112-2 Calculus Midterm (I)
Chapter: 7-1~7-3 & 7-5~7-8
Date: 2024/04/10 17:30-18:20 Total: 50 pts

1. $f(x) = \frac{e^{2x}-2}{e^{2x}+2}$ and $e^{2x} \neq 2$, find $f^{-1}(x) = ?$ (10 pts)

2. $g(x) = (\sqrt{x+12})^{\sqrt{4x}}$, find $g'(4) = ?$ (10 pts)

3. Find the following limits. (10 pts)

   $a.\ \lim_{x\to 0^+}(1 + sin4x)^{\cot x}$ (5 pts) $b.\ \lim_{x\to 0}\frac{x-\tan^{-1}x}{\sin^{-1}x-x}$ (5 pts)

4. Verify the integration formulas: $\int \frac{\tan^{-1}x}{x^2}dx = \ln x - \frac{1}{2}\ln(1+x^2) - \frac{\tan^{-1}x}{x} + C$

   (10 pts)

5. Evaluate the integral $\int \frac{e^{sinx}\cos(x)}{\sqrt{e^{2\sin(x)}-1}}dx$. (10 pts)

Formula Table

$$1.\ \int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C \qquad \text{(Valid for } u^2 < a^2\text{)}$$

$$2.\ \int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C \qquad \text{(Valid for all } u\text{)}$$

$$3.\ \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C \qquad \text{(Valid for } |u| > a > 0\text{)}$$

$$1.\ \frac{d(\arcsin u)}{dx} = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \quad |u| < 1$$

$$2.\ \frac{d(\arccos u)}{dx} = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \quad |u| < 1$$

$$3.\ \frac{d(\arctan u)}{dx} = \frac{1}{1+u^2}\frac{du}{dx}$$

$$4.\ \frac{d(\operatorname{arccot} u)}{dx} = -\frac{1}{1+u^2}\frac{du}{dx}$$

$$5.\ \frac{d(\operatorname{arcsec} u)}{dx} = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}, \quad |u| > 1$$

$$6.\ \frac{d(\operatorname{arccsc} u)}{dx} = -\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}, \quad |u| > 1$$

$$\cosh^2 x - \sinh^2 x = 1$$
$$\sinh 2x = 2\sinh x \cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$
$$\cosh^2 x = \frac{\cosh 2x + 1}{2}$$
$$\sinh^2 x = \frac{\cosh 2x - 1}{2}$$
$$\tanh^2 x = 1 - \operatorname{sech}^2 x$$
$$\coth^2 x = 1 + \operatorname{csch}^2 x$$

$$\int \sinh u\, du = \cosh u + C$$
$$\int \cosh u\, du = \sinh u + C$$
$$\int \operatorname{sech}^2 u\, du = \tanh u + C$$
$$\int \operatorname{csch}^2 u\, du = -\coth u + C$$
$$\int \operatorname{sech} u \tanh u\, du = -\operatorname{sech} u + C$$
$$\int \operatorname{csch} u \coth u\, du = -\operatorname{csch} u + C$$

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right), \quad -\infty < x < \infty$$
$$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right), \quad x \geq 1$$
$$\tanh^{-1} x = \frac{1}{2}\ln\frac{1+x}{1-x}, \quad |x| < 1$$
$$\operatorname{sech}^{-1} x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right), \quad 0 < x \leq 1$$
$$\operatorname{csch}^{-1} x = \ln\left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|}\right), \quad x \neq 0$$
$$\coth^{-1} x = \frac{1}{2}\ln\frac{x+1}{x-1}, \quad |x| > 1$$

$$\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C, \quad a > 0$$
$$\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C, \quad u > a > 0$$
$$\int \frac{du}{a^2-u^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{u}{a}\right) + C, & u^2 < a^2 \\ \frac{1}{a}\coth^{-1}\left(\frac{u}{a}\right) + C, & u^2 > a^2 \end{cases}$$
$$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{u}{a}\right) + C, \quad 0 < u < a$$
$$\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\operatorname{csch}^{-1}\left|\frac{u}{a}\right| + C, \quad u \neq 0 \text{ and } a > 0$$