

1 Find the arc length of the curve  $x(t) = e^t \cos t, y(t) = e^t \sin t, 0 \leq t \leq \frac{\pi}{2}$ . (10 points)

2 Find the area common(共同部分的面積) to the circle  $r = 3 \cos \theta$  and cardioid(心臟線)  $r = 1 + \cos \theta$ . (10 points)

3 If  $z = f(x, y)$ , where  $x = r \cos \theta, y = r \sin \theta$ , show that

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial z}{\partial \theta}\right)^2. \quad (10 \text{ points})$$

4 The power consumed(消耗)in an electrical resistor(電阻器)is given by  $P = \frac{E^2}{R}$  watts. If  $E = 100$  volts and  $R = 8$  ohms, use total differential to estimate the change of  $P$  if  $E$  is increased by 1 volts and  $R$  is increased by 0.2 ohms. (10 points)

5 If  $F(x, y, z) = xe^{yz} + ye^{xz} + xyz = 0$ , find  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$ . (10 points)

6 Let  $f(x, y, z) = z \tan^{-1}(\frac{y}{x})$ . (a) Find the direction of  $f$  at  $(1, 1, 3)$  along which  $f$  increases most rapidly. (10 points) (b) What is the directional derivative in this direction? (5 points)

7 Find the tangent plane and normal line to the surface  $\sqrt{xy} + \sqrt{yz} + \sqrt{xz} = 11$  at the point  $(1, 4, 9)$ . (10 points)

8 Let  $f(x, y) = x^3 - 12xy + y^3$ . Find the local extreme values and saddle point(s) (if any). (12 points)

9 The volume of a rectangular box(長方體盒子)without top(沒有頂部)is to be  $32 m^3$ . Use Lagrange Multiplier to find the minimum surface area(表面積). (12 points)

10 Evaluate  $\int_0^9 \int_{\sqrt{x}}^3 \sin(y^3) dy dx$ . (10 points) (Hint: change the order of integration)

11 Evaluate  $\int_0^1 \int_0^{\sqrt{1-x^2}} \cos(x^2 + y^2) dy dx$ . (10 points) (Hint: change to polar coordinates)

12 Evaluate  $\iiint_S \sqrt{x^2 + y^2} dV$ , where  $S$  is the solid inside the cylinder(柱面)  $r = 2$  between  $z = 0$  and  $z = 2$ . (10 points) (Hint: Use cylindrical coordinates)