

# Introduction to Analog Integrated Circuit Design

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## Short Channel Effects

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# Outline

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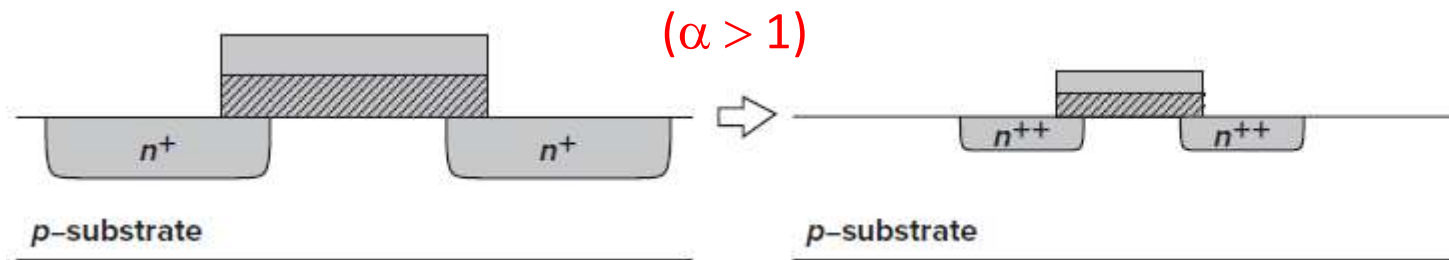
- Scaling Theory on CMOS VLSI
- Short Channel Effects
- MOS Device Models
- Process Corners

*Referred to Textbook, chapter 17*

# MOS Transistor Scaling (1)

The two principal reasons for the dominance of CMOS technology in today's semiconductor industry are

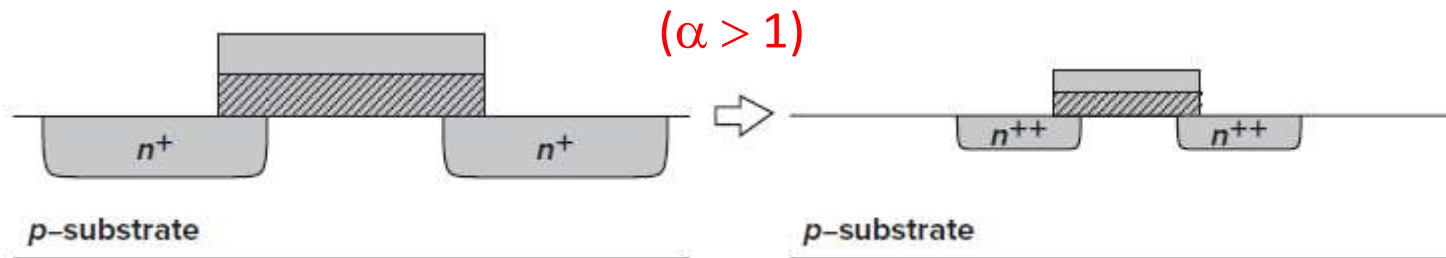
- (1) the **zero static power** dissipation of CMOS logic and
- (2) the scalability of MOSFETs



- **Definition:** Scaling factor  $\alpha$
- **The ideal scaling theory follows three rules:**
  - (1) To reduce all lateral and vertical dimensions by  $\alpha (> 1)$
  - (2) To reduce the threshold voltage and the supply voltage by  $\alpha$
  - (3) To increase all of the doping levels by  $\alpha$  (Fig. 17.1)
- The above is called “**Constant-Field Scaling**”

# MOS Transistor Scaling (2)

- **CMOS: Zero** static power consumption and **Scalability** of MOSFETs



$$I_{D,scaled} = \frac{1}{2} \mu_n (\alpha C_{ox}) \left( \frac{W/\alpha}{L/\alpha} \right) \left( \frac{V_{GS}}{\alpha} - \frac{V_{TH}}{\alpha} \right)^2$$

$$= \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 \frac{1}{\alpha}$$

The total channel capacitance,

$$C_{ch,scaled} = \frac{W}{\alpha} \frac{L}{\alpha} (\alpha C_{ox})$$

$$= \frac{1}{\alpha} W L C_{ox}$$

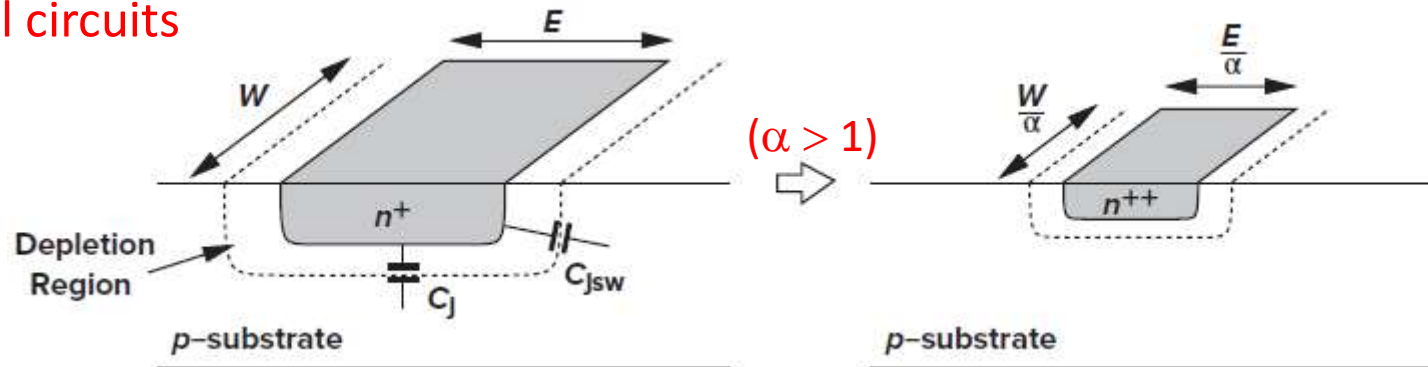
$$W_d = \sqrt{\frac{2\epsilon_{si}}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (\phi_B + V_R)} \quad \xrightarrow{V_R \gg \phi_B} \quad W_{d,scaled} \approx \sqrt{\frac{2\epsilon_{si}}{q} \left( \frac{1}{\alpha N_A} + \frac{1}{\alpha N_D} \right) \frac{V_R}{\alpha}}$$

$$\approx \frac{1}{\alpha} \sqrt{\frac{2\epsilon_{si}}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) V_R}$$

