考試時間是一節課，容易寫不完，得寫快一點
公式有一部分考卷沒有附，作業有寫到的需要記一下

1. (25%) A causal linear time-invariant filter has an impulse response $h[n]$, whose z transform is given by

$$H(z) = \frac{1}{\left(1+\frac{3}{4}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)}$$

(a) Plot the region of convergence of $H(z)$ on the $z-plane$.

(b) Is the filter stable?

(c) Find $h[n]$

(d) Find the output when the input is

$$x[n] = \begin{cases} 1, n \geq 0 \\ 0, n < 0 \end{cases}$$

1(a)

The system is casual ⇒ Right-sided ⇒ ROC: $|z| > |a|$

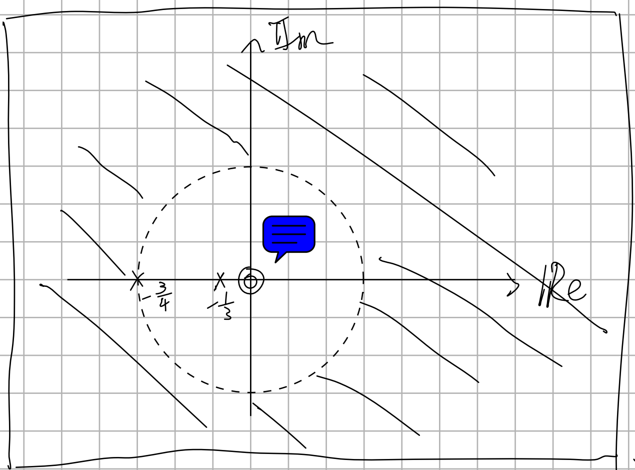
$$H(z) = \frac{1}{(1+\frac{3}{4}z^{-1})(1+\frac{1}{3}z^{-1})} \begin{matrix} \times z^2 \\ \times z^2 \end{matrix} = \frac{z^2}{(z+\frac{3}{4})(z+\frac{1}{3})} \begin{matrix} \text{zeros} \\ \text{poles} \end{matrix}$$

$|-\frac{3}{4}z^{-1}| < 1$ ⇒ $\frac{3}{4} < |z|$

$|-\frac{1}{3}z^{-1}| < 1$ ⇒ $\frac{1}{3} < |z|$

⇒ ROC is the overlapping of these two

⇒ ROC: $|z| > \frac{3}{4}$

⇒ 

#1(a)

1(b) Because the ROC contains the unit circle ⇒ $\boxed{\text{the filter is stable}}$ #1(b)

1(c)

$$H(z) = \frac{1}{(1+\frac{3}{4}z^{-1})(1+\frac{1}{3}z^{-1})} = \frac{A}{1+\frac{3}{4}z^{-1}} + \frac{B}{1+\frac{1}{3}z^{-1}}$$

To solve A, multiply the both sides by $(1+\frac{3}{4}z^{-1})$

$$\frac{1}{(1+\frac{3}{4}z^{-1})(1+\frac{1}{3}z^{-1})} \times(1+\frac{3}{4}z^{-1}) = \frac{A}{1+\frac{3}{4}z^{-1}} + \frac{B}{1+\frac{1}{3}z^{-1}} \times(1+\frac{3}{4}z^{-1})$$

$$\Rightarrow \frac{1}{(1+\frac{1}{3}z^{-1})} = A + \frac{B}{1+\frac{1}{3}z^{-1}}\cdot\left(1+\frac{3}{4}z^{-1}\right)$$

Then set $z^{-1} = -\frac{4}{3}$ to eliminate B:

$$\frac{1}{(1+\frac{1}{3}\cdot(-\frac{4}{3}))} = A + \frac{B}{1+\frac{1}{3}(-\frac{4}{3})}\cdot\underbrace{\left(1+\frac{3}{4}\left(-\frac{4}{3}\right)\right)}_{\to 0}$$

$$\Rightarrow \frac{1}{1-\frac{4}{9}} = A + 0 \Rightarrow A = \frac{9}{5}$$

Use the same approach to solve for B:

