1. $f(x) = 3 - \frac{6}{\sqrt{x}} = 3 - 6x^{-\frac{1}{2}}$ $\qquad f'(x) = 3x^{-\frac{3}{2}} = \frac{3}{\sqrt{x^3}}$

$f'(x) = 0 \Rightarrow \frac{x}{1} = \frac{6^2}{\sqrt{x}}$, $x = 4$

$\begin{cases} x = 4 \Rightarrow f'(x) = 0 \\ x = 0 \Rightarrow f'(x) \text{ is undefined} \end{cases}$

a. $\therefore$ critical point: $x = 0$, $x = 4$ #

b. by the sign graph

$f' = \ (\text{-----} \mid \text{++++}$, with 0 under "(" and 4 under "|"

$f'(x) < 0$ when $0 < x < 4 \Rightarrow$ decreasing #

$f'(x) > 0$ when $4 < x < \infty \Rightarrow$ increasing #

c. $\because f'(4) = 0$ and $f''(4) > 0$

$f(x)$ has local minimum at $x = 4$ #

2. $f(x) = \sqrt{25 - x^2} = (25 - x^2)^{\frac{1}{2}}$

$f'(x) = \frac{-x}{\sqrt{25 - x^2}} \Rightarrow \begin{cases} x = 0 \Rightarrow f'(x) = 0 \\ x = \pm 5 \Rightarrow f'(x) \text{ is undefined} \end{cases} >$ critical point: $x = 0$, $x = 5$, $x = -5$

$f''(x) = \frac{-25}{\sqrt{(25 - x^2)^3}}$

(a) $f(-5) = 0$, $f(5) = 0$ $>$ $f(x)$ has local minimum at $x = 5$ and $x = -5$ #

$f(0) = 5 \rightarrow f(x)$ has local maximum at $x = 0$ #

(b) $f(x)$ has absolute minimum at $x = 5$ and $x = 5$ #

$f(x)$ has absolute maximum at $x = 0$ #

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3. sketch the graph of $y = x^4 - 2x^2 = x^2(x^2-2)$

$y=0,\ x=0,\ \pm\sqrt{2}$

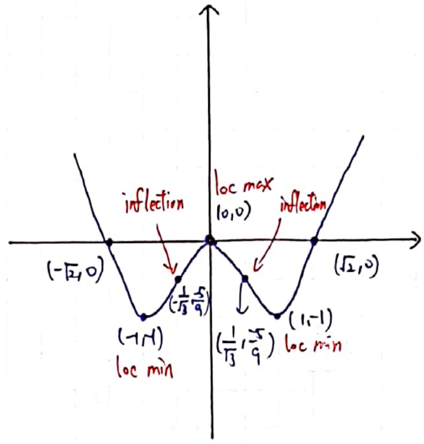
$-1,\ \frac{-1}{\sqrt{3}},\ 0,\ \frac{1}{\sqrt{3}},\ 1$

$f\left(\frac{-1}{\sqrt{3}}\right) = \frac{1}{9} - 2\cdot\frac{1}{3} = \frac{1-6}{9} = \frac{-5}{9}$

$y' = 4x^3 - 4x = 4x(x^2-1) = 4x(x+1)(x-1)$

$y'' = 12x^2 - 4 = 4(3x^2-1) = 4(\sqrt{3}x+1)(\sqrt{3}x-1)$
$= 12\left(x+\frac{1}{\sqrt{3}}\right)\left(x-\frac{1}{\sqrt{3}}\right)$

x	$(-\infty,-1)$	-1	$(-1,\frac{-1}{\sqrt{3}})$	$\frac{-1}{\sqrt{3}}$	$(\frac{-1}{\sqrt{3}},0)$	0	$(0,\frac{1}{\sqrt{3}})$	$\frac{1}{\sqrt{3}}$	$(\frac{1}{\sqrt{3}},1)$	1	$(1,\infty)$
$f(x)$		-1		$\frac{-5}{9}$		0		$\frac{-5}{9}$		-1	
$f'(x)$	$-$	0	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$
$f''(x)$	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$	$+$	$+$
	↘	相對極小值	↗	反曲點	↗	相對極大值	↘	反曲點	↘	相對極小值	↗



$f'(x)<0$ when x is on $(-\infty,-1) \Rightarrow$ decreasing
$f'(x)>0$ when x is on $(-1,0) \Rightarrow$ increasing
$f'(x)<0$ when x is on $(0,1) \Rightarrow$ decreasing
$f'(x)>0$ when x is on $(1,\infty) \Rightarrow$ increasing

