(課 EX.4-8)

1. Consider a very long coaxial cable. The inner conductor has a radius ***a*** and is maintained at a potential $\boldsymbol{V_0}$. The outer conductor has an inner radius ***b*** and is grounded. Determine the potential distribution in the space between the conductors. (10%)

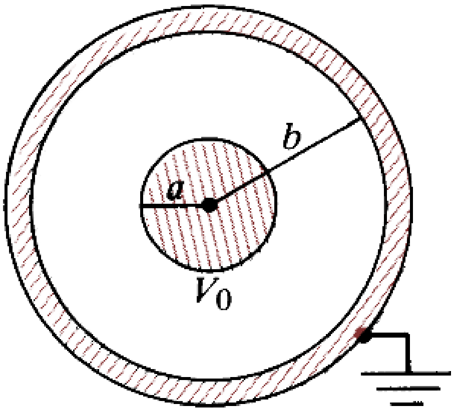


Fig. 1

Ans:

Solution Figure 4–18 shows a cross section of the coaxial cable. We assume no z-dependence and, by symmetry, also no ϕ-dependence ($k = 0$). Therefore, the electric potential is a function of r only and is given by Eq. (4–130).

The boundary conditions are

$$V(b) = 0, \tag{4-131a}$$

$$V(a) = V_0. \tag{4-131b}$$

Substitution of Eqs. (4–131a) and (4–131b) in Eq. (4–130) leads to two relations:

$$C_1 \ln b + C_2 = 0, \tag{4-132a}$$

$$C_1 \ln a + C_2 = V_0. \tag{4-132b}$$

From Eqs. (4–132a) and (4–132b), C_1 and C_2 are readily determined:

$$C_1 = -\frac{V_0}{\ln (b/a)}, \qquad C_2 = \frac{V_0 \ln b}{\ln (b/a)}.$$

Therefore, the potential distribution in the space $a \le r \le b$ is

$$V(r) = \frac{V_0}{\ln (b/a)} \ln \left(\frac{b}{r}\right). \tag{4-133}$$

Obviously, equipotential surfaces are coaxial cylindrical surfaces. ∎

(習 EX.4-7)

2. A point charge ***Q*** exists at a distance ***d*** above a large grounded conducting plane. Determine

   (a) the surface charge density ρ_s. (5%)

   (b) the total charge induced on the conducting plane. (5%)

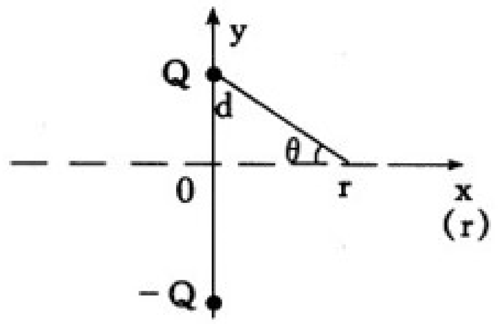


Fig. 2

Ans:

$$\bar{E}|_{y=0} = -\bar{a}_y \frac{Q}{4\pi\epsilon R^2}(2\sin\theta) = -\bar{a}_y \frac{Q_d}{2\pi\epsilon\ (d^2+r^2)^{3/2}}.$$

a) $\rho_s = \bar{a}_y \cdot \epsilon\bar{E}|_{y=0} = \dfrac{Q_d}{2\pi\ (d^2+r^2)^{3/2}}.$

b) $\int_0^{\infty} \rho_s 2\pi r dr = -Q$.

(課 EX.5-6 類似題)

3. A conducting material of uniform thickness ***h*** and conductivity σ has the shape of a quarter of a flat circular washer, with inner radius *a* and outer radius ***b***, as shown in Fig.3. Determine the resistance between the end faces. (20%)

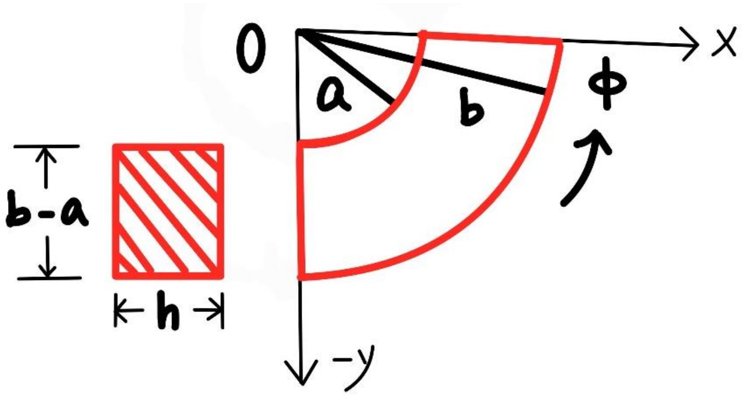
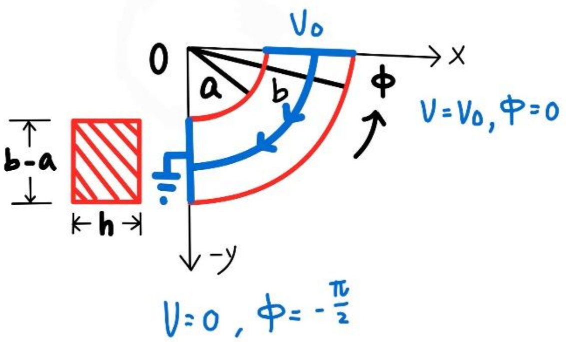


