

- ① The relative number of gas molecules (氣體分子) in a container (容器) that travel at a velocity of  $v$  cm/sec is  $f(v) = cv^2 \exp(-\frac{mv^2}{2kT})$ , where  $T$  is the temperature in  $^{\circ}K$ ,  $m$  is the mass of a molecule and  $c, k$  are positive constants. Find the maximum value of  $f$ . (10 pts)  $V=0 \Rightarrow f(0)=0$

- 2 Find the derivatives of the following functions (a)  $f(x) = (\sqrt{x})^{\sqrt{x}}$  (8 pts)  
 $\rightarrow \sqrt{3+\cos^2(x)} \cdot \ln 2 \cdot 2^x - \sqrt{3+\cos^2(\ln x)} \cdot \frac{1}{x} \rightarrow (\sqrt{x})^{\sqrt{x}} + \ln(\sqrt{x}) \cdot \frac{1}{2} \cdot (\sqrt{x})^{\sqrt{x}-1}$   
 (b)  $g(x) = \int_{\ln x}^{2x} \sqrt{3+\cos^2 t} dt$  (8 pts) (c)  $h(x) = \ln |\log_2(\sec x + \tan x)|$  (8 pts)  
 $\times$  化簡不寫  $\rightarrow \frac{\sec x}{\log_2(\sec x + \tan x) \cdot \ln 2}$

- 3 Find the integrals of the following functions  
 $-\frac{1}{2}(\ln 1 - \ln 2)$   $\frac{1}{2} \ln 2$   $\frac{\pi}{8} = -\frac{1}{2} \ln(\frac{\sqrt{2}}{2})$   $-\ln(1 - \ln 2)$  換底拆開  
 (a)  $\int_0^{\frac{\pi}{2}} \frac{\cos x \sin x}{\cos^2 x + 1} dx$  (8 pts) (b)  $\int_0^{\frac{\pi}{8}} \tan 2x dx$  (8 pts) (c)  $\int_1^2 \frac{dx}{x - x \ln x}$  (8 pts)

- 4  $f(x) = e^x + \ln(x+1)$ ,  $x > -1$

- (a) Show that  $f$  has inverse (5 pts) (b) Find  $(f^{-1})'(1)$  (5 pts)  
 $\frac{1}{x+1}$   $x=0 \rightarrow f(0)=1$

- 5 Find the area of the region between  $y^2 = 4x$  and  $4x - 3y = 4$  by integration with respect to  $y$ . (8 pts)  $\frac{125}{24}$

- 6 Find the volume of the solid generated by revolving the region bounded by the curve  $y = x^3$ , the  $y$ -axis and the line  $y = 1$  about the  $y$ -axis (a) by disc method (8 pts) (b) by cylindrical shell method. (8 pts)  $\frac{3}{5}\pi$

- 7 Let the base (底部) of a solid be the first quadrant (第一象限) plane region bounded by  $y = \sqrt{x}$  and  $y = x^2$ . Suppose that the cross sections (截面) perpendicular (垂直) to the  $x$ -axis are squares (正方形). Find the volume of the solid. (10 pts)  $\frac{9}{70}$

- 8 Find the length of the curve  $y = (4 - \sqrt[3]{x^2})^{\frac{3}{2}}$  between  $x=1$  and  $x=27$ . (10 pts)

- 9 Find the area of the surface generated by revolving the curve  $y = \frac{x^3}{3}$ ,  $1 \leq x \leq \sqrt{7}$  about the  $x$ -axis. (8 pts)  $\frac{27}{5}\pi$