Analog Integrated Circuit Design and Applications
Spring 2024

Nonlinearity and Mismatch

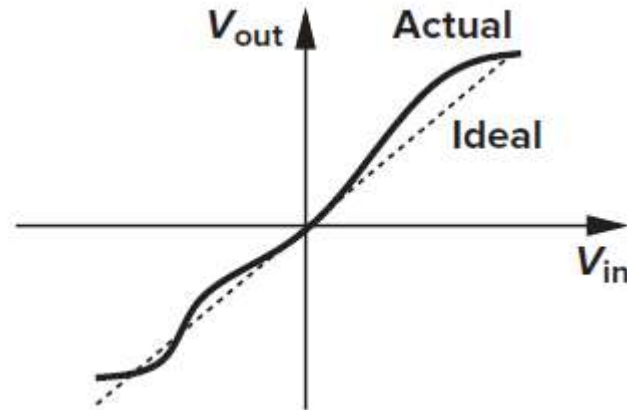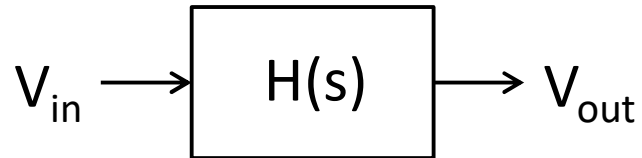Yung-Hui Chung

MSIC Lab

DECE, NTUST

Refer to ch. 14, Razavi's Design of Analog CMOS ICs, 2nd ed.

Mixed-Signal
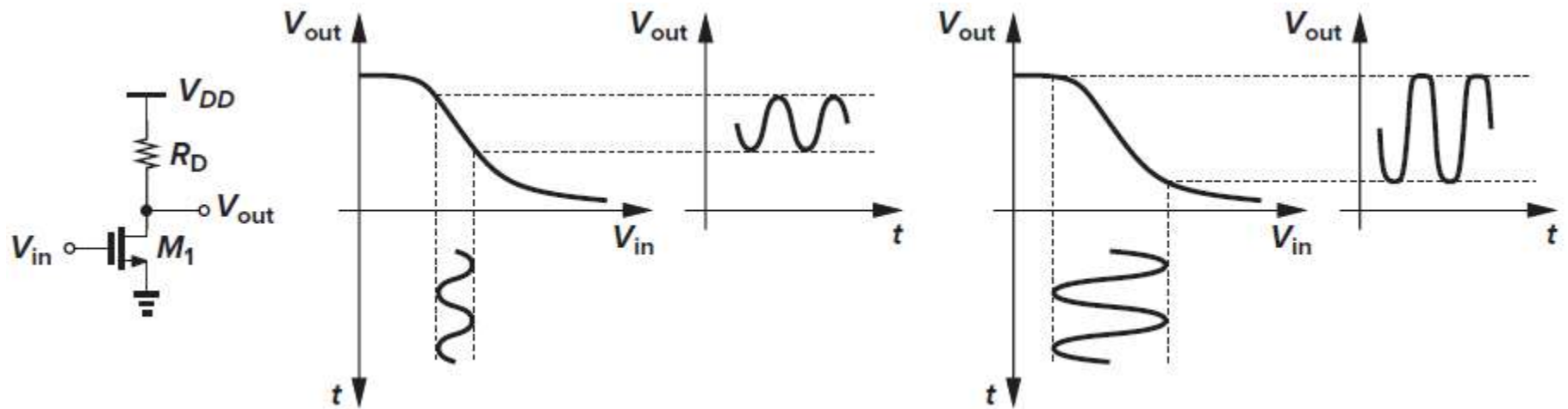IC Laboratory NTUST

Outline

- ## Nonlinearity
  - THD: Total Harmonic Distortion
  - Linearization Techniques

- ## Mismatch
  - Random Mismatch
  - Systematic Mismatch
  - Input-Referred Offset
  - Current Mirror Deviation
  - THD Degradation
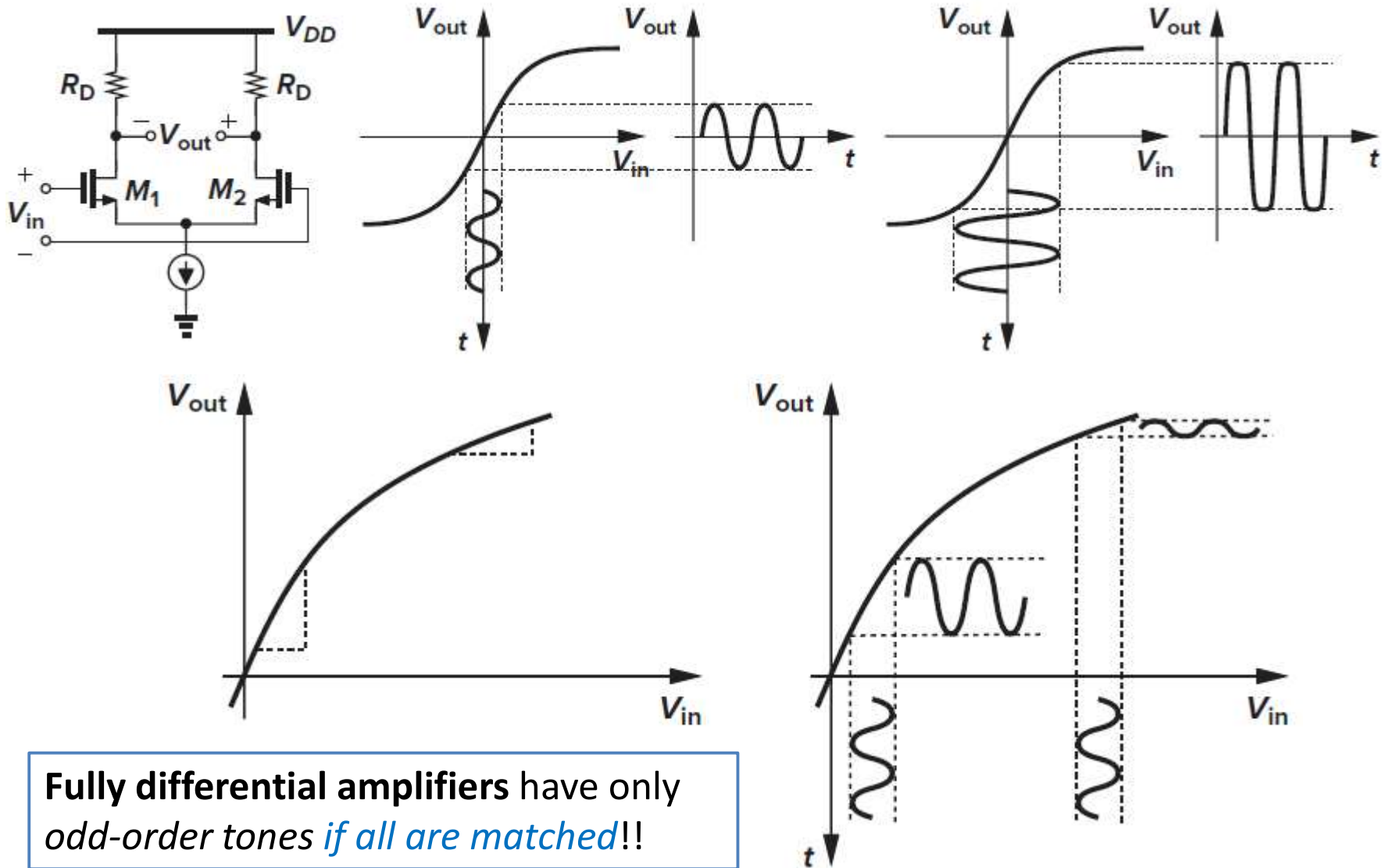
- ## Noise

Nonlinearity

Single-ended amplifier

- Nonlinearity: the amplifier gain is not constant but varies with the input amplitude

Nonlinearity

Fully differential amplifiers have only *odd-order tones* *if all are matched*!!

Total Harmonic Distortion (THD)

$$y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \cdots$$

Input: $x(t) = A\cos\omega t$

$$y(t) = \alpha_1 A\cos\omega t + \alpha_2 A^2\cos^2\omega t + \alpha_3\cos^3\omega t + \cdots$$

$$= \alpha_1 A\cos\omega t + \frac{\alpha_2 A^2}{2}[1+\cos(2\omega t)] + \frac{\alpha_3 A^3}{4}[3\cos\omega t + \cos(3\omega t)] + \cdots$$

$$= \frac{\alpha_2 A^2}{2} + \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right)\cos\omega t + \frac{\alpha_2 A^2}{2}\cos 2\omega t + \frac{\alpha_3 A^3}{4}\cos 3\omega t$$

If considering only HD2 and HD3, THD = HD2+HD3 is

Single-ended:
$$\text{THD} = \frac{(\alpha_2 A^2/2)^2 + (\alpha_3 A^3/4)^2}{(\alpha_1 A + 3\alpha_3 A^3/4)^2}$$

Harmonic distortion is undesirable in most signal processing applications, including audio and video systems. High-quality audio products such as compact disc (CD) players require a THD of about 0.01% (−80 dB), and video products, about 0.1% (−60 dB).

