$f(x) = x^2 + 2x + 3$

Engineering mathematics II

Midterm exam., 4/21/2020

This is an open-book test. The total score is 110 points. Please show your computations.

1.(10%) Assume that the matrix

$$\underline{\underline{A}} = \begin{bmatrix} 1 & -3 & 5 & 7 \\ -2 & 6 & -6 & 4 \end{bmatrix}.$$

is the augmented matrix corresponding to a system of linear equations. Write down the corresponding system of linear equations, and then solve it.

2. Consider the matrix

$$\underline{\underline{A}} = \begin{bmatrix} 1 & -3 & 0 \\ -2 & -1 & 4 \\ 4 & 1 & 3 \end{bmatrix}.$$

(a). (10%) Find $\underline{\underline{A}}^{-1}$ (i.e. the inverse of $\underline{\underline{A}}$).

(b). (5%) Find the determinant of $\underline{\underline{A}}$.

(c). (5%) Find the determinant of $\underline{\underline{A}}^{-1}$.

3. Consider the matrices below:

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 3 \\ 1 & 0 & 2 \end{bmatrix}, \quad \underline{\underline{B}} = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 3 \\ 2 & 2 & 6 \end{bmatrix}, \quad \underline{\underline{C}} = \begin{bmatrix} 1 & 2 & 4 \\ 0 & -1 & -3 \\ 2 & 2 & 6 \end{bmatrix}.$$

(a). (5%) Find an elementary matrix $\underline{\underline{E}}$ such that $\underline{\underline{E}}\,\underline{\underline{A}} = \underline{\underline{B}}$.

(b). (5%) Find an elementary matrix $\underline{\underline{F}}$ such that $\underline{\underline{F}}\,\underline{\underline{B}} = \underline{\underline{C}}$.

(c). (5%) Find a matrix $\underline{\underline{G}}$ such that $\underline{\underline{G}}\,\underline{\underline{A}} = \underline{\underline{C}}$.

4.(10%) It is known that when two rows of a square matrix are swapped, the determinant of the resultant matrix is equal to the negative of the original matrix's determinant. Based on this fact, please show that if a matrix has two identical rows, then its determinant is equal to 0.
<Hint:> After the row swapping, what does the resultant matrix look like, as compared to the original matrix?

5.(10%) The set $W = \{(x, y, z) | x - 3 \cdot a \cdot y + 2 \cdot b \cdot z = c; x, y, z \in \mathcal{R}\}$ is a subset of $\mathcal{R}^3$. Find the values of a, b, and c such that W is a subspace of $\mathcal{R}^3$.
<Hint:> Apply the sub-space test.

6. Let us consider the matrix

$$\underline{\underline{A}} = \begin{bmatrix} 1 & -2 & 1 & 1 & 2 \\ -1 & 3 & 0 & 2 & -2 \\ 0 & 1 & 1 & 3 & 4 \\ 1 & 2 & 5 & 13 & 5 \end{bmatrix} .$$

Its rref (reduced row echelon form) is

$$\begin{bmatrix} 1 & 0 & 3 & 7 & 0 \\ 0 & 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} .$$

(a). (5%) Find a basis for row-spsce($\underline{\underline{A}}$).

(b). (5%) Find a basis for column-spsce($\underline{\underline{A}}$).

(c). (5%) row-rank($\underline{\underline{A}}$) = ?

(d). (5%) column-rank($\underline{\underline{A}}$) = ?

(e). (5%) rank($\underline{\underline{A}}$) = ?

(f). (5%) Find a basis for null-spsce($\underline{\underline{A}}$).

(g). (5%) nullity($\underline{\underline{A}}$) = ?

7.(10%) Let us consider the linear transformation $T : \mathcal{R}^3 \mapsto \mathcal{R}^2$ which is defined by

$$T(x, y, z) = (x + 2y, -x + z) \quad .$$

Find a basis for the null space of T.