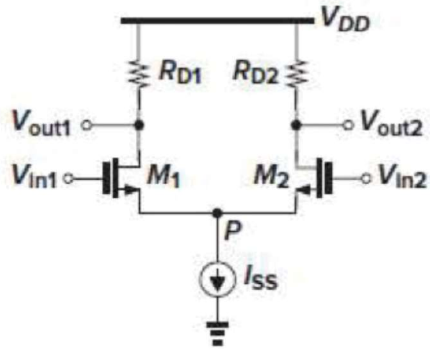
Introduction to Analog Integrated Circuits (111), DECE, NTUST

Homework 3 (Due date: 10/19)

HW3.1: (20 points)

Fig. 3.1 shows a fully differential amplifier and its transfer curves. Assume Vin1 and Vin2 are differential signals, for $I_{D1}=I_{D2}=I_{SS}/2$, $V_{GS1} = V_{GS2} = V_{TH}$+200mV. ΔVin1 is a specified voltage means M1 or M2 is turned off. Please identify ΔVin1 and describe Gm = *f*(ΔVin). Note: *channel length modulation* and *body effect* are ignored.



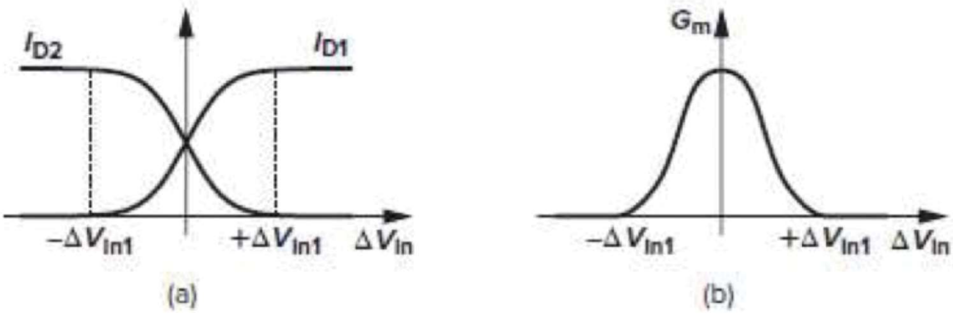


Fig. 3.1

①

$$V_{in1}-V_{in2}=V_{GS1}-V_{GS2}$$

$$(V_{GS}-V_{th})^2=\frac{I_D}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}$$

求 $I_{D1}-I_{D2}$

$$V_{GS}=\sqrt{\frac{2I_D}{\mu_n C_{ox}\frac{W}{L}}}+V_{th} \Rightarrow V_{GS1}=\sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\frac{W}{L}}}+V_{th}$$

$$\Rightarrow V_{in1}-V_{in2}=\sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\frac{W}{L}}}-\sqrt{\frac{2I_{D2}}{\mu_n C_{ox}\frac{W}{L}}}$$

$$(V_{in1}-V_{in2})^2=\frac{2(I_{D1}+I_{D2})}{\mu_n C_{ox}\frac{W}{L}}-2\sqrt{\frac{4I_{D1}I_{D2}}{(\mu_n C_{ox}\frac{W}{L})^2}}$$ ，又 $I_{D1}+I_{D2}=I_{SS}$

$$=\frac{2}{\mu_n C_{ox}\frac{W}{L}}\left(I_{SS}-2\sqrt{I_{D1}I_{D2}}\right)$$

$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1}-V_{in2})^2-I_{SS}=-2\sqrt{I_{D1}I_{D2}}$$ ， $\begin{cases} V_{in1}=V_{GS1}-V_{th} \\ V_{in2}=V_{GS2}-V_{th} \end{cases}$ ，

$$\xrightarrow{\text{2边平方}} \frac{1}{4}(\mu_n C_{ox}\frac{W}{L})^2(V_{in1}-V_{in2})^4+I_{SS}^2-\mu_n C_{ox}\frac{W}{L}(V_{in1}-V_{in2})^2\cdot I_{SS}=4I_{D1}I_{D2}$$

