1 Regular Surfaces; Inverse Images of Regular Values

Definition (Regular Surface)**.** A subset $S \subseteq \mathbb{R}^3$ is a **regular surface** if, for each $p \in S$, there exists a neighborhood V in $\mathbb{R}^3$ and a map $\mathbf{x} : U \to V \cap S$ of an open set $U \subseteq \mathbb{R}^3$ onto $V \cap S \subseteq \mathbb{R}^3$ such that

(1) $\mathbf{x}$ is differentiable. This means that if we write

$$\mathbf{x}(u, v) = (x(u, v), y(u, v), z(u, v)), \quad (u, v) \in U,$$

the functions $x(u, v), y(u, v)$, and $z(u, v)$ have continuous partial derivatives of all orders in U.

(2) $\mathbf{x}$ is a homeomorphism. Since $\mathbf{x}$ is continuous by condition (1), it follows that $\mathbf{x}$ has an inverse $\mathbf{x}^{-1} : V \cap S \to U$ which is continuous.

(3) (Regularity Condition) For each $q \in U$, the differential $d\mathbf{x}_q : \mathbb{R}^2 \to \mathbb{R}^3$ is injective.

Definition (Parametrization)**.** The mapping $\mathbf{x}$ is called a **Parametrization** or **a system of (local) coordinates** in (a neighborhood of) p. The neighborhood $V \cap S$ of p is S is called a **coordinate neighborhood**.