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$\frac{d}{dx}\tan x = \frac{\cos^2 x+\sin^2 x}{\cos^2 x} = \sec^2 x$

$\frac{d}{dx}\cot x = \frac{-\sin^2 x-\cos^2 x}{\sin^2 x} = -\csc^2 x$

$1-2\sin^2 x = \cos 2x$

$\sin^2 x = \frac{1-\cos 2x}{2}$

1101 Calculus Quiz2 Date:2021/12/15 Total:100

1. $f(x) = x^3 + ax^2 + bx - 3$ has relative extremums at $x = 1, -3$, find a and b.

$a=3$; $b=\cancel{-4}\ -9$

$f'(x) = 3x^2+2ax+b = n(x-1)(x+3) = n(x^2+2x-3)$; $n=3 \Rightarrow f'(x) = 3x^2+6x-9$

$f(x) = x^3+\boxed{3}x^2-\boxed{9}x+C$

2. Find the point of inflection $f(x) = x^2 + 1 + |x^3 - 1|$.

$\cancel{(-\frac{1}{3}, \frac{1}{9})}$ $(-\frac{1}{3}, \frac{58}{27})$, $(1,2)$ 背面

$y = f(-\frac{1}{3}) = \frac{1}{9}+1+|-\frac{1}{27}-1| = \frac{10}{9}+\frac{28}{27}$

$f(x) = \begin{cases} x^2+1+x^3-1 = x^3+x^2 & x \ge 1 \\ x^2+1-x^3+1 = -x^3+x^2+2 & x<1 \end{cases}$ $f'(x) = \begin{cases} 3x^2+2x & x\ge 1 \\ -3x^2+2x & x<1 \end{cases}$ $f''(x) = \begin{cases} 6x+2 & x \ge 1 \\ -6x+2 & x<1 \end{cases}$

$x = -\frac{1}{3}$ or $\boxed{\frac{1}{3}}$ (不合) ∴ $x \ge 1$

3. A box without a top is to be made by cutting small squares, of equal size from the corners of an 8×5 inch piece of card and then tuning up the sides, find the maximum possible volume for the box.

18

$V(x) = x(5-2x)(8-2x) = 40x-10x^2-16x^2+4x^3 = 4x^3-26x^2+40x$

$V'(x) = 12x^2-52x+40 = 4(3x^2-13x+10)$; $x=1$ or $\frac{10}{3}$ (不合)

$V(x)_{max} = V(1) = 1(3)(6) = \boxed{18}$ #3.

4. (a) $\int \frac{t\sqrt{t}+\sqrt{t}}{t^2}dt = 2t^{\frac{1}{2}} - 2t^{-\frac{1}{2}} + C$

$= \int t^{-\frac{1}{2}} + t^{-\frac{3}{2}} dt = \boxed{2t^{\frac{1}{2}} - 2t^{-\frac{1}{2}} + C}$ #4(a)

(b) $\int \sin 2x - \csc^2 x\, dx = \cot x - \frac{1}{2}\cos 2x + C$

$= \int \sin 2x\, dx - \int \csc^2 x\, dx = \boxed{-\frac{1}{2}\cos 2x + C + \cot x + C}$ #4(b)

5. An object is shot straight upward from sea level with an initial velocity of 400 ft/sec. Assuming that gravity is the only force acting on the object, find the upper estimate for its velocity after 5 sec have elapsed. Use $g = 32\ \text{ft/sec}^2$ for the gravitational acceleration

240 ft/s

每段 max，都是 -32

$\Delta V = (-32)(1-0)+(-32)(2-1)+(-32)(3-2)+(-32)(4-3)+(-32)(5-4) = -32\times 5 = -160$

$V_f = V_i + \Delta V = 400-160 = 240$ ft/s

6. Find $\lim\limits_{n\to\infty} \frac{(\frac{1}{2n})^p+(\frac{2}{2n})^p+\dots+(\frac{n}{2n})^p}{(\frac{1}{2}+\frac{1}{2n})^p+(\frac{1}{2}+\frac{2}{2n})^p+\dots+(\frac{1}{2}+\frac{n}{2n})^p}$, $p > 0$ $= \cancel{\frac{4}{3}}\ \frac{1}{1-(\frac{1}{2})^{p+1}}$ 在背面