$$\text{又 } 4I_{D1}I_{D2}=(I_{D1}+I_{D2})^2-(I_{D1}-I_{D2})^2=I_{SS}^2-(I_{D1}-I_{D2})^2$$

$$\Rightarrow \frac{1}{4}(\mu_n C_{ox}\frac{W}{L})^2(V_{in1}-V_{in2})^4+\cancel{I_{SS}^2}-\mu_n C_{ox}\frac{W}{L}(V_{in1}-V_{in2})^2\cdot I_{SS}=\cancel{I_{SS}^2}-(I_{D1}-I_{D2})^2$$

$$\Rightarrow (I_{D1}-I_{D2})^2=-\frac{1}{4}(\mu_n C_{ox}\frac{W}{L})^2(V_{in1}-V_{in2})^4+\mu_n C_{ox}\frac{W}{L}(V_{in1}-V_{in2})^2\cdot I_{SS}$$

$$=-\frac{1}{4}(\mu_n C_{ox}\frac{W}{L})^2(V_{in1}-V_{in2})^4+\frac{4}{4}I_{SS}\cdot\frac{(\mu_n C_{ox}\frac{W}{L})^2}{\mu_n C_{ox}\frac{W}{L}}\cdot(V_{in1}-V_{in2})^2$$

$$=-\frac{1}{4}(\mu_n C_{ox}\frac{W}{L})^2(V_{in1}-V_{in2})^4+\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}\cdot\frac{1}{4}(\mu_n C_{ox}\frac{W}{L})^2\cdot(V_{in1}-V_{in2})^2$$

$$=\frac{1}{4}(\mu_n C_{ox}\frac{W}{L})^2(V_{in1}-V_{in2})^2\left[\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}-(V_{in1}-V_{in2})^2\right]$$

$$(I_{D1}-I_{D2})=\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1}-V_{in2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}-(V_{in1}-V_{in2})^2}$$

$$=\sqrt{\cancel{\frac{1}{4}}(\mu_n C_{ox})^{\cancel{2}}(\frac{W}{L})^{\cancel{2}}\cdot\frac{\cancel{4}I_{SS}}{\cancel{\mu_n C_{ox}\frac{W}{L}}}}\cdot(V_{in1}-V_{in2})\cdot\sqrt{1-\frac{\mu_n C_{ox}\frac{W}{L}}{4I_{SS}}(V_{in1}-V_{in2})^2}$$

$$=\sqrt{\mu_n C_{ox}\frac{W}{L}\cdot I_{SS}}\cdot(V_{in1}-V_{in2})\cdot\sqrt{1-\frac{\mu_n C_{ox}\frac{W}{L}}{4I_{SS}}(V_{in1}-V_{in2})^2}$$

$$\Rightarrow \Delta I_D=\sqrt{\mu_n C_{ox}\frac{W}{L}\cdot I_{SS}}\cdot\Delta V_{in}\cdot\sqrt{1-\frac{\mu_n C_{ox}\frac{W}{L}}{4I_{SS}}\cdot\Delta V_{in}^2}$$

$$g_m = \frac{\partial \Delta I_D}{\partial \Delta V_{in}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\cdot \frac{\frac{4I_{SS}}{\mu_n C_{ox} W/L} - 2\Delta V_{in}^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox} W/L} - \Delta V_{in}^2}}$$ ※

(2) $\Delta V_{in1} = ?$

M_2 off, $I_{D1} = I_{SS}$, $V_{in1} = V_{GS} - V_{th} = \Delta V_{in1}$

$$I_{SS} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\cdot \Delta V_{in1}^2$$

$$\therefore \Delta V_{in1} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$$ ※

HW3.2: (30 points)

Assuming that all the transistors in the circuits of Figs. 3.2 are saturated and $\lambda \neq 0$, calculate the small-signal differential voltage gain. Please specify their positive and negative input and output nodes.