Distortion of Differential Amplifiers

$$\alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3$$

$x_1(t)$ $y_1(t)$

$x_2(t)$ $y_2(t)$
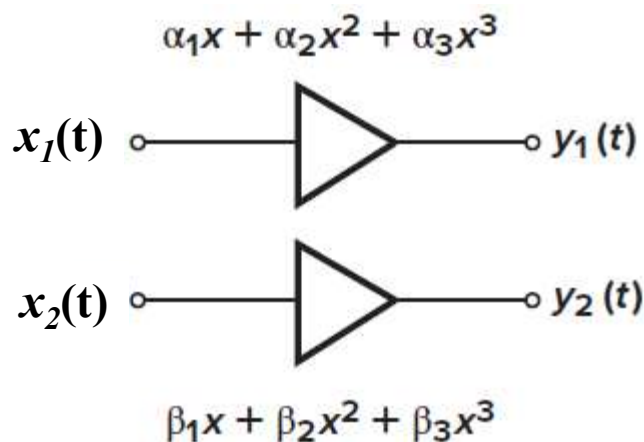
$$\beta_1 x + \beta_2 x^2 + \beta_3 x^3$$

$$y_1 - y_2 = \left(\alpha_1 x_1 - \beta_1 x_2 \right) + \left(\alpha_2 x_1^2 - \beta_2 x_2^2 \right) + \left(\alpha_3 x_1^3 - \beta_3 x_2^3 \right)$$

Assume $x_1 = x$ and $x_2 = -x$,

$$y_d = y_1 - y_2 = \left(\alpha_1 + \beta_1 \right) x + \left(\alpha_2 - \beta_2 \right) x^2 + \left(\alpha_3 + \beta_3 \right) x^3$$

if $\alpha_i = \beta_i, \ \forall i \ \Rightarrow \ \underline{y_d = 2\alpha_1 x + 2\alpha_3 x^3}$

- Single-ended signals have tones of 2nd, 3rd, 4th, …
- Differential signals only have odd-order tones, ideally
- However, the **mismatch** causes even-order tones in differential signals
- In addition, **thermal gradient** also contributes mismatch

Harmonic Distortion: Single-ended
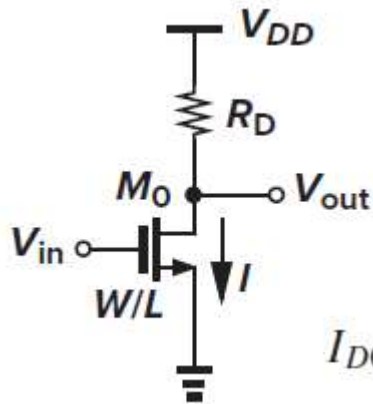
$$|A_v| \approx g_m R_D$$

DC operating point

$$= \mu_n C_{ox} \frac{W}{L} (\underline{V_{GS}} - V_{TH}) R_D$$

$$I_{D0} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH} + \underline{V_m \cos \omega t})^2$$
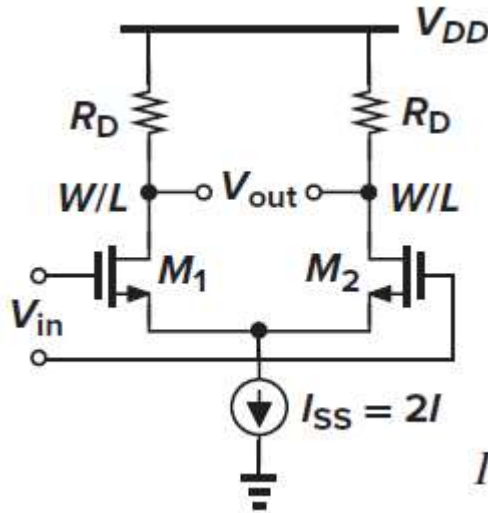
Input signal

$$= \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 + \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) V_m \cos \omega t$$

$$+ \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_m^2 \cos^2 \omega t$$

$$= I + \underbrace{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) V_m \cos \omega t}_{A_F} + \underbrace{\frac{1}{4} \mu_n C_{ox} \frac{W}{L} V_m^2 [1 + \cos(2\omega t)]}_{A_{HD2}}$$

$$\boxed{\frac{A_{HD2}}{A_F} = \frac{V_m}{4(V_{GS} - V_{TH})}}$$

(A very simple version)

Harmonic Distortion: Differential Pair

$$I_{D1} - I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{in}\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - V_{in}^2}$$

(推導見下頁)

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{in}\sqrt{4(V_{GS} - V_{TH})^2 - V_{in}^2}$$

If $|V_{in}| \ll V_{GS} - V_{TH}$, then

$$I_{D1} - I_{D2} = \mu_n C_{ox}\frac{W}{L}V_{in}(V_{GS} - V_{TH})\sqrt{1 - \frac{V_{in}^2}{4(V_{GS} - V_{TH})^2}}$$

$$\approx \mu_n C_{ox}\frac{W}{L}V_{in}(V_{GS} - V_{TH})\left[1 - \frac{V_{in}^2}{8(V_{GS} - V_{TH})^2}\right]$$

$$= \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})\left[V_m\cos\omega t - \frac{V_m^3\cos^3\omega t}{8(V_{GS} - V_{TH})^2}\right]$$
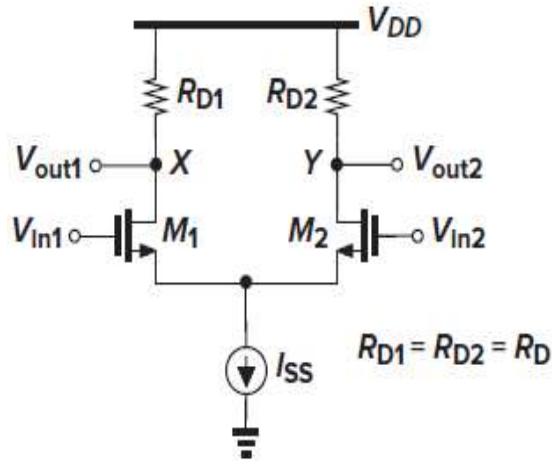
$\cos^3\omega t = [3\cos\omega t + \cos(3\omega t)]/4,$

$$I_{D1} - I_{D2} = g_m\left[V_m - \frac{3V_m^3}{32(V_{GS} - V_{TH})^2}\right]\cos\omega t - g_m\frac{V_m^3\cos(3\omega t)}{32(V_{GS} - V_{TH})^2}$$

If $V_m \gg 3V_m^3/[8(V_{GS} - V_{TH})^2]$, then

$$\boxed{\frac{A_{HD3}}{A_F} \approx \frac{V_m^2}{32(V_{GS} - V_{TH})^2}}$$

Harmonic Distortion: Differential Pair

g_m vs ΔV_{in} 推導之補充

$$\begin{cases} 4I_{D1}I_{D2} = I_{SS}{}^2 - 2I_{SS}K_n\Delta V_{in}{}^2 + K_n{}^2\Delta V_{in}{}^4, \ \Delta V_{in} = V_{in1} - V_{in2} \\ 4I_{D1}I_{D2} = (I_{D1}+I_{D2})^2 - (I_{D1}-I_{D2})^2 = I_{SS}^2 - (I_{D1}-I_{D2})^2 \end{cases}$$

$$\left(I_{D1} - I_{D2}\right)^2 = I_{SS}{}^2 - 4I_{D1}I_{D2}$$
$$= -K_n{}^2\Delta V_{in}{}^4 + 2I_{SS}K_n\Delta V_{in}{}^2$$

$$(I_{D1} - I_{D2})^2 = -\frac{1}{4}\left(\mu_n C_{ox}\frac{W}{L}\right)^2 (V_{in1}-V_{in2})^4 + I_{SS}\mu_n C_{ox}\frac{W}{L}(V_{in1}-V_{in2})^2$$

$$I_{D1} - I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1}-V_{in2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (V_{in1}-V_{in2})^2}$$

$$= \sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}}(V_{in1}-V_{in2})\sqrt{1 - \frac{\mu_n C_{ox}(W/L)}{4I_{SS}}(V_{in1}-V_{in2})^2}$$

$$(V_{GS} - V_{TH})^2 = \frac{I_D}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}$$

$$V_{GS} = \sqrt{\frac{2I_D}{\mu_n C_{ox}\frac{W}{L}}} + V_{TH}$$

$$V_{in1} - V_{in2} = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\frac{W}{L}}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}\frac{W}{L}}}$$

$$\underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}_{=K_n}(V_{in1} - V_{in2})^2 - I_{SS} = -2\sqrt{I_{D1}I_{D2}}$$

求 g_m

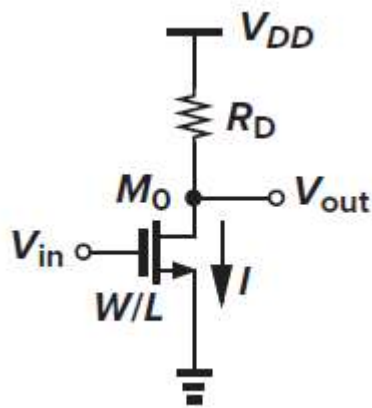$$\boxed{\frac{\partial \Delta I_D}{\partial \Delta V_{in}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\frac{\dfrac{4I_{SS}}{\mu_n C_{ox}W/L} - 2\Delta V_{in}^2}{\sqrt{\dfrac{4I_{SS}}{\mu_n C_{ox}W/L} - \Delta V_{in}^2}}}$$