$W_d$ : Width of depletion region

# MOS Transistor Scaling (3)

Digital circuits



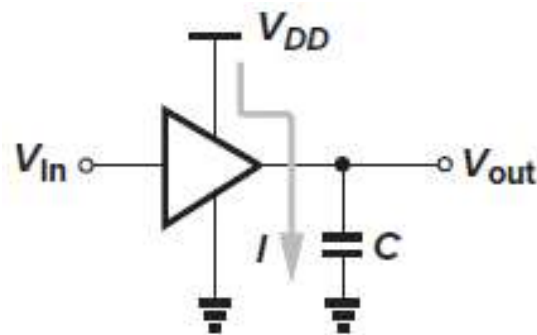
$$C_{S/D,scaled} = \frac{W}{\alpha} \frac{E}{\alpha} (\alpha C_j) + 2 \left( \frac{W}{\alpha} + \frac{E}{\alpha} \right) (C_{jsw})$$

$$= [W E C_j + 2(W + E) C_{jsw}] \frac{1}{\alpha}$$

$$P = f C V^2$$

$$P_{scaled} = f (C / \alpha) (V / \alpha)^2$$

$$= \frac{1}{\alpha^3} f C V^2$$

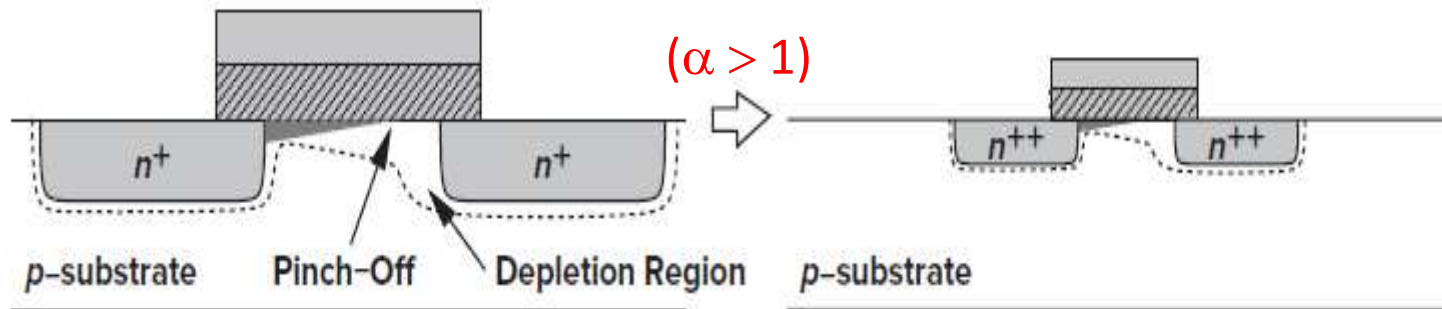


$$T_{d,scaled} = \frac{C / \alpha}{I / \alpha} \frac{V_{DD}}{\alpha}$$

$$= \left( \frac{C}{I} V_{DD} \right) \frac{1}{\alpha}$$

# MOS Transistor Scaling (4)

## Analog circuits



$$g_{m,scaled} = \mu(\alpha C_{ox}) \frac{W/\alpha}{L/\alpha} \frac{V_{GS} - V_{TH}}{\alpha}$$

$$= \mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

$$r_{O,scaled} = \frac{1}{\alpha \lambda \frac{I_D}{\alpha}}$$

$$= \frac{1}{\lambda I_D}$$

$$P = IV$$

$$P_{scaled} = (I / \alpha)(V / \alpha)$$

$$= \frac{1}{\alpha^2} \cdot IV$$

Looks good to analog design, but ...

# MOS Transistor Scaling (5)

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- The reluctance of circuit designers to use *a lower supply voltage* and the fundamental limitations in decreasing the MOS threshold voltage have led to another scaling scenario: **Constant-voltage scaling**.
- In this case, the device dimensions shrink by  $\alpha$ , *the doping levels increase by  $\alpha$ , and the voltages remain constant*, thereby *increasing the electric fields by  $\alpha$* .
- Such *high electric fields* both raise the possibility of device breakdown and exacerbate short-channel effects.

In reality, technology scaling has followed “*a mixture of constant-field and constant-voltage trend*,” thus demanding innovative device design so as to achieve reliability and performance

# Outline

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- Scaling Theory on CMOS VLSI
- Short Channel Effects
  - Threshold Voltage Variation ( $V_{TH}$ )
  - Mobility Degeneration ( $\mu$ )
  - Velocity Saturation ( $g_m$ )
  - Hot Carrier Effects ( $V_{TH}$ )
  - Output Impedance Variation ( $r_{ds}$ )
- MOS Device Models
- Process Corners



# Short-Channel Effects

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- **Constant-Scaling is not easily achieved. Why?**  
(CMOS VLSI can tell you more)
- **Small-geometry effects** arise because of the following five factors
  - The electric fields tend to increase because the **supply voltage has not scaled proportionally**
  - The built-in potential term in Eq. (17.5),  $\phi_B$ , is **neither scalable nor negligible**
$$W_d = \sqrt{\frac{2\epsilon_{si}}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (\phi_B + V_R)}$$
  - The depth of S/D junctions cannot be **reduced easily**
  - The **mobility decreases** as the substrate doping increases
  - The subthreshold slope (described below) is **not scalable**

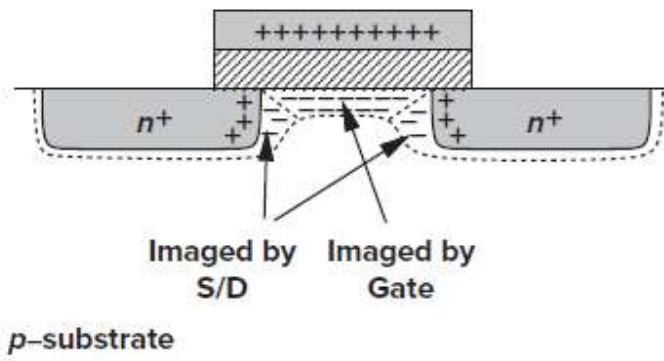
# Short-Channel Effects

## Threshold Voltage Variation

**min  $V_{TH}$  is bounded\*** by

1. Subthreshold behavior
2. Temperature variation
3. Process variation
4. Channel length

\* Max  $V_{TH} < V_{DD}/4$  is for digital circuit speed



## Subthreshold behavior

For long-channel devices, the subthreshold drain current

$$I_D = \mu C_d \frac{W}{L} V_T^2 \left( \exp \frac{V_{GS} - V_{TH}}{\zeta V_T} \right) \left( 1 - \exp \frac{-V_{DS}}{V_T} \right)$$

where

$$C_d = \sqrt{\epsilon_{si} q N_{sub} / (4 \phi_B)}$$

$$V_T = kT/q, \text{ and } \zeta = 1 + C_d / C_{ox}$$

At saturation ( $V_{DS} > 4V_T$ ),  $I_D$  is independent on  $V_{DS}$

$$\frac{\partial(\log_{10} I_D)}{\partial V_{GS}} = (\log_{10} e) \frac{1}{\zeta V_T}$$

