Solutions

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1. $x^{10} \cdot x^{11} = x^{10+11} = x^{21}$.
2. $\frac{y^4}{y^7} = \frac{1}{y^3} = y^{-3}$.
3. $(2x^3)^4 = (2)^4(x^3)^4 = 16x^{12}$.
4. $\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$.
5. $|-12 \div -2| = \left|\frac{-12}{-2}\right| = |6| = 6$.
6. Following pemdas, we have

$$\begin{aligned} \frac{6 + 50 - 4^2}{100 \div (5 + 2^2 \times 5)} &= \frac{40}{\frac{100}{25}} \\ &= \frac{40}{4} \\ &= 10. \end{aligned}$$

7. Via distribution, we can write

$$y(z+7) - 2z(y-6) = yz + 7y - 2zy + 12z$$

8.

$$(4x^2)(9y)(2xy) = 72x^3y^2.$$

9. Simplifying we get

$$\frac{12gk^6n^4}{42g^3k^5n} = \frac{2kn^3}{7g^2} = \frac{2}{7}kn^3g^{-2}.$$

10. Simplifying we get

$$\begin{aligned}\frac{6a^{-5}b^4d^{-1}}{8a^{-3}b^{-2}c^{-2}} &= \frac{6}{8}(a^{-5-3}\cdot b^{4-2})\cdot(d^{-1}\cdot c^{-2})\\ &= \frac{6}{8}(a^{-8}\cdot b^2\cdot d^{-1}\cdot c^{-2})\\ &= \frac{6b^2}{8da^8c^2}.\end{aligned}$$

11. If $a = 3, b = -2$,and $c = 1/3$, then

$$\begin{aligned}12c^2 + 2a(-3b-1) &= 12(1/3)^2 + 2\cdot 3(-3(-2)-1)\\ &= \frac{12}{9} + 6(6-1)\\ &= \frac{12}{9} + 30\\ &= \frac{12+30\cdot 9}{9}\\ &= \frac{12+270}{9}\\ &= \frac{282}{9}\\ &= 31\frac{1}{3} && (\Leftarrow \text{mixed fraction!})\end{aligned}$$

12. Multiplying we get

$$(x+4)(x-7) = x^2 + 4x - 7x - 28 = x^2 - 3x - 28.$$

13. Observe that

$$(3x-5)(4x+2) = 12x^2 - 20x + 6x - 10 = 12x^2 - 14x - 10.$$

We can further simplify by factoring 2

$$12x^2 - 14x - 10 = 2(6x^2 - 7x - 5).$$

14. Observe that

$$\begin{aligned}(y^2-4y+2)(y-3) &= y^3 - 4y^2 + 2y - 3y^2 + 12y - 6\\ &= y^3 - 7y^2 + 14y - 6.\end{aligned}$$

Hence,

$$(y^2-4y+2)(y-3) = y^3 - 7y^2 + 14y - 6.$$

15. What is the **Greatest Common Factor (GCF)** of $16x^2y^3$ and $20x^3y$?

Solution. The **Greatest Common Factor** is $\text{GCF}(16x^2y^3, 20x^3y) = 4x^2y$ ■

16. What is the **Least Common Multiple (LCM)** of $6x^2y$ and $8x^2y$?

    Solution. The **Least Common Multiple (LCM)** is

    $$\text{LCM}(6x^2y, 8x^2y) = \frac{|6x^2y \cdot 8x^2y|}{\text{GCF}(16x^2y^3, 20x^3y)} = \frac{6x^2y \cdot 8x^2y}{4x^2y} = 12x^2y.$$

    ■

Simplify. Assume all variables are positive.

17. $\sqrt{75} + \sqrt{48}$.

    Solution. We can express 75 and 28 in terms of their prime factorizations. Hence, $75 = 5^2 \cdot 3$ and $48 = 4^2 \cdot 3$. Therefore, we can write

    $$\begin{aligned} \sqrt{75} + \sqrt{48} &= \sqrt{5^2 \cdot 3} + \sqrt{4^2 \cdot 3} \\ &= (5+4)\sqrt{3} \\ &= 9\sqrt{3}. \end{aligned}$$

    Hence,

    $$\sqrt{75} + \sqrt{48} = 9\sqrt{3}.$$

    ■

18. $\sqrt{25a^2b^5}$.