Harmonic Distortion

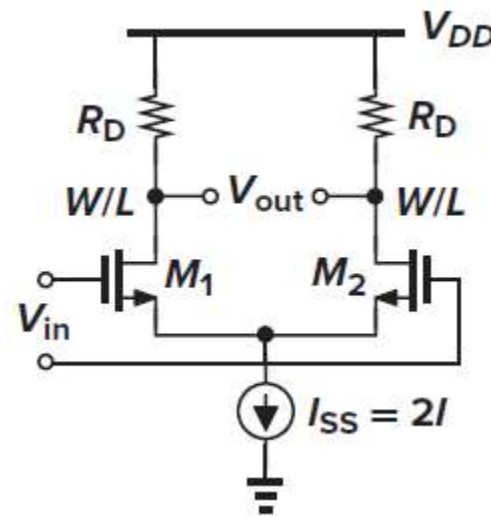if $V_m = 0.2(V_{GS} - V_{TH})$,

Single-ended

$$\frac{A_{HD2}}{A_F} = \frac{V_m}{4(V_{GS} - V_{TH})}$$
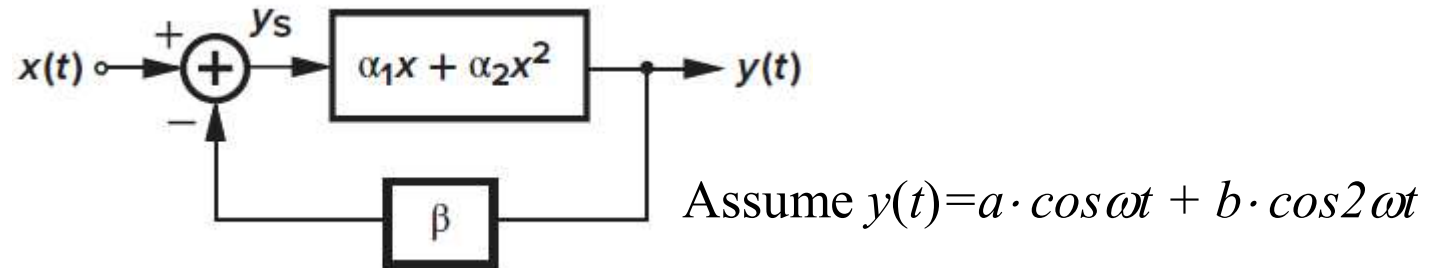
$$= 5\%$$

Fully differential

$$\frac{A_{HD3}}{A_F} \approx \frac{V_m^2}{32(V_{GS} - V_{TH})^2}$$

$$= 0.125\%$$

Keep in mind: How to maintain the advantages of fully differential amplifiers?
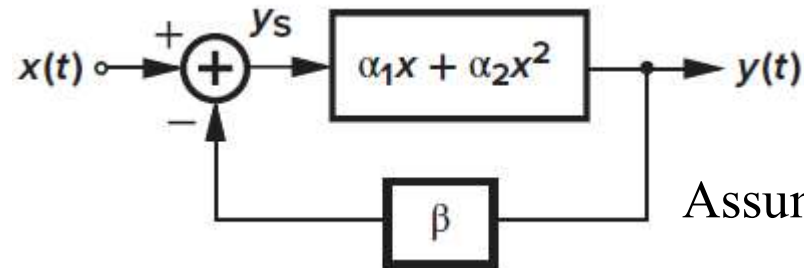
Feedback on THD

Assume $y(t) = a \cdot cos\,\omega t + b \cdot cos\,2\omega t$

$$y_S = x(t) - \beta y(t)$$

$$= V_m \cos \omega t - \beta(a \cos \omega t + b \cos 2\omega t)$$

$$= (V_m - \beta a) \cos \omega t - \beta b \cos 2\omega t$$

$$y(t) = \alpha_1 [(V_m - \beta a \cos \omega t - \beta b \cos 2\omega t]$$

$$+ \alpha_2 [(V_m - \beta a) \cos \omega t - \beta b \cos 2\omega t]^2$$

三角函數

$$= [\alpha_1 (V_m - \beta a) - \alpha_2 (V_m - \beta a)\beta b] \cos \omega t$$

$$+ \left[-\alpha_1 \beta b + \frac{\alpha_2 (V_m - \beta a)^2}{2} \right] \cos 2\omega t + \cdots$$

Feedback on THD

Assume $y(t) = a \cdot \cos \omega t + b \cdot \cos 2\omega t$

$$y(t) = a\cos(\omega t) + b\cos(2\omega t)$$

$$a = (\alpha_1 - \alpha_2 \beta b)(V_m - \beta a)$$

$$b = -\alpha_1 \beta b + \frac{\alpha_2 (V_m - \beta a)^2}{2}$$

$a \approx \alpha_1 (V_m - \beta a)$ If α_2 and b are small

$$a = \frac{\alpha_1}{1 + \beta \alpha_1} V_m$$

To calculate b, we write

$$V_m - \beta a \approx \frac{a}{\alpha_1}$$

$$b = -\alpha_1 \beta b + \frac{1}{2}\alpha_2 \left(\frac{a}{\alpha_1}\right)^2$$

$$b(1 + \alpha_1 \beta) = \frac{\alpha_2}{2}\left(\frac{a}{\alpha_1}\right)^2$$

$$= \frac{\alpha_2}{2\alpha_1^2}\frac{\alpha_1^2}{(1 + \beta \alpha_1)^2}V_m^2$$

$$b = \frac{\alpha_2 V_m^2}{2}\frac{1}{(1 + \beta \alpha_1)^3}$$

$$\frac{b}{a} = \frac{\alpha_2 V_m}{2}\frac{1}{\alpha_1}\frac{1}{(1 + \beta \alpha_1)^2}$$

Improved by feedback

Linearization Technique (1)

A common-source stage biased at a current I_1 experiences an input voltage swing that varies the drain current from $0.75I_1$ to $1.25I_1$. Calculate the variation of the small-signal voltage gain (a) with no degeneration and (b) with degeneration such that $g_m R_S = 2$, where g_m denotes the transconductance at $I_D = I_1$.

[Solution]

Assuming square-law behavior, we have $g_m \propto \sqrt{I_D}$. For the case of no degeneration,

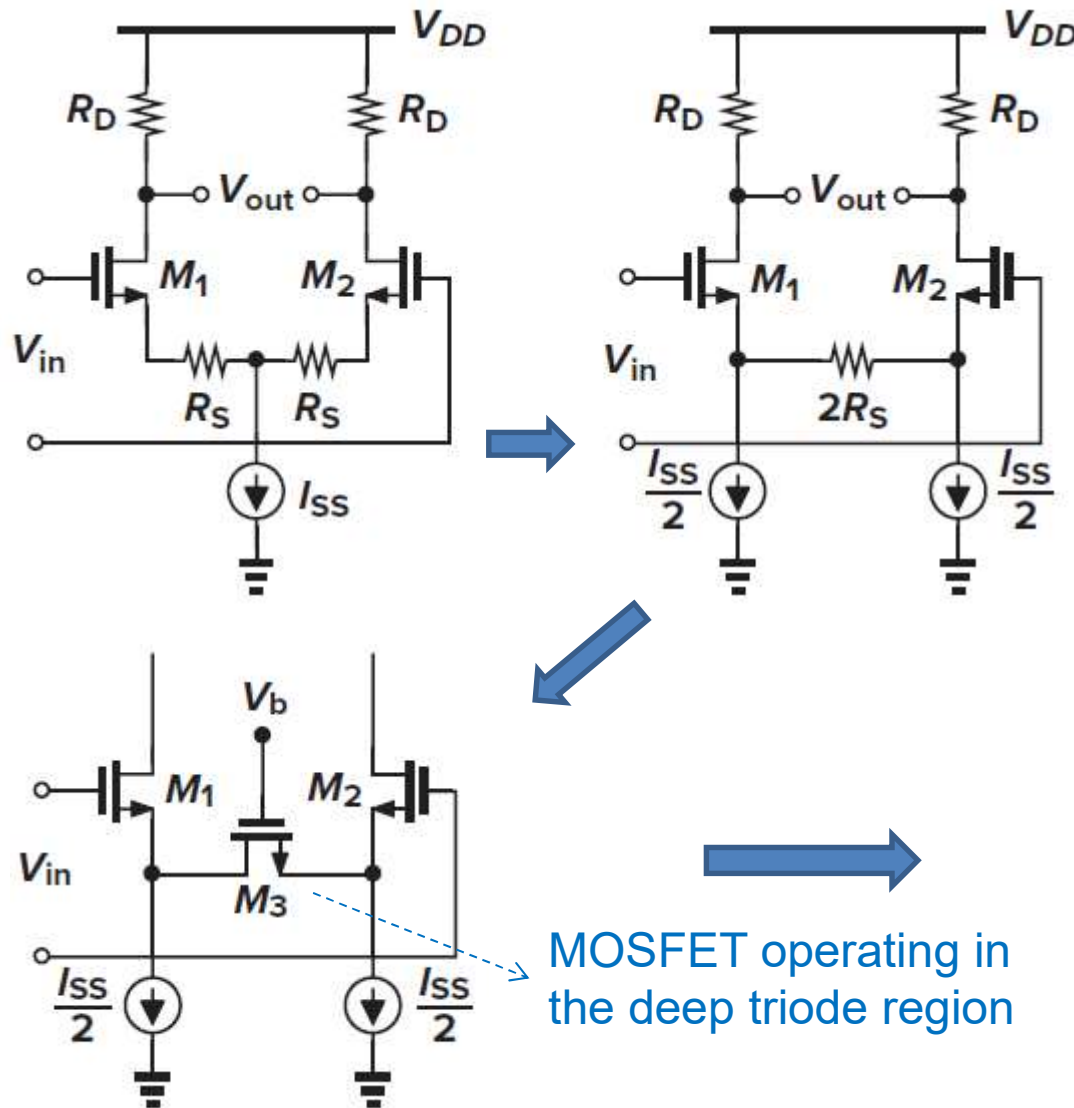$$\frac{g_{m,high}}{g_{m,low}} = \sqrt{\frac{1.25}{0.75}}$$