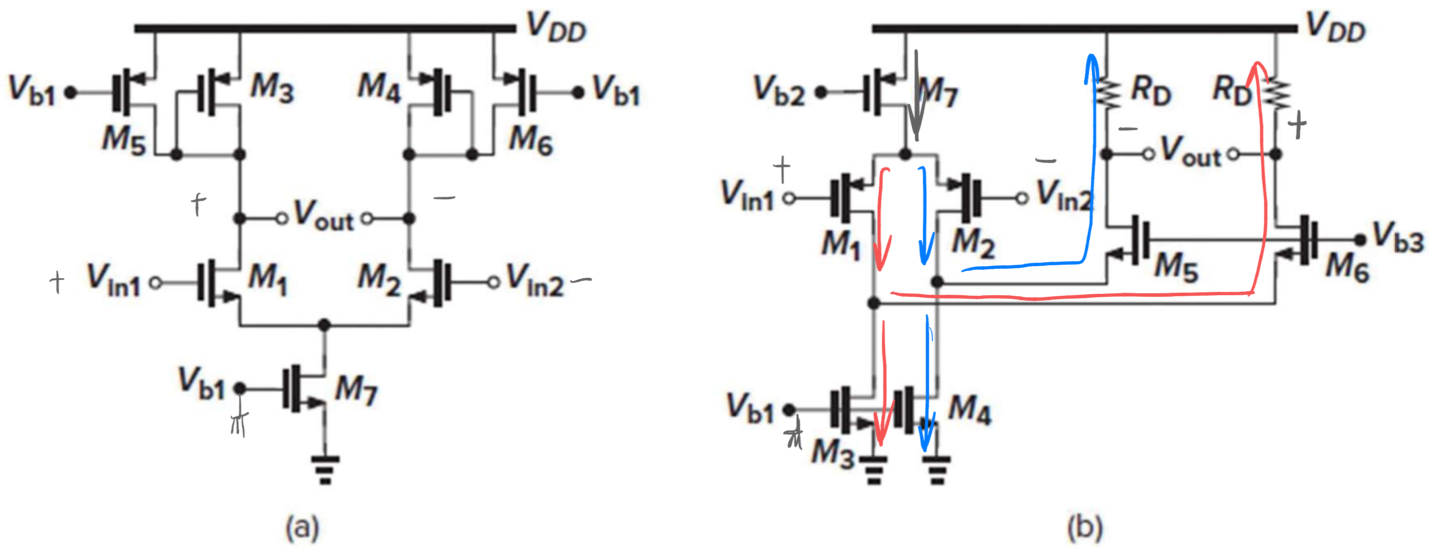
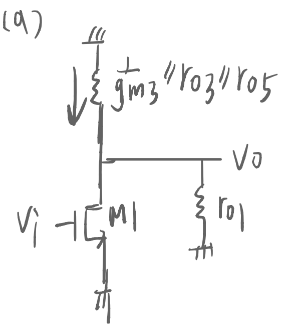


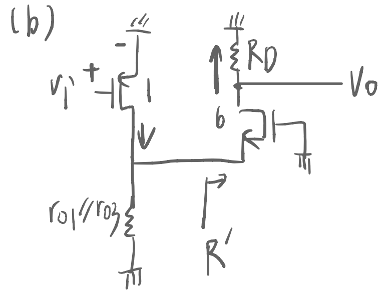
Fig. 3.2

(a)



$$A_V = -g_{m1}\cdot\left(r_{o1} // \frac{1}{g_{m3}} // r_{o3} // r_{o5}\right)$$ ※

(b)



$$R' = \frac{r_{o6} + R_D}{1 + g_{m6} r_{o6}}$$

$$V_o = -V_i \cdot g_{m1} \cdot \frac{r_{o1}//r_{o3}}{r_{o1}//r_{o3} + R'} \cdot R_D$$

$$A_V = \frac{-g_{m1} r_{o3} R_D}{r_{o3} + R'} \quad , \quad R' = \frac{r_{o6} + R_D}{1 + g_{m6} r_{o6}}$$ ※