    Solution. Using our square root properties, we have

    $$\sqrt{25a^2b^5} = 5ab^{5/2}.$$

    ■

19. $\sqrt{14} \cdot \sqrt{2}$.

    Solution. Observe that

    $$\begin{aligned} \sqrt{14} \cdot \sqrt{2} &= \sqrt{2 \cdot 7} \cdot \sqrt{2} \\ &= \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{7} \\ &= 2\sqrt{7}. \end{aligned}$$

    ■

20. $(x^6)^{1/2}$.

    Solution. $(x^6)^{1/2} = x^{6/2} = x^3$. ■

Solve the following equations.

21. $3k - 18 = -9k + 30$.

Solution. Solving for k, we write

$$\begin{aligned} 3k - 18 &= -9k + 30 \\ \implies 12k &= 48 \\ \implies k &= \frac{48}{12} \\ \implies k &= 4. \end{aligned}$$

■

22. $x^2 = 81$.

    Solution. Solving for x by taking the square root, we get $x = \pm\sqrt{81} = \pm 9$. ■

23. $-5(2x - 1) = 15$.

    Solution. Suppose $-5(2x - 1) = 15$. Solving for x, we get

$$\begin{aligned} -5(2x - 1) &= 15 \\ -10x + 5 &= 15 \\ -10x &= 10 \\ x &= -1. \end{aligned}$$

■

24. $|-2x + 5| = 7$.

    Solution. By definition of $|\cdot|$, we have

$$\begin{aligned} |-2x + 5| &= 7 \\ -2x + 5 &= \pm 7. \end{aligned}$$

Here we have two cases, either $-2x + 5 = 7$ or $-2x + 5 = -7$. If $-2x + 5 = 7$, then we have

$$-2x + 5 = 7 \implies x = \frac{7 - 5}{-2} = -\frac{2}{2} = -1.$$

If $-2x + 5 = -7$, then we have

$$-2x + 5 = -7 \implies x = \frac{-7 - 5}{-2} = \frac{-12}{-2} = 6.$$

■

25. Find the value of the variable

$$\frac{5}{7} = \frac{H}{6}.$$

Solution. We have

$$\begin{aligned} \frac{5}{7} &= \frac{H}{6} \\ 7H &= 30 \\ \implies H &= \frac{30}{7}. \end{aligned}$$

■

Use the following expressions to answer the questions below.

$$3x \quad 5 - 2ab \quad 7z^2 + 6xyz - 9 \quad 2x^2 - 3x^5 + 1 \quad x^4.$$

26. Which expressions above are *monomials*?

    Solution. $3x$ and x^4 are *monomials.* ■

27. Which expressions above are *binomials*?

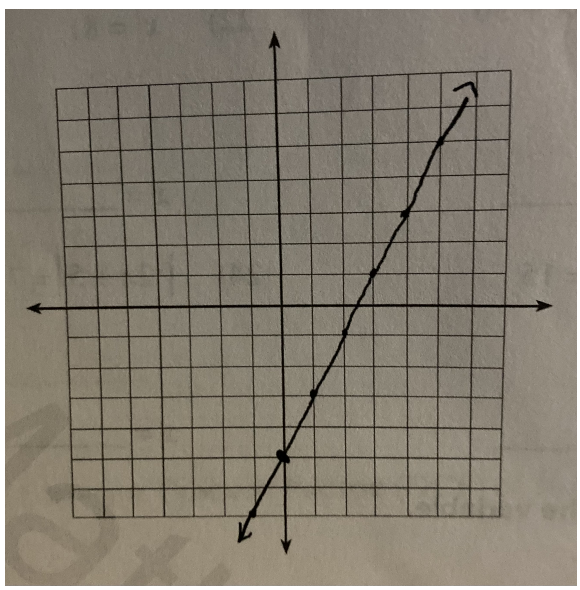
    Solution. Only $5 - 2ab$ is a *binomial.* ■

28. Determine the polynomial with the highest degree. Write it in standard form and determine the degree and leading coefficient.