With $g_m R_S = 2$, (actually, too small)
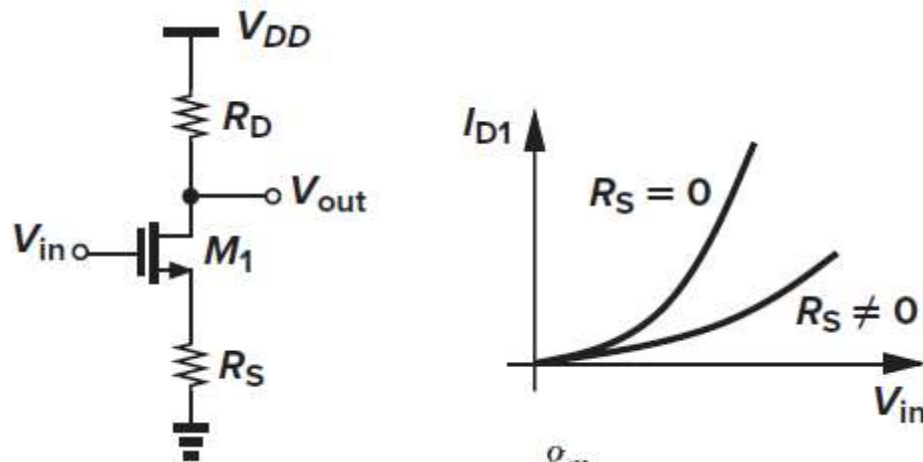
$$G_m = \frac{g_m}{1 + g_m R_S}$$

$$\frac{G_{m,high}}{G_{m,low}} = \frac{\dfrac{\sqrt{1.25}\,g_m}{1 + \sqrt{1.25}\,g_m R_S}}{\dfrac{\sqrt{0.75}\,g_m}{1 + \sqrt{0.75}\,g_m R_S}}$$

$$= \sqrt{\frac{1.25}{0.75}} \cdot \frac{1 + 2\sqrt{0.75}}{1 + 2\sqrt{1.25}}$$

$$= 0.84\sqrt{\frac{1.25}{0.75}}$$

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*Gain variation: $(G_{m,high}-G_m)/G_m$ & $(G_{m,low}-G_m)/G_m$
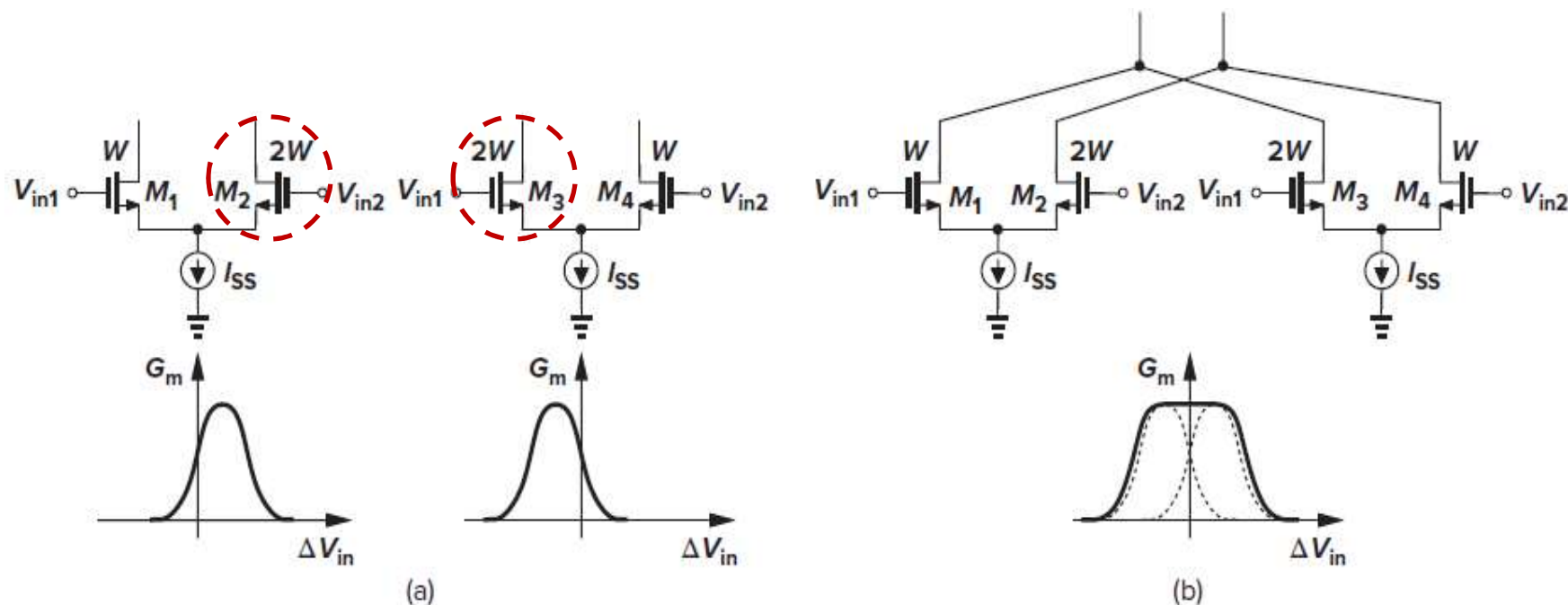
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SCP Degeneration

SCP: Source Couple Pair

$$G_m = \frac{g_m}{1 + g_m R_S}$$

Vb is applied as an input–tracking control

MOSFET operating in the deep triode region

Linearization Technique (2)

(a)

(b)

- Shift g_m by changing the input pair into unbalance conditions: left-shift and right-shift

- The composite final g_m has a flat shape if the optimal design is achieved

- Issue is the g_m flatness is PVT-sensitive

Linearization Technique (3)

Differential pair using input devices operating in the triode region
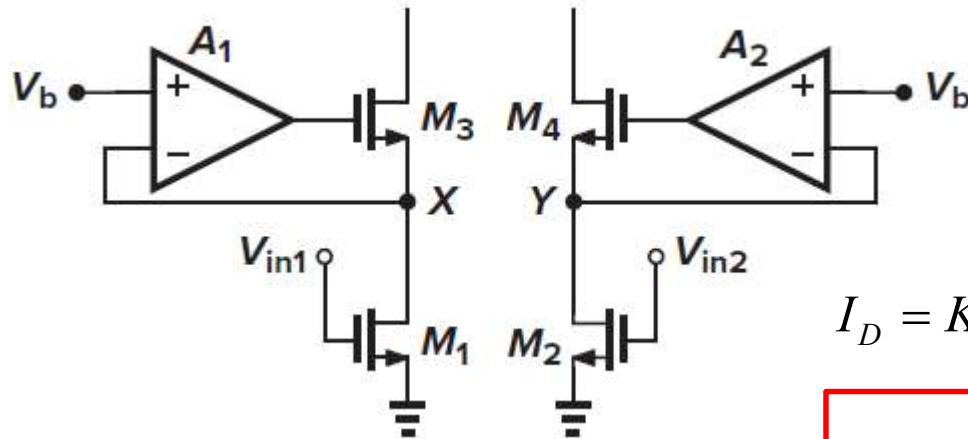
$$I_D = K_N' \left(\frac{W}{L} \right) \left[2\left(V_{GS} - V_{TH}\right)V_{DS} - V_{DS}^2 \right]$$

$$\boxed{g_m = \frac{\partial I_D}{\partial V_{GS}} = 2K_N' \left(\frac{W}{L} \right) V_{DS}}$$

Issues:

- V_{DS} must be low enough to ensure that each input transistor remains in the triode region
- The input common-mode level must be tightly controlled, and it must track V_b so as to define I_{D1} and I_{D2}
- Additional noise contribution from M3, M4, and two amplifiers
- g_m may be too small since V_{DS} is small

Linearization Technique (4)
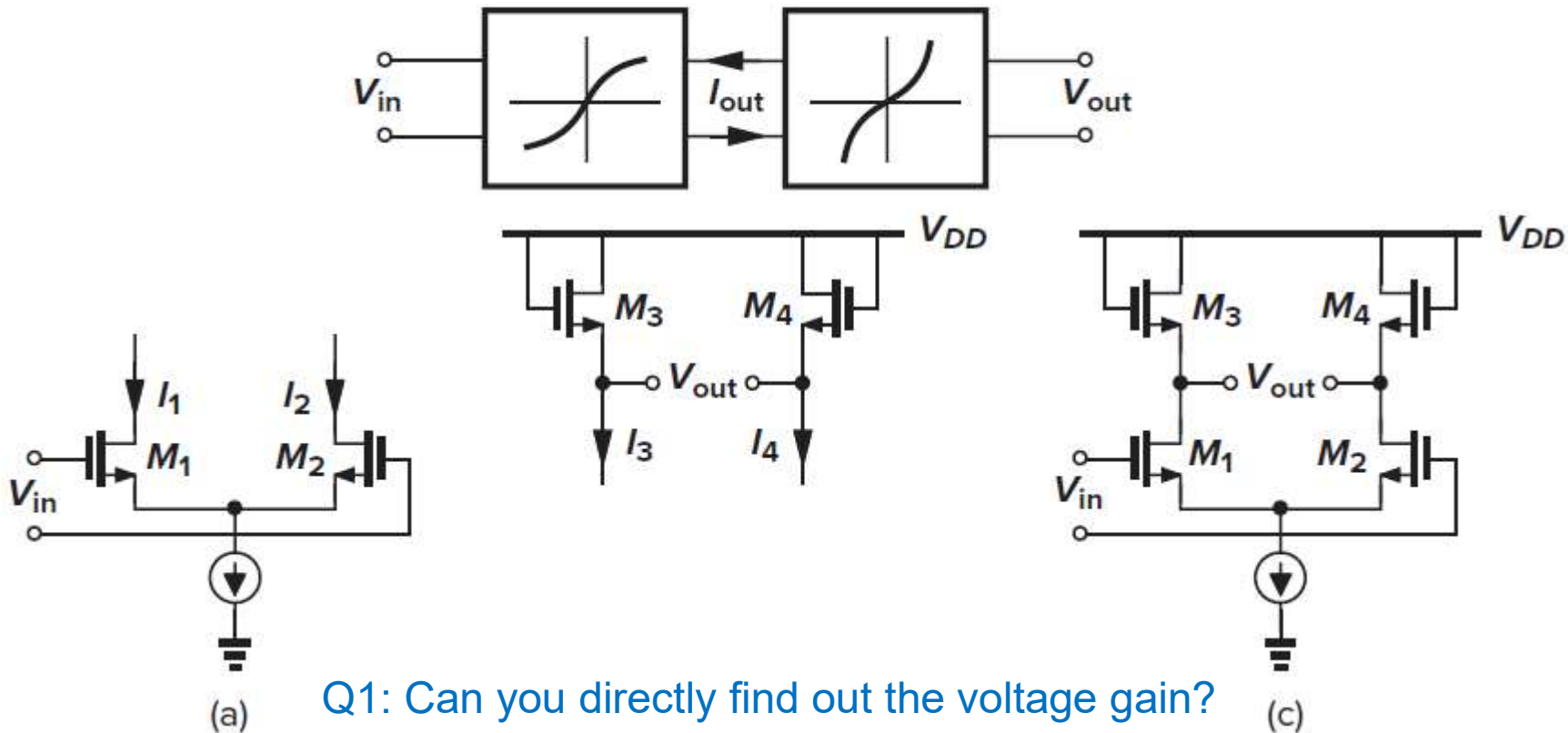
Q1: Can you directly find out the voltage gain?

