Introduction to Analog Integrated Circuits (112), DECE, NTUST

Homework 2 (Due date: 10/04)

HW2.1: (30%)

請參考課本，解釋 CMOS 製程發展(通道長度變小)中，

(a) Constant-Field Scaling 對元件模型的影響。

(b) Constant-Voltage Scaling 對元件模型的影響。

(c) 上述兩種製程微縮(Scaling)準則對類比電路設計的影響。

HW2.2: (40%)

(a) 請寫出課本提到的造成 Threshold Voltage 變化是受到那些效應的影響，並用文字或數學式輔助說明其原因?

(b) 請利用網路上找到的資源作為參考資料(並非是教科書或是上課講義)，寫出 output impedance variation (r_{DS} 會因為 V_{DS} 的大小而改變)是基於那些效應? 本題可以直接抄寫參考資料，但請註明出處。

HW2.3: (30%)

利用課本 Eq. (17.30), 計算一個金氧半場效電晶體(MOSFET) 的轉導 (Transconductance)。 證明下式為轉導之表示式。

$$g_m = \frac{I_D}{V_{GS} - V_{TH}} \left[1 + \frac{1}{1 + \left(\frac{\mu_0}{2 v_{sat} L} + \theta \right) (V_{GS} - V_{TH})} \right]$$

Eq. (17.30)

$$I_D = \frac{1}{2} \mu_0 C_{ox} \frac{W}{L} \frac{(V_{GS} - V_{TH})^2}{1 + \left(\frac{\mu_0}{2 v_{sat} L} + \theta \right) (V_{GS} - V_{TH})}$$

HW 2.1

a.

1.電晶體的尺寸微縮（包含長度、寬度、深度）
2.減少Power supply 及Vth的大小
3.增加摻雜的濃度 (基於要在體積變小的情況下維持相同的電荷量，因此得增加摻雜濃度)

假設 scaling factor α

$$I_{D,scale} = \frac{1}{2}u(\alpha C_{ox})\left(\frac{W/\alpha}{L/\alpha}\right)\left(\frac{V_{GS}}{\alpha}-\frac{V_{TH}}{\alpha}\right)^2$$

$$= \frac{1}{\alpha}\frac{1}{2}uC_{ox}(W/L)(V_{GS}-V_{th})^2$$ 电流变小 $\frac{1}{\alpha}$ 倍

$$W_{d,scale} \overset{(V_R \gg \phi_B)}{\simeq} \sqrt{\frac{2\epsilon_{si}}{q}\left(\frac{1}{\alpha N_A}+\frac{1}{\alpha N_D}\right)\frac{V_R}{\alpha}} = \frac{1}{\alpha}\sqrt{\frac{2\epsilon_{si}}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)V_R}$$

空乏區寬度缩小 $\frac{1}{\alpha}$ 倍

$$C_{ch,scale} = \frac{W}{\alpha}\frac{L}{\alpha}(\alpha C_{ox}) = \frac{1}{\alpha}WLC_{ox}$$

通道电容变小 $\frac{1}{\alpha}$ 倍

也就是說 C_{GS}, C_{GD} 会一起变小 $\frac{1}{\alpha}$ 倍

$$C_{DB}/C_{SB,scale} = \frac{W}{\alpha}\frac{E}{\alpha}(\alpha C_j) + 2\left(\frac{W}{\alpha}+\frac{E}{\alpha}\right)C_{jsw}$$

$$= \frac{1}{\alpha}\left[WE\,C_j + 2(W+E)C_{jsw}\right]$$

空乏电容 C_{DB} 及 C_{SB} 缩小 $\frac{1}{\alpha}$ 倍

$$P_{,scale} = fCV^2 = f\left(\frac{C}{\alpha}\right)\left(\frac{V}{\alpha}\right)^2 = \frac{1}{\alpha^3}fCV^2$$

对於数位电路而言功耗减少 $\frac{1}{\alpha^3}$ 倍

$$T_{d,scale} = \frac{C/\alpha \times \frac{V}{\alpha}}{I/\alpha} = \frac{1}{\alpha}\frac{CV}{I}$$

delay time 缩短为原来的 $\frac{1}{\alpha}$ 倍

$$g_{m,scale} = u(\alpha C_{ox})\frac{W/\alpha}{L/\alpha}\frac{V_{ov}}{\alpha} = uC_{ox}\frac{W}{L}V_{ov}$$

$$r_{o,scale} = \frac{1}{(\alpha\lambda)\frac{I_D}{\alpha}} = \frac{1}{\lambda I_D}$$

$$P_{scale} = IV = \left(\frac{I}{\alpha}\right)\left(\frac{V}{\alpha}\right) = \frac{1}{\alpha^2}VI$$