$$\frac{1}{(1+\frac{3}{4}z^{-1})(1+\frac{1}{3}z^{-1})} \times(1+\tfrac{1}{3}z^{-1}) = \frac{A}{1+\frac{3}{4}z^{-1}} + \frac{B}{1+\frac{1}{3}z^{-1}} \times(1+\tfrac{1}{3}z^{-1})$$

$$\Rightarrow \left.\frac{1}{1+\frac{3}{4}z^{-1}}\right|_{z^{-1}=-3} = \left.\frac{A}{1+\frac{3}{4}z^{-1}}\cdot\left(1+\frac{1}{3}z^{-1}\right) + B\right|_{z^{-1}=-3}$$

$$\Rightarrow \frac{1}{1+\frac{-9}{4}} = B \Rightarrow B = -\frac{4}{5}$$

$$\Rightarrow H(z) = \frac{\frac{9}{5}}{1+\frac{3}{4}z^{-1}} + \frac{-\frac{4}{5}}{1+\frac{1}{3}z^{-1}}$$

5. $a^n u[n]$ $\quad \frac{1}{1-az^{-1}}$ $\quad |z| > |a|$ All z exc

$$\Rightarrow \boxed{h[n] = \frac{9}{5}\left(-\frac{3}{4}\right)^n u[n] - \frac{4}{5}\left(-\frac{1}{3}\right)^n u[n]}$$ #1(c)

1(d) $Y(z) = X(z)H(z)$

5. $a^n u[n]$ $\quad \frac{1}{1-az^{-1}}$ $\quad |z| > |a|$ All z exc

$$= \left(\frac{1}{1-z^{-1}}\right)\left(\frac{1}{(1+\frac{3}{4}z^{-1})(1+\frac{1}{3}z^{-1})}\right)$$

$$u[n] \overset{z}{\longleftrightarrow} \frac{1}{1-z^{-1}}$$

$$= \frac{A}{1-z^{-1}} + \frac{B}{1+\frac{3}{4}z^{-1}} + \frac{C}{1+\frac{1}{3}z^{-1}}$$

$$\left(\frac{1}{1-z^{-1}}\right)\left(\frac{1}{(1+\frac{3}{4}z^{-1})(1+\frac{1}{3}z^{-1})}\right) = \frac{A}{1-z^{-1}} + \frac{B}{1+\frac{3}{4}z^{-1}} + \frac{C}{1+\frac{1}{3}z^{-1}}$$

$\times(1+\frac{3}{4}z^{-1})(1+\frac{1}{3}z^{-1})(1-z^{-1})$ $\times(1+\frac{3}{4}z^{-1})(1+\frac{1}{3}z^{-1})(1-z^{-1})$

$$\Rightarrow 1 = A(1+\tfrac{3}{4}z^{-1})(1+\tfrac{1}{3}z^{-1}) + B(1-z^{-1})(1+\tfrac{1}{3}z^{-1}) + C(1-z^{-1})(1+\tfrac{3}{4}z^{-1})$$

Let $z^{-1}=-3$:

$$1 = C\underbrace{(1-(-3))}_{=4}\underbrace{(1+\tfrac{3}{4}(-3))}_{=-\frac{5}{4}} \Rightarrow 1 = C\cdot(-5) \Rightarrow C = -\frac{1}{5}$$

Let $z^{-1}=-\frac{4}{3}$:

$$1 = B\underbrace{(1-(-\tfrac{4}{3}))}_{=\frac{7}{3}}\underbrace{(1+\tfrac{1}{3}\cdot(-\tfrac{4}{3}))}_{=1-\frac{4}{9}=\frac{5}{9}} \Rightarrow 1 = B\cdot(\tfrac{35}{27}) \Rightarrow B = \frac{27}{35}$$

Let $z^{-1}=1$:

$$1 = A\underbrace{(1+\tfrac{3}{4})}_{=\frac{7}{4}}\underbrace{(1+\tfrac{1}{3})}_{=\frac{4}{3}} \Rightarrow A = \frac{3}{7}$$

$$= \frac{\frac{3}{7}}{1-z^{-1}} + \frac{\frac{27}{35}}{1+\frac{3}{4}z^{-1}} + \frac{-\frac{1}{5}}{1+\frac{1}{3}z^{-1}}$$