The inverse of this quantity is usually called the “subthreshold slope”,  $S$   $( )^{-1}$

$$S = 2.3 V_T \left( 1 + \frac{C_d}{C_{ox}} \right) \text{ V/dec}$$

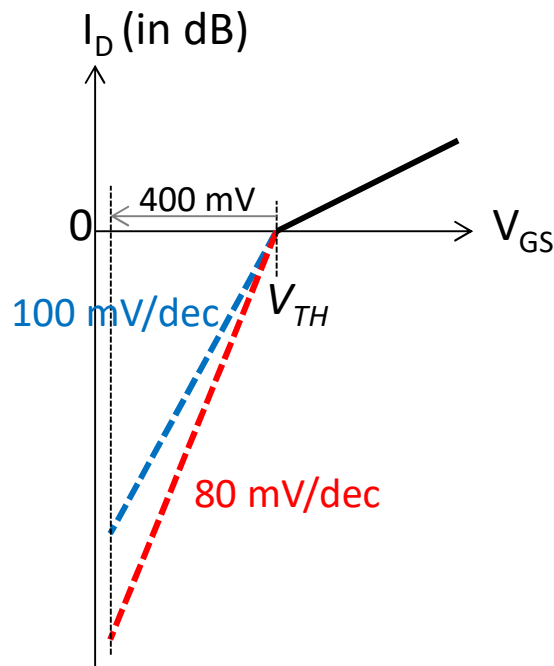
# Short-Channel Effects

## Threshold Voltage Variation

### Subthreshold slope

$$S = 2.3V_T \left( 1 + \frac{C_d}{C_{ox}} \right) \text{ V/dec}$$

where  $C_d = \sqrt{\epsilon_{si} q N_{sub} / (4\phi_B)}$



For example, if  $C_d = 0.67C_{ox}$ , then  $S = 100 \text{ mV/dec}$ , suggesting that a change of 100 mV in  $V_{GS}$  leads to a tenfold reduction in the drain current

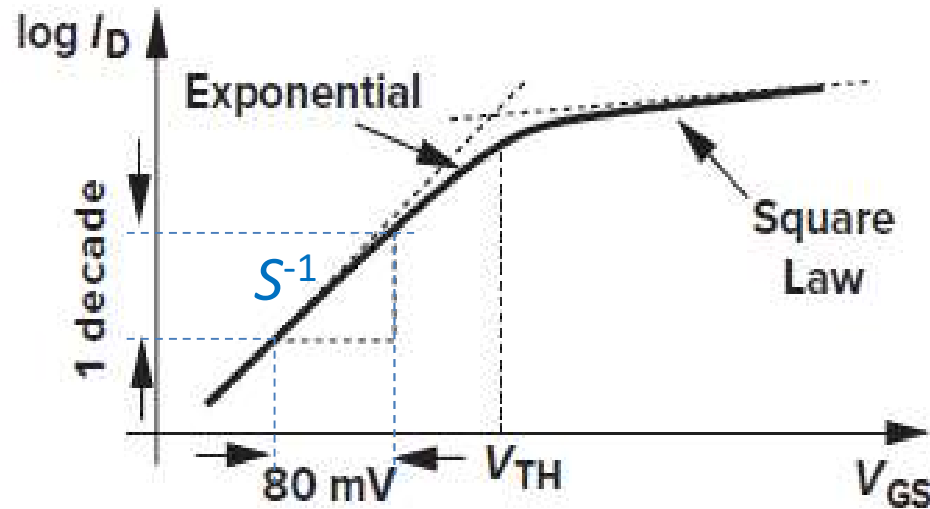
In order to turn off the transistor by lowering  $V_{GS}$  below  $V_{TH}$  (subthreshold operation),  $S$  must be as small as possible, i.e.,  $C_d/C_{ox}$  must be minimized

- $S = 100 \text{ mV/dec} \Rightarrow \Delta V_{GS} = -100 \text{ mV}$  has a current reduction of 1/10
- If  $S$  is smaller, 80 mV/dec,  $\Rightarrow \Delta V_{GS} = -400 \text{ mV}$  can get a current reduction of 10<sup>-5</sup>. This means the on/off current ratio can have a scaling of 10<sup>5</sup>
- That's called the **leakage current**, we care!!

# Short-Channel Effects

## Threshold Voltage Variation

### Leakage Issue



### Subthreshold Current

$$I_D = \mu C_d \frac{W}{L} V_T^2 \left( \exp \frac{V_{GS} - V_{TH}}{\zeta V_T} \right) \left( 1 - \exp \frac{-V_{DS}}{V_T} \right)$$

$$\frac{\partial(\log_{10} I_D)}{\partial V_{GS}} = (\log_{10} e) \frac{1}{\zeta V_T} \quad \longrightarrow \quad S = 2.3 V_T \left( 1 + \frac{C_d}{C_{ox}} \right) \text{ V/dec}$$

Comments:

- If  $V_{TH}$  is too small  $\Rightarrow V_{GS} - V_{TH}$  cannot be more negative  $\Rightarrow$  **OFF** current is not low!!
- Temperature variation on  $V_{TH}$  causes about 100mV over  $-40^\circ\text{C} \sim +125^\circ\text{C}$