Fig. 3

Ans:



$V = V_0$ at $\phi = 0$

$V = 0$ at $\phi = -\frac{\pi}{2}$

$V = C_1\phi + C_2$

$$\begin{cases} V_0 = C_1(0) + C_2 \\ 0 = C_1(-\frac{\pi}{2}) + C_2 \end{cases}$$

$C_1 = +\frac{2V_0}{\pi}, \quad C_2 = 0$

$V = +\frac{2}{\pi}V_0\phi$

$\vec{E} = -\hat{a}_\phi \frac{1}{r}\frac{\partial V}{\partial \phi} \Rightarrow \vec{E} = -\hat{a}_\phi\left(+\frac{2V_0}{\pi r}\right)$

$\vec{J} = \sigma\vec{E} \Rightarrow \vec{J} = -\hat{a}_\phi \frac{2\sigma V_0}{\pi r}$

$d\vec{s} = -\hat{a}_\phi h\,dr$

$I = \int_s \vec{J}\cdot d\vec{s} = \int_a^b \frac{2\sigma V_0}{\pi r} h\,dr$

$\Rightarrow I = +\frac{2\sigma V_0 h}{\pi}\ln\frac{b}{a}$

$R = \frac{V_0}{I} \Rightarrow R = \frac{\pi}{2\sigma h \ln\frac{b}{a}}$ ※

(習 EX. 5-22)

4. Assume a rectangular conducting sheet of conductivity σ, width $\boldsymbol{a}$, and height $\boldsymbol{b}$. A pontential difference $\boldsymbol{V_0}$ is applied to the applied to the side edges, as shown in Fig.4.
   Find
   (a) the potential distribution. (10%)
   (b) the current density everywhere within the sheet. (Hint: Solve Laplace's equation in Cartesian coordinates subject to appriate boundary conditions.) (10%)

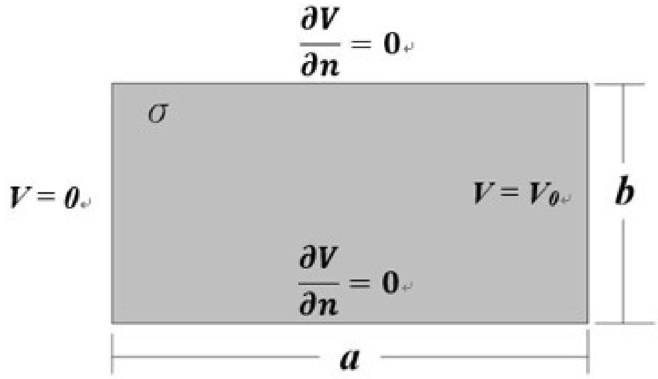


Fig. 4

Ans:

P.5-22 Specified boundary conditions can be satisfied by solutions of Laplace's equation with zero separation constants:

$k_x = k_y = 0 \quad X(x) = A_0x + B_0, \quad Y(y) = C_0y + D_0$.

$B_0 = C_0 = 0 \qquad V(x) = A_0D_0x$

a) At $x = a$, $V(a) = V_0 = A_0D_0a \longrightarrow A_0D_0 = \frac{V_0}{a}$

$\therefore V = \frac{V_0}{a}x$

b) $\overline{E} = -\overline{\nabla}V = -\bar{a}_x\frac{V_0}{a} \longrightarrow \bar{J} = \sigma\overline{E} = -\bar{a}_x\frac{\sigma V_0}{a}$.

(課 EX.6-1)

5. An infinitely long, straight conductor with a circular cross section of radius ***b*** carries a steady current ***I***. Determine the magnetic flux density both inside and outside the conductor. (20%)

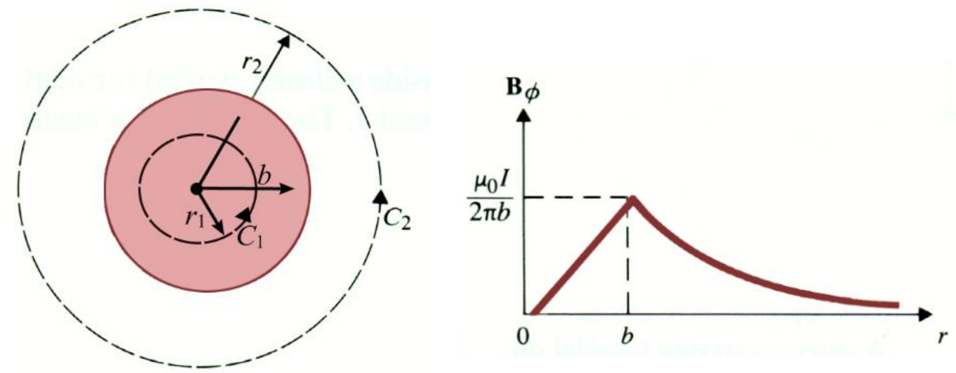


Fig. 5

Ans:

a) *Inside the conductor:*

$$\mathbf{B}_1 = \mathbf{a}_\phi B_{\phi 1}, \qquad d\boldsymbol{\ell} = \mathbf{a}_\phi r_1 d\phi$$

$$\oint_{C_1} \mathbf{B}_1 \cdot d\boldsymbol{\ell} = \int_0^{2\pi} B_{\phi 1} r_1 \, d\phi = 2\pi r_1 B_{\phi 1}.$$

The current through the area enclosed by C_1 is

$$I_1 = \frac{\pi r_1^2}{\pi b^2} I = \left(\frac{r_1}{b}\right)^2 I.$$

Therefore, from Ampère's circuital law,

$$\mathbf{B}_1 = \mathbf{a}_\phi B_{\phi 1} = \mathbf{a}_\phi \frac{\mu_0 r_1 I}{2\pi b^2}, \qquad r_1 \le b. \qquad (6\text{–}11\text{a})$$

b) *Outside the conductor:*

$$\mathbf{B}_2 = \mathbf{a}_\phi B_{\phi 2}, \qquad d\boldsymbol{\ell} = \mathbf{a}_\phi r_2 \, d\phi$$

$$\oint_{C_2} \mathbf{B}_2 \cdot d\boldsymbol{\ell} = 2\pi r_2 B_{\phi 2}.$$

Path C_2 outside the conductor encloses the total current I. Hence

$$\mathbf{B}_2 = \mathbf{a}_\phi B_{\phi 2} = \mathbf{a}_\phi \frac{\mu_0 I}{2\pi r_2}, \qquad r_2 \ge b. \qquad (6\text{–}11\text{b})$$

(習 EX.6-15)

6. An off-center cylindrical cavity that is cut into a very long cylindrical conductor carrying a uniform current density. Refer to the cross section in Fig.6. The uniform axial current density is **J** = $\mathbf{a}_z J$. Find the magnitude and direction of **B** in the cylindrical cavity whose axis is displaced from that of the conducting part by a distance *d*. (*Hint*: Use principle of superposition and consider **B** in the cavity as that due to two long cylindrical conductors with radius b and a and current densities **J** and –**J**, respectively.) (20%)

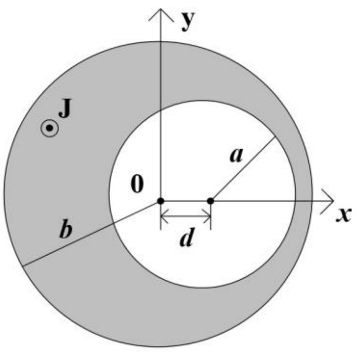
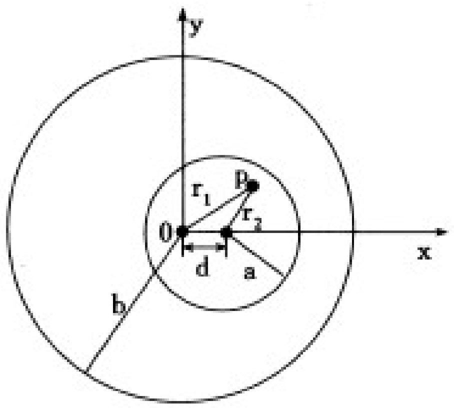


Fig. 6

Ans:

P.6-15 $\bar{J} = \bar{a}_z J$. $\oint \bar{B}\cdot d\bar{l} = \mu_0 I$.



If there is no hole,

$$2\pi r_1 B_{\phi 1} = \mu_0 \pi r_1^2 J$$

$$\longrightarrow B_{\phi 1} = \frac{\mu_0 r_1}{2} J \longrightarrow \begin{cases} B_{x_1} = -\dfrac{\mu_0 J}{2} y_1, \\ B_{y_1} = -\dfrac{\mu_0 J}{2} x_1 . \end{cases}$$

For $-\bar{J}$ in the hole portion:

$$B_{\phi 2} = -\frac{\mu_0 r_2}{2} J \longrightarrow \begin{cases} B_{x_2} = -\dfrac{\mu_0 J}{2} y_2, \\ B_{y_2} = -\dfrac{\mu_0 J}{2} x_2 . \end{cases}$$

Superposing $B_{\phi 1}$ and $B_{\phi 2}$ and noting that $y_1 = y_2$ and $x_1 = x_2 + d$,

we have $B_x = B_{x1} + B_{x2} = 0$, and $B_y = B_{y1} + B_{y2} = \frac{\mu_0 J}{2} d$.