$\xrightarrow{z^{-1}}$ $\boxed{y[n] = \frac{3}{7}u[n] + \frac{27}{35}\cdot(-\frac{3}{4})^n u[n] - \frac{1}{5}(-\frac{1}{3})^n u[n]}$ #1(d)

2. (25%) Consider the following difference equation:

$$y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = 3x[n].$$

(a) Find $H(e^{j\omega})$

(b) Plot block diagram of system

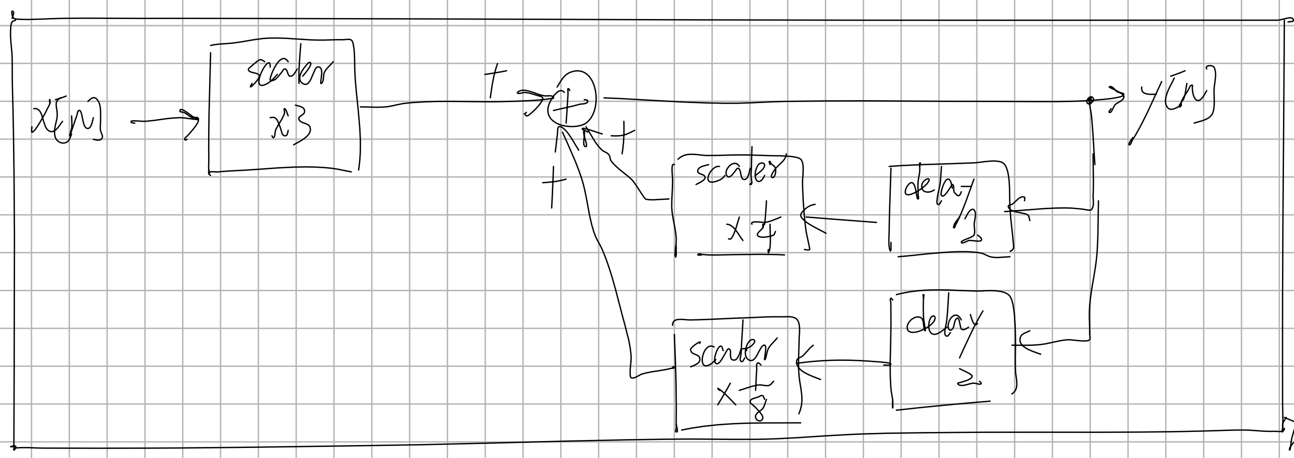
2(a) $Y(e^{j\omega}) - \frac{1}{4}e^{-j\omega}Y(e^{j\omega}) - \frac{1}{8}e^{-j2\omega}Y(e^{j\omega}) = 3X(e^{j\omega})$ $\quad x[n-n_0] \xleftrightarrow{F} e^{-j\omega n_0}X(e^{j\omega})$

Let $x[n]=\delta[n]$, so $X(e^{j\omega})=1$ and $Y(e^{j\omega})=H(e^{j\omega})$ $\quad \delta[n] \xleftrightarrow{F} 1$

$$H(e^{j\omega})\left(1-\tfrac{1}{4}e^{-j\omega}-\tfrac{1}{8}e^{-j2\omega}\right) = 3$$

