Calculus Final Exam. 2011/6/16

1Find the area of the region inside the cardiod $r = 2(1+\cos\theta)$ and outside the circle $r=1$ (10%)

2 Let $V = f(x+ct) + g(x-ct)$,where f and g are any functions possessing continuous second derivatives, c is a constant.
Show that $\frac{\partial^2 V}{\partial t^2} = c^2 \frac{\partial^2 V}{\partial x^2}$ (8%)

3 The temperature at a point in space is $T = 100 - x^2 - y^2 - 2z^2$. In what direction should one move from the point (2,1,1) in order to cool off as rapidly as possible? (6%)

4 Find the equation of the tangent plane and the normal line to the surface $\tan^{-1}(\frac{y}{x}) - z = 0$ at the point $(1,1,\frac{\pi}{4})$ (10%)

5 In electrostatics, the force $\vec{F}$ of attraction between two particles of opposite charge is given by $\vec{F} = \frac{k\vec{r}}{\|\vec{r}\|^3}$ (Coulomb's law), where k is a constant and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}, \|\vec{r}\| = \sqrt{x^2+y^2+z^2}$.
Show that $\vec{F} = \nabla(\frac{-k}{\|\vec{r}\|})$ (10%)

6 Use linear approximation to approximate $f(x,y) = \sin(\pi xy + \ln y)$ at the point (0.01,1.05) (10%)

7 Determine the values of m and b so that the sum S of the squares of the vertical distances of the points (0,2),(1,3) and (2,5) from the line $y = mx + b$ shall be a minimum. (10%)

8 Evaluate $\iiint_D x\,dV$, where D is the region bounded by the planes $x=0, y=0, z=0, x+\frac{y}{2}+\frac{z}{3}=1$ (12%)

9 Evaluate $\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln(x^2+y^2+1)\,dxdy$ (12%)

10 Find the volume of the region bounded above by the paraboloid $z = 5 - x^2 - y^2$ and below by the paraboloid $z = 4x^2 + 4y^2$ (12%)

11 Find the volume of the "ice-cream cone" that is bounded by the cone $\phi = \frac{\pi}{6}$ and the sphere $\rho = 2\cos\phi$ (12%)