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Homework 3 (Due date: 10/19)

HW3.3 (30 points)

Table 2.1 Level 1 SPICE models for NMOS and PMOS devices.

body effect $2\phi_f$ channel length

NMOS Model			
LEVEL = 1	VTO = 0.7	GAMMA = 0.45	PHI = 0.9
NSUB = 9e+14	LD = 0.08e−6	UO = 350	LAMBDA = 0.1
TOX = 9e−9	PB = 0.9	CJ = 0.56e−3	CJSW = 0.35e−11
MJ = 0.45	MJSW = 0.2	CGDO = 0.4e−9	JS = 1.0e−8
PMOS Model			
LEVEL = 1	VTO = −0.8	GAMMA = 0.4	PHI = 0.8
NSUB = 5e+14	LD = 0.09e−6	UO = 100	LAMBDA = 0.2
TOX = 9e−9	PB = 0.9	CJ = 0.94e−3	CJSW = 0.32e−11
MJ = 0.5	MJSW = 0.3	CGDO = 0.3e−9	JS = 0.5e−8

$\varepsilon_{ox} = \varepsilon_{SiO2} \cdot \varepsilon_0$, $\varepsilon_{SiO2} = 3.9$, $\varepsilon_0 = 8.85 * 10^{-14}$ F/cm

VDD=3.3V; VSS=0V

Suppose the differential pair of Fig. 3.3 is designed with $(W/L)_{1,2}=50/0.5$, $(W/L)_{3,4}=10/0.5$, $R_1=R_2=1\text{M}\Omega$, and $I_{SS} = 0.5$ mA. Also, I_{SS} is implemented with an NMOS device having $(W/L)_{SS}=50/0.5$.

(a) What are the maximum and minimum allowable input common-mode levels if the differential swing at the input and output are small? (10 points)

(b) For $V_{in,CM} = 1.2$V, calculate the small-signal differential voltage gain. (10 points)

(c) Suppose M_1 and M_2 have a threshold voltage mismatch of 1mV. What is the CMRR? (10 points)

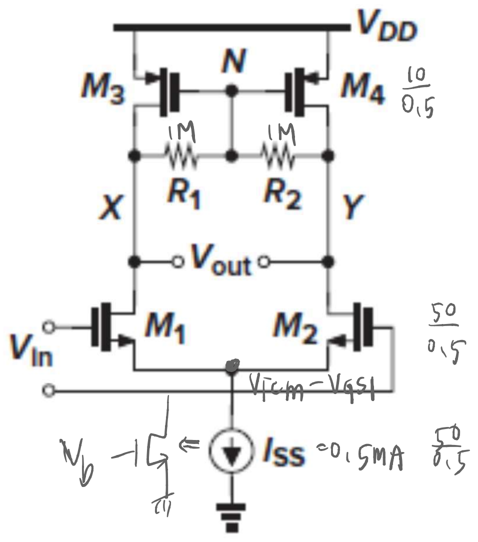


Fig. 3.3

(a) ① $\underline{V_b-(V_{icm}-V_{gs1})\le V_{THN}}$ $\Rightarrow$ $V_{icm}\ge V_b+V_{gs1}-V_{thN}$

$I_{SS}=\frac{1}{2}\mu_nC_{ox}\frac{W}{L}\cdot(V_b-V_{thN})^2$

$\begin{cases} t_{ox}=9\times10^{-9}\,m=9\times10^{-7}\,cm \\ \varepsilon_{ox}=3.9\times8.85\times10^{-14}\ F/cm \end{cases}$

$\cancel{0.5}m=\frac{1}{\cancel{4}}\times350\cdot\frac{3.9\times8.85\times10^{-14}}{9\times10^{-7}}\cdot\frac{50}{0.5}\cdot(V_b-0.7)^2$