    Solution. The polynomial with the highest degree is $2x^2 - 3x^5 + 1$ which is a 5th degree polynomial and its standard form is $-3x^5 + 2x^2 + 1$ with a leading coefficient of -3. ■

29. Graph the equation $y = 2x - 5$.

    Solution. Graphing the equation above looks like



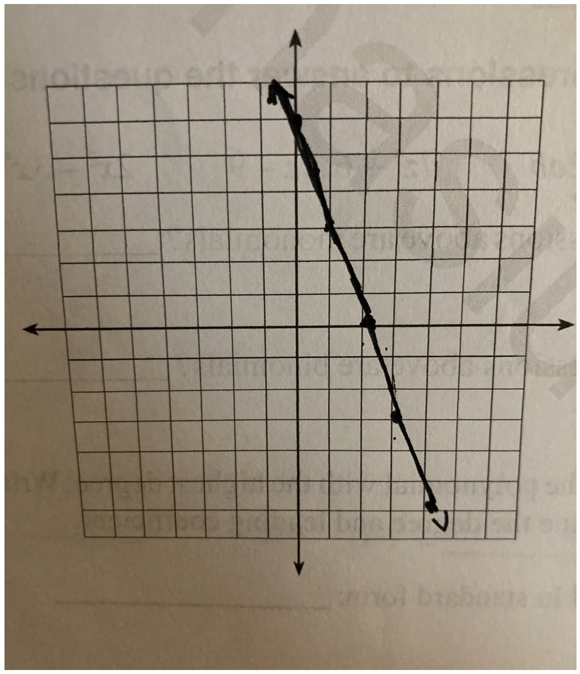
■

30. Graph the equation $6x + 2y = 12$.

    Solution. The equation can be re-written in standard form as

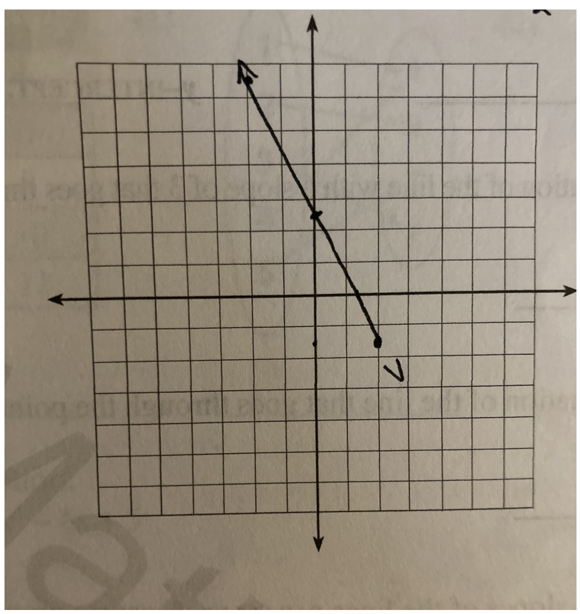
$$y = \frac{1}{2}(12 - 6x) = 6 - 3x$$

which is graphed as follows ■

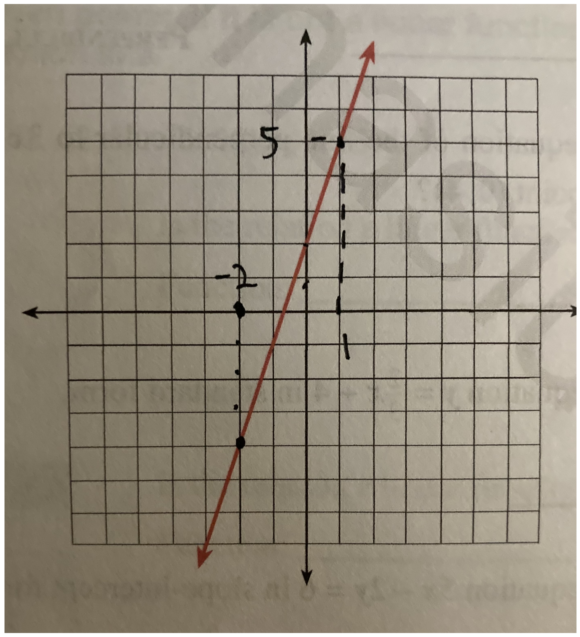


31. Graph the equation $y - 5 = -\frac{5}{2}(x+1)$.

Solution. The equation in slope-intercept form is $y = \frac{-5}{2}x + \frac{5}{2}$. and the graph is ■



