台灣科技大學一百一十學年度上學期期末考

科目名稱：電路學(一)　開課系所：電子系 ET2103301　地點：國際大樓 IB501

考試時間：111 年 12 月 29 日 上午 10:20 至 12:10 (可使用工程計算機)

1. (15%) Determine the equation for the voltage $v_o(t)$ for $t > 0$ in Fig. 1(a) when subjected to the input pulse shown in Fig. 1(b).

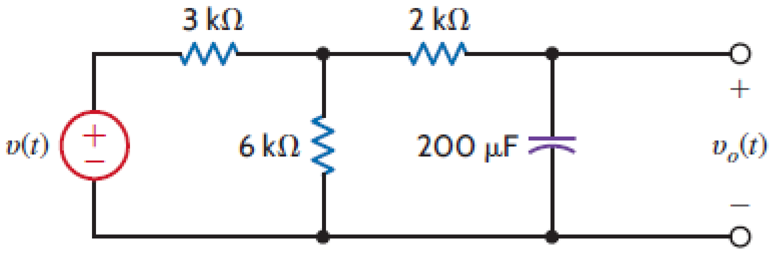


Fig. 1(a)

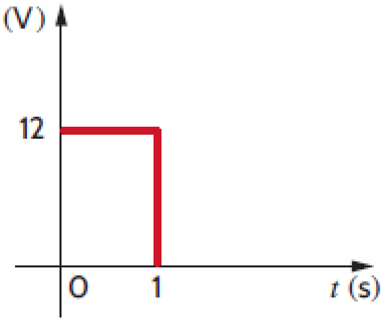


Fig. 1(b)

2. (15%) Please find $v_o(t)$ for $t > 0$.

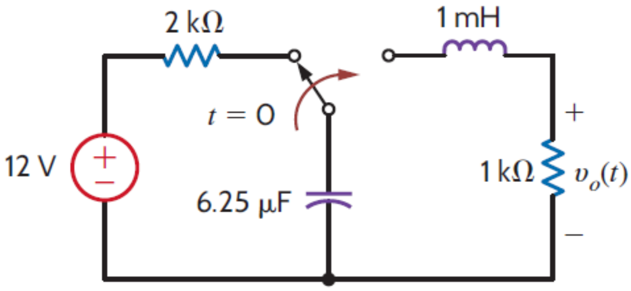


Fig. 2.

3. (20%) Please find $i_L(t)$ for $t > 0$ in Fig. 3 using the step-by-step method.

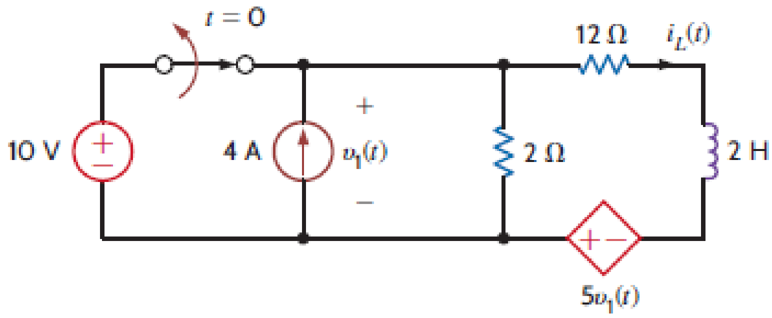


Fig. 3.

4. (20%) Please find $\mathbf{I}_o$ in Fig. 4.

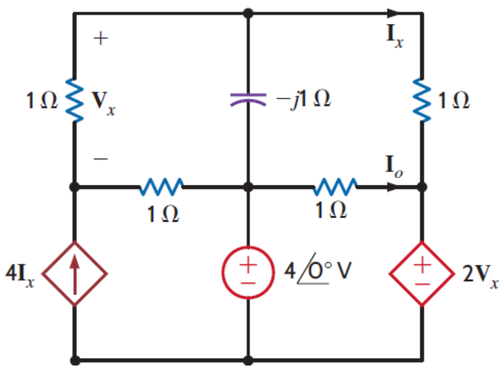


Fig. 4.