# Short-Channel Effects

## Threshold Voltage Variation

### Channel Length Issue

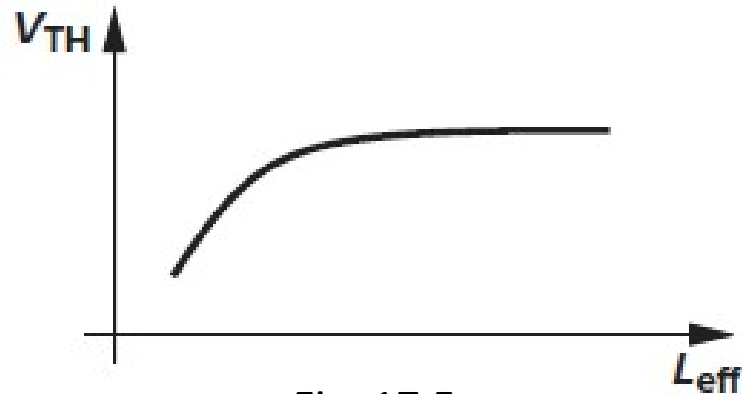


Fig. 17.5

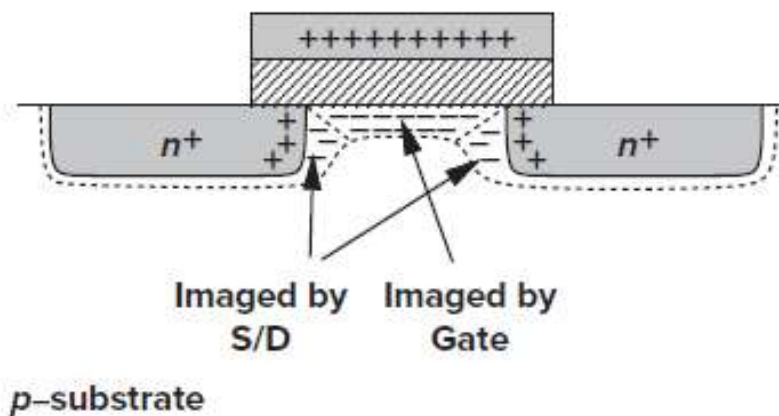


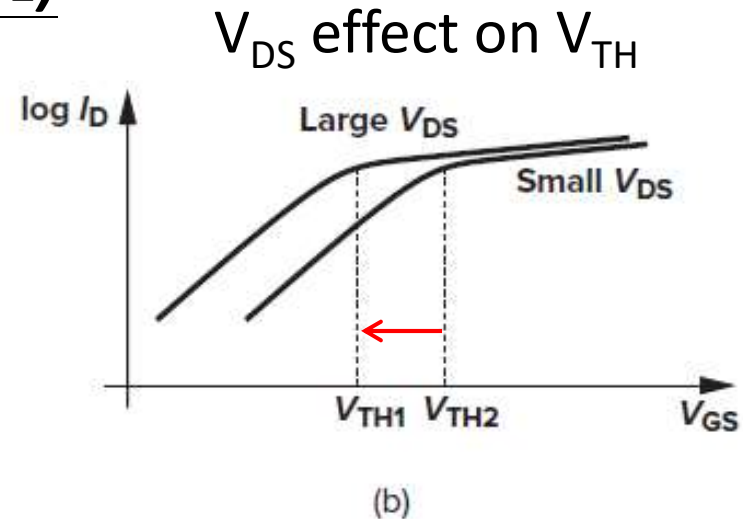
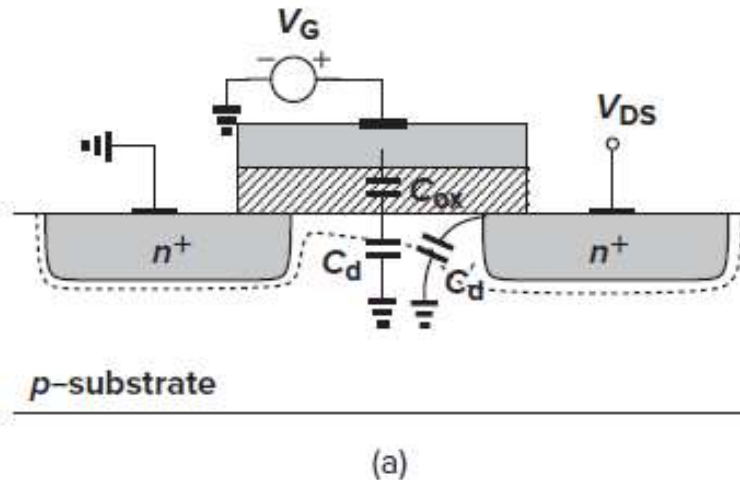
Fig. 17.6

- As shown in Fig. 17.5, transistors fabricated on the same wafer but with different lengths yield lower  $V_{TH}$  as  $L$  decreases
- This is because the depletion regions associated with the source and drain junctions protrude into the channel area considerably, thereby reducing the immobile charge that must be imaged by the charge on the gate (Fig. 17.6)
- In other words, part of the immobile charge in the substrate is now imaged by the charge inside the source and drain areas rather than by the charge on the gate.
- As a result, the gate voltage required to create an inversion layer **decreases**
- However, things cannot be so simple ...

# Short-Channel Effects

## Threshold Voltage Variation

### Drain-induced barrier lowering (DIBL)

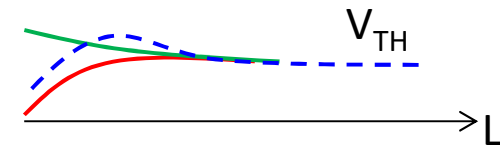


- In short-channel devices, the drain voltage also makes the surface more positive by creating a two-dimensional field in the depletion region
- The principal impact of **DIBL** on circuit design is the degraded output impedance

# Short-Channel Effects

## Threshold Voltage Variation

### Reverse Short-Channel Effect



- In nanometer CMOS technologies, the threshold voltage *decreases as* the channel length increases from its minimum value
- A “**halo**” implant of heavy doping surrounds the source and drain junctions. This implant reduces the penetration of the drain depletion region into the channel area, thereby improving the device characteristics
- Due to the nonuniform substrate doping along the channel in Fig. 17.8, the “local” threshold voltage also varies from the source to the drain
- The channel length increases, the average substrate doping decreases, and so does the threshold voltage ( $L$  is shorter,  $\text{avg}(N_{sub})$  is decreased,  $V_{TH}$  is also lower)

