

Electric Current

- Electric current is the rate at which charges pass through a material ($I = \frac{\text{Coulomb}}{\text{second}} = \text{Amp}$)
- $I = \sum_{i=1}^N \frac{q_i}{t}$ or $\frac{dq}{dt}$; note that current is not a vector
- Although it's not "technically" realistic, we say that I is the flow of positive charge
- Current density, on the other hand, is a vector
- We have that current density $\vec{J} = \frac{I}{A} \hat{r}$, where \hat{r} points in the direction of the electric field; also note $I = \int \vec{J} \cdot d\vec{A}$
- Example: current through wire w/ radius R

$$(\text{Given } \vec{J} = ar^2)$$

$$I = \int \vec{J} \cdot d\vec{A} = \int_0^R ar^2 \hat{z} \cdot 2\pi r dr \hat{z} = 2\pi a \int_0^R r^3 dr = \frac{1}{2} \pi a R^4$$

Drift Speed

- Drift speed: the "speed" of a charge through a material

$$v_d = \frac{L}{t}$$

$Q = nVq$, where n is free charge density, V is volume, q is charge on free charges

$$I = \frac{Q}{t} = \frac{nVq}{t} = \frac{nLAq}{t} = n v_d A q$$

$$v_d = \frac{I}{nAq}$$

□ Note that $n = \frac{\# \text{ of free charges}}{\text{volume}} = \frac{\# \text{ free charges}}{\text{atom}} \times \frac{\text{atom}}{\text{mass}} \times \frac{\text{mass}}{\text{volume}}$

- Example: Find the drift speed of a 1mm copper wire with a 1 Amp current

$$n = \frac{1 \text{ free charge}}{1 \text{ atom}} \times \frac{1 \text{ atom}}{(63.54 \text{ amu})(1.67 \times 10^{-27} \frac{\text{kg}}{\text{amu}})} \times \frac{8.9 \times 10^3 \text{ kg}}{1 \text{ m}^3} = 8.44 \times 10^{28} \frac{\text{charge}}{\text{m}^3}$$

$$v_d = \frac{I}{nAq} = \frac{1}{(8.44 \times 10^{28})(\pi \times (0.5 \times 10^{-3})^2)(1.6 \times 10^{-19})} = 9.4 \times 10^{-5} \frac{\text{m}}{\text{s}}$$

Resistance

- Resistance is the measure of how difficult it is for charges to flow through a wire
- Note that in a wire, \vec{E} is not static; \vec{E} is generated by moving charges and not stationary clumps of charges
- Resistivity, ρ , is equal to $\frac{E}{J}$ and has unit Ωm (ohm \times meter)
- Resistance, Ω , is equal to $\rho \frac{L}{A}$
- Ohm's Law: $R = \rho \frac{L}{A} = \frac{E}{J} \times \frac{L}{A} = \frac{V}{I} \rightarrow V = IR$