对於 g_m、r_o 来說不会改变，实際上会因 Short channel effect (SCE) 有所改变

功耗的部分实際上為了满足 slew 的要求电流依然需要拉高，所以也不会真的是 $\frac{1}{\alpha^2}$ 倍。

以上皆為 constant-field-scaling

b、

其實就是固定電壓後，將電晶體尺寸做微縮，但在
相同電壓下，把通道長度變小將會使得電場太強 $(\uparrow E = \frac{V}{L\downarrow})$
加劇SCE甚至導致Breakdown

$$I_{D,scale} = \frac{1}{2}u(\alpha C_{ox})\frac{W/\alpha}{L/\alpha}(V_{GS}-V_{TH})^2$$
$$= \alpha \frac{1}{2}uC_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2$$

$$g_{m,scale} = u(\alpha C_{ox})\frac{W/\alpha}{L/\alpha}V_{ov} = \alpha uC_{ox}\frac{W}{L}V_{ov}$$

$$r_{o,scale} = \frac{1}{(\alpha\lambda)(\alpha I_D)} = \frac{1}{\alpha^2}\frac{1}{\lambda I_D}$$

$$P_{scale} = IV = (\alpha I)(V) = \alpha IV$$

C、

如果單純以Constant -Field Scaling 的話
會將Power Supply 及Vth 也一起往下拉，將使得Volt
Swing 及 SNR較差，而Vth變小則會使得漏電流變大
但如果以Constant - Voltage Scaling 則可能會因為電場
太大加劇SCE，因此實際上會使用混合Constant Field及
Constant Voltage 的方式來做。

HW2.2

a.

1. Subthreshold behavior

當MOSFET操作於 subthreshold region

$I_D = uC_d \frac{W}{L} V_T^2 e^{\frac{V_{ov}}{nV_T}}\left(1-e^{\frac{-V_{DS}}{V_T}}\right)$ 其中 $n = 1+\frac{C_d}{C_{ox}}$

假設 $V_{DS} > 4V_T$ 則 $I_D \simeq uC_d \frac{W}{L} V_T^2 e^{\frac{V_{ov}}{nV_T}}$

$S = \left[\frac{\partial(\log I_D)}{\partial V_{GS}}\right]^{-1} = 2.3\, V_T \left(1+\frac{C_d}{C_{ox}}\right)$ 越小越好

$V_T \rightarrow \frac{KT}{q}$

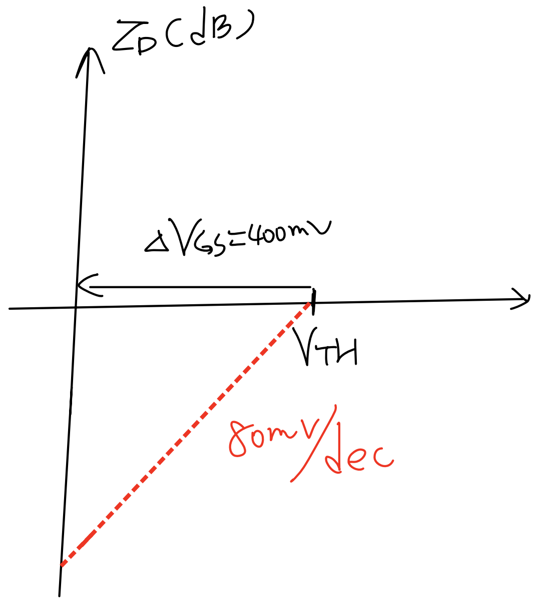
如果 $S = 80mV/dec$ 且 $\Delta V_{GS} = 400mV$

如果希望 $I_{on}/I_{off} = 10^5$

則 $V_{TH(min)} = 400mV$

才可滿足此條件

當S变化時，$V_{TH(min)}$会跟著改变。



2. Process corner variation

因為製程變異產生出不同的Corner ，而使VTH有所改變

例如：SNFP則表示Nmos 慢 Pmos快

因此Nmos的VTH會比原本在TT Corner 下的還要大

Pmos的VTH會比TT Corner 下的小

3. Temperature variation

V_{TH} 会呈負溫度係数

$$\frac{dV_{TH}}{dT} = -\frac{1}{T}\Big[\underbrace{\frac{E_g}{2q} - \phi_f}_{>0}\Big]\Big[2+\frac{\gamma}{\sqrt{2\phi_f}}\Big]$$