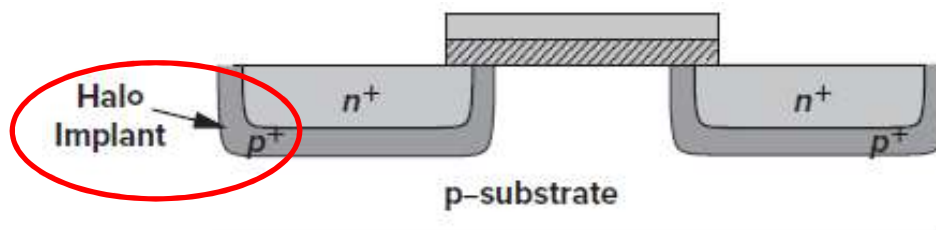


Fig. 17.8

$$V_{TH} = \phi_{MS} + 2\phi_F + \frac{Q_{dep}}{C_{ox}} \uparrow$$

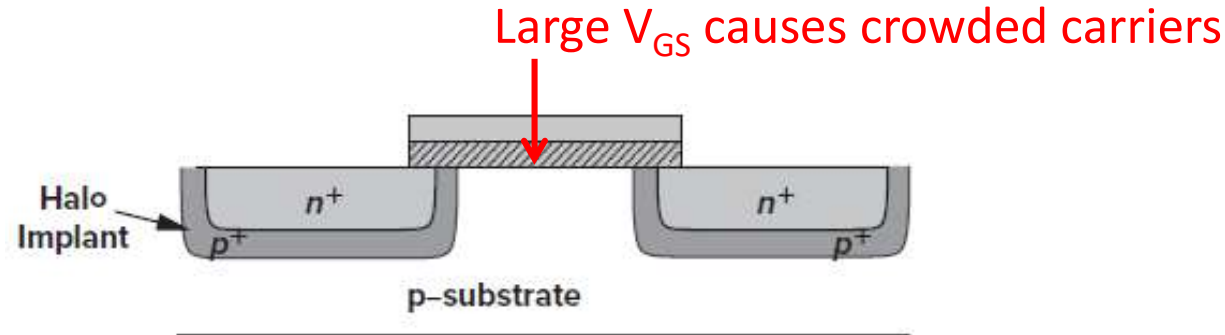
$$\phi_F = (kT/q) \ln(N_{sub}/n_i)$$

$$Q_{dep} = \sqrt{4q\epsilon_{si}|\phi_F|N_{sub}}$$

# Short-Channel Effects

## Mobility Degeneration

### Vertical Field



$$\mu_{eff} = \frac{\mu_0}{1 + \theta(V_{GS} - V_{TH})}$$

$\theta$  is a fitting parameter roughly equal to  $(10^{-7}/t_{ox})$   $V^{-1}$  [7]. For example, if  $t_{ox} = 100 \text{ \AA}$ , then  $\theta \approx 1 \text{ V}^{-1}$

$$I_D = \frac{1}{2} \frac{\mu_0 C_{ox}}{1 + \theta(V_{GS} - V_{TH})} \frac{W}{L} (V_{GS} - V_{TH})^2$$

Assume  $\theta(V_{GS} - V_{TH}) \ll 1$

$$\begin{aligned} I_D &\approx \frac{1}{2} \mu_0 C_{ox} \frac{W}{L} [1 - \theta(V_{GS} - V_{TH})] (V_{GS} - V_{TH})^2 \\ &\approx \frac{1}{2} \mu_0 C_{ox} \frac{W}{L} [(V_{GS} - V_{TH})^2 - \theta(V_{GS} - V_{TH})^3] \end{aligned}$$

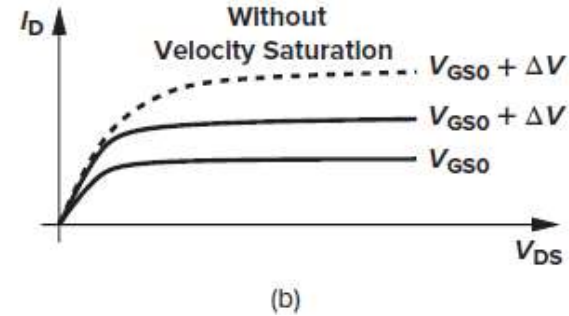
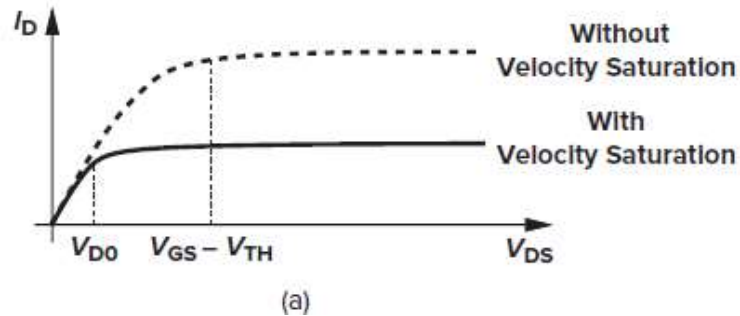
↓  
The mobility begins to fall as the overdrive exceeds 100 mV



# Short-Channel Effects

## Velocity Saturation

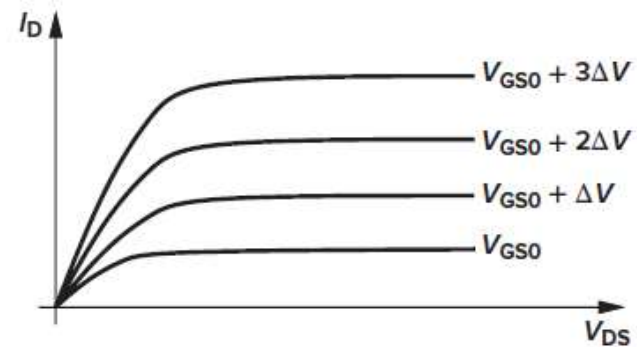
### Lateral Field



$$v = \mu E$$

$v$  approaches a saturated value,  $\sim 10^7$  cm/s

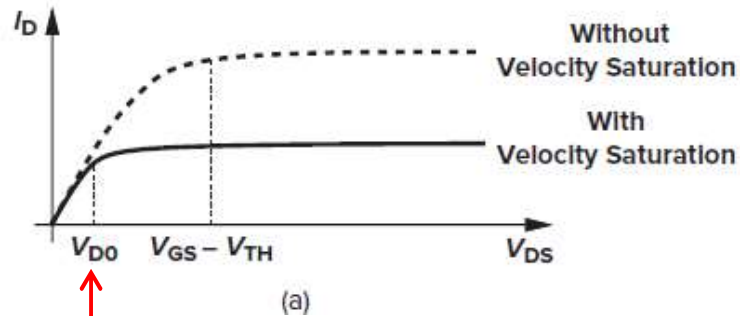
$$\begin{aligned} I_D &= v_{sat} Q_d \\ &= v_{sat} W C_{ox} (V_{GS} - V_{TH}) \\ g_m &= v_{sat} W C_{ox} \end{aligned}$$



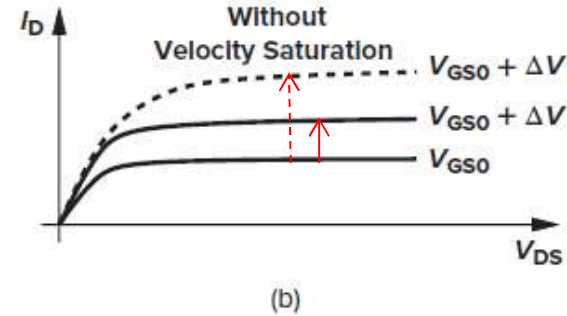
# Short-Channel Effects

## Velocity Saturation

### Lateral Field



$$V_{DS,sat} = \frac{2\mu_{eff}L(V_{GS} - V_{TH})}{2\mu_{eff}L + V_{GS} - V_{TH}}$$



