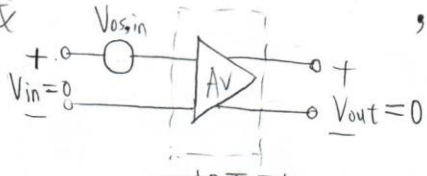
AIC2 期中考

Q1:

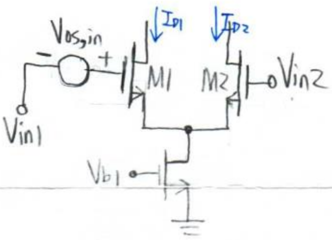
(a) 當電路由於mismatch造成 $V_{in}=0$ 但 $V_{out}\neq 0$ 時，可將其等效成



理想電路

，又由於題目上 $M3=M4$

且 $R_1=R_2$，因此只需考慮 $M1$、$M2$ 的 mismatch 對 $V_{os,in}$ 造成的影響



令 $V_{TH1}=V_{TH}$, $V_{TH2}=V_{TH}+\Delta V_{TH}$

$\left(\frac{W}{L}\right)_1=\left(\frac{W}{L}\right)$, $\left(\frac{W}{L}\right)_2=\left(\frac{W}{L}\right)+\Delta\left(\frac{W}{L}\right)$

因此 $V_{out}=0$ 時，$I_{D1}=I_{D2}=I_D$

$$V_{os,in}=\sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}}+V_{TH1}-\sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}}-V_{TH2}$$

$$=\sqrt{\frac{2I_D}{\mu_n C_{ox}}}\left[\sqrt{\frac{1}{\left(\frac{W}{L}\right)_1}}-\sqrt{\frac{1}{\left(\frac{W}{L}\right)_2}}\right]-\Delta V_{TH}$$

$$=\sqrt{\frac{2I_D}{\mu_n C_{ox}}}\left[\sqrt{\frac{1}{\left(\frac{W}{L}\right)}}-\sqrt{\frac{1}{\left(\frac{W}{L}\right)+\Delta\left(\frac{W}{L}\right)}}\right]-\Delta V_{TH}$$

$$= \sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}}\left[1-\sqrt{\frac{1}{1+\Delta(W/L)/(W/L)}}\right]-\Delta V_{TH}$$

假設 $\Delta(W/L)/(W/L) \ll 1$

且根據Taylor series $\sqrt{1+\varepsilon}\approx 1+\frac{\varepsilon}{2}$, $\left(\sqrt{1+\varepsilon}\right)^{-1}\approx 1-\frac{\varepsilon}{2}$

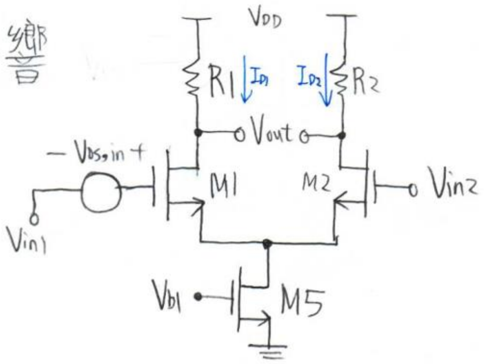
$$V_{os,in}=\sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}}\left[1-1+\frac{\Delta(W/L)/(W/L)}{2}\right]-\Delta V_{TH}$$

$$=\sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}}\;\frac{\Delta(W/L)/(W/L)}{2}-\Delta V_{TH}$$

equilibrium overdrive voltage

$$=\frac{V_{GS}-V_{TH}}{2}\cdot\frac{\Delta(W/L)}{(W/L)}-\Delta V_{TH}$$

(b) 在不使用M3、M4且考慮M1≠M2，R1≠R2對$V_{os,in}$造成的影響



令 $R_1=R_D$, $R_2=R_D+\Delta R_D$

$I_{D1}=I_D$, $I_{D2}=I_D+\Delta I_D$

在 $V_{out}=0$ 時， $I_{D1}R_1=I_{D2}R_2$

$$V_{os,in}=\sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}}+V_{TH1}-\sqrt{\frac{2I_{D2}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}}-V_{TH2}$$

$$=\sqrt{\frac{2}{\mu_n C_{ox}}}\left[\sqrt{\frac{I_D}{\left(\frac{W}{L}\right)}}-\sqrt{\frac{I_D+\Delta I_D}{\left(\frac{W}{L}\right)+\Delta\left(\frac{W}{L}\right)}}\right]-\Delta V_{TH}$$

$$=\sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}}\left[1-\sqrt{\frac{1+\Delta I_D/I_D}{1+\Delta(W/L)/(W/L)}}\right]-\Delta V_{TH}$$

