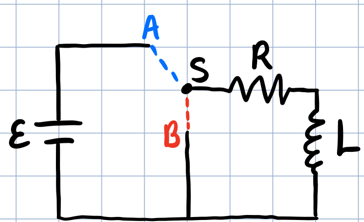
RL Circuit

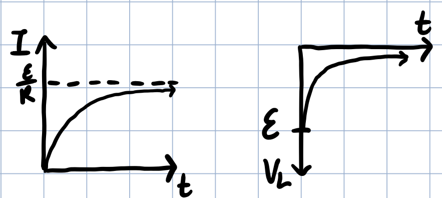


Begin in position A:

$$\mathcal{E}-IR-L\frac{dI}{dt}=0 \rightarrow dt=\frac{L\,dI}{\mathcal{E}-IR} \rightarrow t=-\frac{L}{R}\ln\left(1-\frac{IR}{\mathcal{E}}\right) \rightarrow e^{-Rt/L}=1-\frac{IR}{\mathcal{E}} \rightarrow I=\frac{\mathcal{E}}{R}\left(1-e^{-Rt/L}\right)$$

$$V_L=-L\frac{dI}{dt}=-L\frac{d}{dt}\left[\frac{\mathcal{E}}{R}\left(1-e^{-Rt/L}\right)\right]=-\mathcal{E}e^{-Rt/L}$$

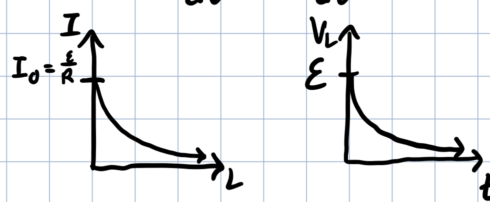
$\tau=\frac{L}{R}$ is time constant



Switch to position B:

$$-IR-L\frac{dI}{dt}=0 \rightarrow dt=-L\frac{dI}{IR} \rightarrow t=-\frac{L}{R}\ln\left(\frac{I}{I_0}\right) \rightarrow I=I_0e^{-Rt/L}=\frac{\mathcal{E}}{R}e^{-Rt/L}$$

$$V_L=-L\frac{dI}{dt}=-L\frac{d}{dt}\left(I_0e^{-Rt/L}\right)=\mathcal{E}e^{-Rt/L}$$

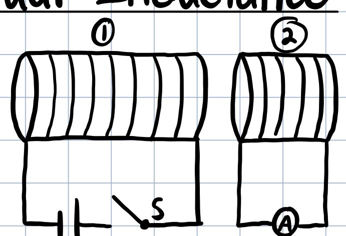


- We say that the energy of an inductor is stored in its magnetic field

$$\mathcal{E}-IR-L\frac{dI}{dt}=0 \rightarrow \mathcal{E}I-I^2R-LI\frac{dI}{dt}=0 \rightarrow P=LI\frac{dI}{dt}=\frac{dU}{dt} \rightarrow LI\,dI=dU \rightarrow U=\frac{1}{2}LI^2$$

- Energy density: $u=\frac{U}{vol}=\frac{U}{\ell A}=\frac{LI^2}{2\ell A}=\frac{\mu_0\lambda^2AI^2}{2A}=\frac{B^2}{2\mu_0}$

Mutual Inductance



Close S: I_1 sets up $\frac{d\Phi_2}{dt}$

\# of coils

Mutual inductance of 2 with respect to I_1: $M_{21}=\frac{N_2\Phi_2}{I_1}$

Mutual inductance of 1 with respect to I_2: $M_{12}=\frac{N_1\Phi_1}{I_2}$

It can be shown that $M_{21}=M_{12}=M$ and $\mathcal{E}_1=-M\frac{dI_2}{dt}$, $\mathcal{E}_2=-M\frac{dI_1}{dt}$

Apparently if you have two inductors close enough, $\mathcal{E}_1=-L\frac{dI_1}{dt}-M\frac{dI_2}{dt}$