### Observed from Lab data

$$I_D = WC_{ox}v_{sat} \frac{(V_{GS} - V_{TH})^2}{V_{GS} - V_{TH} + 2\frac{v_{sat}L}{\mu_{eff}}}$$

where  $\mu_{eff} = \frac{\mu_0}{1 + \theta(V_{GS} - V_{TH})}$

If  $L$  or  $v_{sat}$  is large enough,  
 $I_D = 1/2 \cdot \mu_0 C_{ox} (W/L) \cdot (V_{GS} - V_{TH})^2$

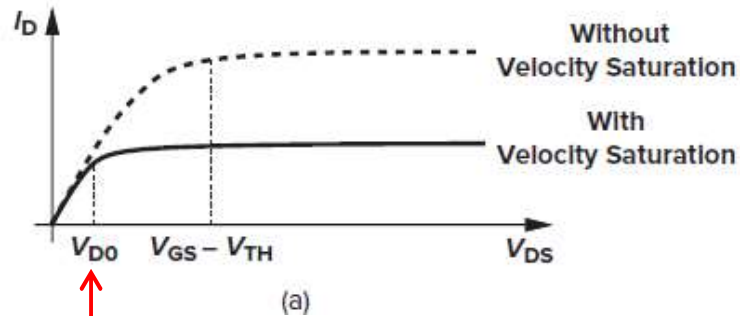
If  $L$  or  $v_{sat}$  is very small,  
 $I_D = WC_{ox}v_{sat} (V_{GS} - V_{TH})$

For example, if  $v_{sat} \approx 10^7$  cm/s,  $L = 0.25 \mu m$ , and  $\mu_0 \approx 350$  cm<sup>2</sup>/V/s, we have  $2v_{sat}L/\mu_0 \approx 1.43$  V, recognizing that for overdrive voltages of a few hundred millivolts, the transistor operation is somewhat close to the square law

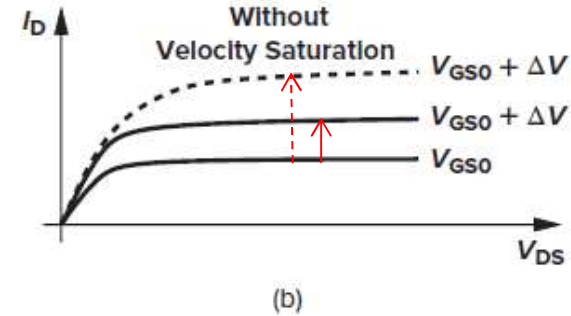
# Short-Channel Effects

## Velocity Saturation

### Lateral Field



$$V_{DS,sat} = \frac{2\mu_{eff}L(V_{GS} - V_{TH})}{2\mu_{eff}L + V_{GS} - V_{TH}}$$



Observed from Lab data

$$I_D = WC_{ox}v_{sat} \frac{(V_{GS} - V_{TH})^2}{V_{GS} - V_{TH} + 2\frac{v_{sat}L}{\mu_{eff}}}$$

where  $\mu_{eff} = \frac{\mu_0}{1 + \theta(V_{GS} - V_{TH})}$

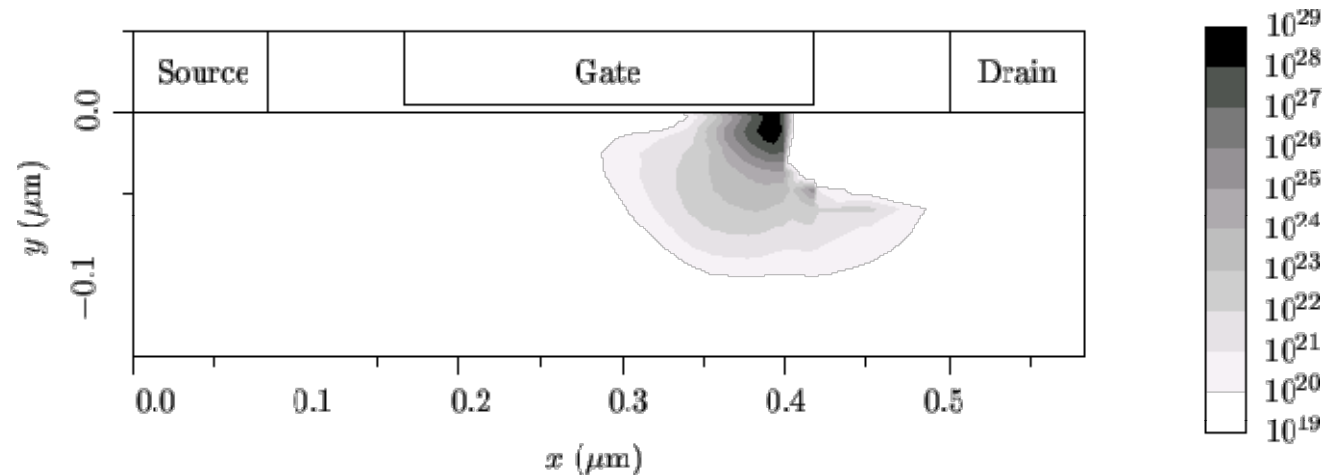


$$\begin{aligned} I_D &= WC_{ox}v_{sat} \frac{(V_{GS} - V_{TH})^2}{V_{GS} - V_{TH} + \frac{2v_{sat}L}{\mu_0}[1 + \theta(V_{GS} - V_{TH})]} \\ &= WC_{ox}v_{sat} \frac{(V_{GS} - V_{TH})^2}{\frac{2v_{sat}L}{\mu_0} + \left(1 + \frac{2v_{sat}L\theta}{\mu_0}\right)(V_{GS} - V_{TH})} \\ &= \frac{1}{2}\mu_0C_{ox}\frac{W}{L} \frac{(V_{GS} - V_{TH})^2}{1 + \left(\frac{\mu_0}{2v_{sat}L} + \theta\right)(V_{GS} - V_{TH})} \end{aligned}$$

➡ For example, the drain current contains “high-order nonlinear terms”

