Polar Representation of Complex Numbers

Let $(\alpha, \beta) \in \mathbb{R}^2$. The polar representation of (α, β) is

$$(\alpha, \beta) = \gamma(\cos\varphi, \sin\varphi)$$

with $\tan\varphi = \frac{\beta}{\alpha}$. Note that if $\psi = 2\pi + \varphi$, then

$$(\alpha, \beta) = \gamma(\cos\psi, \sin\psi)$$

where γ is uniquely defined and φ is defined up to the addition of a multiple of 2π.

Click here for the definition

Argument and Principle Argument

Let $z \in \mathbb{C}^{\cdot}$ and $z = \gamma(\cos\varphi + i\sin\varphi)$ be a polar representation of z. Then φ is called **an argument of** z. If $-\pi < \varphi \leq \pi$, then φ is called **the principal argument of** z and it is denoted by $\mathrm{Arg}(z)$. **Click here for the definition**

Convergence of Sequences

Let $\{z_n\}_{n=1}^{\infty}$ be a sequence in $\mathbb{C}$. We say that $\{z_n\}$ converges to $z \in \mathbb{C}$ if for all $\varepsilon > 0$, we can find $N_\varepsilon \in \mathbb{N}$ such that

$$|z_n - z| < \varepsilon$$

for all $n \geq N_\varepsilon$.
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Convergence of Series

Let $\{z_n\}_{n=1}^{\infty}$ be a sequence in $\mathbb{C}$. Define

$$s_k = \sum_{k=1}^{n} z_k = z_1 + \cdots + z_n$$

where (s_n) is called the **sequence of partial sums** of $\{z_n\}$. If $s_n \to s$, then we say that the series $\sum_{n=1}^{\infty} z_n$ converges and write $\sum_{n=1}^{\infty} z_n = s$.
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Absolute Convergence of Infinite Series

Let (z_n) be a sequence of complex numbers. We say that the series $\sum_{n=1}^{\infty} z_n$ converges absolutely if the series of real numbers

$$\sum_{n=1}^{\infty} |z_n|$$

converges.
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Complex Version of Exponential Function

For $z \in \mathbb{C}$, we define

$$\exp(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!}$$

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Sine and Cosine Series

We define $\sin z$ as

$$\sin z = \sum_{n=0}^{\infty} (-1)^{2n+1} \frac{z^{2n+1}}{(2n+1)!}$$

and $\cos z$ as

$$\cos z = \sum_{n=0}^{\infty} (-1)^{n} \frac{z^{2n}}{(2n)!}.$$

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