Definition 0.0.1 (Open Cover). By an *open cover* of a set E in a metric space X we mean a collection $\{G_\alpha\}$ of open subsets of X such that $E \subset \bigcup_\alpha G_\alpha$.

Definition 0.0.2 (Finite Subcover). A subset K of a metrix space X is said to be *compact* if every open cover of K contains a *finite* subcover. That is, if $\{G_\alpha\}$ is an open cover of K, then there are finitely many indices $\alpha_1, \alpha_2, \ldots, \alpha_n$ such that

$$K \subset \bigcup_{\alpha_i}^{n} G_{\alpha_i}.$$

Theorem 0.0.1. Suppose $K \subset Y \subset X$. Then K is compact relative to X if and only if K is compact relative to Y.

Proof. ∎

Theorem 0.0.2. Compact subsets of metric spaces are closed.

Proof. ∎

Theorem 0.0.3. Closed subsets of compact sets are compact.

Corollary. If F is closed and K is compact, then $F \cap K$ is compact.

Theorem 0.0.4. If $\{K_\alpha\}$ is a collection of compact subsets of a metric space X such that the intersection of every finite subcollection of $\{K_\alpha\}$ is nonempty, then $\bigcap K_\alpha$ is nonempty.

Proof. ∎

Corollary. If $\{K_n\}$ is a countable collection of nonempty compact sets such that $K_n \supset K_{n+1} (n \in \mathbb{N})$, then $\bigcap_{n=1}^{\infty} K_n$ is nonempty.

Theorem 0.0.5. If $\{I_n\}$ is a sequence of intervals in $\mathbb{R}^1$, such that $I_n \supset I_{n+1} (n \in \mathbb{N})$, then $\bigcap_{n=1}^{\infty} I_n$ is nonempty.

Proof. ∎

Theorem 0.0.6. Let k be a positive integer. If $\{I_n\}$ is a sequence of $k-$cells such that $I_n \supset I_{n+1} (n \in \mathbb{N})$, then $\bigcap_{n=1}^{\infty} I_n$ is nonempty.

Proof. ∎

Theorem 0.0.7. Every $k-$cell is compact.

Proof. ∎

Theorem 0.0.8. If a set E in $\mathbb{R}^k$ has one of the following three properties, then it has the other two:

(a) E is closed and bounded.

(b) E is compact.

(c) Every infinite subset of E has a limit point in E.

Proof. ■