5h+4}+2)}$

$=\lim\limits_{h\to 0}\frac{5h+4-2^2}{h(\sqrt{5h+4}+2)}$

$=\lim\limits_{h\to 0}\frac{5h}{h(\sqrt{5h+4}+2)}=\frac{5}{\sqrt{4}+2}=\frac{5}{4}$ ※

(b) $\lim\limits_{x\to 9}\frac{\sqrt{x}-3}{x-9}$

$=\lim\limits_{x\to 9}\frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(x-9)(\sqrt{x}+3)}$

$=\lim\limits_{x\to 9}\frac{1}{\sqrt{x}+3}=\frac{1}{6}$ ※

(c) $\lim\limits_{x\to 0}\frac{\tan 2x}{x}$

$=\lim\limits_{x\to 0}\frac{\sin 2x}{x\cos 2x}$

$=\lim\limits_{x\to 0}\frac{1}{\cos 2x}\cdot\frac{\sin 2x}{2x}\cdot 2$

$=\lim\limits_{x\to 0}\frac{1}{\cos 2x}\cdot\lim\limits_{x\to 0}\frac{\sin 2x}{2x}\cdot\lim\limits_{x\to 0}2$

$=1\cdot 1\cdot 2=2$ ※

(d) $\lim\limits_{h\to 0^+}\frac{\sqrt{h^2+4h+5}-\sqrt{5}}{h}$

$=\lim\limits_{h\to 0^+}\frac{(\sqrt{h^2+4h+5}-\sqrt{5})(\sqrt{h^2+4h+5}+\sqrt{5})}{h(\sqrt{h^2+4h+5}+\sqrt{5})}$

$=\lim\limits_{h\to 0^+}\frac{h^2+4h+5-5}{h(\sqrt{h^2+4h+5}+\sqrt{5})}$

$=\lim\limits_{h\to 0^+}\frac{h+4}{\sqrt{h^2+4h+5}+\sqrt{5}}=\frac{4}{2\sqrt{5}}=\frac{2\sqrt{5}}{5}$ ※

(e) $\lim\limits_{x\to\infty}(\sqrt{x+9}-\sqrt{x+4})$

$=\lim\limits_{x\to\infty}\frac{(\sqrt{x+9}-\sqrt{x+4})(\sqrt{x+9}+\sqrt{x+4})}{\sqrt{x+9}+\sqrt{x+4}}$

$=\lim\limits_{x\to\infty}\frac{(x+9)-(x+4)}{\sqrt{x+9}+\sqrt{x+4}}$

$=\lim\limits_{x\to\infty}\frac{5/\sqrt{x}}{(\sqrt{x+9}+\sqrt{x+4})/\sqrt{x}}=\lim\limits_{x\to\infty}\frac{\frac{5}{\sqrt{x}}}{\sqrt{1+\frac{9}{x}}+\sqrt{1+\frac{4}{x}}}=0$ ※

(f) $\lim\limits_{x\to\infty}\sqrt{\frac{8x^2-3}{2x^2+x}}$

$=\sqrt{\lim\limits_{x\to\infty}\frac{8x^2-3}{2x^2+x}}=\sqrt{4}=2$ ※

$\lim\limits_{x\to\infty}\frac{8x^2-3}{2x^2+x}=\lim\limits_{x\to\infty}\frac{8-\frac{3}{x^2}}{2+\frac{1}{x}}=4$

2、$\lim\limits_{x\to 9}\sqrt{x-5}=2$

$\forall\varepsilon>0,\exists\delta>0$ s.t. $\forall x,\ 0<|x-9|<\delta\Rightarrow|\sqrt{x-5}-2|<\varepsilon$

$|\sqrt{x-5}-2|<\varepsilon$

$\Rightarrow -\varepsilon<\sqrt{x-5}-2<\varepsilon$

$\Rightarrow 2-\varepsilon<\sqrt{x-5}<2+\varepsilon$

$\Rightarrow (2-\varepsilon)^2<x-5<(2+\varepsilon)^2$

$\Rightarrow 4-4\varepsilon+\varepsilon^2<x-5<4+4\varepsilon+\varepsilon^2$

$\Rightarrow -4\varepsilon+\varepsilon^2<x-9<4\varepsilon+\varepsilon^2$

$\therefore$ We take $\delta=\min\{\varepsilon^2+4\varepsilon,4\varepsilon-\varepsilon^2\}=4\varepsilon-\varepsilon^2$

$\forall\varepsilon>0,\exists\delta=4\varepsilon-\varepsilon^2,\ 0<\varepsilon<2$

