CSC320

Introduction to Visual Computing

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Part I

Foundations

CHAPTER

IMAGE GEOMETRY 1

1.1 Linear 2D Transforms

1.1.1 Homogeneous 2D Point Coordinate

Basic Notational Conventions

- Column-Vector Representation

$$\begin{bmatrix} x \\ y \end{bmatrix}$$

- Row-Vector Representation

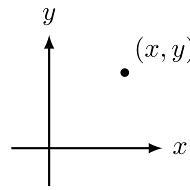
$$\begin{bmatrix} x & y \end{bmatrix}$$

- Matrix Transpose Operation

$$\begin{bmatrix} x \\ y \end{bmatrix}^\top = \begin{bmatrix} x & y \end{bmatrix}$$

Homogeneous Coordinates

In the standard Euclidean coordinate system, a point is represented by a pair of coordinates (x, y), where x and y are the coordinates of the point along the x and y axes, respectively.



In homogeneous coordinates, a point is represented by a triple of coordinates (x, y, w), where x and y are the coordinates of the point along the x and y axes, respectively, and w is a scaling factor that is normally set to 1. The homogeneous coordinates of a point are not unique, since any multiple of the coordinates (x, y, w) represents the same point. For example, $(2, 3, 1)$ and $(4, 6, 2)$ represent the same point. The homogeneous coordinates of a point are unique only up to a scaling factor.

Remark

Two vector of homogeneous coordinates (x, y, w) and (x', y', w') represent the same point if and only if there exists a non-zero scalar λ such that

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \lambda \begin{bmatrix} x' \\ y' \\ w' \end{bmatrix}$$

Example. The homogeneous coordinates of the point $(2, 3)$ are $(2, 3, 1)$, $(4, 6, 2)$, $(6, 9, 3)$, etc. ♢

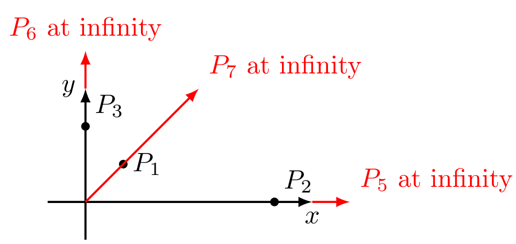
Homogeneous to Euclidean Coordinat

To convert a homogeneous coordinate (x, y, w) to a Euclidean coordinate (x', y'), we divide the first two coordinates of the homogeneous coordinate by the third coordinate, i.e.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x/w \\ y/w \end{bmatrix}$$

Example. Plot the following points in the Euclidean plane:

$$\begin{array}{lll} P_1 = (2, 2, 2) & P_2 = (10, 0, 2) & P_3 = (0, 8, 4) \\ P_4 = (1, 0, 0.01) & P_5 = (1, 0, 0) & P_6 = (0, 1, 0) \\ P_7 = (1, 1, 0) & & \end{array}$$



Note that P_5, P_6, and P_7 are at infinity. This is a very important property of homogeneous coordinates. ♢

Remark

Points infinitely far away from the origin in the Euclidean plane have a finite representation in homogeneous coordinates (i.e. $(x, y, 0)$). These points are sometimes called **ideal points**.

1.1.2 Homogeneous 2D Line Coordinates

Homogeneous 2D Line Coordinates

Remark

The general equation of a line in the Euclidean plane is

$$ax + by + c = 0$$

where a, b, and c are real numbers and a and b are not both zero.
In homogeneous coordinates, using the matrix notation, the general equation of a line is

$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix} = 0$$

Or, equivalently,

$$\ell^\top p = 0$$

where ℓ is the vector holding line coefficients and p is the vector holding point coordinates.

Definition 1.1.1

The **homogeneous coordinates of a line** with the equation

$$ax + by + c = 0$$

is the vector

$$\ell = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Example. The homogeneous coordinates of the line $y = x$ is

$$\ell = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$\diamondsuit$

Line Passing Through Two Points

In this case, ℓ must satisfy the following equations:

$$\ell^\top P_1 = 0 \qquad \ell^\top P_2 = 0$$

where p_1 and p_2 are the homogeneous coordinates of the two points.

We can take the cross product of p_1 and p_2 to obtain ℓ:

$$\ell = P_1 \times P_2$$

> **Remark**
>
> To compute the cross product of two vectors, we can use the following two methods.
>
> $$\ell = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \times \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$$
>
> As matrix multiplication:
>
> $$P_1 \times P_2 = \begin{bmatrix} 0 & -z_1 & y_1 \\ z_1 & 0 & -x_1 \\ -y_1 & x_1 & 0 \end{bmatrix} P_2$$
>
> As determinant:
>
> $$P_1 \times P_2 = \begin{vmatrix} i & j & k \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

Intersection of Two Lines

In this case, p must satisfy the following equations:

$$\ell_1^\top p = 0 \qquad \ell_2^\top p = 0$$

where ℓ_1 and ℓ_2 are the homogeneous coordinates of the two lines.

We can take the cross product of ℓ_1 and ℓ_2 to obtain p:

$$p = \ell_1 \times \ell_2$$

> **Remark**
>
> For parallel lines, we cannot compute its standard Euclidean intersection point. Instead, we can compute the homogeneous intersection point, which will be at infinity.

1.1.3 Affine 2D Transformations

- **Identity**

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} \cong \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- **Stretching** (along x)

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} \cong \begin{bmatrix} sx \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Stretching (along y)

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} \cong \begin{bmatrix} x \\ sy \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- **Shearing** (along y)

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} \cong \begin{bmatrix} x \\ hx + y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ h & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- **Rotation** (about the origin bt angle ϕ)

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} \cong \begin{bmatrix} (\cos\phi)x - (\sin\phi)y \\ (\sin\phi)x + (\cos\phi)y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- **Translation** (along x)

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} \cong \begin{bmatrix} x + d \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & d \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translation (along y)

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} \cong \begin{bmatrix} x \\ y + d \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & d \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

We can combine these transformations to form more complex transformations. For example, we can combine a rotation and a translation to form a **rotation about a point** transformation.

Definition 1.1.2 Affine Transformation (Intuitive)

An **affine transformation** is any combination of scaling, shearing, rotation, and translation.

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & g \end{bmatrix}$$

where $g \neq 0$ is a scaling factor that does not affect the transformation.

Remark

An affine transformation is a transformation that preserves parallelism and ratios of distances along parallel lines.

In the original image, two parallel lines would intersect at some infinity $(x, y, 0)$. After an affine transformation, the two lines would still intersect at some infinity $(x', y', 0)$.

$$\begin{bmatrix} x' \\ y' \\ 0 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & g \end{bmatrix} \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

Definition 1.1.3 Affine Transformation

An affine transformation is any invertible 3×3 matrix that preserves the points at infinity.

But what if the last row of the matrix is not $(0, 0, 1)$?

1.1.4 Homographies

Homographies are also called **projective transformations** or **perspective transformations**. They still preserve linearity, but they do not preserve parallelism.

Definition 1.1.4

A **homography** is any 2D transformation of homogeneous coordinates that is represented by an invertible 3×3 matrix.

$$\begin{bmatrix} a & b & c \\ d & e & f \\ l & m & g \end{bmatrix}$$

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MODEL FITTING

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COLOR IMAGING AND DISPLAYING 4

Part II

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Part III

Appendices

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