假設 $\Delta(W/L)/(W/L) \ll 1$ 且 $\Delta I_D/I_D \ll 1$

根據 Taylor series $\sqrt{1+\varepsilon_1} \approx 1+\frac{\varepsilon_1}{2}$, $\left(\sqrt{1+\varepsilon_2}\right)^{-1} \approx 1-\frac{\varepsilon_2}{2}$

$$=\sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}}\left\{1-\left[\left(1+\frac{(\Delta I_D/I_D)}{2}\right)\cdot\left(1-\frac{\Delta(W/L)/(W/L)}{2}\right)\right]\right\}-\Delta V_{TH}$$

$$\approx\sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}}\left[-\frac{\Delta I_D}{2I_D}+\frac{\Delta(W/L)}{2(W/L)}\right]-\Delta V_{TH}$$

又由於 $I_{D1}R_1 = I_{D2}R_2 \Rightarrow I_D R_D = (I_D+\Delta I_D)\cdot(R_D+\Delta R_D)$

$$= I_D R_D + I_D\Delta R_D + \Delta I_D R_D + \underbrace{\Delta I_D \Delta R_D}_{0}$$

$$\Rightarrow \frac{\Delta I_D}{I_D} \approx -\frac{\Delta R_D}{R_D}$$

$$\approx \frac{1}{2}\sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}}\left[\frac{\Delta R_D}{R_D}+\frac{\Delta(W/L)}{(W/L)}\right]-\Delta V_{TH}$$

$$=\frac{V_{GS}-V_{TH}}{2}\left[\frac{\Delta R_D}{R_D}+\frac{\Delta(W/L)}{(W/L)}\right]-\Delta V_{TH}$$

\#

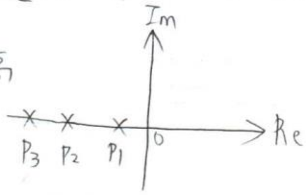
Q2:

(a) 當不考慮 C_c 時，三個pole分別為

$$P_1 = \frac{-1}{(r_{o6}\|r_{o7})\,C_2} \qquad P_2 = \frac{-1}{(r_{o2}\|r_{o4})\,C_1} \qquad P_3 = \frac{-1}{\left(\frac{1}{g_{m3}}\right)\cdot C_3}$$

此 C_2 包含負載電容 C_L

由於 $C_L >> C_1, C_3 \Rightarrow P_1$ 的頻率較低，$g_m r_o >> 1 \Rightarrow P_3$ 的頻率較高

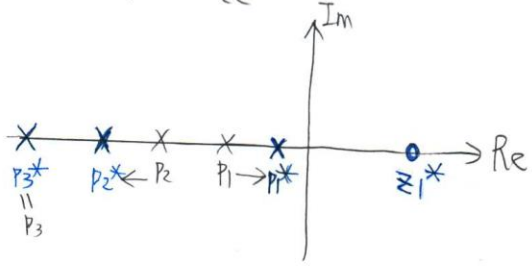


X: pole

(b) 當加上 C_c 時，由於 Miller effect 使第一級輸出(V_{o1})上等效為一個 $g_{m6}(r_{o6}\|r_{o7})C_c$ 的對地電容，因此在 $g_m r_o >> 1$ 時，V_{o1} 上的pole成為主要極點(dominant pole)。C_c 的添加也會造成極點分離，因此pole從原本的 $\frac{-1}{(r_{o6}\|r_{o7})C_2}$ 變為 $\frac{-g_{m6}}{C_1+C_2}$，使pole2外移進而提高 Phase Margin。

$$P_1^* = \frac{-1}{(r_{o2}\|r_{o4})\left[g_{m6}(r_{o6}\|r_{o7})C_c\right]} \qquad P_2^* = \frac{-g_{m6}}{C_1+C_2} \qquad P_3^* = \frac{-g_{m3}}{C_3}$$