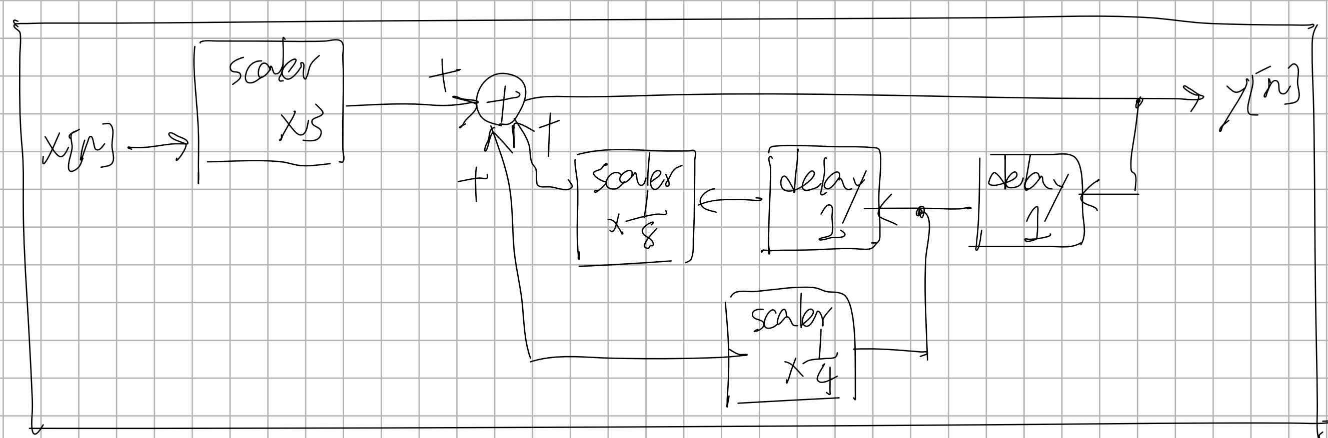
$$\Rightarrow \boxed{H(e^{j\omega}) = \frac{3}{1-\frac{1}{4}e^{-j\omega}-\frac{1}{8}e^{-j2\omega}}}$$ #2(a)

2(b) $y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = 3x[n] \Rightarrow y[n] = 3x[n] + \frac{1}{4}y[n-1] + \frac{1}{8}y[n-2]$



#2(b)

or



#2(b)

3. (25%) A causal LTI system is described by the difference equation:

$$y[n] - 5y[n-1] + 6y[n-2] = 2x[n-1]$$

(a) Determine the homogeneous response of the system, i.e., the possible outputs if $x[n] = 0$ for all n.

(b) Determine the impulse response of the system. $h[n]$

(c) Determine the step response of the system.

3(a)

$y_h[n] - 5y_h[n-1] + 6y_h[n-2] = 0$

$y_h[n] = A_1 z_1^n + A_2 z_2^n$

Solve for z_1, z_2:

$1 - 5z^{-1} + 6z^{-2} = 0 \quad \times z^2$

$\Rightarrow z^2 - 5z + 6 = 0$

$\Rightarrow z_1 = 2, \quad z_2 = 3$

Just as in the case of linear constant-coefficient differential equations for continuous-time systems, without additional constraints or other information, a linear constant-coefficient difference equation for discrete-time systems does not provide a unique specification of the output for a given input. Specifically, suppose that, for a given input $x_p[n]$, we have determined by some means one output sequence $y_p[n]$, so that an equation of the form of Eq. (2.84) is satisfied. Then, the same equation with the same input is satisfied by any output of the form

$$y[n] = y_p[n] + y_h[n], \tag{2.95}$$

where $y_h[n]$ is any solution to Eq. (2.84) with $x[n] = 0$, i.e., a solution to the equation

$$\sum_{k=0}^{N} a_k y_h[n-k] = 0. \tag{2.96}$$

Equation (2.96) is called the *homogeneous difference equation* and $y_h[n]$ the homogeneous solution. The sequence $y_h[n]$ is in fact a member of a family of solutions of the form

$$y_h[n] = \sum_{m=1}^{N} A_m z_m^n, \tag{2.97}$$

where the coefficients A_m can be chosen to satisfy a set of auxiliary conditions on $y[n]$. Substituting Eq. (2.97) into Eq. (2.96) shows that the complex numbers z_m must be roots of the polynomial

$$A(z) = \sum_{k=0}^{N} a_k z^{-k}. \tag{2.98}$$

z_m is the roots of $A(z)$

i.e., $A(z_m) = 0$ for $m = 1, 2, \ldots, N$. Equation (2.97) assumes that all N roots of the polynomial in Eq. (2.98) are distinct. The form of terms associated with multiple roots is slightly different, but there are always N undetermined coefficients. An example of the homogeneous solution with multiple roots is considered in Problem 2.50.

Since $y_h[n]$ has N undetermined coefficients, a set of N auxiliary conditions is required for the unique specification of $y[n]$ for a given $x[n]$. These auxiliary conditions might consist of specifying fixed values of $y[n]$ at specific values of n, such as $y[-1]$, $y[-2], \ldots, y[-N]$, and then solving a set of N linear equations for the N undetermined coefficients.