Q2: How does it achieve the linearization?

$$\Delta I = I_1 - I_2 = g_{m1,2}\Delta V_{in} \qquad \Delta V_{out} = \frac{\Delta I}{g_{m3,4}} \qquad \Delta V_{out} = \frac{g_{m1,2}}{g_{m3,4}}\Delta V_{in}$$

Linearization Technique (4)

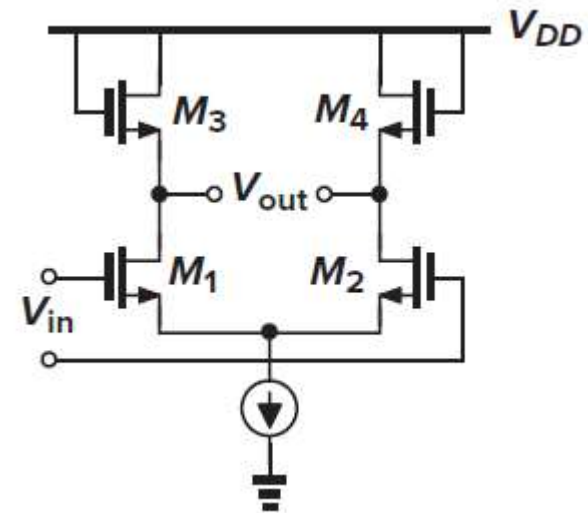$$V_{in1} - V_{in2} = V_{GS1} - V_{GS2}$$

$$= \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_{1,2}}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_{1,2}}}$$

$$V_{out} = V_{GS3} - V_{GS4}$$

$$= \sqrt{\frac{2I_3}{\mu_n C_{ox} \left(\frac{W}{L}\right)_{3,4}}} - \sqrt{\frac{2I_4}{\mu_n C_{ox} \left(\frac{W}{L}\right)_{3,4}}}$$
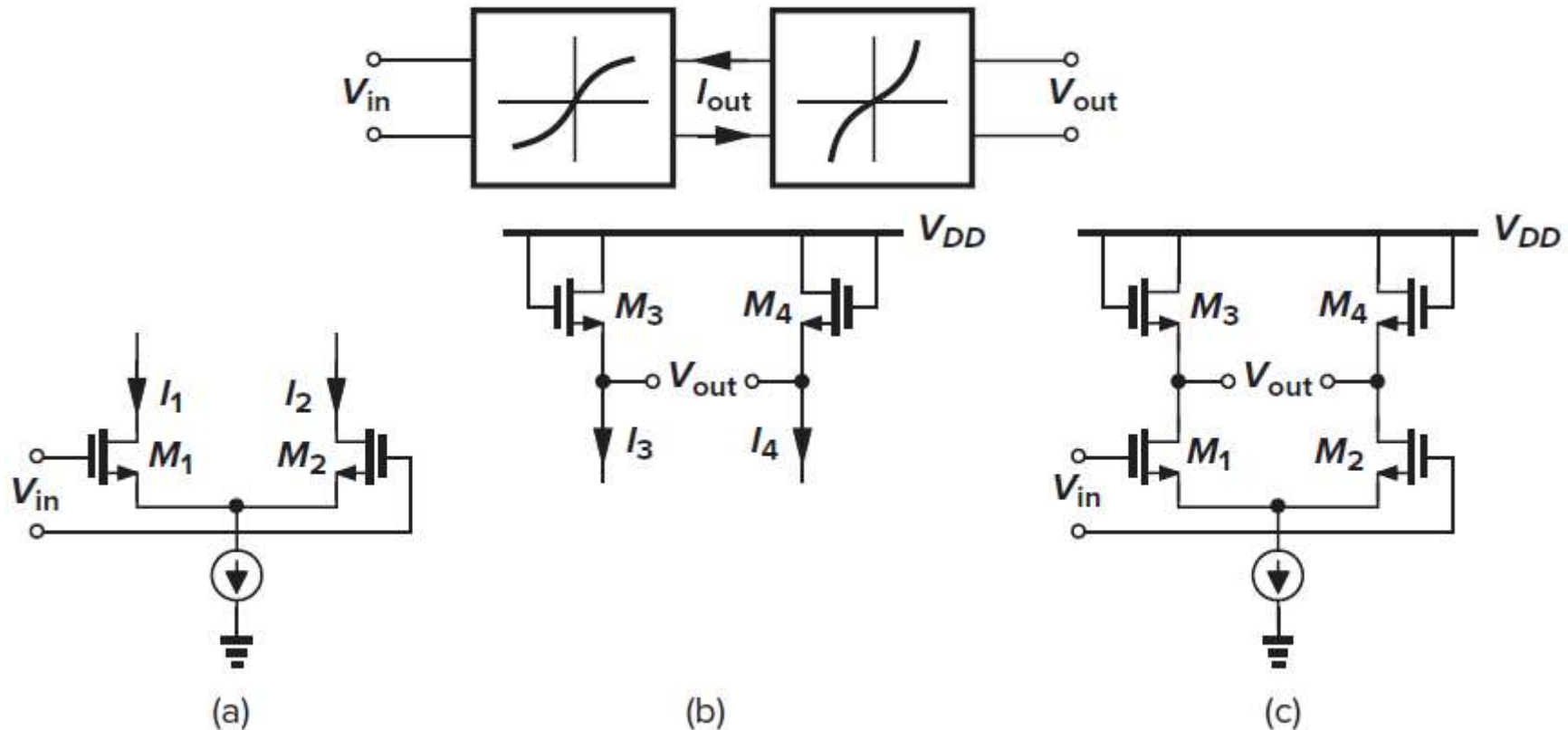
$$V_{out} = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_{3,4}}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_{3,4}}}$$

$$= \frac{1}{\sqrt{\left(\frac{W}{L}\right)_{3,4}}} (V_{in1} - V_{in2}) \, sqrt\left(\frac{W}{L}\right)_{1,2}$$

$$A_v = \sqrt{\frac{\left(\frac{W}{L}\right)_{1,2}}{\left(\frac{W}{L}\right)_{3,4}}}$$

Linearization Technique (4)

(a)   (b)   (c)

- In practice, **body effect** and other nonidealities in short-channel devices will further give rise to nonlinearity in this circuit.
- Furthermore, as the differential input level increases, driving *M1 or M2* into the triode region, the linearization is no longer hold and the gain drops sharply

=> Body effect can be solved by deep-nwell nMOSTs

Linearization Technique (5)
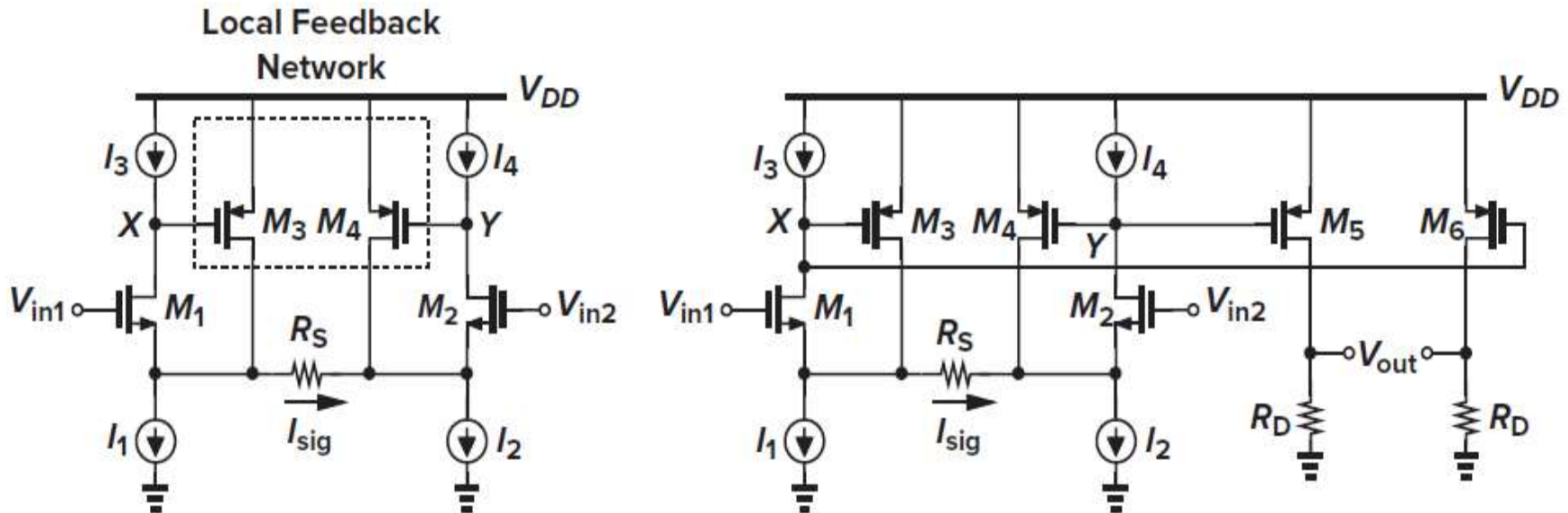
$$V_{in1} = V_{cm} + V_{in}/2$$