32. What is the equation of the given line in slope-intercept form?



Solution. Suppose we start with the slope-intercept form for a line $y = mx+b$. Pick the point $(1, 5)$ from the graph. Since the $y-$ intercept is $b = 2$. We can solve for m to get the slope of the line. Hence,

$$5 = m + 2 \iff m = 3$$

so the equation of the line is

$$y = 3x + 2.$$

■

33. Determine the slope and the coordinate of the $y-$intercept of the line given by the equation $y = 4x - 7$.

    Solution. The *slope* of $y = 4x-7$ is $m = 4$ and the *coordinate* of the $y-$intercept is $(0, -7)$. ■

34. Write the equation of the line with a slope of 3 that goes through the point $(2, 1)$.

    Solution. Using the slope-intercept form $y = mx + b$ where $m = 3$ is the slope, we can write

$$y = 3x + b. \tag{1}$$

We can solve for b to get the $y-$intercept by substituting the coordinate pair $(2, 1)$ into (1) by writing

$$1 = 3(2) + b \iff b = 1 - 6 = -5$$

Hence, we have the linear equation

$$y = 3x - 5.$$

■

35. Write the equation of the line that goes through the points $(1, 2)$ and $(3, -5)$.

Solution. We can take the average rate of change $y_2 - y_1 = m(x_2 - x_1)$ to find the slope m of the linear equation

$$y = mx + b. \tag{1}$$

Plugging the points $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (3, -5)$ and solving for m, we get

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 2}{3 - 1} = \frac{-7}{2}.$$

We can take the $y-$intercept of (1) by plugging in either of the points above. We choose $(1, 2)$ and solving for b in (1), we have

$$2 = \frac{-7}{2}x + b \iff b = 2 + \frac{7}{2} = \frac{11}{2}.$$

Therefore, we have

$$y = \frac{-7}{2}x + \frac{11}{2}.$$

■

36. What are the slopes of the lines *parallel* and *perpendicular* to

$$y = 2x - 3. \tag{1}$$

Solution. The slope of lines that are parallel to (1) is $m = 2$ and slope of lines that are *perpendicular* to (1) is $m = -1/2$. ■

37. What is the equation of the line perpendicular to

$$3x + 5y = 15 \tag{1}$$

that goes through the point $(0, 4)$?

Solution. First we put (1) in slope-intercept form by solving for y. Observe that

$$\begin{aligned} &3x + 5y = 15 \\ &\implies y = \frac{1}{5}(15 - 3x) \\ &\implies y = 3 - \frac{3}{5}x. \end{aligned}$$

Since the slope of (1) is $m = -3/5$, we must have that the slope of a line perpendicular to it is $m' = 5/3$. Since this perpendicular line goes through $(0, 4)$, we just have

$$y = \frac{5}{3}x + 4.$$

■

38. Rewrite the equation

$$y = \frac{2}{3}x + 4 \tag{1}$$

in standard form.

Solution. Our goal is to write (1) into the following form

$$ax + by = d.$$

Hence, (1) becomes

$$3y = 2x + 12 \iff 3y - 2x = 12.$$

Therefore, the standard form of (1) is

$$-2x + 3y = 12.$$

■

39. Rewrite the equation

$$5x - 2y = 6 \tag{1}$$

in slope-intercept form.

Solution. To put (1) into slope-intercept form, we solve (1) for y in terms of x. Hence, we have

$$5x - 2y = 6 \iff y = -\frac{1}{2}(6 - 5x) = \frac{5}{2}x - 3.$$

Hence, the slope-intercept form of

$$y = \frac{5}{2}x - 3.$$

■

Determine if the relation is a function.

40. This is **NOT** a function.
41. This is a function.
42. This is **NOT** a function.
43. Evaluate the function

$$f(x) = 3x^2 - 4x + 5$$

at $x = 2$.

Solution.

$$f(2) = 3(2)^2 - 4(2) + 5 = 12 - 8 + 5 = 9.$$

So $f(2) = 9$. ■

Determine whether the relation is a linear function. If it is, determine the linear function from the given points. If it is not a linear function, write "not a linear function" on the function line.

44. $\{(0, 3), (1, 6), (2, 9), (3, 12), (4, 15)\}$

Solution. Yes, this is a linear function and the function is defined as follows:

$$y = 3x + 3$$

■

45. *Solution.* This is **NOT** a linear function. ■

Determine the domain and range for the relations.

