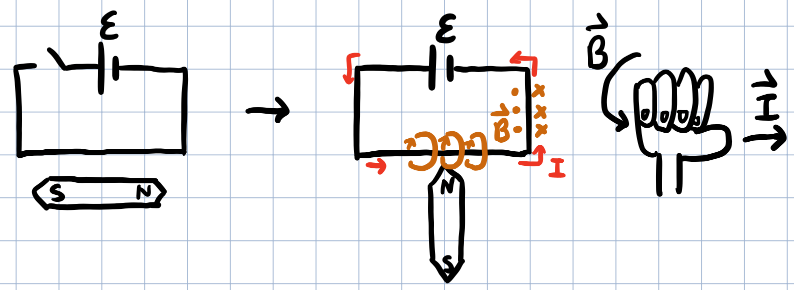
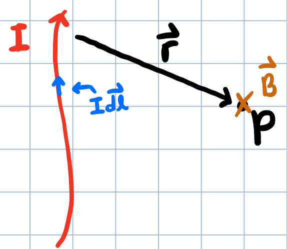
Magnetic Fields

- Magnetism is generated by electric currents!
- In a circuit,

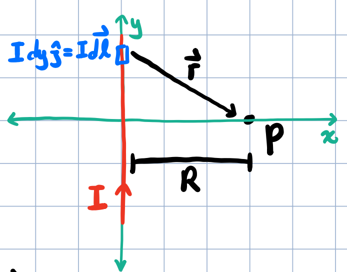


- Biot-Savant Law



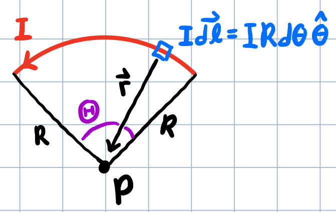
$$\vec{B} = \int \frac{\mu_0}{4\pi} \times \frac{I\,d\vec{l} \times \hat{r}}{r^2} \qquad \mu_0 = 4\pi \times 10^{-7}\,\frac{Tm}{A}$$

- $\vec{B}$ due to an ∞ straight wire



$$\vec{B} = \int \frac{\mu_0}{4\pi} \times \frac{I\,d\vec{l}\,\hat{r}}{r^2} = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{dy\,\hat{j} \times (y\hat{j} + R\hat{z})}{(y^2+R^2)^{3/2}} = \frac{\mu_0 I}{2\pi R} \times -\hat{k}$$

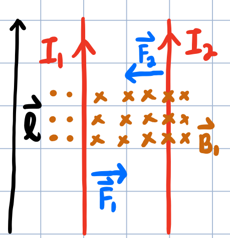
- $\vec{B}$ due to an arc of current



$$\vec{B} = \int \frac{\mu_0}{4\pi} \times \frac{I\,d\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi} \int_0^{\Theta} \frac{R\,d\theta\,\hat{\theta}}{R^2} \times \frac{-R\hat{r}}{R} = \frac{\mu_0 I}{4\pi R} \int_0^{\Theta} d\theta\,\hat{k} = \frac{\mu_0 I}{4\pi R}\,\Theta\,\hat{k}$$

For complete loop, $\Theta = 2\pi \rightarrow \vec{B} = \frac{\mu_0 I}{2R}\,\hat{k}$

- $\vec{F}$ between current-carrying wires



$$\left.\begin{aligned} \vec{F}_2 &= I_2\vec{l}_2 \times \vec{B}_1 = \frac{I_2 l \mu_0 I_1}{2\pi R}(-\hat{z}) = -\frac{\mu_0 l}{2\pi R} I_1 I_2 \hat{z} \\ \vec{F}_1 &= I_1\vec{l}_1 \times \vec{B}_2 = \frac{I_1 l \mu_0 I_2}{2\pi R}\,\hat{z} = \frac{\mu_0 l}{2\pi R} I_1 I_2 \hat{z} \end{aligned}\right\} \text{Newton's Third Law}$$

So parallel currents attract; note that anti-parallel currents repel