Alternatively, if the auxiliary conditions are a set of auxiliary values of $y[n]$, the other values of $y[n]$ can be generated by rewriting Eq. (2.84) as a recurrence formula, i.e., in the form

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{m=0}^{M} b_m x[n-m]. \tag{2.84}$$

$\rightarrow a_0 y[n] + \sum_{k=1}^{N} a_k y[n-k]$

$$y[n] = -\sum_{k=1}^{N} \frac{a_k}{a_0} y[n-k] + \sum_{k=0}^{M} \frac{b_k}{a_0} x[n-k]. \tag{2.99}$$

If the input $x[n]$ for all n, together with a set of auxiliary values, say, $y[-1], y[-2], \ldots, y[-N]$, is specified, then $y[0]$ can be determined from Eq. (2.99). With $y[0], y[-1], \ldots, y[-N+1]$ now available, $y[1]$ can then be calculated, and so on. When this procedure is used, $y[n]$ is said to be computed *recursively;* i.e., the output computation involves not only the input sequence, but also previous values of the output sequence.

Ex.

$y[n] + \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = x[n] + x[n-1]$, $N=2$

$y[-1] = 0, \; y[-2] = 1$

$\Rightarrow y_h[n] = A_1 z_1^n + A_2 z_2^n$

$\Rightarrow 1 \cdot z^0 + \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2} = 0 \quad \times 8z^2$

$\Rightarrow 8z^2 + 6z + 1 = 0$

$\Rightarrow z = -\frac{1}{2}$ or $-\frac{1}{4}$ (z_1, z_2)

$\Rightarrow y_h[n] = A_1(-\frac{1}{2})^n + A_2(-\frac{1}{4})^n$

$y[-1] = A_1(-\frac{1}{2})^{-1} + A_2(-\frac{1}{4})^{-1} = 0$

$\Rightarrow -2A_1 + (-4)A_2 = 0$ —(1)

$y[-2] = A_1(-\frac{1}{2})^{-2} + A_2(-\frac{1}{4})^{-2} = 1$

$\Rightarrow 4A_1 + 16A_2 = 1$ —(2)

(2) + 2·(1):

$-4A_1 - 8A_2 = 0$

$0 + 8A_2 = 1 \Rightarrow A_2 = \frac{1}{8}$

$\Rightarrow 4A_1 + 2 = 1 \Rightarrow A_1 = -\frac{1}{4}$

$\Rightarrow \boxed{y[n] = -\frac{1}{4}(-\frac{1}{2})^n + \frac{1}{8}(-\frac{1}{4})^n}$ #

$$\Rightarrow \boxed{y_h[n] = A_1 \cdot 2^n + A_2 \cdot 3^n}$$ #3(a)

3(b)

By the Fourier Transform:

Let $x[n] = \delta[n]$ to obtain the impulse response:

$$h[n] - 5h[n-1] + 6h[n-2] = 2\delta[n-1]$$

$$\overset{\mathcal{F}}{\Rightarrow} H(e^{j\omega}) - 5e^{-j\omega}H(e^{j\omega}) + 6e^{-j2\omega}H(e^{j\omega}) = 2e^{-j\omega}$$

$$\Rightarrow H(e^{j\omega}) = \frac{2e^{-j\omega}}{1 - 5e^{-j\omega} + 6e^{-j2\omega}}$$

$$= \frac{A}{1 - ae^{-j\omega}} + \frac{B}{1 - be^{-j\omega}}$$

solve for a, b:

$$1 - 5e^{-j\omega} + 6e^{-j2\omega} = (1 - ae^{-j\omega})(1 - be^{-j\omega}) = 0 \quad \times e^{j2\omega}$$

$$\Rightarrow e^{j2\omega} - 5e^{j\omega} + 6 = 0$$

$$\begin{matrix} e^{j\omega} & -2 \\ e^{j\omega} & -3 \end{matrix}$$

$$\Rightarrow e^{j\omega} = 2 \text{ or } 3$$

$$\Rightarrow (e^{j\omega} - 2)(e^{j\omega} - 3) = 0 \quad \times e^{-j2\omega}$$

$$\Rightarrow (1 - 2e^{-j\omega})(1 - 3e^{-j\omega}) = 0$$

$$= \frac{A}{1 - 2e^{-j\omega}} + \frac{B}{1 - 3e^{-j\omega}}$$ 通分

solve for A, B:

$$H(e^{j\omega}) = \frac{A(1 - 3e^{-j\omega}) + B(1 - 2e^{-j\omega})}{(1 - 2e^{-j\omega})(1 - 3e^{-j\omega})} = \frac{2e^{-j\omega}}{(1 - 2e^{-j\omega})(1 - 3e^{-j\omega})}$$

$$\Rightarrow A(1 - 3e^{-j\omega}) + B(1 - 2e^{-j\omega}) = 2e^{-j\omega}$$

Let $e^{-j\omega} = \frac{1}{3}$:

$$\cancel{A(1 - 3 \cdot \tfrac{1}{3})}^{\,0} + B(1 - 2 \cdot \tfrac{1}{3}) = \frac{2}{3} \Rightarrow B = 2$$

Let $e^{-j\omega} = \frac{1}{2}$

$$A(1 - \tfrac{3}{2}) = 1 \Rightarrow A = -2$$

$$= \frac{-2}{1-2e^{-j\omega}} + \frac{2}{1-3e^{-j\omega}}$$

> 1 > 1

$k=-\infty$

4. $a^n u[n]$ ($|a| < 1$) $\dfrac{1}{1-ae^{-j\omega}}$

⇒ cannot use the Fourier Transform to obtain $h[n]$ when $|\alpha|>1$

⇒ Try to use the Z-Transform:

$$Y(z) - 5z^{-1}Y(z) + 6z^{-2}Y(z) = 2z^{-1}X(z)$$

$$\Rightarrow Y(z) = \underbrace{\frac{2z^{-1}}{6z^{-2}-5z^{-1}+1}} X(z) = H(z)\cdot X(z)$$

$$\Rightarrow H(z) = \left(\frac{A}{(1-az^{-1})} + \frac{B}{(1-bz^{-1})}\right)$$

solve for a, b:

$$6z^{-2} - 5z^{-1} + 1 = 0 \quad \times z^2 \quad \times z^2 \quad \Rightarrow z^2 - 5z + 6 = 0$$

$$\begin{matrix} 1 & -2 \\ 1 & -3 \end{matrix}$$

$\times z^{-2}$ $\times z^{-2}$

$$\Rightarrow z^2 - 5z + 6 = (z-2)(z-3) = 0$$

$$\Rightarrow (1-2z^{-1})(1-3z^{-1}) = 0 \Rightarrow a=2,\ b=3$$

$$= \frac{A}{1-2z^{-1}} + \frac{B}{1-3z^{-1}}$$

$$A(1-3z^{-1}) + B(1-2z^{-1}) = 2z^{-1}$$

$z^{-1} = \frac{1}{3}$:

$$B\underbrace{\left(1-\frac{2}{3}\right)}_{=\frac{1}{3}} = \frac{2}{3} \Rightarrow B = 2$$

$z^{-1} = \frac{1}{2}$

$$A\underbrace{\left(1-\frac{3}{2}\right)}_{=-\frac{1}{2}} = 1 \Rightarrow A = -2$$

$$= \frac{-2}{1-2z^{-1}} + \frac{2}{1-3z^{-1}}$$

$$\xrightarrow{\mathcal{Z}^{-1}} \boxed{h[n] = -2\cdot 2^n \cdot u[n] + 3\cdot 3^n \cdot u[n]}$$ #3(b)

3(c)

Use the fact that

$$Y(z)=\frac{2z^{-1}}{6z^{-2}-5z^{-1}+1}\,X(z)$$

a=1 case

5. $a^n u[n]$	$\frac{1}{1-az^{-1}}$	All z exc $\lvert z\rvert > \lvert a\rvert$

and let $X(z)=\mathcal{Z}\{u[n]\} = \frac{1}{1-z^{-1}}$:

$$Y(z) = \frac{2z^{-1}}{6z^{-2}-5z^{-1}+1}\cdot\frac{1}{1-z^{-1}}$$

by the partial fraction result on the last page

$$= \frac{A}{1-2z^{-1}} + \frac{B}{1-3z^{-1}} + \frac{C}{1-z^{-1}}$$

$$\because \frac{2z^{-1}}{(6z^{-2}-5z^{-1}+1)(1-z^{-1})} = \frac{A(1-3z^{-1})(1-z^{-1})+B(1-2z^{-1})(1-z^{-1})+C(1-2z^{-1})(1-3z^{-1})}{(6z^{-2}-5z^{-1}+1)(1-z^{-1})}$$

$$\therefore 2z^{-1} = A(1-3z^{-1})(1-z^{-1}) + B(1-2z^{-1})(1-z^{-1}) + C(1-2z^{-1})(1-3z^{-1})$$