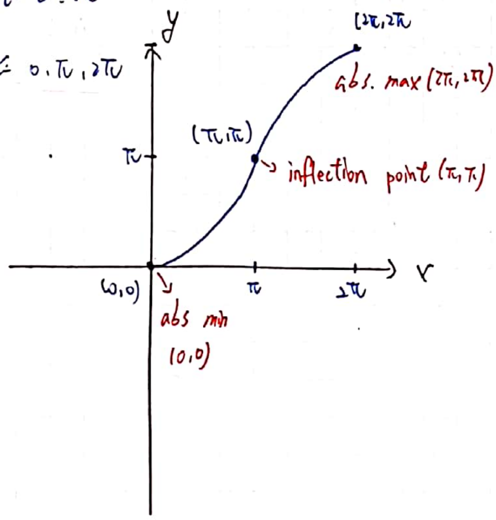
$f''(x)>0$ when x is on $(-\infty,\frac{-1}{\sqrt{3}}) \Rightarrow$ concave up
$f''(x)<0$ when x is on $(\frac{-1}{\sqrt{3}},\frac{1}{\sqrt{3}}) \Rightarrow$ concave down
$f''(x)>0$ when x is on $(\frac{1}{\sqrt{3}},\infty) \Rightarrow$ concave up

4. sketch the graph of $y = x - \sin x$, $0 \le x \le 2\pi$. Include the coordinates of any local and absolute extreme points and inflection point.

$y' = 1 - \cos x$ $\quad y'=0,\ \cos x = 1,\ x = 0, 2\pi$

$y'' = \sin x$ $\quad y''=0,\ \sin x = 0,\ x = 0, \pi, 2\pi$

x	0	$(0,\pi)$	π	$(\pi,2\pi)$	2π
$f(x)$	0		π		2π
$f'(x)$	0	$+$	$+$	$+$	0
$f''(x)$	0	$+$	0	$-$	0
	相對極小值	↗	反曲點	↗	相對極大值



	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0	1	0	-1	0
cos	1	0	-1	0	1

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5. $V = \pi r^2 h = 1000$

$$h = \frac{1000}{\pi r^2}$$

$$A = 8r^2 + 2\pi r h$$

$$= 8r^2 + 2\cancel{\pi r} \cdot \frac{1000}{\pi r^2}$$

$$= 8r^2 + \frac{2000}{r}$$

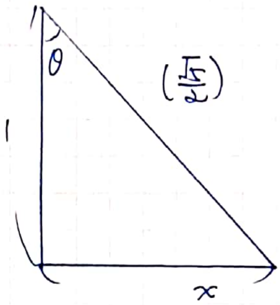
$$\frac{dA}{dr} = 16r + \frac{-2000}{r^2} = 0$$

$$\frac{\cancel{16}r}{1} = \frac{\overset{125}{\cancel{2000}}}{r^2}, \quad r = 5$$

$$\Rightarrow h = \frac{1000}{\pi r^2} = \frac{1000}{\pi 25} = \frac{40}{\pi}$$

$\Rightarrow$ ratio of h to r $\Rightarrow \frac{h}{r} = \frac{\frac{40}{\pi}}{5} = \boxed{\frac{8}{\pi}}$ *

6.



$\left(\frac{1}{2}\right)$

$$\tan\theta = x$$

$$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}(x)$$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{dx}{dt}$$

$$\Rightarrow \left.\frac{dx}{dt}\right|_{x=\frac{1}{2}} = \frac{5}{4} \cdot 3\ \text{rev/min}$$

$$= \frac{5}{4} \cdot 3 \cdot (2\pi)\ \text{rad/min}$$

$$= \frac{15\pi}{2}\ \text{km/min}$$

$$= \frac{15}{2} \times 60\pi\ \text{km/hr}$$

$$= 450\pi\ \text{km/hr}$$ *

$\sec\theta = \frac{\sqrt{5}}{2}$

$\sec^2\theta = \frac{5}{4}$

7.

$$\int \frac{t\sqrt{t}+\sqrt{t}}{t^2}\,dt = \int \frac{t^{\frac{3}{2}}}{t^2} + \frac{t^{\frac{1}{2}}}{t^2}\,dt = \int t^{-\frac{1}{2}} + t^{-\frac{3}{2}}\,dt = 2t^{\frac{1}{2}} + (-2)t^{-\frac{1}{2}} + C = 2\sqrt{t} - \frac{2}{\sqrt{t}} + C$$ *

