Operations on $\mathbb{C}$

Let $z, w \in \mathbb{C}$ and set $z = \alpha_1 + i\beta_1$ and $w = \alpha_2 + i\beta_2$ for any $\alpha_1, \alpha_2, \beta_1, \beta_2 \in \mathbb{R}$. We define the two operations, addition $+$ and multiplication $\cdot$, in the following way:

- Addition:

$$\begin{aligned} z + w &= (\alpha_1 + i\beta_1) + (\alpha_2 + i\beta_2) \\ &= (\alpha_1 + \alpha_2) + i(\beta_1 + \beta_2). \end{aligned}$$

- Multiplication:

$$\begin{aligned} z \cdot w &= (\alpha_1 + i\beta_1) \cdot (\alpha_2 + i\beta_2) \\ &= (\alpha_1\alpha_2 - \beta_1\beta_2) + i(\alpha_1\beta_2 + \beta_1\alpha_2). \end{aligned}$$

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Constructing a Solution for $x^2 + 1 = 0$

Define $i = (0, 1)$ as our imaginary number in $\mathbb{C}$ and let $i^2 = (-1, 0)$.

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Let $z = \alpha + i\beta$ for $\alpha, \beta \in \mathbb{R}$. Then

$$\Re(z) = \alpha \text{ and } \Im(z) = \beta,$$

are the **real and imaginary of** z, respectively. If $\Im(z) = 0$, z is a real number, and if $\Re(z) = 0$, then we call z **purely imaginary**.

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Complex Conjugate

Let $z = \alpha + i\beta$ be a complex number. Its complex conjugate is defined as $\overline{z} = \alpha - i\beta$.

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Modulus of Complex Number

Let $z \in \mathbb{C}$. We define the **modulus** $|z| = \sqrt{z\overline{z}}$.

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Metric Space

For $z, w \in \mathbb{C}$, we call $\mathbb{C}$ a metric space if there exists a function $d : \mathbb{C} \times \mathbb{C} \to \mathbb{R}$ that satisfies the following properties:

(i) For any $z, w \in \mathbb{C}$, we have $d(z, w) \geq 0$.

(ii) For any $z, w \in \mathbb{C}$, $d(z, w) = 0$ if and only if $z = w$.

(ii) For any $z, w \in \mathbb{C}$, we have $d(z, w) = d(w, z)$.

(iii) For any $z, w, u \in \mathbb{C}$, we have

$$d(z, w) \leq d(z, u) + d(u, w).$$