# Short-Channel Effects

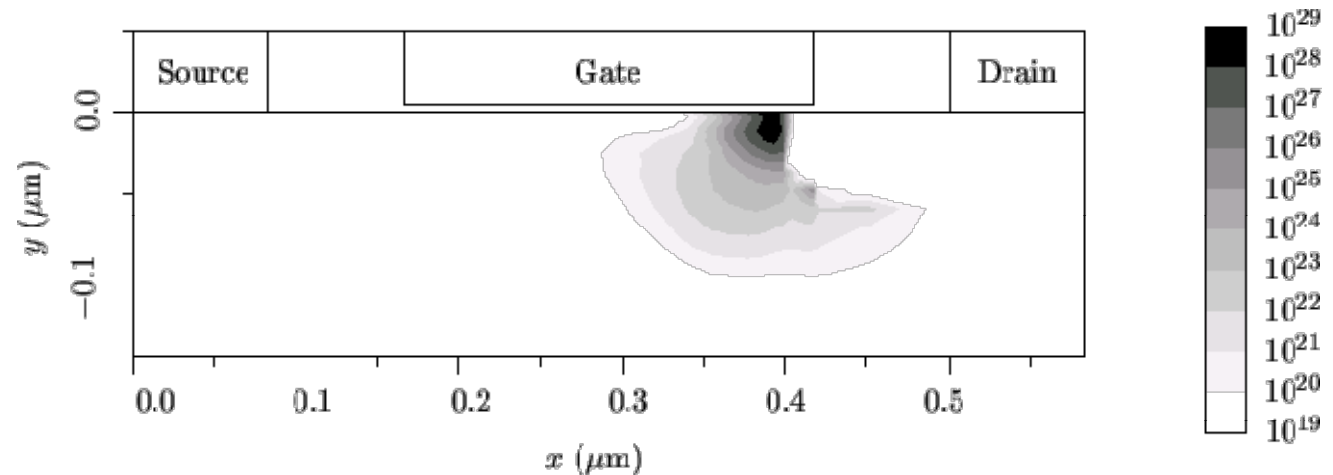
## Hot Carrier Effects



- In the vicinity of the drain region, **hot carriers** may “hit” the silicon atoms at high speeds, thereby creating **impact ionization**. As a result, **new electrons and holes are generated**, with the electrons absorbed by the drain and the holes by the substrate.
- Thus, **a finite drain-substrate current appears**. Also, if the carriers acquire a very high energy, they may be injected into the gate oxide and even flow out the gate terminal, introducing a **gate current**.
- The substrate and gate currents are often measured to study hot carrier effects.
- In nanometer technologies, hot carrier effects have subsided. This is because the energy required to create an electron-hole pair,  $E_g \approx 1.12 \text{ eV}$ , is simply not available if the supply voltage is around 1 V.

# Short-Channel Effects

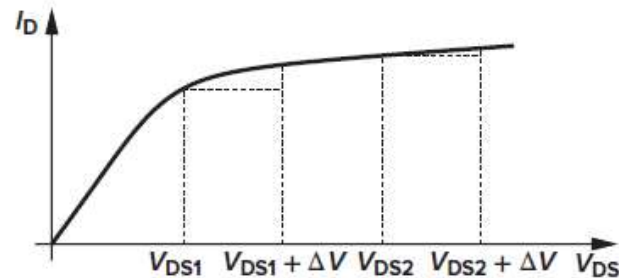
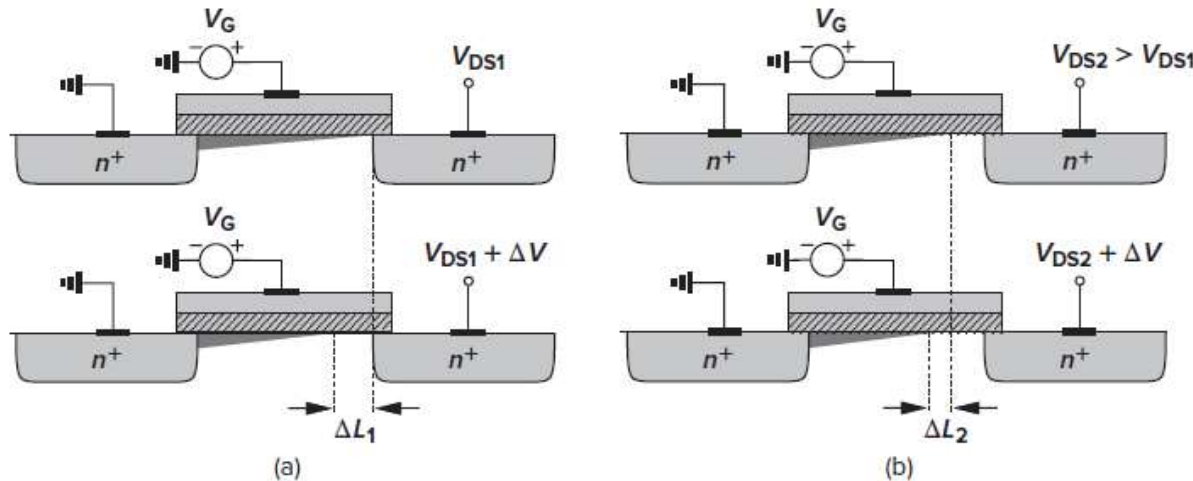
## Hot Carrier Effects



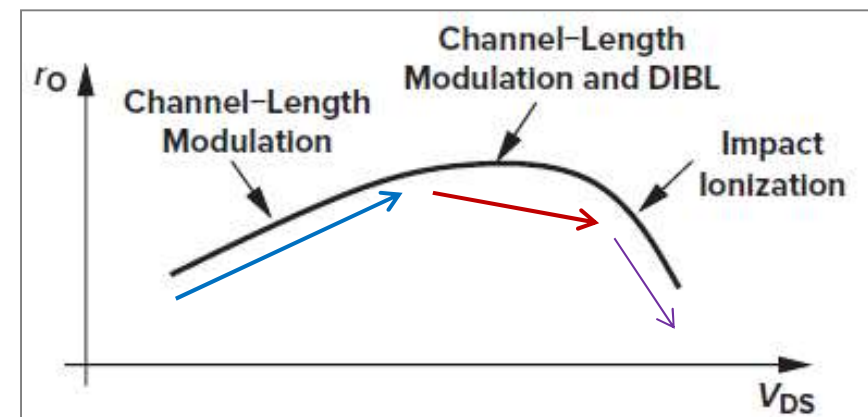
- If a MOS transistor is operated under pinch-off condition, also known as "saturated case", hot carriers traveling with saturation velocity can cause parasitic effects **at the drain side of the channel** known as "**Hot Carrier Effects**" (HCE).
  - **Hot Carrier Effect** causes **threshold voltage reduction** or **latch-up**
  - **Lightly Doped Drain** (LDD) or **Graded Channel** (GC) help to avoid HCE
- Carrier injection into the gate oxide can lead to hot carrier degradation effects such as threshold voltage changes due to occupied traps in the oxide. Hot carriers can also generate traps at the silicon-oxide interface known as "fast surface states" leading to subthreshold swing deterioration and **stress-induced drain leakage**

# Short-Channel Effects

## Output Impedance Variation



$$r_o = \frac{2L}{1 - \frac{\Delta L}{L}} \frac{1}{I_D} \sqrt{\frac{qN_B}{2\epsilon_{si}} (V_{DS} - V_{DS,sat})}$$



In reality, however,  $r_o$  varies with  $V_{DS}$ . As  $V_{DS}$  increases and the pinch-off point moves toward the source, the rate at which the depletion region around the source becomes wider decreases, resulting in a higher incremental output impedance.

