Calculator is not allowed

112-1 Calculus Midterm
Chapter: 2-1-2-6 , 3-1-3-7
Date: 2023/10/25 13:20-15:10 Total: 110 pts

1. Find the following limits. (12 pts)

   $a.\lim_{t\to 0}\frac{cos2t-4cost+3}{t^4}$ (6 pts) $\qquad$ $b.\lim_{x\to\infty}\left(x\sqrt{\frac{x-1}{x+1}}-x\right)$ (6 pts)

2. $\lim_{x\to 0}\frac{\sqrt{1+x+x^2}-(1+ax)}{x^2}=b$ and a,b are constant, find a,b. (10 pts)

3. $f(x)=\begin{cases} cosx\,, x<0 \\ \alpha+x^2\,, 0\le x<1 \\ \beta x\,,\ x\ge 1 \end{cases}$ and is continuous at every x, find $\alpha+\beta$. (10 pts)

4. Find the tangent and normal line of $F(x,y)=2x^2-y^3+4xy-2x=0$ at Point $P(x,y)=(1,-2)$. (10 pts)

5. Is $f(x)=\begin{cases} x^2cos\frac{\pi}{x}\,,\ x\neq 0 \\ 0\,,\ x=0 \end{cases}$ continuous and differentiable at $x=0$? (10 pts)

6. Find the derivatives of the following functions. (24 pts)

   a. $f(x)=(x^2+1)(x+5+\frac{1}{x})$ (6 pts)

   b. $g(z)=\frac{(z-1)(z^2+z+1)}{z^3}$ (6 pts)

   c. $r(\theta)=\sin(\theta^2)\cos(2\theta)$ (6 pts)

   d. $y(t)=(1+tan^4(\frac{t}{12}))^3$ (6 pts)

7. Given that $f'(0)=2$, $\lim_{x\to 0}\frac{f(6x)-f(sinx)}{x}=?$ (10 pts)

8. If $f\left(\frac{x-1}{x+1}\right)=2\cos(\frac{\pi}{2}x)$, find f'(0) = ? (10 pts)

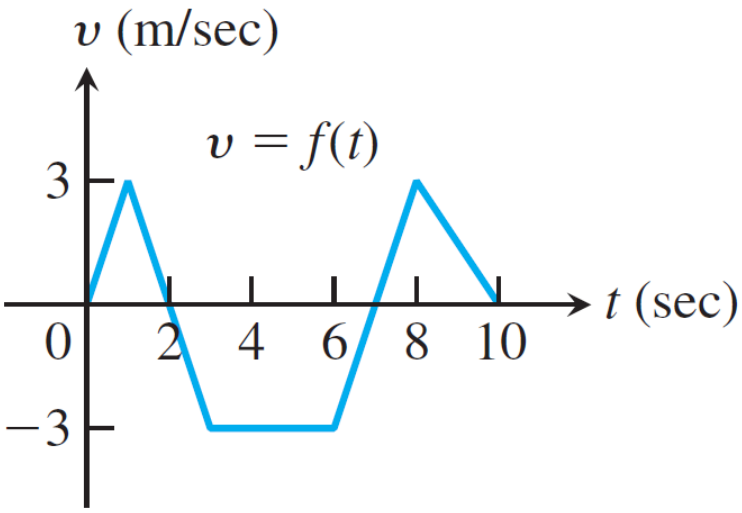
9. The accompanying figure shows the velocity $v=\frac{ds}{dt}=f(t)$ (m/sec) of a body moving along a coordinate line. (14 pts)

(Questions on both sides of the paper)

a. When does the body reverse direction? (4 pts)

b. When (approximately) is the body moving at a constant speed? (4 pts)

c. Graph the body's speed for 0 … t … 10. (6 pts)



(Questions on both sides of the paper)