Homework 4 (Due date: 10/25)

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HW4.1: (20 points)

Using a long-channel model, **prove** that, in strong inversion, the transistor $M_R$ behaves like a resistor ($R_{on,R}$) with its resistance,

$$R_{on,R} = \frac{(W/L)_C}{(W/L)_R} \frac{1}{g_{m,C}}$$

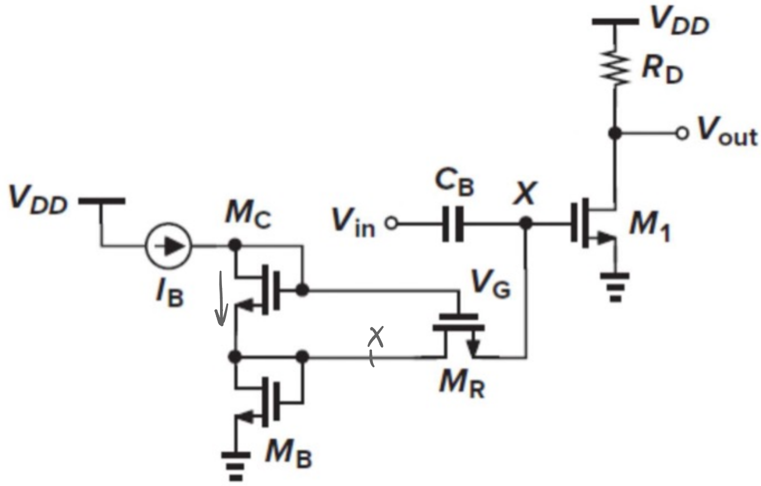



Fig. 4.1

$$R_{onR} = \frac{1}{\mu_n C_{ox} (\frac{W}{L})_R \cdot (V_G - V_X - V_{th})}$$

$$I_B = \frac{1}{2} \mu_n C_{ox} (\frac{W}{L})_C \cdot (V_G - V_X - V_{th})^2$$

$$V_G - V_X - V_{th} = \sqrt{\frac{2I_B}{\mu_n C_{ox} (\frac{W}{L})_C}} = \sqrt{\frac{2\mu_n C_{ox} (\frac{W}{L})_C I_B}{(\mu_n C_{ox} (\frac{W}{L})_C)^2}} = \frac{g_{mC}}{\mu_n C_{ox} (\frac{W}{L})_C}$$

$$\Rightarrow R_{on,R} = \frac{1}{\cancel{\mu_n C_{ox}} (\frac{W}{L})_R \cdot \frac{g_{mC}}{\cancel{\mu_n C_{ox}} (\frac{W}{L})_C}}$$

$$= \frac{(\frac{W}{L})_C}{(\frac{W}{L})_R} \cdot \frac{1}{g_{mC}}$$

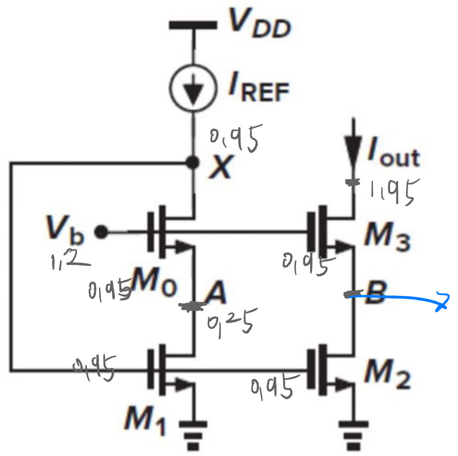
HW4.2: (30 points)

The circuit of Fig. 4.2 is designed with $(W/L)_{1,2} = 8/2$, $(W/L)_{3,0} = 8/2$, and $I_{REF} = 100\ \mu A$. Assume $\mu_n C_{ox}$=800µA/V², VDD=3V and γ=0. $V_{TH}$=0.7V

(a) Determine $V_X$ and the acceptable range of $V_b$.

(b) Estimate the deviation of $I_{out}$ from 100 $\mu$A if the drain voltage of $M_3$ is higher than $V_X$ by 1 V, if λ=0.1V⁻¹.

(c) How to design $V_b$ to have a minimum drain voltage of $M_3$?




