10



Note: To get full points, you should write down the procedure **in detail**.

1. (15 points) Find the derivative of the following functions. (5 points for each)
   (a) $f(x) = (\ln x)^{\cos x}$ $= (\ln x)^{\cos x} \cdot \left[\frac{\cos x}{x \ln x} - (\sin x)(\ln(\ln x))\right]$
   (b) $f(x) = e^x + e^{e^x} + e^{e^{e^x}}$ $= e^x\left[e^{(e^{e^x}+e^x)} + e^{e^x} + 1\right]$
   (c) $f(x) = \ln\left(\tan \frac{x}{2}\right)$ $= \csc x$

2. (15 points) Evaluate the following integrals. (5 points for each)
   (a) $\int_1^2 \frac{e^{1/x}}{x^2}\,dx$ $= e - e^{\frac{1}{2}}$
   (b) $\int \sin\theta \cos(\cos\theta)\,d\theta$ $= -\sin(\cos\theta) + C$
   (c) $\int_0^1 x^2 \sqrt[3]{1-x}\,dx$ $= \frac{27}{140}$

3. (15 points) At which points on the curve $y = 1 + 40x^3 - 3x^5$ does the tangent line have the largest slope? $(2, 225)$ , $(-2, -220)$

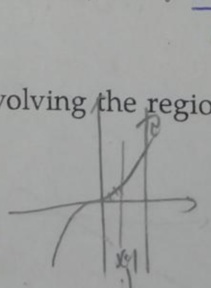
4. (15 points) If $f$ is a continuous function such that

$$\int_1^x f(t)\,dt = (x-1)e^{2x} + \int_1^x e^{-t} f(t)\,dt$$

for all $x$. In order to solve $f(x)$, please answer the following questions. (5 points for each)
   (a) Find $\frac{d}{dx}\left[(x-1)e^{2x}\right]$ $= e^{2x}(2x-1)$
   (b) Express $\frac{d}{dx}\int_1^x e^{-t} f(t)\,dt$ in terms of $f(x)$ and exponential function. $= \frac{1}{e^x} \cdot f(x)$ $e^{-x} f(x)$
   (c) Utilize the results of (a) and (b), find an explicit formula for $f(x)$. $= \frac{e^{3x}(2x-1)}{e^x - 1}$

5. (10 points) Find the volume of the solid generated by revolving the region bounded by $y = x^3$, $y = 0$, and $x = 1$ about $x = 2$. $\frac{6}{10}\pi$

$2\pi \cdot \int_0^1 (2-x)\cdot y\,dx$



$\frac{80\sqrt{10} - 13\sqrt{13}}{27}$

$\frac{8}{27} \cdot \left(10\sqrt{10} - \frac{13\sqrt{13}}{8}\right)$

6. (10 points) If $f(x) = x\sqrt{3+x^2}$, find $(f^{-1})'(-2) = ?$ $\frac{2}{5}$

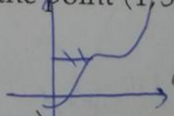
$f'(x) = \sqrt{3+x^2} + x \cdot \frac{1}{2}(3+x^2)^{-\frac{1}{2}} \cdot 2x$ $= x^2(3+x^2)$ $x^4 + 3x^2 = 4$ $f'(x) = \frac{1}{(f^{-1})'(f(x))}$

7. (10 points) Find the arc length of the graph of $x = (y-4)^{3/2}$ from the point $(1, 5)$ to $(8, 8)$.

~~$16\sqrt{2} - 5\sqrt{5}$~~

$\frac{80\sqrt{10} - 13\sqrt{13}}{27}$

$\int \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$



$du = \frac{9}{4}dy$

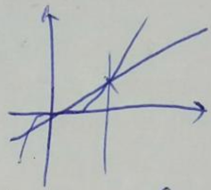
(背面還有題目 / P.T.O. / Please turn over.)

$\frac{3}{2}(y-4)^{\frac{1}{2}}$ $\frac{9}{4}(y-4)$ $1 + \frac{9}{4}y - 9$ $\frac{9}{4}y - 8$ $\int \sqrt{\frac{9}{4}y - 8}$ $\frac{4}{9}\int_{\frac{13}{4}}^{10} u^{\frac{1}{2}}\,du = \frac{4}{9}\cdot\frac{2}{3}\left[u^{\frac{3}{2}}\right]_{\frac{13}{4}}^{10}$

8. (15 points) Find the areas of the region where the region is **under** $y = \frac{4}{\pi}x$ and **above** $y = \tan x$, between $x = 0$ and the first intersection of the curve $(\frac{\pi}{4}, 1)$.

$\frac{1}{8}\pi + \ln\frac{\sqrt{2}}{2}$

$\frac{2}{3}$ $\frac{\pi}{6}$ $= \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$



9. (15 points) Let $f(x) = x - \frac{1}{6}x^2 - \frac{2}{3}\ln x$.

Please answer the following questions. (5 points for each)

(a) Find the intervals of increase or decrease. increa: $(0,1)\cup(2,\infty)$ decrea: $(1,2)$

(b) Find the local maximum and minimum values. max: $\frac{5}{6}$ min: $(2-\frac{2}{3}) - \frac{2}{3}(\ln 2)$

(c) Find the intervals of concavity and the inflection points. $(\sqrt{2}, \sqrt{2} - \frac{1}{3} - \frac{1}{3}\ln 2)$

$\int \tan x \, dx$ $y = \frac{\sin x}{\cos x}$

$= \int \frac{\sin x}{\cos x} dx$

$u \Rightarrow \cos x$

$du = -\sin x$

$= -\int \frac{1}{u} dx = -\ln|\cos x| + C$

0.23
3)0.7

4.333
0.469
97
0.7
0.67
0.49
0.42
0.469

4
0.56
0.94