$$z_1^* = \frac{+g_{m6}}{C_c}$$

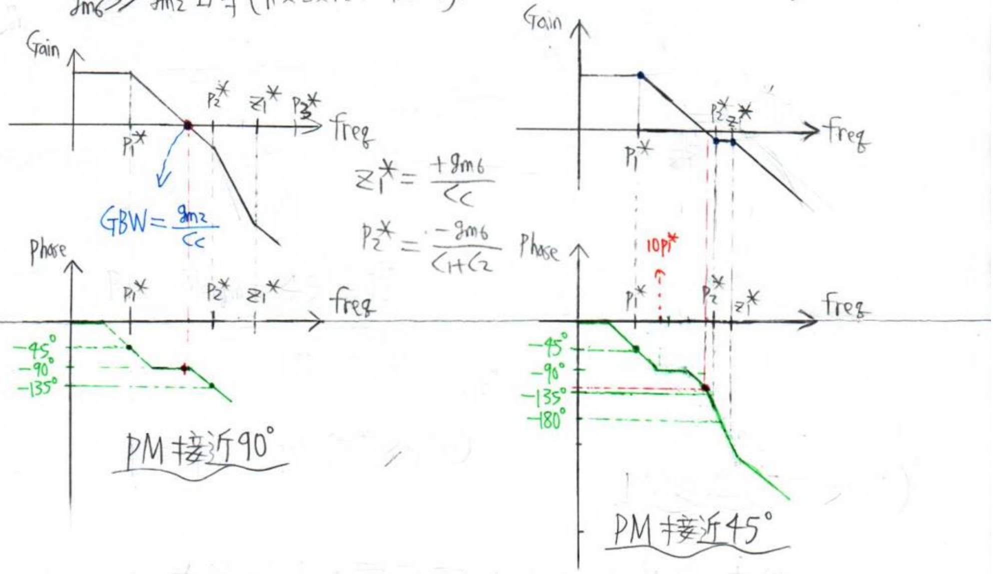


X: pole

O: zero

(c) 如 Q2(b) 的圖中所示，加上 C_c 可將 P_2 移至 P_2^* (P_2 外移) 來改善 Phase Margin，但仍要注意加上 C_c 也引進了"右半平面的 zero ($Z_1^* = \frac{+g_{m6}}{C_c}$)。此時的 OPA 若 Gain Bandwidth (GBW) 內only dominant pole 時，其 $GBW = \frac{g_{m2}}{C_c}$，因此"在固定的 GBW 規格下"，我們需要讓 $\boxed{g_{m6} \gg g_{m2}}$ 來避免 GBW 產生下列情形！

$g_{m6} \gg g_{m2}$ 時 (假設 P_2^* 在 GBW 外)：　　$g_{m6} < g_{m2}$ 時 ($P_2^* \geq Z_1^*$)



若 P_2^* 和 Z_1^* 較近且較靠近 GBW 時，會使 PM 快速下降!!

⇒ 因此在設計上，需要設計較大的 I_{D7} 或 $(W/L)_6$ 來使 P_2^*、Z_1^* 遠離 GBW

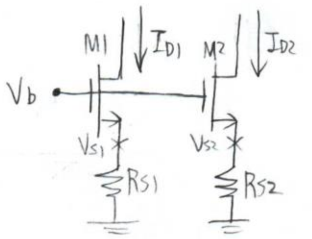
Q3:

傳統 Current Mirror 的 $I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2$

$$\Delta I_D = \underbrace{\frac{\partial I_D}{\partial(\mu_n C_{ox})}\cdot\Delta(\mu_n C_{ox}) + \frac{\partial I_D}{\partial(W/L)}\cdot \Delta(W/L) + \frac{\partial I_D}{\partial V_{TH}}\cdot\Delta V_{TH}}_{微積分}$$

$$\Rightarrow \frac{\Delta I_D}{I_D} = \left[\frac{\Delta(\mu_n C_{ox})}{\mu_n C_{ox}} + \frac{\Delta(W/L)}{(W/L)} - \frac{2\Delta V_{TH}}{V_{GS}-V_{TH}}\right]$$

Source degeneration ($R_{S1}=R_{S2}=R_S$):



