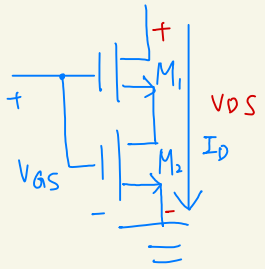
HW1.4



$V_{DS} = V_{DS1} + V_{DS2}$

$V_{GS} = V_{GS2} = V_{DS2} + V_{GS1}$

假設 M_1 導通，$V_{GS1} > V_{th}$

$\underbrace{V_{GS} - V_{DS2}}_{V_{OV}} > V_{th} \Rightarrow V_{DS2} < V_{OV}$ (M_2 必在 triode region)

$I_D = I_{D1} = I_{D2}$：討論兩種情況

① M_1 Sat, M_2 triode

② M_1, M_2 皆 triode

① $I_D = \frac{1}{2} W/L \cdot \underbrace{\mu_n C_{ox}}_{K} \cdot \underbrace{(V_{GS1} - V_{th})^2}_{V_{OV1}} = \frac{1}{2} \cdot W/L \cdot K \cdot [V_{DS2}(2V_{OV2} - V_{DS})]$

$\Rightarrow (\underbrace{V_{GS} - V_{DS2}}_{V_{GS1}} - V_{th})^2 = V_{DS2} \times [2(V_{GS} - V_{th}) - V_{DS2}]$

設 $V_{GS} - V_{th}$ 為 V_{OV} 整理後：$V_{OV}^2 - 2V_{OV} \cdot V_{DS2} + V_{DS}^2 = 2V_{DS2} \cdot V_{OV} - V_{DS2}^2$

$V_{OV}^2 - 4 \cdot V_{OV} \cdot V_{DS2} + 2V_{DS2}^2 = 0 \Rightarrow V_{DS2}^2 = \frac{V_{OV}^2 - 4V_{OV} \cdot V_{DS2}}{2}$

代回 I_{D2}：$\frac{1}{2} \cdot W/L \cdot K \left[2V_{OV} V_{DS2} - \underbrace{V_{DS2}^2}_{②} \right] = \frac{1}{2} W/L \cdot K \left(\frac{V_{OV}^2}{②} \right)$

$= \frac{1}{2} \frac{W}{(2L)} \underbrace{(V_{GS} - V_{th})^2}_{V_{OV}}$

HW1.4 ② M_1, M_2 皆在 triode 假設 $V_{th1}=V_{th2}=V_{th}$

$$\frac{K}{2}\cdot\frac{W}{L}\cdot V_{DS1}\cdot(2V_{ov1}-V_{DS1})=\frac{K}{2}\cdot\frac{W}{L}\cdot V_{DS2}(2V_{ov2}-V_{DS2})$$

$$V_{DS}=V_{DS1}+V_{DS2}\quad V_{ov2}=V_{GS}-V_{th}=V_{OV}$$

$$V_{ov1}=V_{GS}-V_{th}-V_{DS2}=V_{OV}-V_{DS2}$$

$$I_D=I_{D1}=I_{D2}\Rightarrow\frac{I_{D1}+I_{D2}}{2}=I_D$$

$$I_D=\frac{1}{2}\times\frac{K}{2}\times\frac{W}{L}\left[V_{DS1}\cdot(2V_{ov1}-V_{DS1})+V_{DS2}\cdot(2V_{ov2}-V_{DS2})\right]$$

$$=\frac{K}{2}\times\frac{W}{2L}\times\left[V_{DS1}\cdot(2V_{OV}-2V_{DS2}-V_{DS1})+2V_{OV}\cdot V_{DS2}-V_{DS2}^2\right]$$

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$$=\frac{K}{2}\times\frac{W}{2L}\times\left[(V_{DS}-V_{DS2})(2V_{OV}-V_{DS2}-V_{DS})+2V_{OV}\cdot V_{DS2}-V_{DS2}^2\right]$$

$$=\frac{K}{2}\times\frac{W}{2L}\times\left(2V_{OV}V_{DS}-V_{DS}V_{DS2}-V_{DS}^2-2V_{OV}V_{DS2}+V_{DS2}^2+V_{DS}V_{DS2}+2V_{OV}V_{DS2}-V_{DS2}^2\right)$$

$$=\frac{K}{2}\times\frac{W}{2L}\times\left(2V_{OV}V_{DS}-V_{DS}^2\right)$$

由①和②可得出 Fig 1.3 等效為 $\frac{W}{2L}$