$$V_{in2} = V_{cm} - V_{in}/2$$

$$V_{in} = V_{GS1} + I_{sig}R_S - V_{GS2}$$

$$= I_{sig}R_S, \text{ if } \underline{V_{GS1} = V_{GS2}}$$

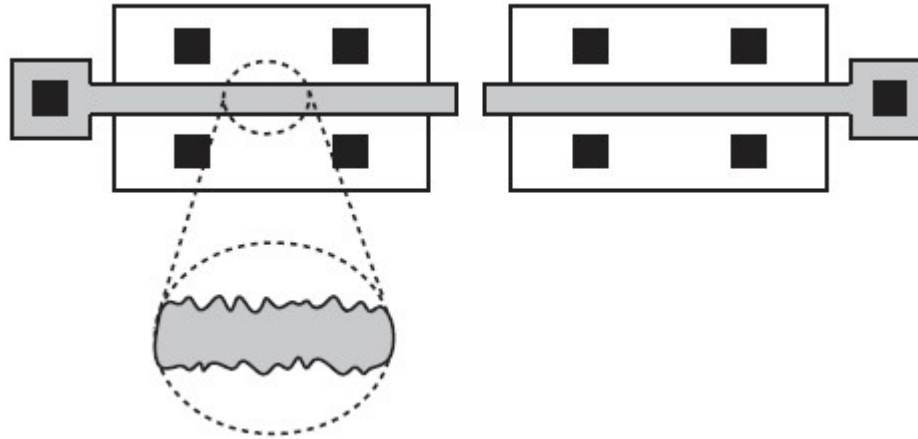where $\underline{I_3 = I_4}$, $I_1 = I_3 + I_{M3}$; $I_2 = I_4 + I_{M4}$

$$V_{out} = (I_6 - I_5)R_D$$

$$I_5 = I_{M4} \text{ and } I_6 = I_{M3}$$

$$V_{out} = (I_{M3} - I_{M4})R_D$$

$$= -I_{sig}R_D = -\frac{R_D}{R_S}V_{in}$$

這與課本不同

Mismatch

- Random mismatches due to microscopic variations in device dimensions（來自晶圓廠，在晶片製造上的非理想因素）

- Study of mismatch consists of two steps:

    (1) To identify and formulate the mechanisms that lead to mismatch between devices, but it is not an easy work. Generally, it depends on the Foundry's process quality control

    (2) To analyze the effect of device mismatches upon the performance of circuits

Mismatch

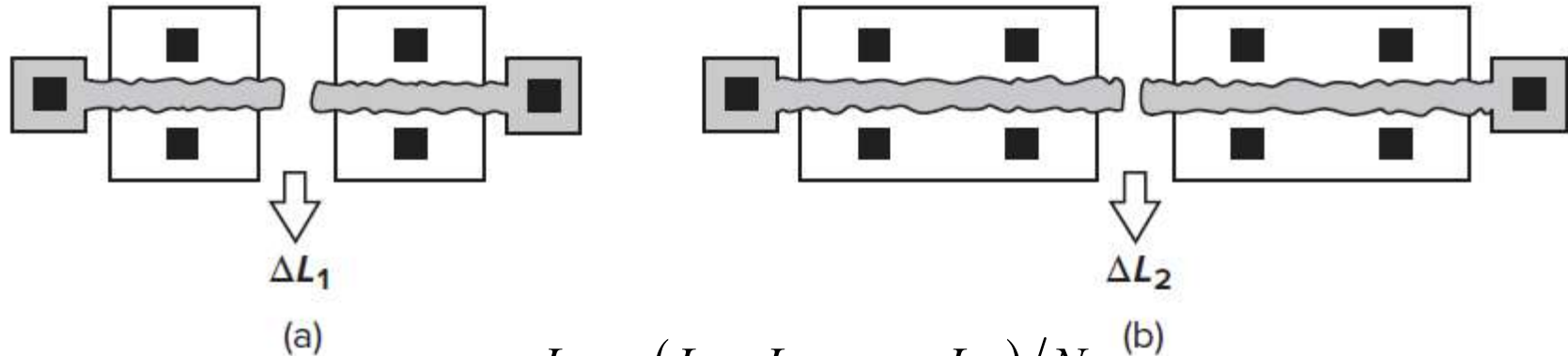(a) ΔL_1

(b) ΔL_2
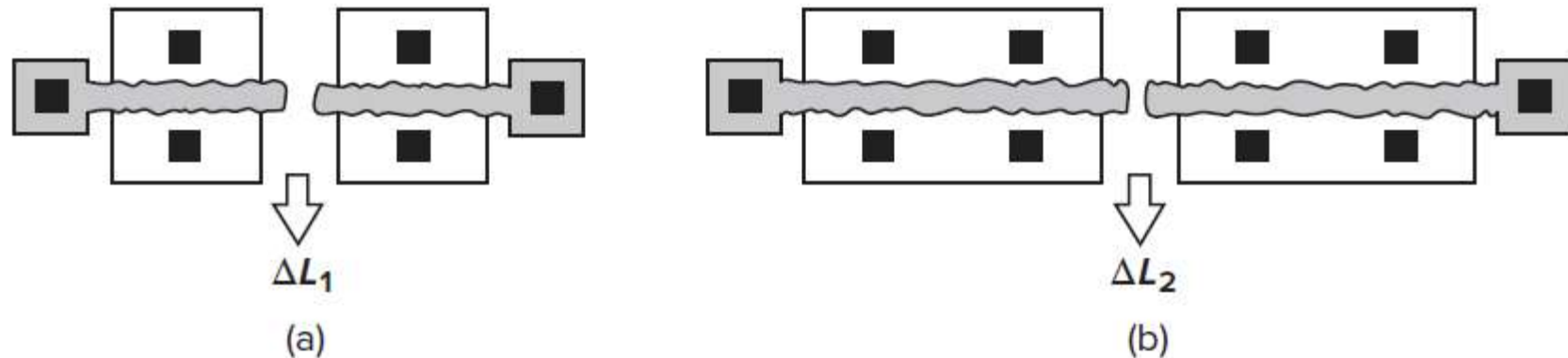
$$I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L} \right) (V_{GS} - V_{TH})^2$$

Mismatch issue on
- Current mirrors
- Bias voltages

- For two nominally-identical transistors
- Mismatches of $\mu_n C_{ox}$, W, L, and V_{TH} result in mismatches between drain currents (for a given V_{GS}) or gate-source voltages (for a given drain current)
- Intuitively, larger devices exhibit smaller mismatches

Mismatch - Concept

(a)

ΔL_1

(b)

ΔL_2

考量通道長度(L)的變異:
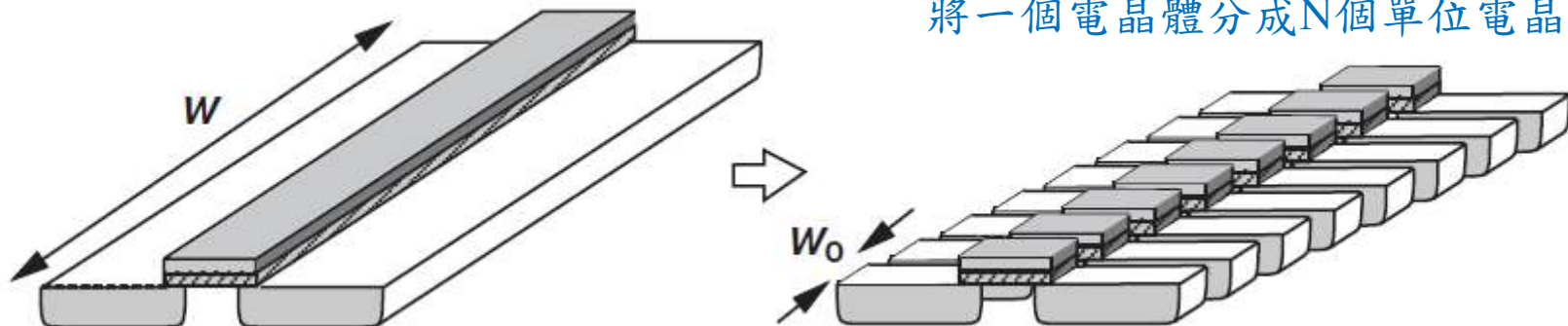$L_i = L_0 + \Delta L_i$
ΔL_i is a random variable

$$L_{eq} \approx \left(L_1 + L_2 + \cdots + L_N \right) / N$$

$$\Delta L_{eq} \approx \frac{\sqrt{\Delta L_1^2 + \Delta L_2^2 + \cdots + \Delta L_N^2}}{N} = \frac{\sqrt{N \Delta L_0^2}}{N} = \frac{\Delta L_0}{\sqrt{N}}$$

where ΔL_0 is the statistical variation of the length
for a transistor with the length of L_0