5. (15%) Please determine $\mathbf{Z_L}$ for maximum average power transfer and the maximum average power transferred to $\mathbf{Z_L}$ in Fig. 5.

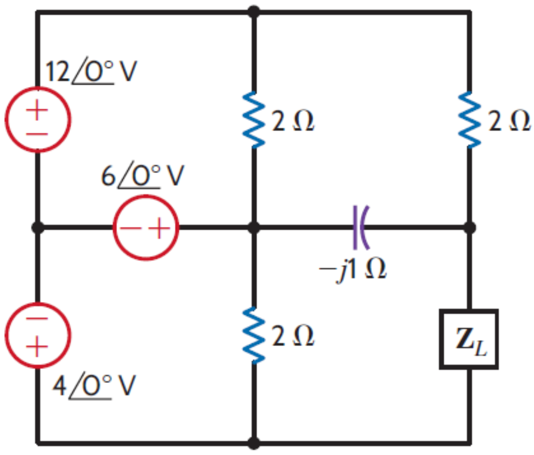


Fig. 5.

6. (12%) Two loads connected in parallel have the following parameters:

LOAD 1: 3 kW with pf = 0.8 lagging

LOAD 2: 5 kW with pf = 0.6 lagging

as shown in Fig. 6.

(a) Determine **I**.

(b) Determine the value of the supply voltage $\mathbf{V_S}$.

(c) Determine the power factor of the source.

(d) Determine the complex power furnished by the source.

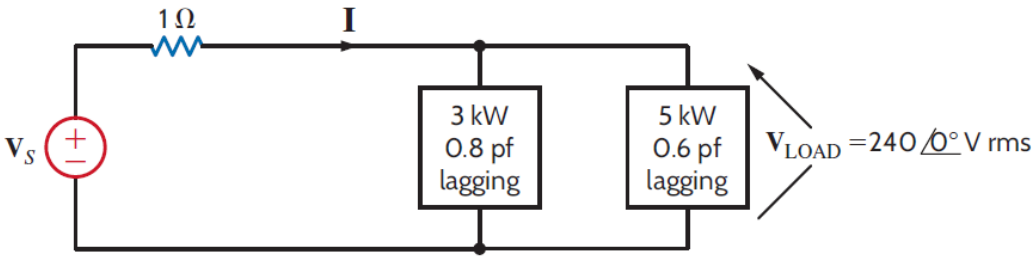


Fig. 6.

7. (13%) A bank of induction motors consumes 50 kW at a PF of 0.7 lagging from a 220 Vrms line. Determine the value of capacitance, which when placed in parallel with the load, will yield a PF of 0.9 lagging.

1.

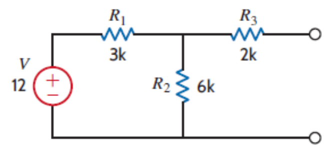
Solution:

For $0 \le t \le 1$ s

$t = 0^-$

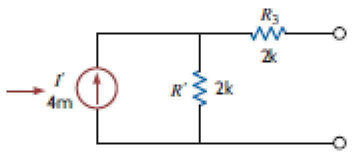
$$v_C(0^-) = v_o(0^-) = 0\text{ V}$$

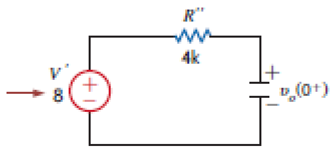
$t = 0^+$



$$I' = \frac{V}{R_1} = 4\text{ mA}$$

$$R' = R_1 \parallel R_2 = 2\text{k}$$



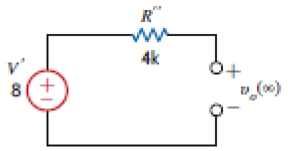


$$V' = I' \cdot R' =$$

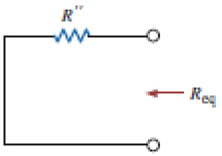
$$R'' = R' + R_3 = 4\text{k}$$

$$v_o(0^+) = v_o(0^-) = 0\text{ V}$$

$t = \infty$



$$v_o(\infty) = V' = 8\text{ V}$$



$$R_{eq} = R'' = 4\text{ k}\Omega$$

$$\tau = R_{eq} \cdot C = 0.8\text{ s}$$

$$v_o(0^+) = 0\text{ V} = K_1 + K_2$$

$$v_o(\infty) = 8\text{ V} = K_1$$

$$K_2 = -8\text{ V}$$

$$v_o(t) = 8 - 8e^{t/-0.8}\text{ V}, \quad 0 \le t \le 1\text{ s}$$

$$v_o(1) = 8 - 8e^{-(1)/0.8}\text{ V} = 5.71\text{ V}$$

For $t \ge 1$ s ($V = 0$ V)

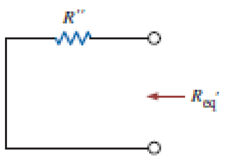
$$t' = t - 1\text{ s}$$

$t = 1^+$

$$v_o(1^-) = v_o(1^+) = 5.71\text{ V}$$

$t = \infty$

$$v_C(\infty) = v_o(\infty) = 0\text{ V}$$



$$R'_{eq} = R'' = 4\text{ k}\Omega$$

$$\tau' = 0.8\text{ s}$$

$$V_o(1^+) = 5.71\text{ V} = K_3 + K_4$$

$$V_o(\infty) = 0\text{ V} = K_3$$

$$K_4 = 5.71\text{ V}$$

$$V'(t) = 5.71e^{-t/0.8}\text{ V}, \quad t > 1\text{ s}$$

$$\therefore V(t) = \begin{cases} 8 - 8e^{-t/0.8}\text{ V}, & 0 \le t \le 1\text{ s} \\ 5.71e^{-(t-1)/0.8}\text{ V}, & t > 1\text{ s} \end{cases}$$

2.

6.3.14 Find $v_o(t)$ for $t > 0$ in the circuit in Fig. P6.3.14 and plot the response, including the time interval just prior to moving the switch.

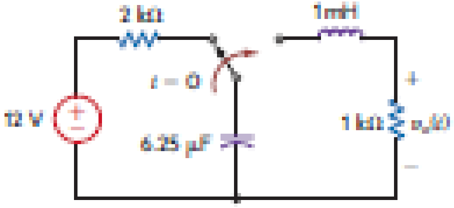
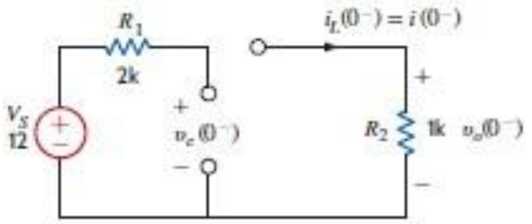


FIGURE P6.3.14

Solution:

$t = 0^-$



$v_C(0^-) = 12\text{ V}$

$i_L(0^-) = 0\text{ A} = i(0^-)$

$V_o(0^-) = 0\text{ V}$

$t > 0$

$$\frac{d^2 i(t)}{dt^2} + \frac{R_2}{L} \cdot \frac{di(t)}{dt} + \frac{1}{LC} \cdot i(t) = 0$$

$$S^2 + (1 \times 10^6)S + (1.6 \times 10^8) = 0$$

$$S = \frac{-(1 \times 10^6) \pm \sqrt{(1 \times 10^6)^2 - 4(1)(1.6 \times 10^8)}}{2(1)}$$