$V_{OV}=0.25$

Fig. 4.2

(a) M0 sat: $V_b - V_{th0} \le V_X$

M1 sat: $V_{gs1} - V_{th1} \le V_b - V_{gs0}$ $\Rightarrow$ $V_{gs0}+V_{gs1}-V_{th1} \le V_b \le V_X + V_{th0}$

$I_1 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \cdot (V_X - V_{th})^2$

$100 = \frac{1}{2}\times 800 \times \frac{8}{2} \cdot (V_X - 0.7)^2 \Rightarrow V_X = 0.95 = V_{gs1}$ #

$100 = \frac{1}{2}\times 800 \times \frac{8}{2} \cdot (V_{gs0} - 0.7)^2 \Rightarrow V_{gs0} = 0.95$

$\Rightarrow 0.95 + 0.95 - 0.7 \le V_b \le 0.95 + 0.7$

$\Rightarrow 1.2 \le V_b \le 1.65$ #

(b) $V_{ov1} = 0{,}25 = V_{ov0}$, $V_{b,min} = 1{,}2$, $V_A = 0{,}25$, $V_{DS0} = 0{,}7$, $V_{D3} = 1{,}7$

假設在M2的drain有 $\Delta V$, 且 $I_{D2} = I_{D3}$

$$I_{D2} = (V_{ov1})^2 [1 + \lambda(V_{DS1} + \Delta V)]$$

$$I_{D3} = (V_{ov0} - \Delta V)^2 [1 + \lambda(V_{DS3} - \Delta V)] \quad \Rightarrow I_{D2} = I_{D3}$$

$$(0{,}25)^2 [1 + 0{,}1(0{,}25 + \Delta V)] = (0{,}25 - \Delta V)^2 [1 + 0{,}1(1{,}7 - \Delta V)]$$

$$0{,}0625 [1 + 0{,}025 + 0{,}1\Delta V] = (0{,}0625 - 0{,}5\Delta V + \Delta V^2)(1 + 0{,}17 - 0{,}1\Delta V)$$

$$0{,}0640625 + 6{,}25m\Delta V = 0{,}073125 - 6{,}25m\Delta V - 0{,}585\Delta V + 0{,}05\Delta V^2 + 1{,}17\Delta V^2 - 0{,}1\Delta V^3$$

$$\Rightarrow 0{,}1\Delta V^3 - 1{,}22\Delta V^2 + 0{,}5975\Delta V - 9{,}0625m = 0$$

$$\Delta V = 11{,}68,\ 0{,}49,\ \underline{0{,}015}$$

$$\Rightarrow \frac{I_{out} = [1 + \lambda(0{,}25 + 0{,}015)]}{I_{ref} = [1 + \lambda(0{,}25)]}$$

$$\Rightarrow I_{out} = \frac{[1 + \lambda \cdot 0{,}265)}{1 + \lambda \cdot 0{,}25} \times 100\mu = \underline{100{,}146\mu A}$$ #

(c) $V_{D3(min)} = V_{ov2} + V_{ov3}$

$$\Rightarrow V_b = V_{ov2} + V_{ov3} + V_{th3} = 0{,}25 + 0{,}25 + 0{,}7 = \underline{1{,}2}$$ #

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HW4.3 (30 points)

In the circuit shown in Fig. 4.3, a source follower using a wide transistor and a small bias current is inserted in series with the gate of $M_3$ so as to bias $M_2$ at the edge of saturation. Assuming $M_0$–$M_3$ are identical and $\lambda \neq 0$, estimate the mismatch between $I_{out}$ and $I_{REF}$ if (a) $\gamma = 0$, (b) $\gamma \neq 0$.

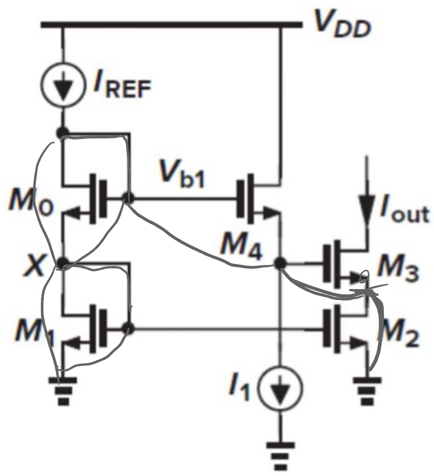



Fig. 4.3

(a) $I_{ref} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{gs1}-V_{th})^2(1+\lambda V_{DS1})$ , $V_{DS1} = V_{gs1}$