$10^{-3}=1.34\times10^{-2}\times(V_b-0.7)^2$

$(V_b-0.7)^2=\frac{10^{-3}}{1.34\times10^{-2}}$ $\Rightarrow V_b=0.97$

• M1, M2 有 body effect：

$V_t=V_{t0}+\gamma\left(\sqrt{2\phi_f+V_{SB}}-\sqrt{2\phi_f}\right)$, $V_{SB}=V_{OV,ISS}=V_b-V_{th}=0.97-0.7=0.27$

$=0.7+0.45\left(\sqrt{0.9+0.27}-\sqrt{0.9}\right)=0.76$

$0.25m=\frac{1}{2}\times350\cdot\frac{3.9\times8.85\times10^{-14}}{9\times10^{-7}}\times\frac{50}{0.5}\times(V_{gs1}-0.76)^2$ $\Rightarrow V_{gs1}=0.953$

$\rightarrow V_{icm}\ge0.97+0.953-0.7=1.22$

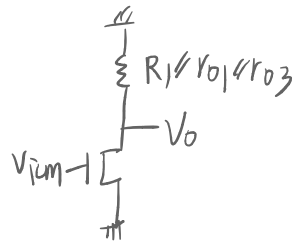
② $\underline{V_{icm}-(V_{DD}-V_{SD3})\le V_{th1}}$ $\Rightarrow$ $V_{icm}\le V_{th1}+V_{DD}-V_{SD3}$ $\rightarrow V_{SG3}$

$0.25m=\frac{1}{2}\times100\times\frac{3.9\times8.85\times10^{-14}}{9\times10^{-7}}\times\frac{10}{0.5}\times(V_{SG3}-0.8)^2$ $\Rightarrow V_{SG3}=1.6$

$\rightarrow V_{icm}\le0.76+3.3-1.6=2.46$

$\Rightarrow$ $1.22\le V_{icm}\le2.46$ #

(b) $V_o=V_{icm}\cdot g_{m1}\cdot(R_1/\!/r_{o1}/\!/r_{o3})\cdot(-1)$



考慮 channel length modulation：

$r_{o1}=\frac{1}{\lambda_n I_D}=\frac{1}{0.1\times0.25m}=40K$, $r_{o2}=\frac{1}{\lambda_p I_D}=\frac{1}{0.2\times0.25m}=20K$

$g_{m1}=\sqrt{2\mu_nC_{ox}\frac{W}{L}\cdot I_{D1}}$

$=\sqrt{2\times350\times\frac{3.9\times8.85\times10^{-14}}{9\times10^{-7}}\times\frac{50}{0.5}\times0.25m}=2.59m$

$A_V=-2.59m\cdot(1M/\!/20k/\!/40k)$

$=-2.59m\times13.1k$

$=34$ #

$1M/\!/13.3K$

$1000K/\!/13.3k$

(c) $CMRR = \dfrac{A_{DM}}{A_{CM-DM}}$, $V_{th1} + 1m = V_{th2}$, $g_m = \mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{th})$, $g_{m1} \neq g_{m2}$

① A_{CM-DM}

$$(g_{m1} + g_{m2}) \cdot (V_{icm} - V_P) R_{SS} = V_P$$

$$V_P = \frac{(g_{m1} + g_{m2}) R_{SS}}{1 + (g_{m1} + g_{m2}) R_{SS}} \cdot V_{icm}$$

$$V_X = -g_{m1} \cdot (V_{icm} - V_P) R_D = -g_{m1} \cdot R_D \cdot \frac{1}{1 + (g_{m1} + g_{m2}) R_{SS}} V_{icm}$$