46. $(2, 3), (4, 5), (6, 7), (8, 9)$

Solution. The domain is $\mathcal{D} = \{x \in \mathbb{R} : 2 \leq x \leq 8\}$ and the range is $\mathcal{R} = \{y \in \mathbb{R} : 3 \leq y \leq 9\}$. ■

47. *Solution.* The domain is $\mathcal{D} = (-\infty, 0]$ and the range is $\mathcal{R} = (-\infty, +\infty)$. ■

48. The amount of water left in a 30 liter tank can be modeled by the equation

$$w = -\frac{1}{2}t + 30,$$

where w is the *amount of water left in the tank* and t is *time in hours* since the water in started leaking. Water leaks from the tank until the tank is empty. Determine the domain and range of this scenario.

Solution. The domain in this scenario is $\mathcal{D} = \{t \in \mathbb{R} : 0 \leq t \leq 60\}$ where t is in hours and the range is $\mathcal{R} = \{w \in \mathbb{R} : 0 \leq w \leq 30\}$ where w is in liters. ■

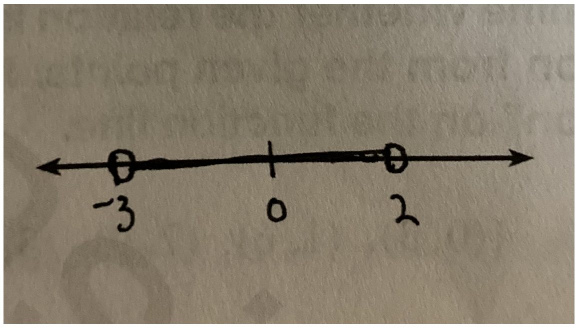
Graph the solutions.

49. $-1 < 2x + 5 < 9$

Solution. First we solve for x. So

$$\begin{aligned} &-1 < 2x + 5 < 9 \\ \implies &-6 < 2x < 4 \\ \implies &-3 < x < 2 \end{aligned}$$

So graphing this on would look like ■



50. $4x + 7 \leq -13$ or $5x + 4 \geq 14$.

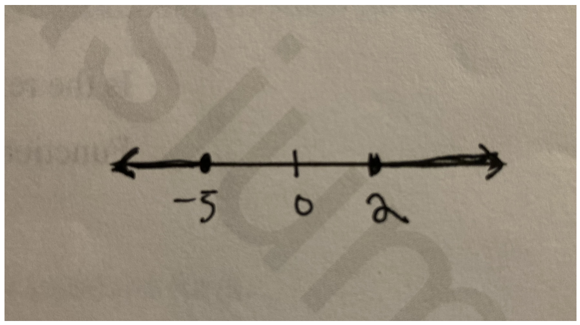
Solution. Again, we solve for x for $4x+7 \leq -13$ and $5x+4 \geq 14$. Starting with the former, we have

$$4x+7 \leq -13 \iff x \leq \frac{-20}{4} = -5.$$

For the latter, we have

$$5x+4 \geq 14 \iff x \geq \frac{10}{5} = 2.$$

Hence, we have $x \leq -5$ or $x \geq 2$. Graphing this looks like



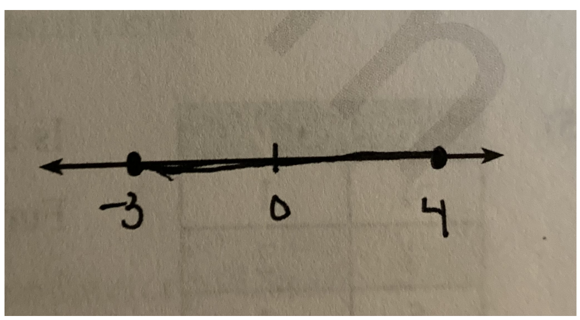
■

51. $|8x-4| \leq 28$

Solution. By definition of absolute value, we have

$$\begin{aligned}
|8x-4| &\leq 28 \\
-28 \leq 8x-4 &\leq 28 \\
-24 \leq 8x &\leq 32 \\
\iff -3 \leq x &\leq 4. \qquad (1)
\end{aligned}$$

Hence, graphing (1) leads to



■

52. The sum of three consecutive odd integers is 195. What are the three integers?

Solution. Let $x \in \mathbb{R}$, then the sum of three consecutive odd numbers can be written as

$$x_1 + x_2 + x_3 = (2k+1) + (2k+3) + (2k+5) = 195. \qquad (1)$$

This implies that

$$6k + 9 = 195 \iff k = \frac{195 - 9}{6} = 31.$$

Plugging this back into (1), yields the following consecutive odd integers

$$63, 65, 67.$$

■

53. Deandra has \$15 more than Elias. Frank has 3 times as much as Deandra. Together they have \$185. How much does each person have?

