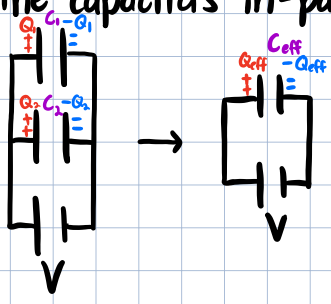
More Capacitors

- Recall the capacitors-in-parallel scenario

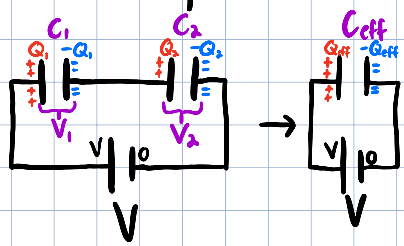


$Q_{eff} = Q_1 + Q_2 \rightarrow C_{eff} = C_1 + C_2$

If $C_1 = 1\mu F$, $C_2 = 2\mu F$, and $V = 10V$, then:

$C_{eff} = 3\mu F$, $Q_{eff} = C_{eff} \times V = 30\mu C$, $Q_1 = C_1 \times V = 10\mu C$, $Q_2 = C_2 \times V = 20\mu C$

- What about capacitors in series?



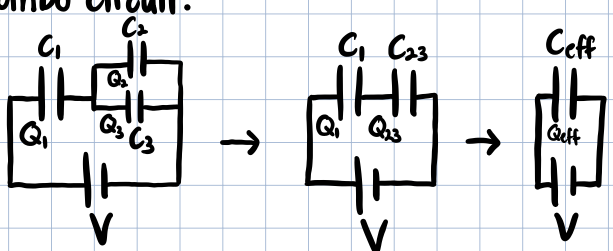
Notice that $Q_1 = Q_2 = Q_{eff}$ and $V_1 + V_2 = V$.

This means $\frac{Q_1}{C_1} + \frac{Q_2}{C_2} = \frac{Q_{eff}}{C_{eff}} \rightarrow \frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2}$ or $\frac{1}{C_{eff}} = \sum_{i=1}^{N} \frac{1}{C_i}$

If $C_1 = 1\mu F$, $C_2 = 2\mu F$, and $V = 10V$, then:

$C_{eff} = 0.67\mu F$, $Q_{eff} = C_{eff} \times V = 6.7\mu C$, $Q_1 = 6.7\mu C$, $Q_2 = 6.7\mu C$, $V_1 = \frac{Q_1}{C_1} = 6.7\,V$, $V_2 = \frac{Q_2}{C_2} = 3.4V$

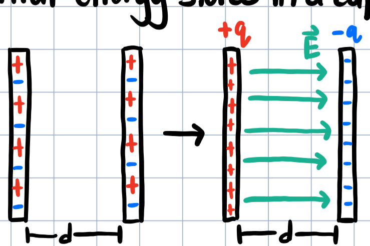
- Combo circuit:



If $C_1 = 1\mu F$, $C_2 = 2\mu F$, $C_3 = 3\mu F$, and $V = 10V$, then:

$C_{eff} = 0.83\mu F$, $Q_{eff} = C_{eff} \times V = 8.3\mu C$, $Q_1 = Q_{eff} = 8.3\mu C$, $V_1 = \frac{Q_1}{C_1} = 8.3V$, $C_{23} = 5\mu F$
$Q_{23} = Q_{eff} = 8.3\mu C$, $V_2 = \frac{Q_{23}}{C_{23}} = 1.66V$, $V_3 = 1.66V$, $Q_2 = 3.32\mu C$, $Q_3 = 4.98\mu C$

- Potential energy stored in a capacitor:



$$W = \int_0^Q dq\,E \times d = \int_0^Q \frac{qd}{A\varepsilon_0}\,dq = \frac{d}{A\varepsilon_0}\int_0^Q q\,dq = \frac{Q^2 d}{2A\varepsilon_0} = \frac{Q^2}{2C}$$

$$\rightarrow U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$

- Energy density:

  $u = \frac{\text{energy}}{\text{volume}} = \frac{\frac{1}{2}\, dQ^2/A\varepsilon_0}{Ad} = \frac{1}{2} \times \frac{Q^2}{A\varepsilon_0} = \frac{1}{2}\varepsilon_0 E^2$, "energy density" $\rightarrow U = \left(\frac{1}{2}\varepsilon_0 E^2\right)(Ad)$