$S =$ $\quad S_1 = -160 \quad , \quad S_2 = -999.8K$

$$i(t) = K_1 e^{-160t} + K_2 e^{-999.8kt}$$

$$i(0^-) = i(0^+) = 0\text{ A} = K_1 + K_2$$

$$12 = L\frac{di(0)}{dt}$$

$$\frac{di(0)}{dt} = 12\text{k}$$

$$\frac{di}{dt} = -160 \cdot K_1 e^{-160t} - 999.8k \cdot K_2 e^{-999.8kt}$$

$$12{,}000 = -160\, K_1 - 999.8k \cdot K_2$$

$$K_1 = 0.012 \quad K_2 = -0.012$$

$$i(t) = 0.012 e^{-160t} - 0.012 e^{-999.8kt}\text{ A}, \quad t > 0$$

$$v_o(t) = R_2 \cdot i(t)$$

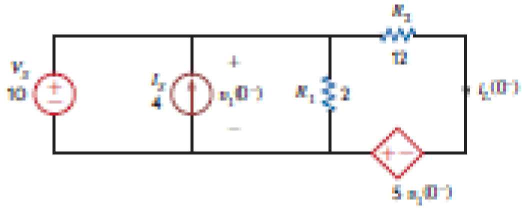
$$v_o(t) = 12(e^{-160t} - e^{-999.8kt})$$

3.

Solution:

$$i_L(t) = K_1 + K_2 e^{-t/\tau}$$

$t = 0^-$



$V_1(0^-) = 10$ V

KVL :

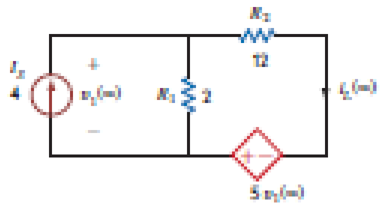
$V_1(0^-) + 5V_1(0^-) = R_2 \cdot i_L(0^-)$

$i_L(0^-) = 5$ A

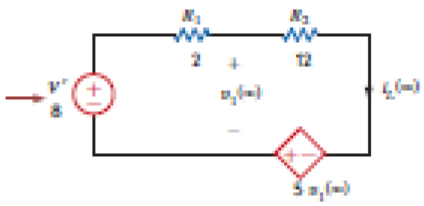
$t = 0^+$

$$i_L(0^+) = i_L(0^+) = 5 \text{ A}$$

$t = \infty$



$V' = I_2 \cdot R_1 = 8$ V



KVL:

$5V_1(\infty) + V_1(\infty) = R_2 \cdot i_L(\infty)$

$V_1(\infty) = \dfrac{R_2 \cdot i_L(\infty)}{6}$

KVL:

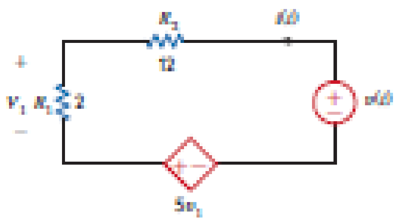
$$5V_1(\infty) + V' = (R_1 + R_2) \cdot i_L(\infty)$$

$$5\left(\frac{R_2 \cdot i_L(\infty)}{6}\right) + V' = (R_1 + R_2) \cdot i_L(\infty)$$

$$i_L(\infty) = \frac{V'}{\left(R_1 + \frac{1}{6}R_2\right)}$$

$$i_L(\infty) = 2 \text{ A}$$

R_{th}



$V_1 = R_1 \cdot i(t),\ R_{th} = \dfrac{v(t)}{i(t)}$

KVL :

$v(t) = (R_1 + R_2) \cdot i(t) + 5V_1$

$v(t) = (R_1 + R_2) \cdot i(t) + 5(R_1 \cdot i(t))$

$\dfrac{v(t)}{i(t)} = 6R_1 + R_2 = 24\ \Omega$

$$R_{th} = 24\ \Omega$$

$$\tau = \frac{L}{R_{th}} = 83.3 \text{ ms}$$

$$i_L(0^+) = 5 \text{ A} = K_1 + K_2$$

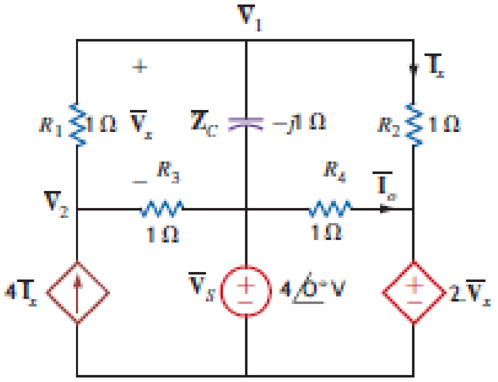
$$i_L(\infty) = 2 \text{ A} = K_1$$

$$K_2 = 3 \text{ A}$$

$$i_L(t) = 2 + 3e^{-t/0.083} \text{ A}, \quad t > 0$$

4.