Solution. Let D, x, F be variables that stand for Deandra, Elias, and Frank respectively. By the problem statement, we have the following equations

$$\begin{aligned} D &= x + 15, \\ F &= 3D = 3(x + 15). \end{aligned}$$

Their total sum is $D + F = 4(x + 15) = \$185$. Since the amount Elias has is unknown, let's solve for x. Hence, we have

$$4(x + 15) = 185 \iff x = \frac{185}{4} - 15 = \$31.25.$$

So Elias has \$31.25, Deandra has

$$D = 31.25 + 15 = \$46.25$$

and Frank has

$$E = 3D = 3(46.25) = \$138.75.$$

■

54. If y varies inversely with x and $y = 2$ when $x = 5$, what is the value of x when $y = 3$?

Solution. Since y varies inversely with x, we have

$$y = \frac{k}{x}. \tag{1}$$

where $k \in \mathbb{R}$ is the constant of proportionality. First we need to find $k \in \mathbb{R}$. Since $y = 2$ when $x = 5$, solving for k yields

$$k = y \cdot x = 2 \cdot 5 = 10.$$

If $y = 3$, the value of x is simply

$$x = \frac{10}{3}.$$

■

Karen can complete 12 coloring book pages in 30 minutes.

55. What is the constant of proportionality in the scenario?

Solution. If Karen can complete 12 coloring book pages in 30 minutes, then the constant of proportionality is

$$k = \frac{12}{30} = \frac{2 \text{ coloring books}}{5 \text{ minutes}}$$

■

56. Write a direct variation equation that represents the scenario.

    Solution. Let $x, y \in \mathbb{R}$ and k constant such that x represents time in minutes and y represents the amount of completed coloring books. The situation can be modeled by the equation:

$$y = \frac{2}{5}x$$

■

57. How long will it take Karen to complete a 96-page coloring book?

    Solution. If we solve for x in the above equation with $y = 96$ coloring books, then the amount of time in minutes is

$$x = \frac{5}{2} \cdot 96 = 240 \text{ min.}$$

■

58. 5 less than a number.

    Solution. This can be written as $5 < x$ where $x \in \mathbb{R}$ ■

59. 7 more than the quotient of x and 3 is 4.

    Solution. This can written as $7 + \frac{x}{3} = 4$ where $x \in \mathbb{R}$. ■

    Write an equation that models the scenario.

60. The price of a pizza costs \$15 plus \$3 for each topping.

    Solution. Let $x \in \mathbb{R}$ represent the amount of toppings and let $y \in \mathbb{R}$ represent the price of the pizza in dollars where $y(0) = \$15$ is the initial value of the pizza. The equation for this scenario is

$$y = 3x + 15.$$

■

    Solve for F.

61. $2H - 3F = 4J + 8K$.

    Solution. Solving for F, we have

$$F = \frac{1}{3}(2H - 4J - 8K).$$

■

62. $\frac{5}{9}(F - 32) = C$.

Solution. Solving for F, we have

$$\frac{5}{9}(F-32) = C$$
$$\implies F - 32 = \frac{9}{5}C$$
$$\implies F = \frac{9}{5}C + 32.$$

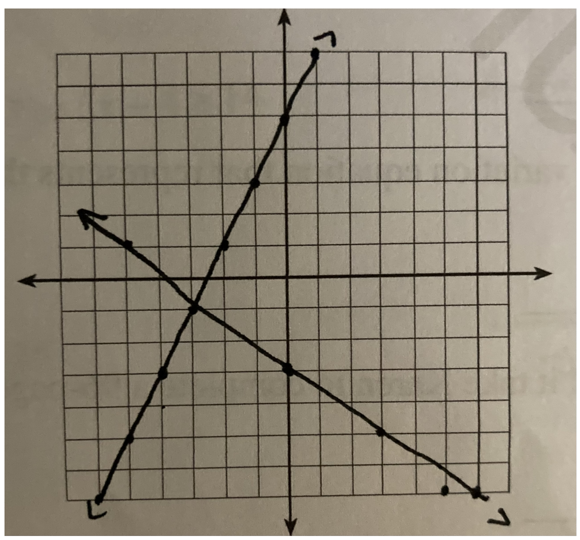
■

63. Find the solution to the system of equations

$$y = -\frac{2}{3}x - 3 \quad (1)$$
$$y = 2x + 5. \quad (2)$$

by graphing. Use a straightedge.

