1. $t = 5\sin\theta,\ -\frac{\pi}{2} < \theta < \frac{\pi}{2},\ dt = 5\cos\theta\, d\theta,\ \sqrt{25 - t^2} = 5\cos\theta;$

$\int \sqrt{25 - t^2}\, dt = \int (5\cos\theta)(5\cos\theta)\, d\theta = 25\int \cos^2\theta\, d\theta = 25\int \frac{1+\cos 2\theta}{2}\, d\theta = 25\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right) + C$

$= \frac{25}{2}(\theta + \sin\theta\cos\theta) + C = \frac{25}{2}\left[\sin^{-1}\left(\frac{t}{5}\right) + \left(\frac{t}{5}\right)\left(\frac{\sqrt{25-t^2}}{5}\right)\right] + C = \frac{25}{2}\sin^{-1}\left(\frac{t}{5}\right) + \frac{t\sqrt{25-t^2}}{2} + C$

2. $v = \sin\theta,\ -\frac{\pi}{2} < \theta < \frac{\pi}{2},\ dv = \cos\theta\, d\theta,\ (1 - v^2)^{5/2} = \cos^5\theta;$

$\int \frac{v^2\, dv}{(1-v^2)^{5/2}} = \int \frac{\sin^2\theta\cos\theta\, d\theta}{\cos^5\theta} = \int \tan^2\theta\sec^2\theta\, d\theta = \frac{\tan^3\theta}{3} + C = \frac{1}{3}\left(\frac{v}{\sqrt{1-v^2}}\right)^3 + C$

3. $x = \sec\theta,\ 0 < \theta < \frac{\pi}{2},\ dx = \sec\theta\tan\theta\, d\theta,\ \sqrt{x^2 - 1} = \sqrt{\sec^2\theta - 1} = \tan\theta;$

$\int \frac{dx}{x\sqrt{x^2-1}} = \int \frac{\sec\theta\tan\theta\, d\theta}{\sec\theta\tan\theta} = \theta + C = \sec^{-1}x + C$

4. $\frac{x^2}{(x-1)(x^2+2x+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2} \Rightarrow x^2 = A(x+1)^2 + B(x-1)(x+1) + C(x-1);\ x = -1$

$\Rightarrow C = -\frac{1}{2};\ x = 1 \Rightarrow A = \frac{1}{4};$ coefficient of $x^2 = A + B \Rightarrow A + B = 1 \Rightarrow B = \frac{3}{4};\ \int \frac{x^2\, dx}{(x-1)(x^2+2x+1)}$

$= \frac{1}{4}\int \frac{dx}{x-1} + \frac{3}{4}\int \frac{dx}{x+1} - \frac{1}{2}\int \frac{dx}{(x+1)^2} = \frac{1}{4}\ln|x-1| + \frac{3}{4}\ln|x+1| + \frac{1}{2(x+1)} + C = \frac{\ln|(x-1)(x+1)^3|}{4} + \frac{1}{2(x+1)} + C$

5. $\int \frac{e^t\, dt}{e^{2t} + 3e^t + 2} = [e^t = y]\int \frac{dy}{y^2 + 3y + 2} = \int \frac{dy}{y+1} - \int \frac{dy}{y+2} = \ln\left|\frac{y+1}{y+2}\right| + C = \ln\left(\frac{e^t+1}{e^t+2}\right) + C$

6. $\int_{-\infty}^{\infty} \frac{x\, dx}{(x^2+4)^{3/2}} = \int_{-\infty}^{0} \frac{x\, dx}{(x^2+4)^{3/2}} + \int_{0}^{\infty} \frac{x\, dx}{(x^2+4)^{3/2}};\ \begin{bmatrix} u = x^2 + 4 \\ du = 2x\, dx \end{bmatrix} \rightarrow \int_{\infty}^{4} \frac{du}{2u^{3/2}} + \int_{4}^{\infty} \frac{du}{2u^{3/2}}$

$= \lim_{b\to\infty}\left[-\frac{1}{\sqrt{u}}\right]_b^4 + \lim_{c\to\infty}\left[-\frac{1}{\sqrt{u}}\right]_4^c = \lim_{b\to\infty}\left(-\frac{1}{2} + \frac{1}{\sqrt{b}}\right) + \lim_{c\to\infty}\left(-\frac{1}{\sqrt{c}} + \frac{1}{2}\right) = \left(-\frac{1}{2} + 0\right) + \left(0 + \frac{1}{2}\right) = 0$

7. $\int_0^2 \frac{ds}{\sqrt{4 - s^2}} = \lim_{b\to 2^-}\left[\sin^{-1}\frac{s}{2}\right]_0^b = \lim_{b\to 2^-}\left(\sin^{-1}\frac{b}{2}\right) - \sin^{-1}0 = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

8. $\int_1^{\infty} \frac{dx}{1+x^3};\ 0 \le \frac{1}{x^3+1} \le \frac{1}{x^3}$ for $1 \le x < \infty$ and $\int_1^{\infty} \frac{dx}{x^3}$ converges $\Rightarrow \int_1^{\infty} \frac{dx}{1+x^3}$ converges by the Direct Comparison Test.

9. $\lim_{n\to\infty} \sqrt{n}\sin\left(\frac{1}{\sqrt{n}}\right) = \lim_{n\to\infty} \frac{\sin\left(\frac{1}{\sqrt{n}}\right)}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\cos\left(\frac{1}{\sqrt{n}}\right)\left(-\frac{1}{2n^{3/2}}\right)}{-\frac{1}{2n^{3/2}}} = \lim_{n\to\infty} \cos\left(\frac{1}{\sqrt{n}}\right) = \cos 0 = 1 \Rightarrow$ converges

10. $\lim_{n\to\infty} \frac{(\ln n)^5}{\sqrt{n}} = \lim_{n\to\infty}\left[\frac{\left(\frac{5(\ln n)^4}{n}\right)}{\left(\frac{1}{2\sqrt{n}}\right)}\right] = \lim_{n\to\infty} \frac{10(\ln n)^4}{\sqrt{n}} = \lim_{n\to\infty} \frac{80(\ln n)^3}{\sqrt{n}} = \ldots = \lim_{n\to\infty} \frac{3840}{\sqrt{n}} = 0 \Rightarrow$ converges

11. $\frac{2n+1}{n^2(n+1)^2} = \frac{1}{n^2} - \frac{1}{(n+1)^2} \Rightarrow s_k = \left(1 - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{9}\right) + \left(\frac{1}{9} - \frac{1}{16}\right) + \ldots + \left[\frac{1}{(k-1)^2} - \frac{1}{k^2}\right] + \left[\frac{1}{k^2} - \frac{1}{(k+1)^2}\right]$

$\Rightarrow \lim_{k\to\infty} s_k = \lim_{k\to\infty}\left[1 - \frac{1}{(k+1)^2}\right] = 1$

12. convergent geometric series with sum $\frac{\left(\frac{3}{2}\right)}{1 - \left(-\frac{1}{2}\right)} = 1$