$\frac{1}{1-(\frac{1}{2})^{p+1}}$

$= \lim\limits_{n\to\infty} \dots = \lim\limits_{n\to\infty} \frac{p(1+2n)}{np+\frac{p(1+n)}{2}} = \lim\limits_{n\to\infty}\frac{p+2np}{np+\frac{p+np}{2}} = \lim\limits_{n\to\infty}\frac{2p+4np}{3np+p} = \boxed{\frac{4}{3}}$ #6

7. Determine the concave upward interval of $y = x^{\frac{1}{3}}(x-4)$

$(-\infty,-2) \cup (0,\infty)$ 見P.221

$y = x^{\frac{4}{3}} - 4x^{\frac{1}{3}}$ $y' = \frac{4}{3}x^{\frac{1}{3}} - \frac{4}{3}x^{-\frac{2}{3}}$ $y'' = \frac{4}{9}x^{-\frac{2}{3}} + \frac{8}{9}x^{-\frac{5}{3}} = \frac{4}{9}x^{-\frac{5}{3}}(x+2)$

up down up
-2 0

8. Find $\int_0^{\frac{\pi}{2}} \frac{3\sin x\cos x}{\sqrt{1+3\sin^2 x}}dx$ $= 1$

$u \Rightarrow 1+3\sin^2 x$, $\frac{du}{dx} = \frac{d}{dx}3\sin^2 x = \frac{3}{2}\left[\frac{d}{dx}1-\cos 2x\right] = \frac{3}{2}\cdot\sin 2x\cdot 2 \Rightarrow du = 6(\sin x\cos x)dx \Rightarrow \int_1^4 \frac{1}{2}\cdot\frac{1}{u^{\frac{1}{2}}}du = \left[\frac{1}{2}\times 2u^{\frac{1}{2}}\right]_1^4 = 2-1 = \boxed{1}$ #8

9. Find $\int \frac{(x-x^3)^{\frac{1}{3}}}{x^4}dx$.

$-\frac{3}{8}(x^{-2}-1)^{\frac{4}{3}}+C$

$= \int \frac{(x^{-2}-1)^{\frac{1}{3}}}{x^3}dx$; $u \Rightarrow x^{-2}-1$, $\frac{du}{dx} = -2x^{-3} \Rightarrow dx = -\frac{1}{2}x^3 du$

$\int -\frac{1}{2}u^{\frac{1}{3}}du = -\frac{1}{2}\times\frac{3}{4}u^{\frac{4}{3}} + C = \boxed{-\frac{3}{8}(x^{-2}-1)^{\frac{4}{3}}+C}$ #9

10. Find the intersections of $f(x) = x^3 - 2x^2 + x - 1$ and $g(x) = -x^2 + 3x - 1$, and the area between two curves.

$(-1,-5), (2,1), (0,-1)$
A: $\frac{37}{12}$

$x^3-2x^2+x-1 = -x^2+3x-1$
$\Rightarrow (x^3-x^2-2x) = 0$
$\Rightarrow x(x^2-x-2) = 0$
$\Rightarrow x = (-1, 2, 0)$
$f(-1) = g(-1) = -1-2-1-1 = -5$
$f(2) = g(2) = 8-8+2-1 = 1$
$f(0) = g(0) = -1$

$\text{Area} = \int_{-1}^0 f(x)-g(x)\,dx + \int_0^2 g(x)-f(x)$
$= \int_{-1}^0 x^3-x^2-2x\,dx + \int_0^2 -x^3+x^2+2x\,dx = \left[\frac{1}{4}x^4-\frac{1}{3}x^3-x^2\right]_{-1}^0 + \left[-\frac{1}{4}x^4+\frac{1}{3}x^3+x^2\right]_0^2$
$= -\frac{1}{4}-\frac{1}{3}+1+(-4)+\frac{8}{3}+4 = \frac{5}{12}+\frac{32}{12} = \boxed{\frac{37}{12}}$ #10.