Solution. The solution is $(x, y) = (-3, -1)$ from the graph below:



■

Solve the following system of equations.

64.

$$y = -3x + 13$$
$$4x + 3y = 29.$$

Solution. Rewriting this system yields

$$3x + y = 13 \quad (1)$$
$$4x + 3y = 29. \quad (2)$$

We can use elimination to to solve for x. Hence, multiply (1) by -3 which yields

$$-3(3x+y) = -3\cdot 13 \iff -9x-3y=-39. \tag{3}$$

Then we can substract (3) and (2) to get

$$-5x=-10 \iff x=2.$$

To find y, we can plug in x to either (1) and (2) above. Suppose we solve for y in (1), then we get

$$y=-3(2)+13=7.$$

Hence, the solution for the system of equations above is $(x,y)=(2,7)$. ■

65.

$$\begin{align} 2x+3y &= 8 \tag{1}\\ 5x-6y &= -7. \tag{2}\end{align}$$

Solution. Multiplying (1) by 2 and adding the result to (2), we get

$$9x=9 \iff x=1.$$

We can plug in $x=1$ into (1) and solve for y by writing

$$\begin{aligned} &2x+3y=8\\ \implies\ & y=\frac{1}{3}(8-2x)\\ \implies\ & y=\frac{1}{3}(8-2)\\ \implies\ & y=2.\end{aligned}$$

Hence, the solution to the system of equations above is $(x,y)=(1,2)$. ■

66. Three T-shirts and four dresses cost \$125. Two T-shirts and three dresses cost \$90$-$. How much does one dress cost, and how much does one T-shirt cost?

Solution. Let $x,y\in\mathbb{R}$ represent T-shirts and dresses respectively. Translating the first sentence and second sentence gives the following system of equations

$$\begin{align} 3x+4y &= 125 \tag{3}\\ 2x+3y &= 90. \tag{4}\end{align}$$

Let us multiply (1) and (2) by 2 and -3 respectively and then add them together

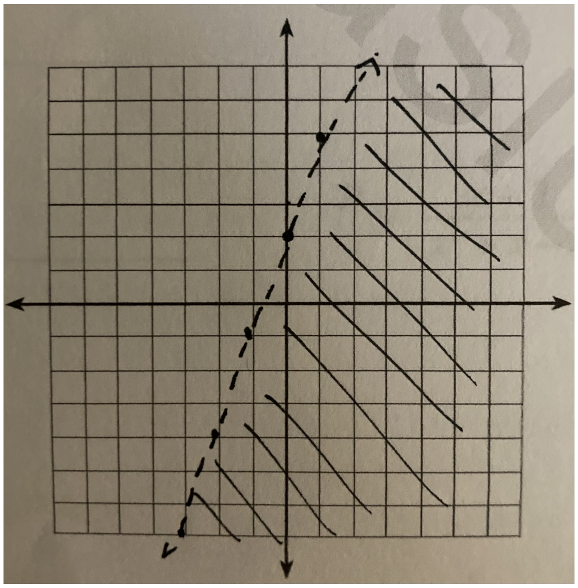
$$\begin{aligned} 2(3x+4y)-3(2x+3y) &= 250-270\\ 6x+8y-6x-9y &= -20\\ 8y-9y &= -20\end{aligned}$$

which implies $y = 20$. Now we solve (1) for x to get

$$x = \frac{1}{3}(125 - 4y) = \frac{1}{3}(125 - 4(20)) = 15.$$

Hence, $x = 15$. **This means that each T-shirt costs \$15 and each dress costs \$20.** ■

67. Graph the inequality $y < 3x + 2$.



today I will be proving the p-series test using the Cauchy Condensation test.