欲求 $\frac{\Delta I_D}{I_D} = \frac{I_{D1}-I_{D2}}{I_D}$ 的推導

$$\begin{cases} I_D = \frac{1}{2}\underbrace{\mu_n C_{ox}}_{令為\beta}\frac{W}{L}\left[(V_b-V_S)-V_{TH}\right]^2 \\ V_S = I_D\cdot R_S \end{cases}$$

$$令\begin{cases} (W/L)_1 = W/L, & (W/L)_2 = (W/L)+\Delta(W/L) \\ \beta_1 = \beta, & \beta_2 = \beta+\Delta\beta \\ V_{TH1} = V_{TH}, & V_{TH2} = V_{TH}+\Delta V_{TH} \\ I_{D1} = I_D, & I_{D2} = I_D+\Delta I_D \end{cases}$$

$$I_{D1}-I_{D2} = \Delta I_D = \frac{1}{2}\beta\left(\frac{W}{L}\right)(V_b - I_D R_S - V_{TH})^2$$
$$-\frac{1}{2}(\beta+\Delta\beta)\left(\frac{W}{L}+\Delta\frac{W}{L}\right)\left[V_b-(I_D+\Delta I_D)R_S-(V_{TH}+\Delta V_{TH})\right]^2$$

二次方難算

用其他方式去避免: $V_b = I_D R_S + V_{GS} \Rightarrow V_b = I_{D1}R_S + \sqrt{\frac{2I_{D1}}{\beta_1 (\frac{W}{L})_1}} + V_{TH1}$

$$= I_{D2}R_S + \sqrt{\frac{2I_{D2}}{\beta_2 (\frac{W}{L})_2}} + V_{TH2}$$

$$(I_{D1}-I_{D2})R_S+\left(\sqrt{\frac{2I_{D1}}{\beta_1\left(\frac{W}{L}\right)_1}}-\sqrt{\frac{2I_{D2}}{\beta_2\left(\frac{W}{L}\right)_2}}\right)+(V_{TH1}-V_{TH2})=0$$

$$-\Delta I_D R_S+\left(\sqrt{\frac{2I_D}{\beta\frac{W}{L}}}-\sqrt{\frac{2(I_D+\Delta I_D)}{(\beta+\Delta\beta)\left(\frac{W}{L}+\Delta\frac{W}{L}\right)}}\right)-\Delta V_{TH}=0$$

$$\sqrt{\frac{2I_D}{\beta\frac{W}{L}}}\left(1-\sqrt{\frac{1+\frac{\Delta I_D}{I_D}}{\left(1+\frac{\Delta\beta}{\beta}\right)\left[1+\frac{\Delta(W/L)}{W/L}\right]}}\right)=\Delta I_D R_S+\Delta V_{TH}$$

by Taylor series $\sqrt{1+\varepsilon_1}\approx 1+\frac{\varepsilon_1}{2}$ $\quad \sqrt{1+\varepsilon_2}^{\,-1}\approx 1-\frac{\varepsilon_2}{2}$

$(\varepsilon_1,\varepsilon_2<<1)$

$$\approx\sqrt{\frac{2I_D}{\beta\frac{W}{L}}}\left[1-\left(1+\frac{\frac{\Delta I_D}{I_D}}{2}\right)\left(1-\frac{\frac{\Delta\beta}{\beta}}{2}\right)\left(1-\frac{\frac{\Delta(W/L)}{W/L}}{2}\right)\right]=\Delta I_D R_S+\Delta V_{TH}$$

$$\approx\sqrt{\frac{2I_D}{\beta\frac{W}{L}}}\left(-\frac{\frac{\Delta I_D}{I_D}}{2}+\frac{\frac{\Delta\beta}{\beta}}{2}+\frac{\frac{\Delta(W/L)}{W/L}}{2}\right)=\Delta I_D R_S+\Delta V_{TH}$$

$$\frac{1}{2}\sqrt{\frac{2I_D}{\beta\frac{W}{L}}}\left(+\frac{\Delta\beta}{\beta}+\frac{\Delta(W/L)}{W/L}\right)=\underline{\frac{+1}{2}\sqrt{\frac{2I_D}{\beta\frac{W}{L}}}\frac{\Delta I_D}{I_D}+\Delta I_D R_S}+\Delta V_{TH}$$

$$\frac{I_D}{I_D}\cdot\sqrt{\frac{I_D}{2\beta\frac{W}{L}}}\left(\frac{\Delta\beta}{\beta}+\frac{\Delta(W/L)}{W/L}\right)=\left(\underbrace{\frac{1}{\sqrt{2\beta\frac{W}{L}I_D}}}_{\frac{1}{g_m}}+R_S\right)\Delta I_D+\Delta V_{TH}$$

$$\frac{I_D}{g_m}\left(\frac{\Delta\beta}{\beta}+\frac{\Delta(W/L)}{W/L}\right)-\Delta V_{TH}=\left(\frac{1}{g_m}+R_S\right)\Delta I_D$$

