Complex Powers

We define $z^w = \exp(w \log z)$. Note that

$$\begin{aligned} \log z &= \operatorname{Log} z + 2\pi i k \\ &= \ln|z| + i(\operatorname{Arg}(z) + 2\pi k),\ k \in \mathbb{Z} \end{aligned}$$

is a multi-valued function.
Click here for the definition

Functional Limit

Let $D \subseteq \mathbb{C}$ and $f : D \to \mathbb{C}$ be a function. Let a be an accumulation point of D. Let $\ell \in \mathbb{C}$. We say that ℓ is **the limit of** $f(z)$ as z approaches to a if for all $\varepsilon > 0$, there is $\delta > 0$ such that for all $z \in D$ with $0 < |z - a| < \delta$, we have

$$|f(z) - \ell| < \varepsilon;$$

that is, $z \in D \cap (B(a, \delta) \setminus \{a\})$ implies $f(z) \in B(\ell, \varepsilon)$.
Click here for the definition

Limits at Infinity

(i) Let $f : D \to \mathbb{C}$ be a function and $a \in \mathbb{C}$ be an accumulation point of D. Then we say $\lim\limits_{z \to a} f(z) = \infty$ if for all $M > 0$, there exists $\delta > 0$ such that

$$z \in D \cap (B(a, \delta) \setminus \{a\})$$

implies $|f(z)| \geq M$; that is, f is unbounded as z approaches to a.

(ii) Let f be a complex function defined on the complement of a ball in $\mathbb{C}$. We say $\lim\limits_{z \to \infty} f(z) = \ell$ if for all $\varepsilon > 0$, there exists $R > 0$ such that $|z| > R$ implies

$$|f(z) - \ell| < \varepsilon.$$

Click here for the definition

Complex Continuity

Let $D \subseteq \mathbb{C}$ and $f : D \to \mathbb{C}$ be a function. We say f **is continuous at** $a \in D$ if for all $\varepsilon > 0$, there exists $\delta > 0$ such that for all $z \in B(a, \delta) \cap D$, we have $f(z) \in B(f(a), \varepsilon)$.
Click here for the definition