W

將一個電晶體分成N個單位電晶體

W_0

Random Variables

Assume x_i is a zero mean random variable which is independent to each other

$E[x_i] = 0$, and $E[x_i^2] = \sigma^2$, $\forall i$

Assume a function $y = x_1 + x_2 + x_3 + x_4$,

$E[y] = E[x_1] + E[x_2] + E[x_3] + E[x_4] = 0$

$\sigma_y^2 = E[y^2] = E[x_1^2] + E[x_2^2] + E[x_3^2] + E[x_4^2] = 4\sigma^2$
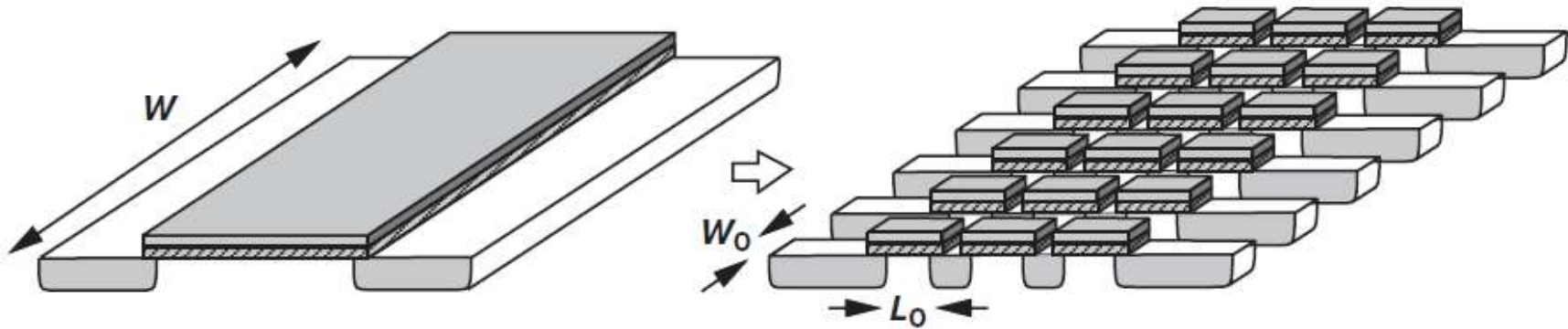
$\Rightarrow \sigma_y = 2\sigma$

Considering two new random variables,

$$y_1 = \sum_{i=1}^{N} x_i \Rightarrow \sigma_{y_1} = \sqrt{N}\sigma$$

$$y_2 = \frac{1}{N}\sum_{i=1}^{N} x_i \Rightarrow \sigma_{y_2}^2 = \frac{1}{N^2}\sum_{i=1}^{N} E[x_i^2] = \frac{\sigma^2}{N} \Rightarrow \sigma_{y_2} = \frac{\sigma}{\sqrt{N}}$$
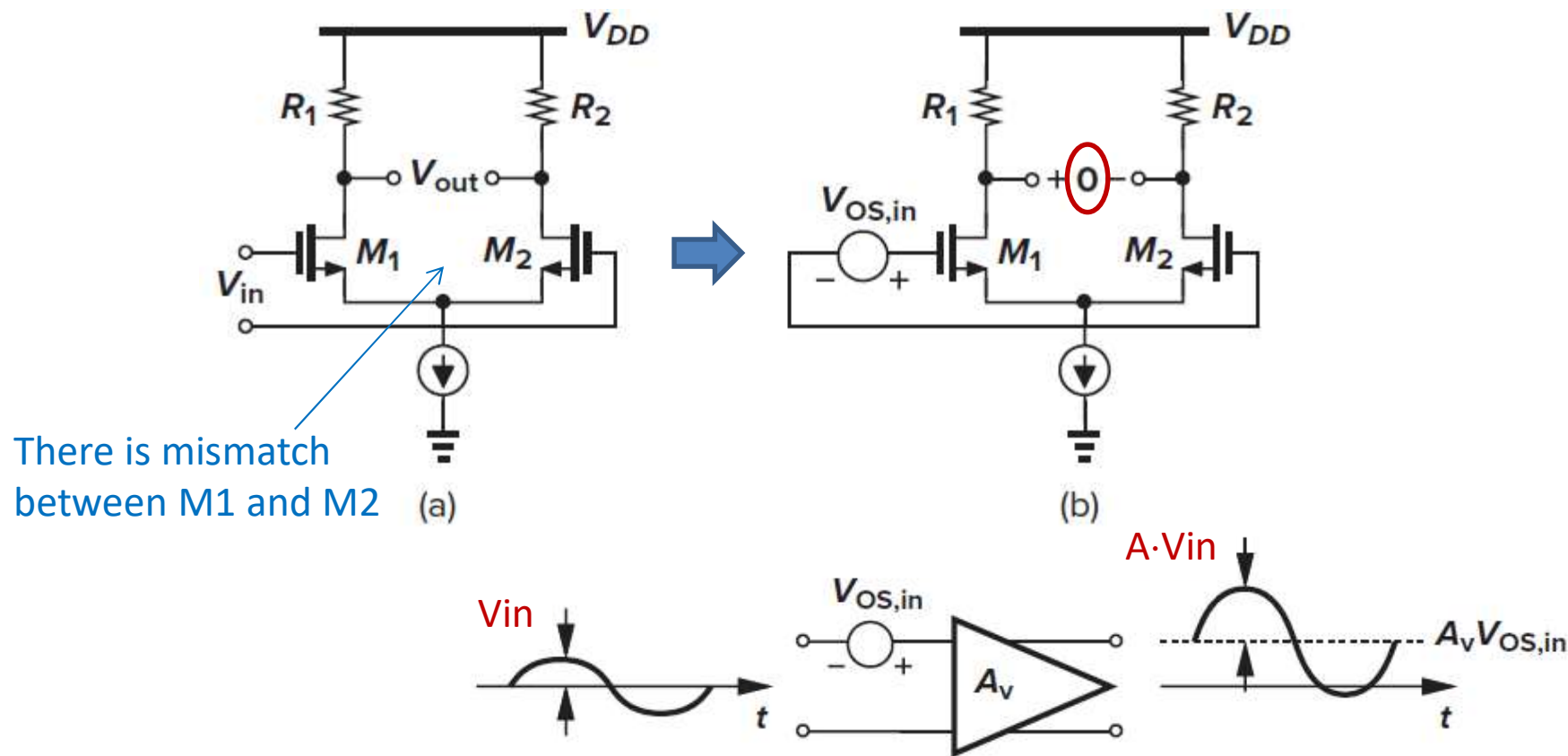
Mismatch

$$\Delta V_{TH} = \frac{A_{V_{TH}}}{\sqrt{WL}}$$

$$\frac{\Delta \beta}{\beta} = \Delta \left(\mu C_{ox} \frac{W}{L} \right) \bigg/ \beta = \frac{A_{\beta}}{\sqrt{WL}}$$

- We postulate that $\beta = \mu C_{ox}(W/L)$ and V_{TH} suffer from less mismatch if the device area increases (larger transistor dimension)

- For given W_0 and L_0, as the number of unit transistors increases, β and V_{TH} experience greater averaging, leading to smaller mismatch between two large transistors
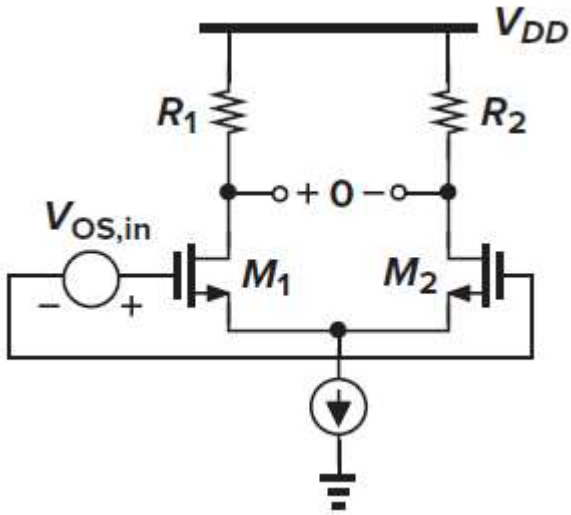
Mismatch of Input Pair

There is mismatch between M1 and M2 (a)

(b)

- **DC Offset** (V_{OS})
  - Two conditions: (1) Vin=0 but Vout$\neq$0; (2) Vin$\neq$0 but Vout=0
  - Figure (b) presents the input-referred offset voltage ($V_{OS,in}$), caused by the device mismatch

Input-Referred Offset

$$V_{TH1} = V_{TH}, \; V_{TH2} = V_{TH} + \Delta V_{TH}$$

$$\left(\frac{W}{L}\right)_1 = \frac{W}{L}, \; \left(\frac{W}{L}\right)_2 = \frac{W}{L} + \Delta\left(\frac{W}{L}\right)$$

$$R_1 = R_D, \; R_2 = R_D + \Delta R$$

Assume $\lambda = \gamma = 0$, $\Delta(\mu_n C_{ox}) = 0$

$$V_{out} = 0, \; I_{D1}R_1 = I_{D2}R_2$$
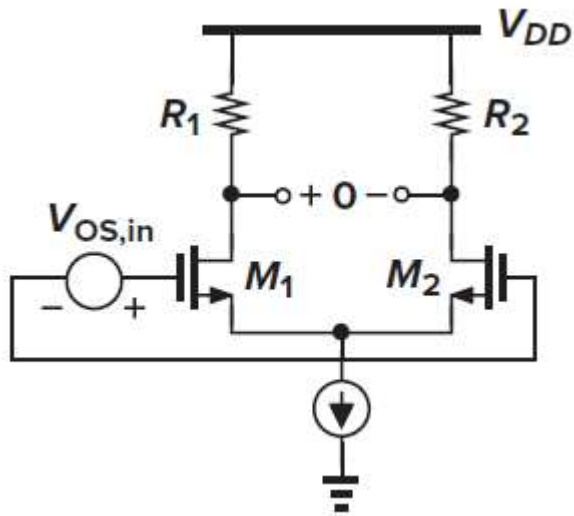
$$I_{D1} = I_D, \; I_{D2} = I_D + \Delta I_D$$

$$V_{OS,in} = V_{GS1} - V_{GS2}$$

$$= \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\left(\dfrac{W}{L}\right)_1}} + V_{TH1} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}\left(\dfrac{W}{L}\right)_2}} - V_{TH2}$$

$$= \sqrt{\frac{2}{\mu_n C_{ox}}}\left[\sqrt{\frac{I_D}{W/L}} - \sqrt{\frac{I_D + \Delta I_D}{W/L + \Delta(W/L)}}\right] - \Delta V_{TH}$$

$$= \sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}}\left[1 - \sqrt{\frac{1 + \Delta I_D/I_D}{1 + \Delta(W/L)/(W/L)}}\right] - \Delta V_{TH}$$