$$\Delta I_D=\frac{\frac{I_D}{g_m}\left(\frac{\Delta\beta}{\beta}+\frac{\Delta(W/L)}{W/L}\right)-\Delta V_{TH}}{\frac{1}{g_m}+R_S}=\frac{I_D\left(\frac{\Delta\beta}{\beta}+\frac{\Delta(W/L)}{W/L}\right)-g_m\Delta V_{TH}}{1+g_mR_S}$$

$$\Rightarrow\frac{\Delta I_D}{I_D}=\frac{1}{1+g_mR_S}\left[\frac{\Delta(\mu_nC_{ox})}{\mu_nC_{ox}}+\frac{\Delta(W/L)}{W/L}-\frac{2\Delta V_{TH}}{V_{GS}-V_{TH}}\right]$$

Q3 (b):

$$\begin{cases} \lambda = \gamma = 0 \\ g_{m3,4} = 0.5\, g_{m5,6} \end{cases}$$

$$\overline{V_{n,out}^2} = 4kT\gamma\,(g_{m1} + g_{m3} + g_{m5}) \times 2 \times R_{out}^2$$

$$|A_v|^2 = (g_{m1,2})^2 R_{out}^2$$

$$\overline{V_{in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2}$$

$$= \frac{4kT\gamma\,(g_{m1}+g_{m3}+g_{m5}) \times 2 \times \cancel{R_{out}^2}}{g_{m1}^2 \cdot \cancel{R_{out}^2}}$$

$$= 4kT\gamma \cdot 2 \cdot \left(\frac{1}{g_{m1}} + \frac{g_{m3}}{g_{m1}^2} + \frac{2g_{m3}}{g_{m1}^2}\right)$$

$$= \frac{8kT\gamma}{g_{m1}} + \frac{24\, g_{m3} \cdot kT\gamma}{g_{m1}^2}$$

Q4 (a):

$I_D : I_{D1} : I_{D2} = 1 : 2.5 : 4$ 且 M0 的parameter為 $\frac{W}{L} = \frac{1\mu}{0.5\mu}$, M=2

又由於 current mirror 要複製不同倍數的電流

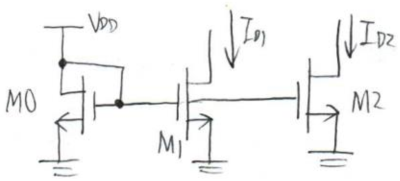
$\Rightarrow$ {
unit element 的元件尺寸要一樣

改變並聯個數(M)來決定複製幾倍

M0、M1、M2的M調整為 2 : 5 : 8，即可複制1:2.5:4倍的電流。

M2 M2 M2 M2 M1 M1 M0 M1 M0 M1 M1 M2 M2 M2 M2

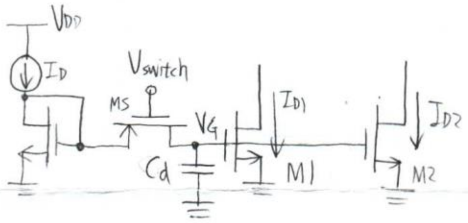
common centroid



(b) 從 $\frac{\Delta I_D}{I_D} = \frac{\Delta(\mu_n C_{ox})}{\mu_n C_{ox}} + \frac{\Delta(W/L)}{W/L} - \frac{2\Delta V_{TH}}{V_{GS}-V_{TH}}$ 的公式中我們可以得知

為了減少 current deviation，我們除了增加 W和L的尺寸來降低 $\Delta(\mu_n C_{ox}\frac{W}{L}) = \frac{A_K}{\sqrt{WL}}$、$\Delta V_{TH} = \frac{A_{VTH}}{\sqrt{WL}}$ 外，我們還可以藉由增加 $V_{GS}-V_{TH}$ 來降低 ΔV_{TH} 對 $\frac{\Delta I_D}{I_D}$ 的影響，但 $V_{GS}-V_{TH}$ 的增加也會減小輸出級的 headroom。

(c) 除了 M1、M2 本身的 thermal noise current 以外，current mirror 還可能受到M0、I_D的noise影響，因此可將 current mirror 調整成



其中MS是用做為一個開關 可以視做R ，而Cd則是 decoupling capacitor，因此從 RC Low-pass Filter 的 Transfer function $H(s)=\frac{1}{1+sRC}$ 可以得知，高頻noise可以被濾除，使 I_{D1}、I_{D2} 上的 thermal noise 下降。