Math 230A: Homework 4

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1. Let A and B be subsets of a metric space (X, d).

   (a) If $A \subseteq B$, then $A' \subseteq B'$.

   Proof. ■

   (b) If $A \subseteq B$, then $\overline{A} \subseteq \overline{B}$.

   Proof. ■

   (c) Prove that $\overline{A \cap B} \subseteq \overline{A} \cap \overline{B}$.

   Proof. ■

2. Let (X, d) be a metric space and let $E \subseteq X$.

   (a) Prove that E' is closed.

   Proof. ■

   (b) Prove that E and $\overline{E}$ have the same limit points.

   Proof. ■

   (c) Construct an example that shows E and E' do not necessarily have the same limit points?

   Proof. Suppose we have the following set $E = \{x < \frac{1}{n} : n \in \mathbb{N}, x \in \mathbb{R}\}$ and its set of limit points $E' = \{\frac{1}{n} : n \in \mathbb{N}\}$. Do these two sets necessarily have the same limit points? ■

3. Construct a bounded set of real numbers with exactly three limit points.

   Solution. Consider $(a, b) \subseteq \mathbb{R}$. Then the three limit points of this set are a, b and a $p \in \mathbb{Q}$ with $a < p < b$ since $\mathbb{Q}$ is dense in $\mathbb{R}$. ■

Let A and B be two sets. If an element $x \notin A \cup B$, then is it the case that $x \notin A$ and $x \notin B$?

Let A be a subset of X. Suppose (X, d) is a metric space. If $x \in A$, then can we construct a neighborhood $N_\varepsilon(x)$ such that $N_\varepsilon(x) \subseteq A$?

Suppose we have a finite intersection of sets $\cap_{i=1}^{n} A_i$. Then is $A_i \subseteq \cap_{i=1}^{n} A_i$?