Incremental Evolutionary Algorithm for the Eight-Queens Problem

Introduction

The Eight-Queens problem is a classic combinatorial optimization challenge in which eight queens must be placed on an 8×8 chessboard such that no two queens attack each other. In our approach, we use an incremental evolutionary algorithm where queens are placed on the board one by one, rather than encoding full board states from the start. This method makes the search process more structured, guiding the evolutionary process toward feasible solutions. By introducing incremental placement, we can avoid many infeasible board configurations and improve the efficiency of the search algorithm.

Search Space Analysis

In the incremental approach, a solution is constructed by sequentially placing queens in columns while ensuring minimal conflicts at each step. Unlike traditional evolutionary algorithms that encode a full permutation of queen placements from the outset, our method gradually builds solutions by making localized decisions. To analyze the search space, it is essential to consider the number of choices available at each step. The first queen can be placed in any of eight positions, and the second queen can also be placed in any of the eight positions, though some may be constrained by the placement of the first queen. The third queen is placed while considering the constraints imposed by the previous two queens, and this pattern continues for all eight queens.

If no pruning were applied, meaning all placements were allowed regardless of conflicts, the total number of possible placements would follow the formula $8^8 = 16,777,216$. However, with pruning techniques and an evolutionary approach that selects placements based on a fitness function to minimize queen conflicts, the effective search space is significantly reduced. The actual size of this pruned space depends on how effectively the algorithm directs placements away from conflicting states. By strategically limiting queen placements based on fitness evaluation, the algorithm effectively narrows the possible board configurations, making the search much more computationally feasible.

Evolutionary Strategy

The evolutionary algorithm operates incrementally by initializing a partial board state and adding queens one by one. Fitness is evaluated based on the number of non-attacking queen pairs, ensuring that better configurations are favored. Selection mechanisms such as tournament selection are applied to choose the most promising candidates for further evolution. Crossover operations merge different board states, allowing for the recombination of partial solutions to explore new possibilities. Additionally, mutation is introduced to reposition a queen, preventing premature convergence and encouraging diversity in the search. The process continues until a valid solution, where fitness equals 28, is found or until a predefined number of generations is reached.

Mutation plays a crucial role in ensuring the diversity of candidate solutions, preventing the algorithm from becoming stuck in local optima. While incremental placement significantly reduces the number of infeasible solutions explored, mutation ensures that alternative configurations can still be tested, avoiding overly restrictive searches. The interplay between mutation and selection enables a balance between exploration and exploitation, key components of any effective evolutionary search process.

Implications of the Search Space Size

By constructing solutions incrementally, the evolutionary algorithm reduces the likelihood of exploring infeasible regions of the search space. This structured placement approach ensures that the algorithm does not waste computational resources on incorrect configurations, making it significantly more efficient compared to traditional methods that randomly shuffle entire board configurations. The guided nature of the search improves convergence rates and increases the probability of finding optimal solutions. The reduction in the search space not only makes the algorithm more efficient but also allows for better scalability when applied to larger versions of the problem, such as the N-Queens problem for arbitrary board sizes.

The efficiency of the search space reduction is largely influenced by the quality of fitness evaluations and selection mechanisms. If the fitness function is well-designed to prioritize placements that avoid early conflicts, then the evolutionary process will be more effective in reaching optimal solutions. In contrast, a poorly designed fitness function could lead to an unproductive search, requiring more generations to reach viable configurations. Therefore, refining the way fitness is evaluated is a critical component of ensuring the success of an incremental evolutionary approach.

Results

The incremental evolutionary algorithm demonstrated strong performance in solving the Eight-Queens problem, consistently reaching optimal solutions across multiple runs. In 100 experimental trials with 1,000 generations each, the algorithm was able to achieve the global optimum (a conflict-free board with a fitness score of 28) in the majority of runs. Notably, most successful runs converged within the first 50 generations, indicating the effectiveness of guided, incremental placement and evolutionary operations in accelerating solution discovery.

