Definition 0.0.1 (Separated).

- Two subsets A and B of a metric space X are said to be *separated* if both $A \cap \overline{B}$ and $\overline{A} \cap B$ are empty. That is, if no point of A lies in the closure of B and no point of B lies in the closure of A.
- A set $E \subset X$ is said to be *connected* if E is *not* a union of two nonempty separated sets.

Remark. Separated sets are disjoint, but disjoint sets are not always separated. For example, suppose we have two sets $[0, 1]$ and $(1, 2)$. They are both disjoint since 1 is not an element of both sets. But if we take the closure of $(1, 2)$, then the intersection with $[0, 1]$ is nonempty.

Theorem 0.0.1. A subset of E of the real line $\mathbb{R}^1$ is connected if and only if it has the following property:

$$\text{If } x \in E, y \in E, \text{ and } x < z < y, \text{ then } z \in E.$$