1091 Calculus Midterm Exam ET161A001/002 Date:2020/11/11 Total:120

Note: Don't use the L'Hopital's rule and calculator. To get full points, you should write down the procedure **in detail**.

1. Find the following limits. (5 points for each)

   (a) $\lim_{x\to 9} \dfrac{\sin(\sqrt{x}-3)}{x-9}$

   (b) $\lim_{x\to\infty} \dfrac{\sin x}{\lfloor x \rfloor}$

   (c) $\lim_{x\to 0} \dfrac{\tan 3x}{2x^2+5x}$

   (d) $\lim_{x\to\infty} \sqrt{2x^2+1}\sin(\dfrac{1}{x})$

2. Prove/Disprove $\lim_{x\to c}\sqrt{x} = \sqrt{c}, c > 0$ by the precise definition. (10 points)

3. Prove/Disprove $\lim_{x\to -5} \dfrac{1}{(x+5)^2} = \infty$ by the precise definition. (10 points)

4. Show how to extend the function $f(x) = \dfrac{\tan x}{\tan(\tan x)}$ to be continuous at the origin. i.e. Define a new function and consider $f(0)$. (10 points)

5. Let $f$ be defined on an interval $(a, b)$ and suppose that $f(c) \neq 0$ at some $c$ where $f$ is continuous. Show that there is an interval $(c-\delta, c+\delta)$ about $c$ where $f$ has the same sign as $f(c)$. (10 points)

6. Find all asymptotes for $y = \dfrac{x^3-3x^2+3x-1}{x^2+x-2}$. Indicate the type of asympotes. (10 points)

7. Find $f'(x)$, if $f(x) = \sqrt{5x\sqrt{5x\sqrt{5x\sqrt{.....}}}}$. (10 points)

8. The impedance $Z$ (ohms) in a series circuit is related to the resistance $R$ (ohms) and reactance $X$ (ohms) by the equation $Z = \sqrt{R^2+X^2}$. If $R$ is increasing at 3 ohms/sec and $X$ is decreasing at 2 ohms/sec, at what rate is $Z$ changing when $R$=10 ohms and $X$=20 ohms? (10 points)

9. Use the linearization of differential to approximately evaluate $\sqrt[3]{27.12}$. (10 points)

10. Find the local extrema and absolute extrema of $f(x) = \dfrac{x}{x^2+1}$. (10 points)

11. Show that the equation $6x^5 + 13x + 1 = 0$ has exactly one real root. (10 points)