

1 Let  $f(x) = \int_1^x \sqrt{e^t + \ln(t^2 + 1)} dt$ , find  $(f^{-1})'(0)$  (8%)

2 Find the derivative of the following functions: (a)  $f(x) = (\log_3 x)^{\ln x}$  (8%)

(b)  $g(x) = \log_x(e^{3x} + 10^x)$  (8%) (c)  $h(x) = \int_2^{\exp(x^2)} \frac{dt}{\sqrt{\ln t}}$  (8%)

3 Evaluate the following integrals: (a)  $\int_0^{\sqrt{3}} x^3 \sqrt{x^2 + 1} dx$  (8%)

$u = 2^{\sin x}$   
 $du = 2^{\sin x} \cos x dx$

(b)  $\int_1^{\sqrt{e}} \frac{\sin(\pi \ln x)}{x} dx$  (8%) (c)  $\int_{\frac{\pi^2}{16}}^{\frac{\pi^2}{9}} \frac{2^{\sin \sqrt{x}} \cos(\sqrt{x})}{\sqrt{x}} dx$  (8%)

4 Find the area of the region bounded by  $y^2 = 4x$  and  $x^2 = 4y$  (8%)

5 Find the volume of the solid obtained by revolving the region in the first quadrant (第一象限) bounded by  $y = 2 - x^2$  and  $y = x^2$  about the  $y$ -axis (8%)

6 Find the volume of the solid generated by revolving the region bounded by  $y = 4x - x^2$ ,  $y = 0$  about the line  $x = -1$  (8%)

7 The base (底部) of a solid is the first-quadrant bounded by  $4x + 5y = 20$

and the coordinate axes (座標軸). Find the volume if every plane

section perpendicular (垂直) to the  $x$ -axis is a semicircle (半圓).

(10%)

8 Find the arc length of the catenary (懸鍊線)  $y = \frac{a}{2} (e^{\frac{x}{a}} + e^{-\frac{x}{a}})$  from  $x = 0$

to  $x = a$ , where  $a > 0$  is a constant. (10%)

9 Find the area of the surface of revolution generated by revolving

the arc of  $y^2 = 12x$ ,  $0 \leq x \leq 3$  about the  $x$ -axis (10%)