

Math 299 Seminar Notes

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Chapter 1

Week 11

1.1 Regular Surfaces; Inverse Images of Regular Values

Definition (Regular Surface). A subset $S \subseteq \mathbb{R}^3$ is a **regular surface** if, for each $p \in S$, there exists a neighborhood V in \mathbb{R}^3 and a map $\mathbf{x} : U \rightarrow V \cap S$ of an open set $U \subseteq \mathbb{R}^2$ onto $V \cap S \subseteq \mathbb{R}^3$ such that

- (1) \mathbf{x} is differentiable. This means that if we write

$$\mathbf{x}(u, v) = (x(u, v), y(u, v), z(u, v)), \quad (u, v) \in U,$$

the functions $x(u, v)$, $y(u, v)$, and $z(u, v)$ have continuous partial derivatives of all orders in U .

- (2) \mathbf{x} is a homeomorphism. Since \mathbf{x} is continuous by condition (1), it follows that \mathbf{x} has an inverse $\mathbf{x}^{-1} : V \cap S \rightarrow U$ which is continuous.
- (3) (Regularity Condition) For each $q \in U$, the differential $d\mathbf{x}_q : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ is injective.

Definition (Parametrization). The mapping \mathbf{x} is called a **Parametrization** or a **system of (local) coordinates** in (a neighborhood of) p . The neighborhood $V \cap S$ of p is called a **coordinate neighborhood**.