Solution:



$$\mathbf{V}_x = \mathbf{V}_1 - \mathbf{V}_2$$

$$\mathbf{I}_x = \frac{\mathbf{V}_1 - 2\cdot\mathbf{V}_x}{R_2} = \frac{-\mathbf{V}_1}{R_2} + \frac{2\mathbf{V}_2}{R_2}$$

KCL:

1. $$\frac{\mathbf{V}_1 - \mathbf{V}_2}{R_1} + \frac{\mathbf{V}_1 - \mathbf{V}_S}{\mathbf{Z}_C} + \mathbf{I}_x = 0$$

$$\frac{\mathbf{V}_1}{R_1} - \frac{\mathbf{V}_2}{R_1} + \frac{\mathbf{V}_1}{\mathbf{Z}_C} - \frac{\mathbf{V}_1}{R_2} + \frac{2\mathbf{V}_2}{R_2} = \frac{\mathbf{V}_S}{\mathbf{Z}_C}$$

2. $$\frac{\mathbf{V}_2 - \mathbf{V}_1}{R_1} + \frac{\mathbf{V}_2 - \mathbf{V}_S}{R_3} - 4\mathbf{I}_x = 0$$

$$\frac{\mathbf{V}_2}{R_1} - \frac{\mathbf{V}_1}{R_1} + \frac{\mathbf{V}_2}{R_3} + \frac{4\mathbf{V}_1}{R_2} - \frac{8\mathbf{V}_2}{R_2} = \frac{\mathbf{V}_S}{R_3}$$

$$\begin{bmatrix} \left(\frac{1}{R_1} + \frac{1}{\mathbf{Z}_C} - \frac{1}{R_2}\right) & \left(\frac{-1}{R_1} + \frac{2}{R_2}\right) \\ \left(\frac{-1}{R_1} + \frac{4}{R_2}\right) & \left(\frac{1}{R_1} + \frac{1}{R_3} - \frac{8}{R_2}\right) \end{bmatrix} \times \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{V}_S}{\mathbf{Z}_C} \\ \frac{\mathbf{V}_S}{R_3} \end{bmatrix}$$

$$\mathbf{V}_1 = 3.267\underline{/17.103°}\text{ V}$$

$$\mathbf{V}_2 = 1.193\underline{/26.565°}\text{ V}$$

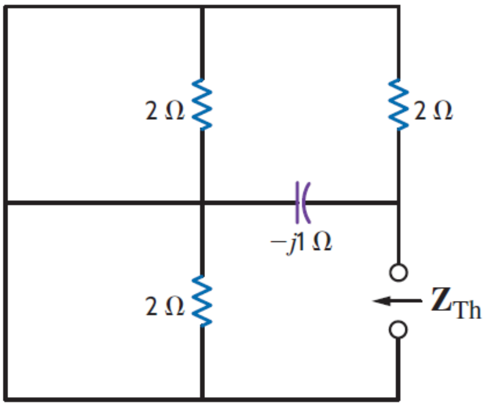
$$\mathbf{I}_o = \frac{\mathbf{V}_S - 2\mathbf{V}_x}{R_4}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_S - 2\mathbf{V}_1 + 2\mathbf{V}_2}{R_4}$$