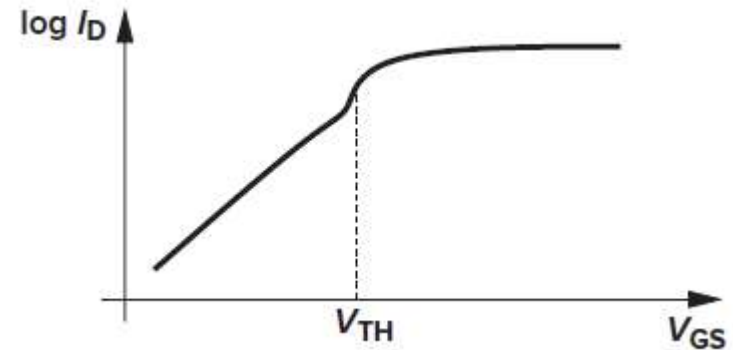
Impact ionization limits the maximum gain that can be obtained from cascode structures because it introduces a small-signal resistance from the drain to the substrate rather than to the source.

# Modeling

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- Level 1 (You have learned in Microelectronics)
- Level 2 (high-order effects considered)

- Level 3



- **BSIM** 1, 2, 3, 4
- **HSPICE Level 28**
- MOS9
- **EKV** (Subthreshold)

# Modeling (Detailed in the textbook)

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## Level 1 Model

$$I_D = \frac{1}{2} K_P \frac{W}{L - 2L_D} [2(V_{GS} - V_{TH})V_{DS} - V_{DS}^2] (1 + \lambda V_{DS}) \quad \text{Triode Region}$$

$$I_D = \frac{1}{2} K_P \frac{W}{L - 2L_D} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS}) \quad \text{Saturation Region}$$

$$C_{GS} = \frac{2}{3} W L C_{ox} \left\{ 1 - \frac{(V_{GS} - V_{DS} - V_{TH})^2}{[2(V_{GS} - V_{TH}) - V_{DS}]^2} \right\} + W C_{ov}$$

$$C_{GD} = \frac{2}{3} W L C_{ox} \left\{ 1 - \frac{(V_{GS} - V_{TH})^2}{[2(V_{GS} - V_{TH}) - V_{DS}]^2} \right\} + W C_{ov}$$

$$C_{GB} = 0.$$

## Level 2 Model

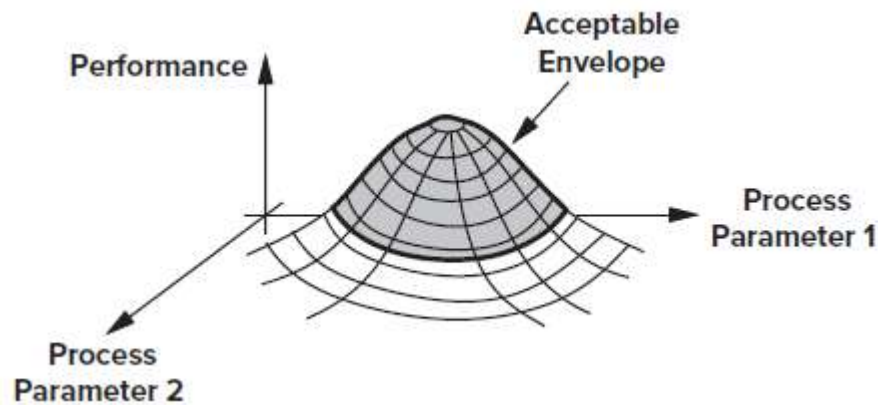
$$I_D = \mu C_{ox} \frac{W}{L} \left\{ (V_{GS} - V_{TH0})V_{DS} - \frac{V_{DS}^2}{2} - \frac{2}{3} \gamma [(V_{DS} - V_{BS} + 2\phi_F)^{3/2} - (-V_{BS} + 2\phi_F)^{3/2}] \right\}$$

$$V_{D,sat} = V_{GS} - V_{TH0} - \phi_F + \gamma^2 \left[ 1 - \sqrt{1 + \frac{2}{\gamma^2} (V_{GS} - V_{TH0} + \phi_F)} \right]$$

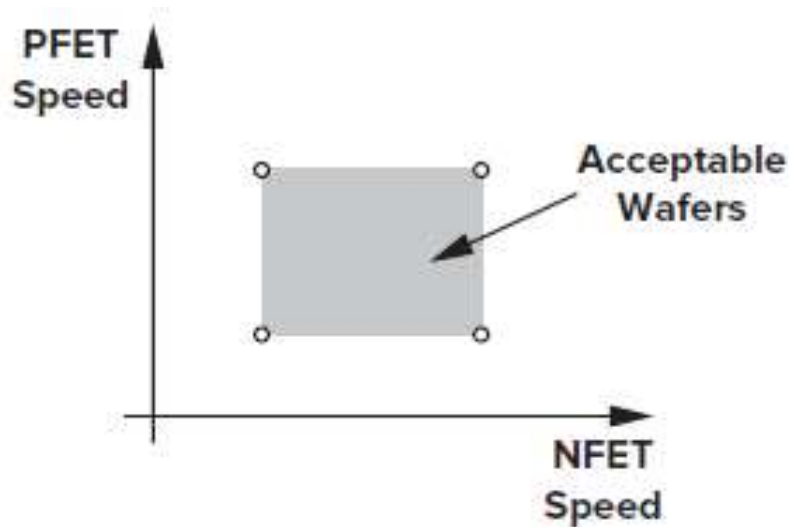
$$I_{DS} = I_{D,sat} \frac{1}{1 - \lambda V_{DS}}$$



# Process Corners



In order to facilitate the task of circuit design to some extent, process engineers guarantee a [performance envelope](#) for the devices, in essence tightening the anticipated parameter variations by discarding wafers that fall out of the envelope (Fig. 17.17).



Illustrated in Fig. 17.18, the idea is to constrain [the speed envelope of the NMOS and PMOS transistors](#) to a rectangle defined by four corners:

- fast NFET and fast PFET
- slow NFET and slow PFET
- fast NFET and slow PFET
- slow NFET and fast PFET