Fitness distribution analysis revealed that while many runs reached the optimal configuration, a subset of runs converged to a near-optimal fitness score of 27, suggesting occasional entrapment in local optima. This outcome emphasizes the need for diversity-preserving mechanisms to overcome stagnation. Mutation played a key role in maintaining population variety; however, small mutation rates sometimes limited the exploration of alternative solutions.

A heatmap of queen placements across successful runs showed no discernible bias, confirming the algorithm's reliance on randomized placement strategies and its robustness against positional favoritism. Despite its success, the algorithm exhibited some limitations, including slow convergence in edge cases and a high computational cost due to the incremental nature of the search process.

Overall, these results validate the effectiveness of the incremental evolutionary strategy in solving constraint satisfaction problems like the Eight-Queens puzzle, while also highlighting areas for potential improvement such as adaptive mutation, diversity control, and hybridization with global search techniques.

Related Work

The Eight-Queens problem has long served as a benchmark for constraint satisfaction and optimization algorithms. Traditional solutions often rely on backtracking or heuristic search methods such as hill climbing and simulated annealing, which explore the full permutation space of queen placements. While effective, these methods can suffer from performance issues due to the vastness of the search space and the potential for getting trapped in local optima.

Evolutionary algorithms (EAs) have emerged as a powerful alternative for solving the N-Queens problem, offering robustness and adaptability. Early EA implementations typically encoded entire board states as permutations, using genetic operators such as crossover and mutation to evolve complete solutions. For example, Ortiz-Bayliss and Terashima-Marín (2014) explored a standard genetic algorithm with positional encoding, showing promising results but also noting issues with scalability and convergence time.

Several enhancements have since been proposed to improve EA performance. Incremental and constructive approaches, similar to the one adopted in this report, have been studied in the context of constraint satisfaction problems to reduce infeasibility in early generations. These methods guide the evolutionary process through partial solutions and localized decisions, reducing the probability of producing invalid configurations.

Other studies have also examined hybrid algorithms that combine evolutionary search with local repair strategies to enhance solution feasibility. For instance, Zhang and Sun (2009) integrated local search heuristics into evolutionary frameworks to improve convergence. Additionally, diversity-preserving techniques such as tournament selection and adaptive mutation have been used to maintain population variety and avoid premature convergence.

Compared to these methods, the incremental evolutionary strategy used in this work emphasizes structure and guidance during the construction of candidate solutions, significantly reducing the effective search space. This approach distinguishes itself by balancing the global search capabilities of genetic algorithms with the precision of step-wise constraint evaluation, providing a scalable and efficient alternative to traditional N-Queens solvers.

Conclusion

The incremental evolutionary algorithm provided a promising and structured approach to solving the Eight-Queens problem. By building solutions one queen at a time and evaluating partial configurations through a fitness function, the algorithm consistently found optimal solutions in most test runs. This guided strategy significantly reduced the search space compared to full-board random configurations and enabled faster convergence, with many successful runs reaching the global optimum within the first 50 generations.

However, the results also highlighted several limitations. Some runs became trapped in local optima, and small mutation rates occasionally failed to introduce enough diversity to escape these traps. Additionally, while the incremental approach improved solution quality, it sometimes required many generations to make marginal progress, resulting in a relatively high computational cost.

These findings suggest that while the current implementation is effective, there is substantial room for enhancement. Future improvements could include adaptive mutation rates to dynamically respond to stagnation, stronger diversity preservation techniques to prevent premature convergence, and hybrid strategies that incorporate global search or local repair mechanisms. Implementing parallel processing could also improve efficiency by exploring multiple evolutionary paths simultaneously.

In summary, our approach successfully balances structure and flexibility in evolutionary search, and with further refinement, it has strong potential for application to more complex constraint satisfaction and optimization problems beyond the Eight-Queens domain.