陳柏辰

Total: 100 points

$\nabla f(0, \frac{\pi}{3}) = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

c. decrease most rapidly direction : $\vec{v} = -\nabla f(0, \frac{\pi}{3}) = -\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}$

1. (**20** points) Let $f(x, y) = e^x \sin y$:
   (a) Find ∇f. $\nabla f = e^x \sin y\,\hat{i} + \cos y \cdot e^x\,\hat{j}$
   (b) Find the directional derivative of f at the point $(0, \pi/3)$ in the direction of $\vec{\mathbf{v}} = -6\hat{\mathbf{i}} + 8\hat{\mathbf{j}}$.
   (c) In which direction does f **decrease most rapidly** at $(0, \pi/3)$.
   (d) In which direction does f have **zero change** at $(0, \pi/3)$.

2. (**20** points) Find all the local maxima, local minima, and saddle point(s) of the function $f(x, y) = x^3 + y^3 - 3xy$.

$\frac{\partial f}{\partial x} = 3x^2 - 3y \Rightarrow x^2 = y$, $\frac{\partial^2 f}{\partial x^2} = 6x$, $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = -3$

$\frac{\partial f}{\partial y} = 3y^2 - 3x \Rightarrow y^2 = x$, $\frac{\partial^2 f}{\partial y^2} = 6y$

$(0,0)$: $f_{xx} = 0$, $f_{xx} \cdot f_{yy} - f_{xy}^2 = 0 - 9 = -9$ $(0,0)$ saddle point

$(1,1)$: $f_{xx} = 6$, $f_{xx} \cdot f_{yy} - f_{xy}^2 = 36 - 9 = 27$, $f(1,1) = -1$ local min

3. (**20** points) Find an equation for the plane tangent to the level surface $f(x, y, z) = \ln(x - 2y) - z = 0$ at the point $P_0(3, 1, 0)$. Also, find parametric equations for the line that is normal to the surface at P_0.

$1 \cdot (x-3) - 2 \cdot (y-1) - z = 0$

$x - 2y - z - 1 = 0$

$\frac{\partial f}{\partial x} = \frac{1}{x-2y} \cdot 1$, $\frac{\partial f}{\partial y} = \frac{1}{x-2y} \cdot -2$, $\frac{\partial f}{\partial z} = -1$

no local max

local min on $(1,1) = -1$

saddle point on $(0,0)$

4. (**10** points) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $e^z - xyz = 0$.

$\frac{\partial z}{\partial x} = \frac{yz}{e^z - xy}$

$\frac{\partial z}{\partial y} = \frac{xz}{e^z - xy}$

5. (**10** points) Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ if $z = (x - y)^5$ and $x = s^2 t$, $y = st^2$.

$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$

$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$

6. (**20** points) A function $f(x, y) = xe^{y+x^2}$.
   (a) Find the linearization $L(x, y)$ of the function $f(x, y)$ at the point $(2, -4)$.
   (b) Utilize the result in (a) to estimate the value of $f(x, y)$ when $x = 2.05, y = -3.92$.

$\frac{\partial f}{\partial x} = e^{y+x^2} + xe^{y+x^2} \cdot (2x) = e^{y+x^2} \cdot (1 + 2x^2)$

$L(x,y) = f(2,-4) + \frac{\partial f}{\partial x}\big|_{(2,-4)} \cdot (x-2) + \frac{\partial f}{\partial y}\big|_{(2,-4)} (y+4)$

$= 2 + 9 \cdot (x-2) + 2 \cdot (y+4)$

$\frac{\partial f}{\partial y} = xe^{y+x^2}$

$f(2,-4) = 2 \cdot e^{(4-4)} = 2$

$f(x,y)$, $x = 2.05$, $y = -3.92$

$f(2.05, -3.92) \approx L(2.05, -3.92) = 2.61$

$\frac{\partial f}{\partial x}\big|_{(2,-4)} = e^0 \cdot (1+8) = 9$

$L(x,y) = 2 + 9 \cdot (\frac{5}{100}) + 2 \cdot (\frac{8}{100})$

$= 2 + \frac{45}{100} + \frac{16}{100}$

$= 2 + \frac{61}{100}$

$= \frac{261}{100}$ #

$\frac{\partial f}{\partial y}\big|_{(2,-4)} = 2 \cdot e^0 = 2$