# Electromagnetics midterm exam

2023-04 ET3103301、ET3103302

1. (10%) Express the base vector $\hat{a}_R$ , $\hat{a}_\Theta$ and $\hat{a}_\emptyset$ of a spherical coordinate system in terms of the cylindrical base vector $\hat{a}_r$ , $\hat{a}_\emptyset$ , $\hat{a}_z$ and coordinate r , Ø and z .

2. (10%) In Fig.1 , verify the divergence theorem by the vector field $\overrightarrow{F} = \hat{a}_R cos^2\emptyset/R^3$ existing in the region between two spherical shells defined by R=2 and R=3.

3. (15%) Given a vector function E=$a_x y$+$a_y x$ , evaluate the scalar line integral $\int E \cdot d\ell$ from $P_1$(2,1,-1) to $P_2$(8,2,-1)
   a) alone the parabola x=2$y^2$,
   b) alone the straight line joining the two points.
   c) Evaluate $\int E \cdot d\ell$ from $P_3$(3,4,-1) to $P_4$(4,3,-1) by converting both E and the positions of $P_3$ and $P_4$ into cylindrical coordinates.

4. (16%) Given three vectors A , B and C as follows,
   A=$a_x$+$a_y 2$-$a_z 3$ , B=$-a_y 4$+$a_z$ , C=$a_x 5$-$a_z 2$
   Find
   a) $a_A$
   b) |A-B|
   c) A · B
   d) $\Theta_{AB}$
   e) the component of A in the direction of C
   f) A×C
   g) A · (B×C) and (A×B) ·C
   h) (A×B)×C and A× (B×C)

5. (10%) In Fig.3 , calculate the electric field *E* at the center of an equilateral triangle .

6. (15%) In Fig.4 , verify Stokes's Theorem with vector function $\overrightarrow{F}$ =$\hat{a}_\emptyset 3\sin(\frac{\emptyset}{2})$ for a hemispherical and the boundary of hemispherical with radius r=4 .

hint: $\nabla \times \overrightarrow{F} = \hat{a}_R \frac{3cos\Theta sin\frac{\emptyset}{2}}{Rsin\Theta}$ - $\hat{a}_\Theta \frac{3sin\frac{\emptyset}{2}}{R}$

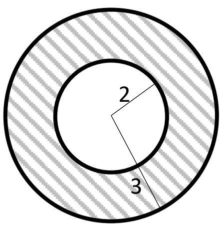

Fig.1

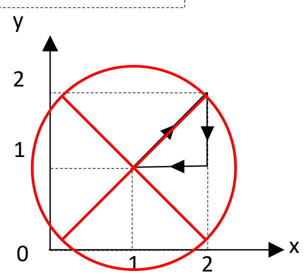

Fig.2

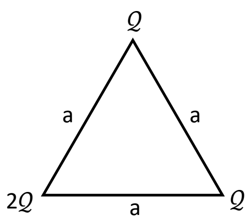

Fig.3

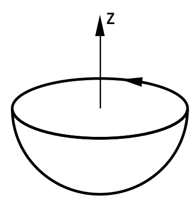

Fig.4

7. (12%) An uniform electron cloud (Fig.5) with density $\rho_{(r)}$= $\rho_0(1-\frac{r^2}{a^2})$ , find the electric field *E* at :
   a) r<a
   b) r>a
   c) Write the integral expression of charge $Q$.

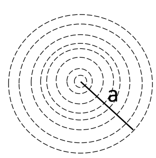

Fig.5

8. (12%) Proof :
   a) $\nabla \cdot (\nabla \times \overrightarrow{A})$ = 0
   b) $\nabla \times (\nabla V)$ = 0