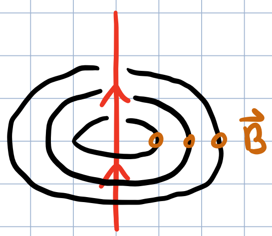
Ampere's Law

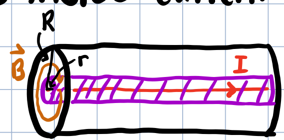
- Ampere's Law:



$$\oint \vec{B}\cdot d\vec{s} = \oint \vec{B}\cdot r\,d\theta\,\hat{\theta} = Br\int_0^{2\pi} d\theta = Br\times 2\pi = \frac{\mu_0 I}{2\pi r}\times 2\pi r = \mu_0 I$$

$$\rightarrow \oint \vec{B} d\vec{s} = \mu_0 I_{encl}$$

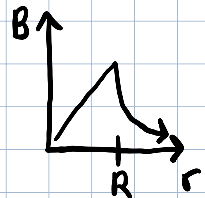
- $\vec{B}$ inside current-carrying wire:



If J is constant ($I/\pi R^2$), $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}$

$$\rightarrow B\times 2\pi r = \mu_0 \frac{I}{\pi R^2}\times \pi r^2 \rightarrow B = \frac{\mu_0 I r}{2\pi R^2}$$

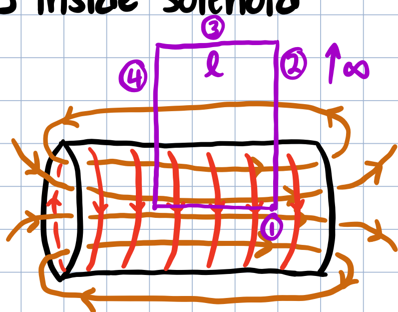
If $r=R$, $B = \frac{\mu_0 I}{2\pi R}$;



If J is not constant (αr),

$$I_{encl} = \int \vec{J}\cdot d\vec{A} = \int_0^r \alpha r\times 2\pi r\,dr = \frac{2\pi\alpha r^3}{3} \rightarrow B = \frac{\mu_0 \alpha r^3}{3}$$

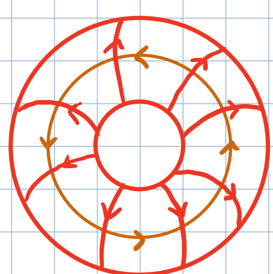
- $\vec{B}$ inside solenoid



$$\oint_{①} \vec{B}\cdot d\vec{s} + \oint_{②} \vec{B}\cdot d\vec{s} + \oint_{③} \vec{B}\cdot d\vec{s} + \oint_{④} \vec{B}\cdot d\vec{s}$$

$$= B\ell + 0 + 0 + 0 = B\ell = \mu_0 I_{encl} \rightarrow B = \mu_0 \lambda I,\quad \lambda = \frac{\#\text{ of coils}}{\text{length}}$$

- $\vec{B}$ inside torroid



$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl} \rightarrow B\times 2\pi r\times \mu_0 NI \rightarrow B = \frac{\mu_0 NI}{2\pi r}$$