$z^{-1}=\frac{1}{3}$:

$$\frac{2}{3} = B\left(1-\frac{2}{3}\right)\left(1-\frac{1}{3}\right) \Rightarrow \frac{2}{3} = B\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) \Rightarrow B = \frac{\cancel{2}}{\cancel{3}}\cdot\frac{\cancel{9}^{3}}{\cancel{2}} = 3$$

$z^{-1}=1$:

$$2 = C\underbrace{(1-2)(1-3)}_{=(-1)(-2)} \Rightarrow C=1$$

$z^{-1}=\frac{1}{2}$:

$$1 = A\left(1-\frac{3}{2}\right)\left(1-\frac{1}{2}\right) \Rightarrow 1 = A\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) \Rightarrow A=-4$$

$$= \frac{-4}{1-2z^{-1}} + \frac{3}{1-3z^{-1}} + \frac{1}{1-z^{-1}}$$

$$\xrightarrow{\mathcal{Z}^{-1}} \boxed{y[n] = -4\cdot 2^n \cdot u[n] + 3\cdot 3^n \cdot u[n] + u[n]}$$ #3(c)

4. (25%) Let $x[n]$ be the sequence with the pole–zero plot shown in Figure P38. Sketch the pole–zero plot for:

(a) $y[n] = \left(\frac{1}{2}\right)^n x[n]$

(b) $\omega[n] = \cos\left(\frac{\pi n}{2}\right) x[n]$

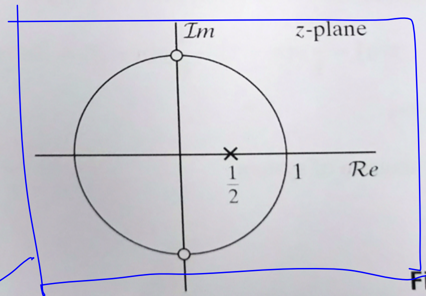


Figure P38

$$X(z) = \frac{(z+j)(z-j)}{(z-\frac{1}{2})}$$

4(a)

$$y[n] = \left(\frac{1}{2}\right)^n x[n]$$

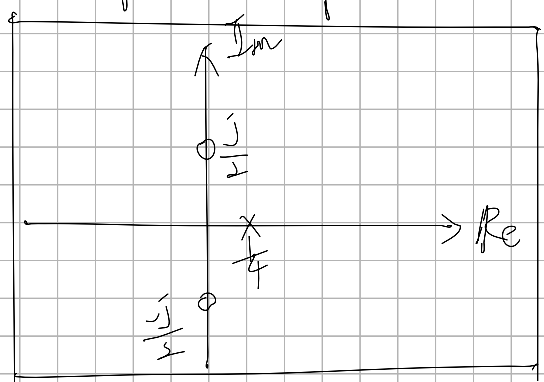
$$z_0^n x[n] \xleftrightarrow{z} X\left(\frac{z}{z_0}\right)$$

$$\xrightarrow{z} Y(z) = X\left(\frac{z}{\frac{1}{2}}\right) = X(2z)$$

$$= \frac{(2z+j)(2z-j)}{(2z-\frac{1}{2})} \begin{matrix} \div 2 \\ \\ \div 2 \end{matrix}$$

$$= \frac{(z+\frac{j}{2})(z-\frac{j}{2})}{z-\frac{1}{4}}$$

$\Rightarrow$ The pole-zero plot is:



#4(a)

4(b)

$$w[n] = \cos\left(\frac{\pi n}{2}\right) x[n]$$

$$= \frac{e^{j\frac{\pi n}{2}} + e^{-j\frac{\pi n}{2}}}{2} \cdot x[n]$$

$$= \frac{1}{2}\left(e^{j\frac{\pi n}{2}} \cdot x[n] + e^{-j\frac{\pi n}{2}} \cdot x[n]\right)$$