Assume $\Delta I_D/I_D$ and $\Delta(W/L)/(W/L) \ll 1$,

for $\varepsilon \ll 1$, $\sqrt{1+\varepsilon} \approx 1 + \varepsilon/2$

$$V_{OS,in} = \sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}}\left\{1 - \left(1 + \frac{\Delta I_D}{2I_D}\right)\left(1 - \frac{\Delta(W/L)}{2(W/L)}\right)\right\} - \Delta V_{TH}$$

$$V_{OS,in} = \sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}}\left[\frac{-\Delta I_D}{2I_D} + \frac{\Delta(W/L)}{2(W/L)}\right] - \Delta V_{TH}$$

Input-Referred Offset

$$\Delta I_D / I_D \approx -\Delta R_D / R_D$$

$$V_{OS,in} = \frac{1}{2}\sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}}\left[\frac{\Delta R_D}{R_D} + \frac{\Delta(W/L)}{(W/L)}\right] - \Delta V_{TH}$$

$$= \frac{V_{GS} - V_{TH}}{2}\left[\frac{\Delta R_D}{R_D} + \frac{\Delta(W/L)}{(W/L)}\right] - \Delta V_{TH}$$

$\Rightarrow V_{OS,in}$ is also a random variable

$$V_{OS,in}^2 = \left(\frac{V_{GS} - V_{TH}}{2}\right)^2\left[\left(\frac{\Delta R_D}{R_D}\right)^2 + \left(\frac{\Delta(W/L)}{(W/L)}\right)^2\right] + \Delta V_{TH}^2$$

$$\sigma_{OS,in}^2 = \left(\frac{V_{GS} - V_{TH}}{2}\right)^2\left[\sigma\left(\frac{\Delta R_D}{R_D}\right)^2 + \sigma\left(\frac{\Delta(W/L)}{(W/L)}\right)^2\right] + \sigma_{V_{TH}}^2$$

Q: From this result, to get a small offset, how to design V_{GS}-V_{TH}?
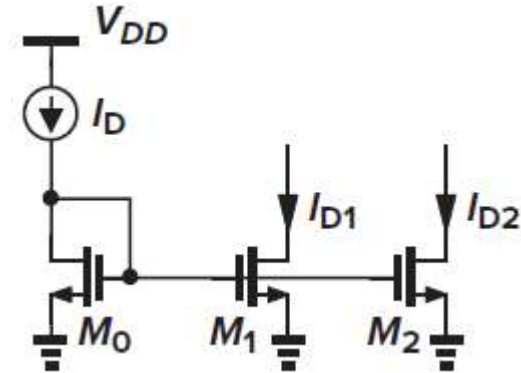
Mismatch of Current Mirror

Recall $y = f(x_1, x_2, \ldots)$

$$\Delta y = \frac{\partial f}{\partial x_1} \Delta x_1 + \frac{\partial f}{\partial x_2} \Delta x_2 + \cdots$$

$$\Delta I_D = \frac{\partial I_D}{\partial (W/L)} \Delta \left(\frac{W}{L}\right) + \frac{\partial I_D}{\partial (V_{GS} - V_{TH})} \Delta (V_{GS} - V_{TH})$$
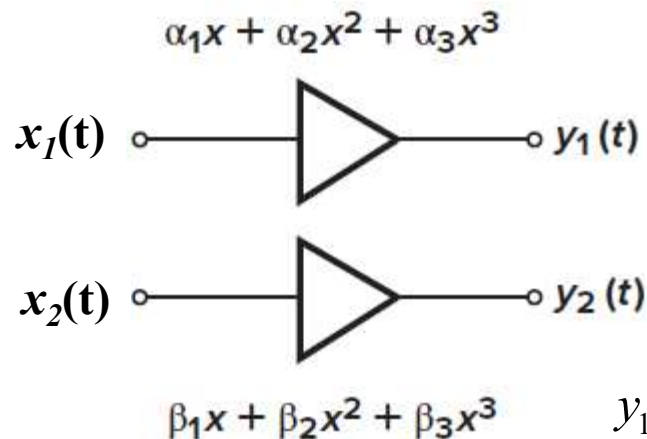
$$\Delta I_D = \frac{1}{2} \mu_n C_{ox} (V_{GS} - V_{TH})^2 \Delta \left(\frac{W}{L}\right) - \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_{TH}) \Delta V_{TH}$$

$$\frac{\Delta I_D}{I_D} = \frac{\Delta (W/L)}{W/L} - \left(\frac{V_{GS} - V_{TH}}{2}\right)^{-1} \Delta V_{TH}$$

- Different from $V_{OS,in}$, the current mismatch is inversely proportional to gate-overdrive voltage ($V_{GS} - V_{TH}$)

Even-Order Distortion: Mismatch

$$\alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3$$

$x_1(t)$ ○————▷———○ $y_1(t)$

$x_2(t)$ ○————▷———○ $y_2(t)$

$$\beta_1 x + \beta_2 x^2 + \beta_3 x^3$$

$$y_1 - y_2 = \left(\alpha_1 x_1 - \beta_1 x_2\right) + \left(\alpha_2 x_1^2 - \beta_2 x_2^2\right) + \left(\alpha_3 x_1^3 - \beta_3 x_2^3\right)$$

Assume $x_1 = x$ and $x_2 = -x$,

if $\alpha_i \neq \beta_i$, $\forall i$

$$y_d = y_1 - y_2 = \left(\alpha_1 + \beta_1\right)x + \underline{\left(\alpha_2 - \beta_2\right)x^2} + \left(\alpha_3 + \beta_3\right)x^3$$

- Single-ended signals have tones of 2nd, 3rd, 4th, …

- Fully differential circuits ideally have only odd-order tones

- However, **mismatch** causes even-order tones in differential signals

- In addition, the (thermal) gradient also contributes mismatch
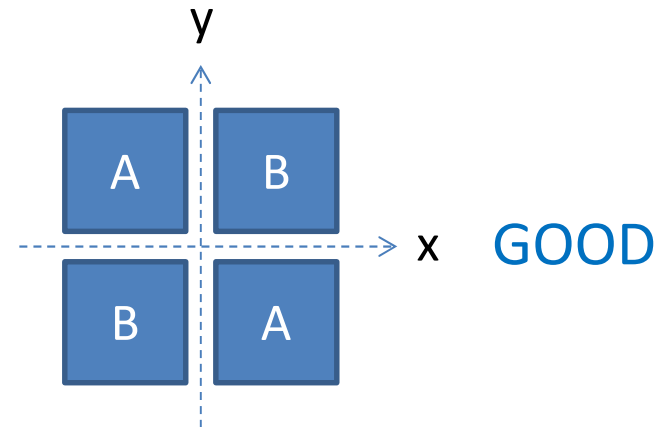
Gradient: Systematic Issue

Random mismatch

Layout pattern for gradient
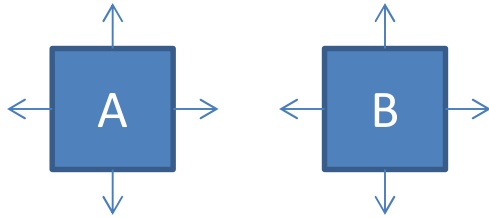
Thermal gradient

Gradient is a deterministic error which causes additional mismatch errors
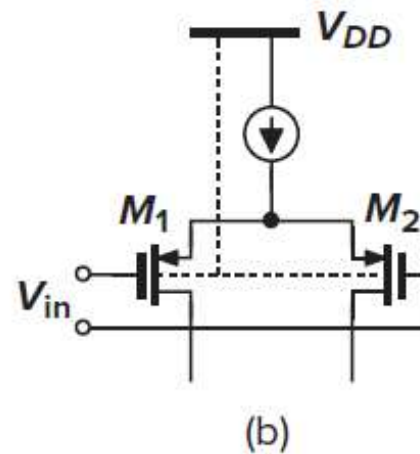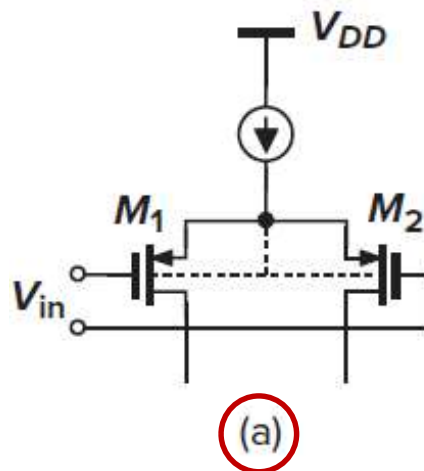
The solution is **common-centroid** layout patterns

CMRR Considerations

$$A_{CM-DM} = \frac{\Delta V_{OS,out}}{\Delta V_{CM,in}}, \quad A_{DM} = \frac{\Delta V_{OS,out}}{\Delta V_{OS,in}}$$

$$CMRR = \frac{A_{DM}}{A_{CM-DM}} = \frac{A_{DM}}{\dfrac{\Delta V_{OS,out}}{\Delta V_{CM,in}}} = \frac{\Delta V_{CM,in}}{\dfrac{\Delta V_{OS,out}}{A_{DM}}} = \frac{\Delta V_{CM,in}}{\Delta V_{OS,in}}$$

Less mismatch, higher CMRR

(a)   (b)

In Fig. (a), body effect is eliminated and the threshold voltages of *M1 and M2* are independent of the input CM level. *But, large layout area is introduced!!*

In Fig. (b), if they suffer from mismatches in their body effect coefficients, the input offset voltage varies with the input CM level