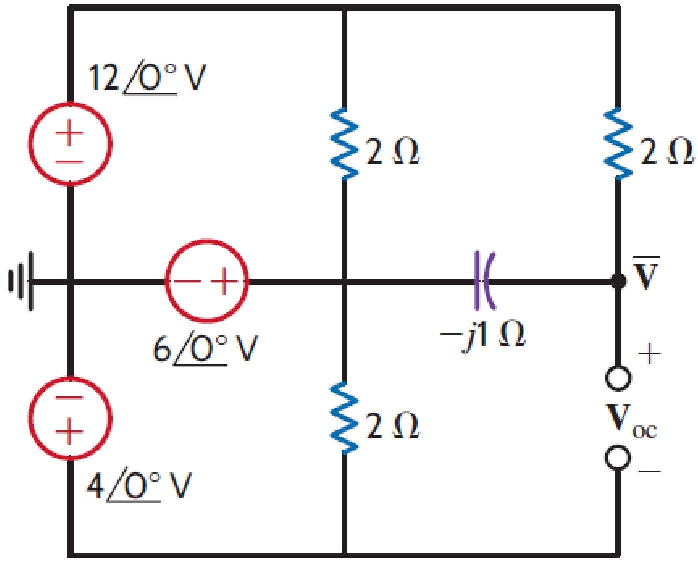
$$\mathbf{I}_o = 1.33\underline{/-126.87°}\text{ A}$$

5.

Solution:



$$\overline{\mathbf{Z}}_{Th} = 2 \parallel -j1 = 0.4 - j0.8\ \Omega$$
$$\overline{\mathbf{Z}}_{L} = \overline{\mathbf{Z}}^{*}_{Th} = 0.4 + j0.8\ \Omega$$



$$\frac{\overline{\mathbf{V}} - 6\angle 0°}{-j1} + \frac{\overline{\mathbf{V}} - 12\angle 0°}{2} = 0$$

$$\Rightarrow \overline{\mathbf{V}} = 7.59\angle -18.43°\ \mathrm{V}$$

$$\overline{\mathbf{V}}_{OC} = \overline{\mathbf{V}} - 4\angle 0° = 4\angle 36.87°\ \mathrm{V}$$

$$P_{\max} = \frac{(V_{OC})^2}{8\,R_{Th}} = \frac{4^2}{8(0.4)} = 5\ \mathrm{W}$$

6.

Solution:

$$\bar{I}_1 = \frac{\left(\frac{3}{0.8}\right)\text{ kVA}}{240\text{ V}_{\text{rms}}}\underline{/-\cos^{-1}(0.8)} = 15.63\underline{/-36.87^\circ}\text{ A}_{\text{rms}}$$

$$\bar{I}_2 = \frac{\left(\frac{5}{0.6}\right)\text{ kVA}}{240\text{ V}_{\text{rms}}}\underline{/-\cos^{-1}(0.6)} = 34.72\underline{/-53.13^\circ}\text{ A}_{\text{rms}}$$

a. $\bar{I} = \bar{I}_1 + \bar{I}_2 = 49.91\underline{/-48.1^\circ}\text{ A}_{\text{rms}}$

b. $\overline{V}_S = 240\underline{/0^\circ} + (\bar{I})(1) = 275.8\underline{/-7.74^\circ}\text{ V}_{\text{rms}}$

c. $pf_S = \cos(-7.74^\circ + 48.1^\circ) = 0.762$ lagging

d. $\overline{S} = \overline{V}_S\bar{I}^* = 13.77\underline{/40.36^\circ}$ kVA

7.

Solution:

$$\theta_{\text{old}} = \cos^{-1}(0.7) = 45.57^\circ$$

$$Q_{\text{old}} = 50\tan(\theta_{\text{old}}) = 51.01\text{ kvar}$$

$$\theta_{\text{new}} = \cos^{-1}(0.9) = 25.84^\circ$$

$$Q_{\text{new}} = 50\tan(\theta_{\text{new}}) = 24.22\text{ kvar}$$

$$Q_C = Q_{\text{old}} - Q_{\text{new}} = 26.79\text{ kvar}$$

$$C = \frac{Q_C}{2\pi(60)(220^2)} = 1.468\text{ mF}$$