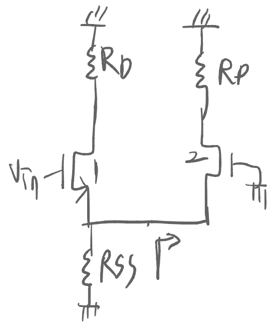
$$V_Y = -g_{m2} \cdot (V_{icm} - V_P) \cdot R_D = -g_{m2} \cdot R_D \cdot \frac{1}{1 + (g_{m1} + g_{m2}) R_{SS}} \cdot V_{icm}$$

$$\Rightarrow V_X - V_Y = -(g_{m1} - g_{m2}) \cdot R_D \cdot \frac{1}{1 + (g_{m1} + g_{m2}) R_{SS}} \cdot V_{icm}$$

$$\Rightarrow A_{CM-DM} = \frac{-\Delta g_m R_D}{1 + (g_{m1} + g_{m2}) R_{SS}}$$

② A_{DM}

Fig. 4.17



$$A_{DM} = \frac{R_D}{2} \cdot \frac{g_{m1} + g_{m2} + 4 g_{m1} g_{m2} R_{SS}}{1 + (g_{m1} + g_{m2}) R_{SS}}$$

③ $CMRR = \dfrac{A_{DM}}{A_{CM-DM}}$

$$= \frac{g_{m1} + g_{m2} + 4 g_{m1} g_{m2} R_{SS}}{2 \Delta g_m} \approx \frac{g_m}{\Delta g_m}(1 + 2 g_m R_{SS})$$

$$g_m = \frac{g_{m1} + g_{m2}}{2} = \frac{\mu_n C_{ox} \frac{W}{L}\left[V_{GS} - V_{th1} + V_{GS} - V_{th2}\right]}{2} = \frac{\mu_n C_{ox} \frac{W}{L}\left[2V_{GS} - (V_{th1} + V_{th2})\right]}{2}$$

由(a) $\begin{cases} V_{GS} = 0.953 \\ V_{th} = 0.76 \end{cases}$

$$\Rightarrow g_m = 350 \times \frac{3.9 \times 8.85 \times 10^{-14}}{9 \times 10^{-7}} \times \frac{50}{0.5} \times \left[2 \times 0.953 - (2 \times 0.76 + 10^{-3})\right] \times \frac{1}{2}$$

$$= 5.16m \times \frac{1}{2} = 2.58m$$

$$\Delta g_m = g_{m1} - g_{m2} = \mu_n C_{ox} \frac{W}{L}\left[V_{GS} - V_{th1} - V_{GS} + V_{th2}\right] = \mu_n C_{ox} \frac{W}{L}(\underbrace{V_{th2} - V_{th1}}_{\Delta V_{th}})$$

$$= 350 \times \frac{3.9 \times 8.85 \times 10^{-14}}{9 \times 10^{-7}} \times \frac{50}{0.5} \times 10^{-3} = 13.4\mu$$

$$R_{SS} = \frac{1}{\lambda I_{SS}} = \frac{1}{0.1 \times 0.5m} = 20k$$

$$\Rightarrow CMRR = \frac{2.58 \times 10^{-3}}{13.4 \times 10^{-6}} \times (1 + 2 \times 2.58m \times 20k) = 20062$$

$$\Rightarrow 20\log 20062 = 86\ dB$$

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Homework 3 (Due date: 10/19)

HW3.4: (20 points)

Consider the circuit shown in Fig. 3.4. VDD=3.3V and VSS=0.

(a) Sketch V_{out} as V_{in1} and V_{in2} vary differentially from zero to VDD.

(b) If $\lambda = 0$, obtain an expression for the voltage gain. What is the voltage gain if $W_{3,4} = 0.8W_{5,6}$?