$0<|x-9|<4\varepsilon-\varepsilon^2$

$\Rightarrow \varepsilon^2-4\varepsilon<x-9<4\varepsilon-\varepsilon^2$

$\Rightarrow \varepsilon^2-4\varepsilon+4<x-5<-\varepsilon^2+4\varepsilon+4<\varepsilon^2+4\varepsilon+4$

$\Rightarrow (2-\varepsilon)^2<x-5<(2+\varepsilon)^2$

$\Rightarrow 2-\varepsilon<\sqrt{x-5}<2+\varepsilon$

$\Rightarrow -\varepsilon<\sqrt{x-5}-2<\varepsilon\Rightarrow|\sqrt{x-5}-2|<\varepsilon$ ※

3、$\lim\limits_{x\to 2^+}\frac{1}{x-2}=\infty$

$\forall M>0,\exists\delta>0$ s.t. $\forall x,\ 0<x-2<\delta\Rightarrow\frac{1}{x-2}>M$

$\frac{1}{x-2}>M$

$\Rightarrow x-2<\frac{1}{M}$

$\therefore$ We take $\delta=\frac{1}{M}$

$\forall M>0,\exists\delta=\frac{1}{M}>0$

$0<x-2<\delta=\frac{1}{M}$

$\Rightarrow \frac{1}{x-2}>M>0$ ※

記分欄

轉頁從此開始寫起。

4. $f(x)=\begin{cases} a^2x-2a, & x\geq 2 \\ 12, & x<2 \end{cases}$

$\lim\limits_{x\to 2^+} f(x)=\lim\limits_{x\to 2^+}(a^2x-2a)=2a^2-2a$

$f(2)=2a^2-2a=12$ $\quad 2a(a-1)=12 \Rightarrow a(a-1)=6$

$\lim\limits_{x\to 2^-} f(x)=\lim\limits_{x\to 2^-} 12=12$ $\quad \Rightarrow a^2-a-6=0$

$a=3$ or -2 ※

• Continuity Test:

$\lim\limits_{x\to x_0} f(x)$ exist

$f(x_0)$ exist

$\lim\limits_{x\to x_0} f(x)=f(x_0)$

6. $y=\sqrt{x}$ $\quad y-2=\frac{1}{4}(x-4)$ ※

$\frac{dy}{dx}=\frac{1}{2}x^{-\frac{1}{2}}$

$\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{4}$

$x^{-\frac{1}{2}}=\frac{1}{2}$

$x=4$

7. $y=1+\sqrt{x}$

$\frac{dy}{dx}=\lim\limits_{h\to 0}\frac{(1+\sqrt{x+h})-(1+\sqrt{x})}{h}$

$=\lim\limits_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$

$=\lim\limits_{h\to 0}\frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{(\sqrt{x+h}+\sqrt{x})}$

$=\lim\limits_{h\to 0}\frac{x+h-x}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$ ※

5. $y=\frac{x^2-1}{2x+4}=\frac{(x+1)(x-1)}{2(x+2)}=(\frac{1}{2}x-1)+\frac{3}{2x+4}$

$\lim\limits_{x\to -2^+}\frac{x^2-1}{2x+4}=+\infty$ $\quad y=-2$ is a vertical asymptote.

$\lim\limits_{x\to -2^-}\frac{x^2-1}{2x+4}=-\infty$

$\lim\limits_{x\to \infty}\frac{x^2-1}{2x+4}=\infty$, $\lim\limits_{x\to -\infty}\frac{x^2-1}{2x+4}=-\infty \Rightarrow$ no horizontal asymptote

$$\begin{array}{r} \frac{1}{2}x-1 \\ 2x+4\,\overline{)\,x^2+0x-1} \\ \underline{x^2+2x} \\ -2x-1 \\ \underline{-2x-4} \\ 3 \end{array}$$

$y=\frac{1}{2}x-1$ is a slant line asymptote.

8. $f(x)=x+\frac{9}{x}$

$\frac{d}{dx}f(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim\limits_{h\to 0}\frac{(x+h)+\frac{9}{x+h}-(x+\frac{9}{x})}{h}$

$=\lim\limits_{h\to 0}\frac{h+\frac{9x-9(x+h)}{x(x+h)}}{h}$

$=\lim\limits_{h\to 0}\frac{h+\frac{-9h}{x(x+h)}}{h}=\lim\limits_{h\to 0}(1+\frac{-9}{x(x+h)})$

$=1-\frac{9}{x^2}$ ※

$\frac{d}{dx}f(x)\Big|_{x=3}=1-\frac{9}{3^2}=0$ ※