Assume that the source-body voltage is zero. Substituting (1.138) into (1.139) gives

$$V_t = \frac{\sqrt{2qN_A\epsilon(2\phi_f)}}{C_{ox}} + 2\phi_f + \phi_{ms} - \frac{Q_{ss}}{C_{ox}} \tag{1.167}$$

Assume that ϕ_{ms}, Q_{ss}, and C_{ox} are independent of temperature.[19] Then differentiating (1.167) gives

$$\frac{dV_t}{dT} = \frac{\sqrt{2qN_A\epsilon(2)}}{2C_{ox}\sqrt{\phi_f}}\frac{d\phi_f}{dT} + 2\frac{d\phi_f}{dT} = \frac{d\phi_f}{dT}\left[2 + \frac{1}{C_{ox}}\sqrt{\frac{qN_A\epsilon}{\phi_f}}\right] \tag{1.168}$$

Substituting (1.136) into (1.135) gives

$$\phi_f = \frac{kT}{q}\ln\left[\frac{N_A\exp\left(\frac{E_g}{2kT}\right)}{\sqrt{N_cN_v}}\right] \tag{1.169}$$

Assume both N_c and N_v are independent of temperature.[20] Then differentiating (1.169) gives

$$\frac{d\phi_f}{dT} = \frac{kT}{q}\left[-\frac{E_g}{2kT^2}\right] + \frac{k}{q}\ln\left[\frac{N_A\exp\left(\frac{E_g}{2kT}\right)}{\sqrt{N_cN_v}}\right] \tag{1.170}$$

Substituting (1.169) into (1.170) and simplifying gives

$$\frac{d\phi_f}{dT} = -\frac{E_g}{2qT} + \frac{\phi_f}{T} = -\frac{1}{T}\left[\frac{E_g}{2q} - \phi_f\right] \tag{1.171}$$

$$\frac{dV_t}{dT} = -\frac{1}{T}\left[\frac{E_g}{2q} - \phi_f\right]\left[2 + \frac{\gamma}{\sqrt{2\phi_f}}\right] \tag{1.172}$$

Equation 1.172 shows that the threshold voltage falls with increasing temperature if $\phi_f < E_g/(2q)$. The slope is usually in the range of -0.5 mV/°C to -4 mV/°C.[21]

4、 Body effect

當逆向偏压(VSB)加大時，使得空乏區變寬

因此在空乏區中的电荷將会增加 Qdep↑

由 $V_{TH} = \phi_{MS} + 2\phi_f + \frac{Q_{dep}}{C_{ox}}$

5、

Drain-Induced Barrier lowing (DIBL)
當L較小時且當VDS上升時，會使得Drain 的空乏區深入通道下方幫忙建立空乏區，因此對於產生相同的空乏區電荷所需的VGS下降了，因為有Drain 端的空乏區電荷共享(Charge Sharing)，因此等同VTH下降了。

HW2.3

$$I_D = \underbrace{\frac{1}{2}\mu_0 C_{ox}\frac{W}{L}}_{K} \frac{(V_{GS}-V_{TH})^2}{1+\Big(\underbrace{\frac{\mu_0}{2v_{sat}L}+\theta}_{a}\Big)(V_{GS}-V_{TH})}$$

$$I_D = K\frac{V_{ov}^2}{1+aV_{ov}}$$

$$\frac{\partial I_D}{\partial V_{GS}} = K\frac{2V_{ov}(1+aV_{ov})-aV_{ov}^2}{(1+aV_{ov})^2}$$

$$g_m = \frac{KV_{ov}^2}{(1+aV_{ov})}\frac{\frac{2}{V_{ov}}(1+aV_{ov})-a}{(1+aV_{ov})} = I_D\frac{\frac{2}{V_{ov}}(1+aV_{ov})-a}{(1+aV_{ov})}$$

$$\Rightarrow g_m = \frac{I_D}{V_{ov}}\frac{2(1+aV_{ov})-aV_{ov}}{(1+aV_{ov})} = \frac{I_D}{V_{ov}}\frac{2+aV_{ov}}{(1+aV_{ov})}$$

$$\Rightarrow g_m = \frac{I_D}{V_{ov}}\left[1+\frac{1}{(1+aV_{ov})}\right]$$

$$a = \frac{u_0}{2V_{sat}L}+\theta$$

$$\Rightarrow g_m = \frac{I_D}{(V_{GS}-V_{TH})}\left[1+\frac{1}{1+\left(\frac{u_0}{2V_{sat}L}+\theta\right)(V_{GS}-V_{TH})}\right]$$