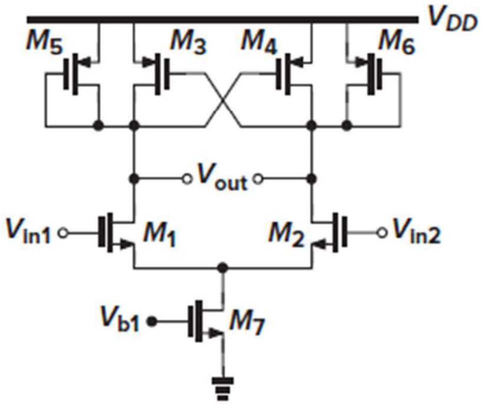
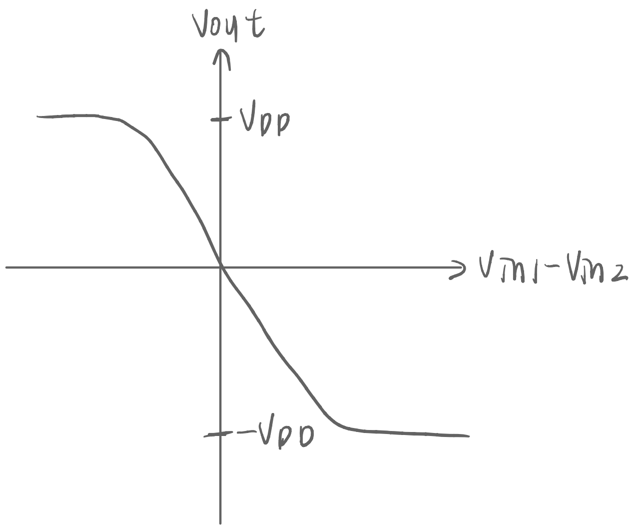


Fig. 3.4

(a)



① $V_{in1} = V_{DD}$, $V_{in2} = 0$

$\rightarrow V_{out1} = 0$, $V_{out2} = V_{DD}$

② $V_{in1} = 0$, $V_{in2} = V_{DD}$

$\rightarrow V_{out1} = V_{DD}$, $V_{out2} = 0$

③ $V_{in1} = V_{in2}$

$\rightarrow V_{out} = 0$

(b) $I_X = -V_3 \cdot g_{m3} \cdot (-1) = -V_4 g_{m4}$

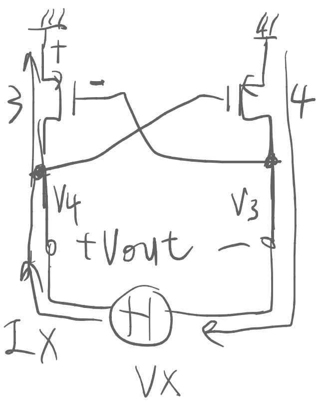
$I_X = V_3 g_{m3} = -V_4 g_{m4}$

$V_3 = -V_4$

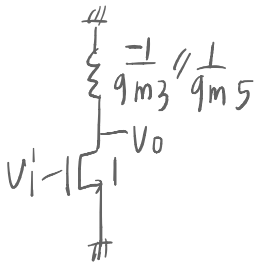
又 $V_3 + V_X = V_4$

$V_X = V_4 - V_3 = 2V_4$

$\frac{V_X}{I_X} = \frac{2V_4}{-V_4 g_{m4}} = \frac{-2}{g_{m4}}$



$W_{3,4} = 0.8\,W_{5,6}$, $g_m = \mu_n C_{ox} \frac{W}{L}(V_{GS}-V_{th}) \Rightarrow g_{m3} = 0.8\,g_{m5}$



$$A_v = -g_{m1}\left(\frac{-1}{g_{m3}} // \frac{1}{g_{m5}}\right)$$

$$= -g_{m1}\left(\frac{-1}{0.8\,g_{m5}} // \frac{1}{g_{m5}}\right)$$

$$= \frac{-g_{m1}}{0.2\,g_{m5}} \ \#$$

$$\frac{\frac{-1}{0.8\,g_{m5}^2}}{\frac{-1}{0.8\,g_{m5}} + \frac{0.8}{0.8\,g_{m5}}}$$

$$\frac{\frac{+1}{g_{m5}}}{+0.2} = \frac{1}{0.2\,g_{m5}}$$