# Homework-2 – Math 45

**Problem-1:** Prove that $(\mathbb{R}^n, d_\infty)$ is complete.

**Problem-2:** (i) Prove that $\pi_i : \mathbb{R}^n \longrightarrow \mathbb{R}$

$$\pi_i\left(\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}\right) = x_i \quad (i^{th} \text{ coordinate})$$

are continuous maps with respect to $d_\infty$ on $\mathbb{R}^n$ and the standard metric on $\mathbb{R}$.

(ii) Prove that $\pi_i$ in (i) are continuous maps with respect to $d_{Euclid}$ on $\mathbb{R}^n$ and the standard metric on $\mathbb{R}$. (Hint: Think about using relation between $d_{Euclid}$, $d_\infty$ and (i))

**Problem-3:** (i) Define $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ by $d(x, y) = |e^x - e^y|$. Prove that $d$ is a metric on $\mathbb{R}$.

(ii) Prove or disprove: $(\mathbb{R}, d)$ is complete.

**Problem-4:** Let $X = \mathbb{N} = \{1, 2, 3, \dots, \dots\}$ – the set of positive integers.

(i) Let $d(m, n) = |m - n|$. Prove that $(X, d)$ is complete.

(ii) Let $d(m, n) = \left|\frac{1}{m} - \frac{1}{n}\right|$. Prove that $(X, d)$ is not complete.

**Problem-5:** Let $X = \{ f : [0,1] \to \mathbb{R} \,/\, f \text{ is continuous} \}$

standard metric

(i) Define $d(f,g) = \int_0^1 |f(t) - g(t)|\, dt$

Prove that $d$ is a metric on $X$.

( You can use the following fact without any proof:
Assume that $f : [0,1] \to \mathbb{R}$ continuous and $f(t) \neq 0$ for some $t \in [0,1]$, then $f(t) \neq 0$ on an interval $J$ such that $t \in J \subseteq [0,1]$. In fact, it follows from continuity, proof is not complicated)

(ii) Prove that $(X, d)$ is not complete.
( This is done in the book, please read it carefully and write in your own words).