ENGINEERING MATHEMATICS (II): LINEAR ALGEBRA MIDTERM

Winter 2022

Note: Provide clear derivations or brief explanations of your answers. You will not receive the credits if only the final results are given.

PROBLEM 1 (35 pts, each question worths 5 pts)

(a) If you type in the following MATLAB commands

A=[1 2 3 5; ones(1,4); 9 -3 2 6; 1 3 8 5]; $D = A([1,3],[2,4])$

then what does that show on your screen?

(b) Write down the MATLAB code to generate a matrix $\mathbf{C}$ which has 2 rows. The first row of $\mathbf{C}$ is equal to the summation of the first row and the second row of $\mathbf{A}$, and the second row of $\mathbf{C}$ is equal to 2 times the third row of $\mathbf{A}$.

(c) Suppose $\mathbf{A}$ is an $m \times n$ matrix and $\mathbf{Ax} = \mathbf{0}$ for ALL $n \times 1$ column vector $\mathbf{x}$. Is it true that $\mathbf{A} = \mathbf{0}$?

(d) Suppose

$$\mathbf{C} = \begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \\ 9 \end{bmatrix} [2, 4, 6, 8, 10]$$

Then is $\mathbf{C}$ invertible?

(e) Let S be the set of ordered pair of real number with addition and scalar multiplication defined, respectively, as

$$\begin{aligned} (x_1, y_1) + (x_2, y_2) &= (x_1 + y_2, y_1 + x_2) \\ c(x, y) &= (cx, cy) \end{aligned}$$

Is S a vector space with these two operations?

(f) Suppose that $\mathbf{G}$ is a 5×4 matrix, where $\mathbf{g}_1 - 2\mathbf{g}_2 + \mathbf{g}_4 = \mathbf{0}$, in which $\mathbf{g}_i$ denotes the i^{th} column of $\mathbf{G}$. Then how many solution(s) does the linear system $\mathbf{Gx} = \mathbf{0}$ have?

(g) Let $\mathbf{A}$ be a 4×4 matrix with reduced row echelon form given by

$$\mathbf{U} = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

If $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ -2 \end{bmatrix}$ and $\mathbf{a}_2 = \begin{bmatrix} 2 \\ -1 \\ 3 \\ 1 \end{bmatrix}$, then determine $\mathbf{a}_3$, where $\mathbf{a}_i$ denotes the i^{th} column of $\mathbf{A}$.

PROBLEM 2 (25 pts)

Consider the following system of linear equations

$$\begin{aligned} x + 4y - 2z &= 1 \\ x + 7y - 5z &= 5 \\ 2x + 5y + \lambda z &= \gamma \end{aligned}$$

Determine the values of λ and γ such that the above system of linear equations has no solution, one solution, and infinitely many solutions. Also **determine the corresponding solution set** when this system of linear equations is consistent.

PROBLEM 3 (20 pts)

Consider two matrices

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -3 & 5 \\ 3 & -1 & 2 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(a) (5 pts) Determine the inverse of $\mathbf{B}$.

(b) (5 pts) Determine $det(2\mathbf{A}^2\mathbf{B}) + det(\mathbf{A}^{-1}\mathbf{B}^T)$.

(c) (5 pts) Determine the nullspace of $\mathbf{B}^3$.

(d) (5 pts) Determine $adj(\mathbf{A}^{-1})$.

PROBLEM 4 (20 pts)

Consider two subspaces V and W of $\mathbf{P}_5$, where $\mathbf{P}_5$ denotes the set of all polynomials of degrees less than 5. V and W are defined, respectively, as

$$\mathsf{V} = \{p(x) : p(x) = p(-x)\}$$

and

$$\mathsf{W} = \{q(x) : q(1) = 0\}$$

(a) (10 pts) Determine $\dim(\mathsf{V})$.

(b) (10 pts) Determine $\dim(\mathsf{V} \cap \mathsf{W})$.