$1-2+1-1 = -1$
$-1+3-1 = 1$

訂正

$f(x)>g(x)$ $g(x)>f(x)$
-1 0 2

$f(x)$, $g(x)$ 交點: $(-1,-5)$, $(0,-1)$, $(2,1)$

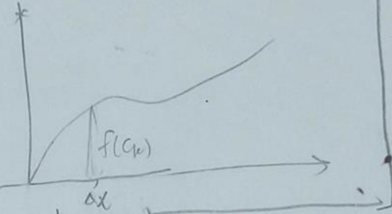
6.

$$\lim_{n\to\infty} \frac{\left(\frac{1}{2n}\right)^p + \left(\frac{2}{2n}\right)^p + \dots \left(\frac{2n}{2n}\right)^p}{\left(\frac{1}{2}+\frac{1}{2n}\right)^p + \left(\frac{1}{2}+\frac{2}{2n}\right)^p + \dots + \left(\frac{1}{2}+\frac{n}{2n}\right)^p} \begin{matrix} \times \frac{1}{2n} \\ \times \frac{1}{2n} \end{matrix}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{2n}\sum_{k=1}^{2n}\left(\frac{k}{2n}\right)^p}{\frac{1}{2n}\sum_{k=1}^{n}\left(\frac{1}{2}+\frac{k}{2n}\right)^p}$$

$$= \lim_{n\to\infty} \frac{\sum_{k=1}^{2n}\left(\frac{k}{2n}\right)^p \times \frac{1}{2n}}{\sum_{k=1}^{n}\left(\frac{1}{2}+\frac{k}{2n}\right)^p \times \frac{1}{2n}} =$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(x)\cdot\Delta x$$



$$\Rightarrow \lim_{n\to\infty} \sum_{k=1}^{n} f\left(a + \frac{b-a}{n}\times k\right)\cdot\frac{b-a}{n}$$

$$\Rightarrow \int_a^b f(x)\,dx$$

$$= \lim_{n\to\infty} \frac{\sum_{k=1}^{2n}\left(0+\frac{1-0}{2n}\times k\right)^p \times \frac{1-0}{2n}}{\sum_{k=1}^{n}\left(\frac{1}{2}+\frac{1-\frac{1}{2}}{n}\times k\right)^p \times \frac{1-\frac{1}{2}}{n}}$$

$$= \frac{\int_0^1 x^p\,dx}{\int_{\frac{1}{2}}^1 x^p\,dx} = \frac{\left[\frac{1}{p+1}x^{p+1}\right]_0^1}{\left[\frac{1}{p+1}x^{p+1}\right]_{\frac{1}{2}}^1}$$

$$= \frac{\cancel{\frac{1}{p+1}}}{\cancel{\frac{1}{p+1}}\left(1-\left(\frac{1}{2}\right)^{p+1}\right)} = \boxed{\frac{1}{1-\frac{1}{2^{p+1}}}}$$ ※

7.

$$f(x) = \begin{cases} x^2+1+x^3-1 = x^3+x^2 & \text{if } x \ge 1 \\ x^2+1-x^3+1 = -x^3+x^2+2 & \text{if } x < 1 \end{cases}$$

$$f'(x) = \begin{cases} 3x^2+2x & \text{if } x \ge 1 \\ -3x^2+2x & \text{if } x < 1 \end{cases}$$

$$f''(x) = \begin{cases} 6x+2 & \text{if } x \ge 1 \\ -6x+2 & \text{if } x < 1 \end{cases}$$

$\lim_{x\to1^+} f''(x) = 8$

$\Rightarrow$ The concavity of $f(x)$ changes sign at $x=1$ $\Rightarrow$ There is a inflection point.

$\lim_{x\to1^-} f''(x) = -4$

(The second derivative of a function at its inflection points can be zero or not exist.)

$\lim_{x\to1^+} f'(x) = 5$

$\ne$ $\Rightarrow$ $\lim_{x\to1} f'(x)$ doesn't exit $\Rightarrow$ $f(x)$ has a corner, there.

$\lim_{x\to1^-} f'(x) = -1$