$e^{jx} = \cos(x) + j\sin(x)$ —(1)

$e^{-jx} = \cos(x) - j\sin(x)$ —(2)

$$\xrightarrow{\frac{(1)+(2)}{2}} \cos(x) = \frac{e^{jx} + e^{-jx}}{2}$$

$$= \frac{1}{2}\left(\left(e^{j\frac{\pi}{2}}\right)^n x[n] + \left(e^{-j\frac{\pi}{2}}\right)^n \cdot x[n]\right) \qquad z_0^n x[n] \xleftrightarrow{\ Z\ } X\left(\frac{z}{z_0}\right)$$

$$\overset{Z}{\Rightarrow} W(z) = \frac{1}{2}\left(\frac{\left(\frac{z}{e^{j\frac{\pi}{2}}}+j\right)\left(\frac{z}{e^{j\frac{\pi}{2}}}-j\right)}{\left(\frac{z}{e^{j\frac{\pi}{2}}}-\frac{1}{2}\right)} + \frac{\left(\frac{z}{e^{-j\frac{\pi}{2}}}+j\right)\left(\frac{z}{e^{-j\frac{\pi}{2}}}-j\right)}{\left(\frac{z}{e^{-j\frac{\pi}{2}}}-\frac{1}{2}\right)}\right)$$

$$e^{j\frac{\pi}{2}} = \cos\left(\frac{\pi}{2}\right) + j\sin\left(\frac{\pi}{2}\right) = j$$

$$e^{-j\frac{\pi}{2}} = -j$$

$$\cancel{\times j}\ \times (j)^2 \qquad \cancel{\times(-j)}\ \times(-j)^2$$

$$= \frac{1}{2}\left(\frac{\left(\frac{z}{j}+j\right)\left(\frac{z}{j}-j\right)}{\frac{z}{j}-\frac{1}{2}} + \frac{\left(\frac{z}{-j}+j\right)\left(\frac{z}{-j}-j\right)}{\frac{z}{-j}-\frac{1}{2}}\right)$$

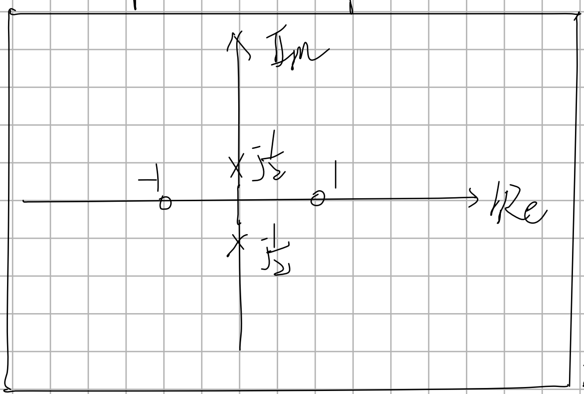
$$\cancel{\times j}\ \times (j)^2 \qquad \cancel{\times(-j)}\ \times(-j)^2$$

$$= \frac{1}{2}\left(\frac{(z-1)(z+1)}{(z-j\frac{1}{2})j} + \frac{(z+1)(z-1)}{(z+j\frac{1}{2})\cdot(-j)}\right)$$

$$= \frac{1}{2}\left(\frac{(z-1)(z+1)\underbrace{(z+j\tfrac{1}{2})\cdot(-j)}_{-jz+\frac{1}{2}} + (z-1)(z+1)\underbrace{(z-j\tfrac{1}{2})\cdot j}_{jz+\frac{1}{2}}}{(z-j\frac{1}{2})(z+j\frac{1}{2})\cdot\underbrace{(j)\cdot(-j)}_{=1}}\right)$$

$$= \frac{1}{2}\,\frac{(z-1)(z+1)\left(\cancel{-jz}+\tfrac{1}{2}\cancel{+jz}+\tfrac{1}{2}\right)}{(z-j\frac{1}{2})(z+j\frac{1}{2})} \rightarrow 1$$

$\Rightarrow$ The pole-zero plot is:



#4(b)