$I_{out} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{gs1}-V_{th})^2(1+\lambda V_{DS2})$ , $V_{DS2} = V_{gs1} + V_{gs0} - V_{gs4} - V_{gs3} \approx V_{gs1} - V_{gs4}$

$$\rightarrow \frac{I_{out}}{I_{ref}} = \frac{1+\lambda(V_{gs1}-V_{gs4})}{1+\lambda V_{gs1}}$$

$$\begin{cases} I_{ref} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{gs1}-V_{th})^2 \Rightarrow V_{gs1} = \sqrt{\frac{2I_{ref}}{\mu_n C_{ox}}\cdot\frac{L}{W}} + V_{th} \\ I_1 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{gs4}-V_{th})^2 \Rightarrow V_{gs4} = \sqrt{\frac{2I_1}{\mu_n C_{ox}}\cdot\frac{L}{W}} + V_{th} \end{cases}$$

(b) $V_{th} = V_{th0} + \gamma\left[\sqrt{|2\phi_f - V_{BS}|} - \sqrt{2\phi_f}\right]$

$$\begin{cases} V_{gs1} = \sqrt{\frac{2I_{ref}}{\mu_n C_{ox}}\cdot\frac{L}{W}} + V_{th0} \\ V_{gs0} = \sqrt{\frac{2I_{ref}}{\mu_n C_{ox}}\cdot\frac{L}{W}} + V_{th0} + \gamma\left[\sqrt{|2\phi_f + V_{gs1}|} - \sqrt{2\phi_f}\right] \\ V_{gs3} = \sqrt{\frac{2I_{ref}}{\mu_n C_{ox}}\cdot\frac{L}{W}} + V_{th0} + \gamma\left[\sqrt{|2\phi_f + V_{DS2}|} - \sqrt{2\phi_f}\right] \\ V_{gs4} = \sqrt{\frac{2I_1}{\mu_n C_{ox}}\cdot\frac{L}{W}} + V_{th0} + \gamma\left[\sqrt{|2\phi_f\ V_{gs3}+V_{DS2}|} - \sqrt{2\phi_f}\right] \end{cases}$$

$$\rightarrow \frac{I_{out}}{I_{ref}} = \frac{1+\lambda(V_{gs1}+V_{gs0}-V_{gs4}-V_{gs3})}{1+\lambda V_{gs1}}$$

HW4.4: (20 points)

The circuit shown in Fig. 4.4 exhibits a *negative* input inductance. Calculate the input impedance of the circuit and identify the inductive component.

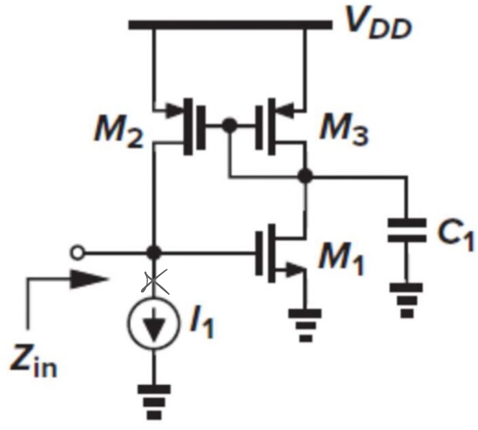
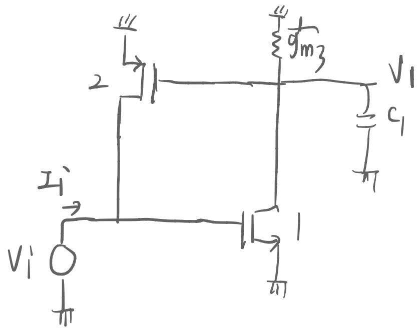



Fig. 4.4




$z_{in} = \frac{V_i'}{I_i'}$ , $I_i' = V_1 \cdot g_{m2}$

$V_1 = V_i' \cdot g_{m1} \cdot \left(\frac{1}{g_{m3}} // \frac{1}{sC_1}\right) = V_i' \cdot g_{m1} \cdot \frac{-1}{g_{m3}+sC_1}$

$$z_{in} = \frac{\cancel{V_i'}}{g_{m2} \cdot \cancel{V_i'} \cdot g_{m1} \cdot \frac{-1}{g_{m3}+sC_1}}$$

$$z_{in} = \frac{g_{m3}+sC_1}{-g_{m1}g_{m2}}$$ #