$\Rightarrow$ check the answers by differention $\Rightarrow \frac{d}{dt}\left(2t^{\frac{1}{2}} - 2t^{-\frac{1}{2}} + C\right) = t^{-\frac{1}{2}} + t^{-\frac{3}{2}} = \frac{t\sqrt{t}+\sqrt{t}}{t^2}$ *

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$\frac{dv}{dt} = 20$

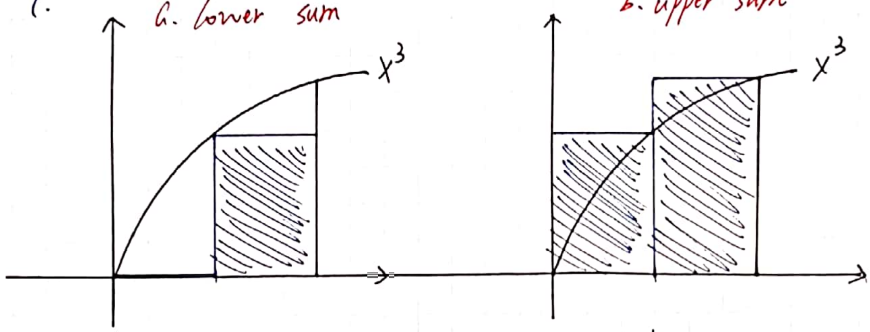
$\therefore V(t) = 20t + c$ and $V(0) = 0$

$\therefore c = 0$

$\Rightarrow V(t) = 20t$

How fast will the rocket be going 1 min later $\Rightarrow V(60) = 1200 \, m/s$ ※

9. a. lower sum　　b. upper sum



$c_k = \frac{k-1}{2}$, $\Delta x_k = \frac{1}{2}$

$\sum_{k=1}^{2} f(c_k) \cdot \Delta x_k = \sum_{k=1}^{2} \left(\frac{k-1}{2}\right)^3 \cdot \frac{1}{2}$

$= \frac{1}{2} \cdot \left(0^3 + \left(\frac{1}{2}\right)^3\right)$

$= \frac{1}{16}$ ※

$c_k = \frac{k}{2}$, $\Delta x_k = \frac{1}{2}$

$\sum_{k=1}^{2} f(c_k) \cdot \Delta x_k = \sum_{k=1}^{2} \left(\frac{k}{2}\right)^3 \cdot \frac{1}{2}$

$= \frac{1}{2} \cdot \left(\left(\frac{1}{2}\right)^3 + 1^3\right)$

$= \frac{1}{2} \cdot \left(\frac{1}{8} + 1\right)$

$= \frac{9}{16}$ ※

10.

a. $\sum_{k=1}^{n}\left(\frac{1}{n} + 2n\right) = \sum_{k=1}^{n}\left(\frac{1}{n}\right) + \sum_{k=1}^{n} 2n = \frac{1}{n} \cdot \sum_{k=1}^{n} 1 + 2n \cdot \sum_{k=1}^{n} 1 = \frac{1}{n} \cdot n + 2n \cdot n = 1 + 2n^2$ ※

b. $\sum_{k=1}^{n}\left(\frac{c}{n}\right) = \frac{c}{n} \sum_{k=1}^{n} 1 = \frac{c}{\cancel{n}} \cdot \cancel{n} = c$ ※

c. $\sum_{k=1}^{n}\left(\frac{k}{n^2}\right) = \frac{1}{n^2} \cdot \sum_{k=1}^{n} k = \frac{1}{n^2} \times \frac{n(n+1)}{2} = \frac{n+1}{2n}$ ※

11. $P = \{\underset{x_0}{-2}, \underset{x_1}{-1.6}, \underset{x_2}{-0.5}, \underset{x_3}{0}, \underset{x_4}{0.8}, \underset{x_5}{1}\}$

$\Rightarrow \begin{cases} |x_1 - x_0| = |-1.6+2| = 0.4 \\ |x_2 - x_1| = |-0.5+1.6| = 1.1 \\ |x_3 - x_2| = |0+0.5| = 0.5 \\ |x_4 - x_3| = |0.8-0| = 0.8 \\ |x_5 - x_4| = |1-0.8| = 0.2 \end{cases}$

the largest is $\|P\| = 1.1$ ※

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