Proof. Let $p > 1$ and define the p-series as the following

$$\sum_{n=1}^{\infty} \frac{1}{n^p}.$$

Note that $b_n = 1/n^p$ is decreasing and $b_n \geq 0$. We can use the Cauchy condensation test to prove the analogous series

$$\sum_{n=1}^{\infty} 2^n \left(\frac{1}{2^n}\right)^p.$$

Since $p > 1$, this is equivalent to

$$\sum_{n=1}^{\infty} 2^n \left(\frac{1}{2^n}\right)^{p-1} = \sum_{n=1}^{\infty} \left(\frac{1}{2^p}\right)^n.$$

But notice that this is a *geometric series* since $p > 1$ and $|r| = 1/2^p < 1$. Hence, the p-series converges.

■

Suppose we try and prove

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}[m\vec{v}] = \frac{d\vec{p}}{dt}.$$

$$\begin{aligned} |x - y| &= |x - x_n + x_n - y| \\ &\leq |x - x_n| + |x_n - y| \\ &< \frac{\epsilon}{2} + \frac{\epsilon}{2} \\ &= \epsilon. \end{aligned}$$

Hence, the uniqueness of limits is proven. I like writing latex because the typesetting is very beautiful. This is to show how fast this auto preview is at compiling latex docs. I can say that I will only be using this when solving exercises for textbooks. Absolute convergence

$$\begin{aligned} |t_n - t_m| &= \Big| \sum_{k=m+1}^{n} a_k \Big| \\ &\leq \sum_{k=m+1}^{n} |a_k| \\ &< \epsilon. \end{aligned}$$

Prove that $\sup(A + B) = \sup A + \sup B$.

Proof. By the Axiom of Completeness, the sets A, B are non empty and *bounded above.* Hence, $\sup A, \sup B$ exists and hence $\sup(A+B)$ exists. Our goal is to show that $\sup(A+B) \leq \sup A + \sup B$ and $\sup(A+B) \geq \sup A + \sup B$.
Our first step is to show that $\sup(A+B) \leq \sup A + \sup B$. Let $a+b \in A+B$. By definition of $\sup(A+B)$, we have that $a+b \leq \sup(A+B)$. Suppose we add $a \in A$ to both sides of this inequality. Hence, we have

$$b \leq \sup(A+B) - a.$$

Since $b \leq \sup B$ for all $b \in B$, we have that

$$\sup B \leq \sup(A+B) - a.$$

Hence, we have

$$a \leq \sup(A+B) - \sup B.$$

Again, we have $a \leq \sup A$ for all $a \in A$. Hence,

$$\sup A \leq \sup(A+B) - \sup B.$$

Hence,

$$\sup A + \sup B \leq \sup(A+B)$$

Now to show that $\sup A + \sup B \geq \sup(A+B)$. By lemma 1.3.8, let $\epsilon > 0$. There exist $\alpha \in A$ and $\beta \in B$ such that

$$\sup A - \frac{\epsilon}{2} \leq \alpha$$
$$\sup B - \frac{\epsilon}{2} \leq \beta.$$

Adding these two inequalities together, we have

$$(\sup A + \sup B) - \epsilon \leq \alpha + \beta$$

Since $\epsilon > 0$ is arbitrary, we have that $\sup(A+B) \geq \sup A + \sup B$. ■

Suppose we have a sequence (a_n) such that $(a_n) \to 0$ and we have a bounded sequence (b_n). Show that $(a_n b_n) \to 0$.

Proof. Let $\epsilon > 0$. Choose $N \in \mathbb{N}$. Assume $n \geq N$ and for some $M > 0$, we have

$$\begin{aligned} |a_n b_n - 0| &= |a_n b_n| \\ &= |a_n||b_n| \\ &< \frac{\epsilon}{M} \cdot M && ((b_n) \text{ is bounded}) \\ &= \epsilon. \end{aligned}$$

Hence, $(a_n b_n) \to 0$. ■

Assume $(x_n) \to x$. Show that $\sqrt{x_n} \to \sqrt{x}$ where $x_n \geq 0$ for all $n \in \mathbb{N}$.

Proof. Let $\epsilon > 0$. Then

$$\begin{aligned} |\sqrt{x_n} - \sqrt{x}| &= \Big| \frac{x_n - x}{\sqrt{x_n} + \sqrt{x}} \Big| \\ &= \frac{|x_n - x|}{|\sqrt{x_n} + \sqrt{x}|} \\ &= \end{aligned}$$

■